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Young children's notations for representing ordinal position and quantity: a longitudinal study of cross-task and within-task qualitative variations

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Abstract
To support children's transition to school mathematics, it is important to maintain sensitivity to notations that children produce in the domain of number and support connections between their informal number knowledge and written symbols. This paper presents an exploratory longitudinal study of the notations that 3–5-year-old children produced within two tasks that prompted them to communicate the ordinal position of an object in a sequence and quantity. Thirty-three children participated in three individual, task-based interviews, over one year. The frequency of numeral use in the quantity task was almost double the frequency of numerals used in the ordinal position task. A considerable cross-task difference in children's numeral use was maintained for all data collection cycles. Notational behaviour was characterised by qualitative variability within and across the two tasks. Longitudinal individual pathways of notation production revealed that, in contrast to the quantity task, children's movement towards use of numerals for indicating ordinal position was often direct, from using other notations to using exclusively numerals, rather than characterised by a gradual emergence of some numerals amongst other notations. We discuss the observed behaviours in relation to different parameters, as illustrated by qualitative data. We propose that children's notational choices can be an indication of intentional intertextual engagement that enables them to draw from across their literacies, to complete a task. The original evidence from this study provides the basis for further theorisation of the role that intertextual engagement may play in the development of symbolic function.

Keywords Early childhood mathematics · Children's notations · Numerals · Ordinal position · Quantity · Qualitative variations

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1 Introduction

Children's understanding and production of mathematical notations are fundamental elements underpinning their mathematics learning (Duval, 1999). In contrast to first-order symbol systems (e.g., a picture of an object that resembles the object), mathematical notations constitute a second-order symbol system, used to represent abstract meanings (Vygotsky, 1978). Second-order symbol systems involve symbols that do not resemble what they stand for, and therefore, the link between conventional symbols and the idea that they represent is arbitrary and established through a process of socially developed shared meaning (Bialystok, 2000).

Hindu-Arabic numerals (e.g., 9, 28) that are used as conventional symbols for representing numbers (McCloskey & Macaruso, 1995) constitute a sub-set of mathematical notations. Numerals can be used for representing the cardinal aspect of number, for indicating ordinal position, for representing measures (e.g., speed), and for carrying out arithmetical operations. Numerals can also be used for non-quantitative purposes (e.g., bus numbers) (Tolchinsky, 2003). Children appreciate that numerals have different functions early in their lives (Sinclair, 1991; Voutsina & Stott, 2023). However, research on children's own notations for representing aspects of number has predominantly examined children's representation of quantities (e.g., Bialystok & Codd, 1996; Hughes, 1986; Munn, 1994; Sinclair et al., 1983; Zhou & Wang, 2004), associations between notations for recording quantity and counting skills (e.g., Munn, 1994), and children's notations alongside their understanding of the written number system (Brizuela, 2004).

Research in young children's notational behaviour lags behind research in the development of mathematical concepts (Tolchinsky, 2003; Worthington et al., 2023), and no previous research has examined children's notations for the communication of quantity and ordinal position as they develop over time. We aim to address this research gap by seeking an answer for the following research questions:

1. What notation types do 3–5-year-old children produce to communicate ordinal position and quantity, and what changes in children's notations can be observed over one year, across and within tasks?
2. What changes can be observed, if any, in the ranges of notations that individual children use to communicate ordinal position and quantity, and how do their individual pathways of notational production unfold over time?

In the early years, important developmental changes take place in relation to children's use of symbolic function and form (Munn, 1998). Furthermore, numerical symbol competence in preschool and the early school years is reliably associated with later mathematics achievement (Malone et al., 2019; Merkley & Ansari, 2016). Examining children's notation production across tasks that elicit the use of numerals for the communication of different concepts, and changes that occur over time, is essential, for informing theorisation for the development of symbolic function, which can, in turn, inform educational practice that can best support children to recognise the 'affordances' (Teubal & Dockrell, 2005) of numerical notations for different purposes.

2 Background

2.1 Children's notations for representing quantity

Research has shown that although children may know numerals, they will not necessarily use them to represent quantity (e.g., Bialystok & Codd, 1996; Zhou & Wang, 2004). For example, in a cross-sectional study with 3-, 4-, and 5-year-old children, Bialystok and Codd (1996) examined children's use of notational forms to represent the quantity of items in a box. Children's notations were classified using three categories: 'Numerals' included conventional Hindu-Arabic digits, 'Analogue notations' included discrete marks used to represent each object in the displayed set (e.g., five lines to represent five objects in the set), 'Global notations' included idiosyncratic representations of "a wholistic indication of the existence or identity of the objects rather than their quantity" (Bialystok, 1992, p. 312). Across the three age groups, children's notational production was increasingly conventional and indicated a move away from global notations. Although there was a significant difference in the use of 'Global notations' between 3- and 5-year-old children, there was no reliable difference across the three age groups in the frequency with which children used 'Analogue notations' to represent quantity. The study concluded that children who had comparable knowledge and skill in counting and number recognition did not always show appreciation of how quantity can be best represented and communicated, suggesting that counting capabilities and number recognition do not provide strong evidence for how children understand the meaning of the written number system. These findings align with earlier research by Hughes (1986) and Sinclair (1991) who, in their studies with 5–7-year-old and 3–6-year-old children respectively, found that the use of one-to-one correspondence analogue representations, either iconic (i.e., tally marks) or pictorial (i.e., drawings of each object in the set), persisted and were employed even by children who had knowledge of and capability to write numerals 0–9.

Observations related to the frequent and persistent use of analogue notations for representing quantity underpin the view that young children's understanding of written notations lags behind their counting skills and appreciation of quantity, as they may not clearly understand the cardinal and "symbolic properties of a numeral as a written notation" (Bialystok & Codd, 1996, p. 289). However, based on evidence from research showing that production of non-conventional notations could be explained by the more demanding nature of certain tasks or by the fact that certain tasks have no clear purpose for children, Teubal and Dockrell (2005) argue that findings showing children's limitations in using numerical notations may be due to task characteristics rather than conceptual limitations. They posit that tasks may "fail to make 'sense' to the child and therefore the poor performance of the task cannot be seen as a reflection of the child's competence" (Teubal & Dockrell, 2005, p. 277).

Based on the view that there should be a need of a communicative purpose in tasks that require children to produce notations, research has also examined preschool children's notations within tasks that are presented with an everyday life scenario. For example, Ewers-Rogers and Cowan (1996) examined 3–4-year-old children's notations using tasks that involved: representing the quantity of toys within a game that made the notation necessary; writing a note for the milk man about the number of bottles of milk needed; and completing a birthday invitation. No child used numerals across all tasks, but some consistency was observed in that, children tended to use the same notations in the two quantity tasks (game and milkman). However, the notations that were intended for the milkman were no more transparent than children's notations in the game task, even though the milkman task was framed by the need of a note that would be read by someone else. Therefore, the researchers suggested that more effective variation of who was to read the notation may affect the transparency of children's notations. These findings, alongside studies by Dockrell and Teubal (2007) and Tolchinsky (2003) who have used similar tasks connected to everyday activities, demonstrate that children as young as three are sensitive to differential use of notations, however, different forms of notations exist in parallel.

2.2 Children's notations for denoting ordinal position

Ordinality is "a property of individual members of a set (the *first* runner, the *second* runner, etc.) in relation to the other members of that set" and is crucial for understanding how individuals process mathematical relations (Lyons et al., 2016, p. 188). It also relates to how cardinal values are ordered (whether 5 stickers are more than 4 stickers) and how a number relates to other numbers in a sequence (5 comes after 4 and before 6) (Fuson, 1988; Lyons & Beilock, 2013). Research has shown the highly predictive power of symbolic number-ordering capabilities for complex mathematics skills children exhibit (Lyons et al., 2014). Studies on children's ordinal understanding (e.g., Bruce & Threlfall, 2004; Colomé & Noël, 2012) have indicated that the cardinal and ordinal aspects of number are distinct, develop separately, and ordinality lags behind cardinality. However, evidence is mixed. Contextual factors, such as the symbolic or non-symbolic nature of tasks, appear to affect distinctions between cardinal and ordinal processing of numbers (Lyons et al., 2016). Research on ordinal numerical knowledge is limited and has primarily focused on symbolic numeral order skills (Gilmore & Batchelor, 2021) and non-symbolic numerical ordering (Harju et al., 2024). In contrast, this study focuses on children's own notations within an ordinal position situation (Fuson, 1988) that "concerns the relative position of one entity with respect to all the other entities in an ordered situation" and "where an ordinal number word (first, second, third, etc.) will apply" (p. 14).

A teaching experiment carried out by Margolinas and Wozniak (2014) is the only previous research that we are aware of that has examined children's production of representations to communicate the position of an object in a sequence. The study employed a didactic design methodology and involved 5–6-year-old children, an older age group than the group involved in our study. The task situations invited children to communicate the position of a coloured bead amongst other uncoloured beads of a model necklace, so that the model could be reproduced later by the children themselves or others. Children's written representations were considered as effective if they included information on the origin and orientation of the necklace, and the position of the coloured bead in relation to the origin. In 76% of 62 productions that included numerals, numerals were used to communicate quantity (e.g., the number of uncoloured beads on either side of the coloured bead). In some of the representations that were considered as relevant, numerals were written in the order of quantities depicted in accompanying drawings of the necklace. Twenty-six percent of 62 messages involved numerals expressing position. Forty-four percent of the messages that included "some kind of ordinal number were non-functional writings" (Margolinas & Wozniak, 2014, p. 38). For example, some children wrote the sequence from 1 to 10, without indicating the position of the coloured bead. Only four messages indicated position in a pertinent way with a numeral.

3 Theoretical perspectives

Children may be able to recognise numerals and recite a number sequence, but these capabilities alone are not sufficient for symbolic thought (Bialystok, 1992). Rather, numbers "achieve symbolic status" once they "are associated with written forms and used to signify specific meanings" (p. 304), based on a second-order relation (Vygotsky, 1978). Children's recognition and use of numbers as symbolic representations depend on early experiences, including symbolic play (Bialystok, 1992).

We align with the position that written numbers are an outcome but also a source of numeral knowledge, and therefore, the meanings that children gather from numerical notations, and information that they produce through their own notations, "is not simply a supplement to previously acquired notions that serves to amplify these notions. Instead, it is a permanent ingredient of developmental processes" (Tolchinsky, 2003, p.98). Radford (2000) posits that signs, as artefacts that are framed by social meanings and norms of use, are not merely "mirrors of internal cognitive processes" but "tools or prostheses of the mind to accomplish actions as required by the contextual activities in which the individuals engage" (p. 241). Therefore, he proposes a shift from what signs represent to what they enable individuals to do within the interactive context of their activities. This highlights that both mental and social capabilities play a key role in understanding and using symbol systems, as it is only prior socially shared experience of the idea that a symbol represents, that supports the inclusion of a symbol in the child's "cognitive model of the world that is shared with those who write or read these same symbols" (Munn, 1998, p. 52).

The role of experience in how learners understand symbols and use them is also highlighted by Rojano et al. (2014), who view a mathematics task that may be used in a class-based or interview situation, as a "textual space" that opens relations with other tasks and texts that individuals know from previous experiences. They define "mathematical texts," as sets of heterogenous signs that students may produce and which can include signs that are considered as being specific to mathematics, but also natural language, diagrams and figures. Thus, they view the act of approaching and making sense of a task as being "intertextual" in that, the individual is drawing from relations with other, previous texts and tasks to make sense of it. This conceptualisation builds on the notion of intertextuality (Kristeva, 1980) which encapsulates the idea that, to produce or make sense of a new text or task, authors and readers of text build on relations with systems, signs and meanings that exist in previous texts that they have encountered in their previous experiences.

The concept of intertextuality has been drawn on more recently by Worthington and colleagues (2023) in a study of seven 3–4-year-old children's communicative production of written signs in relation to mathematical ideas, as part of a longitudinal, ethnographic study. They found evidence of intertextual use of signs as children drew from across all their literacies, signs created by themselves or borrowed by others, including letters, shapes, tallies, numeral-like and scribble-like notations, as well as many conventional numerals, to fulfil their communicative purposes. They concluded that the development of children's notations for communicating mathematical meanings is underpinned by a process of intertextual sharing of signs through which "formal signs (borrowed from others) are woven into the children's personal texts" (Worthington et al., 2023, p. 17), as young children increasingly choose to integrate conventional signs into their own notational production.

Based on the previously discussed evidence and theory that indicate the co-existence of different notational forms, we anticipated that, in our study, children would produce and combine varied notational forms to represent different number-related meanings across tasks. Taking into account that the extent to which tasks are meaningful for children or not may be a factor influencing children's recognition of "the 'affordances' of numerical notations in the context of a relevant task" (Teubal & Dockrell, 2005, p. 260), we used tasks with scenarios that were likely to be familiar to most children (i.e., line of static cars in front of a traffic light, keeping the score in a game), and framed around a need for notations.

4 Research design and methods

We employed an exploratory longitudinal qualitative research design (Saldaña, 2003), to examine within-task and cross-task qualitative variations in the type and range of notations that children produced, and any changes occurring in children's notations over one year. Cycle-one of data collection (C1) took place in June–July 2021. Cycle-two (C2) took place in January–February 2022, and Cycle-three (C3) took place in June–July 2022.

Thirty-three preschool children participated in individual task-based interviews (Maher & Sigley, 2014) that were video-recorded, to capture children's notational behaviour, verbalisations and other gestures, to support analysis. The second author conducted the interviews at the authors' institution. The volunteer sample included children from 26 preschools located in a South of England County. Table 1 presents the participants' average age at each data collection cycle.

Parental written consent and children's verbal assent were sought through all data collection cycles. The research strictly followed the British Education Research Association Ethical Guidelines for Educational Research (BERA, 2024).

Table 1 Participants' average age at each of the three data collection cycles

Cycle of data collection	Average age of participants
C1	49 months
C2	56 months
C3	61 months

4.1 Tasks

4.1.1 *Traffic Queue (TQ) task (ordinal position)*

We adapted a task used by Colomé and Noël (2012). In our study, children were presented with a line of six toy cars in front of a traffic light and a toy police person standing next to the traffic light (Fig. 1).

The task was introduced with the following scenario: "The traffic light is broken. The police person needs some help to remember the order of cars, in case a driver tries to pass in front of other cars." The interviewer indicated the direction through which the cars had to move to pass through the traffic light. Children were prompted to show, by pointing, which car was first, second, fourth and sixth in the line. If a child did not identify one or more of the positions correctly, the interviewer indicated the positions.

After the correct positions had been established, the researcher placed a sticky label next to the cars under focus, on the side facing the child's seated position. Placing the labels in this way, rather than on the cars themselves, provided more writing space. This was necessary because, in an earlier trial, with children who did not participate in the main study, we had observed that some children found it difficult to form notations in a confined writing area. The children were asked: "So, you showed this one was going to go first [pointing at the green car]. Can you put something on there [indicating a label that the researcher placed next to the green car on the side of the child's seated position] to help him remember that this will be first?" This type of prompt was repeated for each position. At the end of the task, the interviewer took a photograph of the line of cars with the child's notations, saying that this would help the police person.

We purposefully placed the traffic light on the right side of the car queue (as seen from the child's seated position, Fig. 1) and skipped the third and fifth position. This was to avoid creating a situation that would prompt children to write numbers in a sequence from left to right, potentially emulating a written number line. The number line is a familiar model to many children and frequently displayed in preschool classes.

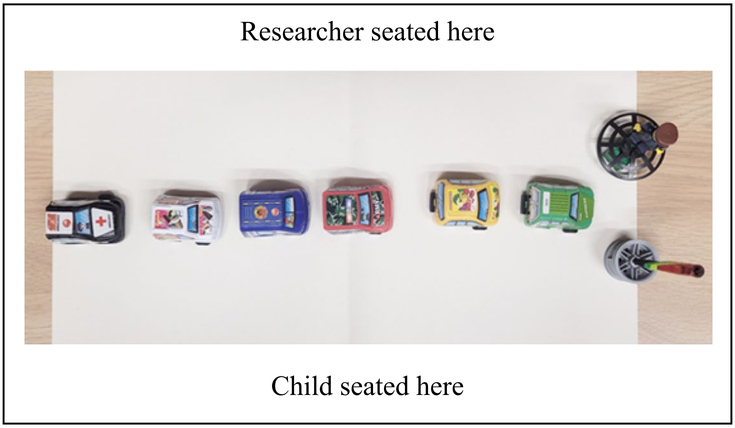


Fig. 1 Traffic Queue task (adapted from Colomé & Noël, 2012)

In an ordinal situation, the direction of the line is significant. If children use counting to ascertain the position of an object in a sequence, their counting "must follow the direction of the ordered series," which, in this task, was determined by the traffic light position (Colomé & Noël, 2012, p. 237) and indicated by the researcher.

In presenting the task, we used ordinal number words ("first," "second," "fourth," and "sixth"), as opposed to phrases such as the following: "car number one, car number two." This was to avoid using number names that children may perceive as cues to note down the number spoken by the researcher, although we recognised that some children may or may not know ordinal number words. We included two positions ("first" and "second") that we assumed would be more readily familiar to children from everyday experiences but which are verbally referenced in English using words that do not have a direct verbal connection to the numeral names "one" and "two," and two positions ("fourth" and "sixth") for which the ordinal number word in English includes the numeral names "four" and "six," respectively.

4.1.2 Snap Game (SG) task (quantity)

Sixteen cards consisting of pairs of animal figures were shared between the child and the researcher. Each player took turns to place one card on the table. When two consecutive cards were matching, the player who called "snap!" faster won all the cards on the table. The task included two rounds. A second round was not played if a child did not wish to. At the end of each round, children were asked to count the cards and make a note that showed how many cards each player had in total, at the end of the round. If a child found it difficult to count, the interviewer helped them to complete their counting. The task elicited up to five notations: Initial number of cards (8) for each player at the start of round one (children were not asked to note this down again for round two), number of cards that each of the two players had in total, at the end of each round.

Although we consider that both task scenarios were familiar to most children, the "need" for the notations was more authentic within the quantity task. In that task, the child's notations could be used by them and the other player, to ascertain the winner of each round, as in many game situations. The ordinal position task involved a fictional character represented by a toy, who "needed" to refer to the child's notations. Using a character represented by a toy, or puppet, is usual practice in research involving young children (e.g., Hughes, 1986; Palmér & Björklund, 2024; Sophian, 1992). This "'neutralises' the task in relation to a child's possible presumptions about the purposes of adults in such contexts" (Bruce & Threlfall, 2004, p. 8). Acknowledging that the two tasks were different in terms of who would need to refer to the produced notation, we consider that both tasks allowed us to study notations that children produced to fulfil a communicative purpose.

4.2 Data analysis

Within our interpretive analysis approach, we have been mindful that our interpretation of children's notational intentions needed to be approached with critical awareness of the potentially differing meanings and intentions that we, as researchers, and children, as participants, bring to the task, in our dual roles as task "authors" and "readers," that are informed by our own previous experiences (Rojano et al., 2014).

Table 2 Notation categories: Analytical framework

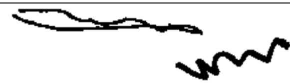
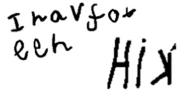
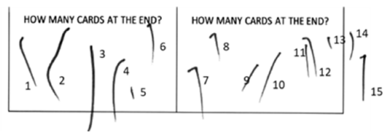
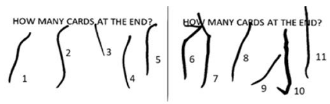
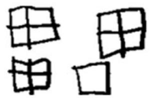
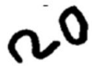
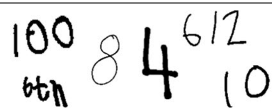
Number and Name of Notation Category		Description	Example notations from our data
1	**Idiosyncratic**	Researchers unable to determine any regularities in the forms produced, that could relate to the task (with regards to quantity, ordinal position). Includes scribbles and zig-zag type marks. The form may be meaningful to the child but not to external observers	
2	**Drawing**	The child intentionally draws a picture of what is in front of them; something from the object or something associated with elements of the task that they referred to in their response (without explicit reference to number words indicating quantity or ordinal number words)	
3	**Continuous Forms**	The child produces continuous linear, pretend writing forms, without any similarity to graphemes or numbers (typically written in left to right direction). The form may be meaningful to the child but not to external observers	
4	**Graphemes**	The child produces a recognisable by the researchers letter or a series of letters or words to signify quantity or ordinal position. There is **no** one-to-one correspondence with quantity	
5	**Other Symbolic Forms**	The child produces common, discrete symbols (such as ticks, crosses, arrows, circles, dots, single vertical or horizontal lines) to signify quantity, ordinal position	
6a	**Iconic**	**a) Iconic Misaligned:** Using various forms (bars, dots etc.), the child produces an inaccurate correspondence of quantity or indication of numbers used as quantities to indicate ordinal position. The notation does not align with child's spoken response	**Child counted 14 cards and drew 15 lines** 

Table 2 (continued)

Number and Name of Notation Category		Description	Example notations from our data
6b		**b) Iconic Aligned:** Using various forms (bars, dots etc.), the child produces an accurate one-to-one correspondence of quantity or indication of numbers used as quantities to indicate ordinal position. The notation aligns with child's spoken response	 Counts 11 cards. Draws 11 lines.  Counts 13 cards. Says “four, eight, twelve and then the last one.” Draws 3 sets of four blocks, and one extra block.
7a	**Numeral-Misaligned**	**a)** Written numeral that **does not align with spoken number** (may be written in developing or developed form)	 Counts 12 cards out loud. Writes 20.  Counts 14 cards but says and writes 13
7b		**b)** Written numeral that aligns with spoken number (may be written in developing or developed form), but **does not align with quantity or ordinal position in the task**	 Points at yellow car for second position correctly. Produces misaligned numeral for second position.
8	**Numeral-Aligned, in Developing form**	Written numeral **that aligns with spoken quantity or position, and task requirement**, resembles Hindu-Arabic numeral and is recognisable by the researchers, but not yet in fully developed form with regards to appearance and / or place value	
9	**Numeral-Aligned, in Developed form**	Written numeral that is in fully developed form, **aligns with spoken number and with quantity or ordinal position in the task**	

We referred to the interview transcripts and video data iteratively, to support our interpretations and classification of notations and considered both notational accuracy and notational quality (Teubal & Dockrell, 2005). We adopted a deductive approach of constant comparison of notations against pre-existing categories drawn from Hughes (1986) and Lucangeli et al. (2012), as we found these to be appropriately comprehensive when analysing an initial data sub-set. We combined this with an inductive approach, considering notations that did not potentially fit existing categorisations and remaining open to the potential emergence of new categories.

We provide examples of notations from both tasks, to illustrate categories in our analytical framework (Table 2). Exceptions to this are the examples that illustrate Category 3 (Continuous forms) and Category 6 (Iconic notations). These were observed in the quantity task only. Therefore, examples are drawn from that task. We retained the definition of "Iconic" notations included in previous literature (e.g., Hughes, 1986), to capture the use of bars or tallies to represent quantities. For the ordinal position task, we were curious to see whether any children would make an implicit or verbalised association between ordinal words such as "fourth," with the number word "four" and the meaning of "four" as quantity, or whether children who used counting to ascertain position, would think of: "car one," "car two," "car four," etc., and would potentially use Iconic representations, based on the cardinal meaning of number words. As shown in the Findings, such notations did not emerge. However, at the start of the analysis process, we decided to not exclude this possibility.

Category 2 includes intentional drawings that children produced of what was in front of them, and elements of the task. In the ordinal position task, some children drew, for example, a car, wheels, a flower appearing on a car sticker, etc., as identifying labels of a specific car in the line. Only one instance from the quantity task was coded under this category (see drawing of stick person as first example, Category 2). This was when a child drew a stick person, to show that the player had zero cards. Sub-categories for Categories 6 and 7 aim to reflect both notational type and accuracy.

Table 2 does not depict a developmental trajectory in children's notation production. Only categories 8–9 imply development, in that, with these categories, we aimed at capturing changes in the quality and accuracy of numeral formation, in line with symbolic conventions.

Both authors coded data independently and discussed any inconsistencies until agreement was reached, adopting a "negotiated agreement" approach (Campbell et al., 2013). This is recommended "in situations where coding requires great sensitivity not only to obvious meanings but also more subtle meanings" (Campbell et al., 2013, p. 306). This approach was useful for achieving increased coding consistency of data based on young children's notations and expressions that can be subtle and unclear to the adult observer.

5 Findings

5.1 Types of notations that children use to represent ordinal position and quantity

Table 3 presents an exploratory overview of the frequency of qualitatively different notations, for both tasks. Category 0 represents instances where the children did not produce any marks. We consider that Categories 7a–9 suggest children's intention to produce a numeral, irrespective of the accuracy of its formation or alignment with the task

Table 3 Percentages of notations coded under each category for each task and cycle of data collection

Categories		Tasks					
		Traffic Queue (TQ)			Snap Game (SG)		
		C1	C2	C3	C1	C2	C3
No notation	0	2.4	1.5			0.8	1.3
Idiosyncratic	1	46.8	10.9	7.8	44.7	15.0	7.1
Drawing	2	4.8	8.8	17.7			0.6
Continuous forms	3				1.2	1.6	
Other symbolic forms	4	1.6	14.6	8.5	5.9	3.1	1.3
Grapheme	5	21.8	20.4	14.2	4.7	12.6	1.3
Iconic-Misaligned	6a				2.4	2.4	
Iconic-Aligned	6b				1.2	2.4	3.9
Numeral-Misaligned with spoken number	7a	0.8	1.5	1.4	2.4	5.5	5.8
Numeral-Misaligned with task	7b	3.2	8.0	5.7	1.2	1.6	1.9
Developing numeral-Aligned with task	8	8.1	10.2	12.8	23.5	21.3	37.7
Developed numeral-Aligned with task	9	10.5	24.1	31.9	12.9	33.9	39.0
Numerals-Aggregated percentages (Categories 7a-9)		*22.6*	*43.8*	*51.8*	*40*	*62.3*	*84.4*

requirement. Therefore, we present an aggregation of percentages under these categories (italicized) to enable an easier view of how children's intention to produce a numeral varied and changed over time.

For the Traffic Queue task (TQ), in C1, 46.8% of children's notations were coded as Idiosyncratic, while 22.6% of notations were coded under Categories 7a–9. Children's production of Idiosyncratic notations reduced sharply in C2 (10.9%) and further reduced to 7.8% by C3. Interestingly, the production of Other Symbolic Forms increased from 1.6% in C1 to 14.6% in C2 and reduced again in C3 (8.5%). The third most prominent type of notations produced in C1 was Graphemes (21.8%), which reduced gradually by C3 (14.2%). By C3, the three most prominent categories captured the production of some form of numeral (Categories 7a–9, 51.8%), Drawing, (17.7%) and Graphemes (14.2%). Children did not produce Continuous Forms or Iconic notations, as defined within our analytical framework, for this task. Drawings and Graphemes sometimes included elements such as, icons from the stickers on the cars, letters and numbers from a toy car's number plate (examples presented in Sect. 5.2.1). Such notations indicated attempts to produce identifying labels for representing a particular car.

Analogous to the TQ task, in C1, the Snap Game (SG) elicited a similar percentage of Idiosyncratic notations (44.7%) that reduced to 7.1% by C3. However, the percentage of numerals coded under Categories 7a–9 to represent quantity in C1 (40%) was almost double the percentage of numerals produced in the same cycle to communicate ordinal position. Children's production of numerals for representing quantity increased in C2. By C3, 84.4% of children's notations were of some form of numeral. For the SG task, only a small percentage of notations were coded as Iconic and were produced by three children. The production of Continuous Forms that has been previously observed by 3.5-year-old children as part of a dictation task (Lucangeli et al., 2012) was observed in the context of the quantity task only (by two children), suggesting emulation of writing behaviour. As noted in Sect. 3, we anticipated, based on previous research, that some children would mix different notation types. Therefore, we subsequently examined qualitative differences in notational choices that the exploratory analysis revealed, by delving into individual children's range of notation production within each task, and changes over time.

5.2 Individual pathways of notation production

Table 4 presents findings based on each child as the unit of analysis. The 33 participants (pseudonyms used) are ordered based on their age at C1, from the youngest to the oldest. Our aim is not to draw age-related generalisations. Age sub-groups here provide further insight to our observations because our sample was voluntary and not homogenous in terms of age. Our focus is on examining variations and individual pathways of change in the range of notations that children drew from, for each task, across data collection cycles. The 0–6 range indicates that the child produced mixed notations that did not include numerals, drawing from categories that ranged from No marks to Iconic notations. The 0–9 range indicates the use of at least one numeral amongst a mix of other notations. The 7a–9 range indicates that the child consistently produced numerals, of varying accuracy or quality of formation.

In the context of the TQ task, we observe a noticeable dichotomy between the 0–6 notational range and the 7a–9 range, when compared to the SG task, where the wide 0–9 range is used more frequently and by most children. For the TQ task, 13 children used notations that did not include numerals consistently across all three cycles. Six of these children were in the older sub-group. For some children, we observe the use of numerals alongside other notations where the wide, 0–9 range is assigned in C1 (Ian, Rita, Thane, Maegan, Orion). These children varied in their notational pathways across cycles. For example, Ian moved from using some numerals in C1, to using entirely other notations in C2 and C3, while Rita moved from producing some numerals in C1, to using other notations in C2, before shifting to consistently using numerals in C3. Four children (Tommy, Gino, Todd, Bart) consistently produced numerals in C1 for indicating all car positions. From these, three children continued using numerals in C2 and C3.

In the context of the SG task, two children (Jay, Karla) consistently used notations that did not include numerals across cycles, while five children (Mahi, Abi, Gino, Thane, Bart) used numerals consistently to represent quantity across cycles. Six children produced notations of the 0–6 range in C1 and C2, before shifting to a wider range that included some numerals in C3 (John, Matt, Hudson, Ian) or shifting sharply to consistent use of numerals in C3 (Beth, Jasper). Interestingly, we observe this latter pathway in the notations produced by both the youngest and oldest child in our sample.

In the quantity task (SG), there was little difference observed in relation to ranges of notations used and individual pathways of notation production amongst the oldest children of the younger sub-group (Lochlyn, Rita, Sam, Tommy, Mahi, Abi) and children in the older sub-group. In the TQ task too, by C3, the oldest children of the younger sub-group (e.g., Sam, Tommy, Mahi, Abi) exhibited similar notation production and individual pathways of change as some of the children in the older sub-group (e.g., Harper, Gino, Rueul, Samu, Thane). Therefore, one of our observations is that, except for some children on the youngest side of our spectrum of ages, children exhibited notation production behaviours that are difficult to relate to age, aligning with Sinclair's (1991) same conclusion.

Individual notational pathways varied. While some children followed a pathway towards the use of some or all numerals across both tasks (Sam, Harper, Megan, Hudson), four children's notational behaviour was unchanged across both tasks (Gino, Karla, Orion, Bart).

Table 4 Individual pathways of notation production across two tasks and three data collection points

			TQ task			SG task		
		Child	C1	C2	C3	C1	C2	C3
3–4-year-old children at C1	1	Beth	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6	CAT 7a-9
	2	John	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-9
	3	Kyra	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-9	CAT 0-9	CAT 7a-9
	4	Elda	CAT 0-9	CAT 0-9	CAT 0-6	CAT 0-6	CAT 0-9	CAT 0-9
	5	Ivana	CAT 0-6	CAT 0-6	CAT 7a-9	CAT 0-6	CAT 0-9	CAT 0-9
	6	Matt	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-9
	7	Jay	CAT 0-6	CAT 0-9	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6
	8	Henry	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-9	CAT 0-6	CAT 0-9
	9	Hudson	CAT 0-6	CAT 0-6	CAT 0-9	CAT 0-6	CAT 0-6	CAT 0-9
	10	Ian	CAT 0-9	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-9
	11	Lochlyn	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-9	CAT 0-9	CAT 0-9
	12	Rita	CAT 0-9	CAT 0-6	CAT 7a-9	CAT 0-9	CAT 0-9	CAT 7a-9
	13	Sam	CAT 0-6	CAT 7a-9	CAT 7a-9	CAT 0-6	CAT 0-9	CAT 7a-9
	14	Tommy	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 0-6	CAT 7a-9	CAT 7a-9
	15	Mahi	CAT 0-6	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 7a-9
	16	Abi	CAT 0-6	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 7a-9
4–5-year-old children at C1	17	Harper	CAT 0-6	CAT 0-9	CAT 7a-9	CAT 0-6	CAT 0-9	CAT 7a-9
	18	Jack	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-9	CAT 7a-9
	19	Amelia	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-9	CAT 0-9	CAT 0-9
	20	Gino	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 7a-9
	21	Rueul	CAT 0-6	CAT 7a-9	CAT 7a-9	CAT 0-6	CAT 0-9	CAT 7a-9
	22	Samu	CAT 0-6	CAT 7a-9	CAT 7a-9	CAT 0-6	CAT 0-9	CAT 7a-9
	23	Thane	CAT 0-9	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 7a-9
	24	Elin	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-9	CAT 7a-9	CAT 0-9
	25	Karla	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6
	26	Irma	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-9	CAT 0-9	CAT 7a-9
	27	Todd	CAT 7a-9	CAT 7a-9	CAT 0-9	CAT 7a-9	CAT 7a-9	CAT 0-9
	28	Maegan	CAT 0-9	CAT 7a-9	CAT 7a-9	CAT 0-9	CAT 7a-9	CAT 7a-9
	29	Orion	CAT 0-9	CAT 0-9	CAT 0-9	CAT 0-9	CAT 0-9	CAT 0-9
	30	Ziggy	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-6	CAT 0-9	CAT 0-9
	31	Bart	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 7a-9	CAT 7a-9
	32	Peter	CAT 0-6	CAT 7a-9	CAT 0-9	CAT 0-9	CAT 0-9	CAT 0-9
	33	Jasper	CAT 0-6	CAT 0-6	CAT 7a-9	CAT 0-6	CAT 0-6	CAT 7a-9

(lightest shade)	CAT 0-6
(medium shade)	CAT 0-9
(darkest shade)	CAT 7a-9

For other children, a move towards the use of numerals was observed in the quantity task (e.g., Beth, Elin, Irma, Ziggy) while notation behaviour remained unchanged in the ordinal task. Three children (Sam, Rueul, Samu) used numerals consistently for the ordinal task from C2 onwards, before numerals emerged in their notation production in the quantity task, in C3. Two children (Ivana, Peter) produced numerals for the ordinal task in C3 and C2, respectively, when, in the same cycles, they mixed numerals with other notations in response to the quantity task.

5.2.1 Zooming into individual pathways

We zoom into three cases (Elda, Harper, Amelia), selected because they illustrate differing pathways of notation production. Examining closely their notations, alongside what they say, provides further insights into children's pathways and notational choices. In the figures that follow, the child's name is covered (Figs. 2, 3, 4, and 5). In the context of the Snap Game, we have retained the researcher's name or initial (D) so that the reader can distinguish which notations correspond to cards held by each player.

Elda's notations In C1, for the TQ task, Elda produced notations within the 0–9 range (Fig. 2). The position of the first, fourth and sixth car were represented by circles (Category-5). Elda recognised the second position correctly and produced a vertical line which she said was representing "one" (Category-7b).

(R: researcher, C: child).

R. And which one is going to go second?

C. The yellow one.

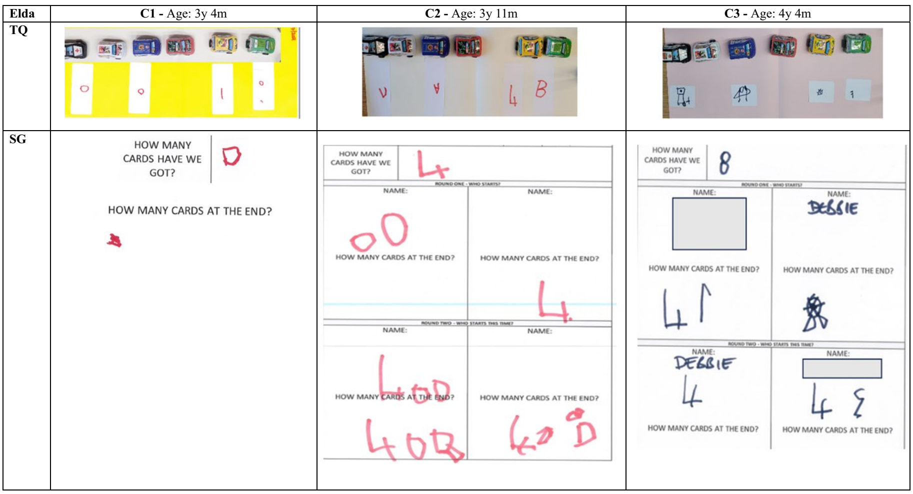


Fig. 2 Elda's notations across two tasks and three cycles of data collection

R. You said it would go second. Can you write something that shows this?

C. [makes single line] One.

It is not clear whether Elda's intention at that point was to represent position, or the number of cars in the second position. For the SG task in C1, she counted correctly her initial eight cards and made a mark to denote this by drawing small lines that she connected into a circular form. She counted independently, correctly, the number of cards that she had up to 12, and continued "… forty, forty, fifty, sixty." She made a single mark to denote the number of cards that she had at the end. Both notations were classified as Category-1.

In C2, for both tasks, Elda shifted to the use of the 0–9 range. In the TQ task, she used the symbol "4," to represent the second position, which she had recognised correctly. She used a letter and other symbols to indicate the other three positions. For the SG task, she produced Misaligned numerals to represent her initial number of cards (writing '4,' after correctly counting eight cards). She combined the numeral "4" with other notations to denote the total number of cards that players had at the end of the round, commenting that "4" was her favourite "cause it's my birthday."

In C3, for the TQ task, Elda produced an Idiosyncratic notation to indicate the first position, Drawings of a wheel and a person to indicate second and fourth position, respectively, and of a car to indicate the sixth position. For the SG task, she wrote "8" to represent her initial number of cards. She produced a Misaligned numeral (that looked like "41") to indicate that she had 17 cards at the end of round one. Before writing anything down she said: "I can't write it" but was encouraged to try. In round two, Elda combined the numeral "4" with an Idiosyncratic notation to indicate that she had 12 cards at the end of the game. She then wrote "4" under the researcher's name, to show that she had four cards, in this case producing a developed numeral, aligned with the spoken number and quantity.

Although Misaligned, mostly, and underpinned by the personal significance of the numeral "4," Elda's use of "4" in C2 suggests some, possibly emerging, recognition of the appropriateness of numerals for signifying position and quantity. It is interesting to note however that for the ordinal position task, Elda moved from a wider range of notations that included at least one numeral in C1 ("1") and C2 ("4"), to a range of notations in C3 that did not include numerals.

Harper's notations In C1, for the TQ task (Fig. 3), Harper indicated correctly the first and second position but not the fourth and sixth. Holding the pen in his fist, he produced vertical lines to mark each position. For the SG task, he drew eight lines to represent his initial eight cards (Iconic-6b). His Iconic notations in both rounds were mostly aligned with the spoken number. When needing to represent 14 cards, he commented: "I can't really draw a four."

In C2, for the TQ task, Harper asked for a green pen to make a green mark to show the position of the first car. As there was no green pen available, he used a red pen and said: "You can pretend it's green though." To indicate the second position, he copied the toy car's number plate. He thus used notations that would allow one to identify the specific car. Within the same task, he shifted to the use of numerals, for denoting the fourth and sixth position. This suggests appreciation of the appropriateness of numerals for communicating ordinal position. It is unclear why he did not use numerals for the first and second position although he identified all positions correctly.

For the SG task in C2, Harper used the numeral "8," in developing form, to denote his initial number of cards. He counted the 16 cards that he had at the end of round one. He

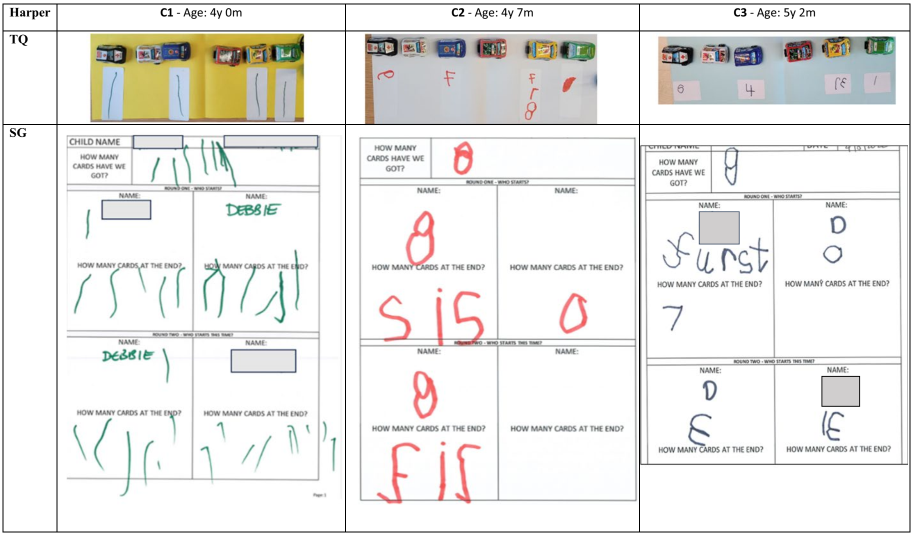
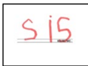
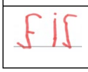


Fig. 3 Harper's notations across two tasks and three cycles of data collection

Fig. 4 Harper's notations denoting 16, 0 and 15 cards won, as part of the SG task in C2

	**End of round 1:** C. Now we do … s-i-x... te-e-n [makes notation on paper, sounding out the word sixteen] Sixteen!
	R. And how many did I get? C. Zero. [makes notation on paper]
	**End of round 2:** R. OK, do you want to put something there that shows how many cards you have? C. Fifteen [makes notations on paper]

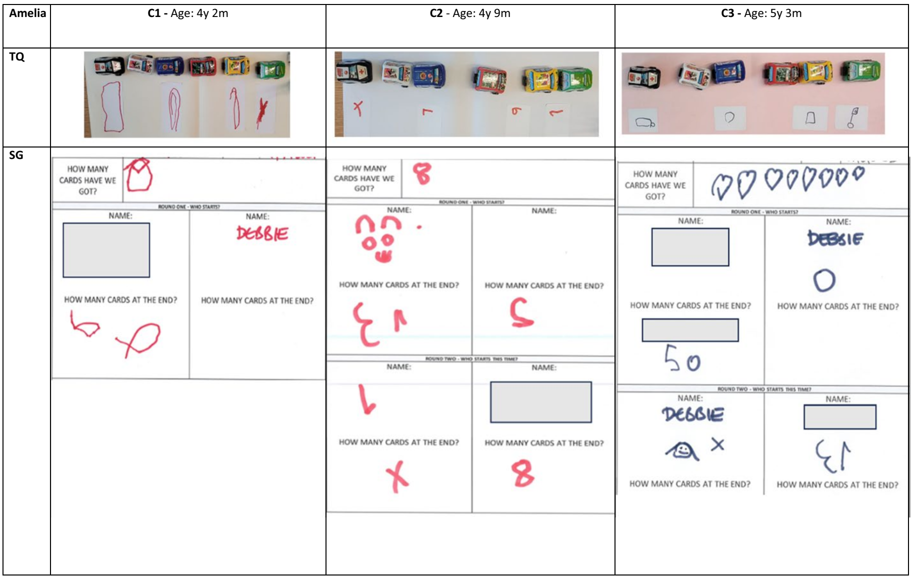


Fig. 5 Amelia's notations across two tasks and three cycles of data collection

attempted to write the words “sixteen” and “fifteen,” to show the number of cards that he had at each round while he used the numeral “0” to indicate the number of cards that the researcher had at the end of the first round (Fig. 4).

In C3, for the TQ task, Harper indicated all car positions correctly and produced numerals in developed or developing form. For the second position, he made two attempts:

C. A two, but I don’t know how to draw a two. It’s just like… No, I don’t know.

I’ll go down there [draws a curved line].

R. That’s okay.

C. I’ll try [makes second notation that resembles a reverse ‘3’].

For the SG task in C3, Harper wrote “furst” under his initial, to show that he would play first in round one. He used numerals consistently to denote the total number of cards that he had.

At different points, Harper expressed verbally that he did not know how to write a numeral. His comments and mixing of Graphemes, colours and numerals suggest that he was aware of the limitations of his skill in forming numerals rather than he did not recognise the “affordances” (Teubal & Dockrell, 2005) of numerals for responding to this task. In C3, his use of “1” to denote the first position in the TQ task and writing of “furst” to show that he played first in SG is an interesting example of drawing from different literacies (Worthington, et al., 2023) for representing ordinal position within the same task.

Amelia’s notations In C1, for the TQ task, Amelia, similar to Elda and Harper, used Idiosyncratic notations for all car positions (Fig. 5). As part of the SG task, she produced a developing numeral to represent her initial eight cards. She formed two Idiosyncratic notations to show that she had “13” cards at the end.

In C2, for the TQ task, Amelia combined Other Symbolic forms and Graphemes.

R. So, you said this one was going to go first. Can you put something there to remind the police lady that this car is going to go first?

C. A tick.

R. You said the yellow one was going to be second. Can you put something there to say that it is going to be second?

C. A ‘b.’

R. We’re going to do the blue one, which you said was fourth. What can you put there to help the police lady remember?

C. An ‘L.’

R. And we’re going to look at the car at the end here, which you said was sixth. What can you put there?

C. An 'X.'

For the SG task, Amelia used numerals to denote her initial eight cards, the quantity of cards that each player had at the end of round one, and the eight cards that she had at the end of round two. She used a cross (Other Symbolic form) to show that the researcher had zero cards at the end of round two.

In C3, for the TQ task, Amelia combined Drawing (of the traffic light and of a car) to indicate the first and sixth position, respectively, and Other Symbolic forms (she did not verbalise what these symbols represented), when prompted to indicate the second and fourth position. For the SG task, in C3, she chose to draw eight hearts to represent her initial eight cards. She used a numeral in round one and an "X" in round two to indicate that the researcher had zero cards. She produced Misaligned numerals ("50" and "31" with a reverse "3") to represent 15 and 13 cards, respectively.

Amelia's pathway across three cycles indicates consistent use of notations that did not include numerals for denoting ordinal position alongside the emergent use of numerals for representing quantities. In C3, we observed a notable difference in her approach to signifying quantity when she used Iconic notations for representing the same quantity (eight) that she had previously represented with a well-formed numeral in C2.

6 Discussion and conclusion

We examined cross-task and within-task variations and individual pathways of notation production over one year. An exploratory overview of the frequency of notational types across tasks indicated that, in the first cycle of data collection, the percentage of numeral use in the quantity task was almost double the percentage of numerals used in the ordinal task. This was maintained for all data collection cycles, while similar percentages of Idiosyncratic notations and a gradual decrease of these were observed over cycles, for both tasks.

Close examination of the range of notations that children combined showed that a wide range of mixed notations that included some numerals amongst other notations was used more prominently for representing quantity rather than ordinal position. In the quantity task, a move towards numeral use was observed by most children, across the three cycles. This is aligned with previous research on children's representations of quantity (e.g., Bialystok & Codd, 1996; Hughes, 1986; Munn, 1998) and numeral production in dictation tasks (e.g., Lucangeli et al., 2012). For the communication of ordinal position, children's notation production followed a similar trajectory, in that, children's use of numerals increased over time. However, in contrast to the quantity task, longitudinal pathways show that the move towards consistent use of numerals for indicating ordinal position was often direct (from using other notations to using exclusively numerals) rather than gradual (as evidenced by a gradual emergence of some numerals amongst other notations).

Previous research too has found that children merge numerals with letters and idiosyncratic signs (Carruthers & Worthington, 2005; Papandreou, 2022). Our findings reveal that the range of notations that the same group of children draw from, and their move towards integrating numerals alongside other notations, or using numerals consistently, can differ across tasks that elicit the production of numerals for different functions. In the absence of previous longitudinal evidence on children's notations in ordinal situations, against which we can compare our findings, this observation merits further research examination.

In contrast to previous research (Hughes, 1986; Sinclair, 1991), children in our sample made limited use of one-to-one correspondence, iconic notations for representing quantity. One possible reason for this may be that children were prompted to make a note of the number of cards that each player had, without all cards being on display as separate items. Tolchinsky (2003) notes that numerals represent quantity in a "compiled way" and to use them, children need to overcome "the idea that multiplicity must be explicitly displayed" (p. 113). Tasks where the multiplicity of items is explicitly displayed may lend themselves for the use of iconic, analogue representations more than tasks (like the one used here) where quantity is not overtly displayed.

Interestingly, Iconic notations were used for the first time by two children in C3, after they had previously used a numeral for representing the same quantity in the previous cycle (Amelia, previously discussed; Todd, Table 4). Iconic representations can be useful for recording quantity throughout life (e.g., tally marks for recording the score in a game or other situations) (Biallystok & Codd, 1996). Rather than being interpreted as a backward shift to a non-numerical notation, the emergence of notations other than numerals in C3 in these cases can be seen as examples of intertextual behaviour (Worthington, 2023), indicative of the expansion of a child's notational repertoire and the introduction of a notation type that the child considers as appropriate or effective in the particular context. This is also exemplified by the use of the numeral "0" and of an "X" by Amelia, within the same task, for showing that a player had no cards at the end of a round.

In the ordinal position task, some children used drawings, colour and graphemes representing colour (e.g., "green") as identifying labels for a specific car. Such notations may have been underpinned by interpretations of the situation as a task that requires representation of "what comes next" in an ordered sequence, so that the sequence can be remembered and replicated. We cannot exclude this potential interpretation by some children, particularly for those who used identifying elements for all positions under focus. However, this potential interpretation does not provide an entirely satisfactory account for cases where notations used as identifying labels were mixed with numerals (e.g., Fig. 3). An alternative interpretation of notations used as identifying labels or the use of other symbolic forms may be that children attempted to represent the spatial rather than ordinal position of cars (see Margolinas & Bessot, 2024). Further examination of this potential interpretation for some children's notational choices is part of our ongoing analysis of our data and could be further explored by research employing tasks that are specifically designed to distinguish children's understanding and representation of spatial and ordinal position.

In both tasks, the use of the wide notational range (0–9) involved the emergence of some numerals, amongst other notation types. Although not consistent and sometimes misaligned with the spoken number or the task requirements, the use of numerals within this range suggests at least some recognition of the appropriateness of a numeral as a symbol for representing ordinal position or quantity. In some cases, misaligned numeral production or the choice to not produce a numeral was based on which numerals children knew, which numerals they knew how to write, or even children's familiarity with a numeral of personal significance, irrespective of its alignment with the task requirement. Some children's comments explicitly revealed that notational choices were influenced by their awareness of constraints in their skill in forming specific numerals, rather than limited awareness of the "affordances" of numerals as symbols for communicating specific meanings (Teubal & Dockrell, 2005). In such instances, children introduced other forms of notations, drawing from their wider experience of symbols (Munn, 1998).

We propose that mixed notational behaviour is an indication of "intertextual engagement" (Worthington, 2020, p. 71) that is intentional, in that, children intentionally draw from across their literacies and notational repertoire (including language, drawing, other symbols), to select or combine forms of notation that allow them to complete an action (Radford, 2000; Worthington, 2020), or fulfil their communicative intentions (Worthington, et al., 2023). Our study shows that children's intertextual notational choices are a complex and multifaceted phenomenon that needs to be further studied by research in relation to the following: task parameters, children's interpretation and meaning making within a task, their symbol knowledge and skills, as well as the goals that children formulate, which may or may not be aligned with the numerical goals that a task aims to elicit.

Munn (1998) notes that "progression lies in the function that the symbol has for the child's action" (p. 47). She posits that children, as 'novices,' likely form different cognitive models based on interaction with objects and interaction with numeric symbols. While children may deduce "the varying meanings of the same number word by attending to the context of the utterance, it is not so simple to deduce the many meanings of the written numeral" (Munn, 1998, p. 53). We propose that there is a need to further explore the role that children's intertextual engagement has as an indicative or even, potentially, necessary, element as children develop their cognitive model of symbolic function that integrates number words, abstract number referents, and number symbols, for serving different goals (Munn, 1998).

Our observation that the older children of the 3–4-year-old age range appeared to exhibit similar pathways as some of the children in the older sub-group is also worth investigating further, with a larger sample and chronological homogeneity within age groups, to examine potential critical developmental points. Future research could also employ tasks that prompt children to produce as well as interpret their own notations at a later point, to provide further insight into the meaning that children ascribe to their notations across ordinal and quantity tasks and the way they use them for specific purposes.

The theoretical notion of intertextuality postulates that individuals interpret tasks by connecting them with previous experiences (Rojano, et al., 2014). In line with this postulation, Bruce and Threlfall (2004) argue that "tardy development of ordinal number may be mostly the result of the attention given by adults, and the nature of the common experiences of young children in a pre-school setting" (p. 24). Extending this point, we propose that emphasis on the cardinal use of numerals in children's experiences, at preschool or at home, may explain differences observed in children's notational choices for the communication of quantity and ordinal position. Such explanation needs to be further examined, also taking into consideration arguments about the higher complexity of ordinal situations in comparison to cardinal situations (e.g., Fuson, 1988), as well as evidence related to challenges associated with the structure of ordinal number words in the English language (Miller et al., 2000). In particular, future research could examine children's exposure to and experiences of cardinal and ordinal situations, as well as cardinal and ordinal number words and numerals, in their preschool and family environments. Though methodologically challenging, this could be done through observational approaches and analysis of numeral meanings that children are exposed most prominently to, as part of early years teaching and curricula, in interactions with adults and peers, as well as interaction with mediating tools, such as story books.

Children's learning of socially developed conventions related to mathematical signs is tightly linked to their interaction with others (Sinclair, 1991), who "frequently model signs (within multi-sign utterances)" (Worthington et al., 2019, p. 95). To support preschool children's transition to formal school mathematics, it is important to maintain sensitivity to their idiosyncratic notations (Munn, 1994) and support connections between their informal knowledge of number and written symbols (Purpura et al., 2013). Providing increased opportunities for children to observe numeral use by others and use numerals themselves, purposefully, to communicate a range of different number-related ideas, is essential, for supporting children's notational development and understanding of the affordances of numerals as referential-communicative symbols that serve different purposes.

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Data availability Raw data not available due to ethical restrictions. The participants of this study did not give written consent for their data to be shared publicly.

Declarations

Ethics approval The research followed the British Educational Research Association Ethical Guidelines for Educational Research (2024).

Competing interests The authors declare no competing interests.

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