

Joint Angle and Velocity-Estimation for Target Localization in Bistatic mmWave MIMO Radar in the Presence of Clutter

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Abstract—Sparse Bayesian learning (SBL)-aided target localization is conceived for a bistatic mmWave MIMO radar system in the presence of unknown clutter, followed by the development of an angle-Doppler (AD)-domain representation of the target-plus-clutter echo model for accurate target parameter estimation. The proposed algorithm exploits the three-dimensional (3D) sparsity arising in the AD domain of the scattering scene and employs the powerful SBL framework for the estimation of target parameters, such as the angle-of-departure (AoD), angle-of-arrival (AoA) and velocity. To handle a practical scenario where the actual target parameters typically deviate from their finite-resolution grid, a super-resolution-based improved off-grid SBL framework is developed for recursively updating the parameter grid, thereby progressively refining the estimates. We also determine the Cramér-Rao bound (CRB) and Bayesian CRB for target parameter estimation in order to benchmark the estimation performance. Our simulation results corroborate the superior performance of the proposed approach in comparison to the existing algorithms, and also their ability to approach the bounds derived.

Index Terms—Target imaging, sparsity, localization, Bayesian learning, parameter estimation, bistatic MIMO radar, clutter.

I. INTRODUCTION

Next-generation (NG) wireless systems are expected to support high-accuracy localization services in indoors for robot navigation [1], human sensing [2], Wi-Fi sensing for smart homes [3], radar sensing for autonomous vehicles [4], location-assisted communication [5], [6], user positioning [7],

[8] and location-aware applications [9], [10], among others. Estimation of distance and angle is also crucial for distance-aware resource allocation and precoding in NG wireless systems [11]. The primary objective of localization is to identify the position and velocity of a target via estimating the direction of arrival, Doppler shift, and time delay. As a further advance, multiple-input multiple-output (MIMO) radar promises significant diversity and multiplexing gains [12], [13]. MIMO radar systems can be classified into colocated and distributed MIMO [14], wherein the transmitter and receiver are at the same position in the former, while they are distributed in different locations in the latter.

Millimetre wave (mmWave) carriers, which belong to the extremely high frequency (EHF) range spanning from 30 GHz to 300 GHz, have extraordinarily short wavelength. This enables the packing of a large number of antennas in devices having compact form factor [15], which in turn enables ultra-high angular resolutions that make them ideally suited for radar operation [16]. Millimetre wave (mmWave) technology is expected to become increasingly dominant in the near future due to the availability of sizeable unlicensed frequency blocks [17], which makes it eminently suitable for demanding applications like ultra-high-definition (UHD) 3D video, virtual and augmented realities, internet-of-things (IoT), satellite communications, unmanned aerial vehicles (UAV), V2X, among others [18]. The MIMO technology [19] relying on a large number of transmit and receive antennas, coupled with the huge bandwidth of the mmWave regime, has the potential of significantly improving radar performance.

A mmWave distributed MIMO radar, composed of multiple bistatic transmitter/ receiver pairs, offers several advantages over its colocated counterpart, such as spatial diversity, high resolution, reduced transmitter-receiver antenna interference, while dispensing with switches or duplexers [20]. Exploiting the angular and temporal sparsity of the mmWave channel in such a system may lead to a significant improvement in target detection and localization performance [21], [22]. Therefore, we conceive techniques for joint parameter estimation in target localization of bistatic mmWave MIMO radar. An overview of the existing contributions in this area is presented next.

A. Importance of location-aware Interaction in the mmWave Band

A specific drawback of operating at high frequencies, such as in the mmWave band, is that propagation losses increase

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with the carrier frequency. Potential solutions to overcome such losses in 5G networks is to utilize accurate location information [5]. Firstly, knowing the positions and thus the distances between nodes can help estimate received power and interference levels. In dense networks, the shortest-distance multihop path between a source and destination is often the most efficient. Additionally, distance and angle estimation helps in location-aware resource allocation [11]. Traditional resource allocation relies heavily on real-time channel state information (CSI), which must be frequently updated through pilot signals and feedback leading to overhead and delay. However, location-aware systems can predict future channel conditions by using the known or estimated position and mobility pattern of a device [23], [24]. Since the path loss, interference, and even shadowing can be correlated with position, the system can estimate the CSI ahead of time—especially in slowly varying environments or when user trajectories are predictable (e.g., vehicles on roads, users on foot). The angle and distance estimation also helps attain improved energy and spectral efficiency by steering beams accurately in directional mmWave communication and turning off the antennas or reduce transmit power when coverage is known to be sufficient [25]. Location-aware precoding is also beneficial in near-field communications which must account for both distance and angle information [26], [27].

B. Literature Review

He *et al.* [28] proposed a two-dimensional multiple signal classification (2D-MUSIC) algorithm for joint angle-of-arrival (AoA) and angle-of-departure (AoD) estimation in MIMO radar that imposes high computational cost. To reduce this, Zhang *et al.* [29] developed a reduced-dimensional MUSIC (RD-MUSIC) algorithm that searches for peaks over a single dimension. Another novel subspace-based method disseminated by Zheng *et al.* [30] is the estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm that is significantly faster than the RD-MUSIC since a closed-form angle estimation solution is obtained by exploiting the invariance property between the transmit and receive arrays. Nevertheless, their algorithm is unable to achieve the same level of accuracy as the family of MUSIC-related techniques. For precisely determining the signal/noise subspaces, both the ESPRIT and MUSIC-based approaches necessitate an excessively large number of snapshots, whose accuracy significantly degrades in scenarios of low signal-to-noise ratios (SNR) and/or insufficient snapshots. With objective of joint AoD and AoA estimation, Tang *et al.* [31] developed an innovative maximum likelihood scheme, wherein an alternating projection algorithm was devised for resolving the associated high-dimensional nonlinear optimization. While it yields improved performance, its convergence is not guaranteed. The treatises [32], [33] predominantly focus on the localization of targets in distributed radar based on time-of-arrival (ToA) and time difference of arrival (TDOA) measurements, respectively. Qin *et al.* [32] exploit the bistatic range (BR) and TDOA measurements for developing an interesting two-step least squares algorithm for target localization. By contrast Nguyen *et al.*

[33] conceive a ToA based geometric model for minimizing the estimation confidence area by maximizing the determinant of the pertinent Fisher information matrix (FIM). However, the above contributions do not include velocity or Doppler shift estimation in the localization problem. As a further advance, the authors of [34], [35] designed potent schemes for location and Doppler estimation based on BR, TDOA and Doppler shift measurements. Nevertheless, it is imperative to mention that TOA-based schemes require accurate time synchronization between the transmitters and receivers, which is challenging to achieve in practical systems. Furthermore, the scattering environment at mmWave frequencies is inherently sparse, a feature that can be exploited for bolstering performance, and one which none of the aforementioned studies fully exploit. The literature specifically related to compressive sensing (CS)-based localization is reviewed next. CS-based methods exhibit several key advantages over both the conventional signal processing methods and over the existing subspace based schemes. For example, they require a remarkably reduced number of snapshots for reliable signal recovery even for an unknown number of sources. Furthermore, they exhibit substantially improved robustness to noise [40]. Chen *et al.* [41] developed a sparse Bayesian learning (SBL)-based technique for direction of arrival (DOA) estimation in colocated MIMO radar. The joint estimation of the AoA and AoD parameters of point targets in MIMO-aided radar systems using SBL was successfully carried out in [36]. As a further development, Maity *et al.* [37] proposed an SBL scheme to jointly estimate the AoA, range and velocity of the targets in mmWave MIMO radar systems. Traditional SBL approaches require the targets to lie on a predefined grid of the AoD/AoA parameters. This in turn mandates the fixed parameter grid to be finely spaced in order for the sparse assumption to be reliable, which could lead to a high computational cost and violate the restricted isometric property (RIP) to be satisfied for sparse recovery [42]. The pioneering work by Yang *et al.* [38] addressed the grid mismatch problem by proposing an off-grid model obtained by linearly approximating the dictionary and updating it in every iteration. However, it does not fully eliminate the unavoidable off-grid performance erosion. To address this impediment, Cao *et al.* [39] jointly estimated the AoDs and AoAs in bistatic MIMO radar by introducing a modified linear approximation model, where the grid points are directly updated in every iteration for ensuring that the updated estimates approach the true AoAs and AoDs upon convergence. However, the exhaustive list of papers discussed above ignore the mobile targets and ground clutter. The undesired echo imposed by the ground clutter such as land, buildings, roads, etc [43], [44] further degrade the sensing performance, if not mitigated. This limits the localization performance in practical integrated sensing and communication (ISAC) systems.

Several clutter suppression techniques [45], [46], [47] identify and eliminate clutter by relying on time-domain and space-time-domain approaches. However, these do not exploit the angular domain sparsity. Mishra *et al.* [48] leveraged the target sparsity and estimated the target parameters using the SBL framework in the presence of ground clutter for colocated radar systems. However, they assumed the clutter covariance matrix

TABLE I: Boldly contrasting our new contributions against the state-of-the-art

Features	[1]	[28], [29], [31]	[33]	[35]	[36]	[37]	[38]	[39]	[48]	Proposed
mmWave band						✓				✓
Distributed radar	✓	✓	✓	✓	✓			✓		✓
Off-grid SBL					✓		✓	✓		✓
Multiple targets		✓			✓		✓	✓		✓
Multiple parameter estimation		θ, ϕ	ϕ	\mathbf{p}, v	θ	RCS, θ, v	θ	θ, ϕ	RCS, θ, v	RCS, θ, ϕ, v
Target localization	✓		✓	✓		✓				✓
CRB for joint parameter estimation									✓	✓
3D sparsity						✓	✓	✓	✓	✓
Unknown clutter	✓					✓				✓
Geometric calculation of location from AoA and AoD										✓

to be known, which is challenging to estimate in real-world systems. To fill these research gaps, this treatise conceives a Bayesian learning-based localization scheme that is capable of jointly estimating the AoA, AoD, velocity and radar cross-section (RCS) parameters of multiple targets in the presence of clutter having unknown statistics, while exploiting the 3D sparsity arising in the angular and Doppler (AD)-domain. The main contributions of this study are itemized next and they are also boldly contrasted to the literature in Table-I. The symbols ϕ, θ, v used in the table denote the target AoA, AoD and velocity, respectively.

C. Contributions of our work

- A novel bistatic mmWave MIMO radar system is developed for the localization of multiple mobile targets and joint estimation of target parameters such as AoA, AoD, Doppler shift and RCS coefficients in the presence of unknown noise-plus-clutter components.
- The localization problem is viewed from a fresh perspective of the emerging field of compressive sensing (CS) that exploits the 3D sparsity in the angle-Doppler (AD)-domain. To overcome the drawback of uniform sampling of the grids toward dictionary matrix formulation, this work presents a framework for non-uniform sampling of the 3D grid, where no two grid points share the same AoA, AoD and Doppler shift values.
- As a result of non-uniform sampling, modeling error arises due to the gap between the true target parameter and its closest grid point. In order to address the off-grid problem, the adjustable grid points are updated recursively, until convergence is reached. Subsequently, a novel block majorization-minimization (MM) procedure is developed for Bayesian inference.
- Furthermore, the proposed algorithm helps filter out the unknown clutter components from the estimated target components. Subsequently, a localization algorithm is devised for determining the position coordinates and velocity of the targets from the estimated AoAs, AoDs and Doppler shifts.
- Next, the Cramér Rao bounds (CRB) are determined for joint AoA, AoD and velocity vector estimation, which benchmark the estimation performance of the proposed design. Furthermore, the Bayesian CRB (BCRB) is derived for RCS coefficient estimation in order to characterize its estimation performance.

- Lastly, our simulation findings comprehensively validate the performance of the proposed schemes and clearly evidence their enhanced performance over the existing methods, even at low signal to clutter-plus-noise ratio (SCNR).

The rest of the paper is outlined as follows, The system model of bistatic mmWave MIMO radar is developed in Section I in presence of clutter. Section III discusses the off-grid SBL-based algorithm conceived for joint AoA-AoD-Doppler shift estimation, followed by our localization algorithm in Section III-A and complexity analysis in Section III-B. Section IV derives the CRB whereas our simulation results are discussed in Section V. Section VI concludes the paper.

D. Notation

The operator $\Re\{\cdot\}$ denotes the real part of a complex-valued argument, while $|\mathbf{a}|$, $\|\mathbf{a}\|_0$ and $\|\mathbf{a}\|_2$ denote the element-wise absolute value, the ℓ_0 norm and the Euclidean norm of the vector \mathbf{a} , respectively. For a matrix \mathbf{A} , $|\mathbf{A}|$, $\text{Tr}(\cdot)$ and $\|\mathbf{A}\|_F$ represent its determinant, trace and the Frobenius norm, respectively. The notations $\text{rect}(\cdot)$ and $\text{sign}(\cdot)$ stand for the rectangular pulse and signum function, respectively. Furthermore, $\mathbf{1}_N$ and \mathbf{I}_N represent an $N \times 1$ vector with entries as 1 and $N \times N$ identity matrix, respectively. The operator $\text{diag}(\cdot)$ results in a diagonal matrix having the elements of its argument vector on the principal diagonal. Operator $\text{vec}(\cdot)$ performs vectorization of its argument matrix by stacking its columns. The operators $(\cdot)^H$, $(\cdot)^*$, $(\cdot)^T$ denote the Hermitian, conjugate and transpose operation applied to the argument, respectively. The operations $A \odot B$ and $A \otimes B$ denote the Hadamard and Kronecker matrix products, respectively. The operator $\mathbb{E}(\cdot)$ represents the statistical expectation, while $\int(\cdot)dw$ and $\frac{\partial}{\partial w}(\cdot)$ are the integration and partial derivative of the argument with respect to w .

II. SYSTEM MODEL FOR BISTATIC MMWAVE MIMO RADAR

A. Signal Model

A bistatic mmWave MIMO radar setup comprising a transmitter and a receiver is considered, which are equipped with N_t transmit and N_r receive antennas, respectively, as shown in Fig. 1. The transmitter is situated at the origin and the receiver's coordinates are given $[R_r, \varphi_r] \in \mathbb{R}^2$, where R_r is the distance between the receiver and transmitter, while φ_r is the

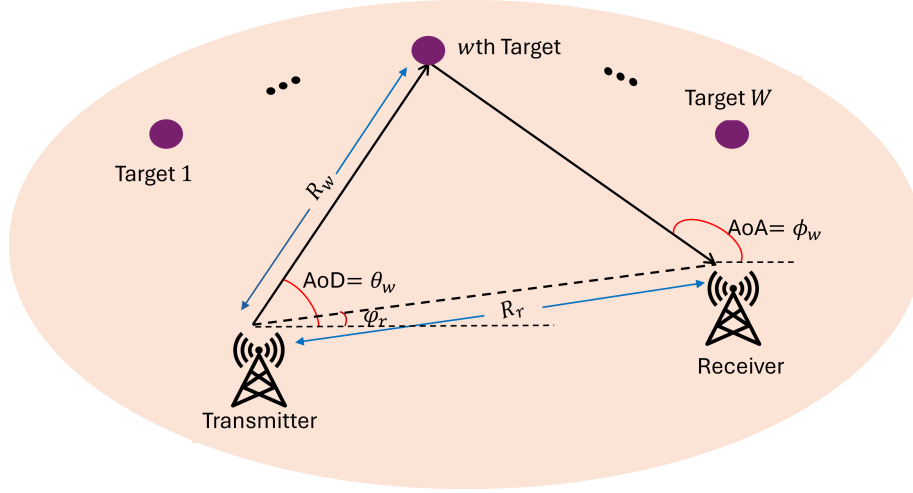


Fig. 1: System model for target localization in bistatic mmWave MIMO radar

orientation. The w th target's unknown polar coordinates are $[R_w, \varphi_w] \in \mathbb{R}^2$. Note that this model can be easily extended to a 3D geometry, where the w th target's unknown polar coordinates can be identified as $[R_w, \theta_w, \phi_w]$. Here, θ_w and ϕ_w represent the azimuth and elevation angle, respectively. Furthermore, the proposed localization framework can be easily adapted to a 3D scenario where the sparsity in the azimuth angle-elevation angle-Doppler domain will be leveraged. For simplicity, the study is limited to the 2D geometry. Let W denote the number of unknown moving targets present in the network and let the velocity of the w th target be v_w . The signal impinging from the t th transmit antenna, $1 \leq t \leq N_t$, can be represented as

$$\mathbf{x}_t^T = [x_t(1), x_t(2), \dots, x_t(U)] \in \mathbb{C}^{1 \times U}, \quad (1)$$

where U is the total number of sub-pulses. Thus, $\mathbf{X} = [\mathbf{x}_1 \ \mathbf{x}_2 \ \dots \ \mathbf{x}_{N_t}]^T \in \mathbb{C}^{N_t \times U}$ denotes the overall transmit signal matrix. The Doppler-shifted signal corresponding to the t th antenna and w th target is defined as $\mathbf{x}_t^T(\omega_w) = \mathbf{x}_t^T \odot \phi^T(\omega_w)$, where $\phi^T(\omega_w) = [1, e^{j\omega_w}, \dots, e^{j\omega_w(U-1)}] \in \mathbb{C}^{1 \times U}$ is a vector corresponding to the Doppler shift ω_w of the w th target, which is defined as

$$\omega_w = 2\pi f_w T_p, \quad (2)$$

where $f_w = \frac{v_w}{\lambda}$ denotes the Doppler frequency and T_p is the sub-pulse interval. We define the matrix $\mathbf{X}_w = [\mathbf{x}_1(\omega_w), \mathbf{x}_2(\omega_w), \dots, \mathbf{x}_{N_t}(\omega_w)]^T \in \mathbb{C}^{N_t \times U}$, which represents the transmitted signal matrix associated with the motion of the w th target. Upon considering N_{cl} ground clutter components, the signal $\mathbf{Y} \in \mathbb{C}^{N_r \times U}$ reflected from the W targets and arriving at the receiver is modeled as

$$\mathbf{Y} = \sum_{w=1}^W e^{\frac{-j2\pi R_w^B}{\lambda}} \beta_w \mathbf{d}_w \mathbf{c}_w^T \mathbf{X}_w + \sum_{z=1}^{N_{cl}} e^{\frac{-j2\pi R_z^B}{\lambda}} \beta_z \mathbf{d}_z \mathbf{c}_z^T \mathbf{X} + \mathbf{Q}, \quad (3)$$

where $\beta_w \in \mathbb{C}$ and $\beta_z \in \mathbb{C}$ are the RCS coefficients of the w th target and z th clutter, respectively. The factors $e^{\frac{-j2\pi R_w^B}{\lambda}}$ and $e^{\frac{-j2\pi R_z^B}{\lambda}}$ represent the roundtrip delay corresponding to

the w th target and z th clutter, respectively, where R_w^B and R_z^B are the corresponding sum distances of the transmitter- w th target-receiver pair and transmitter- z th clutter-receiver pair, respectively. The quantities $\mathbf{c}_w \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{d}_w \in \mathbb{C}^{N_r \times 1}$ are the transmit and receive array response vectors, respectively, corresponding to the w th target, which are given as

$$\mathbf{c}_w = \left[1, e^{\frac{-j2\pi d_t}{\lambda} \sin(\theta_w)}, \dots, e^{\frac{-j2\pi d_t}{\lambda} (N_t-1) \sin(\theta_w)} \right]^T, \quad (4)$$

$$\mathbf{d}_w = \left[1, e^{\frac{-j2\pi d_r}{\lambda} \sin(\phi_w)}, \dots, e^{\frac{-j2\pi d_r}{\lambda} (N_r-1) \sin(\phi_w)} \right]^T, \quad (5)$$

where d_t and d_r denote the transmit and receive antenna spacings, respectively, whereas θ_w, ϕ_w denote the AoD and AoA corresponding to the w th target, respectively. The quantities $\mathbf{c}_z \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{d}_z \in \mathbb{C}^{N_r \times 1}$ are the transmit and receive array response vectors, respectively, corresponding to the z th clutter. Finally, the quantity $\mathbf{Q} \in \mathbb{C}^{N_r \times U}$ is the complex additive white Gaussian noise (AWGN) matrix, the elements of which are independently and identically distributed with zero mean and variance σ^2 . Upon vectorizing the expression in (3), the system model can be recast as

$$\mathbf{y} = \tilde{\Psi} \tilde{\beta} + \mathbf{q}, \quad (6)$$

where $\mathbf{y} = \text{vec}(\mathbf{Y}) \in \mathbb{C}^{N_r U \times 1}$, $\mathbf{q} = \text{vec}(\mathbf{Q}) \in \mathbb{C}^{N_r U \times 1}$, $\tilde{\Psi} = [\tilde{\psi}_1, \dots, \tilde{\psi}_{W'}] \in \mathbb{C}^{N_r U \times W'}$ is the sensing matrix, $\tilde{\beta} = [e^{\frac{-j2\pi R_1^B}{\lambda}} \beta_1, \dots, e^{\frac{-j2\pi R_{W'}^B}{\lambda}} \beta_{W'}] \in \mathbb{C}^{W' \times 1}$ is the RCS coefficient vector and $W' = W + N_{cl}$. Each column of the sensing matrix is given by $\tilde{\psi}_{w'} = \text{vec}(\mathbf{d}_{w'} \mathbf{c}_{w'}^T \mathbf{X}_{w'})$. As observed from Fig. 1, the parameters to be estimated for localization of the w th mobile target in the bistatic scenario are AoD θ_w , AoA ϕ_w and Doppler shift ω_w .

Remark. Note that one can also utilize the classic cyclic prefix-orthogonal frequency division multiplexing (CP-OFDM) waveform for target localization [49], [50] in bistatic mmWave MIMO ISAC systems in the presence of clutter. The l th received sample at the r th receive antenna corresponding to the m th

OFDM symbol can then be expressed as

$$y_m^r[l] = \sum_{w=1}^W \beta_w [\mathbf{d}_w]_r e^{j2\pi f_w m T_s} e^{j2\pi T_s \frac{1}{N_c} f_w} \mathbf{c}_w^T \times \\ \frac{1}{\sqrt{N_c}} \sum_{n=0}^{N_c-1} \mathbf{x}_{n,m} e^{j2\pi n \frac{1}{N_c}} e^{-j2\pi n \Delta f \tau_w} + \\ \sum_{z=1}^{N_{cl}} \beta_z [\mathbf{d}_z]_r \mathbf{c}_z^T \frac{1}{\sqrt{N_c}} \sum_{n=0}^{N_c-1} \mathbf{x}_{n,m} e^{j2\pi n \frac{1}{N_c}} e^{-j2\pi n \Delta f \tau_z}, \quad (7)$$

where $\mathbf{x}_{n,m} \in \mathbb{C}^{N_t \times 1}$ is the complex transmit data vector corresponding to the n th subcarrier of the m th OFDM symbol, T is the total OFDM symbol duration given by $T = T_s + T_{cp}$, T_s is the OFDM symbol duration, T_{cp} is the cyclic prefix duration and $\Delta f = \frac{1}{T_s}$ is the subcarrier spacing. The spectral steering vector and inter-carrier interference (ICI) phase rotation vector are defined as follows:

$$\mathbf{b}_{w'} = [1, e^{-j2\pi \Delta f \tau_{w'}}, \dots, e^{-j2\pi (N_c-1) \Delta f \tau_{w'}}]^T \in \mathbb{C}^{N_c \times 1}, \\ \mathbf{a}_{w'} = [1, e^{j2\pi T_s \frac{1}{N_c} f_{w'}}, \dots, e^{j2\pi T_s \frac{N_c-1}{N_c} f_{w'}}]^T \in \mathbb{C}^{N_c \times 1}. \quad (8)$$

As shown in our previous work [37], upon stacking the received samples $y_m^r[l]$, $1 \leq l \leq N_c$, the received vector at the receiver in the m th OFDM symbol is given as

$$\mathbf{y}_m = \sum_{w'=1}^{W'} \beta_{w'} \left\{ \mathbf{d}_{w'} \otimes \left\{ \mathbf{a}_{w'} \odot \left(\mathbf{F}_{N_c}^H (\mathbf{b}_{w'} \odot (\mathbf{X}_m \mathbf{c}_{w'})) \right) \right\} \right\} \\ + \mathbf{q}_m = \tilde{\Psi} \tilde{\beta} + \mathbf{q}_m \in \mathbb{C}^{N_c N_r \times 1}, \quad (9)$$

where each column of the sensing matrix $\tilde{\Psi}$ is given as $\tilde{\psi}_{w'} = \left\{ \mathbf{d}_{w'} \otimes \left\{ \mathbf{a}_{w'} \odot \left(\mathbf{F}_{N_c}^H (\mathbf{b}_{w'} \odot (\mathbf{X}_m \mathbf{c}_{w'})) \right) \right\} \right\}$ and $\tilde{\beta} = [\beta_1, \beta_2, \dots, \beta_{W'}] \in \mathbb{C}^{W' \times 1}$ is the RCS coefficient vector. It can be readily inferred that (9) is similar to (6).

B. Sparse Problem Formulation

To formulate the joint AoA, AoD and Doppler shift estimation as a sparse problem, let the imaging scene be divided into G AoA bins, G AoD bins and G Doppler bins. The uniform sampling of the 3D grid leads to G^3 3-tuples: $\{(\theta_1, \phi_1, \omega_1), (\theta_1, \phi_1, \omega_2), \dots, (\theta_1, \phi_1, \omega_G), (\theta_1, \phi_2, \omega_1), \dots, (\theta_G, \phi_G, \omega_G)\}$. However, this leads to high computational complexity for the ensuing search and a high correlation between the adjacent grid points [39]. For example, two nearby 3-tuples $(30^\circ, 50^\circ, 0.075)$, $(25^\circ, 50^\circ, 0.25)$ share the same AoA leading to a high correlation between these two bases of the dictionary matrix. This significantly degrades the angle and Doppler shift estimation accuracies. This motivates us to perform non-uniform sampling of the 3D grid, where no two grid points share the same AoA, AoD and Doppler shift, i.e. we have

$$\theta_{g_i} \neq \theta_{g_j}, \phi_{g_i} \neq \phi_{g_j}, \omega_{g_i} \neq \omega_{g_j}, \forall g_i \neq g_j. \quad (10)$$

Therefore, the received signal in (6) can be expressed as

$$\mathbf{y} = \Psi \beta + \mathbf{q}, \quad (11)$$

where $\Psi = [\psi_1, \psi_2, \dots, \psi_g, \dots, \psi_G] \in \mathbb{C}^{N_r U \times G}$ is the dictionary matrix. Each column vector $\psi_g \in \mathbb{C}^{N_r U \times 1}$ of the dictionary matrix Ψ is expressed as

$$\psi_g = \text{vec}(\mathbf{d}_g \mathbf{c}_g^T \mathbf{X}_g). \quad (12)$$

Furthermore, $\beta = [\beta_1, \beta_2, \dots, \beta_G]^T \in \mathbb{C}^{G \times 1}$ is the vector of the complex RCS coefficients. Note that, the vector β is sparse in nature since it has a support of only W' non-zero elements within its support length G , with $G \gg W'$. The target parameters $\eta = [\theta, \phi, \omega] \in \mathbb{R}^{3W' \times 1}$ typically do not lie on the grid of the dictionary matrix Ψ due to the non-uniform sampling of the 3D grid. This grid mismatch often leads to large estimation errors, thus, degrading the performance of the system. This off-grid gap can be handled by linearly approximating the steering vectors using a first-order Taylor expansion as follows

$$\psi(\theta_{w'}, \phi_{w'}, \omega_{w'}) = \psi(\tilde{\theta}_g, \tilde{\phi}_g, \tilde{\omega}_g) + \xi_t(\tilde{\theta}_g, \tilde{\phi}_g, \tilde{\omega}_g) \times \\ (\theta_{w'} - \tilde{\theta}_g) + \xi_r(\tilde{\theta}_g, \tilde{\phi}_g, \tilde{\omega}_g) (\phi_{w'} - \tilde{\phi}_g) \\ + \xi_\omega(\tilde{\theta}_g, \tilde{\phi}_g, \tilde{\omega}_g) (\omega_{w'} - \tilde{\omega}_g), \quad (13)$$

where we have:

$$\xi_t(\tilde{\theta}_g, \tilde{\phi}_g, \tilde{\omega}_g) = \left. \frac{\partial \psi(\theta_{w'}, \tilde{\phi}_g, \tilde{\omega}_g)}{\partial \theta_{w'}} \right|_{\theta_{w'} = \tilde{\theta}_g}, \quad (14)$$

$$\xi_r(\tilde{\theta}_g, \tilde{\phi}_g, \tilde{\omega}_g) = \left. \frac{\partial \psi(\tilde{\theta}_g, \phi_{w'}, \tilde{\omega}_g)}{\partial \phi_{w'}} \right|_{\phi_{w'} = \tilde{\phi}_g}, \quad (15)$$

$$\xi_\omega(\tilde{\theta}_g, \tilde{\phi}_g, \tilde{\omega}_g) = \left. \frac{\partial \psi(\tilde{\theta}_g, \tilde{\phi}_g, \omega_{w'})}{\partial \omega_{w'}} \right|_{\omega_{w'} = \tilde{\omega}_g}. \quad (16)$$

Here, $\tilde{\theta}_g, \tilde{\phi}_g, \tilde{\omega}_g$ are the nearest grid points to the w th target $(\theta_w, \phi_w, \omega_w)$. Upon assuming $\Xi_t = [\xi_t(\tilde{\theta}_1, \tilde{\phi}_1, \tilde{\omega}_1), \dots, \xi_t(\tilde{\theta}_G, \tilde{\phi}_G, \tilde{\omega}_G)]$, $\Xi_r = [\xi_r(\tilde{\theta}_1, \tilde{\phi}_1, \tilde{\omega}_1), \dots, \xi_r(\tilde{\theta}_G, \tilde{\phi}_G, \tilde{\omega}_G)]$, $\Xi_\omega = [\xi_\omega(\tilde{\theta}_1, \tilde{\phi}_1, \tilde{\omega}_1), \dots, \xi_\omega(\tilde{\theta}_G, \tilde{\phi}_G, \tilde{\omega}_G)]$, the off-grid model for \mathbf{y} is formulated as

$$\mathbf{y} = (\Psi(\tilde{\theta}, \tilde{\phi}, \tilde{\omega}) + \Xi_t \text{diag}(\epsilon_t) + \Xi_r \text{diag}(\epsilon_r) + \Xi_\omega \text{diag}(\epsilon_\omega)) \beta \\ + \mathbf{q} = \mathbf{D}(\epsilon_t, \epsilon_r, \epsilon_\omega) \beta + \mathbf{q}, \quad (17)$$

where

$$\epsilon_t = \begin{cases} \theta_w - \tilde{\theta}_g, & w = 1, 2, \dots, W' \\ 0, & \text{otherwise} \end{cases}, \quad (18)$$

$$\epsilon_r = \begin{cases} \phi_w - \tilde{\phi}_g, & w = 1, 2, \dots, W' \\ 0, & \text{otherwise} \end{cases}, \quad (19)$$

$$\epsilon_\omega = \begin{cases} \omega_w - \tilde{\omega}_g, & w = 1, 2, \dots, W' \\ 0, & \text{otherwise} \end{cases}. \quad (20)$$

Here ϵ denotes the gap between the actual parameter value and its closest grid point. Next, we formulate a row sparse estimation model for the mmWave MIMO radar considered

with the aid of multiple transmission snapshots [48]. The received vector corresponding to the l th snapshot, $1 \leq l \leq L$, where L denotes the total number of snapshots, is expressed as

$$\mathbf{y}_l = \mathbf{D}\beta_l + \mathbf{q}_l. \quad (21)$$

Upon concatenating the received vector over L snapshots, so that $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_L] \in \mathbb{C}^{N_r U \times L}$, the output signal matrix is given by

$$\mathbf{Y} = \mathbf{D}\mathbf{B} + \mathbf{Q}. \quad (22)$$

The targets are assumed to be stationary with respect to the tuple for the duration $T_c = LUT_p$, where T_p is the sub-pulse interval. Under this assumption, it can be observed that the vectors β_l have a common sparsity profile, which in turn makes the matrix $\mathbf{B} = [\beta_1, \beta_2, \dots, \beta_L] \in \mathbb{C}^{G \times L}$ row-sparse in nature. The next section describes the SBL algorithm conceived for our joint angle-velocity-RCS estimation problem.

III. BAYESIAN INFERENCE FOR JOINT TARGET PARAMETER ESTIMATION AND LOCALIZATION

The powerful SBL scheme is employed for exploiting the 3D sparsity and for the estimation of the sparse RCS matrix \mathbf{B} . The SBL framework imposes a parameterized Gaussian prior on each g th row of the unknown RCS sparse matrix \mathbf{B} , denoted by β_g , as follows:

$$p(\beta_g; z_g) \sim \mathcal{CN}(\mathbf{0}_{1 \times L}, z_g^{-1} \mathbf{I}_L) \\ = \prod_{l=1}^L \frac{z_g}{\pi} \exp(-z_g |\beta_{g,l}|^2), \quad (23)$$

where the hyperparameter $z_g, 0 \leq g \leq G-1$, specifies the precision matrix of the multivariate prior associated with β_g . The prior for the matrix \mathbf{B} is given by:

$$p(\mathbf{B}; \mathbf{z}) = \prod_{g=1}^G p(\beta_g; z_g). \quad (24)$$

Observe from (23) that as the hyperparameter obeys $z_g \rightarrow \infty$, the associated g th row of the RCS coefficient matrix follows $\beta_g \rightarrow \mathbf{0}_{L \times 1}$ [51]. Therefore, the estimation of the hyperparameter matrix is crucial for the estimation of the sparse matrix \mathbf{B} . However, the received signal model is over-parameterized, since the number of observations is close to the total number of parameters to be estimated. For this purpose, the hyperparameter vector $\mathbf{z} = (z_0, \dots, z_{G-1})^T$ is constrained by imposing a Gamma prior, that is,

$$p(\mathbf{z}) = \prod_{g=0}^{G-1} \text{Gamma}(z_g | a+1, b) = \prod_{g=0}^{G-1} \frac{b^a z_g^a e^{-bz_g}}{\Gamma(a+1)}, \quad (25)$$

where the Gamma function obeys $\Gamma(a) = \int_0^\infty t^{a-1} e^{-t} dt$. To render these priors non-informative (i.e. flat), we assign these parameters very small values, e.g. $a = b = 10^{-4}$, and assume that $\epsilon_t, \epsilon_r, \epsilon_\omega$ have non-informative uniform priors. One can now maximize the posterior distribution of the hidden

variables $\Omega_h = \{\mathbf{B}, \mathbf{z}, \epsilon_t, \epsilon_r, \epsilon_\omega\}$ conditioned on the observed data \mathbf{Y} as follows

$$\Omega_h = \arg \max_{\Omega_h} \ln p(\Omega_h | \mathbf{Y}) \equiv \arg \max_{\Omega_h} \ln p(\mathbf{Y}, \Omega_h). \quad (26)$$

Bayesian inference requires the computation of the posterior distribution

$$p(\Omega_h | \mathbf{Y}) = \frac{p(\mathbf{Y}, \Omega_h)}{\int p(\mathbf{Y}, \Omega_h) d\Omega_h} \\ = \frac{p(\mathbf{Y} | \mathbf{z}, \epsilon_t, \epsilon_r, \epsilon_\omega) p(\mathbf{z})}{p(\mathbf{Y})}. \quad (27)$$

Here, the numerator denotes the joint probability distribution, while the denominator, termed marginal likelihood or Bayesian evidence, serves as the normalization constant. We further define the likelihood of the observation as

$$p(\mathbf{Y} | \mathbf{z}, \epsilon_t, \epsilon_r, \epsilon_\omega) = \mathcal{CN}(\mathbf{Y} | \mathbf{D}\mathbf{B}, \sigma^2). \quad (28)$$

However, since the multidimensional integration in (27) cannot be carried out analytically, a closed-form expression of the posterior $p(\Omega_h | \mathbf{Y})$ is challenging to obtain. To circumvent this issue, we harness the block MM algorithm of [52]. A surrogate (lower bound) function is computed for the objective function $\ln p(\mathbf{Y}, \Omega_h)$, which is maximized with respect to the parameters $\mathbf{z}, \epsilon_t, \epsilon_r, \epsilon_\omega$. The surrogate function of $\ln p(\mathbf{Y}, \mathbf{z}, \epsilon_t, \epsilon_r, \epsilon_\omega)$, at a fixed point $(\dot{\mathbf{z}}, \dot{\epsilon}_t, \dot{\epsilon}_r, \dot{\epsilon}_\omega)$, is given as

$$\mathcal{L}(\mathbf{z}, \epsilon_t, \epsilon_r, \epsilon_\omega | \dot{\mathbf{z}}, \dot{\epsilon}_t, \dot{\epsilon}_r, \dot{\epsilon}_\omega) = \\ \int p(\mathbf{B} | \mathbf{Y}, \dot{\mathbf{z}}, \dot{\epsilon}_t, \dot{\epsilon}_r, \dot{\epsilon}_\omega) \ln \frac{p(\mathbf{B}, \mathbf{Y}, \mathbf{z}, \epsilon_t, \epsilon_r, \epsilon_\omega)}{p(\mathbf{B} | \mathbf{Y}, \dot{\mathbf{z}}, \dot{\epsilon}_t, \dot{\epsilon}_r, \dot{\epsilon}_\omega)} d\mathbf{B}. \quad (29)$$

The parameters $\mathbf{z}, \epsilon_t, \epsilon_r, \epsilon_\omega$ are updated as

$$\mathbf{z}^{(k+1)} = \arg \max_{\mathbf{z}} \mathcal{L}(\mathbf{z}, \epsilon_t^{(k)}, \epsilon_r^{(k)}, \epsilon_\omega^{(k)} | \mathbf{z}^{(k)}, \epsilon_t^{(k)}, \epsilon_r^{(k)}, \epsilon_\omega^{(k)}), \quad (30)$$

$$\epsilon_t^{(k+1)} = \arg \max_{\epsilon_t} \mathcal{L}(\mathbf{z}^{(k+1)}, \epsilon_t, \epsilon_r^{(k)}, \epsilon_\omega^{(k)} | \mathbf{z}^{(k+1)}, \epsilon_t^{(k)}, \epsilon_r^{(k)}, \epsilon_\omega^{(k)}), \quad (31)$$

$$\epsilon_r^{(k+1)} = \arg \max_{\epsilon_r} \mathcal{L}(\mathbf{z}^{(k+1)}, \epsilon_t^{(k+1)}, \epsilon_r, \epsilon_\omega^{(k)} | \mathbf{z}^{(k+1)}, \epsilon_t^{(k+1)}, \epsilon_r^{(k)}, \epsilon_\omega^{(k)}), \quad (32)$$

$$\epsilon_\omega^{(k+1)} = \arg \max_{\epsilon_\omega} \mathcal{L}(\mathbf{z}^{(k+1)}, \epsilon_t^{(k+1)}, \epsilon_r^{(k+1)}, \epsilon_\omega | \mathbf{z}^{(k+1)}, \epsilon_t^{(k+1)}, \epsilon_r^{(k+1)}, \epsilon_\omega^{(k)}). \quad (33)$$

The posterior distribution of each column of \mathbf{B} is given by $q(\beta_l) \sim \mathcal{CN}(\mu_l^{(k+1)}, \Sigma^{(k+1)})$, where we have

$$\Sigma^{(k+1)} = \left(\frac{1}{\sigma^2} \mathbf{D}^H \mathbf{D} + \mathbf{Z}^{(k+1)} \right)^{-1}, \quad (34)$$

$$\mathbf{U}^{(k+1)} = [\mu_1^{(k+1)}, \dots, \mu_L^{(k+1)}] = \frac{1}{\sigma^2} \Sigma^{(k+1)} \mathbf{D}^H \mathbf{Y}. \quad (35)$$

The update expressions of the parameters $\mathbf{z}, \epsilon_t, \epsilon_r, \epsilon_\omega$ are

Step 1. Main Objective

↓ Maximize the *a posteriori* distribution $p(\mathbf{\Omega}_h|\mathbf{Y})$ of the hidden variables given the observed data given in Eq. (25)

Step 2. Optimization Problem

↓ Difficult to obtain closed-form expression for multidimensional integration in (26). Employ the MM algorithm to obtain the closed-form expressions of $\mathbf{B}, \mathbf{z}, \epsilon_t, \epsilon_r, \epsilon_w$

Step 3. Unique solution for hyperparameter and off-grid errors

↓ The hyperparameter \mathbf{z} and the off-grid errors $\epsilon_t, \epsilon_r, \epsilon_w$ are updated as Eq. (29-32) and the closed-form expressions are given in Eq. (35-38)

Step 4. Grid refinement

↓ Update the grid points θ, ϕ, ω as given in Eq. (39-41). Repeat the steps 3-4 till convergence

Step 5. RCS coefficient, AoA, AoD, Doppler shift estimation

↓ Compare the estimated hyperparameters with a suitable threshold z_{th} and select the indices above the threshold to obtain the target parameter estimates given in Eq. (42)

Step 6. Clutter identification

↓ Identify and filter out the clutter components using Eq. (43)

Step 7. Localization of targets

↓ Estimate the 2D-coordinates and the velocities of the targets from AoAs, AoDs, Doppler shifts using Eqs. (46), (52)

Fig. 2: Flow of the proposed algorithm

given as follows:

$$z_g^{(k+1)} = \frac{a + L}{b + \sum_{l=1}^L \Upsilon_l(\epsilon_t^{(k)}, \epsilon_r^{(k)}, \epsilon_w^{(k)})}, \quad (36)$$

$$\epsilon_t^{(k+1)} = \left(\Re \left\{ \Xi_t^H \Xi_t \odot (\mathbf{U}\mathbf{U}^H + L\Sigma) \right\} \right)^\dagger \mathbf{p}_t, \quad (37)$$

$$\epsilon_r^{(k+1)} = \left(\Re \left\{ \Xi_r^H \Xi_r \odot (\mathbf{U}\mathbf{U}^H + L\Sigma) \right\} \right)^\dagger \mathbf{p}_r, \quad (38)$$

$$\epsilon_w^{(k+1)} = \left(\Re \left\{ \Xi_w^H \Xi_w \odot (\mathbf{U}\mathbf{U}^H + L\Sigma) \right\} \right)^\dagger \mathbf{p}_w, \quad (39)$$

where $\Upsilon_l(\epsilon_t^{(k)}, \epsilon_r^{(k)}, \epsilon_w^{(k)}) = \mu_l \mu_l^H + \Sigma$,
 $\mathbf{p}_t = \Re \left\{ \sum_{l=1}^L \text{diag}(\mu_l) \Xi_t^H (\mathbf{y}_l - \Psi_t \mu_l) \right\} - L \Re \left\{ \text{diag}(\Xi_t^H \Psi_t \Sigma) \right\}$ with $\Psi_t = \Psi(\tilde{\theta}, \tilde{\phi}, \tilde{\omega}) + \Xi_r \text{diag}(\epsilon_r) + \Xi_w \text{diag}(\epsilon_w)$,
 $\mathbf{p}_r = \Re \left\{ \sum_{l=1}^L \text{diag}(\mu_l) \Xi_r^H (\mathbf{y}_l - \Psi_r \mu_l) \right\} - L \Re \left\{ \text{diag}(\Xi_r^H \Psi_r \Sigma) \right\}$ with $\Psi_r = \Psi(\tilde{\theta}, \tilde{\phi}, \tilde{\omega}) + \Xi_t \text{diag}(\epsilon_t) + \Xi_w \text{diag}(\epsilon_w)$,
 $\mathbf{p}_w = \Re \left\{ \sum_{l=1}^L \text{diag}(\mu_l) \Xi_w^H (\mathbf{y}_l - \Psi_w \mu_l) \right\} - L \Re \left\{ \text{diag}(\Xi_w^H \Psi_w \Sigma) \right\}$ with $\Psi_w = \Psi(\tilde{\theta}, \tilde{\phi}, \tilde{\omega}) + \Xi_t \text{diag}(\epsilon_t) + \Xi_r \text{diag}(\epsilon_r)$. The detailed derivations are presented in Appendix A. The grid points are then updated

as follows

$$\tilde{\theta}^{(k+1)} = \tilde{\theta}^{(k)} + \epsilon_t, \quad (40)$$

$$\tilde{\phi}^{(k+1)} = \tilde{\phi}^{(k)} + \epsilon_r, \quad (41)$$

$$\tilde{\omega}^{(k+1)} = \tilde{\omega}^{(k)} + \epsilon_w. \quad (42)$$

The steps described above are repeated until convergence. Since the hyperparameters control the sparsity in $\hat{\mathbf{B}}$, the performance attained can be further enhanced by thresholding the hyperparameter estimates with respect to a suitably chosen threshold z_{th} . The rows $\hat{\mathbf{B}}(g, :)$ with hyperparameters z_g higher than z_{th} are set to zero. The pruned estimate $\hat{\mathbf{B}}$ is given by

$$\hat{\mathbf{B}}(g, :) = \begin{cases} \mathbf{U}^{(k+1)}(g, :), & \hat{z}_g^{(k+1)} < z_{th} \\ \mathbf{0}_{1 \times L}, & \text{otherwise.} \end{cases} \quad (43)$$

Subsequently, instead of updating all the G grid points in (40), (41), (42), one can only update those corresponding to the significant rows of $\hat{\mathbf{B}}$. Next, we exploit an interesting property to identify the clutter components in the environment. The ground clutter strength decays with the increase in Doppler frequency [53], [54]. Thus, the significant clutter components can be assumed to be confined to a maximum Doppler shift of ω_{cl}^* , beyond which the clutter effect can be neglected. In the above formulation, the parameter δ is set for ensuring that

$$|\omega_{cl}^*| \leq \delta \frac{\omega_{\max}}{G}, \quad (44)$$

where ω_{\max} is the maximum Doppler shift of any target in the sensing environment. This implies that if the estimated Doppler shift $\hat{\omega}_{w'}^{(k+1)}$ is lower than or equal to the parameter $\delta + 1$, i.e. $\hat{\omega}_{w'}^{(k+1)} \leq \delta + 1$, then the estimated AoA-AoD-Doppler tuple $(\hat{\theta}_{w'}^{(k+1)}, \hat{\phi}_{w'}^{(k+1)}, \hat{\omega}_{w'}^{(k+1)})$ belongs to the clutter. This step filters out the N_{cl} clutter components from the W' signal-plus-clutter components to yield the estimated AoA, AoD and Doppler shift values corresponding to the W targets. Next, we devise an innovative algorithm for localizing the targets by computing their 2D coordinates and velocities from the estimated AoDs, AoAs and Doppler shifts.

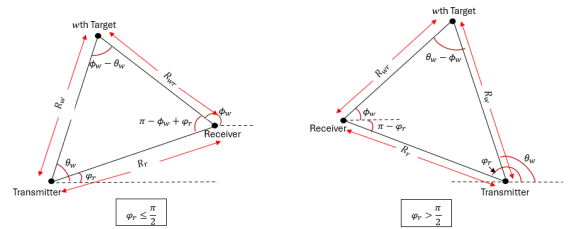


Fig. 3: Point representation of transmitter-with target-receiver pair under different conditions of receiver orientation φ_r

A. Localization Algorithm

2D Coordinate estimation: Fig. 3 represents the point based representation of the transmitter-with target-receiver pair under both conditions of receiver orientation $\varphi_r \leq \frac{\pi}{2}$ and

$\varphi_r > \frac{\pi}{2}$. Using the law of sines for triangles, the distance of the w th target from the transmitter \hat{R}_w can be formulated as

$$\hat{R}_w = \begin{cases} R_r \frac{\sin(\pi - \hat{\phi}_w + \varphi_r)}{\sin(\hat{\phi}_w - \theta_w)}, & \varphi_r \leq \frac{\pi}{2} \\ R_r \frac{\sin(\hat{\phi}_w + \pi - \varphi_r)}{\sin(\theta_w - \hat{\phi}_w)}, & \varphi_r > \frac{\pi}{2}. \end{cases} \quad (45)$$

Furthermore, since the transmitter is assumed to be located at the origin, the orientation of the w th target φ_w equals the estimated AoD $\hat{\theta}_w$

$$\hat{\varphi}_w = \hat{\theta}_w. \quad (46)$$

Subsequently, the x and y coordinates of the w th target are given respectively as

$$\hat{x}_w = \hat{R}_w \cos(\hat{\varphi}_w), \hat{y}_w = \hat{R}_w \sin(\hat{\varphi}_w). \quad (47)$$

Sum distance Estimation: Using geometry, the following expressions can be deduced for $\varphi_r \leq \frac{\pi}{2}$

$$\begin{aligned} R_w \cos(\theta_w - \varphi_r) + R_{wr} \cos(\pi - \phi_w + \varphi_r) &= R_r \\ -R_w \sin(\theta_w - \varphi_r) + R_{wr} \sin(\pi - \phi_w + \varphi_r) &= 0. \end{aligned} \quad (48)$$

The unknown vector $\mathbf{r} \triangleq [R_w, R_{wr}]^T$ can be determined by solving the pair of linear equations above, and it is given as

$$\hat{\mathbf{r}} = \mathbf{P}^{-1} \mathbf{r}_d, \quad (49)$$

where

$$\mathbf{P} = \begin{bmatrix} \cos(\theta_w - \varphi_r) & \cos(\pi - \phi_w + \varphi_r) \\ -\sin(\theta_w - \varphi_r) & \sin(\pi - \phi_w + \varphi_r) \end{bmatrix}, \mathbf{r}_d = \begin{bmatrix} R_r \\ 0 \end{bmatrix}.$$

The estimated range for the transmitter- w th target-receiver pair is thus given as

$$\hat{R}_{wr} = R_r \frac{\sin(\theta_w - \varphi_r)}{\sin(\pi - \phi_w + \theta_w)}, \hat{R}_w = R_r \frac{\sin(\pi - \phi_w + \varphi_r)}{\sin(\pi - \phi_w + \theta_w)}. \quad (50)$$

The estimated sum distance $\hat{R}_w^B = \hat{R}_{wr} + \hat{R}_w$ can now be derived as

$$\hat{R}_w^B = R_r \frac{[\sin(\theta_w - \varphi_r) + \sin(\pi - \phi_w + \varphi_r)]}{\sin(\pi - \phi_w + \theta_w)}. \quad (51)$$

Similarly, for $\varphi_r > \frac{\pi}{2}$, the sum distance \hat{R}_w^B is given as

$$\hat{R}_w^B = R_r \frac{[\sin(\varphi_r - \theta_w) + \sin(\pi - \varphi_r + \phi_w)]}{\sin(\pi + \phi_w - \theta_w)}. \quad (52)$$

Velocity estimation: The estimated velocity \hat{v}_w of the w th target is expressed as

$$\hat{v}_w = \frac{\hat{\omega}_w \lambda}{4\pi T_p}. \quad (53)$$

The flow of the proposed algorithm is summarized in Fig. 2.

B. Complexity Analysis

The computational complexity of the proposed algorithm is as follows:

- The complexities of calculating \mathbf{U} and Σ are $\mathcal{O}(LG^2)$ and $\mathcal{O}[G^2 \cdot \max(G, N_r U)]$, respectively.
- The complexity in updating the hyperparameter vector \mathbf{z} is $\mathcal{O}(LG)$.

- Since ϵ_t, ϵ_r and ϵ_ω are jointly sparse with \mathbf{B} , the complexity in updating ϵ_t, ϵ_r and ϵ_ω can be ignored.

Therefore, the overall computational complexity of the proposed design is $\mathcal{O}[NG^2 \times \max(G, N_r U)]$, where N is the total number of SBL iterations. The computational complexity of

TABLE II: Computational Complexities of Various Algorithms

Algorithm	Computational Complexity
Proposed Scheme	$\mathcal{O}[NG^2 \times \max(G, N_r U)]$
OMP [55]	$\mathcal{O}(LN N_r U G)$
MUSIC [56]	$\mathcal{O}(N_r^3 U^3 + G^2 N_r^2 U^2)$

the proposed SBL algorithm is also compared with the existing OMP [55] and MUSIC [56] schemes and summarized in Table II. Our method and MUSIC have comparable computational complexities, whereas OMP has a lower computational burden than both SBL and MUSIC. However, it can be observed in Section V that the proposed SBL scheme achieves a significant improvement in estimation performance, especially in the case of low SCNR.

IV. CRAMÉR RAO BOUND FOR AOD, AOA, VELOCITY ESTIMATION AND BAYESIAN CRAMÉR RAO BOUND FOR RCS COEFFICIENT MATRIX ESTIMATION

The CRBs for the AoDs, AoAs, velocity estimation and Bayesian CRB (BCRB) for RCS coefficient estimation of the multiple targets are derived next.

A. CRBs for joint AoD, AoA, Velocity Estimation

The unknown channel domain parameter vector can be constructed as $\boldsymbol{\eta} = [\boldsymbol{\theta}^T, \boldsymbol{\phi}^T, \boldsymbol{\omega}^T]^T$, where we have $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_W] \in \mathbb{R}^{W \times 1}$, $\boldsymbol{\phi} = [\phi_1, \phi_2, \dots, \phi_W] \in \mathbb{R}^{W \times 1}$ and $\boldsymbol{\omega} = [\omega_1, \omega_2, \dots, \omega_W] \in \mathbb{R}^{W \times 1}$. The Fisher information matrix (FIM) $\mathbf{J}_\xi \in \mathbb{R}^{3W \times 3W}$ satisfies the information inequality of [57]

$$\mathbb{E} \left\{ (\hat{\boldsymbol{\eta}} - \boldsymbol{\eta}) (\hat{\boldsymbol{\eta}} - \boldsymbol{\eta})^T \right\} \succeq \mathbf{J}_\eta^{-1}, \quad (54)$$

for any unbiased estimator $\hat{\boldsymbol{\eta}}$ of $\boldsymbol{\eta}$. The FIM \mathbf{J}_η can be further written as

$$\mathbf{J}_\eta = \begin{bmatrix} \mathbf{J}_\theta & \mathbf{J}_{\theta\phi} & \mathbf{J}_{\theta\omega} \\ \mathbf{J}_{\phi\theta} & \mathbf{J}_\phi & \mathbf{J}_{\phi\omega} \\ \mathbf{J}_{\omega\theta} & \mathbf{J}_{\omega\phi} & \mathbf{J}_\omega \end{bmatrix}. \quad (55)$$

The log-likelihood function $\mathcal{L}(\mathbf{Y}; \boldsymbol{\eta}) = \ln p(\mathbf{Y}; \boldsymbol{\eta})$ is given by

$$\mathcal{L}(\mathbf{Y}; \boldsymbol{\eta}) = c_1 - \sum_{l=1}^L (\mathbf{y}_l - \mathbf{D}\boldsymbol{\beta}_l)^H (\mathbf{y}_l - \mathbf{D}\boldsymbol{\beta}_l), \quad (56)$$

where the constant obeys $c_1 = -N_y \log(\pi) - \log(\Sigma_c^{-1})$. The diagonal entries of the FIM \mathbf{J}_ξ are computed next. For a

parameter vector, $\zeta = \theta$ or ϕ or ω , the (e, f) th element of the FIM \mathbf{J}_ζ is given by

$$\begin{aligned} [\mathbf{J}_\zeta]_{ef} &= -\mathbb{E}_{\mathbf{Y}} \left\{ \frac{\partial^2 \mathcal{L}(\mathbf{Y}; \boldsymbol{\eta})}{\partial \zeta_e \partial \zeta_f} \right\} \\ &= 2\Re \left\{ \sum_{l=1}^L \left(\frac{\partial (\mathbf{D}\boldsymbol{\beta}_l)^H}{\partial \zeta_e} \right) \left(\frac{\partial (\mathbf{D}\boldsymbol{\beta}_l)}{\partial \zeta_f} \right) \right\} \\ &= 2\Re \left\{ \sum_{l=1}^L \left(\beta_{e,l}^* \frac{\partial \psi_e^H}{\partial \zeta_e} \right) \left(\frac{\partial \psi_f}{\partial \zeta_f} \beta_{f,l} \right) \right\}. \end{aligned} \quad (57)$$

Let us now evaluate the off-diagonal entries of the FIM \mathbf{J}_η . For $1 \leq e, f \leq W$, the (e, f) th element of the FIMs $\mathbf{J}_{\theta\phi}$, $\mathbf{J}_{\theta\omega}$ and $\mathbf{J}_{\phi\omega}$ can be written as

$$[\mathbf{J}_{\theta\phi}]_{ef} = 2\Re \left\{ \sum_{l=1}^L \left(\beta_{e,l}^* \frac{\partial \psi_e^H}{\partial \theta_e} \right) \left(\frac{\partial \psi_f}{\partial \phi_f} \beta_{f,l} \right) \right\}, \quad (58)$$

$$[\mathbf{J}_{\theta\omega}]_{ef} = 2\Re \left\{ \sum_{l=1}^L \left(\beta_{e,l}^* \frac{\partial \psi_e^H}{\partial \theta_e} \right) \left(\frac{\partial \psi_f}{\partial \omega_f} \beta_{f,l} \right) \right\}, \quad (59)$$

$$[\mathbf{J}_{\phi\omega}]_{ef} = 2\Re \left\{ \sum_{l=1}^L \left(\beta_{e,l}^* \frac{\partial \psi_e^H}{\partial \phi_e} \right) \left(\frac{\partial \psi_f}{\partial \omega_f} \beta_{f,l} \right) \right\}. \quad (60)$$

Furthermore, we have:

$$\mathbf{J}_{\theta\phi} = \mathbf{J}_{\theta\phi}^H, \quad \mathbf{J}_{\omega\theta} = \mathbf{J}_{\theta\omega}^H, \quad \mathbf{J}_{\omega\phi} = \mathbf{J}_{\phi\omega}^H.$$

- *CRB of θ* : Let us introduce the vector $\boldsymbol{\rho} = [\phi^T, \omega^T]^T$. The FIM in (55) can be rewritten as

$$\mathbf{J}_\eta = \begin{bmatrix} \mathbf{J}_\theta & \mathbf{J}_{\theta\rho} \\ \mathbf{J}_{\rho\theta} & \mathbf{J}_\rho \end{bmatrix}, \quad (61)$$

where we have:

$$\mathbf{J}_{\theta\rho} = [\mathbf{J}_{\theta\phi} \quad \mathbf{J}_{\theta\omega}] = \mathbf{J}_{\rho\theta}^H, \quad \mathbf{J}_\rho = \begin{bmatrix} \mathbf{J}_\phi & \mathbf{J}_{\phi\omega} \\ \mathbf{J}_{\omega\phi} & \mathbf{J}_\omega \end{bmatrix}. \quad (62)$$

Invoking the Woodbury matrix identity for the partitioned matrices, we eventually obtain the CRB of θ as

$$\mathbb{E} \left\{ \|\hat{\theta} - \theta\|^2 \right\} \geq \text{Tr} \left((\mathbf{J}_\theta - \mathbf{J}_{\theta\rho} \mathbf{J}_\rho^{-1} \mathbf{J}_{\rho\theta})^{-1} \right), \quad (63)$$

where \mathbf{J}_ρ^{-1} can be obtained by once again invoking the Woodbury matrix identity for partitioned matrices as follows

$$\mathbf{J}_\rho^{-1} = \begin{bmatrix} \mathbf{W} & -\mathbf{W}\mathbf{J}_{\phi\omega}\mathbf{J}_\omega^{-1} \\ -\mathbf{J}_\omega^{-1}\mathbf{J}_{\omega\phi}\mathbf{W} & (\mathbf{J}_\omega - \mathbf{J}_{\omega\phi}\mathbf{J}_\phi^{-1}\mathbf{J}_{\phi\omega})^{-1} \end{bmatrix}, \quad (64)$$

where we have $\mathbf{W} = (\mathbf{J}_\phi - \mathbf{J}_{\phi\omega}\mathbf{J}_\omega^{-1}\mathbf{J}_{\omega\phi})^{-1}$.

- *CRB of ϕ* : Similar to (63), the CRB of the delay vector ϕ can be obtained as

$$\mathbb{E} \left\{ \|\hat{\phi} - \phi\|^2 \right\} \geq \text{Tr} \left((\mathbf{J}_\phi - \mathbf{J}_{\phi\rho} \mathbf{J}_\rho^{-1} \mathbf{J}_{\rho\phi})^{-1} \right), \quad (65)$$

where, $\boldsymbol{\rho} = [\theta^T, \omega^T]^T$ and correspondingly, we have

$$\mathbf{J}_{\phi\rho} = [\mathbf{J}_{\phi\theta} \quad \mathbf{J}_{\phi\omega}] = \mathbf{J}_{\rho\phi}^H, \quad \mathbf{J}_\rho = \begin{bmatrix} \mathbf{J}_\theta & \mathbf{J}_{\theta\omega} \\ \mathbf{J}_{\omega\theta} & \mathbf{J}_\omega \end{bmatrix}. \quad (66)$$

The term \mathbf{J}_ρ^{-1} can be derived in a similar fashion as shown in (64).

- *CRB of \mathbf{v}* : The structure of the FIM \mathbf{J}_η for the Doppler shift vector ω can be obtained similar to (65) and it is expressed as

$$\mathbb{E} \left\{ \|\hat{\omega} - \omega\|^2 \right\} \geq \text{Tr} \left((\mathbf{J}_\omega - \mathbf{J}_{\omega\rho} \mathbf{J}_\rho^{-1} \mathbf{J}_{\rho\omega})^{-1} \right), \quad (67)$$

where $\boldsymbol{\rho} = [\theta^T, \phi^T]^T$. The terms $\mathbf{J}_{\omega\rho}$ and \mathbf{J}_ρ are similar to (66), while the evaluation of \mathbf{J}_ρ^{-1} follows (64). For obtaining the CRB of the velocity estimate, one can use (2) for defining the transformation matrix $\mathbf{T} = \frac{\partial \boldsymbol{\omega}^T}{\partial \mathbf{v}} \in \mathbb{R}^{W \times W}$ as

$$\mathbf{T} = \frac{\partial \boldsymbol{\omega}^T}{\partial \mathbf{v}} = \begin{bmatrix} \frac{\partial \omega_1}{\partial v_1} & \frac{\partial \omega_2}{\partial v_1} & \dots & \frac{\partial \omega_W}{\partial v_1} \\ \frac{\partial \omega_1}{\partial v_2} & \frac{\partial \omega_2}{\partial v_2} & \dots & \frac{\partial \omega_W}{\partial v_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \omega_1}{\partial v_W} & \frac{\partial \omega_2}{\partial v_W} & \dots & \frac{\partial \omega_W}{\partial v_W} \end{bmatrix} = \frac{4\pi T_p}{\lambda} \mathbf{I}. \quad (68)$$

Therefore, the FIM of the velocity vector can be determined as

$$\mathbf{J}_\mathbf{v} = \mathbf{T} \mathbf{J}_\omega \mathbf{T}^T. \quad (69)$$

Finally, we have the CRB of the velocity estimate given as

$$\mathbb{E} \left\{ \|\hat{\mathbf{v}} - \mathbf{v}\|^2 \right\} \geq \text{Tr} \left((\mathbf{J}_\mathbf{v} - \mathbf{J}_{\mathbf{v}\rho} \mathbf{J}_\rho^{-1} \mathbf{J}_{\rho\mathbf{v}})^{-1} \right). \quad (70)$$

B. Bayesian CRB (BCRB) for RCS Coefficient Matrix

This section derives the BCRB in the mean squared error (MSE) of estimation of the RCS coefficient matrix \mathbf{B} . Upon vectorizing (22), the received signal $\mathbf{y} = \text{vec}(\mathbf{Y}) \in \mathbb{C}^{LN_r U \times 1}$ can be expressed as

$$\mathbf{y} = \underbrace{(\mathbf{I}_L \otimes \mathbf{D})}_{\mathbf{D}} \boldsymbol{\beta} + \mathbf{q}, \quad (71)$$

where the effective parameter vector obeys $\boldsymbol{\beta} = \text{vec}(\mathbf{B}) \in \mathbb{C}^{LG \times 1}$ and similarly $\mathbf{q} = \text{vec}(\mathbf{Q}) \in \mathbb{C}^{LN_r U \times 1}$. The Bayesian Fisher information matrix (BFIM) $\mathbf{J}_B \in \mathbb{C}^{LG \times LG}$ is defined as [57]

$$\mathbf{J}_B = -\underbrace{\mathbb{E}_{(\mathbf{y}, \boldsymbol{\beta})} \left\{ \frac{\partial^2 \mathcal{L}(\mathbf{y} | \boldsymbol{\beta}; \mathbf{Z})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^H} \right\}}_{\mathbf{J}_\mathcal{D}} - \underbrace{\mathbb{E}_{\boldsymbol{\beta}} \left\{ \frac{\partial^2 \mathcal{L}(\boldsymbol{\beta}; \mathbf{Z})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^H} \right\}}_{\mathbf{J}_\mathcal{P}}, \quad (72)$$

where the matrices $\mathbf{J}_\mathcal{P}$ and $\mathbf{J}_\mathcal{D}$ denote the FIMs with respect to the prior density of the parameter vector $\boldsymbol{\beta}$ and the output vector \mathbf{y} , respectively. After neglecting the constant terms, the FIM $\mathbf{J}_\mathcal{D}$ can be expressed as

$$\mathbf{J}_\mathcal{D} = -\mathbb{E}_{(\mathbf{y}, \boldsymbol{\beta})} \left\{ \frac{\partial^2 \mathcal{L}(\mathbf{y} | \boldsymbol{\beta}; \mathbf{Z})}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^H} \right\} = \mathbf{D}^H \mathbf{D}. \quad (73)$$

The FIM component $\mathbf{J}_\mathcal{P}$ is given by

$$\mathbf{J}_\mathcal{P} = \mathbf{I}_L \otimes \boldsymbol{\Gamma}^{-1}. \quad (74)$$

Thus, the BCRB is given by

$$\begin{aligned} \mathbb{E} \left\{ \|\mathbf{B} - \hat{\mathbf{B}}\|_F^2 \right\} &= \mathbb{E} \left\{ \|\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}\|_2^2 \right\} \\ &= \text{Tr} \left\{ \mathbf{J}_B^{-1} \right\} \\ &= \text{Tr} \left\{ \left(\mathbf{D}^H \mathbf{D} + \mathbf{I}_L \otimes \mathbf{Z}^{-1} \right)^{-1} \right\}. \end{aligned} \quad (75)$$

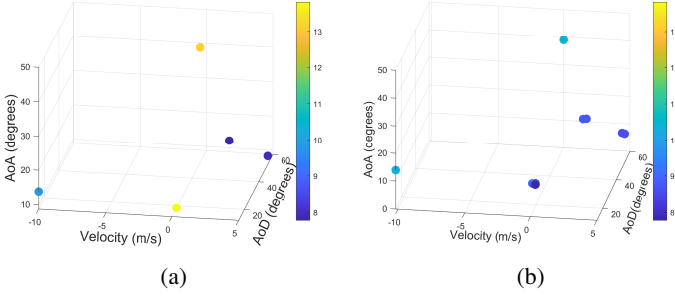


Fig. 4: (a) True target and (b) reconstructed target image of target RCS values in AD domain at SCNR = -3 dB

TABLE III: Simulation Setup

Parameter	Value
Number of grids G	150
Number of targets W	3
Number of clutter N_{cl}	2
Number of snapshots L	6
Number of transmit antennas N_t	6
Number of transmit sub-pulses U	16
Number of receive antennas N_r	6
Signal frequency f_c	30 GHz
Sub-pulse duration T_p	40 μ s
Polar coordinate of receiver $[R_r, \varphi_r]$	(95, 0°)
Polar coordinates of the targets $[R_w, \varphi_w]$ ($\mathbf{u}_w, w = 1, 2, \dots, W$)	(20.66, 52.64°), (25.11, 54.73°), (51.13, 12.08°)
Velocity of targets (\mathbf{v}_w)	(5, 2, -10) m/s

V. SIMULATION RESULTS

For characterizing the algorithm's target localization performance, we consider a mmWave MIMO radar system having the parameters specified in Table III. Each element of the transmit signal matrix $\mathbf{X} \in \mathbb{C}^{N_t \times U}$ is a QPSK symbol of the form $\{\pm\sqrt{E_b} \pm j\sqrt{E_b}\}$, where E_b is the energy per bit of the QPSK symbol. The RCS coefficients of the targets and clutter are generated as i.i.d. samples of a zero-mean symmetric complex Gaussian distribution with target and clutter variances of $\sigma_w^2 = 10\text{dB}$, $\sigma_c^2 = 12\text{dB}$, respectively. The noise samples are also i.i.d. zero-mean symmetric complex Gaussian of unit variance. The radar imaging scene spans the AoA and AoD region of $[0, \pi)$ and the Doppler shift spans in the region $[-1.5, 1.5]$. Furthermore, let us define the target to clutter ratio (TCR) as the ratio of target variance to clutter variance. A high TCR represents a weaker clutter in comparison to the targets, and vice versa. The signal to clutter-plus-noise ratio (SCNR) is defined as $\text{SCNR} = E_b \frac{\sigma_w^2}{\sigma_c^2 + \sigma_n^2}$.

Fig. 4 juxtaposes the image of the radar scattering scene for the true target parameters to that of the respective estimated values. The color bars at the side denote the RCS coefficient. One can observe that the proposed SBL-based algorithm is capable of accurately identifying the locations of all the targets considered. Interestingly, it can be observed from the reconstructed image that the clutter components appear around the zero velocity, thereby, helping in the identification of the target parameter estimates from the clutter components.

We define the *root mean squared error* (RMSE) metric to quantify the algorithm's parameter estimation accuracy as

follows

$$\text{RMSE}_\eta = \sqrt{\frac{1}{M_c W} \sum_{i=1}^{M_c} \sum_{w=1}^W (\hat{\eta}_{w,i} - \eta_w)^2}, \quad (76)$$

where we have $\eta_w \in \{\theta_w, \phi_w, v_w\}$.

Fig. 5 (a) and (b) plot the RMSE of the estimated AoD and AoA values corresponding to different SCNR values both for the proposed SBL method and for the existing techniques. It can be observed that the RMSE values decrease upon increasing the SCNR. The proposed scheme outperforms both the existing CS-based OMP scheme [55] and the subspace-based MUSIC scheme [56] in the low SCNR regime, despite its lower dictionary overhead. The OMP scheme does not optimize the off-grid error between the true parameter and dictionary grid value and hence yields the poorest performance. The MUSIC algorithm deviates from the true values because it frequently suffers from rank deficiency of the covariance matrix for the case of correlated angles formed by closely spaced targets. Moreover, as observed in the low SCNR regime of Fig. 5, the MUSIC algorithm has a poor performance in comparison to OMP. This is due to the fact that at low SCNR values, the signal is heavily corrupted by the clutter-plus-noise components and leads to a lower signal subspace. By contrast, the OMP algorithm exploits the 3D sparsity in the AD domain and yields a marginally improved performance over MUSIC. Additionally, the MUSIC technique is prone to falsely identifying the peaks at clutter positions instead of the real targets, hence leading to poor performance. The proposed scheme also performs close to the root CRB approach, thus validating its supremacy.

Fig. 6 (a), (b) and (c) plot the RMSE of the estimated x-coordinate, y-coordinate and velocity, respectively, corresponding to different SCNR values both for the proposed SBL method and for the existing techniques. Since the position coordinates are directly derived from the angles as given in (47), the location RMSE performance of the proposed scheme is superior to other schemes. Furthermore, the velocity RMSE performance seen in Fig. 6 (c) is also as expected.

Fig. 7 depicts the AoD and velocity RMSE performance achieved versus the number of snapshots L for different values of TCR. The RMSE performance improves upon increasing the number of snapshots. However, the performance of the proposed scheme saturates at approximately $L = 22$ snapshots, which evidences its ability to provide accurate results with fewer snapshots. As a further benefit, the RMSE performance does not suffer at high TCR values, since it is capable of successfully isolating the clutter components from the targets. Both the above properties render it ideally suited for practical scenarios associated with a lower number of snapshots and high clutter.

Fig. 8 portrays the *normalized mean squared error* (NMSE) of the RCS coefficient matrix estimate $\hat{\mathbf{B}}$ corresponding to different receive SCNR values. The NMSE can be defined as, $\text{NMSE} = \frac{1}{M_c} \sum_{i=1}^{M_c} \frac{\|\hat{\mathbf{B}} - \mathbf{B}\|_2^2}{\|\mathbf{B}\|_2^2}$, where M_c is the number of Monte Carlo trials. The proposed SBL technique is observed to provide approximately 5 dB and 10 dB improvements over the OMP [55] and the focal under-determined system solver

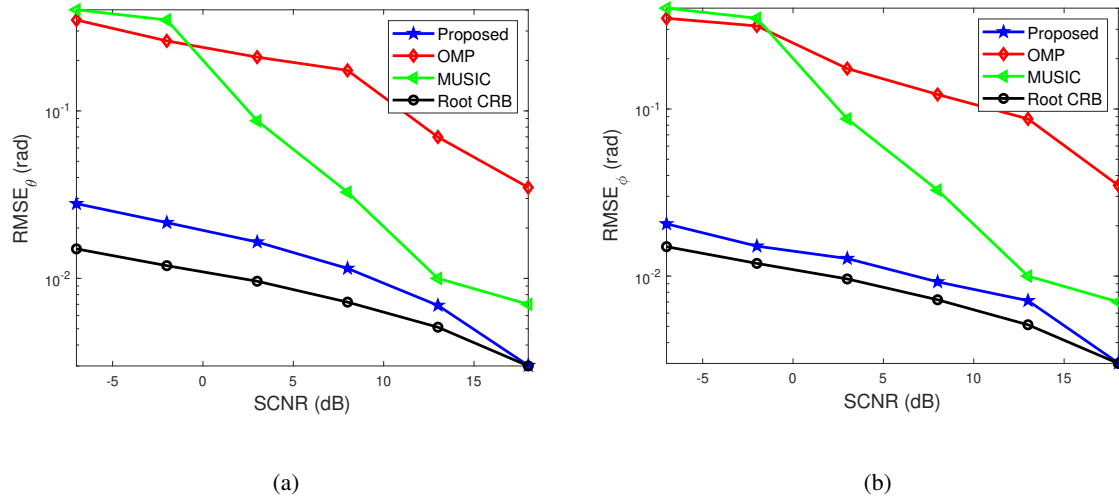


Fig. 5: RMSE versus SCNR for estimation of (a) AoD and (b) AoA of targets for the proposed SBL, MUSIC and OMP algorithms

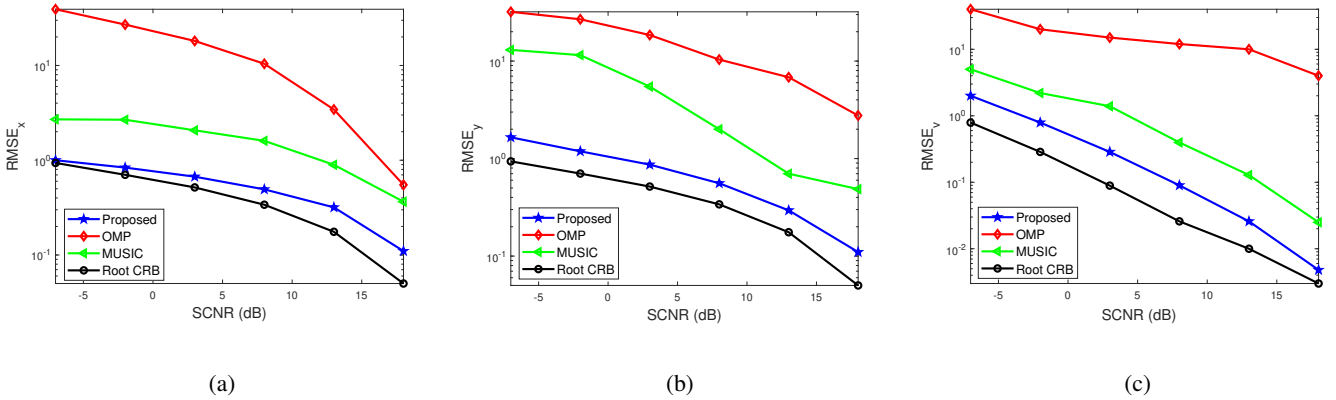


Fig. 6: RMSE versus SCNR for estimation of a) x-coordinate b) y-coordinate c) velocity of targets for the proposed SBL, MUSIC, and OMP algorithms

TABLE IV: True target parameter values for Fig. 9

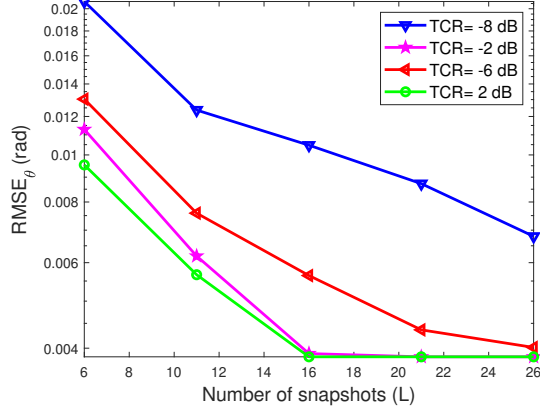
Parameter	Target 1	Target 2	Target 3
x-coordinate (p_x)	12.5 m	12.5 m	12.5 m
y-coordinate (p_y)	16.95 m	17.3 m	16.375 m
velocity (v)	2 m/s	3 m/s	5 m/s

(FOCUSS) [59] schemes, respectively. This is because the FOCUSS scheme is sensitive to the choice of the regularization parameter, whereas the OMP is affected both by the stopping criterion and the choice of the dictionary matrix, while the SBL performance is robust to these scenarios. Additionally, the RMSE of our algorithm is close to the BCRB, which validates its efficiency.

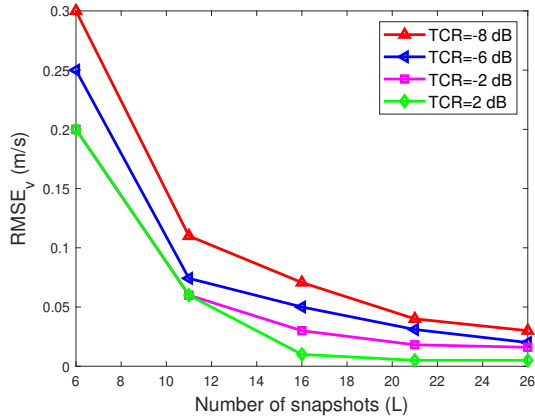
Fig. 9 illustrates the ability of the proposed super-resolution SBL technique to resolve closely spaced targets. The target parameter values are as shown in Table IV. As observed from the figure, the proposed super-resolution SBL scheme is able to successfully distinguish multiple targets with a range resolution of 0.6 m and velocity resolution of approximately 1 m/s.

Fig. 10 depicts the estimation performance of both the proposed scheme and of the on-grid SBL, where the latter is configured by changing the granularity of the grid points chosen. It can be readily deduced that the RMSE performance of our scheme conceived is robust, leading to successful localization of the target. However, the on-grid SBL performance degrades significantly in comparison to the proposed off-grid SBL for coarser grids. The on-grid SBL requires approximately twice the dictionary size compared to the off-grid SBL in order to achieve the same RMSE performance. These results collectively imply that the proposed algorithm offers a higher parameter estimation accuracy at a substantially reduced complexity, rendering it ideally suited for practical implementation.

To substantiate the proposed scheme's contributions in communication systems, we have presented the spectral efficiency (SE) comparison of the hybrid precoding and combining using the proposed positioning scheme with existing multiple measurement vector (MMV)-based sparse Bayesian learning (MSBL)-based hybrid precoding and combining scheme in [58]. We assume that the targets act as scatterers in mmWave



(a)



(b)

Fig. 7: RMSE of (a) AoD and (b) velocity versus number of snapshots L for different values of TCR

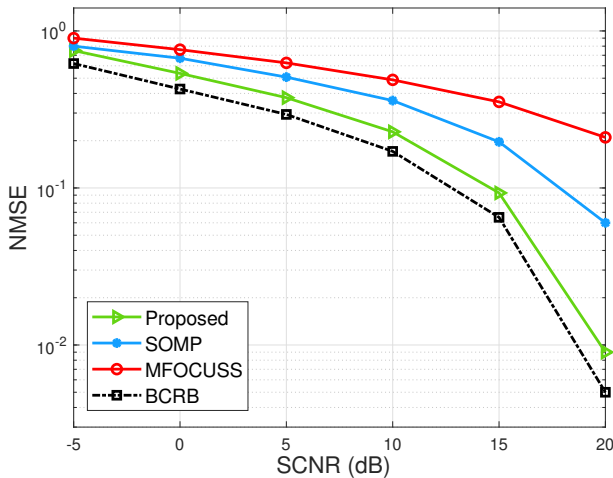


Fig. 8: NMSE versus SCNR for estimation of the RCS coefficient matrix using the proposed SBL, SOMP and MFOCUSS techniques

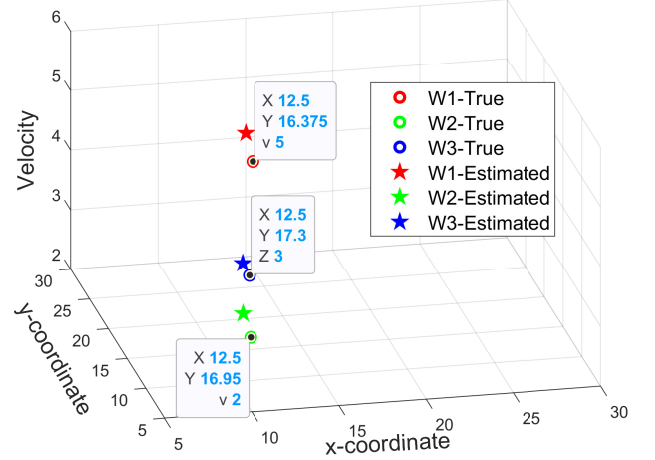


Fig. 9: 3D plot of the true and estimated target in terms of the x-coordinate, y-coordinate and velocity parameters

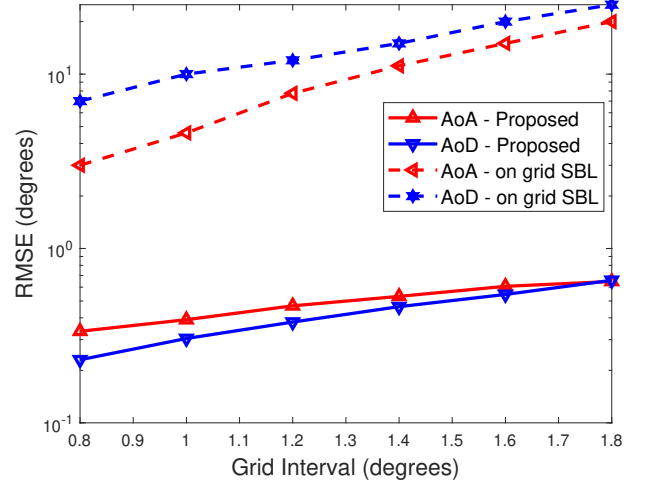


Fig. 10: RMSE versus dictionary grid interval for AoD and AoA estimation at SCNR = -3 dB

MIMO channel $\mathbf{H} = \sum_{w=1}^W \beta_w \mathbf{d}_w \mathbf{c}_w^T e^{j\omega_w} \in \mathbb{C}^{M_r \times M_t}$. Consider a setup with N_s transmit symbols and N_{RF} RF chains. Once one obtains the path state information of the targets (scatterers) such as AoA ϕ , AoD θ , Doppler shift ω and RCS coefficients (channel coefficients) β , the RF precoder $\mathbf{F}_{\text{RF}} \in \mathbb{C}^{M_t \times N_{\text{RF}}}$ and RF combiner $\mathbf{W}_{\text{RF}} \in \mathbb{C}^{M_r \times N_{\text{RF}}}$ can be designed by choosing the dominant transmit and receive steering matrices, respectively. Specifically, we arrange the estimated RCS coefficients (channel coefficients) in a decreasing order, $|\hat{\beta}_{w_1}| \geq |\hat{\beta}_{w_2}| \geq \dots \geq |\hat{\beta}_{w_W}|$. Thereafter, the RF precoder \mathbf{F}_{RF} and RF combiner \mathbf{W}_{RF} are designed as

$$\mathbf{F}_{\text{RF}} = [\mathbf{c}(w_1), \mathbf{c}(w_2), \dots, \mathbf{c}(w_{N_{\text{RF}}})], \quad (77)$$

$$\mathbf{W}_{\text{RF}} = [\mathbf{d}(w_1), \mathbf{d}(w_2), \dots, \mathbf{d}(w_{N_{\text{RF}}})]. \quad (78)$$

Upon performing singular value decomposition (SVD) of the effective channel $\tilde{\mathbf{H}} = \mathbf{W}_{\text{RF}}^H \mathbf{H} \mathbf{F}_{\text{RF}} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H$, the baseband precoder \mathbf{F}_{BB} and combiner \mathbf{W}_{BB} can be designed as $\mathbf{F}_{\text{BB}} =$

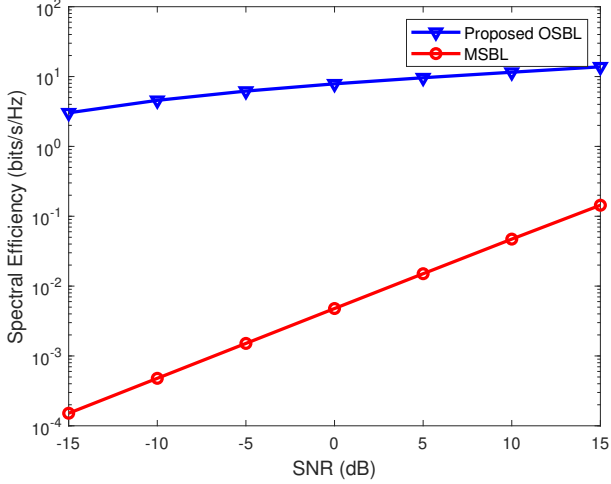


Fig. 11: Spectral Efficiency comparison of the proposed positioning scheme and existing MSBL-based hybrid precoding and combining scheme in [58]

$\mathbf{V}, \mathbf{W}_{\text{BB}} = \mathbf{U}$. One can observe from Fig. 11 a significant SE performance gain using the proposed positioning scheme. The improved SE performance is attributed to the fact that the RF precoder \mathbf{F}_{RF} and RF combiner \mathbf{W}_{RF} designed from the proposed positioning scheme provides significant beamforming gain compared to the scheme proposed in [58] which does not account for the off-grid errors.

VI. CONCLUSIONS

A joint angle-velocity-RCS coefficient estimation algorithm was devised for the localization of multiple targets in a bistatic mmWave MIMO radar system operating in the face of clutter. The proposed scheme exploits the 3D-sparsity of the target in the AD domain for jointly estimating the target parameters, such as its velocity, angles and RCS coefficients. In order to mitigate the estimation errors arising due to grid mismatch, an innovative SBL algorithm was formulated for recursively updating the parameter grid points in each iteration, thereby ensuring that they approach the true target parameter values upon convergence. The AD-domain representation also helps us to distinguish the estimated target parameters from the clutter. We also derived the CRBs for target parameter estimation to benchmark the estimation performance. Our simulation results validated the superiority of the proposed framework compared to the existing solutions. Possible future extension of this work might consider the introduction of reconfigurable intelligent surface (RIS) to tackle the challenging problem of target localization in the blind zone, wherein the transmitted signal is unable to reach the target due to obstacles in the LoS path.

APPENDIX A

PROOFS OF (36), (37), (38), (39)

We present the detailed derivations for (36), (37), (38), (39).

A. Update for \mathbf{z} in (36)

The objective function in (30) can be rewritten as follows after ignoring the independent terms

$$\begin{aligned} \mathcal{L}(\mathbf{z}, \boldsymbol{\epsilon}_t^{(k)}, \boldsymbol{\epsilon}_r^{(k)}, \boldsymbol{\epsilon}_\omega^{(k)} | \cdot, \cdot, \cdot, \cdot) &= \\ &= \int p(\mathbf{B} | \mathbf{Y}, \mathbf{z}^{(k)}, \boldsymbol{\epsilon}_t^{(k)}, \boldsymbol{\epsilon}_r^{(k)}, \boldsymbol{\epsilon}_\omega^{(k)}) \ln p(\mathbf{B} | \mathbf{z}) d\mathbf{B} \\ &+ \int p(\mathbf{B} | \mathbf{Y}, \mathbf{z}^{(k)}, \boldsymbol{\epsilon}_t^{(k)}, \boldsymbol{\epsilon}_r^{(k)}, \boldsymbol{\epsilon}_\omega^{(k)}) \ln p(\mathbf{z}) d\mathbf{B} \\ &= (a + L) \sum_{g=1}^G \ln z_g - b \sum_{g=1}^G z_g - \text{Tr} \left(\sum_{l=1}^L \Upsilon_l \left(\boldsymbol{\epsilon}_t^{(k)}, \boldsymbol{\epsilon}_r^{(k)}, \boldsymbol{\epsilon}_\omega^{(k)} \right) \right), \end{aligned} \quad (79)$$

where we have $\Upsilon_l \left(\boldsymbol{\epsilon}_t^{(k)}, \boldsymbol{\epsilon}_r^{(k)}, \boldsymbol{\epsilon}_\omega^{(k)} \right) = \boldsymbol{\mu}_l \boldsymbol{\mu}_l^H + \boldsymbol{\Sigma}$. Upon differentiating the above expression with respect to z_g and equating it to zero, we obtain (36).

B. Update for $\boldsymbol{\epsilon}_t$ in (37)

After ignoring independent terms, the objective function in (31) can be rewritten as follows

$$\begin{aligned} \mathcal{L}(\mathbf{z}^{(k+1)}, \boldsymbol{\epsilon}_t, \boldsymbol{\epsilon}_r^{(k)}, \boldsymbol{\epsilon}_\omega^{(k)} | \cdot, \cdot, \cdot, \cdot) &= \\ &= \int p(\mathbf{B} | \mathbf{Y}, \cdot, \cdot, \cdot, \cdot) \ln p(\mathbf{Y} | \mathbf{B}, \mathbf{z}^{(k+1)}, \boldsymbol{\epsilon}_t, \boldsymbol{\epsilon}_r^{(k)}, \boldsymbol{\epsilon}_\omega^{(k)}) d\mathbf{B} \\ &= - \int p(\mathbf{B} | \mathbf{Y}, \cdot, \cdot, \cdot, \cdot) \sum_{l=1}^L \left(\mathbf{y}_l - (\boldsymbol{\Psi}_t + \boldsymbol{\Xi}_t \text{diag}(\boldsymbol{\epsilon}_t)) \boldsymbol{\beta}_l \right)^H \\ &\quad \times \left(\mathbf{y}_l - (\boldsymbol{\Psi}_t + \boldsymbol{\Xi}_t \text{diag}(\boldsymbol{\epsilon}_t)) \boldsymbol{\beta}_l \right) d\mathbf{B} \\ &= - \sum_{l=1}^L \left(\mathbf{y}_l - \tilde{\boldsymbol{\Psi}}_t \boldsymbol{\mu}_l \right)^H \left(\mathbf{y}_l - \tilde{\boldsymbol{\Psi}}_t \boldsymbol{\mu}_l \right) - L \text{Tr} \left(\tilde{\boldsymbol{\Psi}}_t \boldsymbol{\Sigma} \tilde{\boldsymbol{\Psi}}_t^H \right) \\ &= - \boldsymbol{\epsilon}_t^H \left(\Re \left\{ \boldsymbol{\Xi}_t^H \boldsymbol{\Xi}_t \odot (\mathbf{U} \mathbf{U}^H + L \boldsymbol{\Sigma}) \right\} \right) \boldsymbol{\epsilon}_t + 2 \mathbf{p}_t^H \boldsymbol{\epsilon}_t + c, \end{aligned} \quad (80)$$

where $\tilde{\boldsymbol{\Psi}}_t = \boldsymbol{\Psi}_t + \boldsymbol{\Xi}_t \text{diag}(\boldsymbol{\epsilon}_t)$ and c is a constant term. Upon differentiating the above expression with respect to the offset vector $\boldsymbol{\epsilon}_t$ and equating it to zero, we obtain (37).

C. Update for $\boldsymbol{\epsilon}_r$ in (38)

Upon ignoring the independent terms, the objective function in (32) can be rewritten as follows

$$\begin{aligned} \mathcal{L}(\mathbf{z}^{(k+1)}, \boldsymbol{\epsilon}_t^{(k+1)}, \boldsymbol{\epsilon}_r, \boldsymbol{\epsilon}_\omega^{(k)} | \cdot, \cdot, \cdot, \cdot) &= \\ &= \int p(\mathbf{B} | \mathbf{Y}, \cdot, \cdot, \cdot, \cdot) \ln p(\mathbf{Y} | \mathbf{B}, \mathbf{z}^{(k+1)}, \boldsymbol{\epsilon}_t^{(k+1)}, \boldsymbol{\epsilon}_r, \boldsymbol{\epsilon}_\omega^{(k)}) d\mathbf{B} \\ &= - \sum_{l=1}^L \left(\mathbf{y}_l - \tilde{\boldsymbol{\Psi}}_r \boldsymbol{\mu}_l \right)^H \left(\mathbf{y}_l - \tilde{\boldsymbol{\Psi}}_r \boldsymbol{\mu}_l \right) - L \text{Tr} \left(\tilde{\boldsymbol{\Psi}}_r \boldsymbol{\Sigma} \tilde{\boldsymbol{\Psi}}_r^H \right) \\ &= - \boldsymbol{\epsilon}_r^H \left(\Re \left\{ \boldsymbol{\Xi}_r^H \boldsymbol{\Xi}_r \odot (\mathbf{U} \mathbf{U}^H + L \boldsymbol{\Sigma}) \right\} \right) \boldsymbol{\epsilon}_r + 2 \mathbf{p}_r^H \boldsymbol{\epsilon}_r + c, \end{aligned} \quad (81)$$

where $\tilde{\boldsymbol{\Psi}}_r = \boldsymbol{\Psi}_r + \boldsymbol{\Xi}_r \text{diag}(\boldsymbol{\epsilon}_r)$, and c is a constant term. Eq. (38) is obtained after differentiating the above expression with respect to the offset vector $\boldsymbol{\epsilon}_r$ and equating it to zero.

D. Update for ϵ_ω in (39)

After ignoring the independent terms, the objective function in (33) can be rewritten as follows

$$\begin{aligned} \mathcal{L}(\mathbf{z}^{(k+1)}, \epsilon_t^{(k+1)}, \epsilon_r^{(k+1)}, \epsilon_\omega | \cdot, \cdot, \cdot, \cdot) &= \\ &\int p(\mathbf{B}|\mathbf{Y}, \cdot, \cdot, \cdot, \cdot) \ln p(\mathbf{Y}|\mathbf{B}, \mathbf{z}^{(k+1)}, \epsilon_t^{(k+1)}, \epsilon_r^{(k+1)}, \epsilon_\omega) d\mathbf{B} \\ &= - \sum_{l=1}^L (\mathbf{y}_l - \tilde{\Psi}_\omega \boldsymbol{\mu}_l)^H (\mathbf{y}_l - \tilde{\Psi}_\omega \boldsymbol{\mu}_l) - L \text{Tr} \left(\tilde{\Psi}_\omega \Sigma \tilde{\Psi}_\omega^H \right) \\ &= -\epsilon_\omega^H \left(\Re \left\{ \Xi_\omega^H \Xi_\omega \odot (\mathbf{U}\mathbf{U}^H + \mathbf{L}\Sigma) \right\} \right) \epsilon_\omega + 2\mathbf{p}_\omega^H \epsilon_\omega + c, \end{aligned} \quad (82)$$

where $\tilde{\Psi}_\omega = \Psi_\omega + \Xi_\omega \text{diag}(\epsilon_\omega)$, and c is a constant term. Upon differentiating the above expression with respect to the offset vector ϵ_ω and equating it to zero, we obtain (39).

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