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SOUND RADIATION FROM A PERFORATED UNBAFFLED PLATE

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Putra, Azma[1]; Thompson, David[1]
[1]Institute of Sound and Vibration Research (ISVR); University of Southampton, Southampton, GB-SO17 1BJ, United Kingdom; ap@isvr.soton.ac.uk, djt@isvr.soton.ac.uk

ABSTRACT

A model to calculate the sound radiation from an unbaffled perforated plate is proposed. This is achieved by modifying an existing model of an unbaffled plate to include the effect of perforation in terms of a continuously distributed surface impedance to represent the holes. Results are compared with those of an idealised situation, a perforated plate in an equally perforated baffle. At low perforation ratios, the radiation efficiency is lower for the unbaffled case but as the perforation ratio increases, the results for both conditions become similar. The effect of perforation increases as the perforation ratio increases and also as the hole diameter reduces. Comparison with existing measurement is also found to give a good agreement.

INTRODUCTION

In attempting to reduce the sound radiated by vibrating plates, several schemes have been developed including introducing perforation over the area of the plate. This technique can be seen in many practical applications, for example product collection hoppers and safety guard enclosures, such as the protective cover over flywheels and belt drives.

Existing models of sound radiation from a perforated plate [1, 2] approximate the acoustic impedance of the holes by the analytical solution for wave propagation in a small tube of circular cross section, as proposed by Maa [3]. For convenience, the holes are then considered to be distributed across the plate surface allowing a continuous surface acoustic impedance to be assumed. Takahashi and Tanaka [1] proposed a model for the sound radiation in one-dimensional case from an infinite, thin elastic plate under a single normal point force excitation. Fahy and Thompson [2] started with a model of radiation by plane bending waves propagating in an unbounded, uniformly perforated plate to calculate the radiation efficiency of a simply supported rectangular plate. The assumptions imply that the plate is effectively mounted in a similarly perforated rigid baffle so that the plate and the baffle have an identical acoustic impedance. The model was then extended to the situation where the plate and the baffle have different specific acoustic impedances. The relation between velocities and pressures was derived in the wavenumber domain as a matrix problem and then solved by matrix inversion. As a preliminary case, this was applied to the radiation by a perforated strip piston, in two dimensions, vibrating in an infinite baffle. Good results were obtained for the case of a perforated strip piston in a perforated baffle and in a rigid baffle. However, problems are found with this model at low frequencies for an unbaffled perforated strip. A very low acoustic impedance of the boundary (relative to the acoustic impedance of the holes) leads to a singular or nearly singular matrix which reduces the quality of the results from the inverted matrix. In addition, expanding this model into the three-dimensional case is found to require intensive computational effort.

In this paper, the sound radiation from a perforated unbaffled plate is calculated by modifying Laulagnet's model [4] to introduce the perforation. At low perforation ratios, it is found that the result is lower than that from the case of a perforated baffle. However, as the perforation ratio increases, the radiation efficiency of the perforated plate in an equally perforated baffle and the perforated unbaffled plate are found to become similar.

THEORY

A brief introduction to the model is given here. More details can be found in [4,5]. Consider a flat thin unbaffled plate in Figure 1 with a surface area S_p located in an infinite medium, excited by a harmonic force distribution $F(x, y)$ at angular frequency ω.

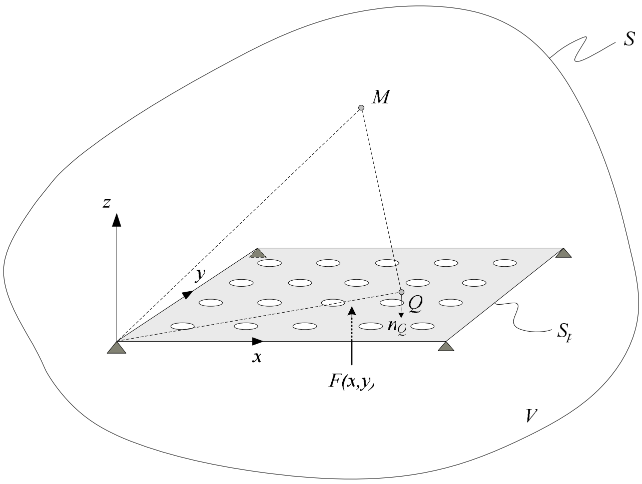


Figure 1.-A perforated unbaffled plate

The equation of motion of the plate excited by the force F and pressure difference Δp is

$$B\nabla^4 w(x, y) - m\omega^2 w(x, y) = F(x, y) + \Delta p(x, y) \quad \text{(Eq. 1)}$$

where B is the bending stiffness, m is the mass per unit area, $w(x, y)$ is the transverse displacement of the plate and $\nabla^4 = \partial^4 / \partial x^4 + 2\partial^4 / \partial x^2 \partial y^2 + \partial^4 / \partial y^4$. The motion of the fluid inside the holes depends on the pressure difference Δp between the two sides of the plate surface. This can be found from the specific acoustic impedance of the distribution of holes z_h,

$$z_h = \frac{\Delta p(x, y)}{v_f(x, y)} \quad \text{(Eq. 2)}$$

where $v_f(x, y)$ is the equivalent velocity of the fluid through the holes, averaged over the plate area. For a circular hole with diameter d_o, the acoustic impedance is inertial provided that d_o is greater than about 1 mm (for smaller holes viscous effects are important). Thus by neglecting the fluid viscosity, Maa's approximation for the impedance of the hole [3], distributed across the plate surface, can then be written as

$$z_h = j\rho c\left[\frac{kt}{\tau}\left(1 + \frac{\pi d_o}{4t}\right)\right] \quad \text{(Eq. 3)}$$

where ρ is the air density, c is the speed of sound, t is the plate thickness, k is the acoustic wavenumber and τ is the perforation ratio. The distributed holes or continuous impedance is indicated by the presence of τ in (Eq. 3). By using Euler's equation on the plate surface at $z = 0$ and substituting (Eq. 1) and (Eq. 2) to eliminate the pressure difference Δp, gives

$$\frac{\partial p}{\partial z}(x, y)|_{z=0} = \rho\,\omega^2 (w(x, y) - w_f(x, y)) = \rho\,\omega^2 w(x, y) + \frac{j\rho\omega}{z_h}(B\nabla^4 w(x, y) - m\omega^2 w(x, y) - F(x, y)) \quad \text{(Eq. 4)}$$

where w_f is the equivalent fluid displacement relative to the plane $z=0$ and $v_f = j\omega w_f$.
The pressure at point M in the fluid can be defined by using Kirchhoff-Helmholtz (K-H) integral

$$p(M) = \int_S \left(p(Q)\partial G(Q,M)/\partial n_Q - G(Q,M)\partial p/\partial n_Q \right) dS \qquad \text{(Eq. 5)}$$

where Q is a point on the plate surface and G is the free-field Green's function. Assuming a thin plate, the normal acceleration ($\partial u_n/\partial t = -(1/\rho)\partial p/\partial n_Q$) is continuous across the plate. Therefore the monopole source contributions in the K-H integral (second term) vanish leaving only the dipole source contributions (first term). For points M on the plate surface, using this and substituting (Eq. 4) yields

$$\rho\,\omega^2 w(M) + \frac{j\rho\omega}{z_h}(B\nabla^4 w(M) - m\omega^2 w(M) - F(M))$$
$$= \int_{S_p} (B\nabla^4 w(Q) - m\omega^2 w(Q) - F(Q)) \frac{\partial^2 G(Q,M)}{\partial z_Q \partial z_M} dS_p \qquad \text{(Eq. 6)}$$

where the second term on the left-hand side has been added by the perforation. The displacement can be considered as the summation of plate modes (m,n). For the case of a simply supported plate, it can be written as

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} d_{mn}\varphi_{mn} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} d_{mn}\sin(m\pi x/a)\sin(n\pi y/b) \qquad \text{(Eq. 7)}$$

where d_{mn} is the modal displacement amplitude, φ_{mn} is the mode shape function and $a \times b$ are the plate dimensions. The force can be written using a similar expression. The orthogonality relationship with mode (p,q) gives

$$\int_{S_p} \varphi_{mn}\,\varphi_{pq}\,dS_p = \frac{S_p}{4}\delta_{mp}\delta_{nq} \qquad \text{(Eq. 8)}$$

where δ is the Kronecker's delta. Applying (Eq. 8) and substituting (Eq. 7) into (Eq. 6) yields

$$\rho\omega^2 \frac{S_p}{4} d_{pq} + \frac{j\omega\rho}{z_h}(m(\omega_{pq}^2 - \omega^2)d_{pq} - F_{pq}) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(m(\omega_{mn}^2 - \omega^2)d_{mn} - F_{mn})C_{pqmn} \qquad \text{(Eq. 9)}$$

where F_{pq} is the modal excitation force and $C_{pqmn} = C_{mnpq}$ are the acoustical cross-modal coupling terms [4] obtained as

$$C_{pqmn} = \frac{2j}{pqmn}\left(\frac{ab}{\pi^3}\right)^2 \int_0^{\infty}\int_0^{\infty} k_z \Upsilon\,\Omega\, dk_x\, dk_y$$
$$\Upsilon = \frac{1-(-1)^p\cos\mu}{((\mu/p\pi)^2 - 1)((\mu/m\pi)^2 - 1)},\quad \Omega = \frac{1-(-1)^q\cos\chi}{((\chi/q\pi)^2 - 1)((\chi/n\pi)^2 - 1)} \qquad \text{(Eq. 10)}$$

where $\mu = k_x a$, $\chi = k_y b$, $k_z = \sqrt{k^2 - k_x^2 - k_y^2}$; k_x, k_y and k_z are component of the wavenumber.

The effect of damping can be included by replacing the plate natural frequency ω_{mn}^2 by $\omega_{mn}^2(1+j\eta)$ where is η the damping loss factor. (Eq. 9) can be simplified by taking only the self-modal coupling terms of (Eq. 10). The inverse of the cross-modal coupling terms $[C_{pqmn}]^{-1}$ can then be replaced by $1/C_{pqpq}$. Such an approximation works well for a baffled plate [6], particularly when the average result over different force positions is considered [7]. It has been shown that this approximation can also be used for the case of an unbaffled plate, perforated or unperforated [4,5]. After re-arranging (Eq. 9), its solution can be written as

$$m_{pq}(\omega_{pq}^2-\omega^2)d_{pq}-\rho\omega^2\left(\frac{S_p}{4}\right)^2\frac{1}{C_{pqpq}}\left(1+\frac{4jm_{pq}}{S_p\omega z_h}(\omega_{pq}^2-\omega^2)\right)d_{pq}=\left(1-\frac{j\omega\rho}{z_h}\frac{S_p}{4}\frac{1}{C_{pqpq}}\right)\hat{F}_{pq} \quad \text{(Eq. 11)}$$

where $m_{pq}=m(S_p/4)$ and $\hat{F}_{pq}=F_{pq}(S_p/4)$ are the generalized mass and the generalized force.

The sound power W and the mean-square spatially average plate velocity $\langle v_{pq}^2\rangle$ are given by

$$W=\frac{\omega S_p}{8}\Re\left\{j\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}(m(\omega_{pq}^2-\omega^2)d_{pq}-F_{pq})d_{pq}^*\right\} \quad \text{(Eq. 12)}$$

$$\langle v_{pq}^2\rangle=\frac{\omega^2}{8}|d_{pq}|^2 \quad \text{(Eq. 13)}$$

Finally the total radiation efficiency σ can be found from

$$\sigma=\frac{W}{\rho c S_p\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\langle v_{pq}^2\rangle} \quad \text{(Eq. 14)}$$

RESULTS AND ANALYSIS

Results are presented for an example $0.65\times0.5\times0.003$ m aluminium plate with η =0.1 excited by point forces averaged over the plate surface. Figure 2 shows the modal and total radiation efficiencies of an unbaffled perforated plate with 40 % perforation ratio and 10 mm diameter holes. The 40 dB/decade slope at very low frequency, i.e. at below the fundamental mode, shows the expected characteristic of dipole source radiation.

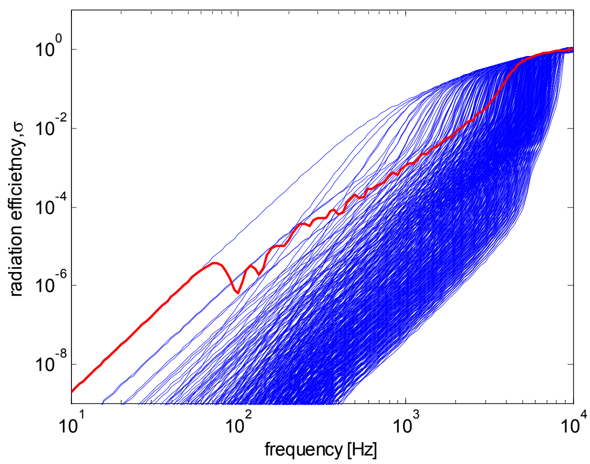
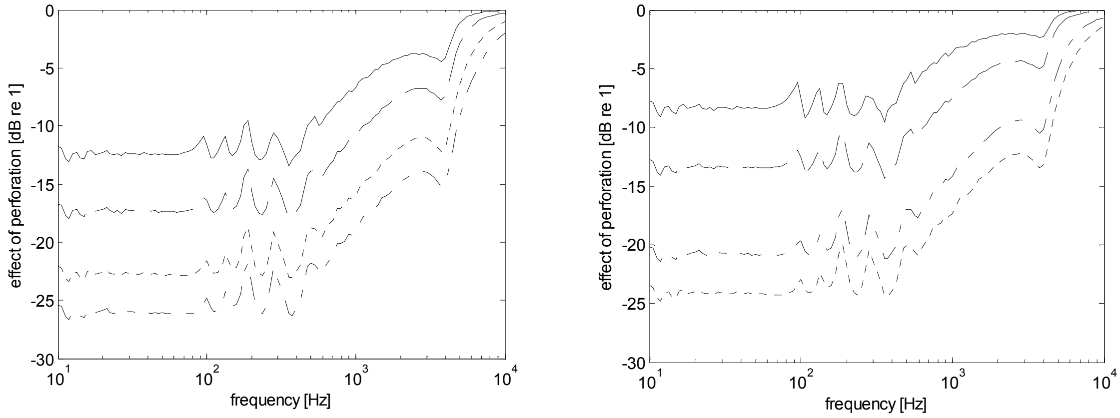


Figure 2.- Modal and average radiation efficiency of an unbaffled perforated plate d_o =10 mm, τ = 40 %: — modal radiation efficiency, —average radiation efficiency.

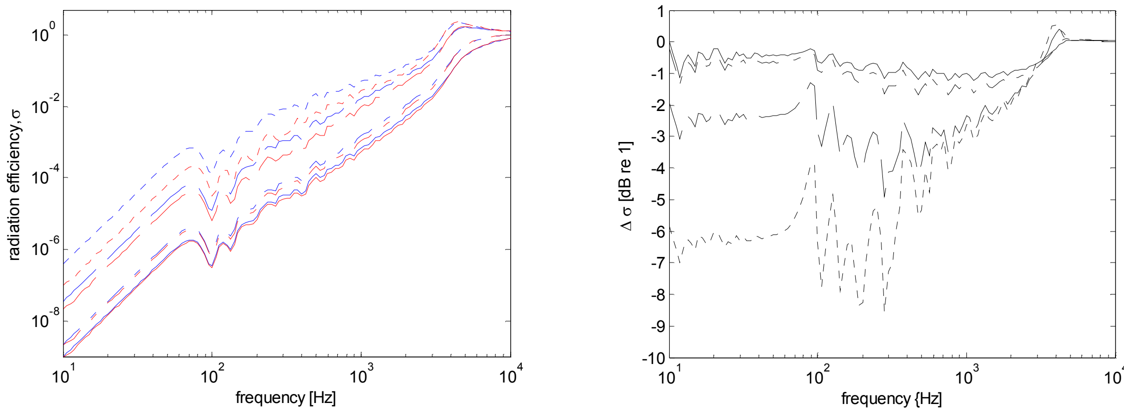
Figure 3(a) and (b) present the effect of perforation defined by $10\log_{10}(W_{\text{perforated}}/W_{\text{unperforated}})$ for constant hole diameter and constant perforation ratio, respectively. These show that, for the case of the unbaffed perforated plate, the effect of perforation is almost independent of the frequency in the fundamental and corner mode regions (< 400 Hz) and then increases in the edge mode region as the frequency increases. The effect of perforation approaches 0 dB at high frequency. Figure 3(a) shows that sound radiation is reduced by increasing the perforation ratio, while Figure 3(b) shows that the radiated sound can be further reduced by reducing the hole size.

Figure 4(a) compares the results with those found using the model of [2] for a perforated plate in an equally perforated baffle. The unbaffled plate has a lower radiation efficiency than for an infinite perforated baffle for most of the frequency range. However, as the perforation ratio increases, the two results become very similar. At high perforation ratio, the contribution of the baffle to the radiated sound is no longer significant. This is obvious in Figure 4(b) which shows the differences of the radiation efficiencies for no baffle and with a perforated baffle on a dB scale. It shows that for a 10 mm hole diameter and 60 % perforation ratio, the difference is only around 1 dB. Therefore with very high perforation ratio, the system with a perforated baffle can then be considered to be equivalent to the unbaffled perforated plate.



(a) (b)

Figure 3.-Effect of perforation on sound power radiation of a perforated unbaffled plate
(a) d_o =10 mm: — τ = 10 %, – – τ = 20 %, --- τ = 40 %, – · – τ = 60 % and
(b) τ = 20 %: — d_o =50 mm, – – d_o =20 mm, --- d_o =5 mm, – · – d_o =2 mm.



(a) (b)

Figure 4.-(a) Comparison of radiation efficiency of a perforated plate in a perforated baffle (blue line) and with that of an unbaffled plate (red line), (b) radiation efficiency difference between both models (d_o =10 mm: --- τ = 3 %, – – τ = 10 %, – · – τ = 40 %, — τ = 60 %)

Figure 5 shows a comparison of the radiation index ($10\log_{10}\sigma$) from the analytical calculation with that from measurements made by Pierri [8] for $0.3 \times 0.3 \times 0.0012$ m steel plates. The results are plotted against 1/3 octave band centre frequencies. The damping loss factor in the calculations is chosen to be very low, i.e. η = 0.001. A very good agreement between the theoretical and the measured results is achieved, although below 800 Hz the measured radiation efficiencies are greater than the predictions by about 3 - 4 dB. This may be due to the background noise from the shaker used to excite the plate.

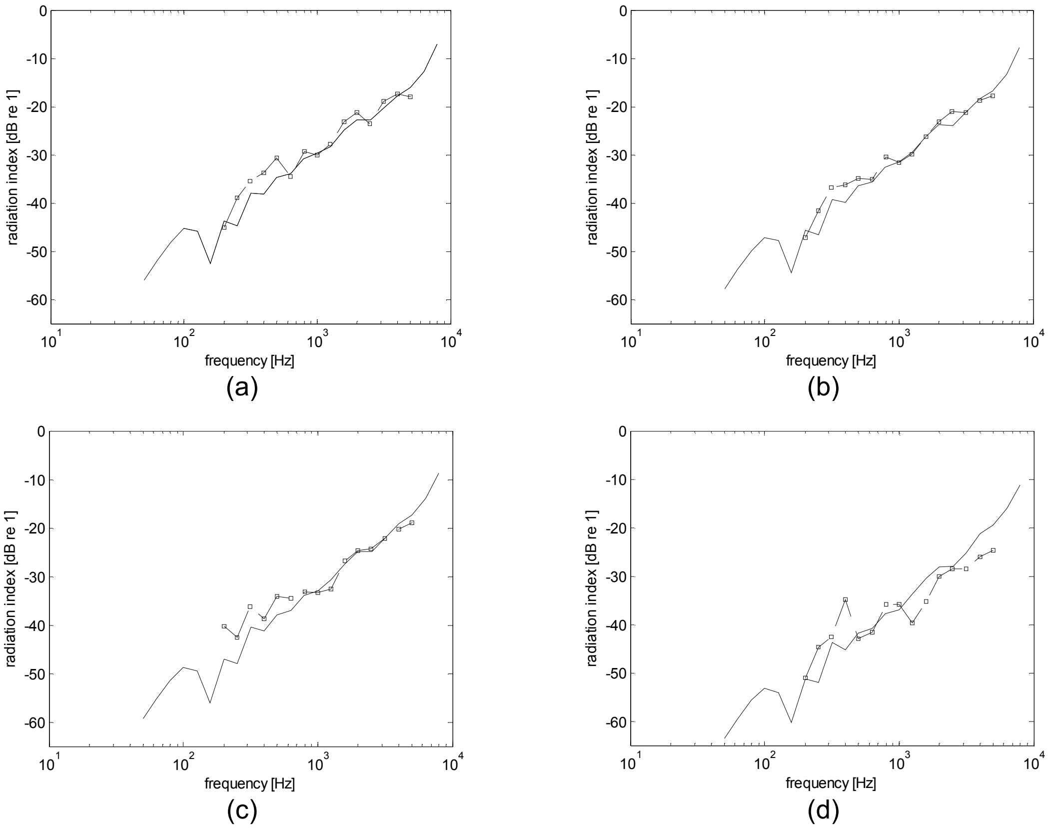


Figure 5.-Comparison of the radiation index (—) with measured data (−□−):
(a) d_o =5.6 mm, τ = 5.7 %; (b) d_o =7.1 mm, τ = 9.4 %; (c) d_o =8.8 mm, τ = 14.1 %;
(d) d_o =15 mm, τ = 41.5 %.

CONCLUSIONS

A model for the sound radiation of a perforated unbaffled plate has been proposed. It has been shown that the model gives the same result as the model of a perforated plate in a perforated baffle at high perforation ratio. The effect of perforation is independent of frequency in the fundamental and corner mode regions. Increasing the perforation ratio increases the reduction of the radiated sound. From the model it is found that the reduction becomes greater by also reducing the hole size. Comparison with existing measured data also shows a very good agreement.

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