Institute of Sound and Vibration Research

Ocean Acoustic Circumpropagation in the Ice Seas of Europa

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FLUID DYNAMICS AND ACOUSTICS GROUP

Ocean Acoustic Circumpropagation in the Ice Seas of Europa

by

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Authorized for issue by
Professor R J Astley

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ABSTRACT

In recent years increased attention has been paid to the potential uses of acoustics for extraterrestrial exploration. A number of important papers have discussed propagation on Europa, primarily with respect to sound in the ice sheet which is believed to cover a salt water ocean. The models used to date assume a flat ice surface and a gravitational acceleration which does not vary with depth. Models of long range acoustic propagation through Europa's ice seas require models which do not make these two assumptions. This report applies such a model to simple Europan geometries to show how observables can be affected by the values of physical parameters: the report considers the specific case of the effect on the travel time of a circumpropagating pulse of the ice and water thicknesses, and water temperature (assumed to be uniform), on a simple spherical planet. Such effects could be the basis of acoustic inversion experiments, although it is recognised that the complexities and uncertainties associated with the actual ice seas of Europa would make the task very much more challenging than the calculations undertaken in this simple study.

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1 Introduction

For much of the history of extraterrestrial exploration, the importance of acoustics has perhaps been underrated (Leighton, 2007). However in recent years interest in the topic has increased (Muir 2007), sparked in part by spectacular successes (such as that achieved by the Huygens probe; Zarnecki et al., 2005) and brave endeavours (Delory et al., 1998). Further impetus has been been found in the development of modelling methods for predicting both extraterrestrial sources of sound (Leighton, 2004; Leighton and White 2004; Leighton *et al.*, 2005) and the propagation of sounds through alien environments (Petculescu and Lueptow, 2007).

Of the many environments where acoustics could provide a useful tool (Leighton 2007; Muir 2007), Jupiter's moon Europa is particularly interesting. Several notable acoustical investigations feature Europa, a main objective being exploration of ways to characterise the ocean which could be present beneath the surface ice. Kovach and Chyba (2001) and Lee *et al.* (2003) produced pioneering studies which modelled acoustic propagation in the ice and the sub-ice ocean of Europa. They opened the way for assessing Europa's ice and ocean using acoustical signals of opportunity, generated by natural processes in the ice, ocean or below the seabed (Crawford and Stevenson, 1988; Schenk and McKinnon, 1989; Hoppa *et al.*, 2000; Greenberg, 2002; Lee *et al.*, 2003, 2005; Nimmo and Schenk, 2006; Panning *et al.*, 2006). However propagation over distances characteristic of the depths of the ocean, or of the separation of major ice cracking events which generate sources of opportunity for acoustic measurements, require use of a model which departs from those developed for terrestrial use. This is because in Europa's ocean, vertical lines are not parallel, and the acceleration due to gravity is not constant with depth (Leighton *et al.*, 2008). Once such features have been included, the model can be used to study propagation in the ocean to great distances from the sources, (long range propagation under Earth's Arctic has been studied with considerable success – Mikhalevsky and Gavrilov, 2001).

Leighton *et al.* (2006) undertook early calculations to examine circumpropagation, that is, the propagation of sound through the liquid ocean, right around the planet under the ice and back to the source. This provided preliminary identification of the expected features, such as the three-dimensional refocusing which occurs at the source and at the point on the planet opposite the source (because, in simple terms, the spreading losses that occur during propagation over one hemisphere are, for ideally spherical conditions, reversed on the broad scale as the waves propagate over the other hemisphere). Because of these key end-points, circumpropagation provides scenarios which can be studied using simple calculations to assess sensitivities in a transparent manner. This report provides improved results for those calculations as input parameter values have been refined, and the processing made sufficient rapid to allow a range of candidate oceans to be studied.

This investigation sets up a simplified model of Europa's ocean, and uses it to demonstrate how the environmental parameters can affect the propagation. A long term goal of such studies is the demonstration of how the inversion of measured acoustic propagation could be used to estimate or monitor the values of those parameters. This is specifically demonstrated with a simple model which assumes uniform temperature throughout the ocean, for the case of propagation of sound around the planet back to the source (circumpropagation). Whilst of course this differs from the likely oceanic conditions on Europa, and from the likely experimental set-up, it serves to demonstrate the sensitivity of the arrival times to the ocean temperature, and to the positions of the seabed and the base of the ice pack. In particular it shows how, if the position of the water/ice interface changes in this spherically-symmetric model of Europa, qualitative changes to the ray paths may result, such that the effect on observables such as the arrival times can be very much greater than a simple linear scaling would predict.

2 Method

Consider a small spherical world of outer radius R_{outer}, where a layer of ice (of thickness h_{ice}) overlies an ocean (of thickness $R_{\mathrm{ice}} - R_{\mathrm{bed}}$) that is assumed to have a uniform temperature. This is taken to be a simple model of Europa. The geometry is shown in Figure 1, and the parameter values for this ocean, are as follows. The outer radius of the world (the distance from the centre to the top of the ice layer) is R_{outer} =1560800 m. The radius at which the seabed occurs R_{bed} = 1440800 m. Let ρ_{w}

and ρ_{ice} be the densities of the water and ice respectively (neglecting any variation of these with depth) and let ρ_{E} be the spatially-averaged density of the seabed, mantle and core: these are assumed to take constant values of ρ_{w} =1000 kg m^{-3}, ρ_{ice} =920 kg m^{-3}, with ρ_{E} =3550 kg m^{-3}. The ocean salinity is assumed to be S=35 parts per thousand (although the exact value is not critical to the trends illustrated in this paper).

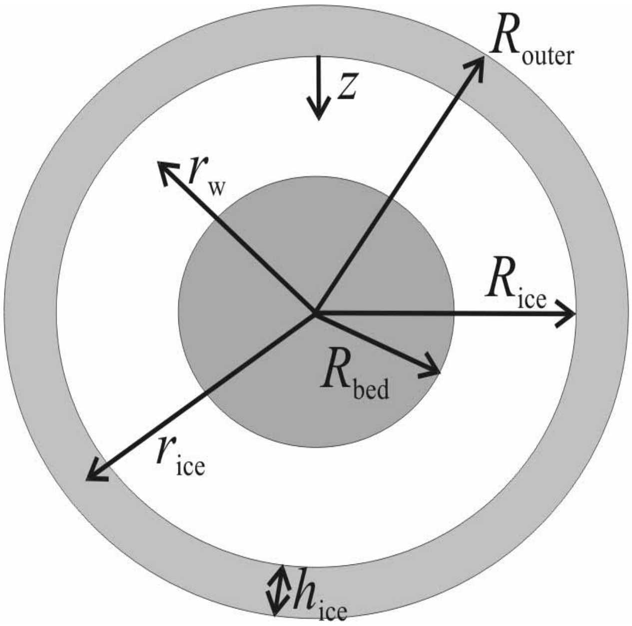

Figure 1. Cross-section of Europa labelling key fixed ($R_{\text{bed}}, R_{\text{ice}}, R_{\text{outer}}$) and variable ($r_{\text{w}}, r_{\text{ice}}$) radii.

Leighton *et al*. (2006, 2008) assumed that the water/ice interface occurred at a radius of R_{ice} =1541 km, such that the water column is 100 km deep (i.e. $R_{\text{ice}} - R_{\text{bed}}$ is constant at 100 km) and the ice layer thickness ($h_{\text{ice}} = R_{\text{outer}} - R_{\text{ice}}$) is fixed at 20 km deep. This paper however will explore the effect of altering the exact position of the water/ice interface (i.e. changing h_{ice} and R_{ice} whilst keeping R_{outer} =1560800 and R_{bed} = 1440800 m constant); and, for fixed values of h_{ice} and R_{ice}, it will explore whether there could be obvious features in the propagation which could be used to infer the ocean temperature (which is assumed to be uniform) in this simple model.

Because the sound speed increases with increasing ocean depth, if ray tracing were to be appropriate for sounds emitted in the ocean, the propagation would be upwardly refracting (Figure 2).

On Earth, the radius of the planet is sufficiently large, and the depth of the ocean is sufficiently shallow, that rectilinear calculations assuming constant gravitational acceleration are accurate over ranges typical of caustic zones. Equivalent dimensions in an isothermal Europan ocean would be those required to describe the path from the source to the first water/ice interface of, say, a ray which is emitted from the water/ice interface, then refracts in such a ways that it skims the seabed before that first bounce off the water/ice interface. A rectilinear approach is outlined in Section 2.1. However on Europa rectilinear techniques may be insufficiently accurate over the path of such a ray, and therefore methods incorporating curvature and variable gravitational acceleration have been developed (Section 2.2).

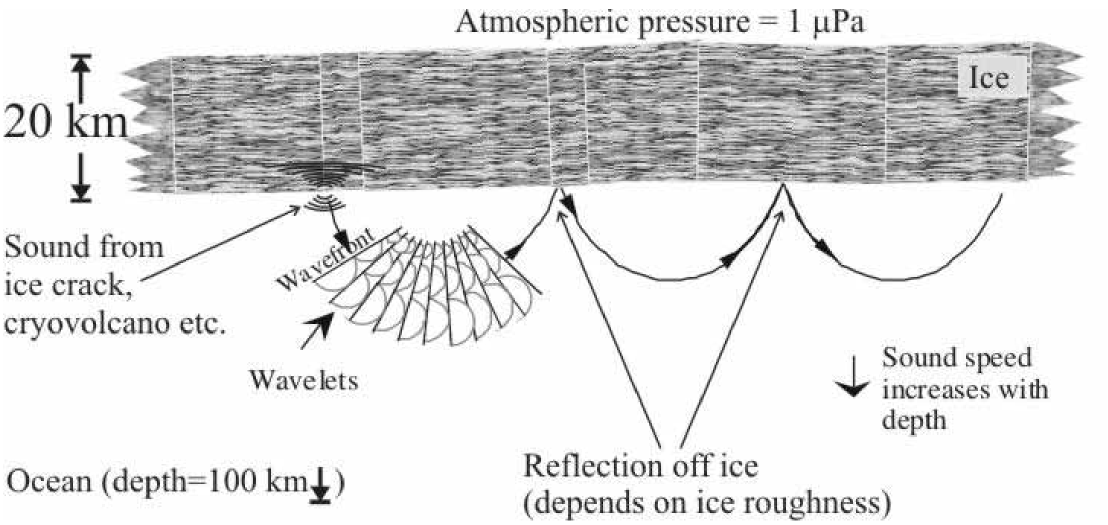

Figure 2. Schematic of the refraction and reflection of sound. A source of sound at the base of the ice field emits into both water and ice. The ocean is upwardly refracting. This can be seen from the Huygens' wavelet construction. As a result, the sound returns to the ice from where it is reflected (with fidelity depending on the smoothness of the ice), to continue along the ocean through subsequent refraction and reflections (From Leighton *et al.*, 2006).

2.1 Rectilinear models

The scenario where the sound speed varies linearly with depth will be very familiar to many acoustical oceanographers. In such circumstances, acoustic rays follow paths which are the arcs of circles. Consider Figure 3. A sound ray is initially travelling horizontally, passing through point X_{i} at a depth d_{i} with speed c_{i}. It then passes through point X_1 at a depth $z = d_1$ with speed c_1, and then passes point X_2 at a depth d_2 with speed c_2. If this is an acoustic ray which follows an arc of a circle, then two conditions must be satisfied.

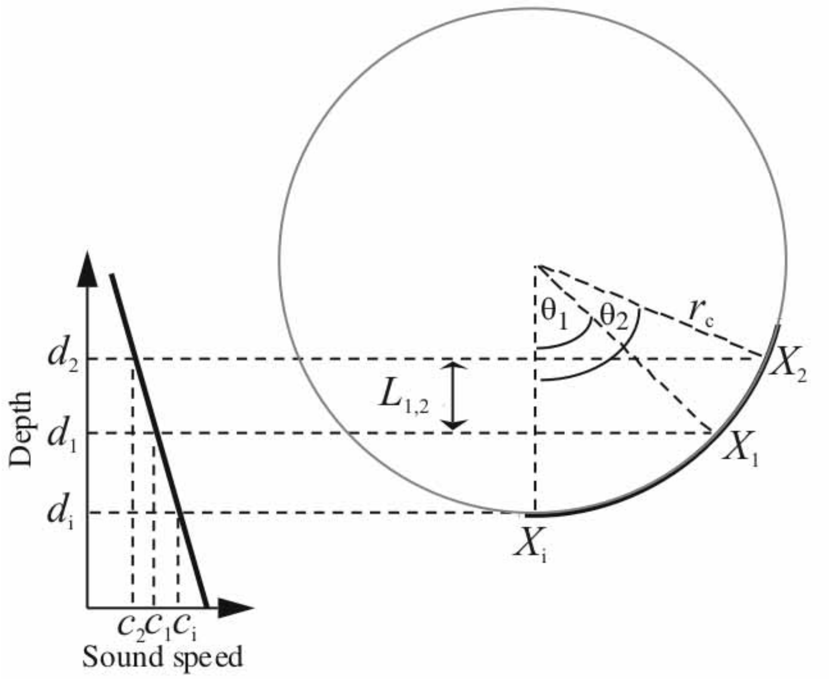

Figure 3. Ray paths in a region of ocean where the sound speed increases linearly with depth.

First, if the ray follows a circle arc, the circle has radius r_{c} such that the distance $L_{1,2}$ on the figure is:

$$L_{1,2} = r_{\mathrm{c}} \cos\theta_1 - r_{\mathrm{c}} \cos\theta_2 = d_1 - d_2 . \qquad (1)$$

Since the sound speed variation is linear with depth, then:

$$\frac{\partial c}{\partial z} = \frac{d_1 - d_2}{c_1 - c_2} . \qquad (2)$$

Second, if the circle arc is a ray, Snell's Law must be satisfied. Snell's law allows statement of the relation between the sound speed at a particular depth and the angle, θ, which a ray at that depth makes locally with the horizontal:

$$\cos\theta_{\mathrm{i}} / c_{\mathrm{i}} = 1/c_{\mathrm{i}} = \cos\theta_1 / c_1 = \cos\theta_2 / c_2 . \qquad (3)$$

Equating $d_1 - d_2$ in (1) and (2), and substituting for the cosine terms using (3), indicates that the ray is following an arc of a circle that has radius

$$r_{\mathrm{c}} = c_{\mathrm{i}} \Big/ \frac{\partial c}{\partial z} . \qquad (4)$$

Equation (4) can be applied to the full ocean depth if it is assumed that the sound speed profile varies linearly with depth throughout the ocean, and this was done by Leighton *et al*. (2006) to produce simple first-order estimates of ray paths in the Europan ocean. However they showed that in Europa’s ocean the sound speed is nonlinear, because vertical lines are not parallel, and because the acceleration due to gravity is not constant with depth (Leighton *et al.*, 2006, 2008).

There are various approaches to the prediction of ray paths through nonlinear sound speed profiles (White, 2004). The approach adopted in this study is based on expressing the ray paths as solutions to a pair of ordinary differential equations (Jensen *et al.*, 2000). These equations can be solved using standard numerical routines. Whilst variations of sound speed in the lateral direction are possible, these are not incorporated into this modelling.

Leighton *et al.* (2006) undertook the preliminary step of performing introductory computational ray tracing in the water column of a spherical Europa. Pioneering studies of wave propagation in the ice had previously been undertaken (Kovach and Chyba, 2001; Lee *et al.*, 2003), where the sea-bed, the sea-ice interface, and the isothermal lines were all straight parallel lines (essentially modelling Europa as a flat surface), and where the approximation that uses a constant gravity was maintained. An example application of such propagation studies is acoustic inversion, where environmental characteristics are inferred from the acoustic signals detected. The location of ice cracks was identified as an early goal for acoustic inversions, for example where ‘the spacing of arrivals in time can be used to robustly estimate source range’ (Makris *et al*., 2001; Makris 2001). Natural emissions from ice cracking provide one specific example of the ‘signals of opportunity’ associated with the ice, ocean or mantle (Crawford and Stevenson, 1988; Schenk and McKinnon, 1989; Hoppa *et al.*, 2000; Greenberg, 2002; Lee *et al.*, 2003, 2005; Nimmo and Schenk, 2006; Panning *et al.*, 2006; Leighton *et al.*, 2006).

2.2 Modelling with planetary curvature and variable acceleration due to gravity

The effect of planetary curvature on rectilinear calculations was demonstrated by Leighton *et al.* (2006) who also took some preliminary circumpropagation calculations, by introducing the curvature of Europa into the computational model. Neither potential deviations from sphericity nor horizontal gradients were incorporated, though such effects have been shown to have a profound effect on Earth (Munk *et al.*, 1988). Considering only those acoustic paths which are confined purely to the liquid ocean, the first tests of Leighton *et al.* (2006) indicated that those rays which just skim the seabed are those which travel most rapidly around the moon and back to their point of origin (applying to circumpropagation the trend that, although the path length is necessarily long to take them to great depth, the sound speed at those depths is very great; see the Appendix). The current paper explores the scenario further, using the methods of Leighton *et al*. (2008).

3 Results and Discussion

3.1 Are the discrepancies in sound speed profile significant?

The method of Leighton *et al.* (2008) can be used to consider the effect on the propagation of using different temperature values for the homogeneous isothermal ocean which is modelled to exist beneath Europa's icy surface. To calculate the oceanic sound speed profile, input values from $0^\circ\mathrm{C}$ to $10^\circ\mathrm{C}$ in steps of $1^\circ\mathrm{C}$ were used for illustrative purposes (and should not be taken to represent our opinion of actual conditions on Europa). Figure 4 shows three of these sound speed profiles ($0^\circ\mathrm{C}$, $4^\circ\mathrm{C}$ and $10^\circ\mathrm{C}$), through an ocean which is assumed to be 100 km deep (i.e. the value of R_{ice} is taken to be 1541 km, such that the ice layer thickness ($h_{\mathrm{ice}} = R_{\mathrm{outer}}$ - R_{ice}) is 20 km. The coordinate z=0 m represents the sea-ice interface, which is assumed to be perfectly smooth and reflecting. For each temperature, two sound speeds are shown: the solid line shows the sound speed calculated after the manner described in Leighton *et al.* (2008); the broken line shows the sound speed calculated if the planet curvature is ignored, and the gravitational acceleration is taken to be equal to that at the planet's surface (1.31 m s^{-1}). It is the solid line profile that is used for the subsequent ray tracing calculations.

The difference between solid and broken lines in the value of the sound speed at the base of the water column, for a given temperature, is around 2%, which raises the immediate question of whether such corrections are significant. It might be argued that this is small compared with the uncertainties in the parameter values which are used as input to the calculations. Such arguments however need to be critically assessed with care, as they are based on a misleading comparison between the systematic sound speed discrepancies shown in Figure 4, and the current uncertainties in the values of other parameters (e.g. ocean temperature and salinity variations) whose physics is correctly incorporated into the models but whose values are estimated with uncertainty. This is because one weakness of acoustic inversion techniques is that it can be difficult to assess the correctness of the result obtained by such an inversion: the fact that the inversion has been regularized sufficiently to converge upon an answer does not mean that the answer is itself correct. If the inversion is based on a model which contains significant systematic errors in the physics, the answer to which it converges may be incorrect and misleading. The 2% error in the broken lines of Figure 4 do not represent random errors, or uncertainties in the value of input parameters, but rather they represent systematic errors in the physical model for propagation.

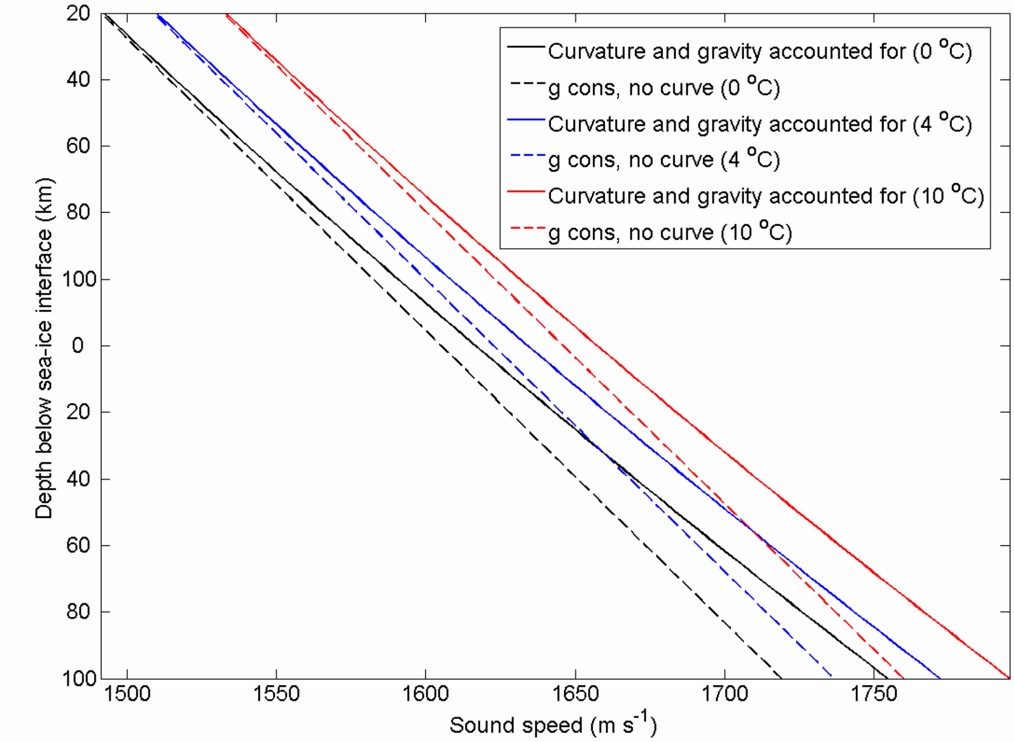

Figure 4. Sound speed profiles for homogeneous ocean temperatures of 0°C (black lines), 4°C (blue lines) and 10°C (red lines). The oceanic sound speed is calculated as a function of depth beneath the base of the ice sheet, assuming a salinity of S=35 ppt, and constant densities for water (ρ_w=1000 kg m^{-3}), ice (ρ_{ice}=920 kg m^{-3}) and using a spatially-averaged density of the seabed, mantle and core of ρ_E=3550 kg m^{-3}. For each temperature, the solid curve plots the sound speed calculated including planet curvature and variable gravitational acceleration (see Leighton *et al.* (2008) for details). Also for each temperature, the dashed curve plots the sound speed calculated ignoring planet curvature and using for the gravitational acceleration a constant value of 1.31 m s^{-2} (the value at the moon's surface). (Original in colour).

Leighton *et al.* (2008) showed that neglect of planetary curvature and the variation of gravitational acceleration generated an error of ~100 km in the distance from the source to the first bounce at the water/ice interface after propagation through the upwardly-refracting water column (the error is half that if planetary curvature is only neglected in the calculation of hydrostatic pressure but included elsewhere, e.g. in the curvature of the water/ice boundary). For the ray in question a 100 km discrepancy represented an error of nearly 20%. This is not an insignificant distance on the scale of the problems considered: as Lee *et al.* (2003) comment “A consistent estimate of the spatial separation between cracking events would be the roughly 100-km scale of a cycloidal feature (Hoppa *et al.*, 1999)”.

Leighton *et al.* (2008) noted that if small discrepancies in the modelling cause the character of the propagation to change qualitatively, the effect can be more significant than the percentage changes implied above. A simple example would be if a ray which would have refracted without intersecting off the seabed were instead (because of a slight change in sound speed profile) to reflect off the seabed. The sequence of acoustic arrivals at a remote location is then changed qualitatively as the pattern of multipaths is affected. The importance of the correct modelling of the qualitative ray paths when quantitatively predicting acoustic travel times is clear if circumstances are such that a small change in the assumed ocean temperature or depth can lead to an unexpectedly greater proportional change in travel time because it alters the number of reflections at the water-ice interface required for a specific return. This effect is demonstrated in a simple circumpropagation example in the next section.

3.2 A simple circumpropagation example

Consider only those ray paths which propagate through the water (rays which reflect off the base of the ice sheet must necessarily be considered for long range acoustic propagation in an upwardly refracting ocean, but rays which interact with the seabed or which travel through the ice are not considered in this simple illustration, and will be neglected throughout this section). Of these rays, the one which travels to great distances in the least time is that ray which penetrates the deepest, because although the path length is the longest, that path takes the ray through the aqueous regions where the sound speed is the greatest. This is illustrated in the Appendix.

The scenario to be considered is the case of rays generated from a given point on the moon, and propagating right around the moon once only to return to the point of origin. The rays which do this most quickly will take only one circumpropagation to return to their point of origin. Clearly only some ray paths will achieve this in one single circumpropagation: others will take more, and arrive later. Given the restrictions cited above on which oceanic waves are being considered, the first arrival will be from the ray which returns to source after a single circumpropagation, and does this whilst penetrating to the deepest ocean depths (see the Appendix). Two cases are shown in the subsections below: (a) the effect on the earliest through-water circumpropagating arrival of changes in the position of the water/ice interface for a fixed ocean temperature; and (b) changes in the ocean temperature for a fixed water/ice interface.

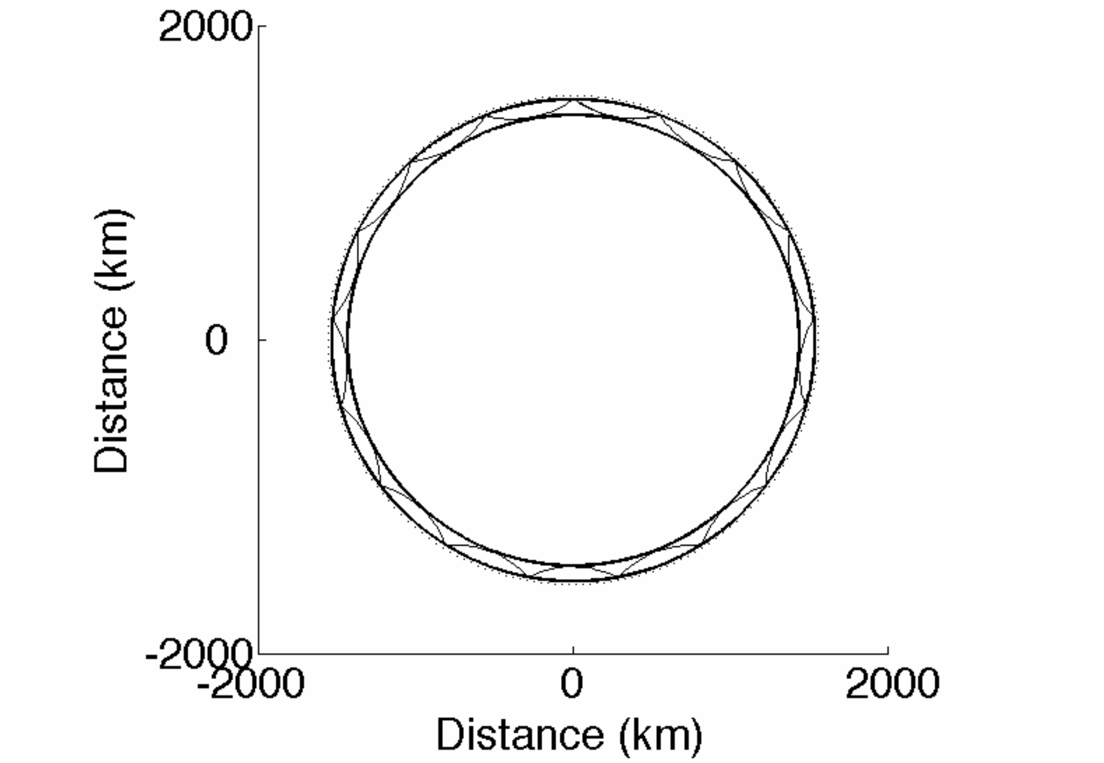

Figure 5. This figure depicts the path followed by the first ray which will circumnavigate the Europan sea were the water to be isotropic at 0° C (considering only those acoustic rays which propagate in the water and which reflect off the water-ice interface). The ray is launched at an angle of 36.1° below the horizontal, and arrives at the starting point after grazing the ocean bottom 17 times. The value of R_{ice} is 1540800 m.

(a) The effect of the position of the water/ice layer

Figure 5 shows one example of a circumpropagating ray which is the first to complete through-water circumpropagation of Europa under ice for the stated conditions. As with all the cases to be discussed in this subsection, the model assumes that the ocean temperature is uniform at $0^{\circ}\mathrm{C}$. This, the first of the rays to return to the point of origin after a single circumpropagation, does so by reflecting off the ice 16 times, and arrives at the starting point after grazing the ocean bottom 17 times. It is the ray which left the source (assumed to be at the base of the ice sheet) at an angle of 36.1° below the local horizontal.

The propagation path lengths and times for the rays which just graze the seabed (and which undertake the most rapid through-water circumpropagation) are shown in Table 1 for constant ocean temperature and constant values for the position of the seabed and the outer radius of the planet (i.e. the top of the ice sheet), as the position of the water/ice layer changes in 1 km increments.

As the ice thickness[1] increases incrementally from 14 to 25 km, the propagation time decreases monotonically (obviously this is an idealised model, and departures from sphericity and environmental inhomogeneities will cause significant perturbations). However between ice thicknesses of 12 and 13 km, the propagation time for the first circumpropagating ray increases by nearly 49 s. This 'toggle point' transition occurs because the propagation path undergoes qualitative changes, with respect to the number of reflections of the water/ice interface which the earliest through-water circumpropagating ray requires. Between ice thicknesses of 26 and 27 km, the travel time for the first through-water arrival undergoes another large discontinuous increase by over 40 s. Like the one which occurred between ice thicknesses of 12 and 13 km, this discontinuity also occurs because of an increase by one in the number of reflections required for circumpropagation by the ray which grazes the seabed. Figure 6 plots the travel time against the ice thickness shown in Table 1. Figure 7 shows, for each ice thickness, what the change in travel time for the most rapid through-water circumpropagation would be were the ice thickness to increase by 1 km.

[1] Note that in this calculation, the effect of changing ice thickness on the through-water propagation time is an indirect one, resulting from the fact that, for fixed R_{bed} and R_{outer}, any change in ice thickness perforce requires a change in the thickness of the water column.

Ice Thickness (km)	Radius of the base of the ice sheet (R_{ice} /km)	Ray angle at release from source (degrees)	Number of times the ray grazes the seafloor during one circumpropagation	Time (mins)	Path length (Mm)
10	1550.8	38.2878	16	100.8849	10.0804
11	1549.8	38.2893	16	100.8157	10.0739
12	1548.8	38.2907	16	100.7466	10.0675
13	1547.8	36.0842	17	101.5599	10.0197
14	1546.8	36.0855	17	101.4902	10.0133
15	1545.8	36.0868	17	101.4205	10.0068
16	1544.8	36.0881	17	101.3508	10.0004
17	1543.8	36.0894	17	101.2811	9.9940
18	1542.8	36.0908	17	101.2114	9.9876
19	1541.8	36.0921	17	101.1417	9.9811
20	1540.8	36.0935	17	101.0719	9.9747
21	1539.8	36.0949	17	101.0022	9.9683
22	1538.8	36.0964	17	100.9325	9.9618
23	1537.8	36.0978	17	100.8627	9.9554
24	1536.8	36.0993	17	100.7929	9.9490
25	1535.8	36.1007	17	100.7232	9.9426
26	1534.8	36.1022	17	100.6534	9.9361
27	1533.8	34.1326	18	101.3223	9.8959
28	1532.8	34.1339	18	101.2520	9.8895
29	1531.8	34.1353	18	101.1818	9.8831
30	1530.8	34.1367	18	101.1115	9.8767

Table 1. Modelling Europa as a spherical world with uniform material properties, for a fixed sea floor (R_{bed} = 1440800 m) and external radius (R_{outer} =1560800 m) and a uniform ocean temperature of 0 $^{\circ}$ C, the characteristics of the earliest-arriving through-water circumpropagating ray are plotted as a function of the radius of the base of the ice sheet (R_{ice}), which is discretely changed in 1 km increments. Two pairs of adjacent rows (for ice thicknesses of 12 and 13 km, and 26 ands 27 km) are shaded, indicating the 'toggle point' where, for these ocean parameters, the 1 km incremental change in the position of the water/ice interface changes the number of reflections required to complete the earliest through-water circumpropagation. The ocean geometry modelled in section 3.2(b) (for ice thickness 26.8 km) is close to the toggle point which occurs between 26 km and 27 km ice thickness. Throughout this paper, the precision to which values are calculated is not meant to reflect their accuracy, but rather the caution used in these preliminary calculations before the magnitude of the effects at the toggle points has been evaluated.

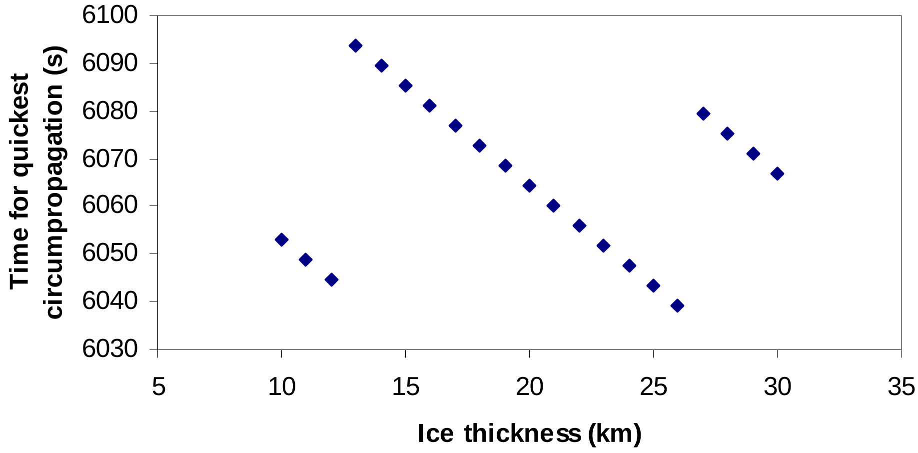

Figure 6. Plot of the travel time for the earliest-arriving through-water circumpropagating ray which returns to the position of the source, as a function of the thickness of the ice (data from Table 1).

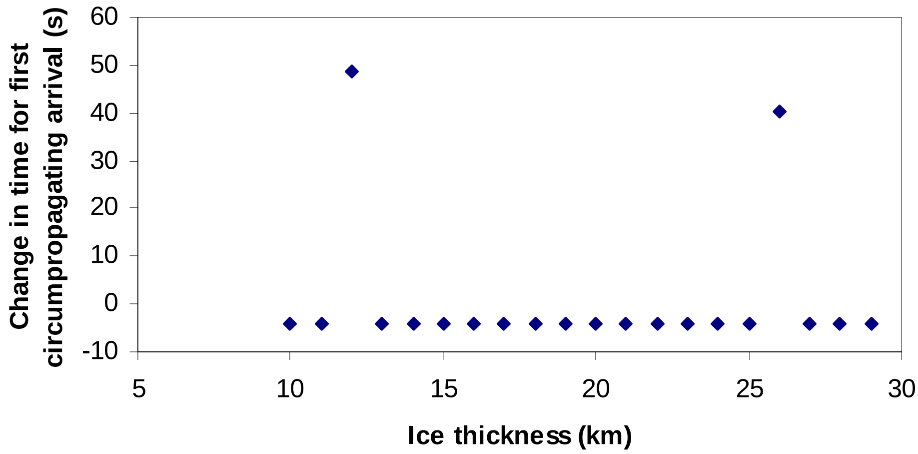

Figure 7. The travel time for the most rapid through-water ray to circumpropagate the moon and return to source, plotted as a function of the ice thickness, using the data of Table 1.

Temperature ($^{\circ}$C)	Number of times the ray grazes the seafloor during one circum-propagation	Launch angle (degrees)	Travel time (mins)	Travel time (s)	Distance (Mm)
0	18	34.13	101.34	6080.4	9.8972
1	18	34.07	101.05	6063	9.8955
2	17	35.97	100.05	6003	9.9272
3	17	35.90	99.79	5987.4	9.9252
4	17	35.84	99.53	5971.8	9.9235
5	17	35.78	99.29	5957.4	9.9218
6	17	35.72	99.05	5943	9.9201
7	17	35.66	98.81	5928.6	9.9184
8	17	35.60	98.59	5915.4	9.9168
9	17	35.55	98.37	5902.2	9.9153
10	17	35.49	98.16	5889.6	9.9137

Table 2. Modelling Europa as a spherical world with uniform material properties (for a fixed sea floor (R_{bed} = 1440800 m), external radius (R_{outer} =1560800 m), and for fixed position of the water/ice interface (R_{ice} =1534000 m) and a fixed ice thickness of 26.8 km)), the propagation characteristics are plotted for the most rapid through-water ray to circumpropagate the moon and return to source. This is done as the uniform ocean temperature varies in increments of 1°C from 0°C to 10 $^{\circ}$C (chosen for the numerical illustration only, and not implying the existence of any particular ocean temperature on Europa).

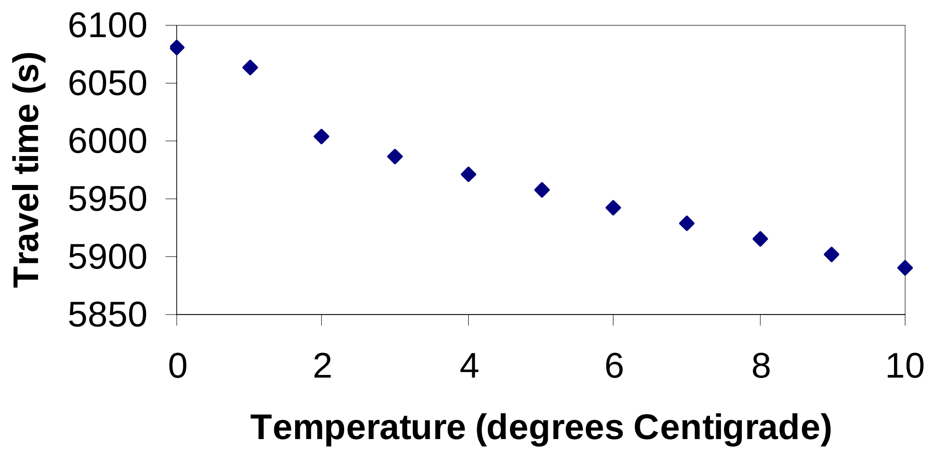

Figure 8. The travel time for the most rapid through-water ray to circumpropagate the moon and return to source, plotted as a function of the ocean temperature, using the data of Table 2.

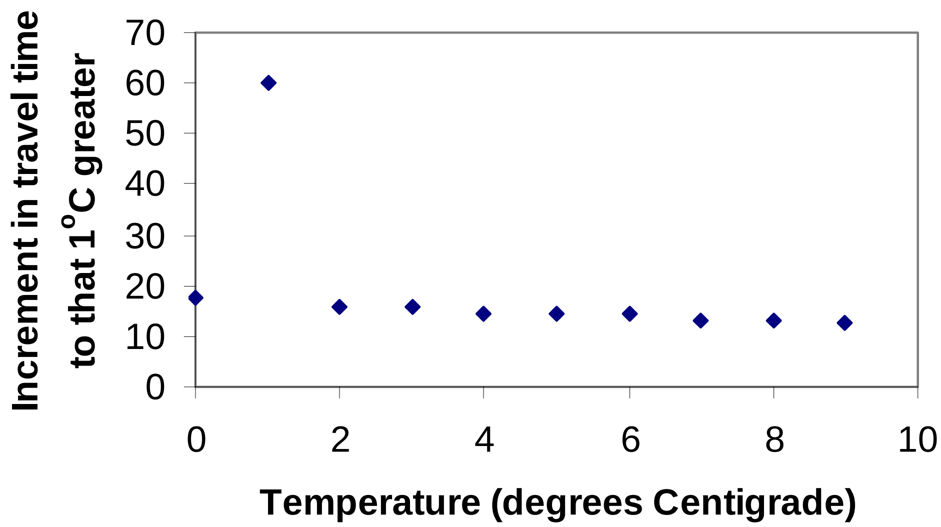

Figure 9. For each ocean temperature cited on the abscissca, the value on the mantissa shows the change in travel time (for the most rapid through-water ray to circumpropagate the moon and return to source) between the travel time at the cited temperature, and the travel time at a temperature of 1 ° C greater. Data obtained from Table 2.

(b) The effect of the ocean temperature

Table 1 plots the propagation characteristics for the most rapid through-water ray to circumpropagate the moon and return to source (modelling Europa as a spherical world with uniform material properties, including a fixed sea floor (R_{bed} = 1440800 m), external radius (R_{outer} =1560800 m), and a fixed position for the water/ice interface (R_{ice} =1534000 m)). This is done as the uniform ocean temperature varies in increments of 1 $^\circ$C from 0 $^\circ$C to 10 $^\circ$C (chosen for the numerical illustration only, and not implying the existence of any particular ocean temperature on Europa).

A ‘toggle point’ occurs between 1 $^\circ$C and 2 $^\circ$C (Figure 8), in that a change occurs in the number of bounces required to circumpropagate the moon and return to source. The qualitative change in the propagation pattern generates an effect which is proportionally much greater than would occur away from a toggle point (Figure 9).

This example illustrates how a change in the qualitative feature in the acoustic propagation can greatly increase the change in an observable far beyond any presupposed linear relationship. Furthermore it points to ways in which such observables could be exploited to provide more sensitive measurements of the ocean environment. Whilst the heterogeneity and bathymetry of the ice seas of both Europa and Earth are vastly more complicated than those assumed in the simple model of this study, the existence of toggle points could provide useful indicators of environmental change even at shorter ranges than those required for circumpropagation.

4 Conclusions

This paper uses the methods of generating sound speed profiles for Europa described by Leighton *et al*. (2008) to consider the paths of those acoustic rays which propagate through the water on Europa, but which only interact with the ice-water interface. The model of the moon itself is very much simplified, in that the moon is assumed to be spherically symmetric, and is divided up into three types of material (water, ice and other material), such that at any given radius only one type of material occurs, and it has uniform density and temperature wherever that material occurs on the moon.

To be specific, this report sets up a simplified model of Europa's ocean, and uses it to demonstrate how the water parameters can affect the propagation, such that inversion of measured acoustic propagation could be used to estimate or monitor the values of those parameters. This is specifically demonstrated with a simple model which assumes uniform temperature throughout the ocean, for the case of propagation of sound around the planet back to the source. Whilst of course this differs from the likely oceanic conditions on Europa, and from the likely experimental set-up, it serves to demonstrate the sensitivity of the arrival times to oceanic characteristics (the example used here is of the ocean temperature and of the position of the water/ice interface).

Taking the somewhat artificial example of circumpropagation (which provides clear goals and end-points for the modelling), the time taken for the earliest through-water acoustic arrival at the position of the source was shown to be an indicator of certain ocean parameters, given knowledge of others. Two specific cases were shown, specifically that of determining ocean temperature if the depth of the seabed and ice were known, and determining the position of the water/ice interface if the ocean temperature and position of the seabed were known. It is important to recall that the scope of any equipment which is eventually placed on Europa will be limited, so that whilst in principle it is possible to determine far more from long-range propagation than these calculations suggest, equipment limitations will restrict the observations. For example, whilst the angle of propagation of each ray in the ocean could be determined using a hydrophone array in the water column, such a deployment would be extremely difficult. This study has restricted itself to simple direct observables, and if the limited equipment payload for Europa is to be optimised, quantitative studies of what can be achieved with limited resources are an important component of planning.

This study also showed that small changes to the ocean environment can produce qualitative changes to the ray path, and thereby generate proportionally much greater changes in the acoustic arrival time. This demonstrates that simple linear scaling is not necessarily applicable, which has implications for the care which must be used when discussing the accuracy of the models required (as discussed in Section 3.1).

It is of course entirely appropriate therefore to question the value of this current study, given the extreme simplifications involved in the current model. The results shown here are intended to be indicative[2], but that does not make them without value. The moon will depart from spherical symmetry, and from the simple model of three homogeneous material types, each material having an unvarying density and temperature wherever it occurs. However the modelling in Tables 1 and 2 is calculated by considering single arcs, and multiplying the effect to obtain results for circumpropagation: departures in spherical symmetry would be revealed by arc-to-arc variations, if sufficient sensors could be deployed to detect these, and could provide a valuable diagnostic tool for the ocean environment.

In similar vein, the example of circumpropagation is somewhat artificial: a sound source which is sufficiently loud to generate a signal that is detectable above the noise and reverberation after circumventing the planet would, in practice, have resulted from such an energetic event that the measurement site itself could conceivably have been destroyed. However a focus point for sound at the moon's point diametrically opposite to the source would occur in the model used here, and in the idealised model the contribution to attenuation caused by geometrical spreading losses will be reversed as signals converge towards the source or its diametric pole.

Another simplification in this model is found in the simple specular reflections from the sea-ice interface, and the lateral homogeneity of the sound speed profile. These are not likely to be found in Europa (the base of the ice sheet will be shaped and may contain bubbles). However the calculations undertaken in this paper were not done to provide accurate predictions for experimental conditions on Europa, but rather to show the trends and potentials for through-water propagation on that moon. If round-planet propagation can be predicted, then shorter ranges which are nevertheless sufficiently great for meaningful inversions, are feasible. Extension of such long-range propagation techniques to include seismic waves in the ice and mantle, and application to other bodies, offer promising opportunities for research.

[2] Repeated calculations, setting up the numerical scheme in different independent ways, showed that the toggle points in Tables 1 and 2 could shift by ± 1 km or $\pm 1^{\circ}$ C. However the object of this study is to draw attention to their existence and trends, not to the exact values of the output.

5 Appendix

This Appendix provides numerical examples to illustrate the assertion that the ray which first circumpropagates the ocean is that which just grazes the seabed. Consider the three rays shown in Table 3, which records the case when R_{ice}=1540.8 km, and R_{bed}=1440.8 km, an ocean temperature of T=0°C. Those rays are launched at angles below the horizontal of 36.1°, 34.1° and 37.5°.

Launch angle	Number of Bounces	Time (Mins)	Distance (Mm)
36.1	17	101.07	9.975
34.1	18	101.82	9.940
37.5	18 (36 “half” bounces)	102.21	10.110

Table 3. Propagation characteristics for three rays for launched at angles below the horizontal of 36.1°, 34.1° and 37.5°, for R_{ice}=1540.8 km, and R_{bed}=1440.8 km, an ocean temperature of T=0°C.

The ray launched at 36.1° would, by the standards used in this report, be taken as being the largest launch angle that completes an integer number of bounces. Of course the ray approach is an inexact analogy for sound propagation, but we may consider this deepest ray as indicating the approximate position of the deepest part of the wavefront to complete circumpropagation without reflecting off the seabed. All calculations in this report are of course approximate because of the limitations of numerical accuracy in such calculations (see footnote 2).

We then explore the properties of the next deepest, and next most shallow, rays to complete circumpropagation once using an integer number of bounces.

The ray launched at 34.1° would be, by the manner of calculation used in this report, the next ray which requires an integer number of bounces (i.e. the one which would be deepest were it not for the ability of the 36.1° ray to complete circumpropagation in an integer number of bounces without reflecting off the seabed). It takes longer to circumpropagate because its path avoids the deeper waters where the sound speed is greatest. Note the 34.1° ray takes a shorter path but through a slower part of the ocean, compared to the 36.1° ray.

The ray launched at 37.5° reflects from the seabed and requires an integer number of complete bounces (a complete bounce is when the ray returns to the surface). Its ’18 bounces’ to complete circumpropagation may be thought of as consisting of 36 half-bounces, a half-bounce consisting of a path between ice and seabed, or vice versa.

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