

Modular-Invariant Random Matrix Theory and AdS_3 Wormholes

Jan Boruch 

Leinweber Institute for Theoretical Physics and Department of Physics, University of California, Berkeley, California 94720, USA

Gabriele Di Ubaldo 

*Leinweber Institute for Theoretical Physics and Department of Physics, University of California, Berkeley, California 94720, USA;
Interdisciplinary Theoretical and Mathematical Sciences Program (iTHEMS), RIKEN, Wako 351-0198, Japan;
and Université Paris-Saclay, CNRS, CEA, Institut de Physique Théorique, 91191, Gif-sur-Yvette, France*

Felix M. Haehl 

School of Mathematical Sciences and STAG Research Centre, University of Southampton, Southampton, SO17 1BJ, United Kingdom

Eric Perlmutter 

*Université Paris-Saclay, CNRS, CEA, Institut de Physique Théorique, 91191 Gif-sur-Yvette, France
and Institut des Hautes Études Scientifiques, 91440 Bures-sur-Yvette, France*

Moshe Rozali 

Department of Physics and Astronomy, University of British Columbia, Vancouver V6T 1Z1, Canada



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We develop a nonperturbative definition of RMT_2 : a generalization of random matrix theory that is compatible with the symmetries of two-dimensional conformal field theory. Given any random matrix ensemble, its n -point spectral correlations admit a prescribed modular-invariant lift to RMT_2 , which moreover reduce to the original random matrix correlators in a near-extremal limit. Central to the prescription is a presentation of random matrix theory in Mellin space, which lifts to two dimensions via the $\text{SL}(2, \mathbb{Z})$ spectral decomposition employed in previous work. As a demonstration we perform the explicit RMT_2 lift of two-point correlations of the GUE Airy model. We propose that in AdS_3 pure gravity, semiclassical amplitudes for off-shell n -boundary torus wormholes with topology $\Sigma_{0,n} \times S^1$ are given by the RMT_2 lift of JT gravity wormhole amplitudes. For the three-boundary case, we identify a gravity calculation which matches the RMT_2 result.

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Introduction—Holographic duality for theories of two-dimensional gravity provides strong evidence that random matrix universality constrains the spectrum of holographic quantum systems [1–4]. The importance of two-dimensional conformal field theories (CFTs), dual to quantum gravity in AdS_3 , for constructing a more robust holographic dictionary for high-energy states motivates the search for an inherently two-dimensional CFT definition of random matrix universality. Conversely, one seeks a framework to uplift any given random matrix theory (RMT) to 2D CFT in a manner that respects modular and conformal invariance. Following [5], we shall refer to such a framework as RMT_2 .

Just as RMT universally quantifies the spectral statistics of chaotic many-body systems in view of the

Bohigas–Giannoni–Schmit conjecture [6,7], RMT_2 should likewise describe the spectra of irrational 2D CFTs in sufficiently high-energy regimes. From a holographic perspective, RMT_2 is expected to quantify the spectral statistics of primary operators dual to black hole states above extremality, generalizing arguments given in [4,8] and built upon in [9,10].

The construction of RMT_2 was initiated in [5,11–13], which focused on the 2D CFT avatar of a central hallmark of random matrix universality, namely, the “linear ramp” of the spectral form factor (SFF). In this Letter, we provide a more complete definition of RMT_2 : a prescription to uplift n -point RMT correlators to modular-invariant objects, without restricting to any regime of times or energies, which moreover reduce to the original RMT correlators in an appropriate near-extremal limit. The two-point case furnishes a modular-invariant SFF preserving a linear ramp and plateau structure. Further details and elaboration will be given in [14].

RMT_2 —We now describe a two-step procedure, which starts with a given RMT and constructs a corresponding

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modular-invariant RMT_2 . Consider the n -point spectral form factor of a given matrix Hamiltonian,

$$K_{\text{RMT}}^{(n)}(\{y_i\}) = \langle \text{Tr}(e^{-y_1 H}) \cdots \text{Tr}(e^{-y_n H}) \rangle_c \quad (1)$$

where $\langle \cdot \rangle_c \equiv Z^{-1} \int dH(\cdot) e^{-N \text{Tr}(H)}$ is the connected matrix integral and y_i are inverse temperatures. The first step consists of computing the Mellin transform:

$$f_{\text{RMT}}^{(n)}(\{\omega_i\}) \equiv \mathcal{M}[K_{\text{RMT}}^{(n)}(\{y_i\})](-i\omega_1, \dots, -i\omega_n), \quad (2)$$

where $\mathcal{M}[K(y)](-i\omega) = \int_0^\infty dy y^{-1-i\omega} K(y)$. This quantity defines *RMT in Mellin space*.

The second step, the conceptual core of the prescription, is to regard the Mellin transform (2) as a spectral overlap in a modular-invariant eigenbasis. The RMT_2 spectral n -point function on n tori with modular parameters $\tau_i := x_i + iy_i$ is defined as follows:

$$\begin{aligned} \mathcal{Z}_{\text{RMT}_2}^{(n)}(\{\tau_i\}) &\equiv \int_{\mathbb{R}^n} \left(\prod_{i=1}^n \frac{d\omega_i}{(2\pi)} E_{\frac{1}{2}+i\omega_i}(\tau_i) \right)_{\text{sym}} f_{\text{RMT}}^{(n)}(\{\omega_i\}) \\ &+ [\text{zero modes}] + [\text{cusp forms}], \end{aligned} \quad (3)$$

where “sym” denotes symmetrization over $\{\tau_i\}$, and

$$E_s(\tau) = \sum_{j \geq 0} (2 - \delta_{j,0}) a_j^{(s)} \cos(2\pi j x) \sqrt{y} K_{s-\frac{1}{2}}(2\pi j y) \quad (4)$$

are nonholomorphic Eisenstein series with spin- j Fourier coefficients $a_j^{(s)}$ left implicit. On the critical line $s \equiv \frac{1}{2} + i\omega \in \mathcal{C}_{\text{crit}}$ with $\omega \in \mathbb{R}$, the Eisenstein series span the continuous part (“scattering states”) of the spectrum of the Laplacian on the fundamental domain $\mathcal{F} = \mathbb{H}/\text{PSL}(2, \mathbb{Z})$ with eigenvalues $s(1-s)$.

The terms in (3) denoted as [cusp forms] refer to the contribution of Maass cusp forms $\phi_n(\tau)$, an infinite set of eigenfunctions which spans the discrete part of the eigen-spectrum (“bound states”). Maass cusp forms are crucial for describing RMT statistics of the spinning spectrum [12] but have no scalar Fourier mode. In this Letter we focus on the Eisenstein sector of RMT_2 , which in turn fully determines the scalar sector.

The terms in (3) denoted as [zeromodes] contain the part of the spectral decomposition that involves the trivial eigenfunction $\phi_0 = \text{constant}$. Because the Eisenstein series has a pole at $s = 1$ with constant residue $\text{vol}(\mathcal{F})^{-1}$, spectral overlaps with the constant function may be obtained by taking iterated residues of the first line of (3) at $\omega_i = 1/2i$: for the n -point correlator, taking m such residues generates terms with $n - m$ Eisenstein factors. For example, the case $n = 2$ is

$$\begin{aligned} \mathcal{Z}_{\text{RMT}_2}^{(2)}(\tau_1, \tau_2) &= \frac{1}{\text{vol}(\mathcal{F})^2} f_{\text{RMT}}^{(2)}\left(\frac{1}{2i}, \frac{1}{2i}\right) \\ &+ \frac{1}{\text{vol}(\mathcal{F})} \int_{\mathbb{R}} \frac{d\omega_1}{2\pi} f_{\text{RMT}}^{(2)}\left(\omega_1, \frac{1}{2i}\right) [E_{s_1}(\tau_1) + E_{s_1}(\tau_2)] \\ &+ \int_{\mathbb{R}^2} \frac{d\omega_1 d\omega_2}{(2\pi)^2} f_{\text{RMT}}^{(2)}(\omega_1, \omega_2) [E_{s_1}(\tau_1) E_{s_2}(\tau_2)]_{\text{sym}} \\ &+ \left[\begin{array}{c} \text{cusp} \\ \text{forms} \end{array} \right]. \end{aligned} \quad (5)$$

As we will see below, following other CFT contexts for the “standard” case $n = 1$ [15], these zero mode terms will in fact be canceled in the genus expansion of RMT_2 .

The two-step procedure that defines RMT_2 yields a modular-invariant “lift” of RMT. Conversely, a crucial property of RMT_2 is that the *near-extremal limit* effectively reduces $E_{s_i}(\tau) \rightarrow \sqrt{y_i} y_i^{i\omega_i}$ in the integral (3): this turns (3) into an inverse Mellin transform, and $\mathcal{Z}_{\text{RMT}_2}^{(n)}(\{\tau_i\})$ reduces to the original $K_{\text{RMT}}(\{y_i\})$,

$$\mathcal{Z}_{\text{RMT}_2}^{(n)}(\{\tau_i\}) \longrightarrow \sqrt{y_1 \cdots y_n} K_{\text{RMT}}^{(n)}(\{y_i\}), \quad (6)$$

where the factor $\sqrt{y_1 \cdots y_n}$ arises because we consider modular-invariant CFT partition functions counting primary states only [16]. The near-extremal limit (for any topology) turns out to be the same as in [17], where one rescales uniformly $y_i \rightarrow \gamma y_i$ and takes $\gamma \rightarrow \infty$. Modular corrections being subleading near extremality are a non-trivial property of RMT_2 , necessary for the self-consistency of the uplift.

Note that we can equivalently write the Eisenstein sector of (3) as

$$\begin{aligned} \mathcal{Z}_{\text{RMT}_2}^{(n)}(\{\tau_i\}) &\supset \langle \text{Tr}(e^{-H}|_{E(\tau_1)}) \cdots \text{Tr}(e^{-H}|_{E(\tau_n)}) \rangle_c \\ e^{-H}|_{E(\tau)} &\equiv \int_{\mathbb{R}} \frac{d\omega}{2\pi} \Gamma(-i\omega) H^{i\omega} E_s(\tau). \end{aligned} \quad (7)$$

This is mathematically equivalent since $\mathcal{M}[e^{-y\lambda}](-i\omega) = \Gamma(-i\omega) \lambda^{i\omega}$ for every eigenvalue $\lambda \in \text{spec}(H)$, but this formulation makes it manifest that the Eisenstein part of RMT_2 is in fact still a matrix integral.

Warmup: AdS₃ wormhole and linear ramp: The simplest prediction of RMT is the leading approximation to the two-point function:

$$K_{\text{RMT}}^{(0,2)}(y_1, y_2) = \frac{\mathcal{C}_{\text{RMT}}}{2\pi} \frac{\sqrt{y_1 y_2}}{y_1 + y_2}, \quad (8)$$

where the constant $\mathcal{C}_{\text{RMT}}$ encodes the RMT universality class (e.g., $\mathcal{C}_{\text{GUE}} = 1$, $\mathcal{C}_{\text{GOE}} = 2$). This contains the linear ramp of the SFF at late Lorentzian times, plus an infinite set

of corrections that resum to the full double-scaled RMT result [2].

In Mellin space,

$$f_{\text{RMT}}^{(0,2)}(\omega_1, \omega_2) = \frac{\mathcal{C}_{\text{RMT}}}{2 \cosh(\pi\omega_1)} \times \pi \delta(\omega_1 + \omega_2). \quad (9)$$

Via (5), this defines the simplest universal contribution to RMT_2 , a modular-invariant completion of (8). To make this more explicit, we recall the result of [4], where the $\mathbb{T}^2 \times I$ wormhole amplitude in AdS_3 pure gravity was found to be

$$\mathcal{Z}_{\text{AdS}_3}^{(0,2)}(\tau_1, \tau_2) = \frac{\mathcal{C}_{\text{RMT}}}{4\pi^2} \sum_{\gamma \in \text{SL}(2, \mathbb{Z})} \frac{\text{Im}(\tau_1) \text{Im}(\gamma\tau_2)}{|\tau_1 + \gamma\tau_2|^2}. \quad (10)$$

In $\text{SL}(2, \mathbb{Z})$ spectral space [5],

$$\begin{aligned} \mathcal{Z}_{\text{AdS}_3}^{(0,2)}(\tau_1, \tau_2) &= \int_{\mathbb{R}} \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} f_{\text{RMT}}^{(0,2)}(\omega_1, \omega_2) E_{s_1}(\tau_1) E_{s_2}(\tau_2) \\ &+ \sum_{n_1, n_2} f_{\text{RMT}}^{(0,2)}(\omega_{n_1}, \omega_{n_2}) \phi_{n_1}(\tau_1) \phi_{n_2}(\tau_2), \end{aligned} \quad (11)$$

where in the cusp form sector one replaces $\pi\delta(\omega_1 + \omega_2) \rightarrow \delta_{n_1, n_2}$. This takes precisely the form of an RMT_2 amplitude. This example illustrates how the RMT_2 formalism reveals essential features of the gravity amplitude [5,13], such as the encoding of the linear ramp in the simple condition $f_{\text{RMT}}^{(0,2)}(\omega_1, \omega_2) \sim \delta(\omega_1 + \omega_2) e^{-\pi\omega_1}$ for large $|\omega_i|$, and the amplitude being the diagonal approximation (*à la* Berry [18]) to a CFT trace formula. In what follows we go “beyond the ramp” by performing the RMT_2 lift of full RMT correlators.

Paradigmatic example: Airy RMT_2 —As a natural starting point for demonstration, we study the topological expansion for the simplest instance of RMT_2 : the lift of the Airy model in the gaussian unitary ensemble (GUE) universality class. The RMT is defined by the spectral density

$$\rho(E) = \rho_0(E) e^{-S_0}, \quad \rho_0(E) = \frac{1}{2\pi} \sqrt{E}, \quad (12)$$

where S_0 is a large parameter. The associated two-point SFF is known exactly [19,20]:

$$K_{\text{Airy}}^{(2)}(y_1, y_2) = \frac{e^{S_0 + \frac{\rho^3}{3}} e^{-2S_0}}{4\sqrt{\pi}(2\beta)^{3/2}} \text{Erf} \left[e^{-S_0} \sqrt{2\beta(\beta^2 + T^2)} \right], \quad (13)$$

where we analytically continued $y_{1,2} = \beta \pm iT$. The prefactor of the error function is the asymptotic “plateau” $\langle Z_{\text{Airy}}(2\beta) \rangle$ at $T \rightarrow \infty$ (with β and S_0 fixed).

For the purposes of this Letter we simplify matters by performing the lift of a “simplified” model in which we drop the doubly exponential prefactor (the full GUE Airy

model will be discussed in [14]). This is equivalent to lifting the Airy model in the τ -scaling limit [19,21] which captures the ramp-plateau transition,

$$T \rightarrow \infty \quad \text{with} \quad \tau \equiv T e^{-S_0} \quad \text{fixed}. \quad (14)$$

To take this limit, we rescale (13) by e^{-S_0} and observe $\beta^2 + T^2 \sim e^{2S_0} \tau^2$.

The RMT_2 lift of the τ -scaled GUE Airy model is performed in the two steps prescribed above. First, the Mellin space formulation of the τ -scaled Airy model is

$$\begin{aligned} f_{\tau\text{-Airy}}^{(2)}(\omega_1, \omega_2) &= \frac{e^{-2i\omega_+ S_0}}{6\pi(\omega_+ + i\epsilon)(2\omega_+ - i)} \\ &\times \Gamma\left(\frac{1}{2} - \frac{i\omega_+}{2} + \frac{3i\omega_-}{2}\right) \\ &\times \Gamma\left(\frac{1}{2} - \frac{i\omega_+}{2} - \frac{3i\omega_-}{2}\right), \end{aligned} \quad (15)$$

where we use the convenient basis of $\omega_{\pm} = \frac{1}{3}(\omega_1 \pm \omega_2)$, and $\epsilon \rightarrow 0$ is a regulator of the pole $\omega_+ = 0$. This then defines the (Eisenstein sector of the) RMT_2 lift, written using $\omega_{\pm}$:

$$\begin{aligned} \mathcal{Z}_{\tau\text{-Airy}}^{(2)}(\{\tau_i\}) &= \frac{9}{8\pi^2} \int d\omega_{\pm} f_{\tau\text{-Airy}}^{(2)}(\{\omega_i\}) [E_{s_1}(\tau_1) E_{s_2}(\tau_2)]_{\text{sym}} \\ &+ \left[\text{zero} \right]_{\text{modes}}. \end{aligned} \quad (16)$$

We suppress the cusp forms, discussed further in [14].

The topological expansion of the Airy model is encoded in the analytic structure of the overlaps (15): in particular, poles correspond to fixed-genus contributions. To illustrate this, we imagine performing the inverse Mellin transform to retrieve the SFF $K_{\tau\text{-Airy}}^{(2)}$, i.e., the integrals (16) with $E_{s_i}(\tau_i) \rightarrow \sqrt{y_i} y_i^{i\omega_i}$. One can start with the ω_- integral by closing the contour in either the upper or lower half complex plane. This picks up an infinite series of residues from the Γ functions, which resum into

$$\begin{aligned} e^{-S_0} K_{\tau\text{-Airy}}^{(2)}(y_1, y_2) &= \frac{1}{4\pi^2} \int_{\mathbb{R}+i\epsilon} d\omega_+ \frac{(y_1 y_2)^{\frac{1}{2}+i\omega_+} (y_1 + y_2)^{i\omega_+-1} \Gamma(-i\omega_+)}{e^{(1+2i\omega_+)S_0} (1+2i\omega_+)}. \end{aligned} \quad (17)$$

The exponential factor implies convergence of the integral as $\text{Im}(\omega_+) \rightarrow -\infty$; we can therefore perform the ω_+ integral via residues in the lower half complex plane. Relevant poles are located at $\omega_+^{(g)} = -ig$ for $g = 0, 1, 2, \dots$,

and the g th pole is suppressed by $e^{-(2g+1)S_0}$: one can check that its residue indeed produces the genus g term in the topological expansion of the τ -scaled Airy model, scaling as $\beta^{g-1}\tau^{2g+1}$.

We stress a noteworthy aspect of this expansion: *the higher-genus terms are increasingly “off-diagonal”*, localized at $\omega_1 + \omega_2 = -3ig$. Note that the $g = 0$ pole, on the diagonal, precisely yields the double-scaled RMT result (8), whose diagonality in Mellin space was previously understood as a CFT₂ avatar of Berry’s approximation in periodic orbit theory [5,18].

Having defined the RMT₂ lift (16) of the τ -scaled Airy model, we can leverage our understanding of its analytic structure to develop the *modular-invariant genus expansion*, thus revealing its inherently two-dimensional nature via modular corrections. This expansion takes the form

$$\begin{aligned} \mathcal{Z}_{\tau\text{-Airy}}^{(2)}(\tau_1, \tau_2) &= \mathcal{Z}^{(0,2)}(\tau_1, \tau_2) \\ &+ \sum_{g=1}^{\infty} \frac{(-1)^g e^{-2gS_0}}{2\pi g(2g+1)} \\ &\times \sum_{k=0}^{g-1} \frac{[E_{2g-k}(\tau_1)E_{g+k+1}(\tau_2)]_{\text{sym}}}{\Gamma(g-k)k!}, \end{aligned} \quad (18)$$

where the genus $g = 0$ contribution $\mathcal{Z}^{(0,2)}$ is universal, i.e., identical to (the Eisenstein part of) (11).

This modular-invariant topological expansion has several interesting features. One of them is the appearance of integer-index Eisenstein series with index bounded above by the genus; in this context, note that replacing $E_n(\tau_i) \rightarrow y_i^{n-1/2}$ in the second line of (18) reproduces exactly the topological expansion of the τ -scaled Airy RMT (13). Another feature is the asymptotic character of the sums (18) at late time: a nontrivial resummation ensures that the late-time plateau in the τ -scaling limit of RMT is preserved in RMT₂ despite the modular corrections at every genus g being larger than the original RMT terms at genus $g + 1$. The τ -scaled limit of the Airy RMT (13) is thus recovered,

$$\lim_{\substack{T \rightarrow \infty \\ \tau \text{ fixed}}} e^{-S_0} \mathcal{Z}_{\tau\text{-Airy}}^{(2)}(\tau_1, \tau_2) = \langle Z_{\text{Airy}}(2\beta) \rangle \text{Erf}(\tau\sqrt{2\beta}), \quad (19)$$

using the analytic continuation $\tau_{1,2} = x_{1,2} + i(\beta \pm iT)$.

Finally, we note that the τ -scaled Airy correlator (15) is paradigmatic for a much larger class of models. In [14] we show that for any spectral density $\rho_0(E)$ with a square root edge the τ -scaled SFF in the GUE ensemble is encoded in

$$f_{\tau\text{-RMT}(\rho)}^{(2)}(\omega_1, \omega_2) = f_{\tau\text{-Airy}}^{(2)}(\omega_1, \omega_2) \times \mathfrak{h}_\rho(\omega_+). \quad (20)$$

In particular, the dependence on ω_- is universal and the way in which the poles encode the genus expansion is structurally identical to the Airy case above.

Application: Off-shell wormholes in AdS₃ pure gravity—By applying this machinery to RMT correlators dual to wormhole amplitudes in two-dimensional gravity, RMT₂ makes predictions for fully connected Euclidean wormhole amplitudes with multiple torus boundaries in AdS₃ gravity. These are off-shell amplitudes, generalizing [4], for three-manifolds $\mathcal{M}_3$ of topology $\Sigma_{0,n} \times S^1$ with trivial fibration and boundary topology $\partial\mathcal{M}_3 = \mathbb{T}^2 \cup \dots \cup \mathbb{T}^2$, the union of n disjoint tori. So far there are no explicit computations or predictions, from either boundary or bulk, for these wormholes with $n > 2$, either for AdS₃ pure gravity or with matter.

Proposal: Our proposal for computing AdS₃ gravity amplitudes for $\Sigma_{0,n} \times S^1$ is simply to plug the appropriate RMT correlators into (2) and compute (3). Let us henceforth focus on AdS₃ pure gravity. To leading order in the semiclassical limit, the proposal is to uplift Jackiw-Teitelboim (JT) gravity amplitudes, i.e. double-scaled RMT correlators with spectral curve $\rho_0(E) = (\gamma/2\pi^2) \sinh(2\pi\sqrt{2\gamma E})$, on topology $\Sigma_{0,n}$:

$$\mathcal{Z}_{\text{AdS}_3}^{(0,n)}(\{\tau_i\}) \propto \mathcal{Z}_{\text{JT-RMT}_2}^{(0,n)}(\{\tau_i\}), \quad (21)$$

where we denote $\mathcal{Z}_{\text{AdS}_3}^{(0,n)}(\{\tau_i\})$ as the pure gravity amplitude on $\Sigma_{0,n} \times S^1$. Moreover, the explicit form of the uplifted JT correlators can be obtained through a simple *replacement rule*, where monomials in the $\{y_i\}$ are replaced by Eisenstein series of appropriate index:

$$\mathcal{Z}_{\text{JT-RMT}_2}^{(0,n)}(\{\tau_i\}) = K_{\text{JT}}^{(0,n)}(\{y_i\}) \Big|_{y_i^a \mapsto E_{\frac{1}{2}+a}(\tau_i)} + \left[\begin{array}{c} \text{cusp} \\ \text{forms} \end{array} \right] \quad (22)$$

with suitable regularization of the $a = 1/2$ case. We derive (22) in the End Matter. The proportionality symbol in (21) signals an unspecified overall τ_i -independent normalization of the gravitational path integral.

This prescription, an n -point instantiation of the MaxRMT proposal [5], is supported by the known emergence of Schwarzian dynamics in the near-extremal limit of 2D CFTs [17,22], the dual emergence of JT dynamics in the dimensional reduction of AdS₃ gravity [8], and the general mechanism of RMT₂ presented in this Letter. As noted earlier, RMT₂ amplitudes contain the RMT seed amplitudes, via the near-extremal limit of $y \propto \gamma$ and $\gamma \rightarrow \infty$; in the dimensional reduction of pure AdS₃ gravity to JT gravity, $\gamma \approx (c/24)$ at large c [8,17,22]. Note that while AdS₃ gravity also contains off-shell wormholes of fixed boundary topology but with “interior” bulk topology, expected to be suppressed by factors exponentially small in G_N , the RMT₂ prescription above gives the leading-order AdS₃ amplitude for a given boundary topology by uplifting the leading-order JT amplitude.

We now demonstrate the RMT_2 prescription for the three-boundary torus wormhole in AdS_3 pure gravity. Because of a special universality of the $n = 3$ case, we are able to perform a heuristic gravity calculation, which is found to match the RMT_2 result. We then describe the RMT_2 lift of JT gravity amplitudes for arbitrary n . We record the explicit results for $n = 4, 5$ in the End Matter.

Three-boundary wormhole from RMT_2 : We start from the three-point RMT correlator, which is universal for any spectral curve $\rho_0(E)$ with a square root edge [23]:

$$K_{\text{RMT}}^{(0,3)}(y_1, y_2, y_3) = \frac{e^{-S_0}}{(2\pi\gamma)^{3/2}} \sqrt{y_1 y_2 y_3}, \quad (23)$$

where the inverse temperatures y_i are measured in units of γ . The Mellin transform of a monomial naively vanishes (e.g. [24]) but admits an ϵ prescription (e.g. Appendix B of [25]). This leads to the spectral overlap

$$f_{\text{RMT}}^{(0,3)}(\omega_1, \omega_2, \omega_3) = \frac{e^{-S_0}}{(2\pi\gamma)^{3/2}} \prod_{i=1}^3 \frac{8\epsilon}{4\epsilon^2 - (1 - 2i\omega_i)^2}, \quad (24)$$

where $\{\omega_i\}$ are constrained to lie on the contours $\text{Im}(\omega_i) = -1/2$. Note that the overlap manifestly factorizes. We now compute the amplitude $\mathcal{Z}_{\text{RMT}_2}^{(0,3)}(\tau_1, \tau_2, \tau_3)$ by plugging this overlap into the spectral integral with the contour specified above. On each boundary we must evaluate the integral (A3), which must be regularized, as the Eisenstein series has a pole at $s = 1$. Using the principal value prescription $\text{PV} \int d\tilde{\omega}/[(\epsilon^2 + \tilde{\omega}^2)\tilde{\omega}] = 0$ leads to the final result:

$$\begin{aligned} \mathcal{Z}_{\text{RMT}_2}^{(0,3)}(\tau_1, \tau_2, \tau_3) &= \frac{e^{-S_0}}{(2\pi\gamma)^{3/2}} \hat{E}_1(\tau_1) \hat{E}_1(\tau_2) \hat{E}_1(\tau_3) \\ &\quad + \left[\begin{array}{c} \text{cusp} \\ \text{forms} \end{array} \right] \end{aligned} \quad (25)$$

where $\hat{E}_1(\tau)$ is the regular part of the Eisenstein series at $s = 1$,

$$\hat{E}_1(\tau) \equiv \lim_{s \rightarrow 1} \left(E_s(\tau) - \frac{3}{\pi(s-1)} \right). \quad (26)$$

We posit that (up to overall normalization) (25) is the AdS_3 pure gravity amplitude for the off-shell three-boundary wormhole $\Sigma_{0,3} \times S^1$. Let us make a few remarks.

First, one should view the principal value prescription as a choice of regularization. While this is a natural choice, more generally there is a possibility of adding a τ_i -independent constant on each boundary component; consistently with our holographic claim, this feature is represented on the gravity side as well (see below).

Second, note that the result does not admit an expansion into a discrete sum over Virasoro characters on each boundary torus, consistent with a coarse-grained interpretation of semiclassical AdS_3 pure gravity in which such amplitudes capture higher moments of dual CFT spectral densities.

Three-boundary wormhole from gravity: We now perform a heuristic gravitational computation of the amplitude for the three-boundary wormhole with topology $\Sigma_{0,3} \times S^1$, utilizing certain simplifications that occur for $n = 3$. We reiterate that this is not a first principles bulk computation, which we understand is being pursued elsewhere [26]. See also the Supplemental Material [27] for more details.

Let us first illustrate the calculation by analogy with the two-boundary wormhole. After summing the seed amplitude in (10) over T transformations $\tau \rightarrow \tau + n$, the resulting seed $g_{\text{AdS}_3}^{(0,2)}(\tau_1, \tau_2)$ can be written as

$$\begin{aligned} \frac{g_{\text{AdS}_3}^{(0,2)}(\tau_1, \tau_2)}{\sqrt{y_1 y_2}} &= 2 \frac{\sqrt{y_1 y_2}}{|\tau_1 \tau_2|} \sum_{j \in \mathbb{Z}} \int_0^\infty dP_1^2 dP_2^2 d\bar{P}_1^2 d\bar{P}_2^2 \\ &\quad \times |Z_{\text{Tr}}(\tau_1, P_1) Z_{\text{Tr}}(\tau_2, P_2)|^2 V_{0,2}^{(j)}(\{P_i\}), \end{aligned} \quad (27)$$

where $Z_{\text{Tr}}(\tau, P)$ is the ‘‘chiral trumpet’’ ($S\tau \equiv -1/\tau$)

$$Z_{\text{Tr}}(\tau, P) \equiv \eta(\tau) \chi_P(S\tau), \quad \chi_P(\tau) = \frac{e^{2\pi i \tau P^2}}{\eta(\tau)}, \quad (28)$$

which reduces to the well-known two-dimensional trumpet by setting $\tau = i\beta$ [4, 28–33]. The trumpets are glued together along geodesic lengths P_1, P_2 by a spin-quantized generalization of the Weil-Petersson volumes:

$$V_{0,2}^{(j)} = \delta(P_1^2 - P_2^2) \delta(\bar{P}_1^2 - \bar{P}_2^2) \delta(P_1^2 - \bar{P}_1^2 - j). \quad (29)$$

The Eisenstein part of the two-boundary wormhole is obtained by considering only the $j = 0$ term and summing over $\text{SL}(2, \mathbb{Z})$. The overall factor is the symplectic volume form on the wormhole moduli space [4]:

$$\sqrt{|\Omega(S\tau_1, S\tau_2)|} = \sqrt{\text{Im}(S\tau_1) \text{Im}(S\tau_2)} = \frac{\sqrt{y_1 y_2}}{|\tau_1 \tau_2|}. \quad (30)$$

This motivates the following gravitational ansatz for the Eisenstein sector of the three-boundary wormhole:

$$\begin{aligned} \frac{g_{\text{AdS}_3}^{(0,3)}(\{\tau_i\})}{\sqrt{y_1 y_2 y_3}} &\supset \sqrt{|\Omega(S\tau_1, S\tau_2, S\tau_3)|} \\ &\quad \times \left| \int_0^\infty dP_1^2 dP_2^2 dP_3^2 Z_{\text{Tr}}(\tau_1, P_1) \right. \\ &\quad \left. \times Z_{\text{Tr}}(\tau_2, P_2) Z_{\text{Tr}}(\tau_3, P_3) V_{0,3}(\{P_i\}) \right|^2. \end{aligned} \quad (31)$$

The object $g_{\text{AdS}_3}^{(0,3)}(\{\tau_i\})$ is defined as the seed of a triple Poincaré sum over $\text{SL}(2, \mathbb{Z})/\Gamma_\infty$, one on each boundary. The prefactor is taken to be $\sqrt{|\Omega(\tau_1, \tau_2, \tau_3)|} = \sqrt{y_1 y_2 y_3}$ in analogy with the two-boundary case. The volume $V_{0,3}(\{P_i\})$ is taken to be the Weil-Petersson volume of a three-holed sphere, namely, $V_{0,3} = 1$. In the Supplemental Material [27] we discuss this choice further; we only mention here that $V_{0,3} = 1$ was obtained explicitly in [34] as a regularized volume within Virasoro topological QFT (TQFT), using a special property of the three-punctured case.

Combining these ingredients, the above expression yields the simple result $g_{\text{AdS}_3}^{(0,3)}(\{\tau_i\}) \supset y_1 y_2 y_3 / (2\pi)^6$. Summing over modular images on each boundary gives the gravity amplitude:

$$\mathcal{Z}_{\text{AdS}_3}^{(0,3)}(\tau_1, \tau_2, \tau_3) \supset \prod_{i=1}^3 \left(\sum_{\gamma \in \text{SL}(2, \mathbb{Z})/\Gamma_\infty} \text{Im}(\gamma \tau_i) \right), \quad (32)$$

where we have dropped the overall constant normalization. Each sum is the Poincaré series representation of the Eisenstein series $E_s(\tau_i)$ at $s = 1$. As this is singular, it must be regularized; this is precisely the same freedom of regularization scheme that appears on the RMT_2 side. A minimal scheme is to subtract off the pole. This yields a match with (25).

n-boundary wormholes from RMT_2 : One can perform the RMT_2 lift of JT gravity amplitudes for arbitrary n . For all $n > 3$, the amplitude $\mathcal{Z}_{\text{JT-RMT}_2}^{(0,n)}$ is a polynomial in Eisenstein series, as governed by the replacement rule (22). As in the $n = 3$ case, it does not admit an expansion into a discrete sum over Virasoro characters on the n boundary tori, and reduces to $K_{\text{JT}}^{(0,n)}$ in the near-extremal limit $y \propto \gamma$ and $\gamma \rightarrow \infty$. The microcanonical spectral statistics of the dual CFT_2 follow from these results via inverse Laplace transform.

We note that for $n > 3$, the most naive extension of the heuristic gravity calculation above does not match the RMT_2 result. This is as expected, from considerations of the $n > 3$ bulk moduli space. A bona fide AdS_3 pure gravity calculation of the n -point amplitudes would be of clear value in learning about random statistics of AdS_3 black hole microstates: either AdS_3 pure gravity does indeed furnish the minimal completion of random matrix statistics, ratifying the proposal (21)–(22); or, perhaps pure gravity is richer than that, containing additional structure that appears only in higher-point correlations.

Outlook—In this Letter we presented a prescription that lifts correlation functions in arbitrary random matrix theories to two-dimensional modular-invariant form factors. We presented the GUE Airy model as the simplest example, and used the formalism to predict multiboundary off-shell wormhole amplitudes in AdS_3 pure gravity. In [14] we will

discuss more involved examples as well as general spectral curves, topological recursion, other universality classes such as the GOE ensemble, and late-time dynamics. We will discuss the cusp form part of RMT_2 and RMT statistics at fixed spin. We will also elaborate on the embedding of RMT_2 into CFT_2 , and on the novel constraints imposed by RMT_2 on the spectra of chaotic CFT_2 .

We recall a different approach toward random matrix universality in CFTs: the matrix-tensor model of [9,10], which describes an ensemble of CFT data which *approximately* solves the CFT bootstrap constraints up to some tolerance. One awaits fully explicit calculations of correlators in that model; these would enable quantitative comparison to both RMT_2 and semiclassical gravity calculations of off-shell wormhole amplitudes. Our perspective suggests that, upon integrating out the tensor degrees of freedom (which encode the operator product expansion (OPE) dynamics of the CFT), the resulting spectral matrix model lies within an RMT_2 universality class after taking a suitable limit of vanishing tolerance. This would be very interesting to pursue.

We end with a comment on the broader point of our proposal. Lacking explicit examples of irrational chaotic CFTs, as well as a first-principles gravity calculation for multiboundary wormholes, we wish to emphasize that RMT_2 predictions should be independently viewed as a way to benchmark chaos in any modular-invariant theory. This rationale is similar to the bootstrap program: RMT_2 encapsulates the constraints that any modular-invariant form factor needs to satisfy if it is to be consistent with random matrix universality. This follows from the construction of RMT_2 using a complete basis of modular-invariant functions and minimal universal input near extremality. Holographically, we envision RMT_2 predictions as a baseline against which to compare future gravity calculations; such a comparison would quantify the degree to which AdS_3 gravity is richer than perhaps expected, containing information that goes beyond a reduction to JT gravity near extremality combined with symmetry constraints.

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End Matter

Wormholes with $n > 3$ boundaries—For wormholes of topology $\Sigma_{0,n} \times S^1$ with $n > 3$ boundaries, the RMT_2 result is sensitive to the choice of spectral curve of the RMT being lifted. As motivated earlier, the pure gravity amplitudes on $\Sigma_{0,n} \times S^1$ to leading order in the semiclassical limit should be given by the RMT_2 lift of the JT gravity amplitudes on $\Sigma_{0,n}$. We emphasize that the procedure can be repeated for any given n , using the genus-0, n -boundary JT correlators in [8,35].

In general, the n -boundary JT wormhole amplitudes take the form of a sum of monomials $\prod_i y_i^{a_i}$. The tool that allows us to find the relevant overlaps for each individual term is the regularized Mellin transform introduced in [25]:

$$\mathcal{M}[y^a](-i\omega) \equiv \int_0^\infty dy y^{a-1-i\omega} = \frac{2\epsilon}{\epsilon^2 - (a - i\omega)^2},$$

$$\text{Im}(\omega) = -a. \quad (\text{A1})$$

The overlaps $f_{\text{JT}}^{(0,n)}(\{\omega_i\})$ are the sum of products of the above expression. Because of this factorized structure, the modular uplift acts individually on each monomial and

reduces to integrals of the form

$$\lim_{\epsilon \rightarrow 0} \left(\int_{\mathbb{R}-ia} \frac{d\omega}{2\pi} \frac{2\epsilon}{\epsilon^2 - (a - i\omega)^2} E_{\frac{1}{2}+i\omega}(\tau) \right) = E_{\frac{1}{2}+a}(\tau),$$

$$\left(a \neq \frac{1}{2} \right). \quad (\text{A2})$$

The case $a = 1/2$ is slightly more subtle because of the Eisenstein pole at $s = 1$: the right-hand side of (A2) should be understood as the pole-subtracted Eisenstein series $\hat{E}_1(\tau)$, defined in (26). To arrive at this result, we expand the integrand near $\omega = 0$ and use the integral representation of the Dirac delta function to obtain

$$\lim_{\epsilon \rightarrow 0} \int_{\mathbb{R}} \frac{d\omega}{2\pi} \frac{2\epsilon}{\epsilon^2 + \omega^2} E_{1+i\omega}(\tau)$$

$$= \hat{E}_1(\tau) + \lim_{\epsilon \rightarrow 0} \int_{\mathbb{R}} \frac{d\omega}{2\pi} \frac{2\epsilon}{\epsilon^2 + \omega^2} \frac{3}{\pi i \omega}. \quad (\text{A3})$$

The second term, a τ -independent constant, must be regularized: a canonical choice is the principal value

prescription,

$$\text{PV} \left[\int_{\mathbb{R}} \frac{d\omega}{(\epsilon^2 + \omega^2)\omega} \right] = 0. \quad (\text{A4})$$

Adopting this choice henceforth allows us to immediately write down the modular uplift of JT gravity amplitudes—indeed, of *any* polynomial RMT amplitude—by using the replacement rule

$$y^a \mapsto E_{\frac{1}{2}+a}(\tau) - \frac{3}{\pi(a - \frac{1}{2})} \delta_{\frac{1}{2},a}. \quad (\text{A5})$$

This proves (22) and also applies to the all-genus expansion of the τ -scaled Airy model, see (18).

For further illustration, we now present the RMT_2 prediction for $n = 4$ and $n = 5$ wormhole amplitudes in AdS_3 pure gravity by lifting corresponding JT gravity expressions.

$n = 4$ boundaries: The JT gravity amplitude with four boundaries is

$$K_{\text{JT}}^{(0,4)}(\{y_i\}) = \frac{e^{-2S_0}}{4\pi^2\gamma^3} \sqrt{y_1 y_2 y_3 y_4} \left(2\pi^2\gamma + \sum_{i=1}^4 y_i \right), \quad (\text{A6})$$

where $\gamma = c/24$ when JT gravity is embedded as a near-extremal sector of AdS_3 pure gravity. Using the replacement rule (A5) leads to the following Eisenstein sector for the four-boundary RMT_2 amplitude:

$$\begin{aligned} \mathcal{Z}_{\text{JT-RMT}_2}^{(0,4)}(\{\tau_i\}) &\supset \frac{e^{-2S_0}}{4\pi^2\gamma^3} \left(\prod_{i=1}^4 \hat{E}_1(\tau_i) \right) \left(2\pi^2\gamma + \sum_{i=1}^4 \frac{E_2(\tau_i)}{\hat{E}_1(\tau_i)} \right) \\ &+ \left[\begin{array}{c} \text{cusp} \\ \text{forms} \end{array} \right]. \end{aligned} \quad (\text{A7})$$

Importantly for the consistency of our proposal, in the near-extremal limit $y_i \propto \gamma$ and $\gamma \rightarrow \infty$, one recovers the JT gravity $n = 4$ wormhole (A6). Note that using instead the Virasoro minimal string (VMS) spectral curve for $n = 4$ amounts simply to having $\gamma_{\text{VMS}} = [(c - 13)/24]$ in the above expressions, with the same near-extremal limit reproducing $K_{\text{VMS}}^{(0,4)}(\{y_i\})$.

$n = 5$ boundaries: The JT gravity amplitude for a wormhole with $n = 5$ boundaries and trivial interior topology is given by

$$\begin{aligned} K_{\text{JT}}^{(0,5)}(\{y_i\}) &= \frac{e^{-3S_0}}{4\sqrt{2}\pi^{5/2}\gamma^{9/2}} \sqrt{y_1 y_2 y_3 y_4 y_5} \\ &\times \left[10\pi^4\gamma^2 + 6\pi^2\gamma \left(\sum_{i=1}^5 y_i \right) + \left(\sum_{i=1}^5 y_i \right)^2 \right]. \end{aligned} \quad (\text{A8})$$

The RMT_2 prediction for the five-boundary amplitude follows by expanding (A8) and applying (A5) term by term:

$$\begin{aligned} \mathcal{Z}_{\text{JT-RMT}_2}^{(0,5)}(\{\tau_i\}) &\supset \frac{e^{-3S_0}}{4\sqrt{2}\pi^{5/2}\gamma^{9/2}} \left(\prod_{i=1}^5 \hat{E}_1(\tau_i) \right) \\ &\times \left[10\pi^4\gamma^2 + \sum_{i=1}^5 \left(6\pi^2\gamma \frac{E_2(\tau_i)}{\hat{E}_1(\tau_i)} + \frac{E_3(\tau_i)}{\hat{E}_1(\tau_i)} \right) \right. \\ &\left. + 2 \sum_{i < j} \frac{E_2(\tau_i) E_2(\tau_j)}{\hat{E}_1(\tau_i) \hat{E}_1(\tau_j)} \right] + \left[\begin{array}{c} \text{cusp} \\ \text{forms} \end{array} \right]. \end{aligned} \quad (\text{A9})$$

For arbitrary n , $K_{\text{JT}}^{(0,n)}(\{y_i\})$ is only known as a generating function, but one may proceed in the above fashion for any given n .