

# Photonic crystal-like scaling behavior of localized Anderson modes in hyperuniform disordered systems

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**Abstract.** Hyperuniform disordered structures have emerged as a promising photonic platform that bridges the benefits of ordered and disordered structures, offering isotropic photonic band gaps, high spectral mode density, and robustness to imperfections. In this work, we provide direct experimental evidence that Anderson-localized dielectric modes in such systems exhibit spectral scaling behavior analogous to that found in photonic crystals. By systematically varying the filling fraction and length scale of hyperuniform disordered dielectric structures, we observe predictable and rigid spectral shifts in localized mode resonances. Hyperspectral scanning near-field optical microscopy enables simultaneous access to spatial and spectral information, confirming that these modes retain their spatial profiles and localization across scale variations. Our results demonstrate that Anderson localization in hyperuniform disordered systems obeys conventional photonic scaling laws despite the absence of long-range periodicity, while maintaining unique advantages such as isotropy, disorder tolerance and large photonic bandgaps. These findings highlight the potential of hyperuniform disordered materials as scalable and flexible platforms for integrated photonics, cavity quantum electrodynamics, and broadband optical devices.

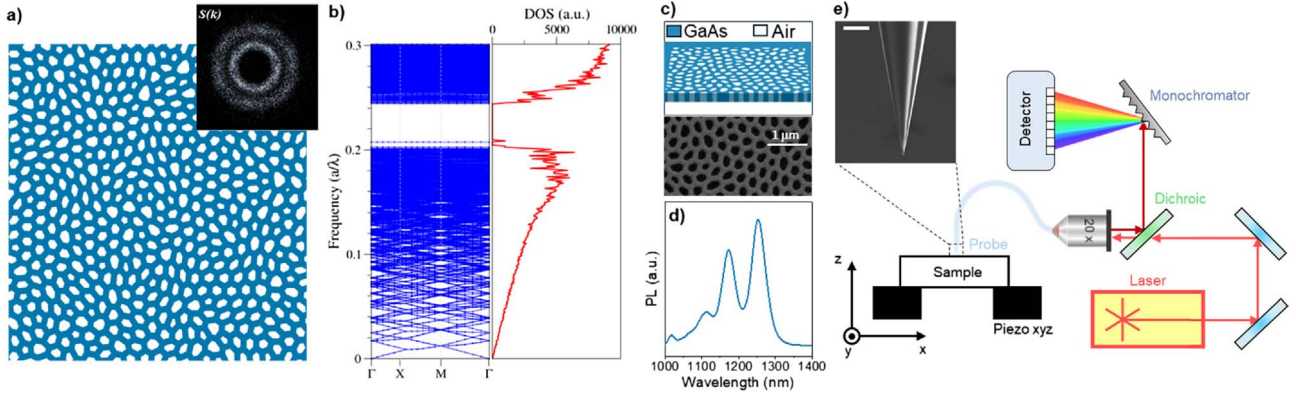
**Keywords:** Photonic-crystals, Anderson localization, Hyperuniform disorder, Near-field Scanning Probe Microscopy.

## 1 Introduction

Photonic structures engineered for precise control over light propagation have historically relied on periodicity to open photonic band gaps (PBGs) and confine electromagnetic modes [1–3]. While photonic crystals (PhCs) offer excellent confinement and spectral control, their performance can be hindered by low spatial and spectral density of states, inherent anisotropies and strict symmetry constraints [1–5]. In contrast, a special class of correlated disorder systems, hyperuniform disordered (HuD) photonic structures [6], filling the gap between perfect periodicity and complete randomness, have emerged as a compelling alternative offering both large, isotropic PBGs and high flexibility in device integration [7–11]. HuD structures are particularly attractive for applications where direction-independent

light manipulation is crucial, such as photovoltaics, sensing, and light-emitting devices [12–21]. Among HuD systems, a peculiar role is played by stealthy HuD structures, characterized by both suppressed long-range density fluctuations and vanishing scattering in a defined range of spatial frequencies [7, 22, 23]. Formally, in reciprocal (Fourier) space, the hallmark of hyperuniformity corresponds to a vanishing structure factor  $S(k)$  in the long wavelength limit. A stealthy hyperuniform system imposes an even stricter condition:  $S(k) = 0$  for all  $|k| < k_c$  for some positive cutoff wavevector  $k_c$ . This implies that the system is completely transparent (i.e., non-scattering) to long-wavelength Fourier components up to that threshold, hence the stealthy term. In the context of photonics, stealthy HuD systems have been shown to display large, complete PBG [9]. As previously demonstrated, this PBG results from interplay of hyperuniformity, short-range geometric order, and uniform local topology [9]; its generation is specifically linked to the Mie resonances [9, 24–27] of the individual

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**Fig. 1.** a) HuD design with corresponding structure factor in the inset. b) Photonic band structure (left) and DOS (right) calculated using 500 k-points randomly distributed in the first Brillouin zone associated with the HuD supercell. c) Sketch of the patterned single membrane (top) and SEM detail of one of the fabricated samples (bottom). d) PL emission spectrum of the QDs embedded in the middle of the membrane. e) Sketch of hyperspectral SNOM setup used in illumination/collection configuration.

scattering centers within the structure, as it occurs when the scattered field is out of phase with the incident field, resulting in the absence of a propagation channel for light. This breakthrough in the field of photonics, by demonstrating that long-range periodicity is not necessary for the opening of a PBG, has showcased the potentiality of HuD systems. Moreover, it opened the path to exploring whether such systems could merge the benefits of structural disorder, such as isotropy and robustness, with the functional advantages of order, including coherent light control and bandgap formation [9, 15]. For instance, high- $Q$  cavities and freeform waveguides have been demonstrated both theoretically and experimentally [17–20]. Moreover, a key indicator of the versatility of HuD photonic systems is their ability to host Anderson dielectric modes at the edges of the PBG [15, 16]. These modes, resulting from structural correlations rather than periodicity breaking, have been shown to exhibit high quality factors and small mode volumes in patterned semiconductor slabs, aspects typically associated with engineered cavities in PhCs. Remarkably, such modes have been found to be more robust to fabrication induced disorder than their periodic counterparts [15, 16].

A central question in the study of HuD photonic materials is the extent to which their optical modes can mimic those of PhCs, particularly in terms of mode scaling and predictability. The ability to control and tune the resonances of a photonic structure is a matter of crucial importance in the framework of light-matter interaction applications, where guaranteeing the spectral and spatial overlap between modes and quantum emitters is mandatory. While it has already been proven that an engineered cavity with resonance inside the PBG of a HuD network exhibits a systematic and predictable spectral shift as expected in correspondence of air or dielectric defects [18], Anderson localized modes arising at the PBG edges remain unexplored under this perspective. In this work, we provide direct experimental evidence that Anderson-localized dielectric modes in HuD networks on slab exhibit systematic spectral shifts when geometrical parameters such as the filling fraction and length scale are varied. This scaling behavior mirrors the conventional scaling laws of PhCs [4],

where the photonic band structure rigidly shifts with the unit cell dimension and refractive index contrast. Our results were obtained through hyperspectral scanning near-field optical microscopy (SNOM), which provides spatially and spectrally resolved maps of the local density of optical states (DOS) with subwavelength resolution. The observation of rigid, predictable spectral tuning in correlated disordered systems further supports the notion that HuD materials, while carrying strategic advantages like isotropy and high spectral density, can replicate the functional capabilities of periodic architectures.

## 2 Material and methods

### 2.1 Theoretical design

The pattern, shown in Figure 1a, is constructed within periodic boundary conditions, using a square domain of side length  $L$ . A characteristic length scale  $a = L/\sqrt{N}$  is then introduced in analogy with the PhCs lattice constant, ensuring a uniform point density of  $1/a^2$ , where  $N$  denotes the number of scattering centers within the supercell.

The 2D HuD network geometry was built according to the protocol reported in Ref. [9] and is defined as stealthy as shown in the inset of Figure 1a: the structure factor  $S(k)$  is exactly zero for a discrete set of wavevectors around the origin [7–9]. While exhibiting the suppression of large-scale density fluctuations as in photonic crystals, the HuD's structure factor resembles the isotropy typical of random systems [6–8, 28, 29]. Through the supercell approach, it is then possible to calculate the 2D photonic bandstructure, showing a large and isotropic PBG (Fig. 1b); the right panel of Figure 1b shows the corresponding photonic Density of States (DOS) calculated using 500 randomly distributed points in the first Brillouin zone associated with the HuD supercell.

### 2.2 Sample fabrication

The fabricated devices are based on GaAs optically active slabs, incorporating high density of InAs quantum dots

(QDs) that emit between 1100 nm–1300 nm. A layer stack of a 250 nm-thick GaAs membrane on top of a 3  $\mu\text{m}$ -thick AlGaAs sacrificial layer that separates the membrane from the GaAs substrate was used to manufacture one wafer. A layer of self-assembled InAs QDs was integrated in the center of the GaAs membrane on a different wafer that was generated with the same layer stack and nominal thicknesses. A molecular-beam epitaxy reactor was used for growth, and the growth parameters were adjusted to guarantee a QD density of  $175\ \mu\text{m}^{-2}$  [30]. The HuD geometry is patterned through electron-beam lithography, followed by reactive-ion etching and a selective removal of the AlGaAs sacrificial layer [15, 16, 18, 31, 32]. A sketch of the single membrane system is reported in Figure 1c, on top of a detailed view of one of the fabricated samples is presented in the scanning electron microscope (SEM) image, while the QDs photoluminescence (PL) emission spectrum is reported in Figure 1d. The two main peaks observed in the PL spectrum correspond to the ground and first excited state of the QDs emitting at room temperature. Each sample was fabricated by changing two structural parameters: 1) The characteristic length scale  $a = 340, 360$  and  $380\ \text{nm}$ ; and 2) The wall thickness  $w = 0.415a, 0.461a$  and  $0.5a$ , i.e. the thickness of the dielectric veins in the networks, that influences the filling fraction. The structural parameters were carefully chosen at the design stage to guarantee the matching between the resonant modes of the HuD geometry and the QDs emission range.

### 2.3 Experimental setup

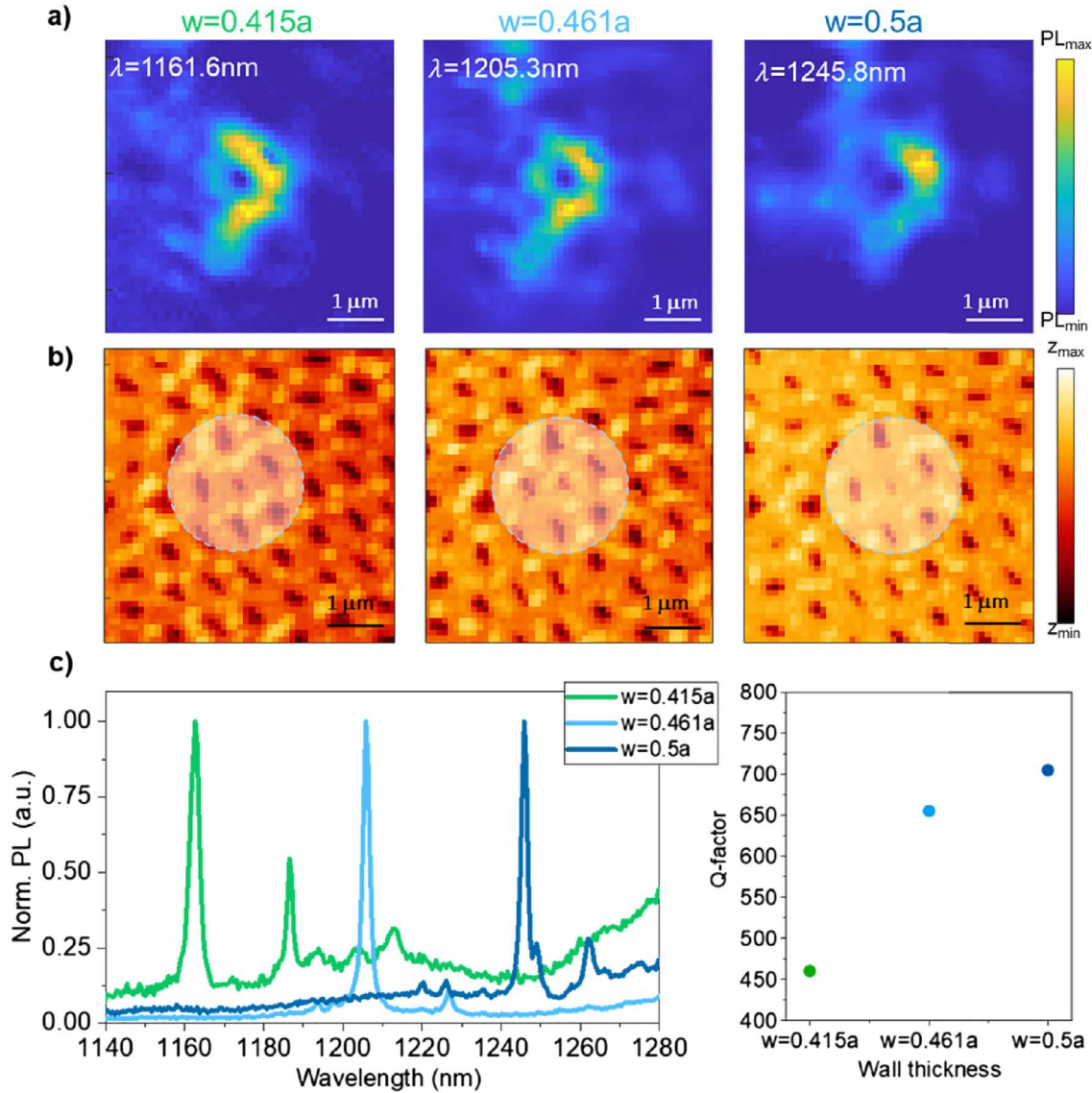
To optically characterize the samples we employ a commercial SNOM (Twinsnom, Omicron) in illumination/collection configuration, as sketched in Figure 1e. The probe is a chemically etched dielectric fiber that, by raster scanning the sample at a fixed distance (few tens of nm) allows to reconstruct both topographic and optical maps. The sample is excited by a laser diode at 785 nm, and both the excitation and the signal collection occur through the tip; the collected signal is dispersed by a spectrometer and collected by a cooled InGaAs array. At every tip position, the entire spectrum of the sample is collected with a spectral resolution of 0.1 nm. The SNOM tip collecting area, which has a lateral size of about 250 nm, is the estimated spatial resolution of our system.

## 3 Results and discussion

As argued above, it has been demonstrated that HuD photonic systems support Anderson-localized modes near the edges of the PBG, closely analogous to the dielectric and air band edge modes observed in photonic crystals [9, 15]. These modes, arising due to multiple scattering and interference effects intrinsic to the short-range order of the HuD architecture, are typically localized around either high- or low-index regions, corresponding to dielectric-like or air-like behavior, respectively. In this work we focus on the dielectric modes of the HuD sample. To investigate the scaling behavior and spectral tunability of

Anderson dielectric modes, we have conducted a systematic near-field optical characterization on the series of samples based on the same underlying stealthy HuD point pattern but with varying length scale and normalized wall thicknesses  $a$  and  $w$  (see Sect. 2). Each structure was probed using the SNOM Hyper-Spectral Imaging (HSI) technique, which allows simultaneous access to both spatial and spectral information with subwavelength resolution. In HSI a full spectrum is collected for any spatial pixel of the near-field map (corresponding to a specific tip position during the scan). This allows us to reconstruct hyperspectral maps that can either be filtered around a single wavelength, for example the central wavelength of a single peak, or around a broad spectral interval. Figure 2 presents a direct comparison of the same Anderson-localized dielectric mode as it appears in three structures with the same  $a = 380\ \text{nm}$  and varying the thickness of the dielectric veins. The SNOM PL intensity maps, filtered around the tuned resonant wavelength of the mode (respectively 1161.6, 1205.3 and 1245.8 nm), are reported in Figure 2a, revealing the spatial profile of the mode across all samples. Despite the variation in wall thickness, the mode exhibits a similar localized spatial distribution in each case, demonstrating its reproducibility and robustness to geometric scaling. The variation in wall thickness is visible in the corresponding SNOM topographies, reported in Figure 2b. Notably, changing  $w$  while keeping  $a$  constant (or the reverse) is analogous to changing the filling fraction of the photonic pattern. Figure 2c displays the corresponding PL spectra, acquired in the points of maximum intensity for each of the three measurements of Figure 2a. A systematic redshift of the resonance peaks of about 40 nm is observed as the wall thickness increases. This behavior is analogue in photonic crystals, where the mode resonant wavelengths of PhC cavities exhibit a redshift (blueshift) in correspondence of an increase (decrease) of dielectric material [4, 18]. In this regard, since in PhC the filling fraction is one of the parameters determining the optimal condition for high quality factors ( $Q$ ) [33], we also evaluate the  $Q$  of the same mode in three different samples with three different values of  $w$  and report them in the right panel of Figure 2c; the values are obtained by fitting with a Lorentzian curve the peaks corresponding to the resonances. We observe that by decreasing the size of the air holes (i.e. increasing  $w$ ), the  $Q$ -factor is increased, meaning that a smaller filling fraction is responsible for a more efficient confinement of light.

To complete this analysis, we fixed the normalized wall thickness at  $w = 0.5a$  and systematically varied the absolute length scale  $a$  across multiple samples, focusing on the same Anderson dielectric mode. This approach isolates the effect of global length scale on the spectral position of the modes, in analogy with 2D photonic crystal on slab scaling laws where eigenfrequencies scale inversely with lattice periodicity. Figure 3a shows the PL spectra acquired from three structures with lattice constants  $a = 340, 360$  and  $380\ \text{nm}$  all with the same relative wall thickness  $w = 0.5a$ . A clear and rigid red shift of the Anderson-localized modes is observed as  $a$  increases, consistent with the expected linear scaling of the photonic band structure. The resonance peaks shift by 40–50 nm over this range,

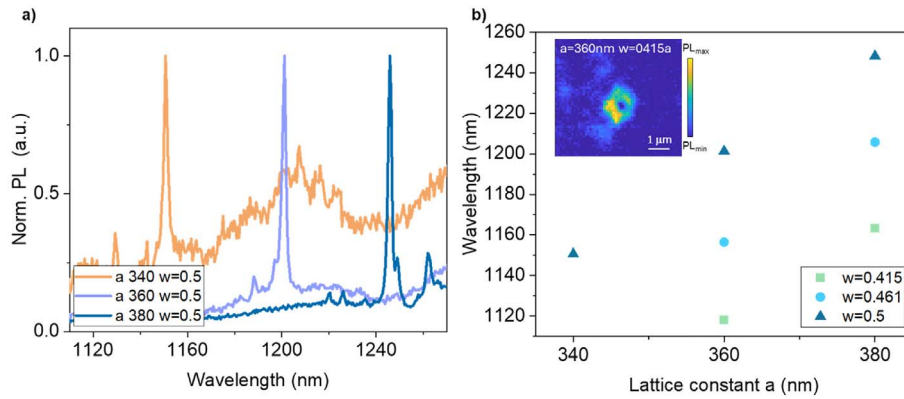


**Fig. 2.** a) PL SNOM maps of the an Anderson-localized mode in three HuD structures with increasing normalized wall thicknesses  $w = 0.415a$ ,  $0.461a$  and  $0.5a$  ( $a = 380$  nm), showing the same mode localization distribution across all three samples. The maps are filtered around the resonant wavelengths of respectively 1161.6, 1205.3 and 1245.8 nm. b) Corresponding SNOM topographies. c) PL spectra collected in the point of maximum intensity of the mode in each structure, showing a systematic red shift in resonance wavelength as wall thickness increases, in agreement with photonic crystal scaling laws. Right panel: experimental  $Q$  factors of the corresponding resonances for different values of  $w$ .

confirming that the localized modes respond predictably to changes in lattice scale, just as in periodic photonic crystals. The inset of Figure 3b shows a near-field PL map of the analyzed mode for  $a = 360$  nm, showcasing the preserved spatial distribution even in the case of different length scales. The graph in Figure 3b summarizes the spectral position of Anderson mode detected in all samples as a function of length scale  $a$  and wall thickness  $w$ . Each set of data points, plotted for the three wall thicknesses follows a consistent monotonic trend, further reinforcing the conclusion that Anderson-localized modes in HuD structures follow conventional photonic scaling laws, despite the absence of long-range periodicity.

## 4 Conclusions

Our results provide direct and compelling experimental evidence that Anderson-localized dielectric modes in HuD photonic structures are both spatially robust and spectrally scalable, in close analogy with the behavior of modes in photonic crystals, despite the absence of long-range periodicity. Using hyperspectral near-field optical microscopy, we demonstrated that systematic variations in geometrical parameters lead to predictable and rigid spectral shifts of localized modes. These findings establish that HuD systems obey the same scaling laws traditionally associated with periodic photonic architecture.



**Fig. 3.** a) SNOM PL spectra collected in the point of maximum intensity of the mode in each with fixed wall thickness  $w = 0.5a$  and varying lattice constants  $a = 340$  nm,  $360$  nm,  $380$  nm. b) Summary plot showing resonance wavelength versus length scale  $a$  for the three different wall thicknesses. The inset shows a SNOM PL map of the studied mode detected in a structure with  $a = 360$  nm and  $w = 0.415a$ .

Importantly, this scaling behavior occurs in structures that retain the key advantages of disorder, such as isotropy, high modal density, and robustness to fabrication imperfections, features that are often unattainable in conventional photonic crystals. This comparison further validates that HuD materials can effectively emulate the spectral functionality of photonic crystals, while surpassing them in terms of design flexibility and direction-independent performance. By confirming that disorder is not always a constraint but can be a tunable photonic design parameter, this work opens a path toward a new class of scalable, disorder-enabled photonic devices, such as broadband emitters, isotropic filters, integrated light sources, and quantum photonic platforms. Our findings not only bridge the gap between order and disorder in nanophotonics but also offer a blueprint for leveraging structurally correlated disorder to engineer optical functionalities with a degree of control previously reserved for periodic systems.

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### Conflicts of interest

The authors have nothing to disclose.

### Data availability statement

Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

### Author contribution statement

Conceptualization, N.G, F.I. and M.F.; Methodology N.G, F.I., M.F. M.L, P.J.V; Software, M.F., K.S.; Investigation, N.G, G.C and C.G; Resources, A.F., M.F. and F.I.; Data Curation, N.G.; Writing – Original Draft Preparation, N.G.; Writing – Review & Editing, N.G, F.I., M.F., A.F.; Supervision, F.I.

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