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Scatter-plots for acoustics lessons

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HIGHLIGHTS

- Scatter plots for fluids are a useful way of examining their acoustic properties.
- Scatter plots for musical-instrument strings are presented.

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ABSTRACT

Material-selection charts, in the form of grouped scatter plots of material properties of solids, are known to be useful in structural engineering, and as a conceptual tool for teaching. In this article similar plots for the properties of liquids and gases are introduced, and their application to the teaching of acoustics is discussed. Analogous charts for musical strings are presented, highlighting the analogy between the properties of wave-bearing fluids and those of wave-bearing strings. Data from one manufacturer show consistently higher tension for bowed strings than plucked strings, with the exception of the bowed erhu and the plucked mandolin, both of which have intermediate tension. Questions that could be discussed in class or form the basis of student investigations are suggested throughout.

1. Introduction

Material-selection charts [1] are scatter plots whose axes represent two different properties of a material, such as Young's modulus E and density ρ. They often show categories of materials, e.g., woods, metals, ceramics, etc. rather than points for individual materials. Logarithmic axes allow a wide range of values to be displayed and mean that contours of power-law relations between the two properties appear as parallel straight lines.

An example of their relevance to acoustics comes via Schelleng [2], who showed that sound radiation from violins should scale with the value of $\sqrt{E/\rho^3}$ for the top plate. Superimposing contours of this function on an E vs ρ chart shows that woods have higher values of this quantity than other available materials. It also shows, surprisingly, that balsa has the highest value among common woods, which inspired Waltham [3] to make a balsa violin and show that, aesthetic considerations aside, its sound radiation was indeed in line with expectations.

The viewpoint provided by this type of plot can be useful in educational settings. In the following brief note I share some examples that might be useful in the teaching of acoustics, and give suggestions for topics of discussion and inquiry that they could lead to.

2. Fluid properties

2.1. Basic acoustic properties

Unlike the solids chosen by structural engineers or luthiers, the liquids and gases that support acoustic waves are not usually chosen by acoustical engineers. Nonetheless, the scatter-plot of isentropic bulk modulus B_0 vs mass-density ρ_0 (zero-subscripts indicating background properties) shown in Fig. 1 for the fluids listed in Table 1 at NTP (Normal Temperature & Pressure, i.e., $T_0 = 20\,°\mathrm{C}$ and $P_0 = 1\,\mathrm{atm} = 101\,325\,\mathrm{Pa}$) reveals some useful information.

The axes show that the densities of the liquids are between two and four orders of magnitude higher than those of the gases, while the bulk moduli of the liquids are three orders of magnitude higher than those of the gases.

Contours of the base-ten log of sound-speed $c_0 = \sqrt{B_0/\rho_0}$ and that of the characteristic specific acoustic impedance $z_0 = \rho_0 c_0^2 = \sqrt{\rho_0 B_0}$ show that although the sound-speeds for the liquids are generally higher, there is some overlap with those of the gases, but the impedances of the liquids are three orders of magnitude higher than those of the gases. Exam-question setters might care to note that it is possible for a gas to have an

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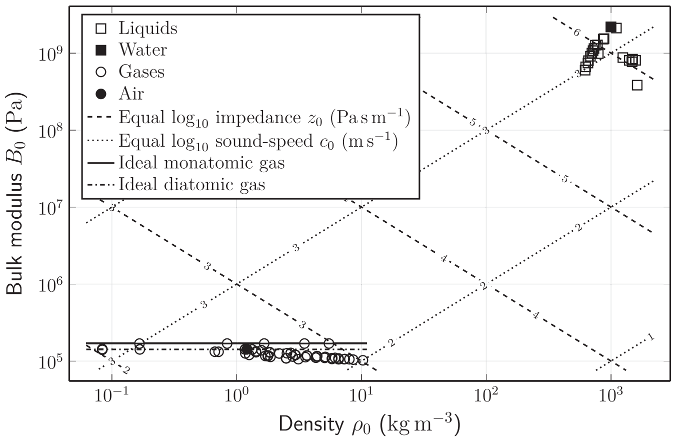

Fig. 1. Scatter plot of bulk modulus B_0 against density ρ_0 for the fluids listed in Table 1.

Table 1
Substances whose properties at NTP are plotted in Figs. 1 & 2. Properties are taken from the NIST Chemistry Webbook [7], with the exception of air, for which the properties are taken from Table A10(c) of Kinsler et al. [5]. Refrigerants are denoted by their ASHRAE number preceded by an R [8].

Gases:	Air, Nitrogen, Hydrogen, Parahydrogen, Orthohydrogen, Deuterium, Oxygen, Fluorine, Carbon Monoxide, Carbon Dioxide, Dinitrogen Monoxide, Methane, Ethane, Ethene, Propane, Propene, Propyne Cyclopropane, Butane Isobutane, 2,2 Dimethylpropane, Helium, Neon, Argon, Krypton, Xenon Ammonia, Nitrogen Trifluoride, R12, R13, R14, R21, R22, R23, R32, R41, R114, R115, R116, R124, R125, R134a, R142b, R143a, R152a, R218, R227ea, R236ea, R236fa, R245fa, RC318, Decafluorobutane, Sulfur Dioxide, Hydrogen Sulfide, Sulfur Hexafluoride, Carbonyl Sulfide
Liquids:	Water, Deuterium Oxide, Methanol, Pentane, 2-Methylbutane, Hexane, 2-Methylpentane, Cyclohexane, Heptane, Octane, Nonane, Decane, Dodecane, R11, R113, R123, R141b, R245ca, Benzene, Toluene, Decafluoropentane

equal or higher sound speed than a liquid, although its impedance will always be much lower.

None of the information in the preceding two paragraphs is likely to be very surprising to the practising acoustician, although I have been unable to find a concise statement of the same information anywhere in my own collection of acoustics textbooks.

The ideal-gas model gives $B_0 = \gamma P_0$, meaning that gases with the same adiabatic index γ will lie on a horizontal line in this plot. Two such lines are shown: one for monatomic gases with $\gamma = 5/3$ and one for diatomic gases with $\gamma = 7/5$.

Like all plots presented here, Fig. 1 has been made monochromatic for reasons of both accessibility and economy. Apart from (dry) air and (fresh) water the individual fluid substances have not been labelled, although the raw data used has been included as supplementary data. Asking students to produce their own plots, however, can be a useful programming exercise, and allows them to explore different ways to present the information, such as hover-over pop-ups to identify particular fluids.

Questions arising from this plot that might lead to useful discussions or, in some cases, follow-up inquiries are given in the following list, with some references to sources of further discussion; interested readers can doubtless add their own.

1. Where would you expect mercury to lie on this chart? It is famously dense, but how would you expect its sound-speed and impedance to compare with those of water?
2. Is it possible to extend the statements above about order-of-magnitude separation of B_0, ρ_0 and z_0 between gases and liquids to *all* liquids and gases, by establishing how far it is possible for substances of either phase to intrude into the unoccupied territory between them? What about at different temperatures and pressures?
3. What trajectory does a particular fluid follow on this chart as its temperature changes? What about boiling liquids and condensing gases?
4. Where on the chart would you expect to find the bulk properties of a bubbly liquid? Can you place a lower limit on its sound-speed in this way? Does the same argument apply to aerosols, and if not why not? A thorough discussion of the physics of such substances can be found in [4].
5. Where would solids, such as those whose properties are given in Table A10(a) of [5], lie on this chart? What is the most appropriate definition of bulk modulus to use when comparing fluids and solids?

It is interesting to note that in the literature of acoustic metamaterials (e.g. [6]) the ‘mechanical’ properties density and bulk modulus are often used to characterize materials, rather than the ‘acoustical’ properties of sound speed and characteristic impedance. It is perhaps surprising, however, that the general acoustics literature almost universally uses one mechanical property (density) and one acoustical property (sound speed) rather than sound speed and characteristic impedance.

2.2. Thermoviscous acoustic properties

For more advanced classes, where thermoviscous acoustic effects are considered, Fig. 2 becomes relevant. This plots the fluid’s momentum diffusivity, given by its kinematic viscosity $\nu = \mu/\rho_0$ with μ being the dynamic shear viscosity, against the thermal diffusivity $\chi = \kappa C_p/\rho_0$ with κ the thermal conductivity and C_p the specific heat at constant pressure. The ratio of these two diffusivities is the fluid’s Prandtl number $\mathrm{Pr} = \nu/\chi$, contours of which are shown, indicating that at NTP $\mathrm{Pr} > 1$ for these liquids and $\mathrm{Pr} < 1$ for these gases. Possible class-discussion questions include:

1. Is it possible to have a liquid with $\mathrm{Pr} < 1$, or a gas with $\mathrm{Pr} > 1$ at NTP, or under any condition?
2. What are the implications for the relative thicknesses of vortical and entropic boundary layers, in liquids and in gases [9, §10.4]?
3. For acoustic propagation away from solid boundaries it becomes appropriate to replace the shear viscosity μ with the longitudinal viscosity $\mu_L = (4/3)\mu + \mu_B$ with μ_B the bulk viscosity [10]. What are typical longitudinal Prandtl numbers for liquids and gases?
4. Why can one not straightforwardly look up values for the bulk viscosity of these substances as one can for the shear viscosity?

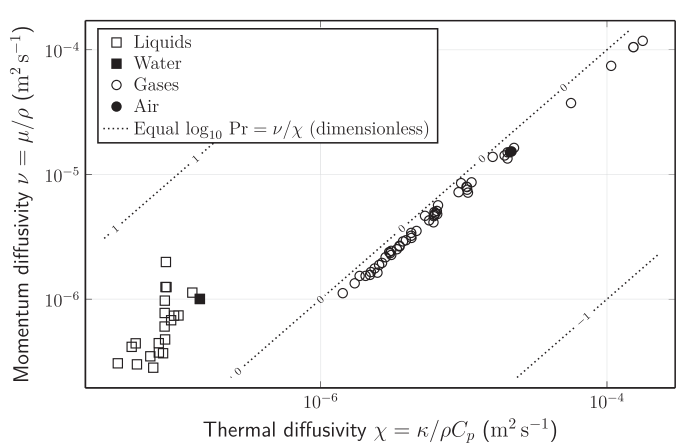

Fig. 2. Scatter plot of momentum diffusivity ν against thermal diffusivity χ for the same fluids.

Interested readers are referred to the last paragraph of section III.B of [11] for a discussion of the implications of having a longitudinal Prandtl number of exactly one.

3. Musical strings

The analogy between longitudinal plane waves in an inviscid fluid and transverse waves on an ideal string, both of which obey the simple wave equation, is a useful one in acoustics lessons, not least because transverse waves are more straightforward to visualise than longitudinal ones. In this analogy the density of the fluid corresponds to the string's mass per unit length μ_s, and the bulk modulus to its tension τ. The wave-speed and mechanical impedance of the string are then $c_s = \sqrt{\tau/\mu_s}$ and $Z = \sqrt{\mu_s \tau}$. Scatter-plots of τ vs μ_s for strings are then analogous to plots of B_0 vs ρ_0 for fluids.

The website of the D'Addario company helpfully provides values of tension and scale length L for each of the strings they supply [12]. From the pitch of the note to which it is to be tuned the wave-speed can be found, since the fundamental frequency will be $f = c_s/2L$, and hence the mass per unit length can be found from the wave-speed and the tension.

3.1. Guitars

Fig. 3 shows a τ-vs-μ_s scatter plot for a selection of electric and classical guitar string sets. The points corresponding to the six strings of each instrument are joined, with the highest notes on the left and the lowest on the right. The electric guitar string-sets are, from lightest to heaviest (i.e. from bottom to top on the chart): Steel Extra Super Light, Nickel Super Light, Nickel Regular Light, Nickel Medium, Pure Nickel Jazz Medium. This ordering applies to all strings except the highest (E4) of the Flat Wound Extra Light set, which is lighter and slacker than that of the Medium set. The three classical string sets are Normal Tension, Hard Tension and Extra Hard Tension. Since L is the same throughout, strings tuned to the same note will have the same wave-speed, and will therefore lie on the same diagonal line.

Questions for class discussions or activities include:

1. The tension varies somewhat across string sets, rather than being uniform as has sometimes been suggested [13, §9.3]. The lowest and highest strings are slacker and tighter respectively than their neighbours. Is this pattern repeated across other D'Addario string sets? Is it present for other manufacturers' strings?
2. Why is the tension variation between classical string sets so much smaller than it is for electric strings?

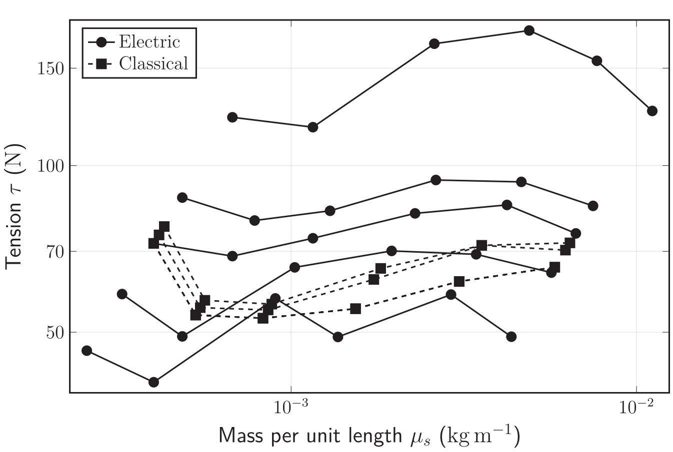

Fig. 3. Scatter plot of tension τ against mass per unit length μ_s for a selection of electric and classical guitars.

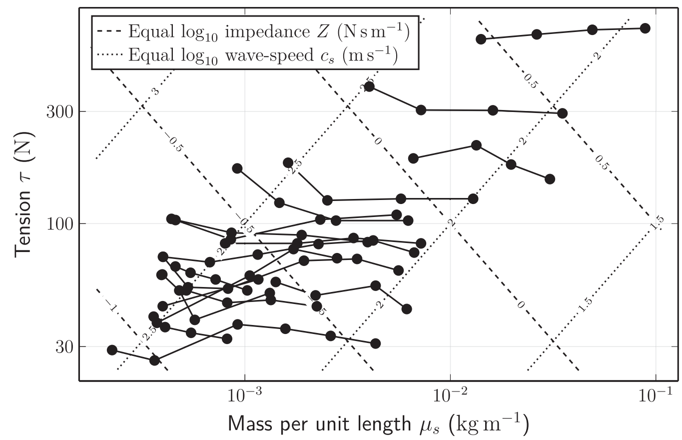

Fig. 4. Scatter plot of tension τ against mass per unit length μ_s for the strings of the instruments listed in Table 2.

Table 2
Instruments whose string properties are plotted in Figs. 4 & 5.

Plucked:	Electric Guitar, Classical Guitar, Electric Bass, Concert Ukulele, Tenor Ukulele, Baritone Ukulele, Bass Ukulele, 5-string Banjo, Mandolin, Pipa, Zhong Ruan, Tenor Guitar, Oud
Bowed:	Violin, Viola, Cello, 3/4-scale Double Bass, Erhu

3. The lightest strings show a slightly different tension profile compared to the other sets. This set also consists of three wound and three plain strings; are the two facts related and if so why?

3.2. Other strings

Fig. 4 shows a similar plot for a wider range of instruments, those included being listed in Table 2. Questions:

1. Harpsichord strings don't vary in weight but they do vary in length. Where would you expect to see them on this chart? Would concert harp strings appear in the same region?
2. Bass ukuleles achieve low-pitched notes with short heavy strings, whereas theorbos (for which I've not yet been able to obtain string data) do so with long strings, originally made of gut. Where would you expect them to lie on this chart? Why was the bass-ukulele approach not adopted at the time when theorbos were being made?
3. Piano-string tension has increased considerably over the history of the instrument. Is it possible to track the path it's followed on this chart? How would the positions of upright-piano—strings and grand-piano—strings differ?

3.3. Plucked and bowed strings

I finish with an interesting observation that arose while examining this data, which contains both plucked and bowed strings. There isn't a clear distinction between the two excitation methods in Fig. 4. Fig. 5, however, scatters the ratio of tension to length, τ/L against impedance Z. While the impedances of the two classes overlap, there appears to be a clear distinction in the values of τ/L, which scale with the static force needed to displace the string from its equilibrium position, with this quantity being consistently higher for bowed strings. The two exceptions that lie in the intermediate zone are the two-string bowed erhu, and the four-string-paired plucked mandolin. Whether there are any other such exceptions, I do not know.

The challenge of gathering string data beyond the D'Addario set should not be underestimated. Piano manufacturers are understandably

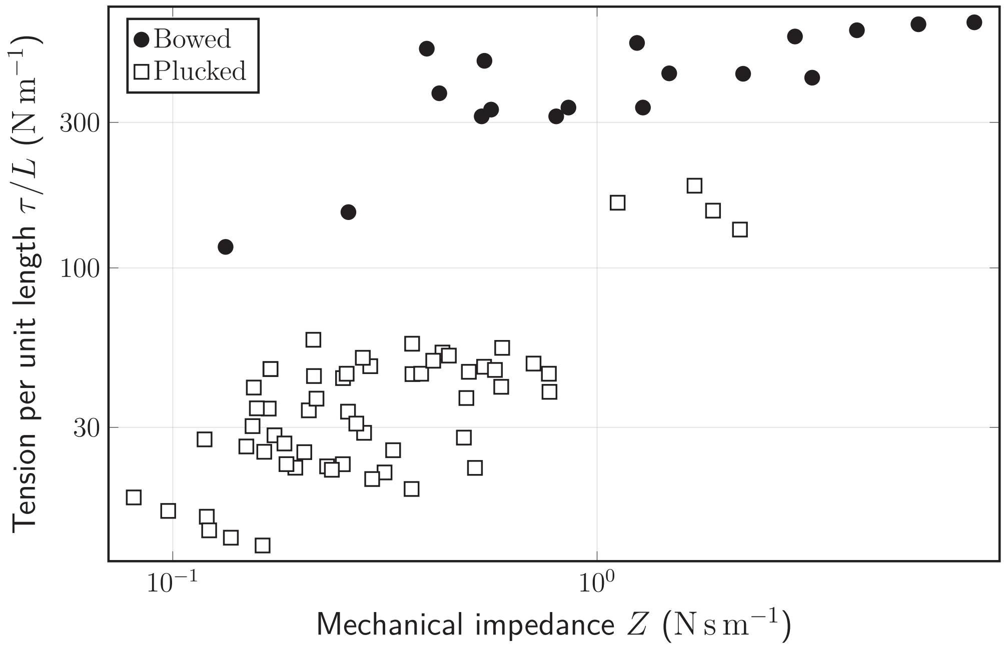

Fig. 5. Tension vs impedance.

guarded about their string data, and estimating the mass of wound strings by measurement *in situ* is not straightforward. If you are able to find additional data I would be grateful to hear about it.

4. Conclusions

The potential use of scatter plots of material properties in acoustics education has been presented and discussed, and a number of suggestions for classroom discussions and activities based on them have been made. Analogous plots for musical-instrument strings have also been presented and discussed, and further questions arising from them have been suggested. A tentative observation (based on data from only one manufacturer) about the tensions of bowed and plucked strings has been made.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data for this article can be found online at doi:10.1016/j.apacoust.2025.111126.

Data availability

Data is provided as supplementary material.

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