An Epistemic Perspective on Agent Awareness

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Abstract

The paper proposes to treat agent awareness as a form of knowledge, breaking the tradition in the existing literature on awareness. It distinguishes the de re and de dicto forms of such knowledge. The work introduces two modalities capturing these forms and formally specifies their meaning using a version of 2D-semantics. The main technical result is a sound and complete logical system describing the interplay between the two proposed modalities and the standard "knowledge of the fact" modality.

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Introduction

Artificial agents are increasingly making important decisions that affect our lives. The choice of the right decision often depends on the *awareness* about other agents' presence at the scene. A war robot must minimise casualties if it is aware of civilians present at the theatre of operations. A self-driving car must stop at the yield (give way) sign if it is aware of an approaching vehicle on the other road. An agent, who is a medical doctor, must offer help if she is aware of someone being sick. An autonomous driving system must stop the vehicle if it is aware of an approaching emergency vehicle. A machine whose values align with humans' must apologise if it is aware of someone being offended. A well-mannered robot should not take the last piece of a cake if it is aware of somebody else wanting this piece.

Awareness is a vague term that could be interpreted in different ways. In the literature, the authors consider awareness of an object (Board and Chung 2022, 2021; Board, Chung, and Schipper 2011) and conceptual awareness (Fagin and Halpern 1987; van Ditmarsch, French, Velázquez-Quesada, and Wáng 2013; van Benthem and Velázquez-Quesada 2010; Grossi and Velázquez-Quesada 2015; Schipper 2015). In this work, we focus on a specific form of the former: "agent awareness" or awareness of one agent about the existence of another agent with a certain property. Although the existing literature on awareness treats it as a distinct concept, the Cambridge Dictionary suggests an epistemic interpretation of awareness by defining it as "knowledge that something exists". In this paper, we give a formal account of this epistemic approach to awareness. While doing this, we observe that there are two distinct ways in which one agent can be aware of another.

Indeed, let us consider the following two sentences:

> *Vehicle's autopilot started to slow down after it became **aware** of a AAAI attendee crossing the road.*

> *Vehicle's autopilot started to slow down after it became **aware** of being followed by a police car.*

Note that, in these sentences, the autopilot is aware in two distinct senses. In the first sentence, it knows that there exists a human on the road ahead of the vehicle. The human happens to be a AAAI attendee, but the autopilot does not necessarily know this. The awareness is about *the physical object* (human on the road) and not her property of being a AAAI attendee. Using first-order epistemic logic, we can write this as

$$\exists x(\text{AAAI-Attendee}(x) \wedge \mathsf{K}_{\text{autopilot}}\text{CrossingRoad}(x)).$$

In the second sentence, the autopilot hears the siren and knows that one of the vehicles driving behind is a police car, but it might not even know which of the vehicles behind belongs to the police. This is awareness of someone with *designator* "police car" being behind:

$$\mathsf{K}_{\text{autopilot}}\,\exists x(\text{PoliceCar}(x) \wedge \text{DrivingBehind}(x)).$$

In the philosophy of language, a distinction between a reference to an object and to a designator of this object is usually referred to as a *de re/de dicto* distinction. Following this tradition, we say that the autopilot is aware of the AAAI attendee *de re* and of the police car *de dicto*. De re/de dicto distinction (without the context of awareness) has been the subject of studies in the philosophy of language (Quine 1956; Lewis 1979; Chisholm 1976; Abusch 1997; Keshet and Schwarz 2019) and law (Anderson 2014; Yaffe 2011). Rebuschi and Tulenheimo (2011) introduced a related notion "de objecto". The existing approaches to formally capturing the distinction mostly rely on quantifiers.

As another example, consider the sentences:

> *A robot series* `i17` *must introduce itself if a security guard becomes **aware** of its presence*

$$\forall x(\texttt{i17}(x) \wedge \mathsf{K}_{\text{guard}}(\text{Present}(x)) \rightarrow \text{MustSelfIntro}(x)),$$

and

*A security guard must report to a supervisor any **sighting** of a series* `i17` *robot*

$$\forall x(\mathsf{K}_{\text{guard}}(\text{Present}(x) \land \texttt{i17}(x)) \to \text{MustReport}(\text{guard})),$$

the first refers to *de re* awareness of the security guard about the robot as a physical object (not necessarily of `i17` series). Otherwise, what would be the point of an introduction if the guard already knows that the robot is of series `i17`? The word "sighting" in the second sentence refers to de dicto awareness. The guard must report to a supervisor if the guard knows that the robot is of that specific series.

Finally, the sentence

A robot of series `i17` *must self-distract if it becomes **aware** of someone **aware** of its presence*

$$\forall x(\texttt{i17}(x) \land \exists y(y \neq x \land \mathsf{K}_x\mathsf{K}_y(\text{Present}(x))) \to \text{MustSelfDistract}(x))$$

mentions awareness twice. The first of them is the awareness of X (robot series `i17`) that some other agent Y nearby has the property of "being aware of X's presence". This is a de dicto awareness. The second of them is Y's de re awareness of X as a physical object, perhaps without knowing that X is a robot series `i17`.

Even without focusing specifically on awareness, very few quantifier-free logical systems for capturing the de re/de dicto distinction have been proposed. Epstein, Naumov, and Tao (2023) considered modalities that capture de re/de dicto versions of "know who". Epistemic Logic with Assignments (Wang and Seligman 2018; Cohen, Tang, and Wang 2021; Wang, Wei, and Seligman 2022) proposes a very general language that can also be used to capture de re and de dicto knowledge of one agent about knowledge of the other. Jiang and Naumov (2025) proposed a modal logical system for reasoning about de re and de dicto knowledge of a property of an agent inferred from a dataset. However, none of these logical systems can express either de re or de dicto awareness.

In this paper, we propose to capture de re and de dicto forms of awareness by two modalities, whose meaning is defined using a 2D-semantics (Schroeter 2021). Our main technical result is a sound and complete logical system capturing the interplay between the traditional "knowledge of the fact" modality and these two new modalities. The completeness proof uses a non-trivial modification of a recently introduced "matrix" technique.

The proposed logical system complements the existing body of literature on other specialised forms of knowledge: know-how (Naumov and Tao 2017; Fervari, Herzig, Li, and Wang 2017; Agotnes and Alechina 2019), know-who (Epstein, Naumov, and Tao 2023), know-whether (Fan, Wang, and van Ditmarsch 2015), know-why (Xu, Wang, and Studer 2019), and know value (Wang and Fan 2013; Baltag 2016).

Running Example

Imagine Ann, who decided to take a break from AAAI-26 meetings at Singapore's *Asian Civilisations Museum*. While in the building, she booked a WeRide self-driving car to take her back to the conference. As Ann leaves the building, she notices a car parked in front of the entrance to the building. Unknown to Ann, the car in front of her is an unmarked police vehicle. Ann is aware of the car in front of her. The car is a police vehicle. Thus, *Ann is de re aware of the police vehicle next to the museum.*

A few seconds after exiting the building, Ann gets a text message that her WeRide has arrived. Ann cannot see the WeRide vehicle because it is parked around the corner, but Ann knows that the WeRide is somewhere near her. Hence, *Ann is de dicto aware of the WeRide vehicle next to the museum*. In this paper, we capture these two forms of awareness by two modalities. We believe that the definitions of these modalities are the most elegant in the *egocentric* logic setting. The semantics of the traditional modal logical systems is defined in terms of a binary relation $w \Vdash \varphi$. In such a setting, formula φ captures a property of possible world w. Prior (1968) proposed to consider logics that capture properties of *agents* rather than possible *worlds*. He called such logics "egocentric". The semantics of egocentric logical systems can be defined in terms of a binary relation $a \Vdash \varphi$ between an agent a and a formula φ. Multiple versions of such systems, not dealing with awareness, have been proposed in the literature (Grove and Halpern 1991, 1993; Grove 1995; Seligman, Liu, and Girard 2011, 2013; Christoff and Hansen 2015; Christoff, Hansen, and Proietti 2016; Jiang and Naumov 2022, 2024).

In order to capture knowledge and awareness, one can extend egocentric semantics by considering a ternary satisfaction relation $w, a \Vdash \varphi$ between possible world w, agent a, and formula φ. In such a setting, formula φ captures property φ of agent a in world w.

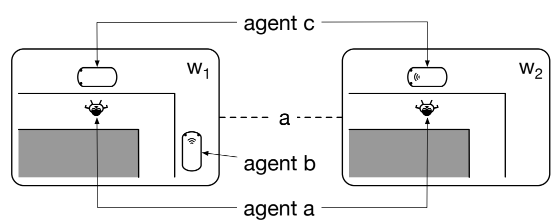

Figure 1: Symbol ᯤ designates a WeRide vehicle.

For example, consider an *epistemic model*, depicted in Figure 1, capturing the setting of our running example. This model has two worlds, w_1 and w_2 with w_1 being the actual world in the example. There are three agents *present* in world w_1: Ann (agent a), WeRide vehicle (agent b), unmarked police vehicle (agent c). In Figure 1, two-directional arrows represent transworld identity between instances of agents in different worlds. Although the nature and the very existence of transworld identity is a widely discussed subject in the philosophy of language (Mackie and Jago 2022), in this work, we assume such identity to be given.

Because agent c is a police vehicle in world w_1 we can write:

$$w_1, c \Vdash \text{"is police vehicle"}.$$

Furthermore, because agent c is located near the museum,

$$w_1, c \Vdash \text{“is police vehicle”} \wedge \text{“is near the museum”}.$$

Ann cannot distinguish world w_1 from world w_2. However, in world w_2 the same agent c is a WeRide vehicle located near the museum:

$$w_2, c \Vdash \text{“is a WeRide vehicle”} \wedge \text{“is near the museum”}.$$

Because the same agent c is present in both worlds that Ann cannot distinguish, Ann is aware of agent c. Since in the actual world agent c is an unmarked police vehicle, in world w_1 Ann is *de re* (as of a physical object) aware of the police vehicle near the museum. We write this as

$$w_1, a \Vdash \mathsf{R}(\text{“is police vehicle”} \wedge \text{“is near the museum”}).$$

Recall that agent b is also present in the world w_1 and, in this world, it is a WeRide vehicle located near the museum:

$$w_1, b \Vdash \text{“is a WeRide”} \wedge \text{“is near the museum”}.$$

Because Ann cannot see agent b, she is not aware of it de re (as a physical object):

$$w_1, a \Vdash \neg\mathsf{R}(\text{“is a WeRide”} \wedge \text{“is near the museum”}).$$

At the same time, because Ann got the message from WeRide that her vehicle had arrived at the museum, there must be a WeRide near the museum in each world that Ann cannot distinguish from the current world. In our example, there is a WeRide near the museum in world w_1 (agent b) as well as in world w_2 (agent c). As a result, Ann is aware of a WeRide, as a concept (de dicto), being present near the museum:

$$w_1, a \Vdash \mathsf{D}(\text{“is a WeRide”} \wedge \text{“is near the museum”}).$$

The rest of the paper is structured as follows. First, we introduce epistemic models and proceed by defining the syntax and semantics of our logical system. Then we propose the axiomatisation. Having informally discussed the axioms, we state the soundness theorem. In the next section, we introduce the notions of general awareness and a λ-***assured*** set. Finally, we prove the completeness of the logical system using the “matrix” technique.

Epistemic Models

In this section, we define the class of models that we use later to define the formal semantics of our logical system. Throughout the paper, we assume a fixed nonempty set of propositional variables.

Definition 1 *A tuple $(W, \mathcal{A}, P, \sim, \pi)$ is an epistemic model if*

1. *W is a (possibly empty) set of all “worlds”,*
2. *$\mathcal{A}$ is a (possibly empty) set of “agents”,*
3. *$P \subseteq \mathcal{A} \times W$ is a “presence” relation,*
4. *$\sim_a$ is an “indistinguishability” equivalence relation on the set $P_a = \{w \in W \mid aPw\}$ for each agent $a \in \mathcal{A}$,*
5. *$\pi(p) \subseteq P$ for each propositional variable p.*

In addition to the notation P_a, introduced above, it is also convenient to use the notation $P_w = \{a \in \mathcal{A} \mid aPw\}$.

In our “museum” running example, set W consists of worlds w_1 and w_2. Set $\mathcal{A}$ is $\{a, b, c\}$. Presence relation P consists of all pairs from the set $\mathcal{A} \times W$ except for (b, w_2) because agent b is not present in world w_2. In the same example, $w_1 \sim_a w_2$. The relations $\sim_b$ and $\sim_c$ are not important for that example. If propositional variable p represents the statement “is a WeRide”, then $\pi(p) = \{(b, w_1), (c, w_2)\}$.

Syntax and Semantics

The language Φ of our logical system is defined by the grammar:

$$\varphi := p \mid \neg\varphi \mid \varphi \to \varphi \mid \mathsf{K}\varphi \mid \mathsf{R}\varphi \mid \mathsf{D}\varphi,$$

where p is a propositional variable. We read $\mathsf{K}\varphi$ as “knows φ about herself”, $\mathsf{R}\varphi$ as “de re aware about someone with property φ”, and $\mathsf{D}\varphi$ as “de dicto aware about someone with property φ”. We assume that conjunction $\wedge$ and disjunction $\vee$ as well as constants truth $\top$ and false $\bot$ are defined in the standard way.

Definition 2 *For any world $w \in W$, any agent $a \in P_w$ of an epistemic model $(W, \mathcal{A}, P, \sim, \pi)$, and any formula $\varphi \in \Phi$, the satisfaction relation $w, a \Vdash \varphi$ is defined recursively as follows:*

1. *$w, a \Vdash p$ if $(a, w) \in \pi(p)$,*
2. *$w, a \Vdash \neg\varphi$ if $w, a \nVdash \varphi$,*
3. *$w, a \Vdash \varphi \to \psi$ if $w, a \nVdash \varphi$ or $w, a \Vdash \psi$,*
4. *$w, a \Vdash \mathsf{K}\varphi$ if $u, a \Vdash \varphi$, for each world $u \in P_a$ such that $w \sim_a u$,*
5. *$w, a \Vdash \mathsf{R}\varphi$ if there is such an agent $b \in P_w$ that*
   (a) $w, b \Vdash \varphi$ and
   (b) for any world $u \in P_a$ if $w \sim_a u$, then $u \in P_b$,
6. *$w, a \Vdash \mathsf{D}\varphi$ if for each world $u \in P_a$ such that $w \sim_a u$ there is an agent $b \in P_u$ such that $u, b \Vdash \varphi$.*

Note that item 4 above requires property φ to be true **about** agent a in all worlds indistinguishable **by** agent a from the current world. Thus, modality $\mathsf{K}\varphi$ captures the knowledge of φ by agent a about herself.

Item 5 above states that agent b has property φ in the current world w and agent b is present in all worlds indistinguishable by agent a from the current world. In other words, agent b has property φ and agent a is aware of agent b. In this case, we say that agent a is *de re* aware of φ. In our example in Figure 1, agent c has the property “is a police vehicle near the museum” in the current world w_1 and agent c is present in all worlds indistinguishable by Ann from the current world.

Item 6 above states that in each world indistinguishable by a from the current world, there is at least one agent with property φ. Thus, agent a is *de dicto* aware of φ. In our running example, there is a WeRide vehicle near the museum in each of the worlds indistinguishable by Ann from the current world w_1.

In the philosophy of language, the type of semantics that we gave in Definition 2 is sometimes called a 2D-semantics (Schroeter 2021). Modality K for such semantics has been studied before (Seligman, Liu, and Girard 2011, 2013; Epstein and Naumov 2021; Epstein, Naumov, and Tao 2023; Naumov and Tao 2023). Modalities R and D are original to this paper.

Axiomatisation

In addition to the tautologies in language Φ, our logical system has the following axioms:

1. Truth: $\mathsf{K}\varphi \to \varphi$,
2. Negative Introspection: $\neg\mathsf{K}\varphi \to \mathsf{K}\neg\mathsf{K}\varphi$,
3. Distributivity: $\mathsf{K}(\varphi \to \psi) \to (\mathsf{K}\varphi \to \mathsf{K}\psi)$,
4. Self-Awareness: $\varphi \to \mathsf{R}\varphi$ and $\mathsf{K}\varphi \to \mathsf{D}\varphi$,
5. Introspection of Awareness: $\mathsf{D}\varphi \to \mathsf{KD}\varphi$,
6. Unawareness of Falsehood: $\neg\mathsf{R}\bot$ and $\neg\mathsf{D}\bot$,
7. Disjunctivity: $\mathsf{R}(\varphi \vee \psi) \to \mathsf{R}\varphi \vee \mathsf{R}\psi$,
8. General Awareness: $\mathsf{D}(\mathsf{R}\varphi \vee \mathsf{D}\varphi) \to \mathsf{D}\varphi$.

The first three axioms are the standard axioms of the epistemic logic. It is easy to see that they hold for "knows about herself" modality K.

Note that Definition 2 requires agent a to be present in the world w each time when $w, a \Vdash \varphi$. Also, by Definition 1, relation $\sim_a$ is defined only on the worlds in which agent a is present. As a result, each agent is present in all worlds that she cannot distinguish from the current world. Thus, each agent is aware of her own presence in the current world. Then, if agent a has property φ, then a is *de re* aware of φ. We capture this observation in the first Self-Awareness axiom. If agent a has property $\mathsf{K}\varphi$, then φ is true about a in all worlds indistinguishable by a from the current world. Hence, if agent a has property $\mathsf{K}\varphi$, then a is *de dicto* aware of φ. We capture this in the second Self-Awareness axiom.

By item 6 of Definition 2, an agent is de dicto aware of φ if each indistinguishable world contains an agent with property φ. Thus, if formula $\mathsf{D}\varphi$ is true, then this formula must also be true in all indistinguishable worlds. We state this in the Introspection of Awareness axiom. A similar axiom for de re modality R, generally speaking, is not valid.

Note that items 5 and 6 of Definition 2 require that formula φ must be true about agent b in the current world. Thus, an agent cannot be either de re or de dicto aware of a falsehood. We state this in the two Unawareness of Falsehood axioms.

Item 5(a) of Definition 2 requires that for $w, a \Vdash \mathsf{R}(\varphi \vee \psi)$ to be true, formula $\varphi \vee \psi$ must be true in the current world about some agent b. Then either φ or ψ must be true about b in *the current world*. As a result, either statement $w, a \Vdash \mathsf{R}\varphi$ or $w, a \Vdash \mathsf{R}\psi$ must be true. This justifies the Disjunctivity axiom.

Note that if, in each indistinguishable world, there is someone who is (either de re or de dicto) aware of a WeRide, then there must be a WeRide in each indistinguishable world. We capture this observation in the General Awareness axiom. We discuss the axiom's name in the section on λ-assured sets.

We write $\vdash \varphi$ and say that formula $\varphi \in \Phi$ is a *theorem* of our logical system if φ is derivable from the above axioms using the Modus Ponens, the Necessitation, and the two forms of the Monotonicity inference rules:

$$\frac{\varphi, \varphi \to \psi}{\psi} \qquad \frac{\varphi}{\mathsf{K}\varphi} \qquad \frac{\varphi \to \psi}{\mathsf{D}\varphi \to \mathsf{D}\psi} \qquad \frac{\varphi \to \psi}{\mathsf{R}\varphi \to \mathsf{R}\psi}.$$

In addition to unary relation $\vdash \varphi$, we also consider a binary relation $X \vdash \varphi$ between a set of formulae $X \subseteq \Phi$ and a formula $\varphi \in \Phi$. We say that $X \vdash \varphi$ is true if formula φ is derivable from the *theorems* of our logical system and the set of additional axioms X using *only* the Modus Ponens inference rule. It is easy to see that the statements $\varnothing \vdash \varphi$ and $\vdash \varphi$ are equivalent. We say that the set of formulae X is consistent if $X \nvdash \bot$. The theorem below captures our informal discussion above. Formally, it follows from Definition 2.

Lemma 1 (Lindenbaum) *Any consistent set of formulae can be extended to a maximal consistent set of formulae.*

Proof. The standard proof of Lindenbaum's lemma (Mendelson 2009, Proposition 2.14) applies here. ⊠

Soundness

Theorem 1 (soundness) *If $\vdash \varphi$, then $w, a \Vdash \varphi$ for each world w and each agent $a \in P_w$ of each epistemic model.*

The soundness of the Truth, the Negative Introspection, the Distributivity, the Self-Awareness, the Introspection of Awareness, the Unawareness of Falsehood, and the Disjunctivity axioms as well as of the inference rules are straightforward. Below, we prove the soundness of the General Awareness axiom as a separate lemma.

Lemma 2 *If $w, a \Vdash \mathsf{D}(\mathsf{R}\varphi \vee \mathsf{D}\varphi)$, then $w, a \Vdash \mathsf{D}\varphi$.*

Proof. Suppose that $w, a \nVdash \mathsf{D}\varphi$. Thus, by item 6 of Definition 2, there exists a world $u \in W$ such that

$$w \sim_a u \tag{1}$$

and

$$\forall b \in P_u \; (u, b \nVdash \varphi). \tag{2}$$

At the same time, by the assumption of the lemma and the same item 6 of Definition 2, there exists an agent $c \in P_u$ such that $u, c \Vdash \mathsf{R}\varphi \vee \mathsf{D}\varphi$. Thus, one of the following two cases takes place:

Case I: $u, c \Vdash \mathsf{R}\varphi$. Hence, by item 5(a) of Definition 2, there exists an agent $d \in P_u$ such that $u, d \Vdash \varphi$, which contradicts statement (2).

Case II: $u, c \Vdash \mathsf{D}\varphi$. Hence, by item 6 of Definition 2 and statement (1), there exists an agent $d \in P_u$ such that $u, d \Vdash \varphi$, which contradicts statement (2). ⊠

In the rest of the paper, we prove the completeness of our logical system.

λ-assured sets

In this section, we introduce a technical notion of a λ-assured set that will be used in the next section. Throughout the rest of the paper, we use the notation $\mathsf{A}\varphi$ to denote the formula $\mathsf{R}\varphi \vee \mathsf{D}\varphi$. We read $\mathsf{A}\varphi$ as "is generally aware of φ". The General awareness axiom is essentially using this modality and is named after it.

By $\mathsf{A}^n\varphi$ we mean the formula $\underbrace{\mathsf{A}\dots\mathsf{A}}_{n \text{ times}}\varphi$. In the special case $n = 0$, the notation $\mathsf{A}^n\varphi$ denotes formula φ.

Definition 3 *A set X of formulae is λ-**assured** if $X \nvdash \mathsf{A}^n\neg\lambda$ for each $n \geq 0$.*

To develop an intuition about λ-assured sets, let X be a maximal consistent set of formulae and λ be the property "is not a spy". Thus, $\neg\lambda$ is the property "is a spy". The formula $\mathsf{A}\neg\lambda$ means the agent is generally aware of a spy. The formula $\neg\mathsf{A}\neg\lambda$ means that the agent is not aware of a spy. The formula $\neg\mathsf{A}\mathsf{A}\neg\lambda$ means that the spy is embedded so well that the agent is not even aware of anyone who is aware of a spy. The formula $\neg\mathsf{A}\mathsf{A}\mathsf{A}\neg\lambda$ allows only the existence of "super spies" who are hidden so well that the agent is not aware of anyone aware of anyone aware of a spy. The notion of λ-assurance captures the fact that only the existence of absolutely undetectable "ghost" spies is consistent with set X. In other words, it says that all "detectable" agents in the setting captured by set X must have property λ.

Completeness

Frames

Traditionally, proofs of the completeness in modal logic use a canonical model in which worlds are defined as maximal consistent sets of formulae. At the core of such proofs is a "truth" lemma stating that $\varphi \in w$ if and only if $w \Vdash \varphi$. This approach is not easy to apply to 2D-semantics as it requires a "decoupling" of a maximal consistent set into a world and an agent. In this paper, we use the "matrix" technique for such decoupling recently proposed by Naumov and Tao (2023). The technique consists of building the canonical model as a matrix, whose rows correspond to worlds and whose columns correspond to agents. The elements of the matrix are maximal consistent sets representing all formulae that are satisfied in a given world-agent combination. (Naumov and Tao 2023) proves the completeness of a logical system for "telling apart" modality. This system contains modality K, but it does not contain awareness modalities and it does not deal with de re/de dicto distinction.

In this paper, we adapt the matrix technique Naumov and Tao (2023) in a novel way: frames include an explicit awareness relation, row labels λ_w, and the requirement that each X_{wa} be λ_w-assured. To emphasise these additions, we refer to our matrices as "frames".

Definition 4 *A frame is a tuple $(\alpha, \beta, \lambda, P, X, \sim, \rightsquigarrow)$, where*

1. *α, β are ordinals and $\lambda_w \in \Phi$ is a formula for each $w < \alpha$,*
2. *$P \subseteq \alpha \times \beta$ is a "presence" relation; we read $(w, a) \in P$ as "agent a is present at world w"; we slightly abuse the notations and for each $w < \alpha$ and each $a < \beta$ by P_w and P_a we denote the set $\{b < \beta \mid (w, b) \in P\}$ and the set $\{u < \alpha \mid (u, a) \in P\}$, respectively,*
3. *X is a function that maps each pair $(w, a) \in P$ into a λ_w-assured maximal consistent set of formulae denoted by X_{wa},*
4. *$\sim_a$ is an "indistinguishability" equivalence relation on the set P_a for each $a < \beta$ such that for any $w, u < \alpha$ and any formula $\varphi \in \Phi$,*
   (a) if $w \sim_a u$, then $\mathsf{K}\varphi \in X_{wa}$ iff $\mathsf{K}\varphi \in X_{ua}$,
5. *$\rightsquigarrow_w$ is a reflexive "awareness" relation on the set P_w for each $w < \alpha$ such that for any $u < \alpha$, any $a, b < \beta$, and any formula $\varphi \in \Phi$,*
   (a) if $a \rightsquigarrow_w b$ and $w \sim_a u$, then $a \rightsquigarrow_u b$,
   (b) if $a \rightsquigarrow_w b$ and $\mathsf{R}\varphi \notin X_{wa}$, then $\varphi \notin X_{wb}$.

In linear algebra, matrices usually have a finite number of rows and a finite number of columns. In this paper, we allow infinite matrices with α rows and β columns, where α and β are two ordinals. Recall that elements of an ordinal α are ordinals smaller than α. For example, $0 = \varnothing$, $1 = \{0\}$, $2 = \{0, 1\}, \dots \omega = \{0, 1, 2, \dots\}, \omega + 1 = \{0, 1, 2, \dots, \omega\}$. If a matrix has α rows, then we assume that the rows are indexed by the elements of ordinal α. For example, a three-row matrix has row 0, row 1, and row 2.

Informally, a "matrix" is usually defined as a table. Formally, a matrix is a function X on the Cartesian product of the set of rows and the set of columns. Following the tradition, we use the notation X_{wa} to denote the value of the matrix function X on the pair (w, a). Note that we define X as a *total* function on the set $\alpha \times \beta$. Thus, $X_{w,a}$ is defined even if $(w, a) \notin P$. This is done only to avoid constant references to the domain of X in the proofs. If $(w, a) \notin P$, then it is not significant for our proof which exactly λ_w-assured maximal consistent set is X_{wa}.

There is a significant difference between the way awareness is treated in epistemic models (Definition 1) and frames (Definition 4). Intuitively, an agent a is "aware" of an agent b in an epistemic model if agent b is present in all worlds that agent a cannot distinguish from the current world. As we will see later, a frame represents a *partially constructed* model. Thus, some of the worlds (rows) might be missing and they will be added later. If we attempt to define awareness in frames the same way as it is done in epistemic models, an agent might become "unaware" of another agent after a new possible world is introduced. To make our construction work properly, we want to avoid this "loss of awareness effect". This problem did not exist in work (Naumov and Tao 2023) that does not deal with awareness. **To achieve this goal, in this paper we equip our frames with an awareness relation $a \rightsquigarrow_w b$.** Intuitively, it means that in world w agent a is aware of agent b. Item 5(a) of Definition 4 states that if agent a is aware of b in the current world, then a is aware of b in each indistinguishable world.

Note that there are two distinct references to awareness in our frame. One of them is semantical: through relation $\rightsquigarrow_w$ on the columns. The other is syntactical, through modalities R and D occurring in the formulae from a set X_{wa}. Item 5(b) of Definition 4 connects these two references to awareness. It states: if $a \rightsquigarrow_w b$ and $\varphi \in X_{wb}$, then $\mathsf{R}\varphi \in X_{wa}$. That is: if a is semantically aware of b and b has property φ, then a is syntactically (de re) aware of someone with property φ. In Definition 4, we state this item in contrapositive form for ease of use.

Complete Frames

As briefly mentioned in the previous subsection, frames represent partially built models. In order to be convertible into a canonical model a frame must be complete.

Definition 5 *A frame* $(\alpha, \beta, \lambda, P, X, \sim, \rightsquigarrow)$ *is **complete** if for each* $(u, b) \in P$ *and each formula* $\varphi \in \Phi$,

1. *if* $\mathsf{K}\varphi \notin X_{ub}$, *then there is* $v \in P_b$ *such that* $u \sim_b v$ *and* $\varphi \notin X_{vb}$,
2. *if* $\mathsf{R}\varphi \in X_{ub}$, *then there is* $c \in P_u$ *such that* $b \rightsquigarrow_u c$ *and* $\varphi \in X_{uc}$,
3. *if* $b \not\rightsquigarrow_u c$, *then there is* $v \in P_b$ *such that* $u \sim_b v$ *and* $c \notin P_v$,
4. *if* $\mathsf{D}\varphi \notin X_{ub}$, *then there is* $v \in P_b$ *such that* $u \sim_b v$, *and* $\lambda_v = \neg\varphi$,
5. *if* $\mathsf{D}\varphi \in X_{ub}$, *then there is* $c \in P_u$ *and* $\varphi \in X_{uc}$.

A frame $(\alpha, \beta, \lambda, P, X, \sim, \rightsquigarrow)$ is **finite** if ordinals α and β are finite. In Lemma 11, we prove that any finite frame can be extended to a complete frame. The formal definition of an extension is below.

Definition 6 *A frame* $(\alpha', \beta', \lambda', P', X', \sim', \rightsquigarrow')$ *is an **extension** of a frame* $(\alpha, \beta, \lambda, P, X, \sim, \rightsquigarrow)$ *if*

1. $\alpha \leq \alpha'$ *and* $\beta \leq \beta'$,
2. $\lambda'_w = \lambda_w$ *for each* $w < \alpha$,
3. $P' \cap (\alpha \times \beta) = P$,
4. $X'_{wa} = X_{wa}$ *for* $(w, a) \in P$,
5. $w_1 \sim_a w_2$ *iff* $w_1 \sim'_a w_2$ *for each* $a < \beta$ *and each* $w_1, w_2 \in P_a$,
6. $a_1 \rightsquigarrow_w a_2$ *iff* $a_1 \rightsquigarrow'_w a_2$ *for each* $w < \alpha$ *and each* $a_1, a_2 \in P_w$.

The following lemmas are proven in the full version.

Lemma 3 *For any finite frame* $(\alpha, \beta, \lambda, P, X, \sim, \rightsquigarrow)$, *any* $(u, b) \in P$, *and any formula* $\mathsf{D}\varphi \in X_{ub}$, *there is an extension* $(\alpha, \beta + 1, \lambda', P', X', \sim', \rightsquigarrow')$ *such that* $\varphi \in X'_{u\beta}$.

Lemma 4 *For any finite frame* $(\alpha, \beta, \lambda, P, X, \sim, \rightsquigarrow)$, *any* $(u, b) \in P$, *and any formula* $\mathsf{K}\varphi \notin X_{ub}$, *there is an extension* $(\alpha + 1, \beta, \lambda', P', X', \sim', \rightsquigarrow')$ *such that (i)* $u \sim'_b \alpha$, *(ii)* $\varphi \notin X'_{\alpha b}$, *and (iii) for each* $c < \beta$, *if* $b \not\rightsquigarrow_u c$, *then* $c \notin P'_\alpha$.

Lemma 5 *For any finite frame* $(\alpha, \beta, \lambda, P, X, \sim, \rightsquigarrow)$, *any* $(u, b) \in P$, *and any formula* $\mathsf{R}\varphi \in X_{ub}$, *there is an extension* $(\alpha, \beta + 1, \lambda', P', X', \sim', \rightsquigarrow')$ *such that* $b \rightsquigarrow'_u \beta$ *and* $\varphi \in X'_{u\beta}$.

Lemma 6 *For any finite frame* $(\alpha, \beta, \lambda, P, X, \sim, \rightsquigarrow)$, *any* $(u, b) \in P$, *and any formula* $\mathsf{D}\varphi \notin X_{ub}$, *there is an extension* $(\alpha + 1, \beta, \lambda', P', X', \sim', \rightsquigarrow')$ *such that* $u \sim'_b \alpha$ *and* λ'_α *is equal to* $\neg\varphi$.

We write $F \sqsubseteq F'$ if frame F' is an extension of the frame F. For any (finite or infinite) chain of frames $F_1 \sqsubseteq F_2 \sqsubseteq F_3 \sqsubseteq F_4 \sqsubseteq \dots$, where $F_i = (\alpha_i, \beta_i, \lambda_i, P_i, X_i, \sim_i, \rightsquigarrow_i)$, the limit $\lim_i F_i$ is the tuple $(\bigcup_i \alpha_i, \bigcup_i \beta_i, \bigcup_i \lambda_i, \bigcup_i P_i, \bigcup_i X_i, \bigcup_i \sim_i, \bigcup_i \rightsquigarrow_i)$. As usual, to compute the union of functions we treat them as functional relations (sets of pairs). The next lemma follows from Definition 6.

Lemma 7 *The limit of a chain of extensions* $F_1 \sqsubseteq F_2 \sqsubseteq F_3 \sqsubseteq \dots$ *is an extension of the frame* F_1.

Definition 7 *A **Type 1** requirement is a tuple* (u, b, φ), *where* $u, b < \omega$ *and* $\varphi \in \Phi$. *In a given frame* $(\alpha, \beta, \lambda, P, X, \sim, \rightsquigarrow)$ *this requirement is*

1. ***active*** *if* $u < \alpha$, $b < \beta$, *and* $(u, b) \in P$,
2. ***fulfilled*** *if it is active and* u, b, *and* φ *satisfy item 1 of Definition 5.*

The definition of **Type 2**, **Type 4**, and **Type 5** requirements are identical to the one above except that they refer to item 2, item 4, and item 5 of Definition 5.

Definition 8 *A **Type 3** requirement is a tuple* (u, b, c), *where* $u, b, c < \omega$. *In a given frame* $(\alpha, \beta, \lambda, P, X, \sim, \rightsquigarrow)$ *this requirement is*

1. ***active*** *if* $u < \alpha$, $b, c < \beta$, $(u, b) \in P$, *and* $(u, c) \in P$,
2. ***fulfilled*** *if it is active and* u, b, *and* c *satisfy item 3 of Definition 5.*

Lemma 8 *Any finite frame that has an active unfulfilled requirement (of any type), can be extended to a finite frame where the same requirement is fulfilled.*

Proof. For requirements of Type 1, Type 2, Type 4, and Type 5, the statement of the lemma follows from Lemma 4, Lemma 5, Lemma 6, and Lemma 3, respectively.

In the case of Type 3 requirement, notice that $\mathsf{K}\bot \to \bot$ is an instance of the Truth axiom. Thus, $\mathsf{K}\bot \notin X_{ub}$ for each $(u, b) \in P$ because set X_{ub} is consistent. Therefore, the statement of the lemma follows from Lemma 4, where φ is $\bot$. ⊠

The next lemma follows from Definition 6 and the definition of a "fulfilled" requirement.

Lemma 9 *If a requirement (of any of the five types) is active and fulfilled in a frame, then it is also fulfilled in any extension of the frame.*

The lemma below follows from Definition 5 and the definition of a "fulfilled" requirement.

Lemma 10 *If all active requirements (of all five types) are fulfilled in a frame, then the frame is complete.*

Lemma 11 *Any finite frame can be extended to a complete frame.*

Proof. Let F be an arbitrary finite frame. Observe that there are countably many requirements of each of the five types. Let $r_1, r_2, r_3, \dots$ be an enumeration of all requirements of all five types (combined). We define a (possibly infinite) chain of *finite* frames $F_1 \sqsubseteq F_2 \sqsubseteq \dots$ recursively:

1. $F_1 = F$,
2. if frame F_n does not contain any active unfulfilled requirements, then F_n is the last element of the chain,
3. otherwise, let r_{min} be the first (in terms of the enumeration $r_1, r_2, r_3, \dots$) active unsatisfied requirement in frame F_n; by Lemma 8, frame F_n can be extended to a finite frame F_{n+1} that fullfils requirement r_{min}.

Claim 1 *Frame* $\lim_n F_n$ *is complete.*

Proof of Claim. Consider any requirement r (of any of the five types). By Lemma 10, it suffices to show that if requirement r is active in frame $\lim_n F_n$, then it is fulfilled.

Indeed, if r is active in $\lim_n F_n$, then (by definition of being "active") r must be active in frame F_i for some $i \geq 0$. Observe that, due to the construction of the chain $F_1 \sqsubseteq$

$F_2 \sqsubseteq \dots$ if requirement r is active in frame F_i, then it is fulfilled in frame F_j for some $j \geq i$. Therefore, requirement r is fulfilled in frame $\lim_n F_n$ by Lemma 7 and Lemma 9. ⊠

Frame $\lim_n F_n$ is an extension of the frame $F_1 = F$ by Lemma 7. ⊠

Canonical Model

For any given frame $(\alpha, \beta, \lambda, P, X, \sim, \rightsquigarrow)$ we consider an epistemic model $(\alpha, \beta, P, \sim, \pi)$, where

$$\pi(p) = \{(w, a) \mid p \in X_{wa}\}. \tag{3}$$

Note that, in particular, worlds of the model are the elements of α and agents are the elements of β. The next lemma connects the epistemic model and the frame on which it is based. This lemma plays the role of a "truth" lemma in the classical proofs of completeness.

Lemma 12 *If frame $(\alpha, \beta, \lambda, P, X, \sim, \rightsquigarrow)$ is complete, then $w, a \Vdash \varphi$ iff $\varphi \in X_{wa}$ for any world $w < \alpha$, any agent $a \in P_w$, and any formula $\varphi \in \Phi$.*

Proof. We prove the lemma by induction on structural complexity of formula φ. If φ is a propositional variable, then the statement of the lemma follows from statement (3) and item 1 of Definition 2. If φ is a negation or an implication then the statement of the lemma follows from either item 2 or item 3 of Definition 2, the induction hypothesis, and the maximality and consistency of set X_{wa} in the standard way.

Suppose that formula φ has the form $\mathsf{K}\psi$.
$(\Rightarrow)$: Assume that $\mathsf{K}\psi \notin X_{wa}$. Thus, by item 1 of Definition 5, there is $u \in P_a$ such that $w \sim_a u$ and $\psi \notin X_{ua}$. Then, $u, a \nVdash \psi$ by the induction hypothesis. Therefore, $w, a \nVdash \mathsf{K}\psi$ by item 4 of Definition 2.
$(\Leftarrow)$: Assume that $\mathsf{K}\psi \in X_{wa}$. Consider any $u < \alpha$ such that $w \sim_a u$. By item 4 of Definition 2 it suffices to show that $u, a \Vdash \psi$. Indeed, the assumptions $\mathsf{K}\psi \in X_{wa}$ and $w \sim_a u$, by item 4(a) of Definition 4, imply that $\mathsf{K}\psi \in X_{ua}$. Then, $X_{ua} \vdash \psi$ by the Truth axiom and the Modus Ponens inference rule. Thus, $\psi \in X_{ua}$ because X_{ua} is a maximal consistent set. Hence, $u, a \Vdash \psi$ by the induction hypothesis.

Suppose that formula φ has the form $\mathsf{R}\psi$.
$(\Rightarrow)$: Assume $w, a \Vdash \mathsf{R}\psi$. Thus, by item 5 of Definition 2, there is an agent $b \in P_w$ such that two facts hold. First,

$$w, b \Vdash \psi. \tag{4}$$

Second, for any world $u \in P_a$, if $w \sim_a u$, then $u \in P_b$. Then, by the contraposition of item 3 of Definition 5,

$$a \rightsquigarrow_w b. \tag{5}$$

At the same time, by the induction hypothesis, statement (4) implies $\psi \in X_{wb}$. Therefore, $\mathsf{R}\psi \in X_{wa}$ by statement (5) and item 5(b) of Definition 4 applied contrapositively.
$(\Leftarrow)$: Assume that $\mathsf{R}\psi \in X_{wa}$. Then, by item 2 of Definition 5, there is $b \in P_w$ such that $a \rightsquigarrow_w b$ and $\psi \in X_{wb}$. Thus, by the induction hypothesis,

$$w, b \Vdash \psi. \tag{6}$$

Furthermore, by item 5(a) of Definition 4, $a \rightsquigarrow_u b$ for every world $u \in P_a$ such that $w \sim_a u$. Note that $\rightsquigarrow_u$ is a relation on set P_u. Thus, $b \in P_u$ for every world $u \in P_a$ such that $w \sim_a u$. In other words, $u \in P_b$ for every $u \in P_a$ such that $w \sim_a u$. Therefore $w, a \Vdash \mathsf{R}\psi$ by equation (6) and item 5 of Definition 2.

Suppose that formula φ has the form $\mathsf{D}\psi$.
$(\Rightarrow)$: Towards contradiction, assume $\mathsf{D}\psi \notin X_{wa}$. Thus, by item 4 of Definition 5, there is world $u \in P_a$ such that

$$w \sim_a u, \tag{7}$$

$$\lambda_u = \neg\psi. \tag{8}$$

By the assumption $w, a \Vdash \mathsf{D}\psi$, statement (7), and item 6 of Definition 2, there exists an agent $b \in P_u$ such that $u, b \Vdash \psi$. Hence, $\psi \in X_{ub}$ by the induction hypothesis. Then, $X_{ub} \vdash \neg\neg\psi$ by the laws of propositional reasoning. Thus, $X_{ub} \vdash \neg\lambda_u$ by equation (8). In other words, $X_{ub} \vdash \mathsf{A}^0\neg\lambda_u$. Therefore, set X_{ub} is not λ_u-assured by Definition 3, which contradicts item 3 of Definition 4.
$(\Leftarrow)$: We need to show that $w, a \Vdash \mathsf{D}\psi$. Consider a world $u \in P_a$ such that

$$w \sim_a u. \tag{9}$$

By item 6 of Definition 2, it suffices to show that there exists an agent $b \in P_u$ such that $u, b \Vdash \psi$.

Assume that $\mathsf{D}\psi \in X_{wa}$. Then, by the Introspection of Awareness axiom and the Modus Ponens inference rule, $X_{wa} \vdash \mathsf{KD}\psi$. Thus, $\mathsf{KD}\psi \in X_{wa}$ because X_{wa} is a maximal consistent set. Hence, $\mathsf{KD}\psi \in X_{ua}$ by item 4 of Definition 4 and statement (9). Then, by the Truth axiom and the Modus Ponens inference rule, $X_{ua} \vdash \mathsf{D}\psi$. Thus, $\mathsf{D}\psi \in X_{ua}$ since X_{ua} is a maximal consistent set. Hence, by item 5 of Definition 5, there exists an agent $b \in P_u$ such that $\psi \in X_{ub}$. Therefore, by the induction hypothesis, $u, b \Vdash \psi$. ⊠

Theorem 2 (strong completeness) *If $X \nvdash \varphi$, then there is a world w and an agent a of an epistemic model such that $w, a \Vdash \chi$ for each formula $\chi \in X$ and $w, a \nVdash \varphi$.*

Proof. Set $\{\neg\varphi\} \cup X$ is consistent by the assumption $X \nvdash \varphi$. By Lemma 1, it can be extended to a maximal consistent set X_{00}. Consider tuple $F = (1, 1, \lambda, P, X, \sim, \rightsquigarrow)$, where

1. $\lambda_0 = \top$,
2. $P = \{(0, 0)\}$,
3. $X(0, 0) = X_{00}$,
4. $\sim_0 = \{(0, 0)\}$,
5. $\rightsquigarrow = \{(0, 0)\}$.

This tuple is a frame by Definition 4. By Lemma 11, frame F can be extended to a complete frame F'. Consider the canonical model corresponding to frame F'. Note that $0, 0 \Vdash \chi$ for each $\chi \in X$ and $0, 0 \Vdash \neg\varphi$ by Lemma 12. Therefore, $0, 0 \nVdash \varphi$ by item 2 of Definition 2. ⊠

Conclusion

We have proposed to interpret "awareness" as knowledge of existence and observed that such knowledge can have two distinct forms: de re and de dicto. Our main technical result is a sound and complete logical system that describes the interplay between two modalities representing these two forms of awareness, as well as the standard "knowledge of the fact" modality usually studied in epistemic logic.

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