

## ABSORBER OF TOPOLOGICALLY STRUCTURED LIGHT: SUPPLEMENTAL DOCUMENT

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### S1. Absorption of light with a phase singularity

Figure S1 compares the absorption of radially polarized light, azimuthally polarized light and spin orbit coupled light with right hand circular polarization and an azimuthal index of +1. We can see that where the radially polarized light is completely absorbed, the azimuthally polarized light is almost completely rejected and the spin orbit coupled beam is 50% absorbed, which is consistent with the qualitative geometric phase model given in the main text.

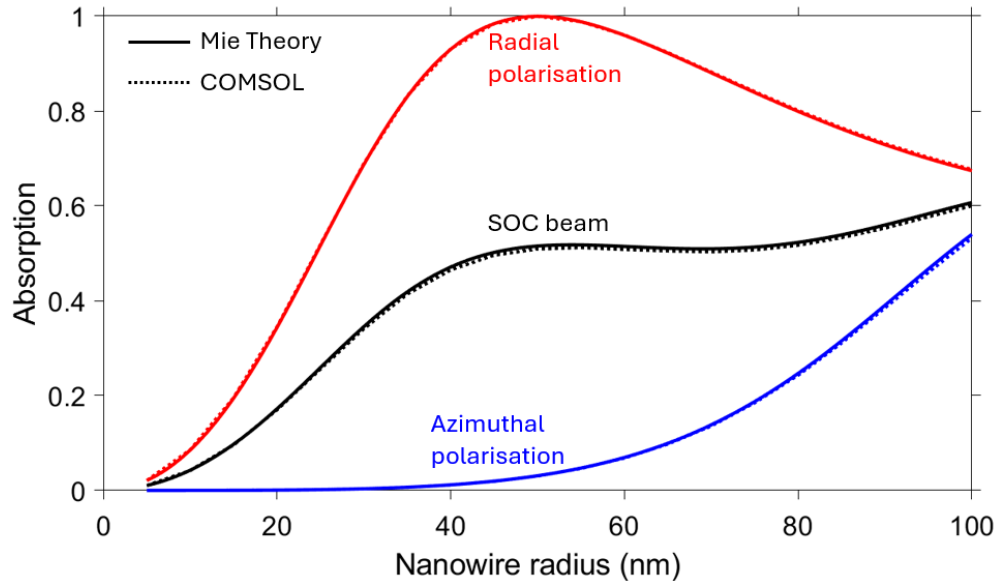


Fig. S 1. Absorption plotted against nanowire radius for radially polarized, azimuthally polarized light and a spin orbit coupled (SOC) beam with right and circular polarization and an azimuthal index of +1.

### S2. Theoretical derivation of absorption

To find the energy absorbed in the nanowire, one must first calculate the field distribution in and around the nanowire, then find the difference between incident and scattered flux.

We begin the derivation with a few simplifying assumptions:

1. The incident beam is either purely radially or azimuthally polarized and perfectly symmetric about the axis of propagation.

2. The beam is well collimated such that, after reflection on the conical mirror, the wave vector of light is purely in the radial direction and the electric field is either along the z axis or along the azimuthal direction.
3. We ignore any edge effects which occur at the ends of the nanowire and the apex of the cone.

The complete field distribution of the system can be obtained using the principles of cylindrical vector harmonics, given in [S1], and the boundary conditions on the surface of the nanowire  $\hat{\mathbf{n}} \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$  and  $\hat{\mathbf{n}} \times (\mathbf{H}_1 - \mathbf{H}_2) = 0$ , where  $\hat{\mathbf{n}}$  is the unit vector normal to the nanowire surface. The cylindrical vector harmonics are obtained by taking the curl of the solution to the Helmholtz equation in cylindrical coordinates and are as follows:

$$\mathbf{M}_s^{(l)} = \sqrt{k^2 - h^2} \left( is \frac{Z_s^{(l)}(nk\rho)}{nk\rho} \hat{\mathbf{e}}_\rho - Z_s^{(l)'}(nk\rho) \hat{\mathbf{e}}_\phi \right) e^{i(s\phi + hz)} \quad (\text{S1})$$

$$\mathbf{N}_s^{(l)} = \frac{\sqrt{k^2 - h^2}}{k} \left( ihZ_s^{(l)'}(nk\rho) \hat{\mathbf{e}}_\rho - hs \frac{Z_s^{(l)}(nk\rho)}{nk\rho} \hat{\mathbf{e}}_\phi + \sqrt{k^2 - h^2} Z_s^{(l)}(nk\rho) \hat{\mathbf{e}}_z \right) e^{i(s\phi + hz)}, \quad (\text{S2})$$

where  $(\rho, \phi, z)$  and  $\hat{\mathbf{e}}_\rho, \hat{\mathbf{e}}_\phi, \hat{\mathbf{e}}_z$  are cylindrical coordinates and corresponding unit vectors,  $s$  and  $h$  are the azimuthal and axial wavevector components, respectively,  $k$  is the free-space wavenumber,  $Z_s^{(l)}(nk\rho)$  is a Bessel function of the  $l^{\text{th}}$  kind and  $n = n' + in''$  is the medium refractive index. These vector harmonics can then be used to express the electric and magnetic fields in the system. Using orthogonality relations, we find the incident field can be expressed as

$$\mathbf{E}_b = \sum_{s=-\infty}^{\infty} E_s \mathbf{N}_s^{(1)}, \quad (\text{S3})$$

$$\mathbf{H}_b = \frac{-ik}{\omega\mu} \sum_{s=-\infty}^{\infty} E_s \mathbf{M}_s^{(1)} \quad (\text{S4})$$

when the polarization is parallel to the axis of the cylinder, where  $\mu$  is the magnetic susceptibility, which we take to be unity in this case,  $\omega$  is the angular frequency of the incident light. Since in the case considered here the wave vector is purely in the radial direction (i.e.  $s = h = 0$ ), then the summations in Eqs. (S3-4) collapse into single terms:

$$\mathbf{E}_b = J_0(n_1 k \rho) \hat{\mathbf{e}}_z, \quad (\text{S5})$$

$$\mathbf{H}_b = -\frac{ik}{\omega} J_0'(n_1 k \rho) \hat{\mathbf{e}}_\phi, \quad (\text{S6})$$

respectively, where  $n_1$  is the refractive index of the medium surrounding the nanowire. Note that the choice of Bessel functions is a consequence of the symmetry of the system. The electric field of ingoing and outgoing cylindrical waves is expressed by the 0-th order Hankel functions  $H_0^{(2)}$  and  $H_0^{(1)}$  respectively, thus the interference of the two waves results in a standing wave given by  $J_0$ . Consequently, the fields inside the nanowire are given by

$$\mathbf{E}_i = f_{TM} J_0(n_2 k \rho) \hat{\mathbf{e}}_z, \quad (\text{S7})$$

$$\mathbf{H}_i = -\frac{in_2k}{\omega} f_{TM} J_0'(n_2k\rho) \hat{\mathbf{e}}_\phi, \quad (\text{S8})$$

where  $n_2$  is the index of the nanowire and  $f_{TM}$  is an unknown complex coefficient. In this case, Bessel functions of the first kind also ensure the electric and magnetic fields are finite at the origin. Since the scattered field only exists outside the nanowire, if present, it is not subject the constraint of being finite at the origin and is purely an outgoing cylindrical wave, therefore it can be expressed using the Hankel functions

$$\mathbf{E}_s = -b_{TM} H_0^{(1)}(n_1k\rho) \hat{\mathbf{e}}_z, \quad (\text{S9})$$

$$\mathbf{H}_s = b_{TM} \frac{in_1k}{\omega} H_0^{(1)'}(n_1k\rho) \hat{\mathbf{e}}_\phi, \quad (\text{S10})$$

where  $b_{TM}$  is a complex coefficient. Using the expressions for the field in and around the nanowire along with the boundary conditions mentioned at the start, a pair of simultaneous equations can be derived, which when solved give the expressions

$$b_{TM} = \frac{n_2 J_0(n_1k\alpha) J_0'(n_2k\alpha) - n_1 J_0(n_2k\alpha) J_0'(n_1k\alpha)}{n_2 H_0^{(1)}(n_1k\alpha) J_0'(n_2k\alpha) - n_1 H_0^{(1)'}(n_1k\alpha) J_0(n_2k\alpha)}, \quad (\text{S11})$$

$$f_{TM} = \frac{n_1 H_0^{(1)}(n_1k\alpha) J_0'(n_1k\alpha) - n_1 H_0^{(1)'}(n_1k\alpha) J_0(n_1k\alpha)}{n_2 H_0^{(1)}(n_1k\alpha) J_0'(n_2k\alpha) - n_1 H_0^{(1)'}(n_1k\alpha) J_0(n_2k\alpha)}. \quad (\text{S12})$$

The total outgoing field is

$$\mathbf{E}_{out} = (0.5 - b_{TM}) H_0^{(1)}(n_1k\rho) \hat{\mathbf{e}}_z. \quad (\text{S13})$$

Consequently, the absorption in the nanowire is given by the following expression

$$A_{TM} = 1 - 4|0.5 - b_{TM}|^2. \quad (\text{S14})$$

For the TE case, absorption can be calculated in the same way by expressing the electric field in terms of the  $\mathbf{M}_s^{(l)}$  vector harmonics. The corresponding scattered electric field is given by

$$b_{TE} = \frac{n_2 J_0(n_2k\alpha) J_0'(n_1k\alpha) - n_1 J_0(n_1k\alpha) J_0'(n_2k\alpha)}{n_2 H_0^{(1)'}(n_1k\alpha) J_0(n_2k\alpha) - n_1 H_0^{(1)}(n_1k\alpha) J_0'(n_2k\alpha)}, \quad (\text{S15})$$

and absorption by

$$A_{TE} = 1 - 4|0.5 - b_{TE}|^2. \quad (\text{S16})$$

Spin orbit coupled beams can be expressed as a superposition of TE and TM modes in phase quadrature with equal weighting, thus their absorption can be expressed by

$$A_{SO} = 1 - 2|0.5 - b_{TE}|^2 - 2|0.5 - b_{TM}|^2 = 0.5(A_{TE} + A_{TM}). \quad (\text{S17})$$

### S3. Absorption of topologically trivial light

Here, we consider the response of the absorbing device to topologically trivial light. By 3D finite element calculations, we obtain the absorption of a focused linearly polarized beam (with wavelength  $\lambda=1 \mu\text{m}$  and focal spot of  $3 \mu\text{m}$ ) by a nanowire of varying radius and refractive index placed along the axis of a perfectly conducting cone of radius of  $4.5 \mu\text{m}$  and half-angle of 45 degrees. Figure S2 shows absorption as a function of nanowire radius and imaginary part

of its refractive index for linearly polarized light (a), radially polarized light (b) and azimuthally polarized light (c). The real part of the refractive index is fixed at  $n'=3$ . At the peak of absorption for radially polarized light ( $\alpha \sim 0.05\lambda$  &  $n'' \sim 1$ ), azimuthally and linearly polarized light is only 6% and 8% absorbed, respectively.

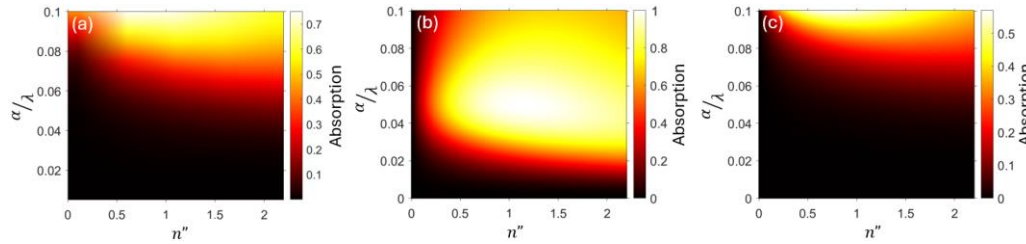


Fig. S2. Absorption as a function of nanowire radius and imaginary component of the refractive index for linearly polarized light (a), radially polarized light (b) and azimuthally polarized light (c). The real part of the refractive index is fixed at  $n'=3$ .

#### S4. Real Materials

Absorption against nanowire radius and illumination wavelength for Zirconium, shown in Fig. S3.a. Absorption of TE and TM light as a function of wavelength for a 40nm radius Zirconium nanowire, demonstrating broadband absorption of TM light (b).

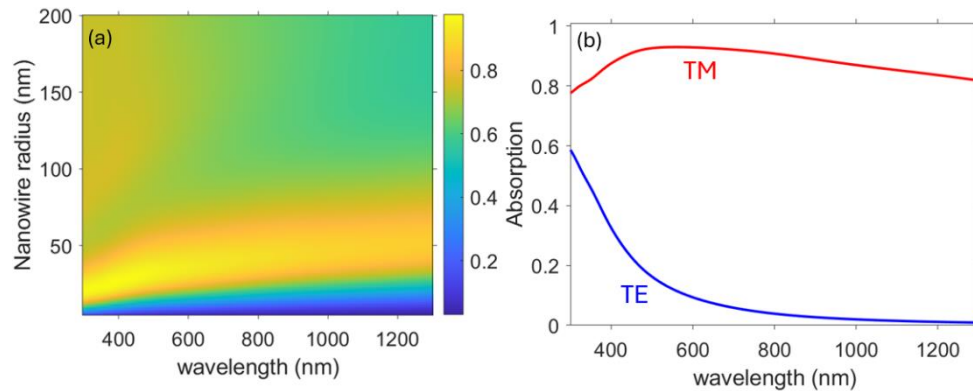


Fig. S3.(a) Absorption of TM light as a function of wavelength and nanowire radius for Zirconium. (b) Absorption as a function of wavelength for a 40nm Zirconium nanowire.

### S5. Topological detector of toroidal light pulses

The considered absorber of toroidal light pulses (TLPs) consisted of a  $WTe_2$  nanowire placed along the axis of a right circular silver cone with a radius of  $25\ \mu\text{m}$  and half-angle of  $45$  degrees. The optical properties of the  $WTe_2$  nanowire used in numerical simulations are shown in Fig. S4 [S2]. The incident TLP was characterized by parameters  $q_1=200\ \text{nm}$  and  $q_2=6,000q_1$ , representing effective wavelength and Rayleigh range, respectively. Its central wavelength was  $800\ \text{nm}$  with a  $200\ \text{nm}$  bandwidth.

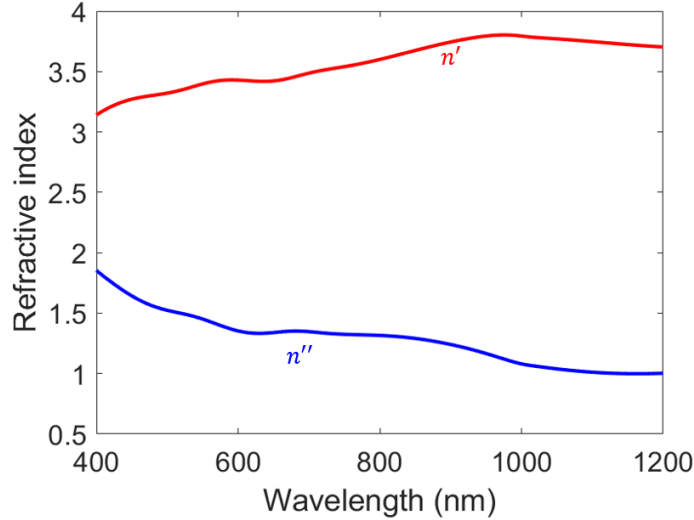


Fig. S4. Real (red) and imaginary (blue) parts of the refractive index for  $WTe_2$ .

The proposed topological detector does not only allow to distinguish between radially (TM) and azimuthally (TE) polarized TLPs (see Fig. S4), but it also rejects topologically trivial light. As an example, we present the absorption spectrum of a linearly polarized pulse in Fig. S5 (black line). Within the  $700$  to  $900\ \text{nm}$  bandwidth of the experimentally demonstrated TLPs, the absorption of TM TLPs is over  $95\%$  while for linearly polarized light (and TE TLPs) it does not exceed  $10\%$ . Finally, we argue that pulses with uniform circular polarization will also be rejected by the topological light detector. Indeed, circular polarized light can be decomposed to two orthogonal linear polarizations, each of which will be rejected by the topological detector. Following the symmetry considerations along the lines of Fig. 1 of the main text, constructive interference on the nanowire is only allowed for incident polarization that results in electric field normal to the nanowire axis, resulting in very weak absorption.

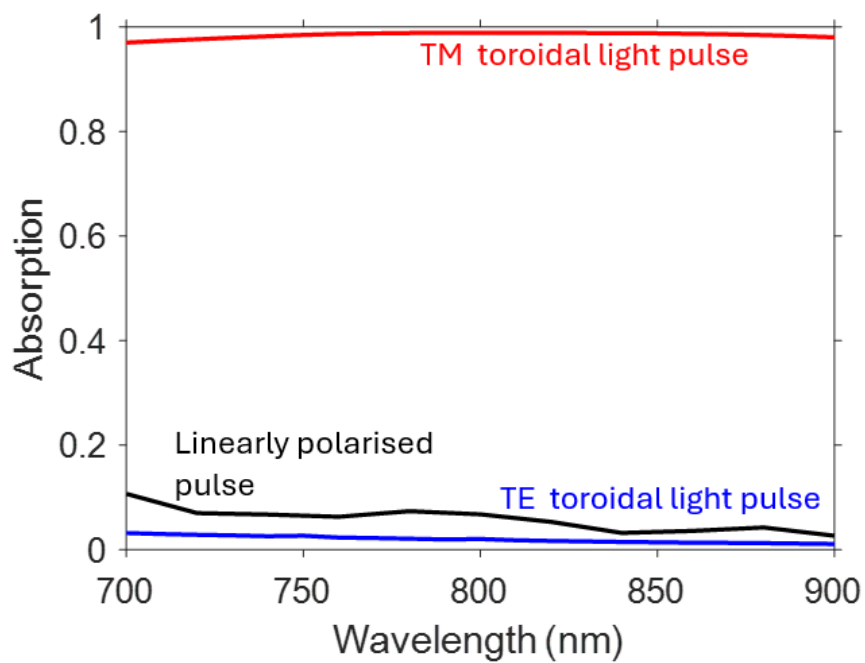


Fig. S5. Absorption spectra for TE (blue) and TM (red) TLPs, and linearly polarized pulses (black).

### References

- S1. C. F. Bohren and D. R. Huffman, "A Potpourri of Particles," in *Absorption and Scattering of Light by Small Particles* (1998), pp. 181-223.
- S2. B. Munkhbat, P. Wróbel, T. J. Antosiewicz, and T. O. Shegai. *ACS Photonics* 9(7), 2398-2407 (2022).