Journal of Physics:
Conference Series

PAPER • OPEN ACCESS

On the Significance of Parameter Uncertainties for Prediction of Leak Noise Wave Speed in Buried Pipes

To cite this article: O. Scussel *et al* 2024 *J. Phys.: Conf. Ser.* **2909** 012009

Journal of Physics: Conference Series **2909** (2024) 012009 doi:10.1088/1742-6596/2909/1/012009

On the Significance of Parameter Uncertainties for Prediction of Leak Noise Wave Speed in Buried Pipes

O. Scussel[1], J M Muggleton[1], M Karimi[2], P Williams[2], M K Kalkowski[1], P F Joseph[1] and P. White[1]

[1] Institute of Sound and Vibration Research, University of Southampton, Highfield, Southampton, SO17 1BJ, UK.
[2] Centre for Audio, Acoustics and Vibration, University of Technology Sydney, NSW 2019, Sydney, Australia.

Corresponding author: oscar.scussel@unesp.br

Abstract. The modern world is facing the challenging issue of water wastage due to leaks, which is causing severe economic, environmental and social impacts. Consequently, the inspection and maintenance of buried water pipes is crucial and there is still a lack of investigations towards the uncertain parameters affecting the wave speed associated with the predominantly fluid-borne wave s=1, the main carrier of leak noise. This study investigates the effects of uncertainties present in the pipe and soil parameters which are affecting the speed of propagation of the leak noise wave. To achieve this, a sensitivity analysis is performed using Monte Carlo simulations and Sobol' indices. Uncertainties are commonly associated with the material and geometrical properties of the pipe along with the surrounding soil characteristics. However, the significance of these parameters varies depending on the type of soil in which the water pipe is buried. In clay soil, the soil-related parameter plays a crucial role compared to sandy soil and this is verified through some experimental work carried out in two water pipe systems with very different properties, one in the UK and the other one in Brazil. This research is of fundamental importance for determining the most critical parameters affecting the leak noise wave, allowing to evaluate and integrate uncertainty information into decision-making of current technologies, such as loggers and leak noise correlators, aiming enhanced detection and location of water leakage in buried plastic water pipes.

1. Introduction

Water leak detection has become an increasingly critical aspect of water resource management, as the modern world faces growing water scarcity and infrastructure challenges [1-3]. Among the various technologies employed, acoustic correlators have proven to be effective tools to pinpoint a leak in buried pipelines and they require the knowledge of the speed at which leak noise wave travels along the pipe-wall [4,5]. Hence, the wave speed can be heavily influenced by the pipe properties and the surrounding soil [6-8].

However, there exists a notable gap in the literature concerning the uncertainties associated with the pipe and soil parameters, and more specifically their influence on the leak noise wave speed. Uncertainties are inherent due to the complexity of a pipe system, which can significantly impact the effectiveness of current leak detection methods. Furthermore, there is a lack of comprehensive exploration of such uncertainties along with a systematic approach to measure the relative importance of different factors, such as pipe material, geometry and soil properties in determining wave speed variations. This motivates the work described in this paper, which aims to fill this gap in knowledge and to provide a useful approach based on Sobol' indices and Monte Carlo simulations [9,10].

Sobol' indices play a crucial role in sensitivity analysis, widely utilized to explore diverse problems [11]. They provide a quantitative measure of the impact of input factors on the overall variability of a model's output. In water leak problems, employing Sobol indices becomes essential for gaining insights into the influence of different parameters and the combination of their effects on the leak noise wave speed.

The rest of the paper is structured as follows. In Section 2, an overview on the stochastic modeling of leak noise wave is given. Section 3 briefly describes the Sobol' indices for sensitivity analysis of the leak noise wave speed. In Section 4, the stochastic wave speed as well as Sobol' indices are estimated for two pipe systems, one in the United Kingdom and the other one in Brazil with very different pipe and soil properties. Some conclusions are then given in Section 5.

2. Uncertainties affecting the speed of the leak noise wave

This section gives an overview of the stochastic model of the leak noise wave speed using the wave dynamic stiffness approach. This facilitates clear interpretation of the wave speed behaviour in terms of the physical properties of the system, which are the pipe material and geometry along with soil properties.

2.1. Overview on the speed of leak noise wave in buried pipes

When there is a leak, internal fluid flows out to generate vibration and it propagates along the pipe and into the soil in the form of an acoustic wave, which is the axisymmetric $(n=0)$ predominantly fluid-borne wave $(s=1)$ and it is heavily influenced by the pipe and soil properties [12,13]. The characteristics and behaviour of this wave have been investigated extensively because of its significance in identifying and locating leaks in water distribution pipes [14,15]. At low frequencies, well below the pipe ring frequency, the simplified wavenumber of $s=1$ wave is calculated using wave dynamic stiffness as follows [16]:

$$k = k_{\text{water}}\left(1+\frac{K_{\text{water}}}{K_{\text{pipe}}+K_{\text{soil}}}\right)^{\frac{1}{2}}, \tag{1}$$

where $K_{\text{water}} = 2B_{\text{water}}/a$ is the stiffness of the water inside the pipe in which B_{water} is the bulk modulus of water and a is the mean radius of the pipe. The term $k_{\text{water}} = \omega/c_{\text{water}}$ is the wavenumber of water, in which c_{water} is the wave speed in water and ω is the angular frequency in rad/s. The terms K_{pipe} and K_{soil} are related to the pipe and soil effects respectively, with $K_{\text{pipe}} = E(1-i\eta)h/a^2 - \rho h\omega^2$ where E, ρ, a, h and η are the Young's modulus, density, mean radius, thickness and loss factor of the pipe respectively, and $\mathrm{i}^2=-1$. The dynamic stiffness for the surrounding soil is given by $K_{\text{soil}} = 2G/a$ where G is the shear modulus of the soil. This paper focuses on the wave speed of $s=1$ wave, which is the real part of k and is given by

$$c = c_{\text{water}}\left(1+\frac{K_{\text{water}}}{K_{\text{pipe}}+K_{\text{soil}}}\right)^{-\frac{1}{2}}, \tag{2}$$

A good estimate of the wave speed is important for the effective functioning of vibro-acoustic correlators in leak detection. These devices rely on analyzing the time it takes for the leak noise to travel between different measurement points in a pipeline. Estimating the wave speed is essential for accurately determining the distance between these points and the leak, and subsequently pinpointing the leak location. Furthermore, formulating the wave speed as a function of wave dynamic stiffnesses of the components of the pipe system facilitates investigations into the way in which the pipe and soil properties affect its behaviour [16]. In the next subsection, an overview of the stochastic model of the wave speed is given.

2.2. Modelling of the stochastic leak noise wave speed

Consider a probability space (Ω, F, P) where Ω is a sample space, F is a $\sigma-$algebra on Ω, which is a non-empty collection of sets that is closed under taking complements, countable unions and intersections, and P is a probability measure on Ω [17-19]. Some parameters from the predominantly fluid-borne wave speed in Eq. (2), such as the pipe and soil properties are described now by random quantities and are defined on the probability space (Ω, F, P). A total of five parameters have been designated as random variables. The wave speed in Eq. (2) is now written as a function of the random variable vector $\boldsymbol{\xi} = \begin{bmatrix} \xi_1 & \xi_2 & \xi_3 & \xi_4 & \xi_5 \end{bmatrix}^T$ and is given by:

$$c(\boldsymbol{\xi}) = c_{\text{water}} \left(1 + \frac{K_{\text{water}}}{\hat{K}_{\text{pipe}} + \hat{K}_{\text{soil}}} \right)^{-\frac{1}{2}}, \tag{3}$$

where ξ_1 is the uncertain variable affecting the Young`s modulus of the pipe with $E(\xi_1): \Omega_1 \to \Re$, ξ_2 is the uncertain variable affecting the density of the pipe material with $\rho(\xi_2): \Omega_2 \to \Re$, ξ_3 is the uncertain variable affecting the mean radius of the pipe with $a(\xi_3): \Omega_3 \to \Re$ and ξ_4 is the uncertain variable influencing the thickness of the pipe-wall with $h(\xi_4): \Omega_4 \to \Re$. In a similar manner, the uncertain variables ξ_5 affects the shear modulus of the soil respectively, with $G(\xi_5): \Omega_5 \to \Re$. Each random variable has its own random space, i.e., $\xi_j \in \Omega_j$ where Ω_j stands for a sample space corresponding to the probability space [17]. The random parameters are then modelled through normal distribution $N(\mu_{\xi_j}, \sigma^2_{\xi_j})$ and probability density function given by $\frac{1}{\sqrt{2\pi\sigma^2_{\xi_j}}} \exp\left(-(\xi_j - \mu_{\xi j})^2 / 2\sigma^2_{\xi_j} \right)$ with $\mathrm{j} \in \{1, 2, \ldots, 5\}$. The terms μ_{ξ_j} and σ_{ξ_j} are the mean and standard deviation of the uncertain variable ξ_j, respectively. Note that the dynamic stiffnesses in the denominator of Eq. (3) $\hat{K}_{\text{pipe}}$ and $\hat{K}_{\text{soil}}$ are simply the stochastic version of the ones given in Eq. (2).

3. Sobol' indices for sensitivity analysis of the leak noise wave speed

The Sobol' indices is an important concept used in sensitivity analysis which aims to quantify how the uncertainty in the output of a given model, wave speed of the axisymmetric predominantly fluid-borne wave, can be attributed to different sources of uncertainties in the input variables, pipe and soil properties. These indices, named after the Russian mathematician Ilya M. Sobol, provide a way to decompose the overall variance of a model's output into contributions from individual input variables and their interactions [9,10]. In other words, they help to understand how changes in each input variable, alone and in combination with others, influence the variability of the output. The Sobol's effect sensitivity indices of the leak noise wave speed are calculated as

$$S_j = D_j / D, \; S_j^{\text{tot}} = D_j^{\text{tot}} / D, \tag{4a,b}$$

where S_j is the first order index for the parameter ξ_j, i.e., it measures the main effect (partial contribution) of ξ_j to the variance on the speed of the leak noise wave described in Section 2. The term S_j^{tot} is the total order index and measures the interaction between all the parameters ξ_j to the total variance of the leak noise wave speed with $\mathrm{j} \in \{1, 2, \ldots, 5\}$. The sum of all terms in Eqs. (4a) and 4(b) is equal to 1. The partial and total variances are calculated in a discrete form as [20]:

$$D_j = D - \frac{1}{2N}\sum_{\iota=1}^{N}\left[c\left(\xi_{j\iota}\right) - c\left(\xi_{j\iota}, \xi_{\sim j\iota}\right)\right]^2,\ D_j^{\mathrm{tot}} = \frac{1}{2N}\sum_{\iota=1}^{N}\left[c\left(\xi_{j\iota}\right) - c\left(\xi_{j\iota}, \xi_{-j\iota}\right)\right]^2, \qquad (5a,b)$$

where the mean $\bar{c}$ and total variance $D = \sigma^2(c)$ are given by $\bar{c} \approx \frac{1}{N}\sum_{\iota=1}^{N} c(\xi_\iota)$ and $\sigma^2(c) \approx \frac{1}{N}\sum_{\iota=1}^{N} c^2(\xi_\iota) - \bar{c}^2$, respectively, N is the sampling size for Monte Carlo simulations and ξ_{-j} is the combination of a set of parameters complementary to ξ_j. Steps for the implementation of the method are summarized as follows [21]:

- Step 1. Initially, set the number of Monte Carlo simulations N to be performed.
- Step 2. Choose the parameters for sensitivity analysis. Here, a total of five parameters are selected $\boldsymbol{\xi} = \begin{bmatrix}\xi_1 & \xi_2 & \xi_3 & \xi_4 & \xi_5\end{bmatrix}^T$ related to the pipe and soil properties.
- Step 3. Assume ranges for test variables: upper and lower values for the chosen parameters.
- Step 4. Choose a distribution for each of the parameters. In this case a random distribution is chosen for all the five parameters.
- Step 5. Calculate the mean and variance of each parameter.
- Step 6. Compute first-order effects (partial variance) for each parameter by fixing the values of that parameter and varying the remaining parameters.
- Step 7. Compute total sensitivity effects.
- Step 8. Sort the parameters according to their sensitivities.

4. Results and discussion

In this section, the stochastic wave speed model of the s=1 wave generated due to a leak in a plastic water pipe is examined in relation to two experimental test rigs, one in the UK which has a MDPE pipe and the soil properties surrounding the pipe are representative of sandy soil, and the other one is located in Brazil, which has a PVC pipe and the surrounding soil has properties that are representative of clay soil. Sensitivity analysis is then conducted through Sobol' indices and Monte Carlo method in each case to measure the importance of pipe and soil parameters by quantifying their individual and combined contribution to the speed of the leak noise wave.

4.1. Description of the test rigs

Two experimental pipe systems, one located in Brazil and the other one located in UK, are considered to predict the stochastic wave speeds in sub-section 4.2, and the corresponding Sobol' indices are presented in the sub-section 4.3. The pipe systems have very different properties as showed in Tab. 1.

Photography and schematic diagram of both test rigs are shown in Fig. 1. The UK pipe system, located in East Anglia, has been described in detail in [13,15]. The test rig consists of a 32 metres long pipe, pressurised by 1.5 m head of water in the termination tanks which are located at each end of the pipe. A photograph and a schematic diagram are shown in Fig. 1(a). Rather than generating a predominantly fluid-borne wave with a leak, an underwater loudspeaker fitted at one end of the pipe was used. It was supplied with a stepped sine signal, via a power amplifier, increasing from 30 Hz to 1 kHz in 1 Hz increments. The dynamic pressure was measured using two hydrophones positioned 2 metres apart as shown in Fig. 1(a). More details about the measurements were carried out and how the wave speed was calculated are given in [13,15].

Table 1. Properties of each experimental test rig.

Pipe-water-soil system properties	UK	Brazil
Young's modulus E (N/m^2)	2×10^9	3×10^9
Density ρ (kg/m^3)	900	1350
Loss factor η	0.06	0.06
Pipe radius a (mm)	84.5	38.5
Pipe-wall thickness h (mm)	11.0	3.40
Bulk modulus of water B_{water} (GN/m^2)	2.25	2.25
Wave speed in water c_{water} (m/s)	1500	1500
Shear modulus of soil G (GN/m^2)	0.02	0.20
Type of soil	Sand	Clay
Dynamical stiff. of water K_{water} (GN/m)	53	160
Dynamical stiff. of pipe-wall K_{pipe} (GN/m)	3.0	12
Dynamical stiff. of soil K_{soil} (GN/m)	0.47	14

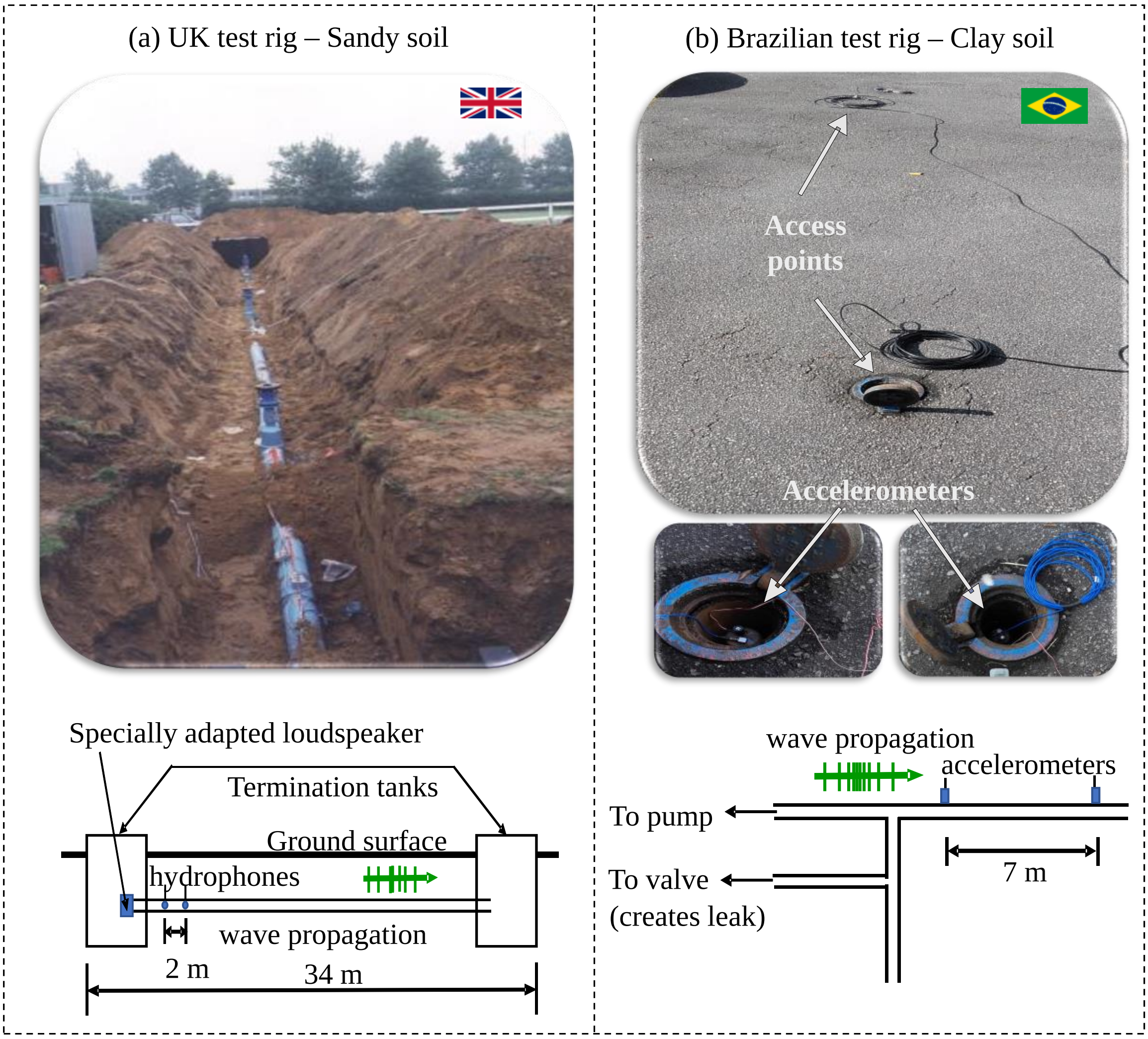

Figure 1. Photographs, experimental setup and schematic diagram (not to scale) of the pipe systems adopted: (a) UK pipe system located in East Anglia, (b) Brazilian pipe system located in São Paulo (from Scussel et al. [15,22]).

The Brazilian pipe system, located in São Paulo city, has been described in detail [7,8]. It consists of a closed-circuit system in which the pipe is much smaller than UK system. A photograph and a schematic diagram of part of the test rig are shown in Fig. 1(b). In this test rig, the pipe was pressurised with a centrifugal pump (3.4 bar), and the predominantly fluid-borne wave was excited by opening a valve and the signals related to this leak noise wave were measured by using two PCB 333B30 piezoelectric accelerometers at two access points 7 m apart. The acquisition system used for data collection was the LMS SCADAS from SIEMENS and the time histories were recorded a sampling frequency of 12.8 kHz for 60 s. The wave speed was determined by calculating the time of flight from the cross-correlation function between the two signals [7,8].

4.2. Stochastic wave speed estimation

The Monte Carlo simulations were performed for $N = 2^{14}$ realizations of the model described by Eq. (3), in which $E,\ \rho,\ a,\ h$, and G were considered to have the mean values given in Table 1 and normal probability density function. The corresponding standard deviations σ_{ξ_j} were taken from [22]. The results are shown in Fig. 2 and compared with measured wave speed as predicted in [22]. The mean $\bar{c}$ and confidence interval $\bar{c} \pm 3\sigma(c)$ (99,7%) estimated via Monte Carlo method are also shown.

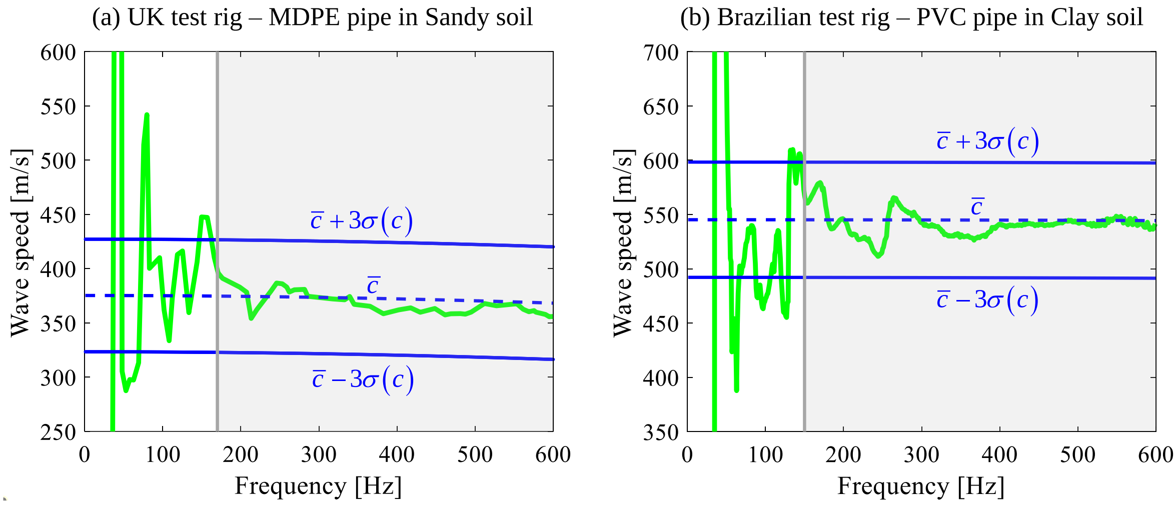

Figure 2. Measured and predicted wave speeds for the buried pipes shown in Fig. 1, and the confidence interval calculated using Monte Carlo Simulations (a) UK pipe system, (b) Brazilian pipe system. The grey shaded area denotes the frequency range where wave speed can be estimated. Continuous green thick line (▬) is the measured wave speed, dashed central blue line (- - -) is the mean wave speed from Eq. (3), continuous blue lines form the confidence interval.

As can be seen in Fig. 2, the predicted wave speed is about 375 m/s for the UK pipe system and 540 m/s for Brazilian pipe system. This difference is mainly due to the larger soil dynamic stiffness of the clay soil compared to the sandy soil as shown in Tab. 1. Furthermore, the wave speed in the UK pipe system marginally decreases within the bandwidth 0-600 Hz whereas the wave speed in the Brazilian pipe system does not change much. This is due to the inertial effect of the pipe-wall since the PVC pipe has a much smaller pipe-wall and higher density compared to the MDPE pipe. Additionally, the wave speeds marginally decrease as frequency increases, primarily because of the mass loading effect of the soil. Fig. 2 also shows a shaded region identifying the low frequency range content where wave speed can be estimated. The fluctuations at low frequencies are due to reasons such as effects from reflections, phase unwrapping and influences exerted by the surrounding soil.

4.3. Which parameter has the most significant impact on the wave speed?

The Sobol`s method uses the decomposition of variance to compute sensitivity indexes, known as Sobol`s indexes [9,10]. Here, the Sobol`s first order and total order indexes are computed for the pipe systems described in sub-section 4.2. The main goal is to perform a sensitivity analysis of the speed in which leak noise wave travels along the pipe-wall due to uncertainties in the pipe material and geometry as well as uncertainties in the surrounding soil. First-order effects and total effects were computed, and the parameters were ranked according to their contribution to the overall variance of the wave speed of the leak noise wave. A total of $N = 2^{14}$ simulations were performed and the Sobol`s indexes from Eqs. (4a,b) and (5a,b) were calculated for 3 case studies at a particular frequency of 300 Hz. In Case 1, standard deviation of 5% of the mean value was adopted in each uncertain parameter and in Case 2 only the soil parameter deviation was increased to 10% while the others remained 5%. In Case 3, it was assumed that a and ρ have standard deviation of 1% of the mean, E with 5% of deviation of the mean, h and G with 10% of deviation of the mean. The nominal values for Case 3 were chosen based on the standard specifications provided by ASTM standard with deviations in the pipe properties selected in accordance with the specifications ASTM 1785-12 [22]. All the results are shown in Figs. 3, 4 and 5.

Examining Fig. 3(a), it can be seen that the density of the pipe material along with shear modulus of the soil are the least sensitive for the UK pipe, which is buried in sandy soil. This is not the case in the Brazilian pipe system, shown Fig. 3(b), where soil parameter is the most sensitive parameter, due to the higher value of shear modulus characterizing clay soil as discussed in sub-section 4.2.

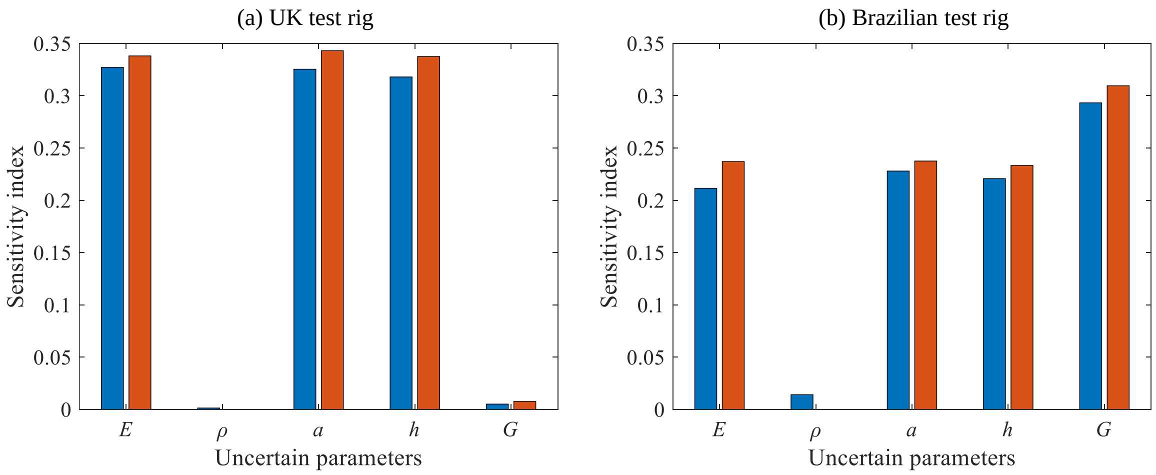

Figure 3. First-order effects (blue bar) and Total effects (orange bar) of the pipe and soil parameters using Sobol`s method and Monte Carlo simulations – Case 1.

Regarding the total effects, a similar behaviour can be observed by comparing it with the first-order effects, and it can be seen overall that the total effect of the density of the pipe is quite close to zero. Note that the the parameters related to the pipe material and geometry have a similar contribution on the partial variance and total variance of the leak noise wave speed. For the case 2 depicted in Fig. 4, it can be seen that by increasing the deviation of the soil parameter to 10% increases its contribution on the wave speed for the UK pipe system whereas in the Brazilian pipe system is much higher, increasing the sensitivity index to values above 0.6. By setting the same deviation in the parameters, it can be observed that the Young’s modulus, mean radius and thickness of the pipe are equally important in the UK pipe system. In the Brazilian system, all parameters are relevant besides the density of the soil.

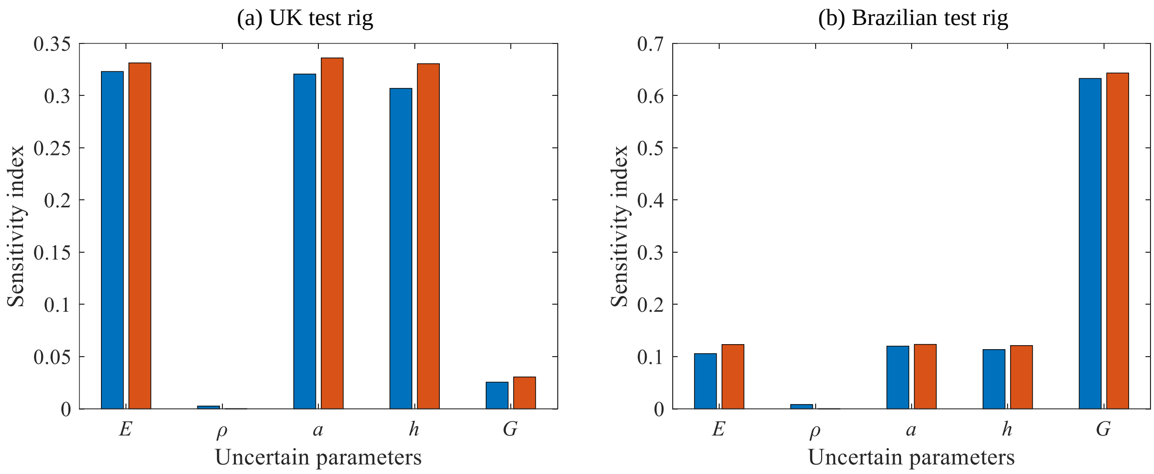

Figure 4. First-order effects (blue bar) and Total effects (orange bar) of the pipe and soil parameters using Sobol`s method and Monte Carlo simulations – Case 2.

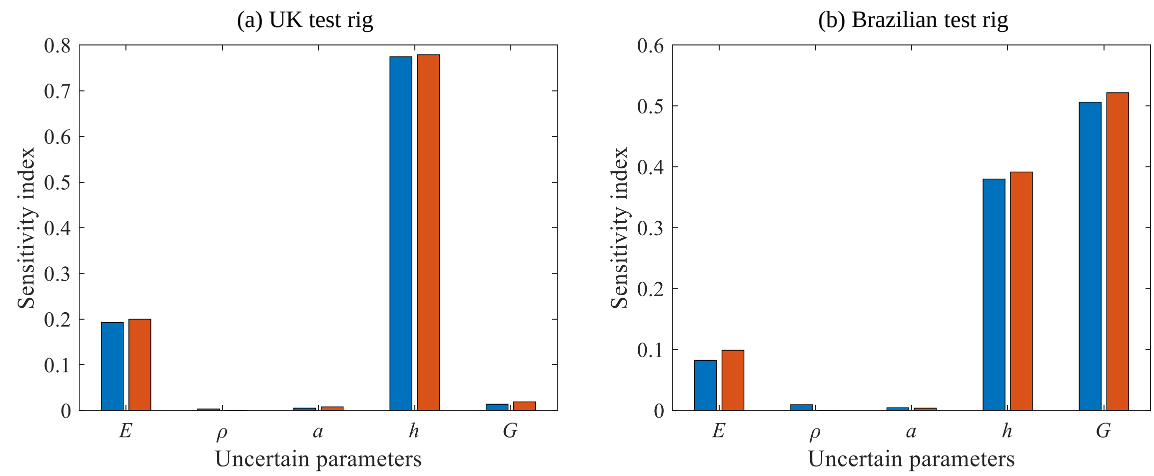

Figure 5. First-order effects (blue bar) and Total effects (orange bar) of the pipe and soil parameters using Sobol`s method and Monte Carlo simulations – Case 3.

In case 3, the thickness of the pipe-wall has dominant effect on the wave speed of the leak noise whereas in the for the Brazilian system both thickness and shear modulus of soil have relevant contribution.

5. Conclusions

This paper has investigated effects of uncertainties present in the pipe and soil parameters which are affecting the speed of propagation of the leak noise wave. Sensitivity analysis was performed using Sobol's indexes and Montel Carlo simulations by selecting five uncertain parameters: the Young's modulus, density, mean radius and thickness of the pipe, as well as the shear modulus of the soil. Experimental work has been carried out in two test rigs with very different pipe and soil properties, one located in the UK and the other one located in Brazil. It has been found that density has very low contribution on the leak noise wave speed in both test rigs. Furthermore, the contribution of the soil parameter is very low in the UK system due to the fact that the pipe was buried in sandy soil. On the other hand, it was found that the soil parameter has much higher influence on the speed of the leak noise wave in the Brazilian system since the pipe was buried in clay soil. The outcomes of this paper enable the assessment and incorporation of uncertainty information into the decision-making processes of existing vibro-acoustic technologies such as loggers and leak noise correlators.

ACKNOWLEDGMENTS
The authors are grateful for the financial support provided by EPSRC under the project RAINDROP (EP/V028111/1). The first author would like to thank the financial support from São Paulo Research Foundation (FAPESP) grant number 2024/13559-0.

References
[1] Romero-Ben L, Alves D, Blesa J, Cembrano G, Puig V and Duviella E 2023 Leak detection and localization in water distribution networks: Review and perspective *Annu. Rev. Control* **55** 392-419
[2] Hu Z, Tariq S and Zayed T 2021 A comprehensive review of acoustic based leak localization method in pressurized pipelines *Mech. Syst. Sig. Process.* **161** 107994
[3] Fan H, Tariq S and Zayed T 2022 Acoustic leak detection approaches for water pipelines *Autom. Constr.* **138** 104226
[4] Puust R, Kapelan Z, Savic DA and Koppel T 2010 A review of methods for leakage management in pipe networks *Urban Water J.* **7** 25–45
[5] Kafle MD, Fong S and Narasimhan S 2022 Active acoustic leak detection and localization in a plastic pipe using time delay estimation *Appl. Acoust.* **187** 108482
[6] Gao Y, Brennan MJ and Joseph PF 2006 A comparison of time delay estimators for the detection of leak noise signals in plastic water distribution pipes *J. Sound Vib.* **292** 552–570
[7] Brennan MJ, Karimi M, Muggleton JM, de Almeida FCL, de Lima FK, Ayala PC, Obata D, Paschoalini AT and Kessissoglou N 2018 On the effects of soil properties on leak noise propagation in plastic water distribution pipes *J. Sound Vib.* **427** 120–133
[8] Scussel O, Brennan MJ, Almeida FCL, Muggleton JM, Joseph PF and Rustighi E 2021 Estimating the spectrum of leak noise in buried plastic water distribution pipes using acoustic or vibration measurements remote from the leak *Mech. Syst. Sig. Process.* **147** 107059
[9] Sobol IM 1993 Sensitivity analysis for nonlinear mathematical models *Math. Model.* **1** 407-414
[10] Sobol IM 2001 Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates *Math. Comput. Simul.* **55** 271-280.
[11] Saltelli A, Chan K and Scott EM 2000 *Sensitivity Analysis* (John Wiley & Sons Ltd)
[12] Pinnington RJ and Briscoe AR 1994 Externally applied sensor for axisymmetric waves in a fluid flieed pipe *J. Sound Vib.* **173**(4) 503-516
[13] Muggleton JM, Brennan MJ and Linford PW 2004 Axisymmetric wave propagation in fluid-filled pipes: wavenumber measurements in in vacuo and buried pipes *J. Sound Vib.* **270** 171–190
[14] Gao Y, Sui F, Muggleton JM, Yang J 2016 Simplified dispersion relationships for fluid-dominated axisymmetric wave motion in buried fluid-filled pipes *J. Sound Vib.* **375** 386-402
[15] Scussel O, Brennan MJ, Muggleton JM, de Almeida FCL and Paschoalini AT 2019 Estimation of the bulk and shear moduli of soil surrounding a plastic water pipe using measurements of the predominantly fluid wave in the pipe *J. Appl. Geophys.* 164 237–246
[16] Scussel O, Brennan MJ, Almeida FCL, Iwanaga MK, Muggleton JM, Joseph PF and Gao Y 2023 Key factors that influence the frequency range of measured leak noise in buried plastic water pipes: Theory and experiment *Acoust.* **5** 490-508
[17] Sepahvand K, Marburg S and Hardtke H-J 2010 Uncertainty quantification in stochastic systems using polynomial chaos expansion *Int. J. Appl. Mech.* **2** 305–353
[18] Kha J, Croaker P, Karimi M and Skvortsov A 2023. Uncertainty Analysis in Airfoil–Turbulence Interaction Noise Using Polynomial Chaos Expansion, *AIAA Journal* 1-11
[19] Datz J, Karimi M, and Marburg S 2021 Effect of uncertainty in the balancing weights on the vibration response of a high-speed rotor *J. Vib. Acous.t* 143(6) 061002.
[20] Zheng Y and Rundell A 2006 Comparative study of parameter sensitivity analyses of the TCR-activated Erk-MAPK Signalling Pathway *Proceedings of Systems Biology* **153**(4) 201-211.
[21] Bilal N 2014 Implementation of Sobol's Method of Global Sensitivity Analysis to a Compressor Simulation Model *Proc. Int. Compressor Eng. Conf.* Paper 2385.

[22] Scussel O, Seçgin A, Brennan MJ, Muggleton JM and Almeida FCL 2021 A stochastic model for the speed of leak noise propagation in plastic water pipes *J. Sound Vib.* **501** 116057