A Torsional Acoustic Black Hole

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Abstract. Torsional vibration in structures such as buildings, wind turbines and drive-shafts can increase structural fatigue, reduce efficiency, and generate unwanted noise. The Acoustic Black Hole (ABH) is a recent development in structural vibration control, where enhanced energy dissipation is achieved though a reduction in the wave speed along the structure. This is usually achieved by introducing a graded reduction in stiffness, either with a tapered thickness or graded material properties. Reducing the wave speed makes viscoelastic damping materials more effective and therefore a broadband reduction in vibration can be achieved. Until now, the ABH has mostly been applied to control flexural waves in beams or plates. The application to torsional vibration requires a different approach as the wavespeed cannot be modified by a tapered cross-section alone. This study presents a graded-metastructure consisting of a tapered and periodically stepped shaft, that is able to realise the ABH effect in torsion.

1 Introduction

In recent years, Acoustic Black Holes (ABHs) have increasingly been investigated for controlling flexural vibration, with interesting performance benefits [1]. ABHs work by gradually reducing the phase velocity of the elastic wave, reducing the wavelength and making the kinetic energy more easily dissipated through material losses. For flexural vibration, this slowing of the wave can be achieved with a gradual reduction in the dynamic stiffness over the dimensions via either a tapered thickness [2], or a graded elastic modulus [3]. The flexural response of an ABH is modal, and therefore in the case of a tapered thickness, a viscoelastic layer is added to achieve a more broadband control of vibration [4]. ABHs can be integrated as part of a structure, rather than as an additional component, and because they do not add mass they could be more suitable in lightweight applications. Due to the broadband performance, they are also suited to applications where there is uncertainty in the structural resonance frequencies, for example in a structure with variable dynamics (cranes, ships, submersibles etc.).
ABH technology has not yet been widely applied to torsional vibration, yet potentially offers significant benefits over traditional approaches such as dual-mass flywheels [5] and tuned vibration absorbers [6, 7], such as: broadband vibration attenuation with no additional mass; no moving parts; rotational symmetry; and no additional friction. This extended abstract introduces a cylindrical shaft with a Torsional ABH termination, utilising graded periodicity to achieve a reduction in wave speed, and therefore increased energy dissipation.

2 Controlling Torsional Wave speed

The benefits of ABHs rely on the ABH effect: a gradual reduction in the phase velocity of vibration along the dimensions of the structure which therefore also reduces the wavelength. The torsional wave speed (phase velocity) in a uniform cylindrical shaft, c, can be expressed as

$$c = \sqrt{\frac{GJ}{I}} \tag{1}$$

where G is the shear modulus, J is the torsional constant / polar moment of inertia, and I is the mass moment of inertia. For a uniform cylinder, $J = \frac{\pi R^4}{2}$ and $I = \frac{\pi \rho R^4}{2}$, where R is the radius and ρ is the density. Equation 1 therefore reduces to

$$c = \sqrt{\frac{G}{\rho}}, \tag{2}$$

and wave speed is independent of radius. However, using the theory of graded-metastructures, graded effective material properties can be achieved as long as the wavelength is much larger than the period. Instead of a continuous uniform cross-section, the cross-section is designed to periodically vary along the length of the structure such that the ratio between the torsional stiffness (polar moment of inertia) and mass moment of inertia gradually decreases, therefore reducing the wavespeed. Considering a cylindrical shaft, this can be achieved by periodically stepping the radius between a tapered "core" of gradually decreasing radius and constant radius sections, as shown in Figure 1. As in flexural ABHs, in order to increase dissipation and provide smooth broadband attenuation, a viscoelastic material with a high damping coefficient is introduced, and in this case is constrained between adjacent sections (as shown in red in Figure 1).

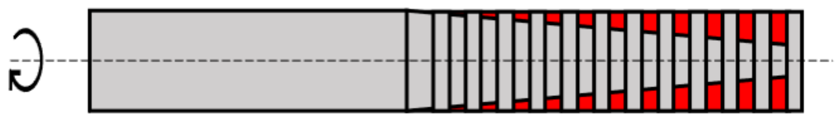

Figure 1: Side view of a shaft with Torsional ABH termination: a graded stiffness-mass ratio by stepping the radius between a central "core" with a radius that decreases over the length, and sections of uniform radius. A viscoelastic material (red) is constrained between adjacent sections, to maximise dissipation.

3 Simulation Study

Using an axisymmetric model, the torsional response of the proposed shaft with Torsional ABH termination is simulated using COMSOL Multiphysics. The structure comprises a 0.5 m long shaft section of constant radius 0.05 m, and a subsequent 0.5 m long Torsional ABH section, which periodically steps between the "core", that decreases in radius linearly over the length of the ABH (to 0.005 m at the tip), and sections with the same radius as the shaft. Figure 2 shows the realised geometry in 3D with a cutaway to highlight the varying radius. The continuous and stepped radius shaft sections (grey) are Aluminium and the constrained viscoelastic material (red) is Silicone Rubber.

3.1 Confirming the ABH Effect

The circular face at the uniform radius end is subject to a periodic total rotational load about the shaft axis of 1 N at frequencies of 1kHz, and 5kHz, and the resulting rotational velocity, $\dot{W}_{\phi}$, at a line along the perimeter of the structure over its length, is shown in Figures 3.a - b respectively. In Figure 3 the wavelength of the oscillating rotational velocity over the length can be clearly seen to decrease along the length of the ABH. It can also be seen that the maximum rotational velocity does not occur at the tip. For the 5 kHz results shown in Figure 3.b, it can be seen that propagation of the wave beyond $x =$ 0.9 m is limited, and appears to stop entirely beyond around $x = 0.95$ m. These frequency-dependent localised maxima are a phenomenon known as "rainbow trapping", and this likely results from the interactions between scattered waves when the wavelength is no longer large compared to the period of the periodically-stepped radius, as demonstrated in acoustic graded metastructures [8] . As frequency increases, the waves will become trapped further and further from the tip of the ABH. This is, however, still beneficial in aiding dissipation of energy, as the displacement maxima occurs in a region with high

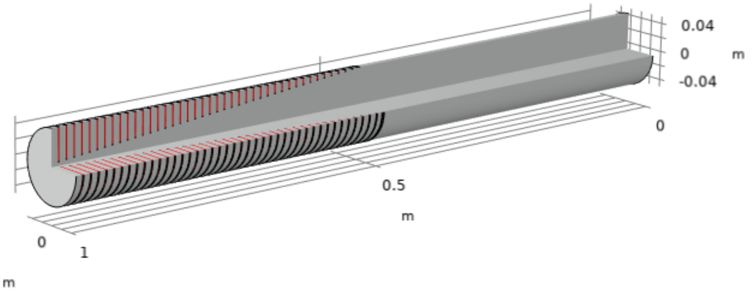

Figure 2: The example shaft with Torsional ABH termination in 3D with a cutaway. Damping material is shown in red.

damping. It also means that the tip of the Torsional ABH is almost completely isolated from the shaft section above a certain frequency. This could have applications in precision machining or measurement.

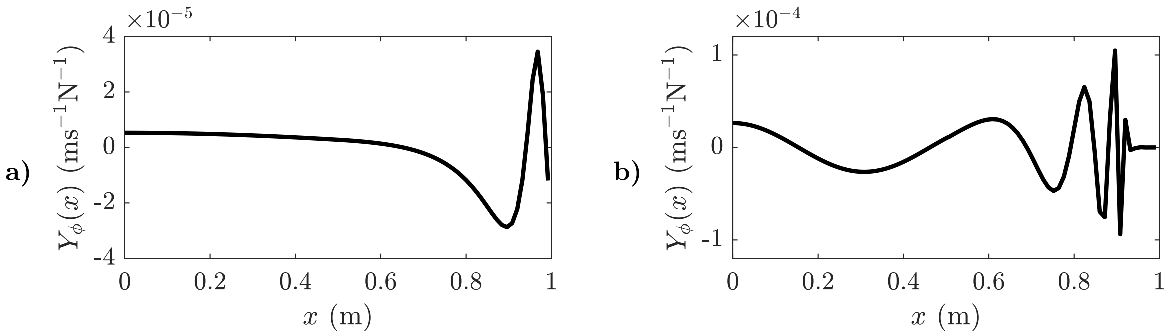

Figure 3: Rotational velocity, $\dot{W}_\phi$, of the perimeter of the structure over its length, at: a) 1 kHz; b) 5 kHz.

4 Simulation of Dynamic Response

With a reduction in wave speed achieved, the attenuating effect of the Torsional ABH is considered. Figure 4 shows the total kinetic energy of a 1 m long shaft with no Torsional ABH and a 0.5 m long shaft with 0.5 m long Torsional ABH termination. The shaft with the Torsional ABH termination has a highly damped response, and the mode shown in the response of the 1 m shaft is completely attenuated. As expected, as the shaft with the Torsional ABH is more compliant and lower in mass, it has slightly greater energy at frequencies outside of the modal peak of the shaft, but the total broadband kinetic energy over 100–2500 Hz (to avoid the dominance of the rigid-body-mode) is 11 dB lower than for the 1 m shaft with no ABH. This is a significant reduction. Were the frequency range to be extended upwards to encompass higher order modes of the 1 m shaft, the difference in energy may be even more significant. However, it is expected that at some point, the rainbow-trapping effect will limit the dissipation possible when waves become localised in areas with very little damping material.

5 Conclusions

A Torsional Acoustic Black Hole, like the flexural counterpart, offers the potential for significant broadband attenuation of torsional vibration. By periodically stepping the radius of a cylindrical shaft between a tapered "core" and a constant radius, a reduction in the torsional wave speed over the length of the taper is possible, and has been demonstrated using simulations. In combination with a viscoelastic material constrained between the sections, this approach has been demonstrated to produce a structure that has a significantly lower dynamic response than a simple shaft of the same overall dimensions.

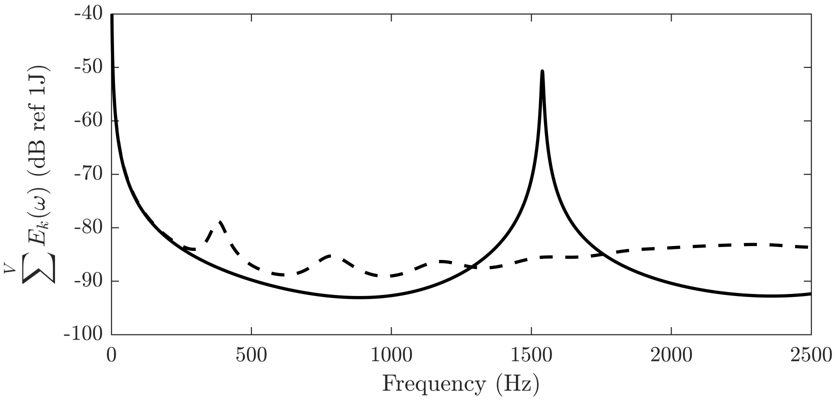

Figure 4: Total kinetic energy, E_k, of the entire volume of: a 1 m long shaft with no Torsional ABH termination (solid line) compared to the proposed 0.5 m long shaft with 0.5 m long Torsional ABH termination (dashed line).

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