# Generation of Self-Dual Lorentz-Invariant Toroidal Light Pulses

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We present the experimental generation of self-dual pulses of toroidal topology, invariant under Lorentz boosts. They will be of interest in telecommunications, spectroscopy, and particle acceleration.

Flying Doughnuts (FDs), also known as Toroidal Light Pulses, are few-cycle, space-time non-separable, broadband pulses of toroidal topology [1]. They are non-chiral and can be radially or azimuthally polarized. They exhibit exotic topological properties, such as self-similar field configurations, vortex ring singularities, and energy backflow, and they are strongly coupled to toroidal and anapole modes in nanoparticles. Here, we demonstrate experimentally that superimposing radially and azimuthally polarized FDs leads to the formation of self-dual toroidal pulses, where the electromagnetic fields are related by $\boldsymbol{E} = \pm i\boldsymbol{B}$. Such pulses are invariant under electromagnetic duality ($\boldsymbol{E} \leftrightarrow \boldsymbol{B}, \boldsymbol{B} \leftrightarrow -\boldsymbol{E}$), as well as under rotation-free Lorentz transformations (boosts) along the propagation direction. In contrast to FDs, their self-dual counterparts exhibit parallel electric and magnetic field components and are strongly non-chiral, as seen in Fig.1a&b, which in turn results in non-zero fundamental Lorentz invariant $\boldsymbol{E} \cdot \boldsymbol{B} \neq \boldsymbol{0}$, drawing parallels with the source term of axion electrodynamics.

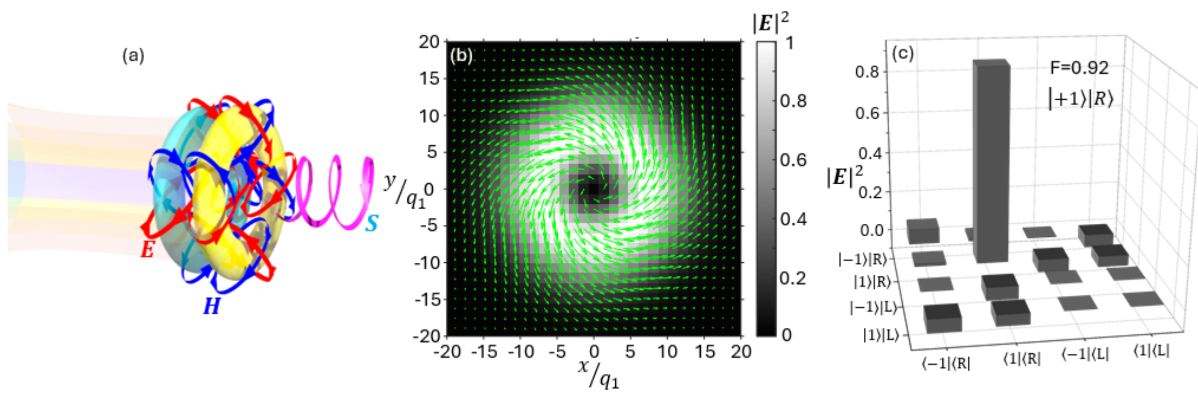


Fig. 1. (a) Conceptual illustration of a self-dual toroidal pulse. Red arrows represent electric field, $\boldsymbol{E}$, blue arrows represent magnetic field, $\boldsymbol{H}$, and the helical arrow represent the Poynting vector, $\boldsymbol{S}$. (b) Intensity of the experimentally generated pulse in the transverse plane. Arrows represent the electric field of the monochromatic component at λ=800 nm. (c) Experimental density matrix for generated pulse with orbital angular momentum $\ell = +1$. The fidelity, 0<F<1, with respect to the ideal pulse is F=0.92, indicating high similarity.

The generation scheme for self-dual toroidal pulses was based on the conversion of near-infrared 10 fs laser pulses upon propagation through a segmented waveplate. The axes of the waveplate crystal vary along its circumference (over 8 segments), which allows the conversion of incident horizontally (vertically) linearly polarized pulses to radially (azimuthally) polarized FDs [1]. On the other hand, incident circularly polarized pulses result in a superposition of radially and azimuthally polarized FDs and thus to the generation of self-dual toroidal pulses. For characterization of the generated pulses space-polarisation tomography was used that allows the pulse to be decomposed into a series of modes with different polarization and orbital angular momentum states. Experimental results (Fig.1c) showed the pulses corresponded very closely to that of ideal self-dual toroidal pulse with a fidelity of F=0.92.

In summary, we have generated and characterized experimentally self-dual toroidal light pulses. We expect that such self-dual toroidal pulses could have potential applications in particle acceleration, spectroscopy and information transfer.

**References:**

[1] A. Zdagkas, Y. Shen, C. McDonnell, J. Deng, G. Li, T. Ellenbogen, N. Papasimakis, and N. I. Zheludev, "Observation of toroidal pulses of light," Nature Photon. **16**, 523-528 (2022)