Nonreciprocal Interactions of Toroidal Charge-Current Configurations

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We show that electromagnetic interactions of toroidal charge-current configurations result in non-reciprocal optical forces that violate the action-reaction principle. Such behaviour is reminiscent of active matter dynamics and can underpin mobilization phase transitions into time crystal state.

According to the action-reaction principle, forces between objects in a closed system have equal magnitude and are directed in opposite directions. In electromagnetism, the action-reaction principle results in the Lorentz or Feld-Tai reciprocity lemmas. These principles were challenged in 2001 by G. N. Afanasiev who predicted that charge-current configurations of toroidal topology can violate both the action-reaction principle and the Lorentz/Feld-Tai reciprocity lemmas [1].

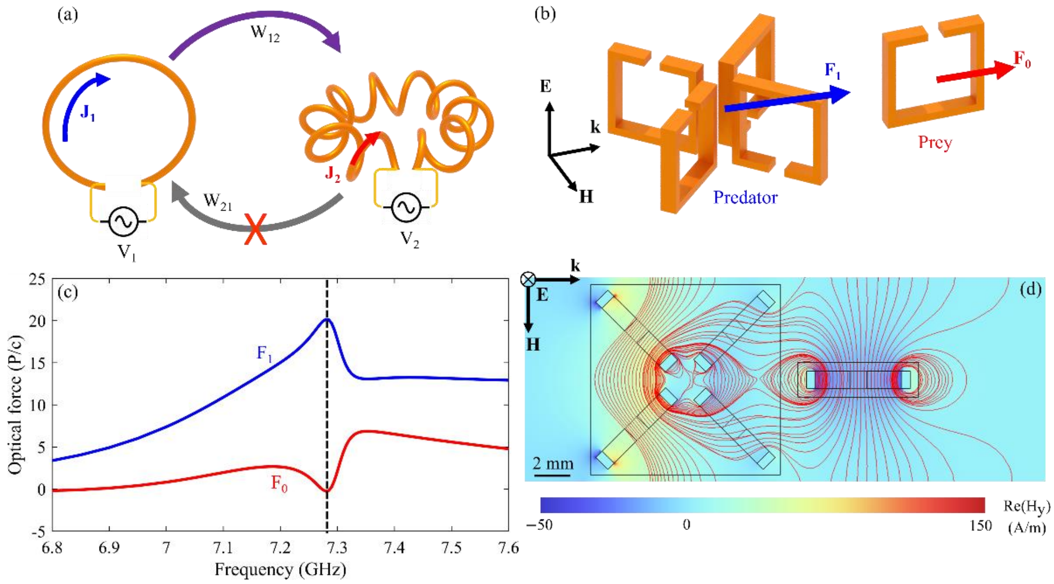

Fig. 1. (a) Nonreciprocal interactions of toroidal charge-current configurations of different order: ring current (left) and poloidal current (right). (b) Predator-prey behaviour between toroidal resonators supporting poloidal currents and split-ring resonators supporting ring currents driven by an external electromagnetic field. (c) Optical forces in a metamaterial array of toroidal metamolecules and split-ring resonators under plane wave illumination with linearly polarized light. Blue and red lines correspond to the optical force component along the propagation direction acting on the toroidal and split-ring resonators, respectively. The vertical dashed line marks the frequency at which the optical force on the split-ring resonators vanishes. (d) Magnetic field map at a frequency of 7.28 GHz (see dashed line in c)). Red lines represent magnetic field lines.

We investigate Afanasiev's predictions by considering the interactions in a metamaterial consisting of toroidal metamolecules [2] and split-ring resonators (see Fig. 1b) driven by an incident linearly polarized plane wave. We employ a finite element solver to retrieve numerically the optical forces between the two different charge-current configurations by integrating Maxwell's stress tensor. The frequency spectra for optical forces along the propagation direction, F_1 and F_0, acting on the toroidal metamolecules and split-ring resonators, respectively, is presented in Fig. 1c. The force acting on the former (latter) is substantially stronger (weaker) throughout the frequency spectrum indicating "predator"-like ("prey"-like) behaviour. Importantly, at ~7.28 GHz the force acting on the 1st order toroidal configuration is at a maximum, while the force on the 0th order configuration vanishes (see vertical dashed line, Fig. 1b), resembling a "captive" state.

In summary, we observe nonreciprocal interactions in charge-current configurations of toroidal topology driven by an external electromagnetic field. Our results will be of interest in the study of time crystals and their phase transitions [3].

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