# Engineering Optical Forces for Time Crystal Transitions

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We show continuous tuning of interactions in ensembles of plasmonic resonator arrays, from reciprocal to fully nonreciprocal. Our results provide a framework for investigating and engineering synchronization phenomena in many-body systems, including topological states and time crystals.

The recent experimental demonstrations of continuous time crystals has resulted in a renewed interest in the many-body physics and synchronization phenomena of coupled oscillator ensembles [1]. While nonreciprocal interactions are recognized as essential for the emergence of time crystal states, a systematic framework for designing and engineering such interactions remains elusive. Here, we address this challenge by developing an analytical framework for tuning nonreciprocity in plasmonic resonator arrays. We demonstrate analytically that light-induced optical forces on interacting plasmonic resonators can be continuously tuned from fully nonreciprocal to fully reciprocal through the introduction of asymmetries in either the illumination conditions or the properties of individual resonators.

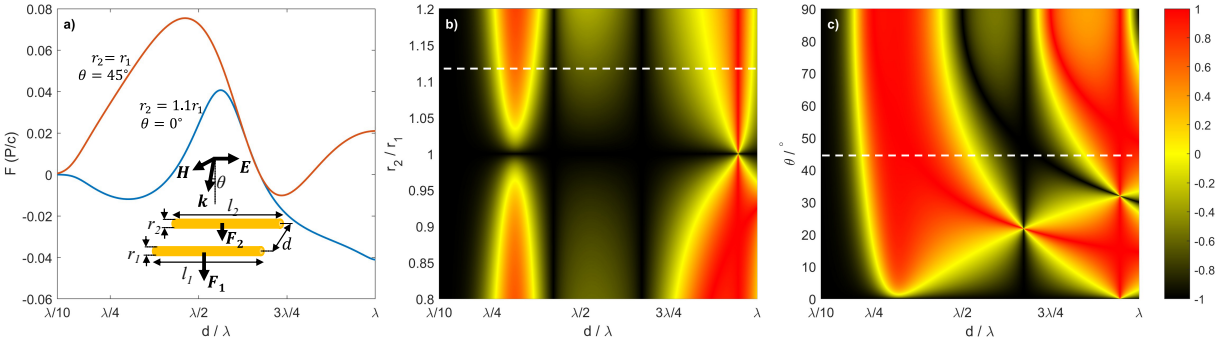


**Fig. 1** (a) Nonreciprocal optical forces in asymmetric plasmonic nanowire systems. The red curve represents two identical nanowires with $l_1 = l_2 = 950$nm and radius with $r_1 = r_2 = 25$nm illuminated by a linearly polarized plane wave at 45° incidence, while the blue curve represents two dissimilar nanowires with $r_1 = 25\,nm$, $r_2 = 1.1r_1$ and under normal incidence ($l_1 = l_2 = 950$nm). (b,c) Ratio of nonreciprocal to reciprocal force components for dissimilar nanowires under normal incidence as a function of the nanowire radii ratio $r_2/r_1$ (b) and for identical nanowires under oblique incidence as a function of angle of incidence (c). The incident wavelength is λ = 1550 nm with the electric field polarized along the nanowire axis. White dashed lines in (b,c) indicate the cross-sections shown in panel (a).

We consider a one-dimensional array of plasmonic gold nanowires with lengths nanowires with lengths $l_1$, $l_2$ , … $l_N$ and radii $r_1$, $r_2$,… $r_N$, respectively, separated by nearest-neighbor distance d, and illuminated by a plane wave polarized along the nanowire axis at wavelength λ = 1550 nm. To first approximation, optical forces between nanowires can be calculated using the dipole approximation, neglecting higher-order multipole contributions. The dipole moments are determined by solving coupled dipole equations projected onto the polarization plane. The nonreciprocal force component between any two nanowires in the array is expressed as $F_{nonrec}^{lm} = \Im\left\{\frac{dG^*}{dx_i}\right\}\Im\{p_l p_m^*\}$, while the reciprocal force is $F_{rec}^{lm} = \Re\left\{\frac{dG^*}{dx_i}\right\}\Re\{p_l p_m^*\}$, where $G$ is the dyadic Green's function and $p_l$, $p_m$ are the corresponding dipole moments.

Figure 1a shows the distance-dependent nonreciprocal optical forces in a nanowire dimer for two configurations: dissimilar nanowires under normal incidence (blue) and identical nanowires under oblique illumination (red). Figures 1b,c map the ratio of nonreciprocal to reciprocal forces across the full parameter space of nanowire radius asymmetry (radius ratio $r_2/r_1$) and illumination angle θ for dissimilar and identical nanowires, respectively. These results demonstrate that directional coupling can be continuously tuned from fully nonreciprocal to fully reciprocal by controlling either geometric asymmetry or illumination angle. Crucially, the direct relationship between the relative phase of the resonators' dipole moments and the nature of their interactions establishes a fundamental connection between collective radiative properties and synchronization dynamics in coupled resonator arrays.

## References

[1] T. Liu, J.-Y. Ou, K. F. MacDonald, and N. I. Zheludev, "Photonic metamaterial analogue of a continuous time crystal," Nat. Phys. 19, 986 (2023).