Convergence of Beam Propagation through Turbulent Air Modelling using Full-3D Statistics

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***Abstract*—By relinquishing the Markov approximation and modelling turbulence in 3D, we are able to show improved convergence behavior of the split-step method for light propagation through turbulent air. We use this improved model to identify step-sizes for which the Markovian model is accurate.**

Keywords—FSOC, atmospheric turbulence, beam propagation

I. Introduction

Driven particularly by demand for higher access data-rates in an era where unused microwave bandwidth is becoming increasingly scarce, free-space optical communication (FSOC) is seeing redoubled interest. Diverse applications (e.g. LIFI, LEO), standardisation activities (SDA OCT, CCSDS 141, etc) and the use of machine learning in FSOC (requiring high quality training data) are all motivating the improvement of simulation [1] capabilities, with turbulence modelling being a major target.

To reduce computational complexity, the modelling of light propagation through turbulent air commonly makes use of the Markov approximation, dividing the beam-path into a number of phase screens that are, importantly, assumed uncorrelated [2]. This puts a lower-bound on step-size that is at odds with the general observation that numerical accuracy increases with decreasing step-size. There are a number of methods used to estimate the appropriate number of phase screens for sufficient accuracy [3,4], but these do not provide the confidence that is normally sought by observing convergence of a simulation at sufficiently small step-size. These constraints become more troublesome for links much shorter than the correlation length.

In this work, the use of modern GPU hardware enables us to forgo the Markov approximation (referred to hereon as the 2D model) and instead consider the full 3D statistical variation of turbulent air (the 3D model), liberating us from the step-size constraints previously adhered to. We observe that the full 3D model does indeed show robust convergence when reducing step-size, unlike the traditional 2D model, which only shows accurate results for very specific step-sizes. We believe this work promotes the adoption of full 3D turbulence models.

II. Technique

We implement the split-step method using the dispersion operator, $\widehat{D}$, and turbulence operator, $\widehat{T}$, shown in Eq. 1 and Eq. 2, respectively, where dz represents the propagation step-size and the thickness of a phase screen, k is the wavenumber of the propagating beam (with k_x and k_y being its transverse components), and $n_m(x,y)$ represents the refractive index of the mth phase screen, at a distance between mdz and $(m+1)dz$. With our focus on accuracy, we implement $\widehat{D}$ as the exact Helmholtz propagator. We adopt the Modified von Kármán spectrum [2] for the power spectral density of refractive index fluctuations, $\Phi_n(\kappa)$, shown in Eq. 3, where $\kappa_0 = 2\pi/L_0$, $\kappa_m = 5.92/l_0$ [4], l_0 and L_0 are the inner and outer scales [2], respectively, and C_n^2 is the refractive index structure parameter.

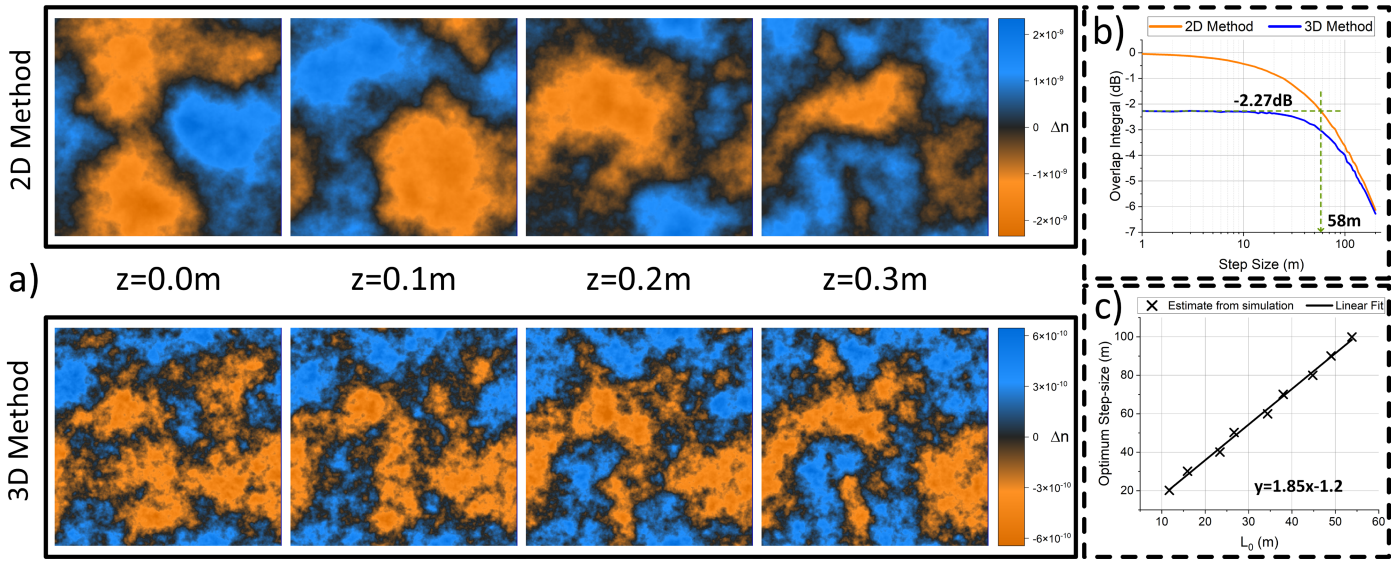

Fig. 1: a) Phase screens using the 2D and 3D model; b) Overlap vs step-size for the 2D and 3D models; c) Optimum step-size vs L_0.

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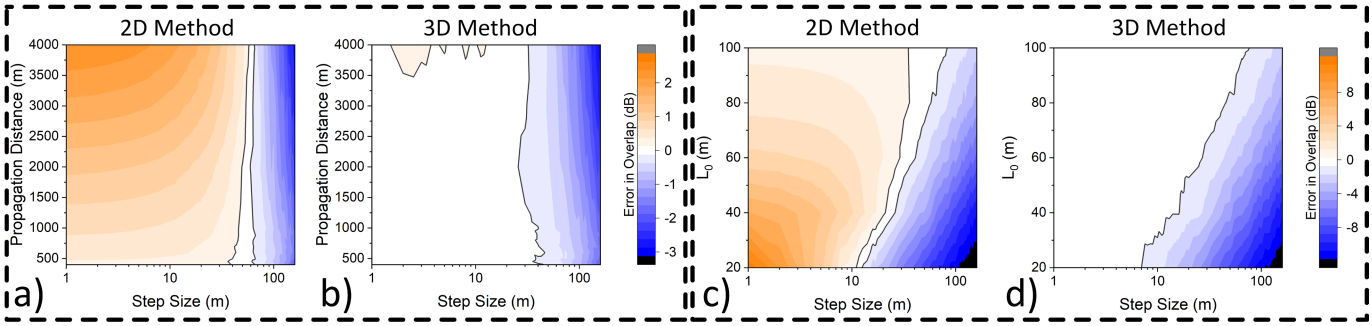

Fig. 2: Overlap error vs propagation distance and step-size for a) 2D b) 3D models. Overlap error vs L_0 and step-size for c) 2D d) 3D models.

$$\hat{D} = \exp\left(i \cdot \mathrm{d}z \cdot \sqrt{k^2 - k_x^2 - k_y^2}\right) \quad (1)$$

$$\hat{T} = \exp\left(i \cdot k \cdot \mathrm{d}z \cdot n_m(x,y)\right) \quad (2)$$

$$\Phi_n(\kappa) = \frac{0.033 C_n^2 \exp\left(-\kappa^2/\kappa_m^2\right)}{\left(\kappa^2+\kappa_0^2\right)^{11/6}} \quad (3)$$

To generate turbulent phase screens, we first fill a cuboidal mesh of refractive index samples with normally distributed random values. For the 3D method investigated in this work, we filter the random samples in 3D Fourier space using Eq. 3 with $\kappa = \kappa_x + \kappa_y + \kappa_z$. For comparative purposes, we also implement the 2D method by filtering each layer, m, of the cuboidal mesh in 2D Fourier space using Eq. 3 with $\kappa = \kappa_x + \kappa_y$. After filtering, the whole refractive index mesh is rescaled to deliver the desired value of C_n^2, which in this paper is fixed at a moderately strong value of 1×10^{-13}. For both the refractive index mesh and propagating beam itself, a 256×256 sample grid is used in x and y and sample rates are fixed in both dimensions at $100\mathrm{m}^{-1}$. The inner scale, l_0 is also held constant at $0.005\mathrm{m}$.

For illustrative purposes, four successive phase screens for the 2D and 3D methods (with $L_0 = 100\mathrm{m}$ and a common seed) are presented in the top and bottom rows of Fig. 1, with a step-size (screen thickness) of 0.1m. True to the Markov approximation, there is no visual correlation between successive screens when using the 2D method. In contrast, strong correlation can be seen when using the 3D method, whose more granular features are lacking from the 2D approximation.

III. Results

Firstly, we investigate the convergence of the two models with decreasing step-size. We adopt the overlap integral as our performance metric, owing to its sensitivity to both phase and power distortion. To isolate the effects of the phase screen method, we choose to overlap the propagated Gaussian beam (with initial beam half-waist of 20cm) with a reference beam propagated over the same distance in the absence of turbulence. Fig. 1-b provides a plot of the overlap integral (in dB), averaged over 10000 realisations of a 4000m long turbulent beam path, versus step-size for both the 2D (orange curve) and 3D (blue curve) approach, with $L_0 = 100\mathrm{m}$. The convergence of the 3D method for step-sizes $< 10\mathrm{m}$ is clear to see and in stark contrast to the curve for the 2D method which can be seen to approach 0dB overlap with reducing step-size. This greatly reduces the predictive ability of the 2D, Markovian model. It is interesting to note, however, that the 2D method gives the same overlap as the 3D method for a step-size of $58\mathrm{m}$ (see Fig. 1-b annotations).

We define the overlap error as the difference between the calculated overlap (in dB) for a particular step-size compared to the converged value of the 3D method (i.e. the result from using the smallest step-size). Fig. 2-a and -b provide contour plots of the overlap error (averaged over 1000 realisations) for a range of propagation distances and step-sizes. Black contour lines demarcate a $\pm 0.125\mathrm{dB}$ region around an error of 0dB. Fig. 2-a shows that the behaviour of the 2D method is largely the same regardless of propagation distance, with a monotonically increasing estimate of overlap with reducing step-size. Importantly, the 3D method shows converged behaviour for step-sizes $< 10\mathrm{m}$, again, regardless of propagation distance. Interestingly, the step-size at which the 2D method matches the converged value of the 3D method (between the contours) is remarkably constant with step-size (sitting at around 58m).

In light of this, we also present the overlap error (averaged over 1000 realisations) for a fixed propagation distance of $2000\mathrm{m}$ but for a range of L_0, with the 2D and 3D results plotted in Fig. 2-c and -d, respectively, with contours at $\pm 1\mathrm{dB}$. Once again, robust converging behaviour is observed for the 3D model when reducing step-size, whilst the step-size for which the 2D model delivers zero overlap error varies with L_0. We plot this optimum step-size for the 2D model against L_0 in Fig. 1-c. This reveals a linear relationship between the optimum step-size and L_0 for the 2D model, which can advise optimum step-size selection in scenarios where use of the 2D model is a must (this value likely depends on the form of $\Phi_n(\kappa)$ used).

IV. Conclusions

Our results show the constrained predictive ability of the traditional 2D (Markovian) model for beam propagation through turbulent air. Enabled by the use of GPU computing, we are able to show the strong convergence of the 3D model and reveal the possibility to identify optimum step-sizes for the 2D model.

Acknowledgment

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References

[1] Daniel A. Paulson, et al. , "Randomized spectral sampling for efficient simulation of laser propagation through optical turbulence," J. Opt. Soc. Am. B 36, 3249-3262 (2019

[2] L. C. Andrews, "Laser beam propagation through random media", 2nd ed., SPIE, 2005, Ch. 3.

[3] Wm. A. Coles, et al., "Simulation of wave propagation in three-dimensional random media," Appl. Opt. 34, 2089-2101 (1995)

[4] Chahine, Y. K., et al., "Beam propagation through atmospheric turbulence using an altitude-dependent structure profile with non-uniformly distributed phase screens", in Free-Space Laser Communications XXXII, 2020, vol. 11272