

# Statistical Methods & Applications

## Two-Way Capture-Recapture Methods with Emphasis on the Bootstrap

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# Two-Way Capture-Recapture Methods with Emphasis on the Bootstrap

## Abstract

The Chapman estimator is widely used in dual system estimation for estimating the size of an elusive target population. Two independent sources are required, delivering a two-by-two table of those units identified by both sources, of those identified only by the first, and those only by the second source. Interest is in the frequency of those identified by neither source that remain hidden. While asymptotic variance estimates exist for the Chapman estimator, they often perform poorly with small sample sizes. In addition, the Chapman estimator may be biased for small sample sizes. This study explores the bias of the Chapman and a bias-corrected Chapman estimator by investigating both imputed and non-imputed (simple) bootstrap methods as alternatives for estimating variance and constructing confidence intervals. Through simulation studies, we assess the reliability of these methods by analyzing confidence interval coverage probabilities. Our findings show that the imputed bootstrap consistently delivers better performance, yielding coverage probabilities closer to the nominal level, even under moderate dependence between sources. We demonstrate the practical application of these methods with two case studies: suicide data in Cambodia and heroin use in Thailand.

**Keywords:** Population size estimator, Chapman estimator, Semi-parametric bootstrap, Imputed bootstrap, Uncertainty

# 1 Introduction

The capture-recapture (CR) method is a powerful statistical technique used to quantify the total population sizes and unobserved units that are missing or difficult to fully observe. It is frequently used in many fields, such as biology, ecology, epidemiology, and social sciences (see McCrea and Morgan 2014; Böhning et al. 2018; Seber and Schofield 2019). For example, scientists in ecology use CR approaches to estimate the population size of animal species or wildlife in a given area (Borchers et al. 2002; 2004; Foley et al. 2025). Social scientists apply CR methods to estimate the population size of illicit drug users, drug offenders, and perpetrators of domestic violence (Böhning et al. 2004; Roberts and Brewer 2006). They are also key for epidemiologists in determining how many people are affected by diseases and estimate the incidence of many diseases and health-related problems, including cancer, stroke, and homeless people (Sukrat et al. 2020; Lerdsuwansri et al. 2022; Domitz et al. 2024). Since the accuracy of population size estimation is an important aspect of the purposes, various methodologies have been developed, depending on count data distributions, inflation in count frequencies, and heterogeneity problems.

The simplest and *most frequently* used CR method uses two trapping sources (or lists) of data to estimate the unobserved count, leading to an estimate of the total population size. We refer to this as *dual system estimation*. Under this method, it is important to note that the duration of the survey is typically conducted over a short period of time, so it is unlikely that the evolution of new cases or the extinction of existing cases will occur during the study. The population of interest must be closed (no changes in the population size during the time period). All members of the population must be independent of each other. Furthermore, all observational occasions must be independent of each other. For details on fundamental and important assumptions in CR studies and their applications, see Böhning et al. (2004); Brittain and Böhning (2009); Mukem et al. (2014); Harris et al. (2016) for example.

Due to the two sources used in dual system estimation, the lists refer to different but overlapping populations and one empty, unknown cell. Estimating the unobserved cell counts, leading to an estimate of the population size, is therefore a matter of interest. The following subsections are detailed and extensive explanations of the traditional types of two-source CR estimation corresponding to this work. They include the Lincoln-Petersen estimator and the Chapman estimator for population size. These are provided to motivate our approaches, which will be described in detail in Section 2.

## 1.1 Settings and notations

Assume that the unknown size  $N$  of a population remains unchanged during the study period and also suppose that there are two sources of the identification mechanism. Both sources can be linked, and each covers some part of the same population. According to the settings shown in Table 1, it leads to the following data constellation:  $n_{11}$  is the frequency of individuals identified by the two sources,  $n_{10}$  is the frequency of those identified by source 1 only but not by source 2,  $n_{01}$  is the frequency of those identified by source 2 only but not by source 1, and  $n_{00}$  is the frequency of individuals identified by neither source. In the marginal of the  $2 \times 2$  contingency table,  $n_{1\cdot}$  is the number observed at source 1,  $n_{\cdot 1}$  is the number observed at source 2. If all

**Table 1** The capture-recapture setting for two-source situation

		Source 2		
		Observed (1)	Not observed (0)	
Source 1	Observed (1)	$n_{11}$	$n_{10}$	$n_{1\cdot}$
	Not observed (0)	$n_{01}$	$n_{00}$	$N$

cell frequencies are given, the population size is simply calculated as  $N = n + n_{00}$ , where  $n = n_{11} + n_{10} + n_{01}$  (Bishop et al. 2007; Baffour et al. 2013). However, the cases known would be  $n$  and an estimate of the quantity  $n_{00}$ , the number of missing cases,

1 is required. So, if  $n_{00}$  is accurately estimated using the appropriate CR methodology,  
2 so would be  $N$ .

## 3 4 5 **1.2 Overview of estimators**

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7 As a simple method for dual system estimation, an estimate  $\hat{N}$  of  $N$  can be constructed  
8 by  $\hat{N} = n + \hat{n}_{00}$ , where  $n$  is the observed sample size and  $\hat{n}_{00}$  is an estimator to estimate  
9 the unknown value  $n_{00}$ . Lincoln (1930) and Petersen (1896) derive the estimator for  
10  $n_{00}$  using maximum likelihood estimation. It is given as  $\hat{n}_{00_{LP}} = n_{10}n_{01}/n_{11}$ . The  
11 Lincoln-Petersen estimator for the true population size  $N$  is then formulated by  
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$$15 \hat{N}_{LP} = n + \hat{n}_{00_{LP}} = \frac{n_{1.}n_{.1}}{n_{11}}$$

16  
17 with the estimated variance

$$18 \widehat{Var}(\hat{N}_{LP}) = \frac{n_{10}n_{01}n_{1.}n_{.1}}{n_{11}^3}.$$

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22 The Lincoln-Petersen estimators given above are often used in applications, as they  
23 provide the simple closed-form solutions and are straightforward. However, this  
24 method requires the following essential conditions:  
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- 27 1. Both sources must be independent. In other words, the odds ratio (OR) between  
28 the sources must be equal to 1. This can be written as  $OR = n_{11}n_{00}/n_{10}n_{01} = 1$ ,  
29 leading to  $n_{00} = n_{10}n_{01}/n_{11}$ .
- 30 2. The frequency  $n_{11}$  must be greater than 0 to avoid the undefinable  $\hat{n}_{00_{LP}}$  of the  
31 denominator.  
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1 According to condition 2, if there is no overlap between sources ( $n_{11} = 0$ ),  $\hat{n}_{00_{LP}}$   
 2 cannot be defined. Chapman (1951) proposed the nearly unbiased modification  
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$$4 \hat{N}_C = \frac{(n_{1\cdot} + 1)(n_{\cdot 1} + 1)}{n_{11} + 1} - 1 = n + \frac{n_{10}n_{01}}{n_{11} + 1}, \quad (1)$$

7 with the implied estimator of the missing cell  
 8

$$9 \hat{n}_{00_C} = \frac{n_{10}n_{01}}{n_{11} + 1}.$$

10 A standard large-sample variance estimator is  
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$$12 \widehat{\text{Var}}(\hat{N}_C) = \frac{(n_{1\cdot} + 1)(n_{\cdot 1} + 1) n_{10} n_{01}}{(n_{11} + 1)^2 (n_{11} + 2)},$$

13 leading to the usual  $(1 - \alpha)100\%$  two-sided Wald interval  
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$$16 \hat{N}_C \pm z_{1-\alpha/2} \sqrt{\widehat{\text{Var}}(\hat{N}_C)}, \quad (2)$$

17 where  $z_{1-\alpha/2}$  is the  $(1 - \alpha/2)100\%$ th percentile of the standard normal distribution.  
 18

19 Chapman's estimator is exactly unbiased whenever the combined sample size meets  
 20 or exceeds the population size ( $n_{1\cdot} + n_{\cdot 1} \geq N$ ); otherwise it has a slight downward bias  
 21 whose magnitude decreases as  $n_{11}$  grows. Moreover, the usual variance estimator above  
 22 is unbiased when  $n_{1\cdot} + n_{\cdot 1} > N$  (Seber 1970). Normal-approximation intervals can  
 23 undercover when  $n_{11}$  is small. Alternatives are thus required (Sadinle 2009). Moreover,  
 24 small sample sizes can bias  $\hat{N}_C$ , so that:  
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$$30 E(\hat{N}_C) = N - N\Delta = N(1 - \Delta),$$

1 where  $\Delta = \exp\{-(n_{1.} + 1)(n_{.1} + 1)/N\}$ . According to this performance, the potential  
2 small-sample bias of the Chapman estimator can be constructed. As given in [Seber](#)  
3 [\(2002\)](#), it is denoted as the bias-corrected estimator and given by  
4

$$5 \hat{N}_{BC} = \frac{\hat{N}_C}{1 - \hat{\Delta}},$$

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10 where  $\hat{\Delta} = \exp\{-(n_{1.} + 1)(n_{.1} + 1)/\hat{N}_C\}$  is an approximation of  $\Delta$ .

11  
12 The Chapman estimator  $\hat{N}_C$  corrects for bias in the Lincoln-Petersen estimator  
13  $\hat{N}_{LP}$ . We have outlined above the conditions under which the Chapman estimator  
14 is unbiased. If these conditions are violated, the Chapman estimator can experience  
15 a negative bias, in other words provides a lower bound for the true population size.  
16 This bias-corrected Chapman estimator  $\hat{N}_{BC}$  might be viewed as a small-sample-bias-  
17 corrected Chapman estimator, although we keep the name bias-corrected Chapman  
18 estimator for simplicity.  
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### 26 **1.3 Our ultimate objective**

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28 The sole use of the asymptotic variance for statistical inference might be problematic,  
29 particularly, in the case of a small observed sample size. Hence, we are interested  
30 in exploring alternative ways to provide uncertainty assessment for the dual systems  
31 estimator. In doing so, we will focus on estimating the population size using the  
32 resampling technique. Since the Chapman estimator has some advantages, it will be  
33 used to perform bootstrap estimates and confidence intervals. Several versions of the  
34 bootstrap are considered. One is the simplest non-parametric bootstrap, in which data  
35 are resampled based on the observed sample. However, as pointed out by [Buckland and](#)  
36 [Garthwaite \(1991\)](#), [Rivest et al. \(1995\)](#), and [Zult et al. \(2025\)](#), the bootstrap should  
37 be applied with an imputed population size to account for the uncertainty introduced  
38 by imputation. The imputed bootstrap and double bootstrap were discussed, but  
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1 the procedures were described somewhat vaguely. Therefore, they are studied and  
2 explained in depth in the next section. A simulation study with different scenarios is  
3 then used to investigate the methods. We expect that bootstrapping can address the  
4 limitation of existing estimators.  
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## 7 **2 Uncertainty assessment of the Chapman estimator** 8 **using bootstrap** 9

10 The bootstrap method is a powerful statistical tool to estimate the parameter of  
11 interest based on a resampling technique. The main idea of this method is to use  
12 sampling with replacement to sample new sets of samples from existing data. The  
13 percentiles of the sample statistics of interest then produce the point estimate and  
14 the confidence limits (Efron 1988; DiCiccio and Efron 1996). To be more precise,  
15 we introduce three types of bootstrap approach for estimating  $N$ . These include the  
16 imputed bootstrap, the double bootstrap with imputation, and the simple bootstrap  
17 without imputation. Since the bootstrap approach can provide the standard error of  
18 the statistics, uncertainty estimates can be derived. The following subsections provide  
19 details of these bootstraps and their computation procedures.  
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### 32 **2.1 Bootstrap I (Bootstrap with imputation)** 33 34

35 From the two-source CR setting, it is reasonable to assume that the sampling distri-  
36 bution of the data follows a multinomial distribution with size parameter  $\hat{N}$  and a  
37  $2 \times 2$  probability parameter. However, observing only three of the four cells in the  
38 two-way table makes it impossible to define the full probabilities. We then employ the  
39 following method to estimate this unknown value.  
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44 In this section, the imputed bootstrap (or semiparametric bootstrap method) uses  
45 a simple imputation method to estimate the probability parameter of the missing cell.  
46 It is modified from the imputation method given in Norris and Pollock (1996) and  
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the parametric bootstrap approach introduced in Zwane and van der Heijden (2005). Although the imputed bootstrap in the CR setting has been applied to estimate the population size in several works (see Anan et al. 2017; Böhning et al. 2023; Böhning and Friedl 2024), the previous works have not been available for the two-source situation. Therefore, we provide the algorithm for the imputed bootstrap method as follows:

1. According to the settings in Table 1, calculate the event probability vector

$$\hat{p} = \left( \frac{n_{11}}{\hat{N}_C}, \frac{n_{10}}{\hat{N}_C}, \frac{n_{01}}{\hat{N}_C}, \frac{\hat{n}_{00C}}{\hat{N}_C} \right),$$

where  $\hat{N}_C = n + \hat{n}_{00C}$  is the Chapman estimator for  $N$  and  $n = n_{11} + n_{10} + n_{01}$ .

2. Draw  $n_{11}^*, n_{10}^*, n_{01}^*$ , and  $n_{00}^*$  from a multinomial distribution with size parameter  $\hat{N}_C$  and parameter vector  $\hat{p}$ . We have attached a \* to the four frequencies to indicate that they are sampled from the observed/imputed sample.
3. Ignore  $n_{00}^*$  and calculate the bootstrap estimates

$$\hat{n}_{00}^* = \frac{n_{10}^* n_{01}^*}{n_{11}^* + 1}$$

and

$$\hat{N}^* = n_{11}^* + n_{10}^* + n_{01}^* + \hat{n}_{00}^*.$$

4. Repeat steps 2 and 3  $B$  times and yield  $B$  Chapman estimates

$$\hat{N}^{*(1)}, \hat{N}^{*(2)}, \dots, \hat{N}^{*(B)}.$$

5. Calculate the  $(1 - \alpha)100\%$  percentile bootstrap confidence interval for  $N$ .

## 2.2 Bootstrap II (Double bootstrap with imputation)

In Bootstrap I, the estimated, unobserved frequency estimate is kept fixed throughout the process. We think this may ignore the uncertainty involved in estimating the unobserved frequency. In a different approach, we try to incorporate this element in a further step. This approach is referred to as the double bootstrap method. We provide the bootstrap algorithm to compute uncertainty measures for the estimator as follows:

1. From the available data, calculate the event probability vector

$$\tilde{p} = \left( \frac{n_{11}}{n}, \frac{n_{10}}{n}, \frac{n_{01}}{n} \right).$$

2. Draw  $n_{11}^*, n_{10}^*, n_{01}^*$  from a multinomial distribution with size parameter  $n$  and event parameter  $\tilde{p}$ .
3. Calculate the bootstrap estimate

$$\hat{n}_{00}^* = \frac{n_{10}^* n_{01}^*}{n_{11}^* + 1}.$$

4. Then, calculate

$$\hat{p}^* = \left( \frac{n_{11}}{\hat{N}^*}, \frac{n_{10}}{\hat{N}^*}, \frac{n_{01}}{\hat{N}^*}, \frac{\hat{n}_{00}^*}{\hat{N}^*} \right),$$

where  $\hat{N}^* = n_{11} + n_{10} + n_{01} + \hat{n}_{00}^*$ .

5. Draw  $n_{11}^{**}, n_{10}^{**}, n_{01}^{**}, n_{00}^{**}$  from a multinomial distribution with size parameter  $\hat{N}^*$  and probability parameter  $\hat{p}^*$ .
6. Ignore  $n_{00}^{**}$  and compute bootstrap estimates

$$\hat{n}_{00}^{**} = \frac{n_{10}^{**} n_{01}^{**}}{n_{11}^{**} + 1}$$

and

$$\hat{N}^{**} = n_{11}^{**} + n_{10}^{**} + n_{01}^{**} + \hat{n}_{00}^{**}.$$

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7. Repeat steps 2 to 6  $B$  times to get  $B$  Chapman estimates

$$\hat{N}^{**(1)}, \hat{N}^{**(2)}, \dots, \hat{N}^{**(B)}.$$

8. Calculate the  $(1 - \alpha)100\%$  percentile bootstrap confidence interval for  $N$  on the basis of estimates given in step 7.

### 2.3 Bootstrap III (Bootstrap without imputation)

In this subsection, we consider a bootstrap that is solely based on the resampling approach, conditional on the observed data of size  $n$ . The major distinction between the bootstrap given in this section, called Bootstrap III, and the previous two bootstraps is that  $n$  is fixed, and there is no imputation involved. Therefore, Bootstrap III is just based on the observed data, which is similar to the simple non-parametric bootstrap (Zwane and van der Heijden 2005). Again, each individual is generated from a multinomial distribution. We outline the process for Bootstrap III as follows:

1. From the available data, calculate the event probability vector

$$\tilde{p} = \left( \frac{n_{11}}{n}, \frac{n_{10}}{n}, \frac{n_{01}}{n} \right).$$

2. Draw  $n_{11}^*, n_{10}^*, n_{01}^*$  from a multinomial distribution with size parameter  $n$  and probability parameter  $\tilde{p}$ .

3. Compute the bootstrap estimates

$$\hat{n}_{00}^* = \frac{n_{10}^* n_{01}^*}{n_{11}^* + 1}$$

and

$$\hat{N}^* = n_{11}^* + n_{10}^* + n_{01}^* + \hat{n}_{00}^*.$$

4. Repeat steps 2 and 3  $B$  times and yield  $B$  Chapman estimates

$$\hat{N}^{*(1)}, \hat{N}^{*(2)}, \dots, \hat{N}^{*(B)}.$$

5. Calculate the  $(1 - \alpha)100\%$  percentile bootstrap confidence interval for  $N$ .

The algorithms described in the previous subsections are not limited to the Chapman estimator, which uses  $\hat{N}_C$ , but can be readily extended to other estimators. In the application and simulation sections, they are applied to calculate the population size using the bias-corrected estimator, which is based on  $\hat{N}_{BC}$ .

### 3 Case studies

Two real data examples are used to illustrate the methods introduced in this work. The first example is related to the suicide rate in Cambodia, obtained from Harris et al. (2016). Since there are no adequate Cambodian suicide statistics, the work focusses on estimating the population size of suicides in this country. The reported suicides were collected in 2012 from the two newspapers, including the Raksmeay Kampuchea Daily News and the Koh Santepheap Daily News, which were the two largest newspapers in the native language (Khmer language) of Cambodia. The frequency counts of suicides from these two sources are shown in Table 2. Here, a total of 158 suicide deaths are observed. However, there are likely hidden units due to their non-reporting in published news or other media. Therefore, the total number of suicides in the population needs to be estimated.

The second example is related to heroin users in Thailand. Although Thailand is not a heroin-producing country, heroin and narcotic abuse have long been a distressing issue in the country and increased in use among age groups (Centre for Addiction

**Table 2** Number of suicides in Cambodia obtained from two newspapers in year 2012 ( $n = 158$ )

Source 1 (Raksmeay Kampuchea News)	Source 2 (Koh Santepheap News)	
	Observed (1)	Not observed (0)
Observed (1)	12	94
Not observed (0)	52	-

Studies 2023). Question: How many hidden heroin users? To demonstrate, we look at data of heroin users in Pathum Thani province of Thailand to estimate the number of hidden users. The Princess Mother National Institute on Drug Abuse Treatment (PMNIDAT) (see <http://www.pmnidat.go.th/thai/>), which is organized under the Department of Medical Services, collected data on heroin addicts who attended the treatment facility during Thailand’s fiscal year 2023, which started in October 2022 and ended in September 2023. Repeated counts occurred over a given period, leading to a total number of 1447 heroin users observed with their visits. As the context of this work is concerned with the two-source situation and the contribution is built upon the Chapman estimator, we divide the data into two distinct lists: Source 1, representing the first half of the fiscal year (October 2022 to March 2023), and Source 2, which represents the second half of the fiscal year (April to September 2023). The number of heroin users on two occasions is provided in Table 3.

**Table 3** Number of heroin users from the health support in Pathum Thani, Thailand, obtained from the fiscal year 2023 ( $n = 1447$ )

Source 1 (First half year)	Source 2 (Second half year)	
	Observed (1)	Not observed (0)
Observed (1)	121	747
Not observed (0)	579	-

The approximate number of cases that are never captured from both sources ( $\hat{n}_{00} = \hat{N} - n$ ) and estimated population sizes ( $\hat{N}$ ) from two examples are given in Table 4.

Note that the numbers in brackets in Table 4 refer to the bias-corrected Chapman estimator. Due to the large sample size, there is almost no difference between  $\hat{N}_C$  and  $\hat{N}_{BC}$ . The formula method refers to parameter estimation based on  $\hat{N}_C$  and the asymptotic normal confidence interval given in (1) and (2), respectively. Since the variance formula for  $\hat{N}_{BC}$  is not available in a closed form, the confidence interval is then only based on the bootstrap method. The bootstrap estimate in each approach is averaged from the median of 10000 bootstrap samples. The statistical analysis is done through the R programming language (R Core Team 2024). From the results, the point estimates from the formula method are similar to those of bootstraps. However, interval estimation reveals an effect. For suicide data, where  $n$  is small, the confidence intervals of all methods are different. For heroin users data, where  $n$  is larger, the confidence intervals based on the formula and Bootstrap III methods behave quite similarly, but are different from Bootstraps I and II. We note that the formula and Bootstrap III methods do not allow imputation of missing cells. Meanwhile, the rest are imputed bootstraps.

**Table 4** Estimated number of suicides in Cambodia and heroin users in Pathum Thani, Thailand using Chapman estimator (and bias-corrected Chapman estimator in bracket) with confidence interval (CI) and length of interval from the formula and bootstrap methods

Dataset	Method	$\hat{n}_{00}$	$\hat{N}$	95% CI for $N$	Interval length
Suicide ( $n = 158$ )	Formula	376 (376)	534 (534)	300-769	469
	Bootstrap I	380 (376)	538 (536)	360-941 (361-935)	581 (574)
	Bootstrap II	376 (381)	534 (539)	358-941 (364-929)	583 (565)
	Bootstrap III	379 (379)	537 (537)	377-926 (374-932)	549 (558)
Heroin ( $n = 1447$ )	Formula	3545 (3545)	4992 (4992)	4248-5735	1487
	Bootstrap I	3542 (3543)	4989 (4990)	4338-5849 (4338-5841)	1511 (1503)
	Bootstrap II	3553 (3552)	5000 (4999)	4341-5868 (4344-5843)	1527 (1499)
	Bootstrap III	3548 (3547)	4995 (4994)	4296-5745 (4364-5795)	1449 (1431)

The results of all population estimators seem to be realistic, as the lower and upper limits for  $N$  are greater than the observed sample size. However, to investigate the performance of these methods, especially the uncertainty of estimation in terms of

confidence interval, we provide a simulation study on the basis of several scenarios in the next section.

## 4 Simulation study

### 4.1 Simulation scenarios

A simulation study is performed to investigate the performance of the confidence intervals for  $N$ . We design the study to cover scenarios in CR studies with two sources. The parameter settings are given as follows. The population sizes  $N$  are 10, 25, 50, 100, and 250. The capture probabilities are varied as shown in Table 5, where in case A, we choose populations with increasing  $p_{00}$  to study this effect, while in case B populations are chosen with increasing dependency, so that the estimator becomes more conservatively biased. The case B simulation is designed with increasing levels of positive dependency, which is the most likely case in reality. See also [Brittain and Böhning \(2009\)](#). The odds ratio can be calculated as  $OR = p_{11}p_{00}/p_{10}p_{01}$ . Since the odds ratio between sources plays a crucial factor in the behaviour of the Chapman estimator, we study the cases where two sources are independent ( $OR = 1$ ) and positive dependent ( $OR > 1$ ). For each of the populations, the frequencies  $\mathbf{f} = (n_{11}, n_{10}, n_{01}, n_{00})$  are generated from a multinomial distribution with parameters  $(N, \mathbf{p})$ , where  $\mathbf{p} = (p_{11}, p_{10}, p_{01}, p_{00})$ . Then, the unobserved count  $n_{00}$  is ignored as the CR data.

The formula method and three bootstrap approaches are used to estimate  $N$ . Each scenario is repeated 10000 times, and 5000 bootstrap samples are used. All are done using R (R Core Team 2024). On average,  $\hat{N}$  obtained from the formula method is computed by

$$\frac{1}{R} \sum_{r=1}^R \hat{N}_r,$$

**Table 5** Capture probabilities of event occurrence in two-source CR studies and odds ratio (OR) for simulations

Case	Population	$p_{11}$	$p_{10}$	$p_{01}$	$p_{00}$	OR
A	A1	0.320	0.480	0.080	0.120	1
	A2	0.250	0.250	0.250	0.250	1
	A3	0.125	0.125	0.375	0.375	1
	A4	0.050	0.050	0.450	0.450	1
	A5	0.040	0.160	0.160	0.640	1
	A6	0.020	0.080	0.180	0.720	1
B	B1	0.200	0.150	0.350	0.300	1.143
	B2	0.200	0.120	0.380	0.300	1.316
	B3	0.300	0.200	0.250	0.250	1.500
	B4	0.250	0.200	0.200	0.350	2.188
	B5	0.300	0.150	0.200	0.350	3.500
	B6	0.350	0.100	0.200	0.350	6.125

where  $\hat{N}_r$  is the estimated population size in the  $r$ -th replication. Furthermore,  $\hat{N}$  based on the bootstrap approach is calculated by

$$\frac{1}{R} \sum_{r=1}^R \left[ \text{median} \left( \hat{N}^{*(1)}, \hat{N}^{*(2)}, \dots, \hat{N}^{*(B)} \right) \right]_r,$$

where  $\hat{N}^{*(b)}$  are the bootstrap estimates in the  $b$ -th sample, for  $b = 1, 2, \dots, B$ , depending on Bootstrap I, II, or III. The performance of the estimator is evaluated in terms of relative bias, approximated by

$$\frac{1}{N} \left( \frac{1}{R} \sum_{r=1}^R \hat{N}_r - N \right).$$

We investigate the performance of the 95% confidence interval by evaluating how often the confidence limits cover the true  $N$ . Therefore, the estimated coverage probability of the confidence interval from simulation is calculated by

$$\frac{1}{R} \sum_{r=1}^R C_r,$$

1 where  $C_r$  is an indicator variable defined as 1 if  $CI_{L,r} \leq N \leq CI_{U,r}$  and 0 otherwise.  
2  $CI_{L,r}$  and  $CI_{U,r}$  are the lower and upper limits of  $N$  in the  $r$ -th iteration. In conclusion,  
3 the confidence interval with a coverage probability close to the nominal level of 0.95,  
4 or the confidence interval that has the largest coverage probability, is more efficient  
5 than the other comparator.  
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## 10 4.2 Simulation results

11 The simulation results show the relative biases of estimated population size and the  
12 coverage probabilities of the confidence intervals for  $N$  using the data generated in  
13 *case A* and *case B*. They are presented in Tables 6-9. The coverage rates are also  
14 shown by graphs displayed in Figures 1 and 2.  
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19 We first examine the simulation results under *case A* (see Tables 6 and 7), which  
20 assumes the independence between two sources. The main findings are summarized  
21 as follows. The estimates of  $N$  obtained from the three bootstraps are quite close to  
22 those of the formula method. They underestimate if  $N \leq 50$  and the probability of  
23 any individual not appearing in either source ( $p_{00}$ ) is large, corresponding to popula-  
24 tions A5 and A6. However, all approaches provide a small estimation bias when  $N$  is  
25 increased. The Chapman and biased-corrected estimators show similar behaviour in  
26 terms of relative bias, except when the population size is small ( $N = 10$ ). As can be  
27 seen in Figure 1, the confidence intervals for  $N$  from Bootstrap I and Bootstrap II have  
28 similar coverage probabilities. They are also close to the target probability of 0.95 for  
29  $N > 50$  and show better coverage than the comparators in all cases of the study. The  
30 difference in coverage probability of the confidence intervals from Bootstraps I and II  
31 to Bootstrap III and the confidence interval using the variance formula is particularly  
32 pronounced if  $p_{00}$  becomes large (see populations A4-A6). In conclusion, Bootstraps  
33 I and II perform equally well in general cases where two sources are independent.  
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1 However, Bootstrap II is computationally more expensive as it takes time in the cal-  
 2 culation. More specifically, a complete simulation run for one setting of the simulation  
 3 design takes approximately two minutes for Bootstraps I and III, whereas it requires  
 4 approximately five minutes for Bootstrap II. Bootstrap I is then recommended in this  
 5 situation.  
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8 We then consider the situation in which the odds ratio between two sources is  
 9 greater than 1 (*case B*). This allows for dependence between sources, which can  
 10 often occur in applications. In this case, we use the expected value of the Chapman  
 11 estimator, called the *estimand*, as the true parameter. It is defined by  
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$$15 E(\hat{N}) \approx N \left( \frac{p_{10}p_{01}}{p_{11}} + p_{11} + p_{10} + p_{01} \right).$$

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 21 Tables 8 and 9 show the relative biases of estimates and 95% coverage probabilities  
 22 for  $E(\hat{N})$  from simulations. All methods provide relative biases close to zero in all sit-  
 23 uations. The coverage probabilities of the confidence intervals are then evaluated and  
 24 shown in Figure 2. Most importantly, the confidence intervals based on Bootstraps I  
 25 and II provide coverage probabilities very close to 0.95 when  $OR < 3.5$  (see popula-  
 26 tions B1-B4). For the higher values of the OR (populations B5 and B6), all interval  
 27 estimators have low performance, as their coverage probabilities are lower than 0.95.  
 28 However, the confidence intervals from Bootstraps I and II still outperform those of  
 29 Bootstrap III and the asymptotic normal confidence interval, in particular for small  
 30  $N$ . The confidence interval based on the biased-corrected estimator provides coverage  
 31 rates smaller than that based on the Chapman method when  $N = 10$ . As  $N$  increases,  
 32 the two become increasingly similar.  
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42 Finally, we consider a simulation study on the performance of the estimation meth-  
 43 ods. The variances of the Chapman estimates from the three bootstrap methods are  
 44 provided and compared to the true variance. The results given in Figure 3 show that  
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1 with increasing  $N$  the variances of all bootstrap methods increase and behave simi-  
2 larly to the true variance. If the OR increases (population B6 has the largest OR), the  
3 variances of the bootstrap methods increase much less than the true variance, with  
4 strong underestimation for large OR. In particular, they are much lower variance than  
5 the true variance when  $OR \geq 3.5$  (see populations B5 and B6). This could lead to a  
6 confidence interval which is too narrow and, consequently, has too low level of cover-  
7 age probability. To clarify, we show in Table 10 the simulated percentiles of Chapman  
8 estimates for the true and bootstrap distributions under the three situations:  $N = 250$   
9 for population A1,  $N = 250$  for population B6, and  $N = 1000$  for population B6.  
10 Under the first situation (independence of sources), the medians of the estimates of  
11 the bootstrap distributions are close to the true  $N$ . This is in contrast to the latter  
12 two situations (dependence setting). The medians of the estimates of the bootstrap  
13 distributions are still correctly estimating the estimand  $E(\hat{N})$ . However, the other rel-  
14 evant percentiles of the estimates from the true and bootstrap distributions clearly  
15 differ, the more the further in the tails of the distribution. Here, the bootstrap meth-  
16 ods have interval lengths narrower than the true distribution. This point corresponds  
17 to the simulation study given in Figure 2, where the bootstrap variance is less than  
18 the true variance, leading to a small interval length and low coverage probability of  
19 confidence interval for the extremely large dependency situation. The reason for this  
20 performance lies in the fact that in case of high association of the two sources, the  
21 imputed bootstrap will consistently sample from a population with a potentially much  
22 smaller population size (the estimand) rather than the true (potentially imputed)  $N$ ,  
23 and hence underestimate the variability of the Chapman estimator. There is no way  
24 that this can be corrected, as the true odds ratio would need to be known to do so.  
25 Therefore, undercoverage of confidence intervals will occur in dependence situations  
26 even if the estimand is used instead of the true population size.  
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## 5 Concluding remarks

The Lincoln-Petersen estimator (1896; 1930) is the simplest tool in capture-recapture studies with two sources used to estimate the hidden population size. Due to a limitation of a zero frequency count of individuals identified at both sources (no overlap between sources), Chapman (1951) modifies the estimator of Lincoln-Petersen and shows that it is less biased than the original one, under certain conditions even being unbiased. Hence, it can be viewed as a bias-corrected Lincoln-Petersen estimator. We have outlined the conditions under which the Chapman estimator is unbiased. If these conditions are violated, the Chapman estimator can experience a negative bias, in other words provides a lower bound for the true population size. We have seen in the simulation work that this underestimation bias is mainly relevant for population sizes below 50. In these cases, one might apply the bias-correction as worked in Section 1.2. Uncertainty assessment of these estimators requires knowledge of their variances, and estimates have been developed based on asymptotic normality. However, these turn out to be poor for small and moderate sample sizes. Hence, an alternative approach based on bootstrapping has been explored here.

Bootstrapping can be implemented in several ways. In fact, bootstrapping methods have already been considered to estimate uncertainty in population size estimation in recent papers including the simple bootstrap and the imputed bootstrap. Here, we have also suggested a double bootstrap to cope with the uncertainty induced by the imputation. The imputed bootstrap, the double bootstrap with imputation, and the simple bootstrap (non-parametric bootstrap without imputation) have not yet studied in depth in two-source CR estimation and compared through simulations. In this paper, we provide novel insights on the behaviour of these methods.

A large-scale simulation study is used to evaluate the performance of our bootstraps and the normal approximation-based variance formula approach in terms of the confidence intervals. What works and what does not work? The findings are divided

1 into two situations. Under the independence of sources situation (assumption holds),  
2 the formula method and bootstrap without imputation do not perform well, as their  
3 coverage probabilities of the confidence intervals are lower than those of the imputed  
4 bootstrap and double bootstrap methods. From the results, the performances of the  
5 imputed bootstrap and double bootstrap are similar. However, as we have noted, the  
6 computational effort for the double bootstrap is larger by a factor of 2.5. We therefore  
7 recommend using the imputed bootstrap in practice due to its reduced computational  
8 effort compared to the double bootstrap. Under the violation of the independence  
9 assumption, all methods do not work well when an extreme dependence between two  
10 sources occurs, such as the odds ratios being 3 or more. For small and moderate depen-  
11 dence between sources setting, the imputed bootstrap and double bootstrap perform  
12 satisfactorily. They provide a coverage probability larger than the simple bootstrap  
13 and the one based on the asymptotic normal approximation. Again, there are only  
14 slight differences in the confidence intervals between these two imputed bootstrap ver-  
15 sions. In conclusion, the imputed bootstrap seems to be the best choice among all the  
16 methods considered here.

## 29 **Conflict of interest**

30 The authors declare no conflict of interest.

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**Table 6** Relative bias of Chapman estimator and biased-corrected Chapman estimator under simulations (Case A: OR = 1)

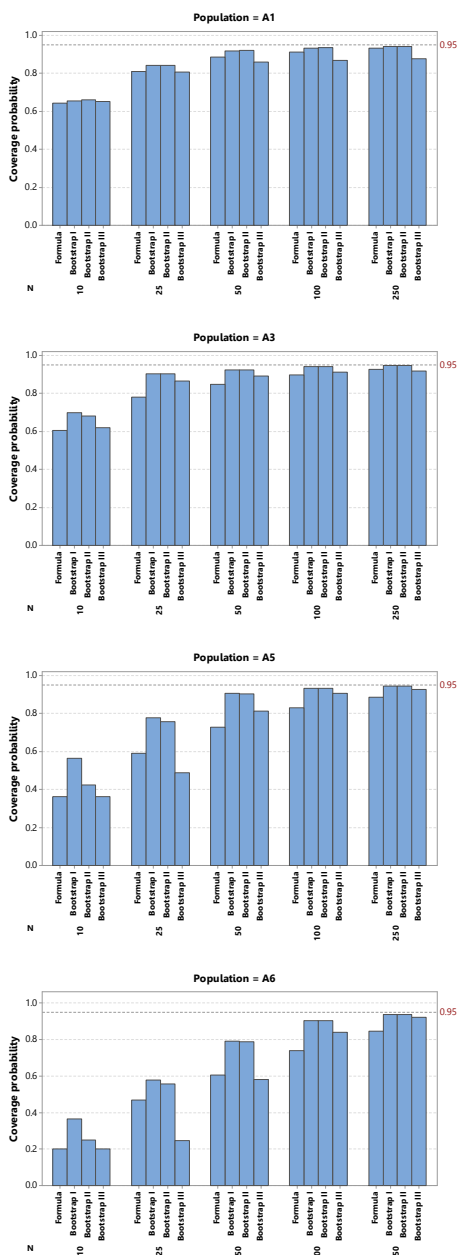
N	Population	Chapman						Biased-corrected Chapman					
		Formula	Bootstrap I	Bootstrap II	Bootstrap III	Formula	Bootstrap I	Bootstrap II	Bootstrap III				
10	A1	-0.0025	-0.0096	-0.0089	-0.0117	0.0217	0.0156	0.0166	0.0134				
	A2	-0.0205	-0.0333	-0.0331	-0.0345	0.0501	0.0334	0.0357	0.0381				
	A3	-0.1148	-0.1511	-0.1327	-0.1236	0.0774	0.0239	0.0535	0.0692				
	A4	-0.2908	-0.3291	-0.3004	-0.2939	-0.0529	-0.1107	-0.0661	-0.0563				
	A5	-0.4658	-0.4726	-0.4726	-0.4757	-0.4096	-0.4129	-0.4139	-0.4150				
	A6	-0.5734	-0.5799	-0.5798	-0.5815	-0.5324	-0.5345	-0.5349	-0.5352				
25	A1	-0.0017	-0.0041	-0.0051	-0.0119	0.0027	-0.0016	-0.0026	-0.0096				
	A2	-0.0020	-0.0024	-0.0031	-0.0106	0.0083	0.0072	0.0063	-0.0030				
	A3	-0.0133	-0.0216	-0.0212	-0.0189	0.0752	0.0655	0.0652	0.0636				
	A4	-0.1396	-0.1701	-0.1577	-0.1419	0.0950	0.0420	0.0559	0.0741				
	A5	-0.2369	-0.2708	-0.2662	-0.2450	0.0019	-0.0460	-0.0394	-0.0096				
	A6	-0.4301	-0.4717	-0.4571	-0.4345	-0.2151	-0.2726	-0.2529	-0.2245				
50	A1	0.0017	0.0003	0.0000	-0.0042	0.0007	0.0004	0.0000	-0.0042				
	A2	-0.0007	-0.0007	-0.0010	-0.0055	0.0011	-0.0005	-0.0008	-0.0054				
	A3	-0.0011	-0.0009	-0.0010	-0.0043	0.0123	0.0137	0.0135	0.0078				
	A4	-0.0357	-0.0488	-0.0457	-0.0380	0.0999	0.0885	0.0910	0.0922				
	A5	-0.0894	-0.0971	-0.0974	-0.0931	0.1029	0.0893	0.0885	0.0831				
	A6	-0.2628	-0.2893	-0.2872	-0.2649	-0.0171	-0.0479	-0.0454	-0.0179				
100	A1	-0.0013	-0.0019	-0.0021	-0.0045	0.0012	-0.0019	-0.0021	-0.0045				
	A2	-0.0004	-0.0004	-0.0005	-0.0029	-0.0006	-0.0004	-0.0005	-0.0029				
	A3	-0.0010	-0.0009	-0.0009	-0.0039	-0.0005	-0.0007	-0.0007	-0.0038				
	A4	0.0100	0.0096	0.0097	0.0092	0.0254	0.0492	0.0491	0.0453				
	A5	-0.0104	0.0026	0.0025	-0.0086	0.0615	0.0721	0.0718	0.0516				
	A6	-0.0942	-0.0920	-0.0919	-0.0855	0.1027	0.1085	0.1085	0.1097				
250	A1	0.0001	-0.0002	-0.0002	-0.0012	0.0012	-0.0002	-0.0002	-0.0012				
	A2	0.0002	0.0002	0.0002	-0.0008	0.0007	0.0002	0.0002	-0.0008				
	A3	0.0012	0.0012	0.0011	-0.0004	0.0029	0.0012	0.0011	-0.0004				
	A4	-0.0010	-0.0009	-0.0008	-0.0017	-0.0027	-0.0005	-0.0004	-0.0015				
	A5	-0.0007	0.0064	0.0064	0.0005	-0.0016	0.0083	0.0083	0.0018				
	A6	0.0006	0.0184	0.0185	0.0130	0.0372	0.0641	0.0643	0.0544				

**Table 7** Coverage probability of the 95% confidence interval for  $N$  using Chapman estimator and biased-corrected Chapman estimator under simulations (Case A: OR = 1)

$N$	Population	Formula	Chapman			Biased-corrected Chapman		
			Bootstrap I	Bootstrap II	Bootstrap III	Bootstrap I	Bootstrap II	Bootstrap III
10	A1	0.6424	0.6552	0.6620	0.6518	0.7232	0.7363	0.5421
	A2	0.7193	0.8036	0.8130	0.7658	0.8636	0.8759	0.7688
	A3	0.6071	0.6986	0.6805	0.6207	0.7700	0.7702	0.6803
	A4	0.3423	0.3840	0.3670	0.3147	0.4467	0.4460	0.3597
	A5	0.3649	0.5654	0.4257	0.3642	0.5670	0.5670	0.5657
	A6	0.2021	0.3658	0.2503	0.2021	0.3677	0.3677	0.3675
25	A1	0.8093	0.8425	0.8434	0.8064	0.8426	0.8435	0.7737
	A2	0.8540	0.9280	0.9302	0.8663	0.9275	0.9302	0.8640
	A3	0.7804	0.9032	0.9046	0.8653	0.9062	0.9073	0.8777
	A4	0.6210	0.7026	0.7006	0.6875	0.7248	0.7246	0.7001
	A5	0.5916	0.7790	0.7577	0.4878	0.9037	0.8986	0.6967
	A6	0.4700	0.5780	0.5580	0.2490	0.7650	0.7490	0.4800
50	A1	0.8858	0.9187	0.9212	0.8599	0.9186	0.9209	0.8577
	A2	0.8921	0.9331	0.9349	0.8748	0.9327	0.9346	0.8733
	A3	0.8469	0.9248	0.9248	0.8928	0.9265	0.9262	0.8941
	A4	0.7450	0.8735	0.8743	0.8540	0.8773	0.8783	0.8682
	A5	0.7280	0.9058	0.9051	0.8126	0.9087	0.9079	0.8388
	A6	0.6061	0.7918	0.7887	0.5813	0.9100	0.9106	0.7619
100	A1	0.9136	0.9331	0.9345	0.8677	0.9331	0.9345	0.8677
	A2	0.9227	0.9422	0.9430	0.8768	0.9422	0.9430	0.8768
	A3	0.8979	0.9430	0.9427	0.9127	0.9430	0.9427	0.9126
	A4	0.8375	0.9256	0.9255	0.9112	0.9275	0.9265	0.9143
	A5	0.8308	0.9327	0.9342	0.9061	0.9351	0.9357	0.9139
	A6	0.7388	0.9036	0.9030	0.8387	0.8899	0.8900	0.8498
250	A1	0.9321	0.9426	0.9430	0.8764	0.9426	0.9430	0.8764
	A2	0.9392	0.9483	0.9492	0.8889	0.9483	0.9492	0.8889
	A3	0.9275	0.9484	0.9480	0.9178	0.9484	0.9480	0.9178
	A4	0.8930	0.9409	0.9397	0.9285	0.9408	0.9398	0.9285
	A5	0.8859	0.9443	0.9442	0.9265	0.9446	0.9442	0.9266
	A6	0.8468	0.9373	0.9372	0.9213	0.9373	0.9374	0.9241

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(a) Chapman estimator



(b) Bias-corrected Chapman estimator

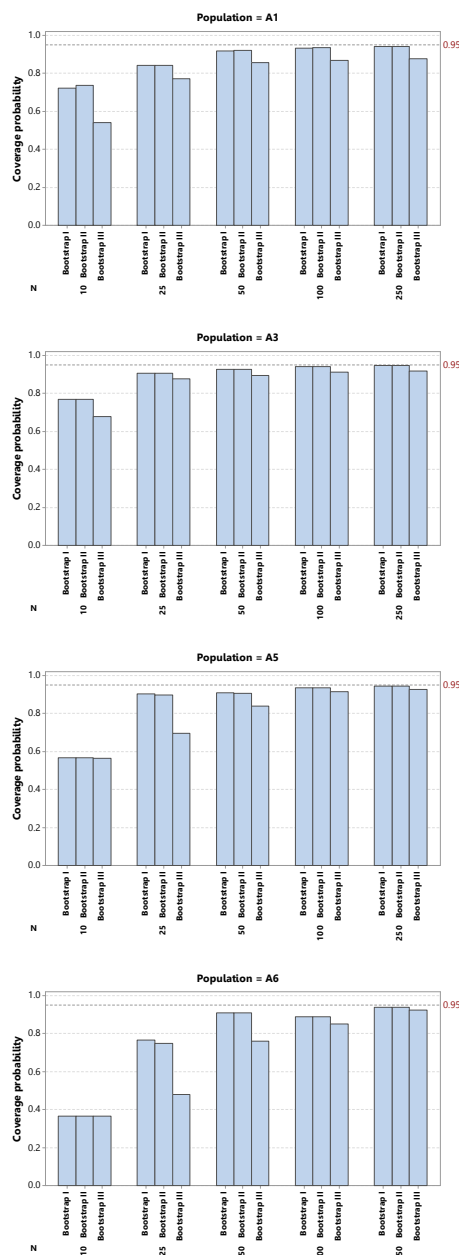


Fig. 1 Coverage probability of the 95% confidence interval for  $N$  using two estimators under simulations (Case A: OR = 1)

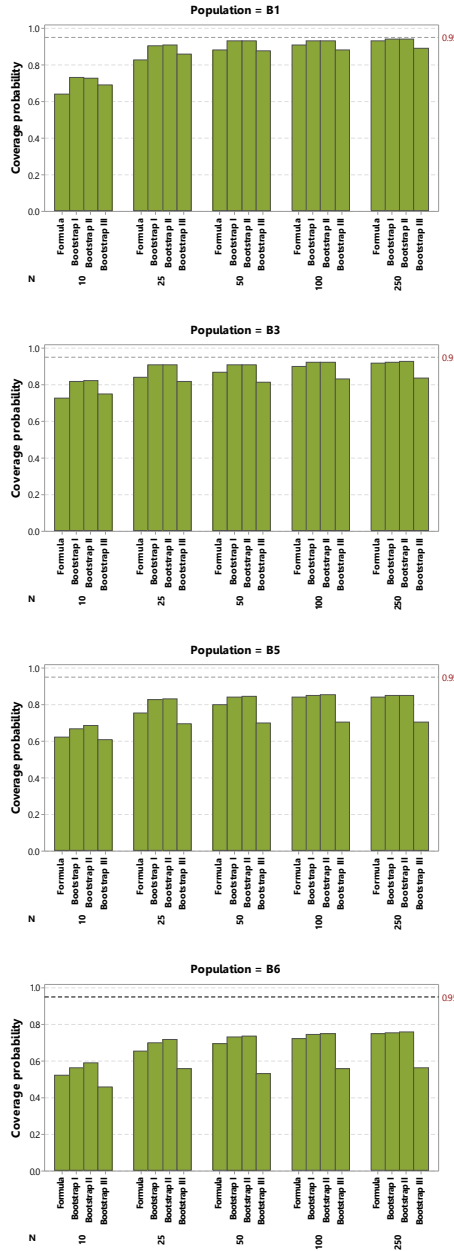
**Table 8** Relative bias of Chapman estimator and biased-corrected Chapman estimator under simulations (Case B:  $OR > 1$ )

N	Estimand	Population	Chapman			Biased-corrected Chapman				
			Formula	Bootstrap I	Bootstrap II	Bootstrap III	Formula	Bootstrap I	Bootstrap II	Bootstrap III
10	10	B1	-0.0738	-0.0939	-0.0882	-0.0852	0.0555	0.0268	0.0379	0.0456
	9	B2	-0.0978	-0.1180	-0.1102	-0.1081	0.0092	-0.0161	-0.0048	0.0006
	9	B3	-0.0900	-0.0980	-0.0990	-0.1044	-0.0378	-0.0460	-0.0460	-0.0497
	8	B4	-0.2005	-0.2126	-0.2102	-0.2154	-0.1374	-0.1516	-0.1474	-0.1490
	8	B5	-0.2529	-0.2611	-0.2607	-0.2678	-0.2297	-0.2372	-0.2361	-0.2408
	7	B6	-0.2926	-0.2987	-0.2989	-0.3040	-0.2582	-0.2610	-0.2616	-0.2622
25	24	B1	-0.0395	-0.0413	-0.0420	-0.0473	-0.0220	-0.0199	-0.0207	-0.0285
	23	B2	-0.0714	-0.0742	-0.0749	-0.0792	-0.0526	-0.0515	-0.0522	-0.0590
	23	B3	-0.0850	-0.0850	-0.0861	-0.0940	-0.0802	-0.0791	-0.0803	-0.0889
	20	B4	-0.1895	-0.1897	-0.1907	-0.1978	-0.1864	-0.1851	-0.1862	-0.1944
	19	B5	-0.2506	-0.2503	-0.2517	-0.2588	-0.2494	-0.2484	-0.2500	-0.2573
	18	B6	-0.2934	-0.2929	-0.2946	-0.3005	-0.2918	-0.2909	-0.2927	-0.2988
50	48	B1	-0.0365	-0.0369	-0.0373	-0.0422	-0.0371	-0.0372	-0.0374	-0.0427
	46	B2	-0.0736	-0.0746	-0.0749	-0.0796	-0.0712	-0.0717	-0.0720	-0.0770
	46	B3	-0.0854	-0.0853	-0.0856	-0.0903	-0.0835	-0.0834	-0.0837	-0.0884
	40	B4	-0.1907	-0.1906	-0.1909	-0.1953	-0.1918	-0.1916	-0.1919	-0.1965
	38	B5	-0.2507	-0.2503	-0.2507	-0.2551	-0.2496	-0.2491	-0.2495	-0.2540
	35	B6	-0.2917	-0.2907	-0.2915	-0.2956	-0.2923	-0.2913	-0.2921	-0.2962
100	96	B1	-0.0370	-0.0372	-0.0373	-0.0406	-0.0383	-0.0386	-0.0387	-0.0419
	93	B2	-0.0708	-0.0713	-0.0714	-0.0742	-0.0724	-0.0729	-0.0730	-0.0759
	92	B3	-0.0821	-0.0820	-0.0821	-0.0846	-0.0829	-0.0829	-0.0829	-0.0855
	81	B4	-0.1891	-0.1891	-0.1891	-0.1914	-0.1897	-0.1896	-0.1897	-0.1921
	75	B5	-0.2485	-0.2482	-0.2483	-0.2508	-0.2501	-0.2498	-0.2499	-0.2524
	71	B6	-0.2925	-0.2920	-0.2921	-0.2946	-0.2925	-0.2920	-0.2921	-0.2945
250	241	B1	-0.0374	-0.0375	-0.0375	-0.0390	-0.0381	-0.0383	-0.0383	-0.0398
	232	B2	-0.0720	-0.0722	-0.0723	-0.0735	-0.0722	-0.0725	-0.0725	-0.0738
	229	B3	-0.0835	-0.0835	-0.0835	-0.0847	-0.0835	-0.0834	-0.0834	-0.0846
	202	B4	-0.1900	-0.1900	-0.1900	-0.1910	-0.1889	-0.1889	-0.1889	-0.1899
	188	B5	-0.2500	-0.2499	-0.2499	-0.2510	-0.2498	-0.2497	-0.2497	-0.2508
	177	B6	-0.2929	-0.2927	-0.2927	-0.2938	-0.2932	-0.2930	-0.2930	-0.2940

**Table 9** Coverage probability of the 95% confidence interval for estimand  $E(N)$  using Chapman estimator and biased-corrected Chapman estimator under simulations (Case B:  $OR > 1$ )

N	Estimand	Population	Formula	Chapman			Biased-corrected Chapman		
				Bootstrap I	Bootstrap II	Bootstrap III	Bootstrap I	Bootstrap II	Bootstrap III
10	10	B1	0.6439	0.7328	0.7311	0.6913	0.8179	0.8234	0.7400
	9	B2	0.6697	0.7260	0.7264	0.6720	0.7555	0.7608	0.6019
	9	B3	0.7310	0.8181	0.8246	0.7536	0.7870	0.8137	0.5782
	8	B4	0.6964	0.7509	0.7601	0.6690	0.7282	0.7536	0.5119
	8	B5	0.6233	0.6690	0.6867	0.6113	0.6237	0.6563	0.4607
	7	B6	0.5217	0.5637	0.5927	0.4580	0.6288	0.6288	0.5867
25	24	B1	0.8276	0.9085	0.9101	0.8621	0.9150	0.9174	0.8650
	23	B2	0.8160	0.9083	0.9094	0.8503	0.9150	0.9154	0.8539
	23	B3	0.8418	0.9117	0.9137	0.8185	0.9114	0.9111	0.8009
	20	B4	0.8339	0.8955	0.9012	0.7704	0.8969	0.9024	0.7557
	19	B5	0.7576	0.8306	0.8340	0.6989	0.8205	0.8246	0.6485
	18	B6	0.6575	0.7024	0.7213	0.5600	0.6736	0.7016	0.4617
50	48	B1	0.8840	0.9354	0.9356	0.8801	0.9329	0.9344	0.8819
	46	B2	0.8713	0.9257	0.9267	0.8665	0.9206	0.9207	0.8621
	46	B3	0.8727	0.9101	0.9115	0.8164	0.9161	0.9156	0.8242
	40	B4	0.8775	0.9057	0.9075	0.7856	0.9035	0.9077	0.7861
	38	B5	0.8034	0.8423	0.8473	0.7030	0.8386	0.8431	0.6940
	35	B6	0.6984	0.7324	0.7388	0.5336	0.7368	0.7448	0.5227
100	96	B1	0.9097	0.9329	0.9336	0.8826	0.9393	0.9403	0.8887
	93	B2	0.8993	0.9311	0.9322	0.8773	0.9318	0.9309	0.8796
	92	B3	0.9026	0.9232	0.9236	0.8338	0.9193	0.9218	0.8309
	81	B4	0.8888	0.9069	0.9093	0.8079	0.9067	0.9064	0.8092
	75	B5	0.8411	0.8536	0.8561	0.7064	0.8552	0.8573	0.7067
	71	B6	0.7251	0.7465	0.7506	0.5588	0.7553	0.7595	0.5637
250	241	B1	0.9341	0.9449	0.9454	0.8938	0.9410	0.9421	0.8919
	232	B2	0.9276	0.9377	0.9381	0.8850	0.9386	0.9401	0.8891
	229	B3	0.9210	0.9271	0.9286	0.8376	0.9301	0.9300	0.8386
	202	B4	0.9050	0.9112	0.9118	0.8171	0.9126	0.9118	0.8110
	188	B5	0.8450	0.8507	0.8518	0.7052	0.8579	0.8578	0.7163
	177	B6	0.7521	0.7579	0.7596	0.5665	0.7468	0.7501	0.5600

(a) Chapman estimator



(b) Bias-corrected Chapman estimator

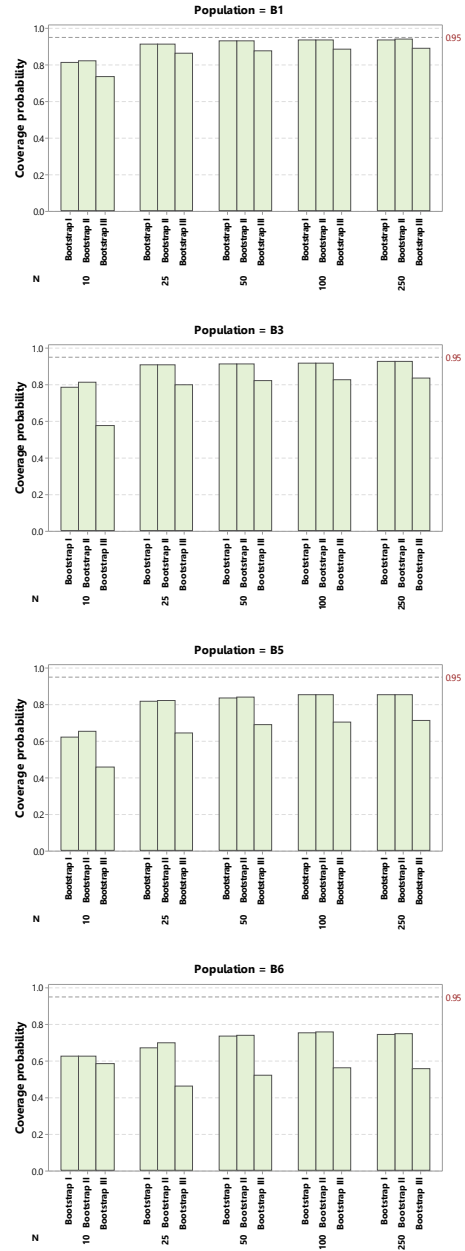
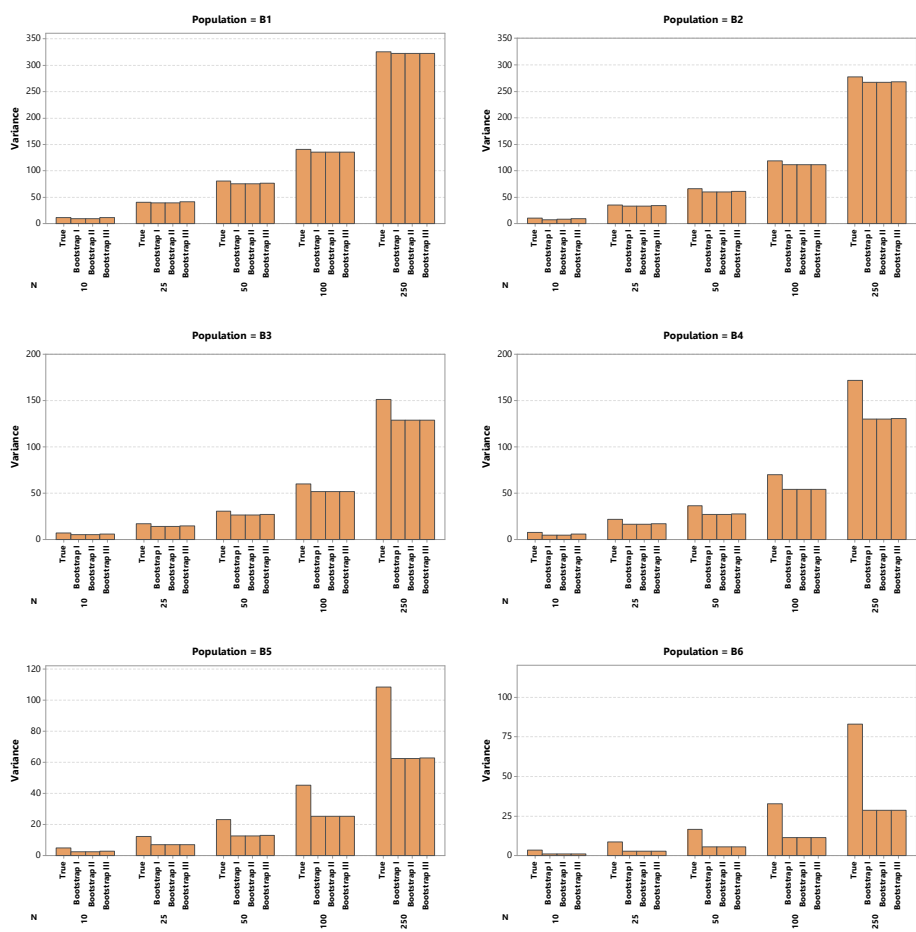


Fig. 2 Coverage probability of the 95% confidence interval for estimand  $E(N)$  using two estimator under simulations (Case B:  $OR > 1$ )

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**Fig. 3** True variance of the Chapman estimator and bootstrap variances of three methods from simulated data under Case B

**Table 10** Simulated percentiles of Chapman estimates from the true distribution and bootstrap distribution based on Bootstrap I, II, and III

Percentile	True distribution	Bootstrap distribution		
		Bootstrap I	Bootstrap II	Bootstrap III
Situation: $N = 250$ and population A1				
2.5th	231.6152	232.4173	232.3927	236.9602
5th	234.5968	235.0416	235.0408	238.6338
25th	243.1481	243.4503	243.4629	244.6365
50th	249.5747	249.6567	249.6667	249.3904
75th	256.0909	256.2191	256.1631	254.7612
95th	266.7746	266.6519	266.6709	263.8889
97.5th	270.5200	270.2577	270.3365	267.1757
Interval length of 95% CI	38.9048	37.8404	37.9438	30.2155
Situation: $N = 250$ and population B6 (Estimand $E(\hat{N}) = 177$ )				
2.5th	159.7861	166.5509	166.5120	170.6292
5th	162.2846	168.1612	168.1526	171.4776
25th	170.6129	173.1921	173.1935	174.2000
50th	176.7826	176.7242	176.6844	176.4382
75th	182.8325	180.2871	180.3046	179.0000
95th	191.7146	185.7734	185.8120	183.4525
97.5th	194.7676	187.6952	187.7344	185.0580
Interval length of 95% CI	34.9815	21.1443	21.2224	14.4288
Situation: $N = 1000$ and population B6 (Estimand $E(\hat{N}) = 707$ )				
2.5th	672.0109	686.5784	686.6159	690.1391
5th	677.2965	689.8878	689.9573	696.0588
25th	694.9621	700.1330	700.1367	702.3058
50th	706.9611	707.2899	707.2914	707.0318
75th	719.1297	714.5415	714.5897	712.1243
95th	736.5640	725.1644	725.2258	720.0085
97.5th	742.7284	728.7827	728.7849	722.7434
Interval length of 95% CI	70.7175	42.2043	42.1690	32.6043

## Response to Reviewers

**Manuscript No.** SMAP-D-25-00164R1

**Title:** Two-Way Capture-Recapture Methods with Emphasis on the Bootstrap

*We sincerely thank the Associate Editor and reviewers for the valuable comments and suggestions. The manuscript has been revised accordingly. Our detailed responses are provided below.*

### COMMENTS TO THE AUTHOR:

**AE:** I am pleased with the responses and the revision. However, there are still a couple of comments from Reviewer #2.

*Thank you for your positive assessment of our responses and the revised manuscript. We have carefully addressed the remaining comments from Reviewer #2. Only new changes to the paper are highlighted in blue.*

### Reviewer #1: Recommendation

I have reviewed the authors' detailed responses and the revised manuscript. I am satisfied that all issues raised in my original review have been addressed thoroughly and constructively. The authors provided clear justifications for their methodological choices, and implemented the necessary corrections and clarifications.

In particular:

- \* The discussion of the small-sample bias of the Chapman estimator is now much clearer, and the authors appropriately contextualise the limitations of the estimator in scenarios where sample sizes are extremely small.
- \* The clarification regarding the use of the estimand rather than the true population size in simulation B is also clearly articulated, and the implications for interpreting coverage probabilities are now explicitly stated. Although it would have been informative to report in the table the coverage rate with respect to the true population size alongside that of the estimand, the additional explanation provided in the revised text adequately clarifies the issue, and the presentation is satisfactory as it stands.
- \* The numerical corrections in Table 4 have been implemented correctly, and the bootstrap results have been re-checked.
- \* The rationale behind the simulation settings has been expanded, and the additional remarks on the computational burden of Bootstrap II provide useful guidance for practitioners.
- \* Stylistic and terminological refinements have been incorporated, enhancing clarity and consistency.

Overall, the revised version is substantially improved. The authors have responded carefully and transparently.

I have no remaining concerns. I recommend that the manuscript be accepted. explicitly stated.

*Thank you very much for your explicit recommendation for acceptance. We sincerely appreciate your careful assessment.*

**Reviewer #2:**

General comment on the revised version of the manuscript with title:  
Two-Way Capture-Recapture Methods with Emphasis on the Bootstrap  
and number: SMAP-D-25-00164R1

In my opinion the author(s) did a good job in improving the content and readability of the paper, it has seriously improved. Also, I am satisfied with the way they have responded to all my suggestions and comments. For this version I only have one very small and one medium comment left. First, the formula of delta on page 6 contains 'n1.' twice, while I suspect it should be 'n1.' and 'n.1'.

*Thank you very much. The notation in the formula of  $\Delta$  and  $\hat{\Delta}$  on page 6 has been corrected. The second occurrence of  $n_{1.}$  has been replaced by  $n_{.1}$ .*

My second comment refers to the names given to the “Chapman estimator” and the “bias-corrected Chapman estimator”. To me this is slightly confusing, because (as the author(s) also mention(s), ) the Chapman estimator is already a bias-corrected estimator, so the BC estimator basically corrects for bias twice. If I understand Table 6 correctly, this may even lead to a slight overcorrection (for example for  $N = 250$  in scenario A6), introducing some small bias. It would help me if both biases would be linked more explicitly to each estimator. For example, by referring to the original Chapman estimator as the “finite-sample-bias-corrected Chapman estimator” and the bias-corrected Chapman estimator as the “small-sample-bias-corrected Chapman estimator”. The authors may also come up with another clearer way of doing this.

*The reviewer is right in saying that both estimators correct for bias. The original Chapman estimator corrects for bias in the Lincoln-Petersen estimator. We have outlined the conditions under which the Chapman estimator is unbiased. If these conditions are violated, the Chapman estimator can experience a negative bias, in other words provides a lower bound for the true population size. We will see in the simulation work that this underestimation bias is mostly relevant for population sizes below 50. In these cases, one might apply the bias-correction as worked out above. (Some practitioners might be happy using this lower bound.) This bias-corrected Chapman estimator might be viewed as a small-sample-bias-corrected Chapman estimator. In our view, we would like to keep just the name Chapman estimator for the original Chapman estimator, as it is a bias-corrected Lincoln-Petersen estimator. We think calling the original Chapman estimator finite-sample-bias-corrected Chapman estimator would send out a confusing message it seems to indicate that there is a bias-correction happening with the Chapman estimator. We added a few sentences for clarification in the paper on page 6 and page 19.*