ILC for stroke rehabilitation: Can current approaches cope with trial-varying initial conditions?

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***Abstract*— Iterative learning control (ILC) has been widely used in rehabilitation, having recently succeeded in obtaining target postures of the hand and wrist. However, it is not designed to deal with the trial-varying initial conditions or disturbances that often occur in biological systems. This paper validates the application of ILC in this context, and shows its equivalence to a particular form of feedback controller. Experimental results then highlight the benefits of this approach. This work provides valuable insight into principled control design within the rehabilitation setting.**

I. INTRODUCTION

Stroke affects nearly 100 million people worldwide, frequently leading to significant long-term impairment. Movement can be regained using rehabilitation, often assisted using technologies such as functional electrical stimulation (FES), in which electrical impulses stimulate muscle contraction and generate movement. Recovering dextrous movement requires precise control, with iterative learning control (ILC) being a widely proposed solution [1]. ILC successively improves tracking accuracy over repeated tasks, and the repetitive nature of rehabilitation has made it a leading application area for the past 20 years. However, ILC relies on assumptions of identical initial conditions and iteration-invariant disturbances, which are difficult to enforce in practice.

This paper takes the first step towards validating existing ILC approaches for trial-varying initial conditions, focusing on the common rehabilitation task of achieving a target posture [2]. Parallels are drawn between this and a modified form of integral controller, motivating a feedback-type ILC formulation that can better reject disturbances. Experimental results demonstrate the clear advantages of this approach.

II. PRELIMINARIES AND PROBLEM STATEMENT

Consider the following discrete-time LTI system:

$$x_k(t+1) = Ax_k(t) + Bu_k(t) \tag{1}$$
$$y_k(t) = Cx_k(t), \tag{2}$$

with time index $t = 0, 1, \dots, N$ and trial index $k \in \mathbb{N}_+$. Here $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, and $y \in \mathbb{R}^q$ denote the state, input, and output vectors of the system respectively. Note that the initial state $x_k(0)$ may vary between trials.

Assumption 1. *The system is stable, i.e. $\rho(A) < 1$, where $\rho(\cdot)$ denotes the spectral radius.*

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The standard ILC objective is to update $u_k(t)$ on each iteration such that the output $y_k(t)$ tracks an iteration-invariant reference $y_d(t)$, and the error $e_k(t) = y_d(t) - y_k(t)$ converges to zero as $k \to \infty$.

However, in rehabilitation achieving a desired final posture $y_d(N)$ is often more critical than precise trajectory tracking. Several authors have therefore modified the control objective to update the final input $u_k(N-1)$ such that

$$\lim_{k\to\infty} \|e_k(N)\| = 0 \tag{3}$$

where $e_k(N) = y_d(N) - y_k(N)$, and

$$y_k(N) = \sum_{i=1}^{N} CA^{i-1}Bu_k(N-i) + CA^N x_k(0). \tag{4}$$

The input is updated as

$$u_{k+1}(N-1) = u_k(N-1) + Le_k(N) \tag{5}$$

in which $L \in \mathbb{R}^{q\times p}$ is a suitable learning operator. This forms a modified point-to-point (P2P) setup with only a single point and with trial-varying initial conditions. In the standard P2P framework the input $u_{k+1}(t)$, $t \in [0, N-2]$ is updated using a system model [1], however in rehabilitation obtaining a sufficiently accurate model can be challenging. Instead many authors simply ramp the input to the final value $u_k(N)$, with this ramp signal selected to ensure patient comfort. Applying this approach in the presence of varying $x_k(0)$ has shown experimental success but lacks theoretical justification.

III. CONVERGENCE ANALYSIS

This section provides theoretical validation for applying update (5) in the presence of iteration-varying initial conditions, along with guidelines on how to select the input signal $u_k(t)$, $t \in [0, N-2]$ without requiring system knowledge. It focuses first on LTI systems, before extending to Hammerstein-Weiner systems.

A. Linear systems

Convergence analysis relies on the following assumption.

Assumption 2. *The trial duration satisfies $N > m$, where m denotes the sample time for which $A^m \approx 0$ for a given precision level. The existence of such m is guaranteed by Assumption 1.*

Theorem 1. *Let ILC update (5) be applied to the system (1)-(2). Then under Assumptions 1 and 2, if input $u_k(t)$ is* *held constant over period $t \in [N-m, N-1]$, monotonic convergence of the error norm $\|e_k(N)\|$ is guaranteed if*

$$\|I - LG\| < 1, \tag{6}$$

where

$$G := \sum_{i=1}^{m} CA^{i-1}B. \tag{7}$$

Proof: Under Assumption 2, the final output on trial k is given by

$$y_k(N) = \sum_{i=1}^{m} CA^{i-1}Bu_k(N-i) \tag{8}$$

$$= Gu_k(N-1) \tag{9}$$

where input-output map $G : u_k(N-1) \mapsto y_k(N-1)$ is independent of the initial state $x_k(0)$. Together this mapping and update (5) form a special case of the standard ILC formulation (with a single time-step and no initial conditions), and thus standard ILC convergence proofs apply directly. □

Remark 1. *The ILC formulation outlined in Thm 1 has several properties that make it well-suited to FES applications. Firstly the choice of $u_k(t), t \in [0, N-m-1]$ is arbitrary. This validates the selection of a slow ramp input, but permits any form of input preferred by the designer. Additionally, if $u_k(t)$ is kept constant over $t \in [N-m-r, N-1]$, $0 < r < N-m$, the output $y_k(t)$ will remain constant over $t \in [N-r, N]$. When applied experimentally in the presence of noise, this can be averaged to compute the value of $y_k(N)$ in the input update.*

Remark 2. *Thm 1 can be extended to reference tracking over timesteps $t \in [N-s, N]$, where $m < N-s$, and used to show monotonic error convergence over this interval.*

B. Extension to Hammerstein-Wiener systems

Results are now extended to Hammerstein-Wiener systems, commonly used to model muscle dynamics [1] and represented by

$$x_k(t+1) = Ax_k(t) + Bh_1(u_k(t)) \tag{10}$$

$$y_k(t) = h_2(x_k(t)) \tag{11}$$

where $h_1(\cdot)$, $h_2(\cdot)$ are static non-linear functions.

Theorem 2. *The application of ILC update (5) to system (10)-(11) will ensure monotonic convergence of $\|e_k(N)\|$ if*

$$\|I - Lg'(u_k(N-1))\| < 1, \quad \forall k \tag{12}$$

where $g'(u_k(N-1))$ is the Jacobian of $g(u_k(N-1)) := h_2(\tilde{G}\, h_1(u_k(N-1)))$, with

$$\tilde{G} := \sum_{i=1}^{m} A^{i-1}B. \tag{13}$$

Proof: The final output of (10)-(11) on trial k is

$$y_k(N) = h_2\left(\tilde{G}h_1(u_k(N-1))\right) \tag{14}$$

$$= g(u_k(N-1)) \tag{15}$$

which enables the non-linear ILC convergence proofs outlined in [3] to be applied. □

IV. LINKS TO FEEDBACK CONTROL

This section now outlines the links between ILC update (5) and feedback control. The update law can be equivalently expressed as

$$u_{k+1}(N-1) = u_0(N-1) + L\sum_{i=0}^{k} e_i(N) \tag{16}$$

where the input at trial $k = 0$ is commonly selected as $u_0(N-1) = 0$. If ILC trials were to be run continuously (i.e. without resetting), viewing this system in the time domain would correspond to a delayed form of integral control, with sample time equal to the ILC trial length. This enables feedback-type controllers to be designed using standard ILC methods, and reduces the impact of trial-varying disturbances. The effectiveness of this is illustrated experimentally in the following section.

V. EXPERIMENTAL RESULTS

Preliminary results on a single healthy individual are now presented. The hardware outlined in [2] was used to apply stimulation and record wrist angles, using $y_d(N) = 75°$. A trial length of 4s and timestep 0.025s was used, and $y_k(N)$ was computed as the mean over the final 0.5s.

Figure 1 demonstrates that when ILC update (5) is applied continuously (fig 1b), the converged error norm is far smaller than when the system is reset to (approximately) identical initial conditions (average of 3.15° vs 6.83° over the final 3 iterations). This suggests that a feedback-type ILC formulation is better suited to the rehabilitation context.

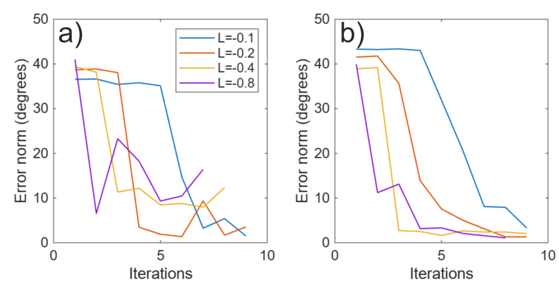

Fig. 1: ILC convergence using a) resetting, $x_k(0) \approx x_{k-1}(0)$ b) continuous operation, $x_k(0) = x_{k-1}(N)$

VI. CONCLUSIONS

This paper has validated the use of ILC to achieve a target posture in the presence of varying initial conditions, and presented a link to feedback control. This feedback-type ILC provided 54% error norm reduction compared to standard ILC. Future work will explore parallels with repetitive control, as well as more general non-linear extensions.

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