UNIVERSITY OF SOUTHAMPTON

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Investigating Statistical Models to Handle Competing Risks with Applications to Mortality

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Abstract

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The aim of this thesis is to develop a framework for the modelling of historic cause-specific mortality rates allowing efficient forecasting of the aforementioned rates. Whilst the International Classification of Diseases and Related Health Problems (ICD) has been used to record the underlying cause of death for all reported deaths since the end of the 19th Century, the decennial revisions to the classification require special attention to ensure continuity of the time series for statistical analysis. While these updates have an influence on all causes of death, the greatest impact of the changes in ICD coding rules applies to the number of deaths attributed to pneumonia and bronchopneumonia. While comparability ratios exist for certain groups of causes of death, their application to the data distorts the overall number of deaths in a country. We propose a Multinomial Logistic extension of the classical Lee-Carter model as well as the Li-Lee model to estimate the effect of age and time on cause-specific mortality rates. The Li-Lee model is a multi-country extension of the Lee-Carter model with an additional bilinear term that pools mortality experience across countries and allows us to borrow strength when there are issues with the data for a single country. While the classical Lee-Carter and Li-Lee models are applied to mortality rates, we apply them to survival and death probabilities to take advantage of the sum to one constraint imposed by the logit transformation. This is the first use of the Multinomial Logistic Li-Lee model and also the first application of a Multinomial Logistic Lee-Carter model to cause-specific mortality as far as we are aware. We sort death counts from England & Wales and France during the period 1968 to 2005 into six groups by cause of death and five-year age groups for the majority of ages using mortality data collected by World Health Organization and made available in their Mortality Database. We estimate model coefficients using maximum likelihood and assess their fits using information criteria. We also compare the standard errors of the coefficient estimates obtained via bootstrap and MCMC. We then re-estimate the probabilities of death using the Hamiltonian Monte Carlo (HMC) algorithm and compare the mean squared errors of the in-sample values. We find that the Multinomial Logistic Li-Lee model outperforms the Multinomial Logistic Lee-Carter model when applied to the 1968-2005 mortality data for England & Wales and for France. Finally, we use the above mentioned models to project the cause-specific mortality probabilities using the HMC algorithm for the years 2006 through 2014.

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Declaration of Authorship

I declare that this thesis and the work presented in it is my own and has been generated by me as the result of my own original research.

I confirm that:

1. This work was done wholly or mainly while in candidature for a research degree at this University;
2. Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;
3. Where I have consulted the published work of others, this is always clearly attributed;
4. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;
5. I have acknowledged all main sources of help;
6. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;
7. None of this work has been published before submission.

Signed:.. Date:..................

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The data concerning registered deaths, the underlying cause of death, and population counts used in this thesis were drawn from the World Health Organization's Mortality Database. The World Health Organisation is the copyright holder of the International Statistical Classification of Diseases and Related Health Problems, 10th Revision, Fifth Edition, 2015. All analysis, interpretations, and conclusions presented in this thesis are solely of the author.

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Definitions and Abbreviations

ADF	**A**ugmented **D**ickey-**F**uller
AIC	**A**kaike **I**nformation **C**riterion
AM	**A**daptive **M**etropolis
APC	**A**ge-**P**eriod-**C**ohort
APCI	**A**ge-**P**eriod-**C**ohort-**I**mprovement
AR	**A**uto**r**egressive
ARIMA	**A**uto**r**egressive **I**ntegrated **M**oving **A**verage
ARMA	**A**uto**r**egressive **M**oving **A**verage
BFGS	**B**royden–**F**letcher–**G**oldfarb–**S**hanno algorithm
BIC	**B**ayesian **I**nformation **C**riterion
BTL	**B**asic **T**abulation **L**ist
CAE	**C**ommon for **A**ge **E**ffect
CBD	**C**airns-**B**lake-**D**owd
CDC	**C**enters for **D**isease **C**ontrol and Prevention
CoD	**C**ause **o**f **D**eath
CoDCM	**C**ause **o**f **D**eath **C**ode **M**apping
CoDQL	**C**ause **o**f **D**eath **Q**uery Online
CM	**C**linical **M**odification
CMI	**C**ontinuous **M**ortality **I**nvestigation
FRA	**Fra**nce
GAM	**G**eneralised **A**dditive **M**odel
HMC	**H**amiltonian **M**onte **C**arlo
IC	**I**nformation **C**riterion
ICD	**I**nternational Statistical **C**lassification of **D**iseases and Related Health Problems
LC	**L**ee-**C**arter
LL	**L**i-**L**ee
MA	**M**oving **A**verage
MAE	**M**ean **A**bsolute **E**rror
MCMC	**M**arkov **C**hain **M**onte **C**arlo
MCCD	**M**edical **C**ertificate of the **C**ause **o**f **D**eath
MD	**M**ortality **D**atabase
ML	**M**aximum **L**ikelihood
MLE	**M**aximum **L**ikelihood **E**stimate
MLG	**M**ultinomial **L**o**g**istic
MLG-LC	**M**ultinomial **L**o**g**istic **L**ee-**C**arter
MLG-LL	**M**ultinomial **L**o**g**istic **L**i-**L**ee
MRSA	**M**eticillin-**R**esistant **S**taphylococcus **A**ureus
MSE	**M**ean **S**quared **E**rror
MVN	**M**ulti**v**ariate **N**ormal distribution

NCHS	**N**ational **C**enter for **H**ealth **S**tatistics
ONS	**O**ffice for **N**ational **S**tatistics
PI	**P**rediction **I**nterval
RW	**R**andom **W**alk
RWD	**R**andom **W**alk with **D**rift
SQL	**S**tucture **Q**uery **L**anguage
StatCan	**Stat**istics **Can**ada
UDD	**U**niform **D**istribution of **D**eaths
UI	**U**ncertainty **I**nterval
VAR	**V**ector **A**uto**r**egressive model
VECM	**V**ector **E**rror **C**orrrection **M**odel
WHA	**W**orld **H**ealth **A**ssembly
WHO	**W**orld **H**ealth **O**rganization
XEW	**E**ngland and **W**ales

Chapter 1

Introduction

Mortality statistics are not only important for national statistics agencies but also for international comparisons. The World Health Organisation (WHO) is an agency of the United Nations that is responsible for promoting health across the globe. It has been responsible for the International Statistical Classification of Diseases and Related Health Problems (ICD) since the classification's 6th revision was adopted by the 1st World Health Assembly (WHA) in 1948 (WHO, 2023). The latest revision, ICD-11, was released on 18 June 2018 and came into effect for mortality coding on 1 January 2022 (The Lancet, 2018).

The aim of this thesis is to develop a framework to model and project mortality rates by cause of death. Whilst forecasting overall mortality rates allows us to predict the number of deaths in a calendar year, the model will lack information regarding the underlying cause of death. Estimating future deaths solely by cause as a fixed percentage of overall deaths can serve as a very crude estimate at best given the year-on-year fluctuations in cause-specific mortality rates. This difference is primarily due to shifting morbidity, e.g. increase in prostate cancer deaths accompanied by a decrease in lung cancer deaths in English males, though changes in mortality coding practices, e.g. increase in the number of recorded deaths due to mental disorders or a decrease of pneumonia deaths following the introduction of ICD-10, do contribute as well (Richards, 2009).

The overall mortality rate is no longer sufficiently representative of the health of the population because of the decline in deaths from infectious diseases along with a rise in deaths due to chronic illnesses (Institute and Faculty of Actuaries, 2012). The Office for National Statistics (2018c, p. 3) defines *healthy life expectancy* as 'an estimate of the number of years lived in "Very good" or "Good" general health, based on how individuals perceive their general health'. This metric relies on survey data to estimate quality of life — more specifically the Annual Population Survey in the United Kingdom (ONS, 2018d). Another approach to quantify the effect of different causes of death on life expectancy involves the idea of cause-elimination. Alai, Arnold, and Sherris (2015) modelled death rates with mortality shocks precisely using this idea of cause-elimination.

Lee and Carter (1992) proposed a method to model and forecast US mortality, which is now known as the Lee-Carter (LC) model. The LC model was then applied by Wilmoth (1993) to fit Japanese data, by Brouhns and Denuit (2001a) to estimate Belgian deaths, and by Lee and Rofman (1994) to model Chilean mortality. Li and Lee (2005) subsequently proposed a multi-country extension of the LC model with the underlying assumption that mortalities of different countries converge to a common trend in the long term. A number of multi-population models, including the above mentioned Li-Lee (LL) model and common age effect (CAE) model explored by Kleinow (2015), are reviewed by Enchev, Kleinow, and Cairns (2017). More recently Lyu, Waegenaere, and Melenberg (2021) extended the LL model to handle multiple causes of death.

A further extension of the LC model – the Poisson log-bilinear model – uses the bilinear specification for the log of the force of mortality instead of the log of the mortality rate. Brouhns, Denuit, and Vermunt (2002a; 2002b) fitted this model using Maximum Likelihood whereas Czado, Delwarde, and Denuit (2005) utilised a Bayesian approach to estimate model coefficients. The Bayesian framework was also used by Wong, Forster, and Smith (2018) who added overdispersion in their versions of the Poisson log-normal LC and Negative binomial LC models.

Dodd et al. (2021) proposed a hybrid Age-Period-Cohort (APC) model for mortality improvements. Their approach used a Negative binomial distribution instead of a Poisson to allow for overdispersion. Due to the linear relationship between age, period, and cohort, it is necessary to impose constraints on the APC model to ensure its identifiability (Carstensen, 2007). Wong, Forster, and Smith (2023) then implemented a Bayesian age-period-cohort-improvement (APCI) model originally proposed by the Continuous Mortality Investigation (2016) as a deterministic model. Richards et al. (2019) developed a fully stochastic version of this model with a Poisson assumption for the number of deaths. Additionally, they compare the APCI model with the performance of the APC and LC models.

Building on the multi-population approach, Girosi and King (2008) presented a method for forecasting using Bayesian techniques that employed the idea of borrowing strength from different mortality experiences. Dodd et al. (2018) described a Bayesian approach for the modelling of all-cause, or overall, mortality using a generalised additive model (GAM) to allow for the smoothing of the age-specific parameters. A GAM model with smooth age effects and discrete period effects was then used by Hilton et al. (2018) to project UK mortality rates.

Lastly, a two factor crude mortality model for higher ages, i.e. ages 60-90, was proposed by Cairns, Blake, and Dowd (2006). We will refer to this stochastic model as the Cairns-Blake-Dowd (CBD) model. Cairns et al. (2009) pointed out the higher flexibility of the LC model in regards to the age effect parameters when compared to the CBD model. This was in contrast to the ability of the P-splines used by Currie, Durban, and Eilers (2004) to smooth age effects. However, the extension of the CBD model with a quadratic age effect presented in Cairns et al. (2009) had the most robust parameter estimates. Further overall mortality models can be found in Booth and Tickle (2008).

Alternatives to overall mortality projection use traditional models, e.g. LC model, to forecast each cause of death separately. Wilmoth (1996) provides a comparison of mortality and life expectancy forecasts created using both the overall mortality and the cause of death approach. Others, such as Zhao et al. (2023), use an APC model to project cancer mortality in Hong Kong. While the Continuous Mortality Investigation (CMI) Working Paper 3 (2004) recommends to forecast overall mortality rates to avoid issues with causes that were eradicated, e.g. smallpox, Tabeau, Berg Jeths, and Heathcote (2001) argue that short term forecasts yield similar estimates regardless of whether the overall or cause-specific rates were used. Furthermore, they argue that for long term forecasts, expert opinion should prevail over trends to take into account mortality shocks. A model that takes into account the overall mortality rate as well as cause specific mortality rates is therefore necessary to ensure that shifts in mortality and morbidity over longer periods of time are reflected; however, Wilmoth

(1995) has previously shown that cause-specific mortality projections for Japan using the LC approach will forecast greater number of deaths than overall mortality models.

The co-integration property of related time series proposed by Granger (1981) and Engle and Granger (1987) has also been explored by many. Arnold and Sherris (2015; 2016) modelled the long-run relationship between five causes of death in major countries using a Vector Error-Correction Model (VECM). This co-integration analysis was further expanded in Arnold and Glushko (2021; 2022).

An alternative solution to the above problem would be to constrain the cause-specific death rates to the overall mortality rate by using a multinomial logistic model. By definition, the logit function restricts the estimated probabilities to be in the interval $[0, 1]$. The sum of the fitted probabilities of death for each cause plus the probability of staying alive for each age group and each year must sum to one. This constraint is necessary to avoid unrealistic zero-death long-term projections — a drawback of poorly specified single-cause mortality models.

Poston and Min (2008) investigated the cause of death of the "oldest old", i.e. those aged 80 years and over, in the USA using a multinomial regression model. Their modelling examined ten major causes of death along with an "other" group. While Zhao (2012) and Zhao et al. (2013) have previously modelled Chinese mortality data using a binary logistic model, this thesis aims to extend the Multinomial Logistic Lee-Carter to model multiple causes of death. The accuracy of the predictions is crucial for reserving purposes in the insurance and pensions industry as incorrect projections would yield inappropriate annuity values leading to inaccurate valuations and disproportionate reserves set aside by life insurance companies and pension schemes.

The ultimate goal of this project is to provide a coherent statistical methodology to estimate mortality rates, both overall and cause-specific, by looking at the underlying causes of morbidity and mortality and to propose a Bayesian approach to correct the deficiencies and inconsistencies in the statistical reporting of the causes of death. WHO (2018) notes that 'it is not possible to convert ICD-9 data sets into ICD-10 data sets or vice versa' and recommends that statistical analysis of mortality rate uses comparability ratios produced during 'bridge coding' studies. Use of these ratios alone, however, would result in fractional numbers of deaths and distort the overall mortality rate. Our approach to the problem at hand is twofold: Instead of solely relying on comparability ratios, we have developed a Cause of Death Code Mapping (CoDCM) database to aggregate individual causes of death into groups that are comparable, though not equivalent, across time. We implement multi-population models that are capable of borrowing strength from mortality experiences in different countries and correct for some of the deficiencies in the coding of underlying causes of death.

To the best of our knowledge, the research presented in this thesis is the first use of the Multinomial Logistic LL model. This is also the first application of a multinomial logistic LC model to cause-specific mortality as far as we are aware.

1.1 Thesis Overview

This thesis is divided into eight chapters. The datasets used for this project and the various issues that arise due to the data collection processes employed by the national statistics agencies are described in Chapter 2.

We explore the theory of competing risks, its application to mortality and morbidity, and definitions of various mortality measures in Chapter 3. Furthermore, Section 3.3 introduces the Lee-Carter and Li-Lee models as well as the Multinomial Logistic model that will be used throughout this thesis. While the classical Lee-Carter and Li-Lee models are applied to mortality rates, we apply them to survival and death probabilities to take advantage of the sum to one constraint imposed by the logit transformation.

Model specifications of the Multinomial Logistic Lee-Carter and the Multinomial Logistic Li-Lee model as well as their constraints are presented in Chapter 4. This is the first use of the Multinomial Logistic Li-Lee model and also the first application of a Multinomial Logistic Lee-Carter model to cause-specific mortality as far as we are aware. In Section 4.2 we derive an expression for the likelihood function used for the initial model estimations. Section 4.3 then builds on the maximum likelihood estimates (MLE) presented in the previous section and outlines sampling methods to quantify uncertainty under the frequentist approach, including Bootstrap and Multivariate Normal Sampler tested on the mortality data for England & Wales. We included deaths in England & Wales only instead of the entire United

Kingdom due to different changeover years for ICD-10 – the classification of the deaths data would therefore not be aggregated across the entire country in a consistent manner.

Chapter 5 offers an overview of the time series methods considered in this thesis including stationarity and ARIMA specifications of the period effects from the cause-specific mortality models.

Chapter 6 revisits the mortality models presented in the previous chapter using a Bayesian approach. In Section 6.1 we run Markov Chain Monte Carlo (MCMC) simulations for a single-country model and compare the estimates with the MLEs from Section 4.4. Due to the complexities and high-dimensionality of the multi-country models, we apply the Hamiltonian Monte Carlo (HMC) algorithm in Section 6.2. We also re-estimate the single-country model to be able to compare the performance of the single- and dual-country models and find that the Multinomial Logistic Li-Lee model outperforms the Multinomial Logistic Lee-Carter model.

A projection framework for mortality probabilities is presented in Chapter 7. Our approach follows on from the Bayesian framework presented in Chapter 6 with time series priors of the Multinomial Logistic Lee-Carter model informed by the analysis from Chapter 5. We test the autoregressive integrated moving average (ARIMA) model for single-country projections and investigate the possibilities as well as complications of using a dual-country model for England & Wales and France. We chose France as the second country due to its population size being similar to England & Wales as well as its geographic proximity to Great Britain.

Lastly, Chapter 8 presents conclusions from our project and outlines some possible avenues for future research.

Chapter 2

Data

Cause of death mortality modelling requires deaths data that are properly coded and categorised into time series that can easily be loaded into statistical software for analysis. This chapter will look at the datasets and classifications used for this project, the historical trends in the time series, and some of the challenges associated with the use of these statistics.

2.1 Cause-Specific Mortality Data

The data used for this project were obtained from the World Health Organization's (WHO) Mortality Database (2017b). The deaths were recorded, coded, and transmitted to WHO by the national authorities, such as the Office for National Statistics (ONS), which is responsible for statistical recording of deaths in England and Wales. All registered deaths are coded using the International Statistical Classification of Diseases and Related Health Problems (ICD) – a methodology for the classification of diseases used across the world for standardised recording of the causes of death. ICD is used not only for mortality data but also morbidity statistics. For example, a Clinical Modification (CM) of ICD developed by the National Center for Health Statistics (NCHS) is used in the United States of America (USA) for morbidity purposes, such as patient records and billing of medical procedures in the healthcare sector (CDC, 2017).

INTERNATIONAL FORM OF MEDICAL CERTIFICATE OF CAUSE OF DEATH

Cause of death		Approximate interval between onset and death
I Disease or condition directly leading to death*	(a)	....
	due to (or as a consequence of)	
Antecedent causes Morbid conditions, if any, giving rise to the above cause, stating the underlying condition last	(b)	....
	due to (or as a consequence of)	
	(c)	....
	due to (or as a consequence of)	
	(d)	....
II Other significant conditions contributing to the death, but not related to the disease or condition causing it	....	....
	....	....

**This does not mean the mode of dying, e.g. heart failure, respiratory failure. It means the disease, injury, or complication that caused death.*

FIGURE 2.1:
International Form of Medical Certificate of Cause of Death
(WHO, 2004)

The cause of death and any other contributing conditions are first recorded by a doctor on the medical certificate of the cause of death (MCCD). The international form of the MCCD is shown in Figure 2.1. Deaths are then coded according to the underlying cause of death, which is defined by WHO as ‘the disease or injury which initiated the train of morbid events leading directly to death, or the circumstances of the accident or violence which produced the fatal injury’ (2004). The codes used for registered deaths will vary according to the revision of ICD in force at the time of the death or at the time the death was registered. A timeline showing the various versions of ICD in force throughout England & Wales is presented in Figure 2.2. In most cases, the underlying cause of death will be the cause of death recorded on the lowest line of Part I of the MCCD; however, a number of selection rules can override this general principle (WHO, 2004). The most notable one is selection Rule 3, which calls for a condition listed in Part II of the MCCD to be used as the underlying cause of death instead if the condition leading to death is an obvious consequence of the condition in Part II (ONS, 2017). Rule 3 has greatest impact on deaths historically classified to respiratory diseases (Rooney and

Smith, 2000). The ICD-10 Instruction Manual states that 'any pneumonia in J12-J18 should be considered an obvious consequence of conditions that impair the immune system' (WHO, 2004). The most notable consequence of Rule 3 occurs when mental and behavioural disorders, e.g. dementia – including Alzheimer's disease – appears in Part II of the MCCD. Other groups that are significantly affected include diseases of the circulatory system and neoplasms (Rooney, Griffiths, and Cook, 2002). Details about other selection rules as well as further examples can be found in the ICD-10 Instruction Manual (WHO, 2004). Furthermore, the 43rd WHA recommended the addition of line (d) to Part I of the death certificate to allow for the recording of more illnesses that may be experienced by individuals as a result of longer lifetimes (WHO, 2004). This has been implemented in the USA for example, but not in England & Wales during the period to which our data relates (Anderson et al., 2001; ONS, 2018b).

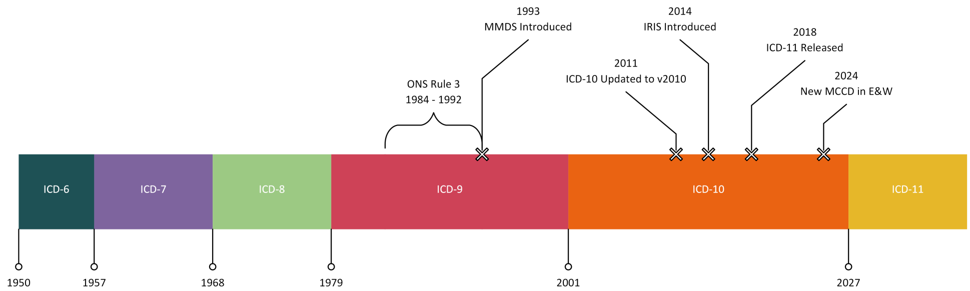


FIGURE 2.2: History of ICD in England & Wales

ICD Revisions 7, 8 and 9 are available from WHO as Basic Tabulation Lists (BTL) of 150 causes of death. BTLs were calculated using death counts recorded using 4-digit ICD codes used by the responsible national authorities. The death statistics are generally aggregated by 5-year age groups. Starting from ICD-10, WHO Mortality Database provides death counts at 4-character code level allowing international comparison at the level of individual causes rather than groups. To ensure consistency of classifications when comparing multiple countries, we use the data provided by the WHO Mortality Database instead of datasets from national statistical offices. This is to ensure that any special national codes are recoded to the internationally recognised codes. For example, ONS uses code U50.9 – 'Event awaiting determination of event' – for accelerated death registrations. These deaths are reported to WHO under code Y33.9 – 'Other specified events, undetermined intent' – though most are eventually reassigned to one of the codes in the 'Assault' group (X85–Y09) (ONS, 2017).

The quality and consistency of the of the coding varies by country. In England and Wales, an automated system is used to match causes of death to an ICD code. In 2014, the NCHS Mortality Medical Data System (MMDS) previously used in England & Wales was replaced by Eurostat-supported software IRIS (ONS, 2017). This has led to changes in the proportion of deaths attributed to chest infections with a notable increase in the proportion of deaths as a result of mental and behavioural disorders (Fearn, 2014; Wells, 2014). These changes continue the trend that was brought in with the introduction of ICD-10.

2.2 Classification of Causes of Death

The WHO Mortality database contains annual records of deaths by cause dating as far back as 1950 depending on the country. Deaths assigned to a each code are aggregated by sex – either male or female – year of death, country where the death occurred, and age at time of death aggregated in five-year age groups. It should be noted that deaths before a child's first birthday, i.e. during age zero, are included in a separate group. The second age group therefore only contains ages 1 through 4 meanwhile the final age group contains ages 85+.

To create a comparable multi-decade time series from 1950 up to 2015 for multiple countries, we have grouped the death counts by cause of death by high-level disease types that make up a large proportion of annual deaths, e.g. neoplasms, diseases of the circulatory system, external causes of morbidity and mortality. We have included infectious and parasitic diseases due to their historic significance as well as the rise is drug-resistant bacteria and fungi, e.g. Candida auris, Clostridium difficile, and Neisseria gonorrhoeae (Centers for Disease Control and Prevention, 2019; UK Health Security Agency, 2024). Causes of death have been divided into six groups as shown in Table 2.1:

TABLE 2.1: CAUSE OF DEATH GROUPS

Cause of Death Group	Description
1	Infectious and Parasitic Diseases
2	Neoplasms
3	Diseases of the Circulatory System
4	Diseases of the Respiratory System
5	External Causes
6	Other Causes
0	All Causes of Death

This allocation is largely in line with ICD-10 and the groupings described in Alai, Arnold, and Sherris (2015). The notable change being the inclusion of ICD-7 BTL code A070 – 'Vascular lesions affecting central nervous system' – with the diseases of the circulatory system. This is to reflect the classification of cerebral haemorrhage, cerebral embolism, and thrombosis under ICD-10 and is consistent with the work of Janssen and Kunst (2004). We did not include mental disorders and diseases of the nervous system as a separate group due to the relatively low number of the deaths recorded using the relevant codes prior to 1984 – the year Rule 3 was implemented in England & Wales. The full breakdown of ICD codes for each cause group is provided in Table 2.2:

TABLE 2.2: INTERNATIONAL CLASSIFICATION OF DISEASES CAUSE-OF-DEATH CODES

Cause-of-Death	Revision of International Classification of Diseases			
Group	ICD 7	ICD 8	ICD 9	ICD 10
1	A001–A043	A001–A044	B01–B07	A00–B99
2	A044–A060, A070	A045–A061	B08–B17	C00–D48
3	A079–A086	A080–A088	B25–B30	I00–I83, I85-I99
4	A087–A097	A089–A096	B31–B32	J00–J98
5	A138–A150	A138–A150	B47–B56	V01–Y89 †‡
6	A061–A069, A071-A078, A098–A137	A062–A079, A097–A137	B18–B24, B33–B46	D50–H93, I84, K00–R99, U04–U06
θ	A000	A000	B00	A00–Y89 *

Once mortality coding using ICD-11 becomes commonplace, the categories will need to be adjusted again to be consistent with the classification of stroke as a diseases of the nervous system rather than the circulatory system as endorsed by the 72nd WHA in May 2019 (Shakir and Norrving, 2017). The number of deaths attributed to diseases of the circulatory system has previously experienced a decrease when the classification of vascular dementia was corrected for in 2010 and deaths moved from code I67.9 (cerebrovascular disease) to code F01 (vascular dementia). The emphasis on deaths due to dementia was further increased in 2014 when chest infection deaths with a mention of dementia on the death certificate were moved from code J98 – to codes F01 or F03 – (ONS, 2016a). The effect of the changes can be seen in Figures 2.3 and 2.4. The focus on the topography of diseases has also resulted in the reclassification of haemorrhoids from the ICD-10 Chapter IX - Diseases of the Circulatory System to ICD-10 Chapter XI - Diseases of the Digestive System with effect from January 2013 (WHO, 2017a). Unfortunately the same corrections cannot be applied retrospectively to coding that was a result of ICD Rule 3 as this relies on information from the MCCD. The implementation of Rule 3 has had biggest effect on pneumonia deaths (Anderson et al., 2001; Geran et al., 2005) and will require a different approach to account for it.

*Y90-Y98, U82-U85, Z00-Z99 are not used for primary mortality coding.

†Includes U50.9 — 'Event awaiting determination of event'.

‡Nature of injury code (Chapter XIX: S00-T98) is not reported for external causes (Chapter XX: V01-Y89, incl. U50.9).

2.3 Cause-of-Death Code Mapping

For the purposes of this project, we have created a local Microsoft Access Mortality Database (MD) using the WHO Mortality Database dataset (2017b). This local database facilitates the creation of cause-of-death groups using Structured Query Language (SQL) based on a specified selection of ICD codes. The use of SQL is a more efficient procedure for creating groupings when compared with the use of WHO's Cause of Death Query online (CoDQL) as groups can be predefined, thereby removing the need to manually select groups by marking a tick box for each desired whenever a new country is added. The hierarchical structure of the local MD also eliminates errors arising from unticked boxes in CoDQL.

TABLE 2.3: CODCM CHAPTERS

Chapter Code	Chapter Description
CH00	All Causes
CH01	Infectious and and Parasitic Diseases
CH02	Neoplasms
CH03	Endocrine, Nutritional and Metabolic Diseases
CH04	Diseases of the Blood and Blood-forming Organs
CH05	Mental Disorders
CH06	Diseases of the Nervous System and Sense Organs
CH07	Diseases of the Circulatory System
CH08	Diseases of the Respiratory System
CH09	Diseases of the Digestive System
CH10	Diseases of the Genitourinary System
CH11	Obstetric Complications
CH12	Diseases of the Skin and Subcutaneous Tissue
CH13	Diseases of the Musculoskeletal System and Connective Tissue
CH14	Congenital Anomalies
CH15	Certain Conditions Originating in the Perinatal Period
CH16	Symptoms, Signs and Ill-defined Conditions
CH17	External Causes

The local MD that we have developed utilises a hierarchically structured series of Cause-of-Death Code Mapping (CoDCM) chapter, subchapter, superhigh, and high codes to enable the creation of comparable time series across revisions of ICD. The basis for the chapter codes was the ICD-9 BTL special list for USSR. (See Table 6 of the WHO Mortality Database (2017b) documentation file for details.) The subchapters were created in order to incorporate the chapter structure of ICD-10. CoDCM is intended for internal use only and is not endorsed by WHO. The revision and update of ICD in force at the time a death certificate was coded should be accepted as the correct classification at the time. The CoDCM chapters and subchapter codes can be found in Tables 2.3 and 2.4, respectively. The full code mapping, including the database relationships, can be found in Appendix A.

TABLE 2.4: CODCM SUBCHAPTERS

Subchapter Code	Subchapter Description
CH01	Infectious and and Parasitic Diseases
CH02-1	Malignant Neoplasms
CH02-2	Benign Neoplasms
CH02-3	Other Neoplasms
CH03	Endocrine, Nutritional and Metabolic Diseases
CH04	Diseases of the Blood and Blood-forming Organs
CH05	Mental Disorders
CH06-1	Diseases of the Nervous System
CH06-2	Diseases of the Eye and Adnexa
CH06-3	Diseases of the Ear and Mastoid Process
CH06-9	Other Diseases of the Nervous System and Sense Organs
CH07	Diseases of the Circulatory System
CH08	Diseases of the Respiratory System
CH09	Diseases of the Digestive System
CH10	Diseases of the Genitourinary System
CH11	Obstetric Complications
CH12	Diseases of the Skin and Subcutaneous Tissue
CH13	Diseases of the Musculoskeletal System and Connective Tissue
CH13-9	Other Diseases of the Musculoskeletal System and Connective Tissue
CH14	Congenital Anomalies
CH15	Certain Conditions originating in the Perinatal Period
CH16	Symptoms, Signs and Ill-defined Conditions
CH17-1	Transport Accidents
CH17-3	Other Accidents
CH17-5	Suicide and Self-Inflicted Injury
CH17-6	Violence
CH17-7	Medical Complications
CH17-9	Other External Causes
CH17-0	All External Causes (Accidents, Poisoning, and Violence)
CH18	Other

A graphical representation of the distribution of deaths in England & Wales from 1950 to 2015 by CoDM subchapters can be found in Figures 2.3 - 2.4. Figure 2.3 illustrates a rise in the percentage of deaths due to neoplasms. While only 17.0% of all male deaths in 1950 was classified as being due to cancer, this percentage has increased to 30.6% by 2015. At the same time there is noticeable fall in the percentage of male deaths in England & Wales attributable to diseases of the circulatory system over time, more specifically a drop from 45.9% in 1950 to 27.4% in 2015. This change, however, does not occur in isolation. We can see that the decrease in the percentage of deaths for this subgroup is accompanied by an increase in the percentage of deaths due to mental disorders from 1984 onwards and in diseases of the nervous system from 2000. Furthermore, a drop in deaths attributable to respiratory diseases between 1984 and 1992 is not true representation of a drop in deaths due to pneumonia and other conditions that fall under this group but rather the result of the ICD Rule 3 mentioned in Section 2.1 (Griffiths, Brock, and Rooney, 2004). The rise and fall of respiratory deaths corresponds almost perfectly to the fall and rise, respectively, in deaths due to diseases of the nervous system, e.g. dementia and Alzheimer disease (Brock, Griffiths, and Rooney, 2006).

The trends for female deaths recorded in England & Wales follow largely the same pattern as those for males (see Figure 2.4). The percentage of female deaths due to neoplasms is equal to 17.2% in 1950 and rises to 25.3% by 2015. Meanwhile the percentage of female deaths due to diseases of the circulatory system drops from 52.7% in 1950 down to 25.1% in 2015. This does not correspond to a decrease in the total number of deaths as the opposite is true – the total number of female deaths rose from 249,149 in 1950 to 272,447 in 2015. Similarly to Figure 2.3, there is also a noticeable rise in deaths due to mental disorders and diseases of the nervous system from 1984 onwards. The larger proportion of female deaths classified under the mental disorders group can be explained by the higher female life expectancy and therefore increased risk for dementia and Alzheimer disease (ONS, 2020).

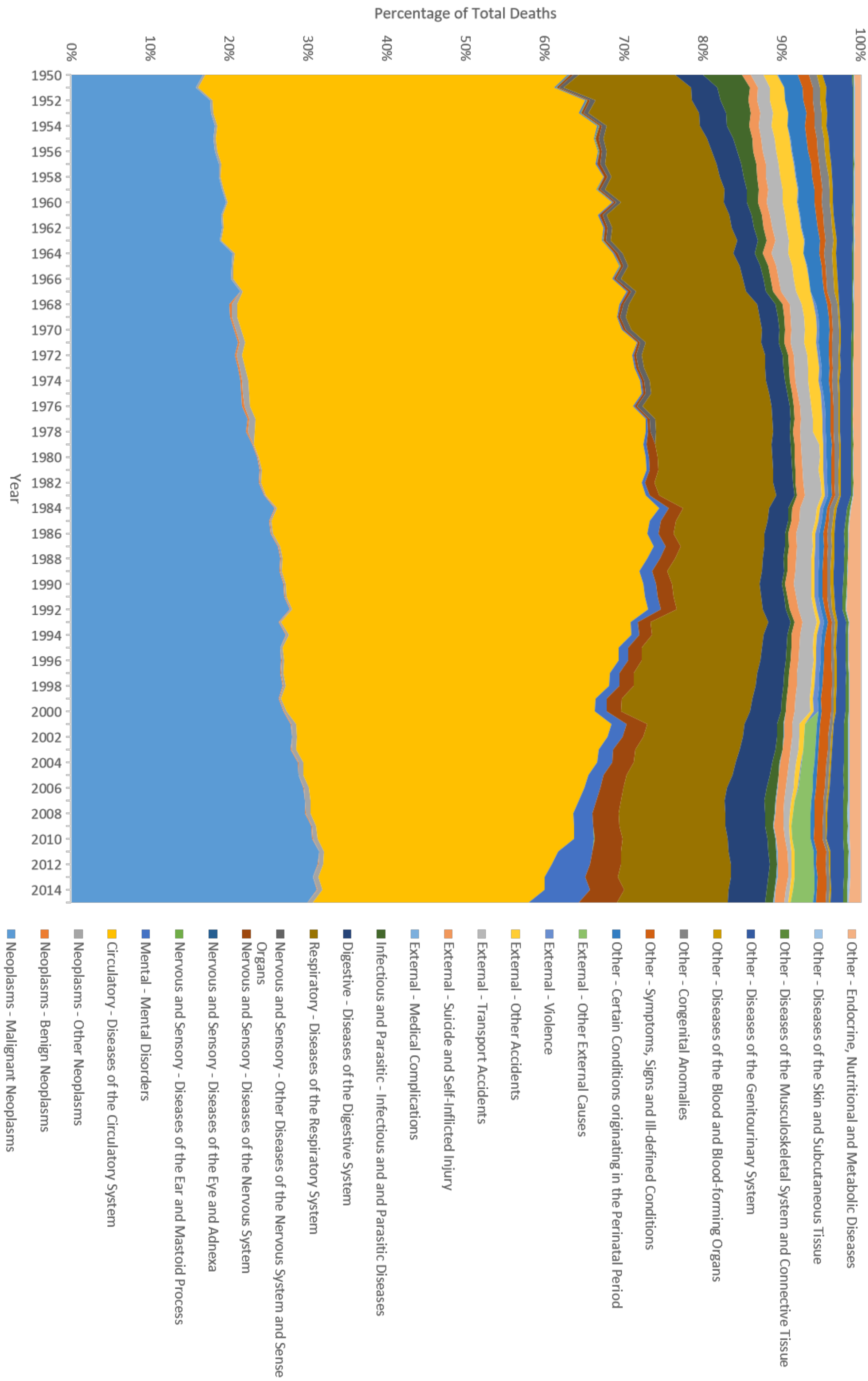


FIGURE 2.3: Percentage of Male Deaths Attributable to Particular Groups of Causes, England and Wales, 1950-2015

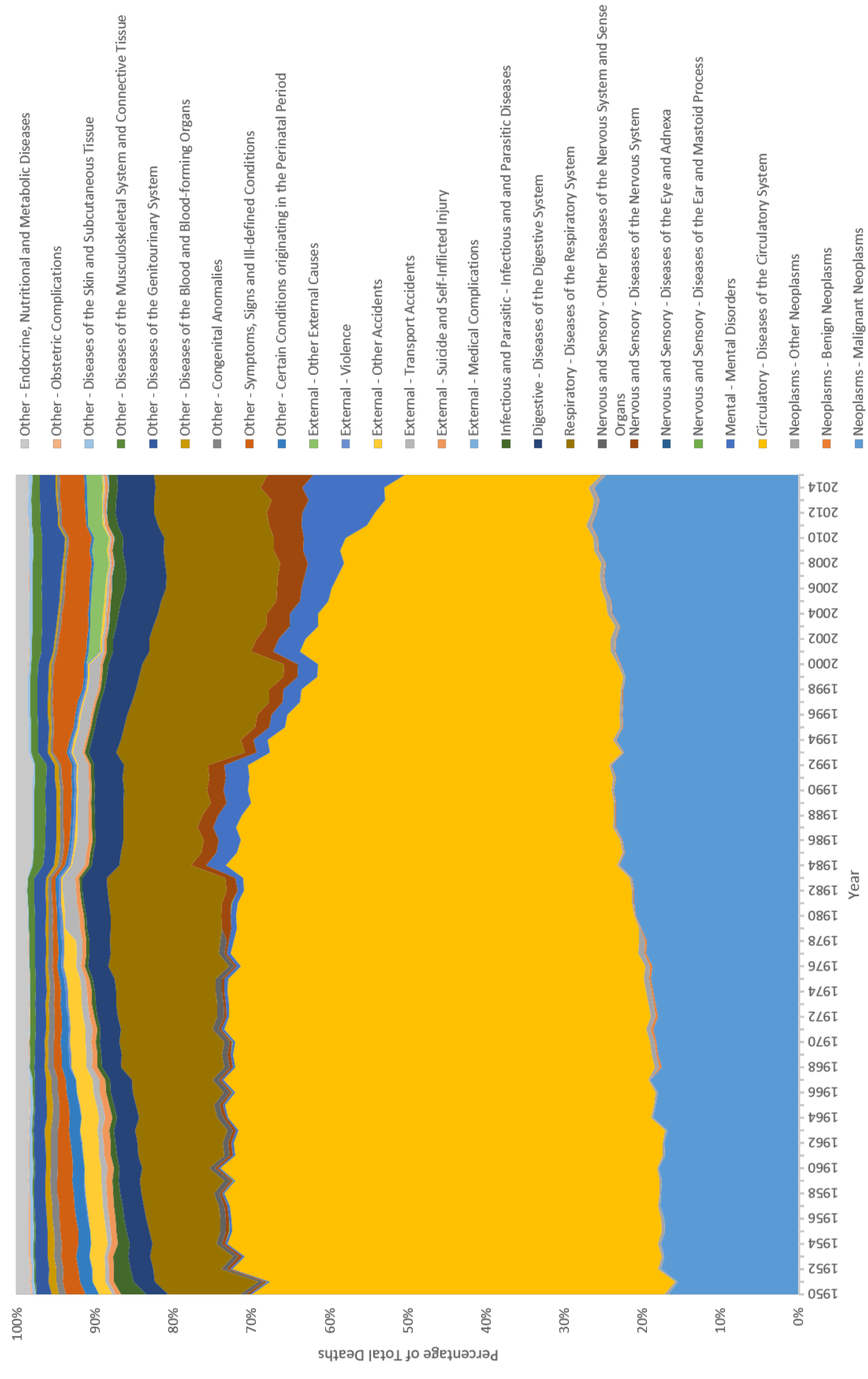


FIGURE 2.4:
Percentage of Female Deaths Attributable to Particular Groups of Causes, England and Wales, 1950-2015

2.4 Comparability Ratios

In the previous section we have seen that not all changes to ICD coding can be corrected for by reallocation of codes to a different group alone. There are instances where a new disease or condition has been identified that was previously assigned to a code that was most reflective of the medical understanding at the time. In these instances a different adjustment method is required to create a time series that is consistent over time.

One possible method is through the use of comparability ratios. Theses are ratios that allow us to compare mortality statistics recorded under different versions of the ICD (Cano-Serral et al., 2006). They are calculated using a sample of death records usually in the year preceding a change in coding practice (ONS, 2016b). These coding studies have taken place whenever there was a change in the ICD version as well as when a different, whether new or updated, version of a coding software was implemented. Some studies used a random sample of deaths, some used a non-random subset of available deaths, while others such as the ONS used a a full set of deaths registered in a particular year (Cano-Serral et al., 2006; Pavillon et al., 2005; ONS, 2002). The choice of which sample selection to use will depend not only on the availability of data but also on the need to manually code some of the deaths using ICD-10. For example Spain used mostly complete 1999 data from five communities plus Barcelona while France used a random sample of a tenth of 1999 deaths (Cano-Serral et al., 2006; Pavillon et al., 2005).

Once comparability ratios are calculated, they are applied to historic data to bring them in line with the current data set as baseline – this is useful for reference purposes and forecasting. While it may be useful to correct past data using these ratios, the ONS only uses comparability ratios for time series data (ONS, 2019). The adjusted number of deaths can be calculated as follows:

$$\check{d}(x,t,j,g) = r(j,g) \cdot d(x,t,j,g) \tag{2.1}$$

where $d(x,t,j,g)$ is the original number of deaths of individuals in age group x who died in year t in country g, $r(j,g)$ is the comparability ratio for cause j and country g, and $\check{d}(x,t,j,g)$ is the adjusted number of deaths of individuals in age group x who died in year t in country g. The comparability ratio is calculated as follows

$$r(j,g) = \frac{d^{ICD-10}(x,t,j,g)}{d^{ICD-9}(x,t,j,g)} \tag{2.2}$$

where $d^{ICD-10}(x,t,j,g)$ is the number of deaths coded to cause j in ICD-10 and $d^{ICD-9}(x,t,j,g)$ is the number of deaths coded to the equivalent cause j in ICD-9. We will not be applying comparability ratios to deaths coded using ICD-8 as the classification and coding was comparable to ICD-9 (Rooney and Smith, 2000).

The ICD-10 vs ICD-9 comparability ratios for males and females in England & Wales from the 2001 ONS bridge coding study using deaths data from 1999 are given in Tables 2.5 and 2.6, respectively.

Group	ICD-10 Codes	ICD-9 Codes	Comparability Ratio	95% Confidence Interval
Infectious and parasitic diseases	A00-B99	001-139	1.056	(1.028, 1.084)
Neoplasms	C00–D48	140–239	1.035	(1.033, 1.036)
Circulatory diseases	I00-I99	390-459	1.034	(1.032, 1.035)
Respiratory diseases	J00-J99	460-519	0.789	(0.785, 0.793)
External causes of mortality	V01-Y89	E800-E999	0.999	(0.994, 1.003)

TABLE 2.5: Comparability Ratios for Males – England & Wales, (ONS, 2002)

The standard errors and corresponding confidence intervals for the comparability ratios were calculated by the ONS based on multinomial sampling theory – see Rooney, Griffiths, and Cook (2002) for details. A comparability ratio interval that includes the value of 1 is not significant at the 5 % significance level and there is therefore insufficient statistical evidence that the number of deaths is different between the two classifications. It should be noted that this dual coding study used all deaths registered in 1999 for the respective calculations rather than a sample (Office for National Statistics, 2002).

Name	ICD-10 Codes	ICD-9 Codes	Comparability Ratio	95% Confidence Interval
Infectious and parasitic diseases	A00-B99	001-139	1.107	(1.075, 1.139)
Neoplasms	C00–D48	140–239	1.031	(1.030, 1.033)
Circulatory diseases	I00-I99	390-459	1.043	(1.041, 1.044)
Respiratory diseases	J00-J99	460-519	0.765	(0.761, 0.769)
External causes of mortality	V01-Y89	E800-E999	1.008	(0.999, 1.017)

TABLE 2.6: Comparability Ratios for Females – England & Wales, (ONS, 2002)

We can see that the 95% confidence intervals for the comparability ratio for external causes of mortality for both males and females include the value 1 so are not statistically significant and a correction was therefore not be applied to any of the time series for this group in our analyses.

For infectious and parasitic diseases, neoplasms, and circulatory diseases, the comparability ratios indicate that the ICD-9 death counts are within 10% of the ICD-10 death counts. We therefore chose not to apply the ratios above to these groups and will instead estimate the uncertainty of the death counts using the models presented in the chapters that follow. The only group for England & Wales where we did apply comparability ratios as per (2.1) was therefore respiratory diseases. This ratio was applied only for years preceding the introduction of ICD-10 where Rule 3 was not in force, i.e. 1968-1983 and 1993-2000. For years 1984-1992, we consider the deaths coded using ICD-9 under Rule 3 to be comparable to deaths coded under ICD-10 (Rooney and Smith, 2000; Rooney, Griffiths, and Cook, 2002). Given the large impact of the coding changes on deaths due to diseases of the nervous system and mental and behavioural disorders, we chose age 50 as the cut off point for the application of comparability ratios (Brock, Griffiths, and Rooney, 2006). This corresponds to the later half of the age period for young-onset dementia (Rossor et al., 2010). The death counts for the 'Other' group (J) are calculated using the adjusted death counts for the remaining groups using

$$\check{d}(x,t,J,g) = d(x,t,g) - \sum_{j=1}^{J-1} \check{d}(x,t,j,g) \tag{2.3}$$

where $d(x,t,g)$ is the number of deaths of individuals in age group x who died in year t in country g. The original death count is used where no comparability ratio is applied, i.e. $\check{d}(x,t,j,g) = d(x,t,j,g)$. The constraint imposed by (2.3) ensures that the total number of deaths for each age group stays fixed as these values are considered correct excepting any delays to death registration, e.g. when a death is referred to a coroner.

The combined ICD-10 vs ICD-9 comparability ratios for both sexes in France as calculated by Pavillon et al., 2005 are presented in Table 2.7. We applied the comparability ratios to the number of deaths due to infectious & parasitic diseases and respiratory diseases as the ratios indicate a discrepancy between ICD-9 and ICD-10 death counts to be greater than 10%.

Group	ICD-10 Codes	ICD-9 Codes	Comparability Ratio
Infectious and parasitic diseases	A00-B99	001-139	1.38
Neoplasms	C00–D48	140–239	1.01
Circulatory diseases	I00-I99	390-459	1.00
Respiratory diseases	J00-J99	460-519	0.86
External causes of mortality	V01-Y89	E800-E999	0.96

TABLE 2.7: Comparability Ratios for Both Sexes – France, (Pavillon et al., 2005)

All models that will be presented in this thesis use data for the period 1968-2005. The starting point was selected to coincide with the introduction of ICD-8 in both England & Wales and France. This time period will provide us with 38 calendar years worth of data while limiting the number of ICD versions to three. While we have aggregated mortality data by each group of causes of death for both sexes, only the mortality experience of females will be explored in the subsequent chapters.

Chapter 3

Competing Risks

General mortality can be considered as a simple stochastic process with two states: Alive or Dead. This basic model can be extended to cause-specific mortality by including an absorbing state for each cause or group of causes of death. We can represent the probability of dying using transition probability $q(t)$, i.e. the probability of moving from the Alive state to the Dead state at time t. The probability of staying in the Alive state, i.e. not transitioning to the Dead state, can then be represented as $p(t)$.

In a basic two-state mortality model, the transition probabilities $p(t)$ and $q(t)$ must sum to one as an individual can either stay alive throughout year t or die. (We are assuming there is no migration and no individual is lost to observation.) Similarly, in a multi-state model the sum of the probabilities of survival and all deaths must equal to one; it is therefore crucial that our model ensures the number of deaths in a single year does not exceed the number of individuals alive.

Recently, Spreeuw, Owadally, and Kashif (2022) proposed a Markov chain model to incorporate mortality improvements. However, this model only focuses on overall mortality and relies on data for single-year age groups. Cause-specific mortality data, on the other hand, tends to be available in five-year age groups, which complicates the estimation of transition intensities especially for ages below 25 due to the 'accident hump'. Alternatively, the sum-to-condition can be achieved by applying the logit transformation and using the Alive state as a baseline.

In Section 3.1 that follows, we look at various measures to quantify both overall and cause-specific mortality. Section 3.2 provides a brief overview of a selection of models for overall mortality and Section 3.3 introduces the logistic model.

3.1 Measurement of Mortality

Hinde (1998) quantifies the mortality experience of a particular population using either what he calls the 'm-type' and 'q-type' mortality rate. The primary difference between these two measures is the value in the denominator – i.e. the population that is at risk of dying – referred to in demographic literature 'exposed to risk' (Elandt-Johnson, 1984). We define the 'm-type' and 'q-type' rates in Subsections 3.1.1 and 3.1.2, respectively.

3.1.1 Mortality Rate

We define the crude overall mortality rate $\tilde{m}(x,t,g)$ as the ratio

$$\tilde{m}(x,t,g) = \frac{d(x,t,g)}{E^C(x,t,g)} \tag{3.1}$$

where $d(x,t,g)$ is the number of deaths of individuals in age group x who died in year t in country g and $E^C(x,t,g)$ is the corresponding central exposed to risk, i.e. the average count of the population at risk of death (Hinde, 1998). We will substitute the mid-year population of each country for $E^C(x,t,g)$ as in Dodd et al. (2018).

The crude cause-specific mortality rate will be defined as the ratio

$$\tilde{m}(x,t,j,g) = \frac{d(x,t,j,g)}{E^C(x,t,g)} \tag{3.2}$$

where $d(x,t,j,g)$ is the number of cause-specific deaths or decrements due to cause j of individuals in age group x who died in the time interval $[t,t+1)$ in country g and $E^C(x,t,g)$ is the corresponding central exposed to risk as in (3.1). Whenever a comparability ratio was used to adjust the number of cause-specific deaths, the adjusted count $\check{d}(x,t,j,g)$ should be used instead of $d(x,t,j,g)$. No adjustment is needed for the overall rates as the adjustments are constrained so that the total number of deaths stays fixed (see Section 2.4 for details).

3.1.2 Probability of Death in Year *t*

Instead of mortality rates, we can also use probability of death to quantify mortality. We calculate the crude probability of death in year t using the ratio of $d(x,t,g)$, i.e. the number of deaths of individuals in age group x who died in the year t in country g, and $E^0(x,t,g)$, the initial exposed to risk at time t for each age group x, such that

$$\tilde{q}(x,t,g) = \frac{d(x,t,g)}{E^0(x,t,g)}. \tag{3.3}$$

The initial exposed to risk $E^0(x,t,g)$ is the count of individuals at risk of death at the start of the reference period – in our case the reference period being the start of the calendar year (1 January). We approximate the initial exposed to risk using the population at the start of the calendar year. As statistical offices mainly provide mid-year population estimates, we average the mid-year population from year $t-1$ and t to calculate the initial population in year t

$$E^0(x,t,g) = \frac{E^C(x,t-1,g) + E^C(x,t,g)}{2}. \tag{3.4}$$

This definition relies on the assumption that deaths are distributed evenly throughout the year, i.e. the uniform distribution of deaths (UDD) (Promislow, 2015).

We then define the cause-specific probability of death $\tilde{q}(x,t,j,g)$ in year t due to cause j, also referred to as the probability of decrement (Seal, 1948), as the proportion of deaths $d(x,t,j,g)$ due to cause j to the initial exposed to risk $E^0(x,t,g)$ at time t for each age group x, such that

$$\tilde{q}(x,t,j,g) = \frac{d(x,t,j,g)}{E^0(x,t,g)}. \tag{3.5}$$

We assume that the number of individuals who stayed alive during year t is $l(x,t,g) = E^0(x,t,g) - \sum_{j=1}^{J} d(x,t,j,g)$. This definition can be thought of as a multinomial extension of the definition of probability of death found in Chiang (1968; 1991) who treats deaths as decrements in a competing risks model for a closed population. In other words, the number of deaths and survivors must add up to the total number of individuals in a single period.

3.2 Mortality Models

There are two approaches to modelling competing risks and mortality – either with an independence assumption or a dependence assumption. The assumption of independence is unrealistic as deaths due to causes are likely to be dependent on each other (Richards, 2009; Arnold and Sherris, 2013; Arnold and Sherris, 2015). For example a death due to an infection as a result of weakened immune system from chemotherapy prescribed for a particular cancer. In practice, it is impossible to measure the survival time for the secondary dependent cause of death as the patient will die of the first cause. The individual's survival time had the death due to the primary cause not occurred therefore cannot be identified.

3.2.1 Lee-Carter Model

The model described by Lee and Carter (1992), hereinafter simply referred to as the Lee-Carter (LC) model, allows for the modelling of a baseline age effect on mortality along with a period effect. The LC model for the natural logarithm of the mortality rate $\log m(x,t)$ is defined as follows:

$$\log m(x,t) = \alpha(x) + \beta(x)\kappa(t) \tag{3.6}$$

where $\alpha(x)$ is the average mortality experience at age x, $\beta(x)$ is the average rate of mortality change at age x, and $\kappa(t)$ is the period effect at time t. The LC model suffers from an identifiability problem, however. Suppose $\dot{\alpha}(x)$, $\dot{\beta}(x)$, and $\dot{\kappa}(t)$ are the solutions to the model and c_1 and c_2 are constants, then

$$\begin{aligned}
\ddot{\alpha}(x) &= \dot{\alpha}(x) + c_1\dot{\beta}(x) \\
\ddot{\beta}(x) &= \dot{\beta}(x) \\
\ddot{\kappa}(t) &= \dot{\kappa}(t) - c_1
\end{aligned}$$

or

$$\ddot{\alpha}(x) = \dot{\alpha}(x)$$
$$\dddot{\beta}(x) = \frac{\dot{\beta}(x)}{c_2}$$
$$\ddot{\kappa}(t) = c_2\dot{\kappa}(t)$$

will also be a solution (Enchev, Kleinow, and Cairns, 2017). To remedy this problem, identifiability constraints will need to be imposed on the model parameters. The constraints proposed by Lee and Carter (1992) are $\sum_x \beta(x) = 1$ and $\sum_t \kappa(t) = 0$. Other constraints for β are possible, for example $\sum \beta^2(x) = 1$ used by Girosi and King (2007) or equivalently $||\boldsymbol{\beta}|| = 1$ where $||\cdot||$ is the Euclidean norm as used by Kleinow (2015). For a list of further constraints, see Beutner, Reese, and Urbain (2017).

3.2.2 Li-Lee Model

Li and Lee (2005) proposed a multi-country extension of the LC model. The underlying assumption is that mortalities of different countries will converge to a common trend in the long term. The log of the mortality rate is then a combination of country and global effects such that

$$\log m(x,t,g) = \alpha(x,g) + \beta(x,g)\kappa(t,g) + B(x)K(t) \tag{3.7}$$

where x denotes age or age group, t denotes the year, and g is the country index. Parameter $\alpha(x,g)$ then denotes the average age effect at age x in country g, $B(x)$ is the global rate of mortality change at age x, $K(t)$ is the global period effect in year t, $\beta(x,g)$ is the country-specific rate of mortality change at age x, and $\kappa(t,g)$ is the country-specific period effect in year t. Do note that constraints similar to those presented in Section 3.2.1 are needed to ensure identifiability of the bilinear terms. As $\alpha(x,g)$ is approximately the average mortality rate for age group x in country g, the countries whose mortality experiences are pooled should be of equal population size so as to not have a single country that would dominate the likelihood (Enchev, Kleinow, and Cairns, 2017). Kleinow (2015) attempts to bypass this issue of differing population sizes by using centralised log mortality rates for each country instead. The centering point is the average log mortality rate for each country over the time period under investigation. This approach allows populations from countries of different sizes to be included in a single model, e.g. United Kingdom (UK) and United States (US). If the death counts for the two countries were pooled instead of their mortality rates, the mortality experience of the US would dominate that of the UK as the US population is nearly five-fold that of the UK.

The Li-Lee (LL) model could be extended to include deaths due to different causes in order to pool the mortality experience across different countries, provided that they are similar enough. This has been done by Lyu, Waegenaere, and Melenberg (2021) for neoplasm, circulatory, and other deaths in Belgium, the Netherlands, and France. A single-country LL model that pools mortality across sexes would however not be possible as the mortality experience for various causes of death, e.g. external causes or circulatory diseases, is different for males and females.

The specifications of both the LC and LL models make future mortality forecasting relatively straightforward as only the period effect(s) need to be projected (Li and Lee, 2005). While the LC model contains only a single $\kappa(t, g)$ for each population g, the LL extension includes an additional common term $K(t)$. There are various time series forecasting strategies, including random walk, random walk with drift, autoregressive process, autoregressive integrated moving average model, and vector autoregressive process reverting to zero (Li and Hardy, 2011; Enchev, Kleinow, and Cairns, 2017). We will explore these models further in Chapter 5.

3.3 Multinomial Logistic Model

The Multinomial Logistic (MLG) model extends the logistic regression model to problems with multiple outcomes such that

$$\log\left(\frac{\pi_i}{\pi_{i^*}}\right) = \eta(\mathbf{z_i}), \quad i \neq i^* \tag{3.8}$$

where π_i is the probability of outcome i, π_{i^*} is the probability of the baseline outcome i^*, and $\eta(\cdot)$ is the predictor function. In a model with N many possible outcomes, the probabilities of all N outcomes will sum to one, i.e. $\sum_{i=1}^{N} \pi_i = 1$; we can therefore rewrite (3.8) as follows

$$\log\left(\frac{\pi_i}{\pi_N}\right) = \log\left(\frac{\pi_i}{1 - \sum_{i=1}^{N-1} \pi_i}\right) = \eta(\mathbf{z_i}) \tag{3.9}$$

where N is the total number of possible outcomes, π_i is the probability of outcome i, π_N is the probability of the baseline outcome N, and $\eta(\cdot)$ is the predictor function. The predictor function is often a linear combination of explanatory variables, i.e.

$$\eta(\mathbf{z_i}) = \beta_0 + \beta_1 z_{1i} + \cdots + \beta_R z_{Ri} \tag{3.10}$$

where R is the number of explanatory variables z_{ri} for each outcome i and β_r denotes their respective coefficients.

The MLG model relates the logit of probabilities to a predictor function with linear predictor being the most common specification. The model allows both age and period, along with other predictor variables, to be included as either a continuous or categorical variable. It is clear from mortality data that the age effect on overall mortality experience is not linear on the log-scale across ages, e.g. the 'accident hump' roughly between ages 10 and 40 caused by the relative increase in deaths due to external causes (Heligman and Pollard, 1980). While there are instances where a log-linear trend could be applied to a specific disease, e.g. ischaemic heart disease (Chang et al., 2017), the majority of models require the age effect to be categorical. We would, however, expect the underlying age effects to be smooth rather than discrete (see Dodd et al., 2018). The period effect is also not linear —Zhao (2012) argues that the time effect can be assumed to be linear for short periods; however, long term forecasts would project unrealistically low mortality rates.

Alai, Arnold, and Sherris (2015) applied the MLG model to deaths of females in France split into six cause groups with categorical age and continuous time predictor. While the application of linear time effect greatly reduces the number of parameters needed to be estimated, especially if an interaction term is present, we do not expect the period effect to be linear or smooth as a model with continuous time periods cannot account for mortality shocks. For example, annual influenza epidemics affecting respiratory diseases or the increasing incidence of infectious diseases previously declared to be eradicated, e.g. measles, cannot be incorporated via a polynomial predictor.

3.3.1 Multinomial Logistic Lee-Carter Model

We propose a model combining the desired properties of the MLG model and a classical stochastic mortality model, such as the LC model, to model cause-specific mortality. The Multinomial Logistic Lee-Carter (MLG-LC) model is a combination of the MLG model with the right hand side of the LC model applied as the predictor function $\eta(\cdot)$. The simplest example of the MLG-LC is the binary case with the logit of the crude probability of death $q(x,t)$ defined as

$$\begin{aligned}\log\left[\frac{q(x,t)}{1-q(x,t)}\right] &= \eta(x,t) \\ &= \alpha(x)+\beta(x)\kappa(t)\end{aligned} \tag{3.11}$$

where $\alpha(x)$ is the average mortality experience at age x, $\beta(x)$ is the average rate of mortality change at age x, and $\kappa(t)$ is the period effect at time t. The baseline survival probability $p(x,t)$ is defined as $p(x,t) = 1 - q(x,t)$ and therefore (3.11) can be rewritten as

$$\log\left[\frac{q(x,t)}{p(x,t)}\right] = \alpha(x)+\beta(x)\kappa(t) \tag{3.12}$$

Extending the definition in (3.11) to the multinomial case yields Equation (3.13) for $q(x,t,j)$, i.e. the probability of an individual in age group x dying due to cause j in time interval t:

$$\begin{aligned} \log\left[\frac{q(x,t,j)}{1-\sum_{j=1}^{J} q(x,t,j)}\right] &= \eta(x,t,j) \\ &= \alpha(x,j)+\beta(x,j)\kappa(t,j) \end{aligned} \tag{3.13}$$

where J is the number of causes of death, $\alpha(x,j)$ is the average mortality experience at age x for cause j, $\beta(x,j)$ is the average rate of mortality change at age x for cause j, and $\kappa(t,j)$ is the period effect at time t for cause j. The baseline survival probability $p(x,t)$ is now defined as $p(x,t) = 1 - \sum_{j=1}^{J} q(x,t,j)$ yielding

$$\log\left[\frac{q(x,t,j)}{p(x,t)}\right] = \alpha(x,j)+\beta(x,j)\kappa(t,j) \tag{3.14}$$

While Zhao (2012) and Zhao et al. (2013) have previously applied the binary logistic model with probability of death as the baseline to Chinese mortality data, we are not aware of any papers implementing the MLG-LC to model multiple causes of death.

3.3.2 Multinomial Logistic Li-Lee Model

We extend the MLG-LC model to include multiple countries and causes of death by creating a Multinomial Logistic Li-Lee (MLG-LL) model. The MLG-LL model combines the specifications of the multinomial logistic model presented by Alai, Arnold, and Sherris (2015) with the multi-country parametrisation of Li and Lee (2005). While the intention of the LL model is to pool similarities in mortality experiences for different countries, our model primarily aims to exploit the similarities between causes of death.

The general form of MLG-LL can be written as

$$\log\left[\frac{q(x,t,j,g)}{p(x,t,g)}\right] = \log\left[\frac{q(x,t,j,g)}{1-\sum_{j=1}^{J} q(x,t,j,g)}\right] = \zeta(x,t,j,g) \tag{3.15}$$

where J is the number of causes of death, $q(x,t,j,g)$ is the probability of an individual in age group x and in country g dying due to cause j in time interval t, $p(x,t,g)$ is the survival probability of an individual in age group x and in country g at the start of time interval t, and $\zeta(x,t,j,g)$ is the MLG-LL predictor function. $\zeta(x,t,j,g)$ can be decomposed further into a country and cause-specific predictor $\eta(x,t,j,g)$ and a pooled predictor $H(x,t,j,g)$, i.e.

$$\zeta(x,t,j,g) = \eta(x,t,j,g) + H(x,t,j,g) \tag{3.16}$$

where

$$\eta(x,t,j,g) = \alpha(x,j,g) + \beta(x,j,g)\kappa(t,j,g). \tag{3.17}$$

$H(x,t,j,g)$ can contain predictors pooled by causes across countries, by countries across causes, globally, or a combination thereof. The full specification of $H(x,t,j,g)$ with all the pooled terms is given below

$$H(x,t,j,g) = b(x,j)k(t,j) + b(x,g)k(t,g) + B(x)K(t). \tag{3.18}$$

By definition of (3.17), setting $H(x,t,j,g)$ equal to zero reduces (3.15) to a multi-country version of MLG-LC given in (3.13). Further possible combinations will be discussed Section 4.1.1.

It is important to note that Li and Lee (2005) imposed the expectation of 'similar socioeconomic conditions' on the countries pooled to create the multi-country model. While this assumption is unlikely to hold for the United States, Bosworth (2018) suggests that there is a convergence in mortality experience among different socio-economic groups in Europe and Canada. The likely explanation for this is the difference between the health insurance models and the resulting access to healthcare. James et al. (2007) demonstrated that the introduction of universal health insurance in Canada led to a large reduction in mortality due to socio-economic differences. Furthermore, ONS (2022) data show that there has been a decline in avoidable mortality for the most deprived areas in England and the rate was slowly converging to that of the least deprived areas. This trend, however, reversed in 2020 as a result of the COVID-19 pandemic. Various other studies have explored the link between deprivation and access to healthcare such as Dixon-Woods et al. (2006) or Peconi et al. (2019). We chose the UK and France as the two countries for our analysis due to their similar population size and geographic proximity.

In the chapters that follow, we will estimate the models described in Section 3.2 using a variety of statistical techniques. In Chapter 4 we present cause-specific mortality as a multi-state stochastic model that can be estimated using maximum likelihood with uncertainty of parameters quantified using bootstrap.

Chapter 4

Stochastic Modelling of Cause-Specific Mortality in England & Wales

The competing risks framework discussed in Chapter 3.2 imposes a notable restriction on the cause-specific death probabilities — the sum of the transition probabilities for each group has to sum to one. This property of the MLG model becomes extremely useful as it allows us to extend the two-state stochastic mortality model to include a different state for each cause or group of causes of death while ensuring the sum of the probabilities of dying or staying alive cannot exceed one.

In Section 4.1 we will define the general MLG model for multiple causes of death and propose a number of predictor functions based on the LC and LL mortality models. Unlike Lee and Carter (1992) who used Singular Value Decomposition, we will use the `optim()` function in the R programming language (R Core Team, 2022) to fit our models using the likelihood function derived in Section 4.2. We explore the use of random sampling, e.g. multivariate normal sampler and parametric bootstrap, to quantify coefficient uncertainty in Section 4.3. We end the chapter by presenting the results of our model estimation in Section 4.4.

4.1 Proposed Model Specification

We proceed to specify the model with the transition probability $q(x,t,j,g)$, i.e. the probability of an individual in age group x from country g dying due to cause j in time interval t. Survival probability $p(x,t,g)$ is the probability of staying alive throughout time interval t. Hence we have

$$\log\left[\frac{q(x,t,j,g)}{p(x,t,g)}\right] = \log\left[\frac{q(x,t,j,g)}{1-\sum_{j=1}^{J} q(x,t,j,g)}\right] = \zeta(x,t,j,g) \tag{4.1}$$

The transition probability $q(x,t,0,g)$ denotes the probability of staying in the recurrent reference state $j=0$, i.e. the survival probability $p(x,t,g)$. This is equal to

$$p(x,t,g) = q(x,t,0,g) = 1-\sum_{j=1}^{J} q(x,t,j,g) = \frac{1}{1+\sum_{j=1}^{J} \exp\left[\zeta(x,t,j,g)\right]} \tag{4.2}$$

By manipulating (4.1) and using the expression for $p(x,t,g)$ from (4.2), the probability $q(x,t,j,g)$ can be shown to be equal to

$$q(x,t,j,g) = \frac{\exp\left[\zeta(x,t,j,g)\right]}{1+\sum_{j=1}^{J} \exp\left[\zeta(x,t,j,g)\right]} \tag{4.3}$$

where $\zeta(x,t,j,g)$ is the predictor function for cause of death j and country g evaluated at age group x and year t.

4.1.1 Predictors

We propose four different parameterisations of the MLG-LL model (M1, M2, M4, and M5) in addition to the single country parametrisation of the MLG-LC model (M0). We have chosen these particular model specifications as they allow for pooling of death counts to borrow strength when there are issues with the data for a particular country or cause of death. The predictors $\zeta(x,t,j,g)$ for each model are given in Table 4.1. Models M1 and M4 could be combined to form a model M3 with both a cause-specific age and period bilinear term pooled across countries and country-specific age and period bilinear term pooled across causes of death. As we did not estimate this model due to the large number of parameters and multiple bilinear terms — its parameterisation is therefore not presented in this thesis.

TABLE 4.1: PROPOSED MODEL PREDICTORS

M0	$\zeta(x,t,j,g)$	=	$\alpha(x,j,g)+\beta(x,j,g)\kappa(t,j,g)$
M1	$\zeta(x,t,j,g)$	=	$\alpha(x,j,g)+\beta(x,j,g)\kappa(t,j,g)+b(x,g)k(t,g)+B(x)K(t)$
M2	$\zeta(x,t,j,g)$	=	$\alpha(x,j,g)+\beta(x,j,g)\kappa(t,j,g)+b(x,g)k(t,g)$
M4	$\zeta(x,t,j,g)$	=	$\alpha(x,j,g)+\beta(x,j,g)\kappa(t,j,g)+b(x,j)k(t,j)+B(x)K(t)$
M5	$\zeta(x,t,j,g)$	=	$\alpha(x,j,g)+\beta(x,j,g)\kappa(t,j,g)+b(x,j)k(t,j)$

Model M1 includes country- and cause-specific age effects $\alpha(x,j,g)$, $\beta(x,j,g)$ and time effect $\kappa(t,j,g)$, country-specific common terms $b(x,g)$ and $k(t,g)$ and global terms $B(x)$ and $K(t)$. Instead of country-specific common terms, model M4 contains cause-specific common terms $b(x,j)$ and $k(t,j)$ and global terms $B(x)$ and $K(t)$. Models M2 and M5 are reduced versions of M1 and M4, respectively, as they do not have global $B(x)$ and $K(t)$ terms. There are therefore two pairs of nested models: M1 & M2 and M4 & M5. Model M2 only pools mortality experience within a single country for multiple causes of death via $b(x,g)$ and $k(t,g)$ while model M5 estimates a common cause-specific age effect $b(x,j)$ and common cause-specific period effect $k(t,j)$. Model M0 is a simple cause-specific MLG-LC estimated separately for each country as there is no global term.

4.1.2 Constraints

To enable the estimation of the model specifications from the previous section, a number of constraints need to be imposed on the model parameters. These constraints are listed in Table 4.2.

TABLE 4.2: PARAMETER CONSTRAINTS

Model 0	Model 1	Model 2	Model 4	Model 5
$\sum_x \beta(x,j,g) = 1$ $\sum_t \kappa(t,j,g) = 0$	$\sum_x \beta(x,j,g) = 1$ $\sum_t \kappa(t,j,g) = 0$	$\sum_x \beta(x,j,g) = 1$ $\sum_t \kappa(t,j,g) = 0$	$\sum_x \beta(x,j,g) = 1$ $\sum_t \kappa(t,j,g) = 0$	$\sum_x \beta(x,j,g) = 1$ $\sum_t \kappa(t,j,g) = 0$
	$\sum_x b(x,g) = 1$ $\sum_t k(t,g) = 0$	$\sum_x b(x,g) = 1$ $\sum_t k(t,g) = 0$	$\sum_x b(x,j) = 1$ $\sum_t k(t,j) = 0$	$\sum_x b(x,j) = 1$ $\sum_t k(t,j) = 0$
	$\sum_x B(x) = 1$ $\sum_t K(t) = 0$		$\sum_x B(x) = 1$ $\sum_t K(t) = 0$	

We implement the constraints for age-specific rate of change parameters $\beta(x,j,g)$, $\beta(x,j)$, $b(x,g)$, and $B(x)$ so that the first age group, i.e. ages 1-4, is deterministic and all other age groups are free to vary. This is so that the rate of change parameter for the ultimate age group 85+, which has the highest number of deaths, is not a linear combination of the others. (Recall from Section 2.2 that deaths during age zero are categorised into a special group. This age is therefore not included in our analysis to ensure consistency and avoid artificial spikes in the number of deaths due to Cause 6 – Other Causes.) The constraint for the period-specific parameters $\kappa(t,j,g)$, $k(t,j)$, $k(t,g)$, and $K(t)$ are implemented so that the first time period, i.e year 1968, is deterministic and all other years are free to vary. This is so that the ultimate year 2005, which will also be the starting point for our projections in Chapter 7, is not a linear combination of other parameters. In the Sections 4.2 and 4.3 that follows, we explore a number of estimation techniques to calculate the cause-specific probability of death $q(x,t,j,g)$.

4.2 Maximum Likelihood Estimation of Parameters

To find the maximum likelihood estimates (MLEs) of the model parameters, we begin by defining the probability mass function of the multinomial logistic model. The probability of $d(x,t,j,g)$, $d(\cdot) \in \mathbb{N}^0$, individuals in country g and age group x dying due to cause j, $j = 0,\dots,J$, during time interval $[t, t+1)$ is given by

$$f(d(x,t,0,g),\dots,d(x,t,J,g)|E^0(x,t,g),\boldsymbol{\zeta}) = \frac{E^0(x,t,g)!}{\prod_{j=0}^{J}(d(x,t,j,g)!)} \cdot \prod_{j=0}^{J}(q(x,t,j,g))^{d(x,t,j,g)} \tag{4.4}$$

where $j \in \{1,\dots,J\}$ is the j^{th} group of causes of death and $j = 0$ denotes the individuals stayed alive, $E^0(x,t,g)$ denotes the population at the start of the year, i.e. initial exposed to risk, in age group x and country g in the time interval $[t, t+1)$, and $\boldsymbol{\zeta}$ is the vector of terms $\zeta(x,t,j,g)$ $\forall x,t,g$ and $j \in \{1,\dots,J\}$. We assume that the number of individuals who stayed alive throughout interval $[t, t+1)$ is $d(x,t,0,g) = E^0(x,t,g) - \sum_{j=1}^{J} d(x,t,j,g)$. The joint probability mass function is therefore

$$f(\mathbf{d}|\mathbf{E^0},\boldsymbol{\zeta}) = \prod_{g=1}^{G}\prod_{t=1}^{T}\prod_{x=1}^{X}\left[\frac{E^0(x,t,g)!}{\prod_{j=0}^{J}(d(x,t,j,g)!)} \cdot \prod_{j=0}^{J}(q(x,t,j,g))^{d(x,t,j,g)}\right] \tag{4.5}$$

where $\mathbf{d}$ is the vector of terms $d(x,t,j,g)$ $\forall x,t,g$ and $j \in \{0,\dots,J\}$ and $\mathbf{E^0}$ is the vector of terms $E^0(x,t,g)$ $\forall x,t,g$.

The likelihood function for the predictor coefficients $\boldsymbol{\zeta}$, where $\zeta(x,t,j,g)$ is the multinomial logistic model predictor for age group x, time period t, cause j and country g, where applicable, is given by:

$$
\begin{aligned}
\mathscr{L}(\boldsymbol{\zeta}|\mathbf{d},\mathbf{E^0}) &= \prod_{g=1}^{G}\prod_{t=1}^{T}\prod_{x=1}^{X}\left\{\frac{E^0(x,t,g)!}{\prod_{j=0}^{J}[d(x,t,j,g)!]}\cdot\prod_{j=0}^{J}[q(x,t,j,g)]^{d(x,t,j,g)}\right\} \\
&\propto \prod_{g=1}^{G}\prod_{t=1}^{T}\prod_{x=1}^{X}\prod_{j=0}^{J} q(x,t,j,g)^{d(x,t,j,g)} \\
&= \prod_{g=1}^{G}\prod_{t=1}^{T}\prod_{x=1}^{X}\left[p(x,t,g)^{E^0(x,t,g)-\sum_{j=1}^{J}d(x,t,j,g)}\cdot\prod_{j=1}^{J}q(x,t,j,g)^{d(x,t,j,g)}\right] \\
&= \prod_{g=1}^{G}\prod_{t=1}^{T}\prod_{x=1}^{X}\left[\frac{p(x,t,g)^{E^0(x,t,g)}}{p(x,t,g)^{\sum_{j=1}^{J}d(x,t,j,g)}}\cdot\prod_{j=1}^{J}q(x,t,j,g)^{d(x,t,j,g)}\right] \\
&= \prod_{g=1}^{G}\prod_{t=1}^{T}\prod_{x=1}^{X}\left[\frac{p(x,t,g)^{E^0(x,t,g)}}{\prod_{j=1}^{J}p(x,t,g)^{d(x,t,j,g)}}\cdot\prod_{j=1}^{J}q(x,t,j,g)^{d(x,t,j,g)}\right] \\
&= \prod_{g=1}^{G}\prod_{t=1}^{T}\prod_{x=1}^{X}\left\{p(x,t,g)^{E^0(x,t,g)}\prod_{j=1}^{J}\left[\frac{q(x,t,j,g)}{p(x,t,g)}\right]^{d(x,t,j,g)}\right\} \\
&= \prod_{g=1}^{G}\prod_{t=1}^{T}\prod_{x=1}^{X}\left(\left\{\frac{1}{1+\sum_{j=1}^{J}\exp\left(\zeta(x,t,j,g)\right)}\right\}^{E^0(x,t,g)}\prod_{j=1}^{J}[\exp\left(\zeta(x,t,j,g)\right)]^{d(x,t,j,g)}\right) \\
&= \prod_{g=1}^{G}\prod_{t=1}^{T}\prod_{x=1}^{X}\left(\left\{1+\sum_{j=1}^{J}\exp\left(\zeta(x,t,j,g)\right)\right\}^{-E^0(x,t,g)}\prod_{j=1}^{J}\exp\left(d(x,t,j,g)\zeta(x,t,j,g)\right)\right).
\end{aligned}
$$

The product

$$
c_1 = \prod_{g=1}^{G}\prod_{t=1}^{T}\prod_{x=1}^{X}\left\{\frac{E^0(x,t,g)!}{\prod_{j=0}^{J}[d(x,t,j,g)!]}\right\} \tag{4.6}
$$

can be ignored as the initial exposed to risk $E^0(x,t,g)$ and the number of individuals $d(x,t,j,g)$ who died due to cause j in age group x in period t and country g are fixed and do not depend on the coefficient estimates. This approach speeds up the optimisation algorithm by eliminating unnecessary calculations. The corresponding log-likelihood up to an additive constant can then be shown to be equal to:

$$
\begin{aligned}
\ell\left(\boldsymbol{\zeta}|\mathbf{d},\mathbf{E^0}\right) &= \log \mathscr{L}(\boldsymbol{\zeta}|\mathbf{d},\mathbf{E^0}) \\
&= \log \prod_{g=1}^{G}\prod_{t=1}^{T}\prod_{x=1}^{X}\left(\left\{1+\sum_{j=1}^{J}\exp\left[\zeta(x,t,j,g)\right]\right\}^{-E^0(x,t,g)}\prod_{j=1}^{J}\exp\left[d(x,t,j,g)\cdot\zeta(x,t,j,g)\right]\right) \\
&\quad + \log c_1 \\
&= \sum_{g=1}^{G}\sum_{t=1}^{T}\sum_{x=1}^{X}\left(\log\left\{\left[1+\sum_{j=1}^{J}\exp\left[\zeta(x,t,j,g)\right]\right]^{-E^0(x,t,g)}\right\}\right. \\
&\quad \left. + \sum_{j=1}^{J}\log\left\{\exp\left[d(x,t,j,g)\cdot\zeta(x,t,j,g)\right]\right\}\right) + c_2 \\
&= \sum_{g=1}^{G}\sum_{t=1}^{T}\sum_{x=1}^{X}\left(-E^0(x,t,g)\cdot\log\left\{1+\sum_{j=1}^{J}\exp\left[\zeta(x,t,j,g)\right]\right\}\right. \\
&\quad \left. + \sum_{j=1}^{J}\left[d(x,t,j,g)\cdot\zeta(x,t,j,g)\right]\right) + c_2
\end{aligned}
$$

where the constant c_2 is the natural logarithm of the constant product c_1 from (4.6), i.e.

$$
c_2 = \log c_1 = \log \prod_{g=1}^{G}\prod_{t=1}^{T}\prod_{x=1}^{X}\left\{\frac{E^0(x,t,g)!}{\prod_{j=0}^{J}\left[d(x,t,j,g)!\right]}\right\}. \tag{4.7}
$$

While the partial log-likelihood is sufficient to estimate a model, use of the full log-likelihood will allow us to compare the various nested models using Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

4.3 Random Sampling and Bootstrap

In this section we will look at techniques that are complementary to the MLE approach from Section 4.2. Recall that the MLG-LC model is a special case of MLG-LL and the predictor function $\zeta(x,t,j,g)$ can therefore be simplified to $\eta(x,t,j,g)$ as per (3.17). The limiting distribution of the Maximum Likelihood Estimator $\hat{\boldsymbol{\eta}}$ of the predictor $\boldsymbol{\eta}$ from the previous sections is then given as

$$\hat{\boldsymbol{\eta}} \rightarrow N\left(\boldsymbol{\eta}, \mathcal{I}(\boldsymbol{\eta})^{-1}\right) \tag{4.8}$$

where $\mathcal{I}(\boldsymbol{\eta})$ is the Fisher information matrix such that

$$\mathcal{I}(\boldsymbol{\eta})_{ir} = -E\left\{\frac{\partial^2 \ell\ (\boldsymbol{\eta}|\mathbf{d}, \mathbf{E^0})}{\partial\eta_i \partial\eta_r}\right\} \tag{4.9}$$

and $\boldsymbol{\eta}$ is the vector of all model parameters to be estimated (Efron and Tibshirani, 1993). The sampling distribution of $\hat{\boldsymbol{\eta}}$ can then be approximated by

$$N\left(\hat{\boldsymbol{\eta}}, \mathcal{I}(\hat{\boldsymbol{\eta}})^{-1}\right). \tag{4.10}$$

In a single-country scenario, for example, the vector of parameters $\boldsymbol{\eta}$ for the MLG-LC model (M0) can be written in matrix form as $\boldsymbol{\eta} = (\boldsymbol{\alpha}^1,\ \boldsymbol{\beta}^1,\ \boldsymbol{\kappa}^1,\ \ldots\ ,\ \boldsymbol{\alpha}^J,\ \boldsymbol{\beta}^J,\ \boldsymbol{\kappa}^J)$ where $\boldsymbol{\alpha}^j = (\alpha(1,j),\ \ldots\ \alpha(X,j))^\top$, $\boldsymbol{\beta}^j = (\beta(1,j),\ \ldots\ \beta(X,j))^\top$, and $\boldsymbol{\kappa}^j = (\kappa(1,j),\ \ldots\ \kappa(T,j))^\top$ for age groups $x \in \{1,\ldots,X\}$, years $t \in \{1,\ldots,T\}$, and causes $j \in \{1,\ldots,J\}$. Due to constraints imposed on $\boldsymbol{\beta}^1,\ldots,\boldsymbol{\beta}^J$ as outlined in Section 4.1.2, using $\beta(X) = 1 - \sum_{x=0}^{X-1}[\beta(x)]$, the last column of each variance-covariance submatrix $\mathrm{V}(\boldsymbol{\beta}^J)$ for $j \in \{1,\ldots,J\}$ will be the variance of the (negative of the) sum of the other parameters. Due to the symmetry of the variance-covariance submatrix, the same property will apply to the last row. A sketch of a proof for a generic $\boldsymbol{\beta}$ is given below:

$$\mathrm{Var}(\boldsymbol{\beta}) = \mathrm{Var}[1 - \sum_{x=0}^{X-1}(\beta(x))] = \mathrm{Var}[\sum_{x=0}^{X-1}\beta(x)]. \tag{4.11}$$

The vector of effective parameters in $\boldsymbol{\beta}$ is therefore of length $X-1$ since the last entry can be calculated deterministically via the constraint.

Similarly for $\boldsymbol{\kappa}$, the first row and first column of each submatrix $\mathrm{V}(\boldsymbol{\kappa}^j)$ for $j \in \{1, \ldots, J\}$ will be a linear combination of the other rows and columns, respectively. The derivation for $\kappa(1)$ follows naturally from (4.11). Were these constrained parameters to be included in the Fisher information, the estimated matrix would be singular and non-invertible.

The large number of effective parameters in the model poses yet another challenge. In order to have uncertainty estimates for the constrained parameters, and more importantly uncertainty for the estimated probabilities of death, we propose two different methods to do this: random sampling and parametric bootstrap.

4.3.1 Random Sampling

The random sampling approach treats the Fisher information as a block diagonal matrix where each block is a square matrix of partial derivatives as defined in (4.9) for parameter blocks $\boldsymbol{\alpha}^1,\ \boldsymbol{\beta}^1,\ \boldsymbol{\kappa}^1,\ \ldots\ ,\ \boldsymbol{\alpha}^J,\ \boldsymbol{\beta}^J,\ \boldsymbol{\kappa}^J$. Each block is then inverted separately from all other blocks once the constrained column and row are removed to create variance-covariance matrix $\boldsymbol{\Lambda}$. This specification forces zero covariance between blocks for the sampling distribution. This is necessary due to the high number of model parameters making the matrix inversion otherwise computationally difficult. This trade-off leads to a loss of covariance information between blocks for different values of j; however, the probabilities of death will continue to be linked via the logit link function. The parameter and variance estimates can be obtained by generating W many samples from Multivariate Normal (MVN) distributions for each block of parameters and evaluating the constrained parameters, where appropriate. Furthermore, the confidence intervals for the death probabilities can be estimated by calculating the probabilities at each sample replication and applying the percentile methodology of Efron and Tibshirani (1986).

4.3.2 Parametric Bootstrap

The Parametric Bootstrap approach uses the following Bootstrap algorithm proposed by Efron and Tibshirani (1986):

1. Draw W bootstrap samples $\boldsymbol{d}^{*1}, \boldsymbol{d}^{*2}, \ldots, \boldsymbol{d}^{*W}$ from the multinomial distribution of deaths data $\boldsymbol{d}$ where $\mathbf{d}$ is a multidimensional array of decrements, i.e. counts of deaths due to cause j and individuals who stayed alive until the end of the interval, for each age group x, year t, and country g, where applicable.
2. Evaluate the Maximum Likelihood (ML) estimate $\hat{\boldsymbol{\theta}}^{*}$ of parameter $\hat{\boldsymbol{\theta}}$ for each w

$$\hat{\boldsymbol{\theta}}^{*w} = \mathrm{ML}(\boldsymbol{d}^{*w}, \mathrm{E}^{\mathbf{0}}) \qquad w = 1, 2, \ldots, W. \tag{4.12}$$

3. Calculate the sample standard deviation of the W maximum likelihood estimates to approximate the standard error $\mathrm{se}(\hat{\boldsymbol{\theta}})$.

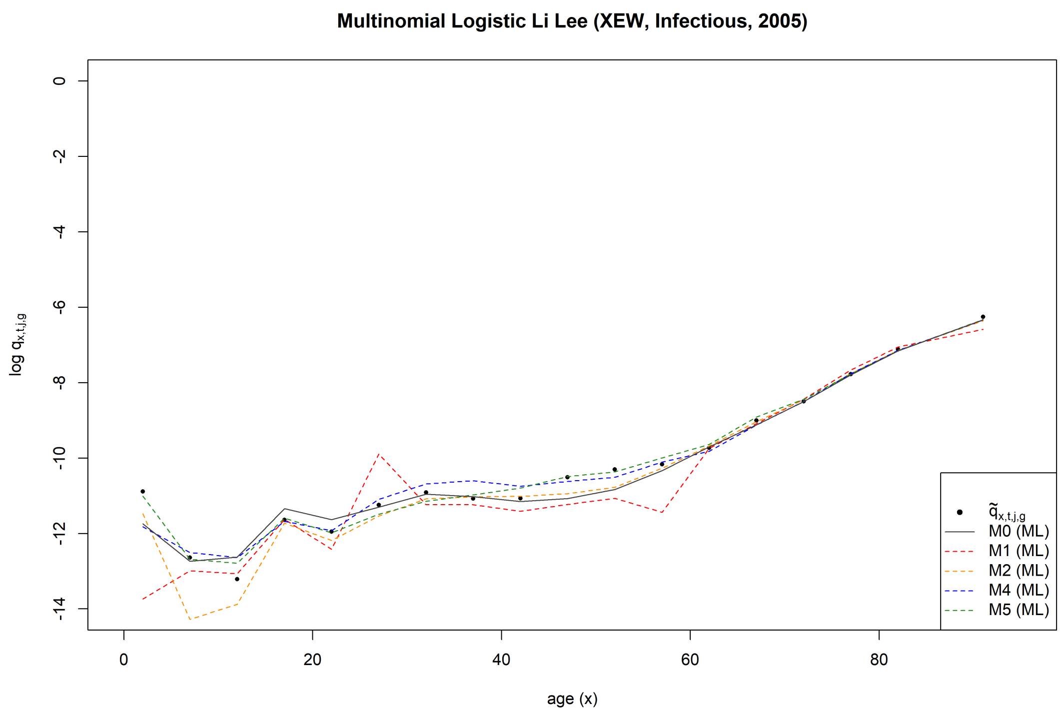
Efron and Tibshirani (1986, 1993) note that choosing W between 50 and 200 should be sufficient. For our simulation we used $W = 500$ to ensure sufficient sampling of the tails of the distribution.

4.4 Results

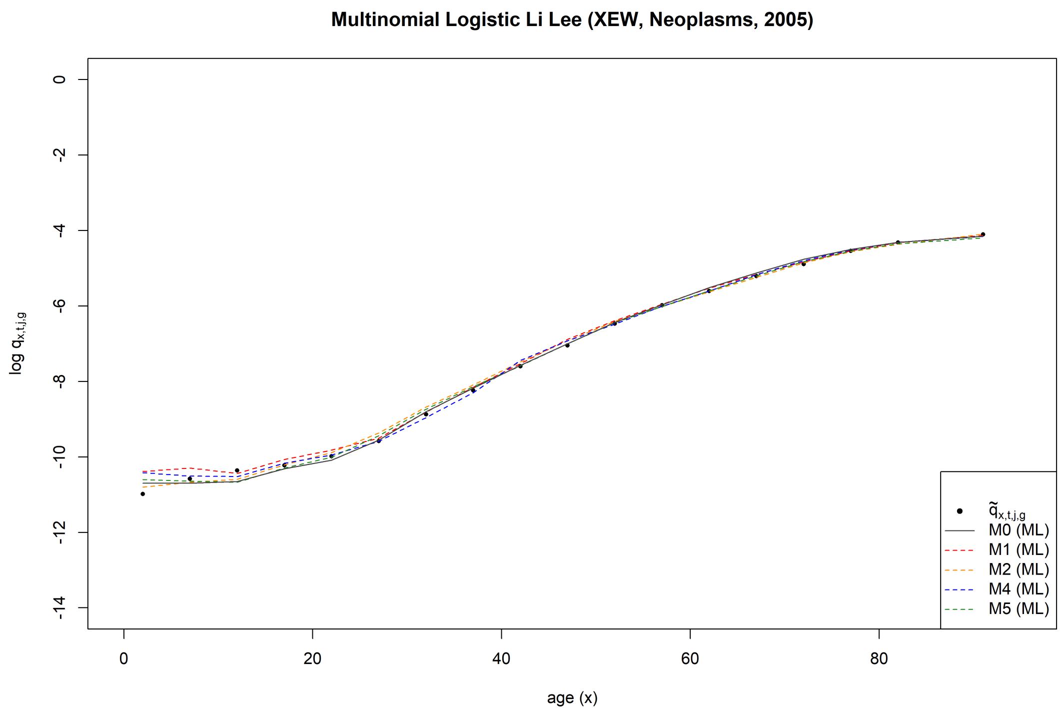
The MLEs of the probability of death $\hat{q}(x,t,j,g)$ using the five different model specifications (M0, M1, M2, M4, and M5) for the six different groups of causes of death for England and Wales in the year 2005 as well as crude probabilities of death $\tilde{q}(x,t,j,g)$ as defined in (3.5) are shown in Figure 4.1. Plots for a selection of other years can be found in Appendix B.

We can see that the models fit the data comparably well for adult ages for neoplasms, circulatory diseases, and external causes of death. The performance of the models for young ages is mixed. We note the dip in $\hat{q}$(5-9, 2005, 3, XEW) fitted using M5 and shown in Figure 4.1.c. This appears to be an isolated anomaly for year 2005 that is not present in other years or when techniques discussed in future chapters are used to estimate mortality for circulatory diseases.

The MLEs for respiratory diseases are affected by the implementation of ONS Rule 3 discussed in Section 2.1. While the categorical $\kappa(t, j, g)$ term is able to correct for this issue for the older age groups, the estimates for the younger ages suffer as a result as the primary impact of ONS Rule 3 was on pneumonia deaths with dementia listed on the MCCD — a cause of death that would not occur at very young ages. This in turn affects the estimates for the deaths in group 6 – Other Causes – as that is where these excess dementia deaths are counted. The $\kappa(t, j, g)$ terms decreased over time for all causes of death except for Cause 1 – infectious and parasitic diseases, where $\kappa(t, 1, g)$ consistently increased in England and Wales. The rise in infectious deaths is the result of the increase in antibiotic resistance across the world leading to the spread of superbugs (CDC, 2013; Zaman et al., 2017). For example the number of deaths attributed to sepsis (ICD-10 A40-A41), which includes sepsis deaths caused by superbugs such as Staphylococcus aureus (ICD-10 A41.0), Clostridium difficile (ICD-10 A41.4), and Escherichia coli (ICD-10 A41.5), has increased by 30% between 2001 and 2005 in England and Wales.

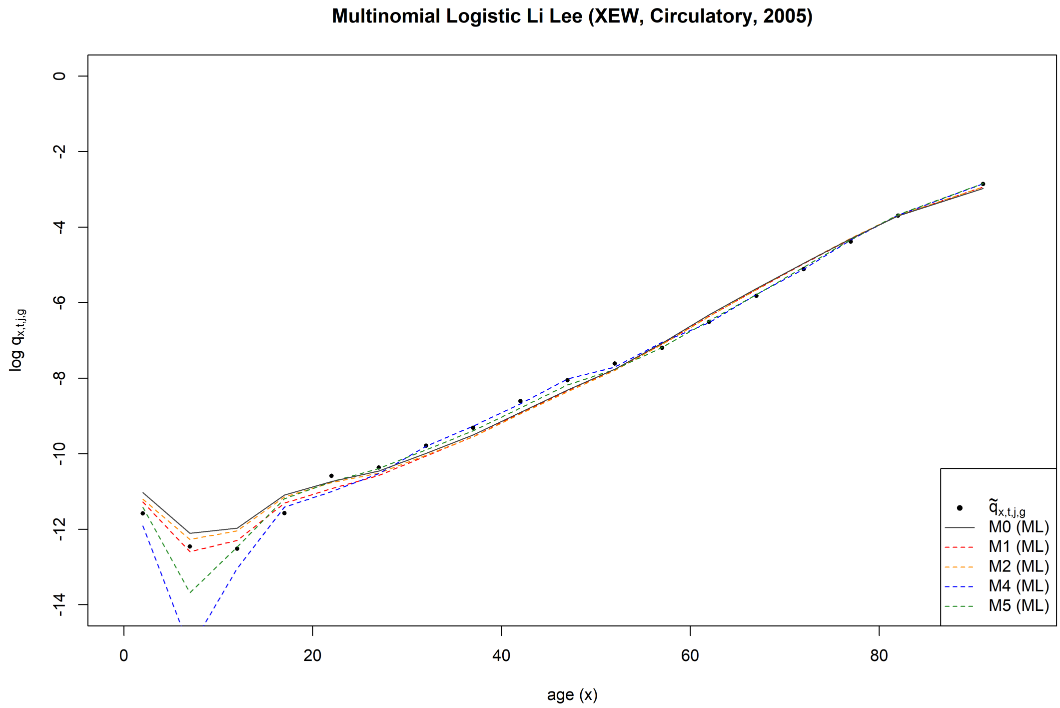


(a) Infectious and Parasitic Diseases

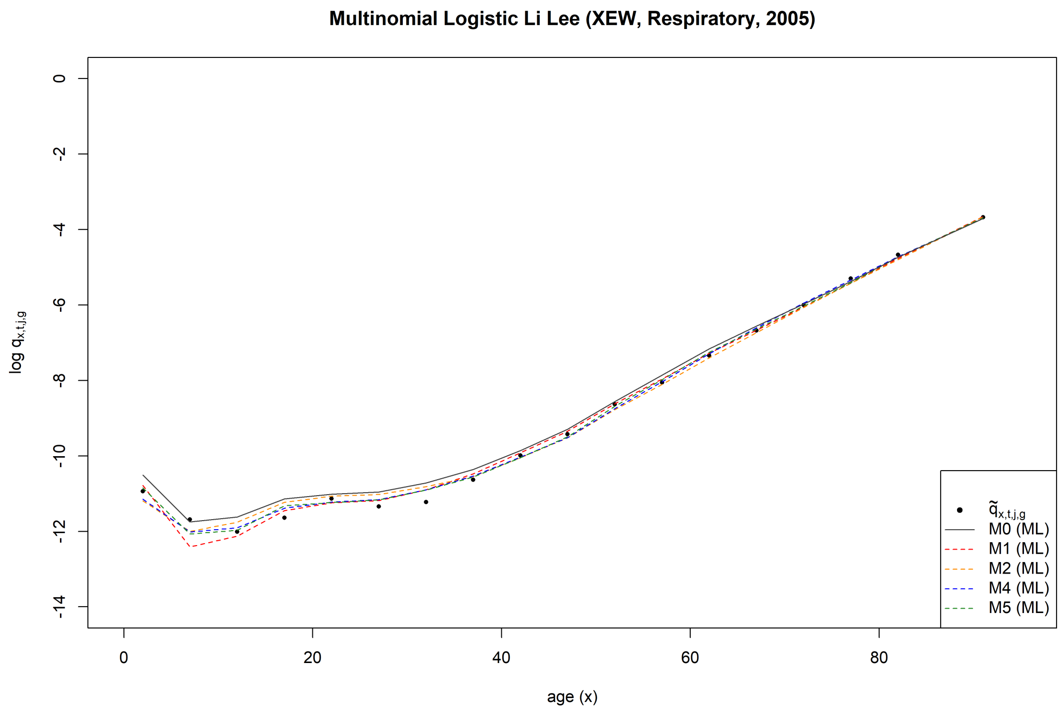


(b) Neoplasms

FIGURE 4.1: Probabilities of Death in Year 2005
(England and Wales)

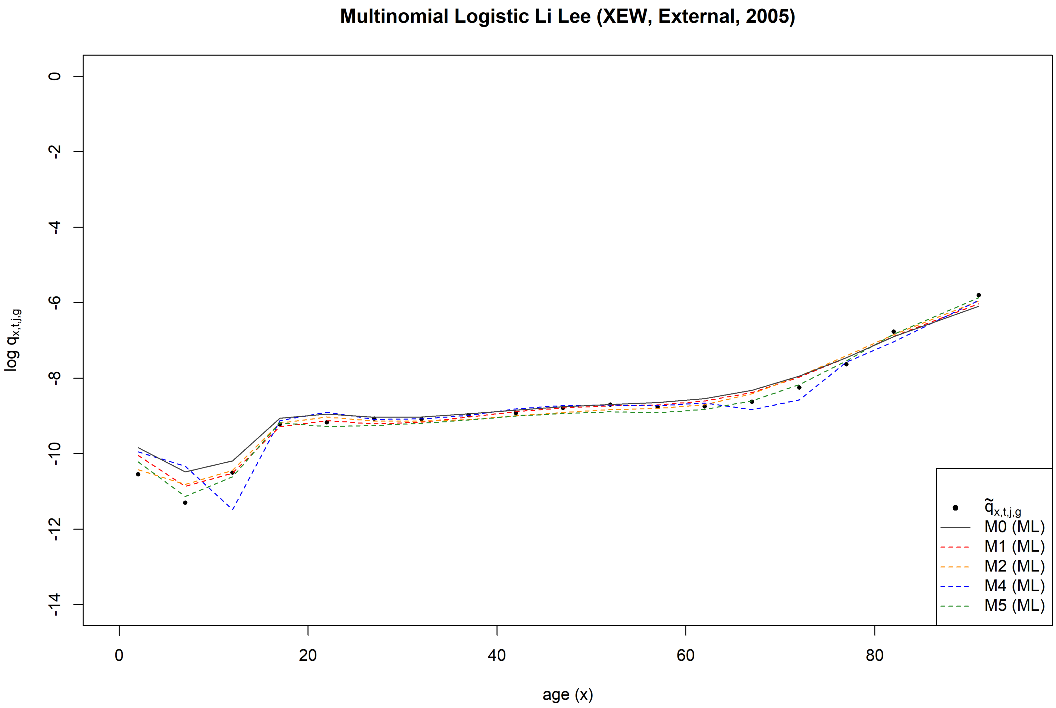


(c) Circulatory Diseases

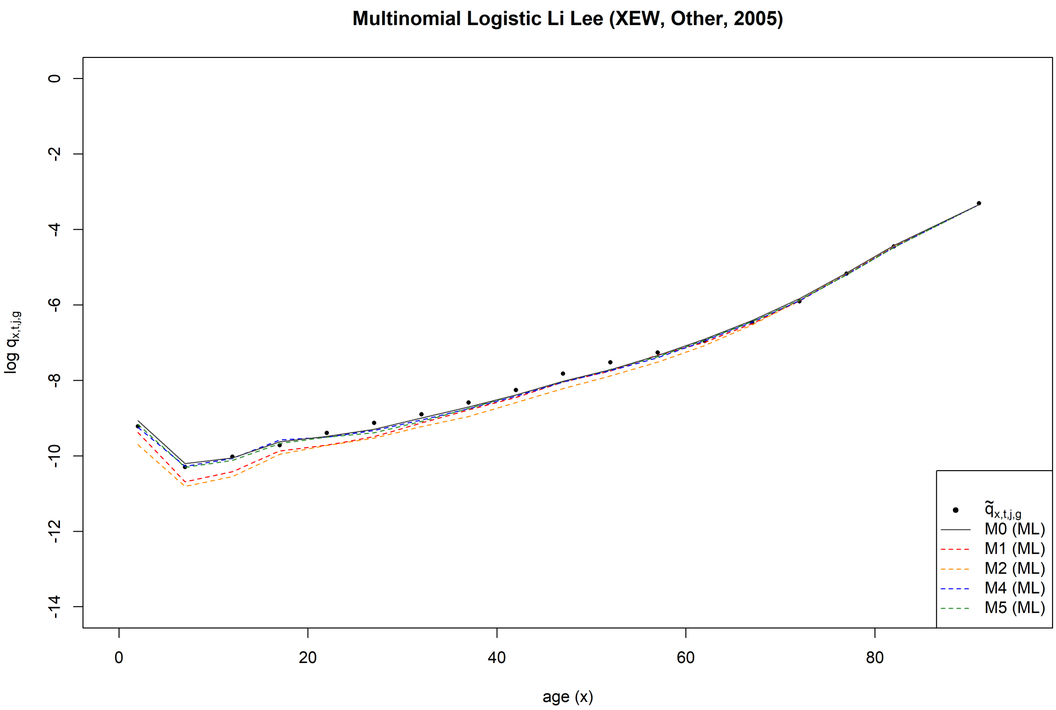


(d) Respiratory Diseases

FIGURE 4.1 (CONT.)



(e) External Causes



(f) Other Causes

FIGURE 4.1 (CONT.)

We evaluate the performance of the various model specifications using the following definitions of AIC and BIC from Burnham and Anderson (2002):

$$\text{AIC} = -2\hat{\ell} + 2k, \tag{4.13}$$

$$\text{BIC} = -2\hat{\ell} + k \log n \tag{4.14}$$

where $\hat{\ell} = \log \hat{\mathscr{L}}$ is the value of the log-likelihood for the model, k is the number of model parameters, and n is the sample size. The values of information criteria (IC) for the five model specifications are given in Table 4.3. We use the total number of cells for n rather than the total exposure as in Cairns et al. (2009) and Enchev, Kleinow, and Cairns (2017), i.e.

$$n = number\ of\ ages \times number\ of\ years \times number\ of\ causes \times number\ of\ countries.$$

The calculation also uses the number of effective – or *free* – parameters for k, i.e. those parameters that are not defined deterministically by a constraint. Out of the multi-country models, specification M5 has the lowest AIC and BIC values. (Do note that this model is only directly comparable with M4 and not other models due to the nesting structure.)

TABLE 4.3: INFORMATION CRITERIA FOR VARIOUS MODEL SPECIFICATIONS

Model	Countries	n	k	$k^{\text{effective}}$	$\hat{\ell}$	AIC	BIC
M0 §	England & Wales and France	12312	888	864	-66332.00	134392.00	140801.44
M1	England & Wales and France	12312	1056	1026	-62549.46	127150.92	134762.13
M2 §	England & Wales and France	12312	1000	972	-54815.58	111575.16	118785.78
M4	England & Wales and France	12312	1280	1242	-51251.18	104986.36	114199.93
M5	England & Wales and France	12312	1224	1188	-46870.83	96117.66	104930.64

The log-likelihood and information criteria for M0 are calculated jointly for both countries even though the models were fitted individually in order to provide comparability. The log-likelihoods for England & Wales and France are -35723.00 and -30609.00 respectively. We note that M2 could also be fitted individually although we coded the model to be estimated jointly using a single optimisation run.

§The values of n, k, $k^{\text{effective}}$ shown in this row are double of those for a single-country model.

The values of the coefficient and uncertainty estimates for M0 under Maximum Likelihood, Multivariate Normal Sampler, and Parametric Bootstrap for Cause 2 – Neoplasms – are presented in Tables 4.4 and 4.5. The estimates for the remainder of the cause groups are presented in Appendix C. The standard errors for the ML approach were calculated as the square root of the diagonal elements of the negative inverse of the Hessian matrix estimated using the `optim()` function (R Core Team, 2022; Pan and Pan, 2022). The standard errors for the Multivariate Normal Sampler and Parametric Bootstrap were calculated using the steps described in Section 4.3. We can see that the coefficient estimates are similar regardless of the approach used. This is not the case for the standard errors, however. While the standard errors for Parametric Bootstrap estimates of $\alpha(x, j, g)$ are generally similar to the ML standard errors, they are wider for $\beta(x, j, g)$ and $\kappa(t, j, g)$. The increase in standard errors can be explained by the increased variability introduced by the Bootstrap procedure and is particular prominent for Cause 1 – infectious and parasitic diseases – where the ratio of Parametric Bootstrap standard errors to the ML standard errors is as high as 514.4 for $\beta(85+, 1, XEW)$ and 33.2 for $\kappa(2005, 1, XEW)$. The significant increase can be attributed to the increase in variability as a result of the small number of deaths that are assigned annually to Cause 1. Meanwhile the ratios of Multivariate Normal Sampler standard errors to those from ML are very similar; in fact all standard errors lie on the interval $(0.98; 1.02)$. This finding offers a possible approach to approximate standard errors for more complex MLG-LL models, although we have not explored this hypothesis further.

TABLE 4.4: PARAMETER ESTIMATES AND STANDARD ERRORS FOR CAUSE 2 — NEOPLASMS, ENGLAND & WALES, FEMALES

	Cause 2 — Neoplasms					
	Maximum Likelihood		Multivariate Normal Sampler		Parametric Bootstrap	
	Estimate	s.e.	Median	s.e.	Median	s.e.
$\alpha(x,2)$						
1 –4	-9.9772	0.020049 ***	-9.9772	0.019997 ***	-9.9778	0.021664
5 –9	-10.1376	0.019368 ***	-10.1374	0.019439 ***	-10.1370	0.019613
10 –14	-10.2165	0.020233 ***	-10.2164	0.020303 ***	-10.2191	0.021500
15 –19	-9.9917	0.018135 ***	-9.9921	0.018057 ***	-9.9925	0.017843
20 –24	-9.7375	0.015657 ***	-9.7372	0.015677 ***	-9.7378	0.016035
25 –29	-9.2113	0.012030 ***	-9.2113	0.012058 ***	-9.2110	0.011660
30 –34	-8.5008	0.008558 ***	-8.5009	0.008552 ***	-8.5002	0.008033
35 –39	-7.8431	0.006297 ***	-7.8432	0.006329 ***	-7.8437	0.005888
40 –44	-7.2177	0.004723 ***	-7.2176	0.004802 ***	-7.2175	0.004326
45 –49	-6.6385	0.003582 ***	-6.6385	0.003620 ***	-6.6390	0.003680
50 –54	-6.1506	0.002859 ***	-6.1506	0.002872 ***	-6.1509	0.002754
55 –59	-5.7501	0.002385 ***	-5.7500	0.002377 ***	-5.7505	0.002399
60 –64	-5.3951	0.002055 ***	-5.3951	0.002061 ***	-5.3955	0.002073
65 –69	-5.0860	0.001821 ***	-5.0860	0.001819 ***	-5.0863	0.001843
70 –74	-4.8057	0.001678 ***	-4.8057	0.001684 ***	-4.8062	0.001691
75 –79	-4.5574	0.001646 ***	-4.5574	0.001657 ***	-4.5579	0.001812
80 –84	-4.3312	0.001764 ***	-4.3311	0.001770 ***	-4.3321	0.001931
85+	-4.1054	0.001758 ***	-4.1055	0.001770 ***	-4.1069	0.001988
$\beta(x,2)$						
1 –4	0.1648	0.008414 ***	0.1648	0.008370 ***	0.1649	0.009408
5 –9	0.1283	0.008070 ***	0.1282	0.007989 ***	0.1288	0.009260
10 –14	0.1002	0.008312 ***	0.0999	0.008414 ***	0.1004	0.009492
15 –19	0.0736	0.007685 ***	0.0736	0.007693 ***	0.0732	0.008759
20 –24	0.0799	0.006889 ***	0.0799	0.006891 ***	0.0798	0.006935
25 –29	0.0790	0.005424 ***	0.0790	0.005399 ***	0.0788	0.005582
30 –34	0.0691	0.003811 ***	0.0692	0.003775 ***	0.0689	0.004071
35 –39	0.0767	0.002798 ***	0.0767	0.002770 ***	0.0767	0.003284
40 –44	0.0813	0.002111 ***	0.0813	0.002116 ***	0.0813	0.002467
45 –49	0.0823	0.001605 ***	0.0823	0.001606 ***	0.0824	0.002519
50 –54	0.0651	0.001259 ***	0.0651	0.001259 ***	0.0652	0.001852
55 –59	0.0487	0.001042 ***	0.0487	0.001031 ***	0.0487	0.001536
60 –64	0.0252	0.000915 ***	0.0251	0.000912 ***	0.0252	0.001309
65 –69	0.0048	0.000810 ***	0.0048	0.000812 ***	0.0049	0.001077
70 –74	-0.0152	0.000738 ***	-0.0152	0.000741 ***	-0.0152	0.000966
75 –79	-0.0220	0.000714 ***	-0.0220	0.000719 ***	-0.0221	0.000956
80 –84	-0.0191	0.000752 ***	-0.0191	0.000753 ***	-0.0192	0.000963
85+	-0.0227	0.000728 ***	-0.0227	0.000727 ***	-0.0233	0.001347

TABLE 4.5: PARAMETER ESTIMATES AND STANDARD ERRORS FOR CAUSE 2 — NEOPLASMS, ENGLAND & WALES, FEMALES (CONTINUED)

	CAUSE 2 — NEOPLASMS					
	Maximum Likelihood		Multivariate Normal Sampler		Parametric Bootstrap	
	Estimate	s.e.	Median	s.e.	Median	s.e.
$\kappa(t,2)$						
1968	2.4401	0.107936 ***	2.4408	0.106719 ***	2.4495	0.135212 ***
1969	2.6130	0.107496 ***	2.6117	0.108261 ***	2.6134	0.123096 ***
1970	2.4249	0.108270 ***	2.4256	0.107719 ***	2.4208	0.126002 ***
1971	2.5950	0.107946 ***	2.5944	0.107738 ***	2.5873	0.129899 ***
1972	2.5756	0.108111 ***	2.5746	0.107235 ***	2.5681	0.120784 ***
1973	2.4781	0.108559 ***	2.4790	0.108984 ***	2.4798	0.123386 ***
1974	2.3824	0.109066 ***	2.3820	0.109062 ***	2.3747	0.117119 ***
1975	2.3160	0.109649 ***	2.3137	0.108874 ***	2.3234	0.117456 ***
1976	2.2496	0.110121 ***	2.2505	0.110749 ***	2.2455	0.122971 ***
1977	2.1444	0.110563 ***	2.1427	0.108997 ***	2.1532	0.122692 ***
1978	2.1727	0.110439 ***	2.1720	0.110277 ***	2.1734	0.114846 ***
1979	1.9286	0.111145 ***	1.9278	0.111687 ***	1.9324	0.112833 ***
1980	1.8987	0.111525 ***	1.8971	0.112342 ***	1.8978	0.112138 ***
1981	1.7274	0.111537 ***	1.7254	0.110688 ***	1.7275	0.114337 ***
1982	1.5831	0.112234 ***	1.5838	0.111815 ***	1.5710	0.114975 ***
1983	1.4262	0.112410 ***	1.4261	0.114052 ***	1.4289	0.120519 ***
1984	1.0100	0.113300 ***	1.0109	0.114211 ***	1.0017	0.124669 ***
1985	0.6530	0.114213 ***	0.6539	0.114648 ***	0.6525	0.129951 ***
1986	0.6255	0.114175 ***	0.6235	0.115315 ***	0.6214	0.121086 ***
1987	0.3412	0.114536 **	0.3438	0.114519 **	0.3296	0.124876 **
1988	0.2475	0.114331 *	0.2474	0.114916 *	0.2382	0.126429
1989	-0.3156	0.115450 **	-0.3146	0.115405 **	-0.3233	0.126595 *
1990	-0.4565	0.115324 ***	-0.4553	0.113784 ***	-0.4584	0.129256 ***
1991	-0.7362	0.115530 ***	-0.7346	0.115994 ***	-0.7443	0.120208 ***
1992	-0.9894	0.115470 ***	-0.9883	0.114308 ***	-0.9874	0.114534 ***
1993	-0.8372	0.114369 ***	-0.8386	0.113859 ***	-0.8439	0.122193 ***
1994	-1.3187	0.115186 ***	-1.3162	0.115166 ***	-1.3120	0.114763 ***
1995	-1.3792	0.114655 ***	-1.3797	0.114308 ***	-1.3617	0.118483 ***
1996	-1.7517	0.115078 ***	-1.7532	0.114473 ***	-1.7385	0.121256 ***
1997	-2.0199	0.115227 ***	-2.0201	0.115538 ***	-2.0226	0.117155 ***
1998	-2.3692	0.115556 ***	-2.3675	0.115867 ***	-2.3675	0.125509 ***
1999	-2.6461	0.115728 ***	-2.6441	0.115382 ***	-2.6409	0.125020 ***
2000	-2.5610	0.114912 ***	-2.5602	0.113864 ***	-2.5539	0.123437 ***
2001	-3.6302	0.117696 ***	-3.6309	0.117520 ***	-3.6277	0.134032 ***
2002	-4.0158	0.118166 ***	-4.0168	0.116337 ***	-4.0075	0.138088 ***
2003	-4.2750	0.118558 ***	-4.2733	0.118582 ***	-4.2615	0.147511 ***
2004	-4.1715	0.117922 ***	-4.1714	0.117673 ***	-4.1666	0.151629 ***
2005	-4.3597	0.117874 ***	-4.3599	0.116654 ***	-4.3577	0.160444 ***

Chapter 5

Time Series

Both the Lee-Carter (LC) and Li-Lee (LL) specifications contain a period-specific term or terms that can easily be modelled using a time series. The LC specification presented in (3.6) contains a single period-specific term $\kappa(t)$ while the LL model from (3.7) contains not only a country-specific period term $\kappa(t, g)$ but also a global period effect $K(t)$. The fact that these terms are categorical and therefore provide a value for discrete calendar years makes the modelling process straightforward.

This chapter serves as an overview of the time series methods considered in this thesis and will underpin the prior specifications for the MLG-LC model presented in Chapter 6 as well as the MLG-LC projections methodology presented in Chapter 7. In Section 5.1, we begin with an overview of classical time series models. Due to the nature of the period terms in our models, only non-seasonal time series are appropriate and hence seasonal time series methods will not be considered here. Section 5.2 looks at methods to assess whether a time series is stationary and offers an extension to the models from the previous section by introducing the concept of order of integration. We perform stationarity tests and ARIMA model selection for the MLG-LC and MLG-LL models in Sections 5.3.3 & 5.4.4, respectively. The resulting selections yield a different specification for various causes as well as variation in the model specification between countries. As stated previously, the annual mortality data used in these models relates to the years between 1968 and 2005. The data for years between 2006 and 2014 was not included and will instead form part of the holdout sample presented in Chapter 7.

5.1 ARMA Model Specifications

We begin with an overview of a number of time series models that we have considered during this project. One of the simplest specifications is an autoregressive model (AR). For a single time lag, the AR(1) model is defined as

$$Y_t = \phi Y_{t-1} + \varepsilon_t \tag{5.1}$$

where ϕ is the lag coefficient, and ε_t is the random noise or error term at time t. Increasing the number of lags to p, the AR(p) model is then defined as

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + \varepsilon_t \tag{5.2}$$

where ϕ_p is the lag coefficient at time p, and ε_t is the random noise or error term at time t.

An alternative to the AR(p) process is the moving average (MA) model. Going back a single time lag, the MA(1) process is given

$$Y_t = \mu + \varepsilon_t + \theta \varepsilon_{t-1} \tag{5.3}$$

where μ is a constant term, ε_t is the random noise or error term at time t, ε_{t-1} is the error term at time $t-1$, and θ is the moving average coefficient. Extending this to more than one time lag, the MA(q) model is given by

$$\begin{aligned} Y_t &= \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} && (5.4) \\ &= \mu + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} && (5.5) \end{aligned}$$

where μ is again the constant term, ε_t is the random noise or error term at time t, ε_{t-i} is the error term at time $t-i$ where $i = 1, 2, \ldots, q$, and θ_i are the corresponding moving average coefficients.

Putting the AR(p) and MA(q) models together yields an autoregressive moving average – ARMA(p,q) – process

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} \tag{5.6}$$

$$= \varepsilon_t + \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}. \tag{5.7}$$

5.2 Order of Integration and ARIMA

Time series that are not stationary can be made stationary by differencing, i.e. by taking the difference between the value at time t and $t-1$ to produce a new time series

$$\Delta Y_t = Y_t - Y_{t-1} \tag{5.8}$$

where Δ is the difference operator. Sometimes this process needs to be repeated a second or even more times to achieve stationarity. A time series thas has been differenced d many times before becoming stationary is said to be integrated of order d, noted as $\mathrm{I}(d)$. For example, a stationary time series Y_t that was not differenced at all will be $\mathrm{I}(d=0)$, whereas a time series $\Delta^2 Y_t$ that was differenced twice before becoming $\mathrm{I}(d=0)$ will be denoted as $I(d=2)$. An $\mathrm{ARMA}(p,q)$ time series that is $\mathrm{I}(0)$ after being differenced d many times is called an autoregressive integrated moving average, $\mathrm{ARIMA}(p,d,q)$, model where p is the number of autoregressive terms, q is the number of moving average terms, and d is the order of differencing (Davidson and MacKinnon, 2004).

5.2.1 Stationarity of Time Series

One of the most commonly used tests for stationarity is the augmented Dickey–Fuller (ADF) test proposed by Said and Dickey (1984). This is an extension of the Dickey–Fuller test described by Dickey and Fuller (1979; 1981) for an $\mathrm{ARMA}(p,q)$ model with unknown parameters (Davidson and MacKinnon, 2004). The ADF test determines whether there is sufficient evidence to reject the null hypothesis of non-stationarity by testing the null hypothesis that $\gamma = 0$ in the model

$$\Delta y_t = \mu + \nu t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \cdots + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon_t \tag{5.9}$$

where μ is a constant term representing the drift, ν is the coefficient of the trend, and ε_t is the error term at time t (Harris, 1992).

An alternative test was proposed by Phillips and Perron (1988); however, this test underperforms in finite samples and will therefore not be discussed further (Davidson and MacKinnon, 2004; Perron and Ng, 1996; Schwert, 1989).

Given the relatively small number of years, we select the upper bound for the number of lags by following the approach described in Schwert (1989). The maximum number of lags is therefore determined using the formula

$$\max lags = \left\lfloor 12 \times \left(\frac{T}{100}\right)^{1/4} \right\rfloor \tag{5.10}$$

where $\lfloor \cdot \rfloor$ denotes the floor function and T is the total number of time periods. Do note that differencing of the time series results in loss of time periods. The actual number of lags used in the ADF test can then be selected algorithmically using an information criterion such as AIC or BIC.

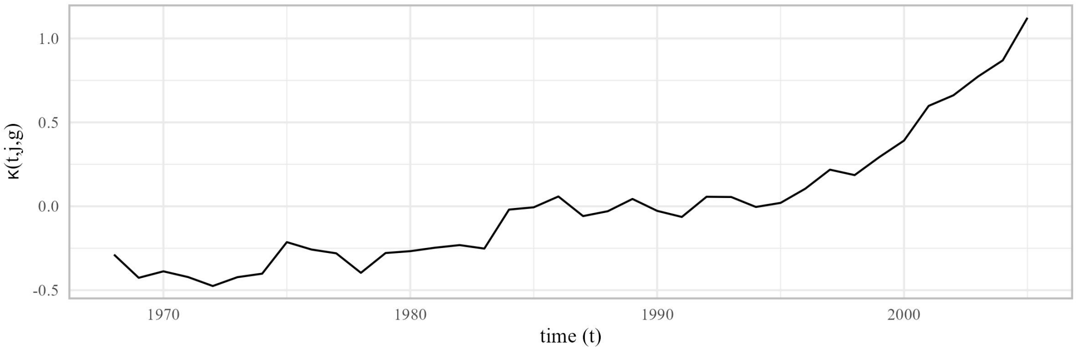
There are many `R` packages that are capable of testing for stationarity. We implemented the ADF test from (5.9) with the maximum number of lags as per (5.10) using the `CADFtest()` function from the `CADFtest` package by Costantini, Lupi, and Popp (2007) and Lupi (2009) as this package was more user-friendly than others. We selected the optimal number of lags based on the values of the BIC (Chakrabarti and Ghosh, 2011).

5.3 Estimation of Period Effects Using the Multinomial Logistic Lee-Carter Model

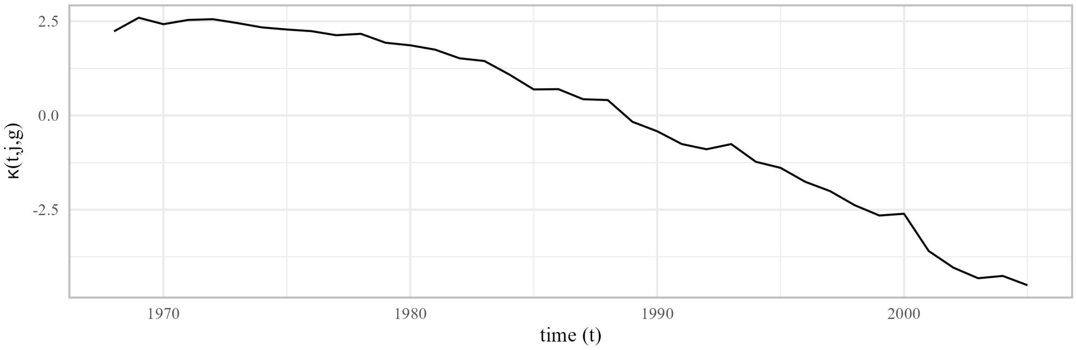
We proceed to apply the procedure from Section 5.2.1 to the Multinomial Logistic Lee-Carter model (M0). In this model, period effects are captured solely by the country-specific $\kappa(t, j, g)$ term for each cause of death and country. We will consider the period effects for England & Wales in Section 5.3.1 and for France in Section 5.3.2.

5.3.1 Stationarity of $\kappa(t, j, g)$ for England & Wales

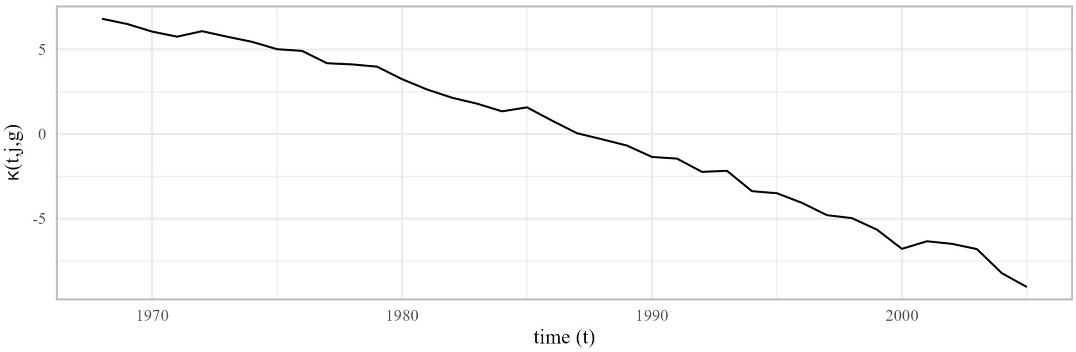
The plots of $\kappa(t, j, g)$ for all six causes in England & Wales are presented in Figure 5.1:



(a) Infectious and Parasitic Diseases

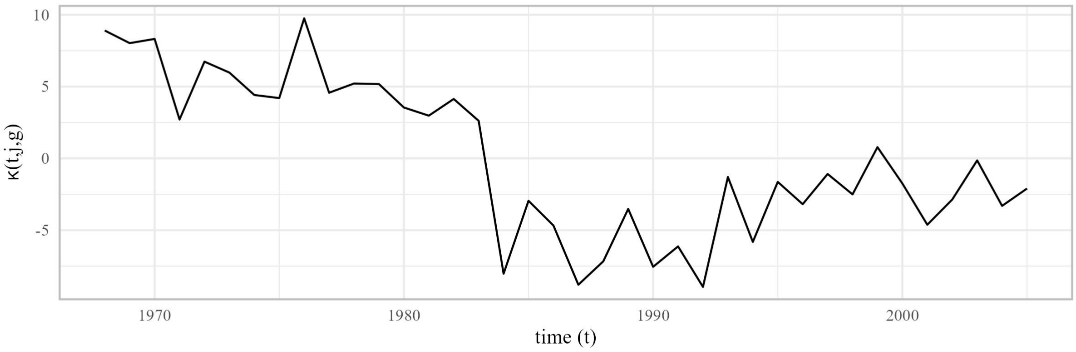


(b) Neoplasms

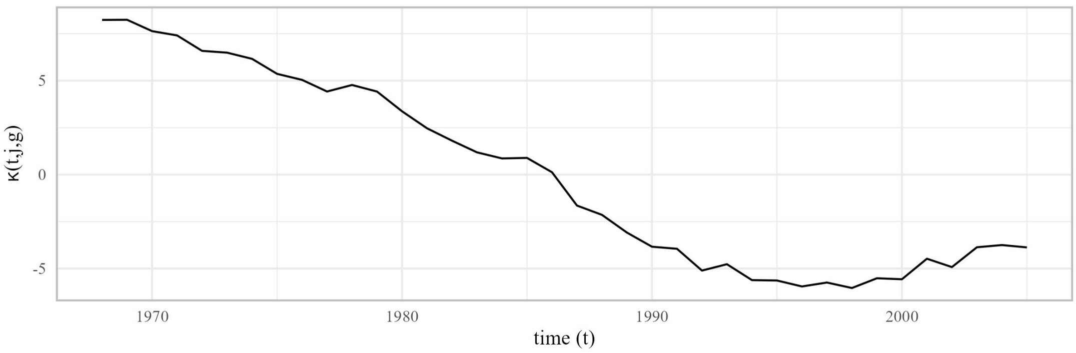


(c) Circulatory Diseases

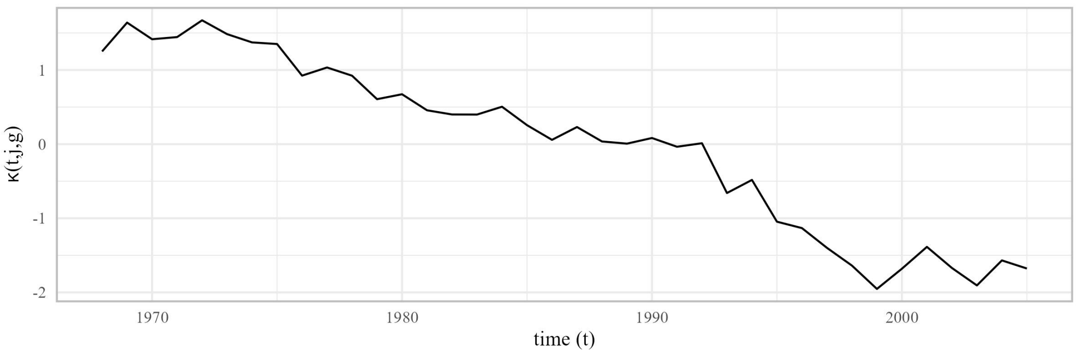
FIGURE 5.1: Time series plots of $\kappa(t, j, g)$
(England & Wales, Females)



(d) Respiratory Diseases



(e) External Causes



(f) Other Causes

FIGURE 5.1 (CONT.)

The plots of $\kappa(t, j, g)$ for all six causes shown in Figure 5.1 suggest that the non-differenced time series are not stationary. We test this formally by running an ADF test as described in Section 5.2.1 and present the values of the ADF test statistic τ and the corresponding p-values in Table 5.1:

TABLE 5.1: ADF TEST FOR STATIONARITY — NO DIFFERENCING, MLG-LC, ENGLAND & WALES, FEMALES

Cause of Death	Lags	τ	p-value
Infectious	0	2.21109	0.99990
Neoplasm	0	0.93160	0.99446
Circulatory	0	0.71022	0.99021
Respiratory	1	-2.16831	0.22148
External	0	-2.26504	0.18958
Other	0	-0.91447	0.76848

We can see that the test statistic τ is not significant for any of the causes of death. We therefore fail to reject the null hypothesis of non-stationarity for all six causes and proceed to difference each time series once before performing the ADF tests again. The plots of $\Delta\kappa(t, j, g)$ can be found in Figure D.1 in the appendix. We present the results of the stationarity tests for $\Delta\kappa(t, j, g)$ in Table 5.2:

TABLE 5.2: ADF TEST FOR STATIONARITY — SINGLE ORDER OF DIFFERENCING, MLG-LC, ENGLAND & WALES, FEMALES

Cause of Death	Lags	τ	p-value
Infectious	0	-4.16678	0.00010
Neoplasm	1	-1.82756	0.06490
Circulatory	1	-1.26779	0.18448
Respiratory	0	-9.69332	0.00000
External	1	-1.91953	0.05351
Other	0	-6.56152	0.00000

As the time series are relatively short, the small sample size is going to affect the statistical power of the ADF test as has been shown by Cheung and Lai (1995). We therefore opt to test the null hypothesis at the 10% level to increase the power of the test. This follows the approach used by Li and Hardy (2011). The test statistic τ in Table 5.2 is significant at the 10% level for all causes of death except for circulatory diseases.

As we failed to reject the null hypothesis of non-stationarity for circulatory diseases after one order of differencing, we difference it again and formally test for stationarity once more using the ADF test. (See Figure D.2.c in the appendix for the plot of $\Delta^2\kappa(t, j = 3, g = 1)$.) The results for the respective $\Delta^2\kappa(t, j, g)$ are presented in Table 5.3.

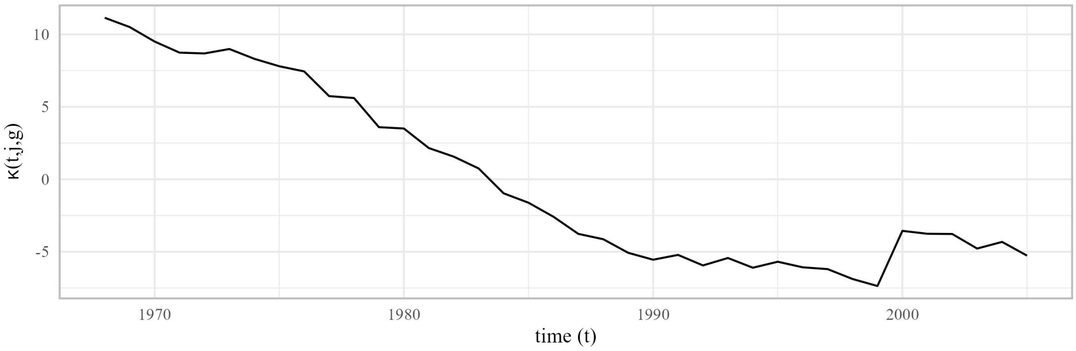
TABLE 5.3: ADF TEST FOR STATIONARITY — SECOND ORDER OF DIFFERENCING, MLG-LC, ENGLAND & WALES, FEMALES

Cause of Death	Lags	τ	p-value
Circulatory	2	-5.86381	0.00000

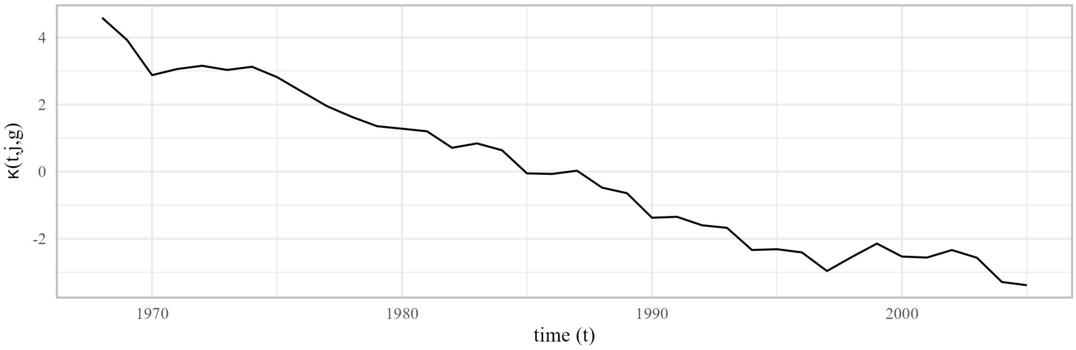
We can see that the test statistic τ is now significant for circulatory diseases. We can therefore reject the null hypothesis of non-stationarity after second order of differencing. In the next section, we will test for stationarity of the time series for France before we estimate the ARIMA parameters p and q in Section 5.3.3.

5.3.2 Stationarity of $\kappa(t, j, g)$ for France

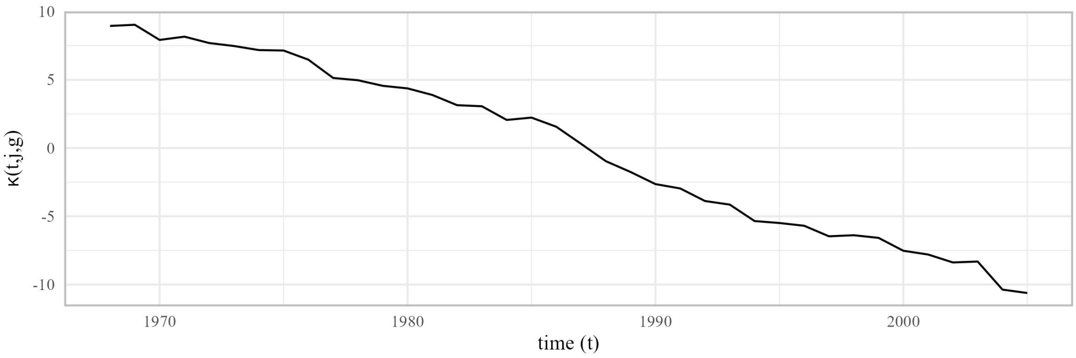
The plots of $\kappa(t, j, g)$ for all six causes in France are presented in Figure 5.2:



(a) Infectious and Parasitic Diseases

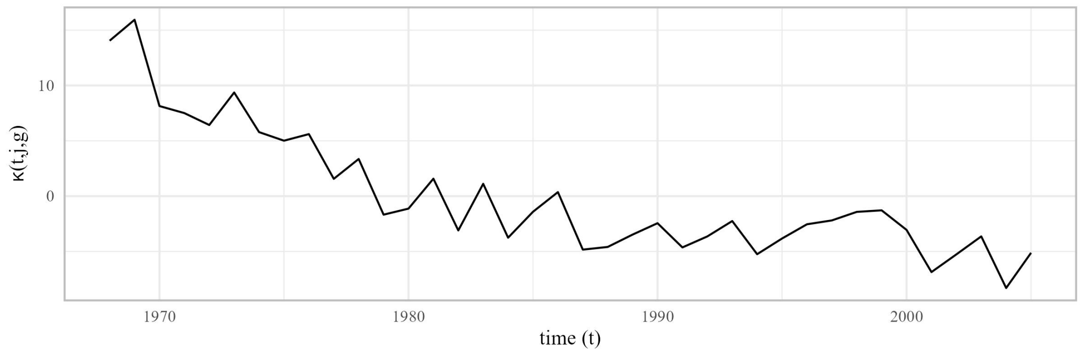


(b) Neoplasms

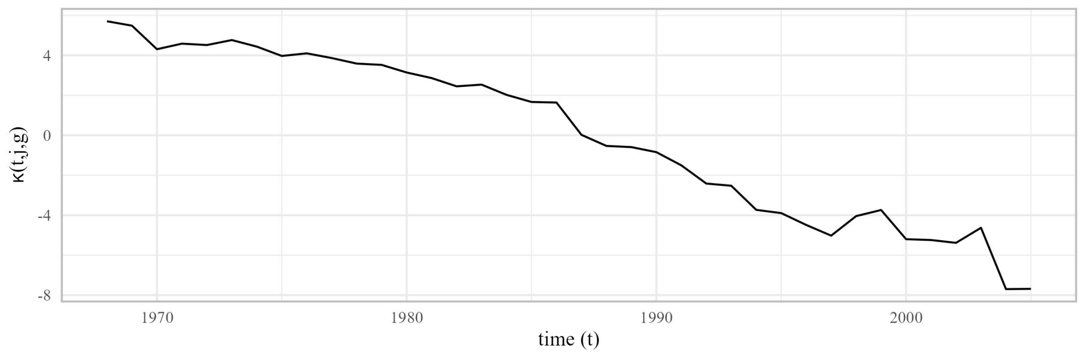


(c) Circulatory Diseases

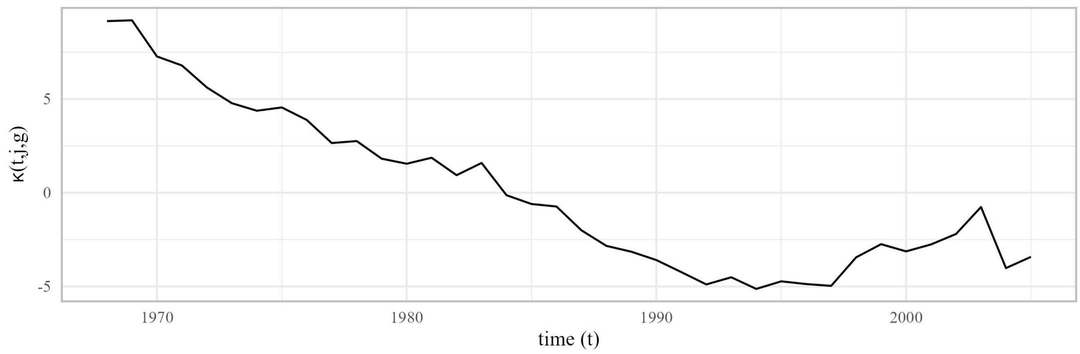
FIGURE 5.2: Time series plots of $\kappa(t, j, g)$
(France, Females)



(d) Respiratory Diseases



(e) External Causes



(f) Other Causes

FIGURE 5.2 (CONT.)

Similarly to England & Wales, the plots of $\kappa(t, j, g)$ in Figure 5.2 show a general downward drift, which suggests that the non-differenced time series for France are not stationary. As the plots of $\Delta\kappa(t, j, g)$ (see Figure D.3 in the appendix) suggest a non-zero mean, we formally test for stationarity of $\kappa(t, j, g)$ by running an ADF test with drift (i.e. $\mu \neq 0$) and present the values of the test statistic τ and the corresponding p-values in Table 5.4.

TABLE 5.4: ADF TEST FOR STATIONARITY — NO DIFFERENCING, MLG-LC, FRANCE, FEMALES

Cause of Death	Lags	τ	p-value
Infectious	0	-2.57849	0.10918
Neoplasm	0	-1.09019	0.70533
Circulatory	0	0.14019	0.96323
Respiratory	2	-2.22525	0.20229
External	0	-0.16175	0.93258
Other	0	-1.76691	0.38832

None of the test statistics are significant and we therefore fail to reject the null hypothesis of non-stationarity for all causes in France. We now difference each time series once and repeat the ADF test. The results of the ADF tests are presented in Table 5.5.

TABLE 5.5: ADF TEST FOR STATIONARITY — SINGLE ORDER OF DIFFERENCING, MLG-LC, FRANCE, FEMALES

Cause of Death	Lags	τ	p-value
Infectious	1	-2.43014	0.01676
Neoplasm	2	-1.40355	0.14625
Circulatory	1	-1.17229	0.21494
Respiratory	0	-9.85298	0.00000
External	0	-5.58674	0.00000
Other	0	-6.40620	0.00000

We find that the test statistic τ is significant at the 10% level for four out of the six causes. The causes for which we fail to reject the null hypothesis of non-stationarity are neoplasms and circulatory diseases. For these causes we therefore need to difference the time series a second time and perform the ADF test once more. (See Figure D.4 in the appendix for the plots of $\Delta^2\kappa(t, j, g)$.) We present the results of the ADF test for neoplasms and circulatory diseases in Table 5.6.

TABLE 5.6: ADF TEST FOR STATIONARITY — SECOND ORDER OF DIFFERENCING, MLG-LC, FRANCE, FEMALES

Cause of Death	Lags	τ	p-value
Neoplasm	1	-9.11695	0.00000
Circulatory	0	-11.94280	0.00000

We can see that the second order of differencing has resulted in the time series for neoplasms and circulatory diseases being stationary. Now that we have identified the necessary order of differencing for each cause, we will proceed with the estimation of the ARIMA parameters p and q in the next section.

5.3.3 ARIMA Model Selection for MLG-LC Period Effects

We proceed to fit the non-seasonal specification of the ARIMA model for each cause of death group using the R function `auto.arima()` from the `forecast` package that is based on the algorithm described by Hyndman and Khandakar (2008). This algorithm selects a model that minimises the value of an information criterion for a specific order of differencing. For consistency, we use BIC throughout to inform the model selection, where the order of differencing needed for each time series was determined in Sections 5.3.1 & 5.3.2. The ARIMA(p, d, q) specifications of the time series for England & Wales are listed in Table 5.7.

TABLE 5.7: ARIMA MODEL SPECIFICATIONS FOR $\kappa(t, j, g)$ SELECTED USING AUTO.ARIMA() FUNCTION IN R — MLG-LC, ENGLAND & WALES, FEMALES

		ARIMA			
Cause of Death	p	d	q	ℓ	BIC
Infectious	0	1	0	35.46	-63.69
Neoplasm	0	1	0	1.45	4.33
Circulatory	0	2	1	-19.95	47.08
Respiratory	0	1	1	-94.31	195.84
External	2	1	0	-28.71	68.26
Other	1	1	0	2.64	5.56

We note that two causes have only autoregressive terms, two causes have only moving average terms, and two causes have neither autoregressive nor moving average terms.

The ARIMA(p, d, q) model selections of the differenced time series for France are listed in Table 5.8.

TABLE 5.8: ARIMA MODEL SPECIFICATIONS FOR $\kappa(t, j, g)$ SELECTED USING AUTO.ARIMA() FUNCTION IN R — MLG-LC, FRANCE, FEMALES

	ARIMA				
Cause of Death	p	d	q	ℓ	BIC
Infectious	0	1	0	-49.48	106.18
Neoplasm	2	2	1	-12.33	38.99
Circulatory	1	2	1	-27.29	65.33
Respiratory	0	1	1	-85.66	182.15
External	0	1	1	-36.58	83.99
Other	0	1	0	-48.68	104.59

The ARIMA specifications for France are different than those for England & Wales for four out of the six causes of death. The time series $\kappa(t, j, g)$ for respiratory diseases for both countries follow ARIMA(0,1,1) despite the time series for England & Wales exhibiting shocks due to ICD and coding changes. The ARIMA specifiations for the 'other causes of death' group are different, however. Interestingly, the selected specification for neoplasms in France is ARIMA(2,2,1) whereas the specification for England & Wales is ARIMA(0,1,0), despite visual similarities between the two $\kappa(t, j, g)$ time series.

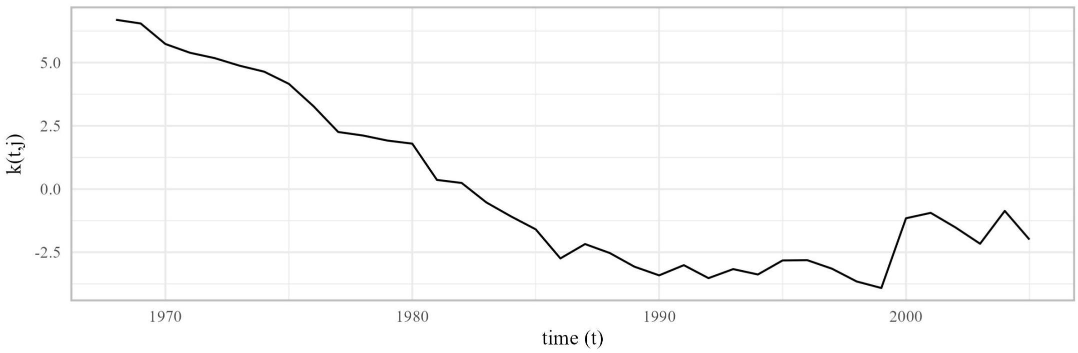
The ARIMA model specifications presented in this section will underpin the prior specifications for the period effects in the next chapter, specifically in Section 6.2.1. In Section 5.4 we look at the specifications of the time series for the period effects from the MLG-LL model.

5.4 Estimation of Period Effects using the Multinomial Logistic Li-Lee Model

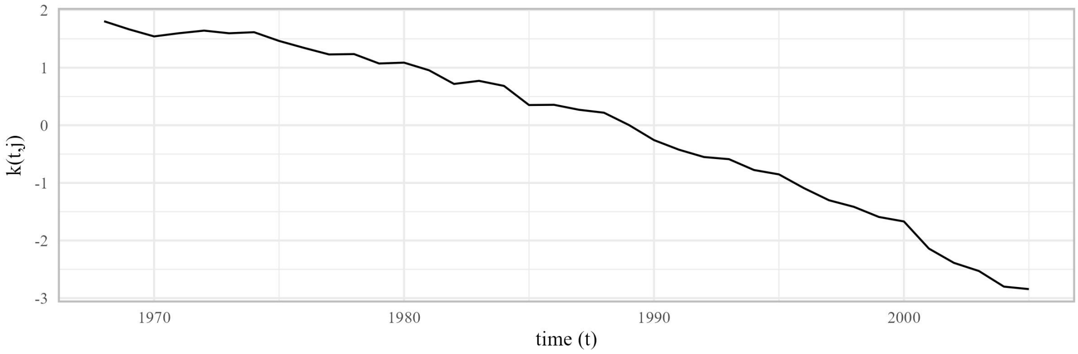
We now repeat the procedure described in Section 5.2.1 for the Multinomial Logistic Li-Lee model (M5). In this model we do not just have the country-specific $\kappa(t, j, g)$ terms but also common cause-specific $k(t, j)$ terms that we will need to assess for stationarity to determine the appropriate order of differencing.

5.4.1 Stationarity of Common Cause-Specific Parameters $k(t, j)$

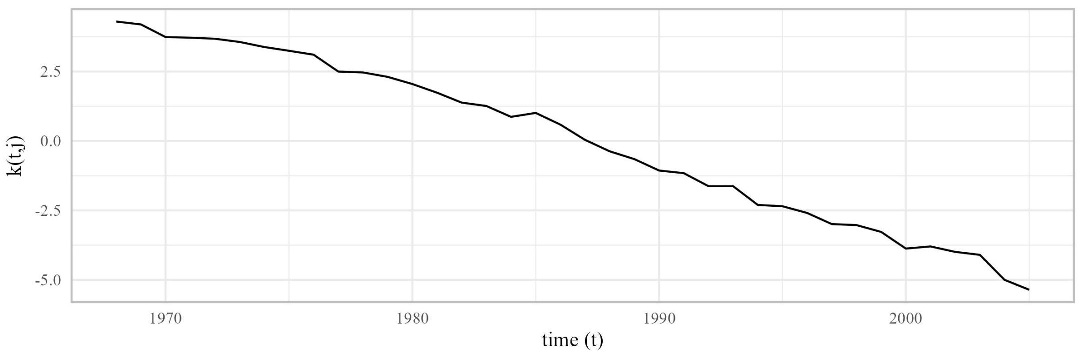
The plots of $k(t, j)$ for all six causes presented in Figure 5.3 suggest that the non-differenced time series are not stationary.



(a) Infectious and Parasitic Diseases

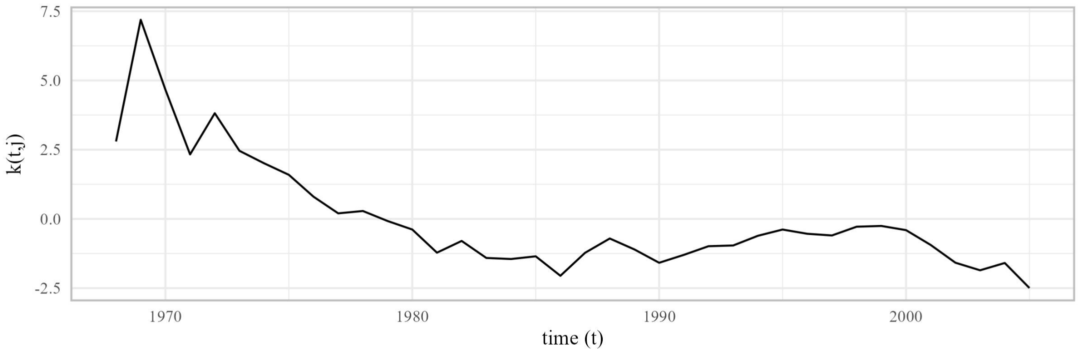


(b) Neoplasms

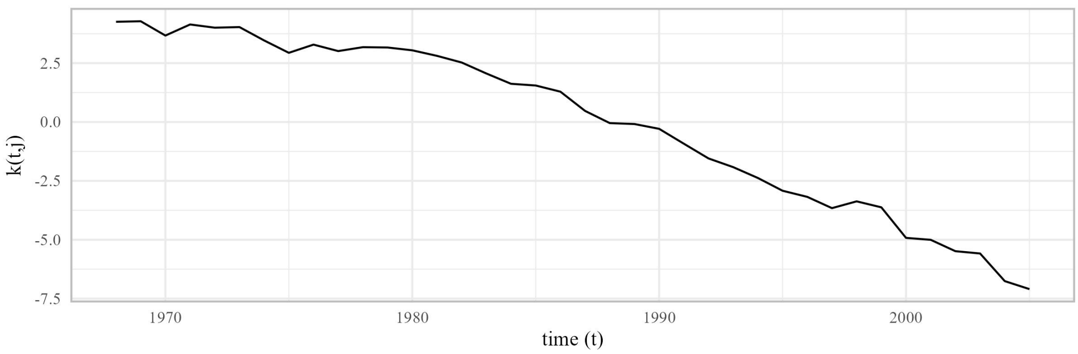


(c) Circulatory Diseases

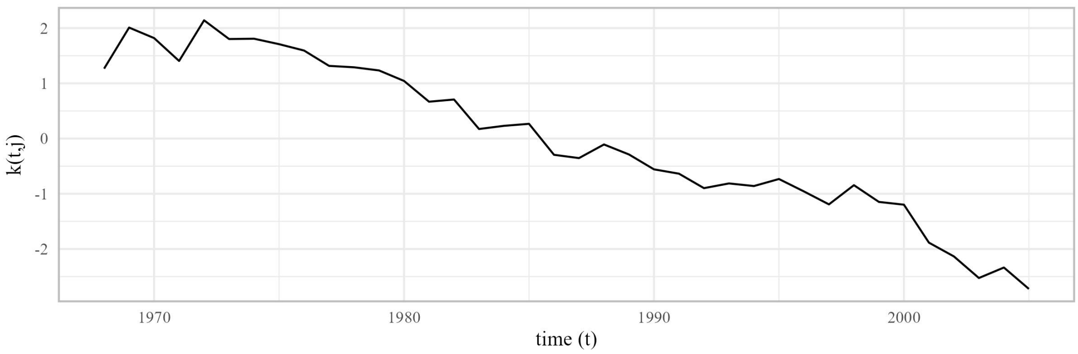
FIGURE 5.3: Time series plots of $k(t, j)$
(MLG-LL, Females)



(d) Respiratory Diseases



(e) External Causes



(f) Other Causes

FIGURE 5.3 (CONT.)

As the plots of $\Delta k(t,j)$ (see Figure D.5 in the appendix) suggest a non-zero mean, we formally test for stationarity of $k(t,j)$ by running an ADF test with drift and present the values of the test statistic τ and the corresponding p-values in Table 5.9:

TABLE 5.9: ADF TEST FOR STATIONARITY OF $k(t,j)$— NO DIFFERENCING (MLG-LL, FEMALES)

Cause of Death	Lags	τ	p-value
Infectious	0	-2.21162	0.20677
Neoplasm	4	4.18375	1.00000
Circulatory	1	0.82097	0.99261
Respiratory	0	-1.52248	0.50779
External	0	1.33187	0.99815
Other	0	-0.09689	0.94049

None of the test statistics are significant and we therefore fail to reject the null hypothesis of non-stationarity for all causes. We now repeat the ADF test for the time series after a single order of differencing, i.e. $\Delta k(t,j)$. The results are presented in Table 5.10.

TABLE 5.10: ADF TEST FOR STATIONARITY OF $k(t,j)$ — SINGLE ORDER OF DIFFERENCING (MLG-LL, FEMALES)

Cause of Death	Lags	τ	p-value
Infectious	0	-5.19376	0.00001
Neoplasm	2	-0.78958	0.36558
Circulatory	1	-0.98199	0.28463
Respiratory	0	-4.64354	0.00004
External	4	0.02199	0.68196
Other	0	-5.01380	0.00001

We find that the test statistic τ is significant at the 10% level for three out of six causes: infectious, respiratory, and other diseases. At the same time we fail to reject the null hypothesis of non-stationarity for neoplasms, circulatory diseases, and external causes of death. For the latter causes we therefore need to difference the time series a second time and perform the ADF test on $\Delta^2 k(t,j)$. (See Figure D.6 in the appendix for the plots of $\Delta^2 k(t,j)$.) We present the results of these ADF tests in Table 5.11.

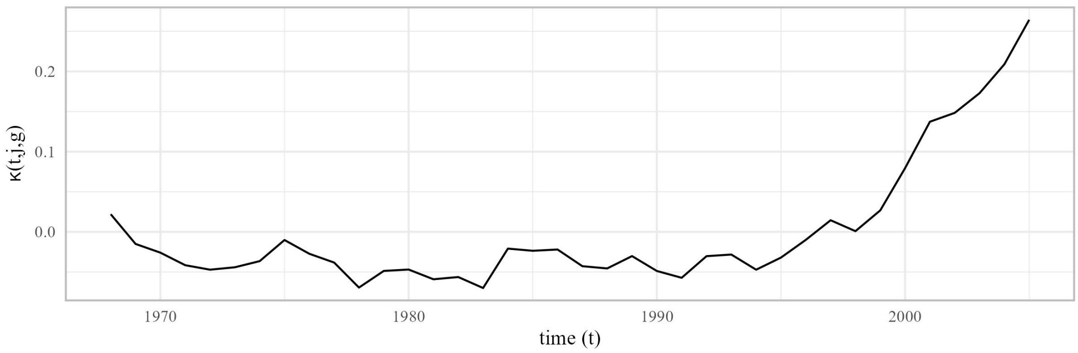
TABLE 5.11: ADF TEST FOR STATIONARITY OF $k(t, j)$ — SECOND ORDER OF DIFFERENCING (MLG-LL, FEMALES)

Cause of Death	Lags	τ	p-value
Neoplasm	3	-5.53964	0.00000
Circulatory	2	-5.63269	0.00000
External	3	-6.35242	0.00000

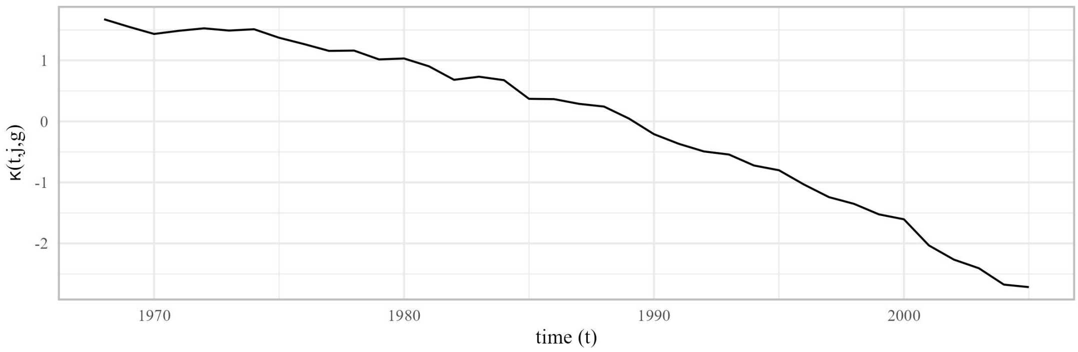
We can see that the second order of differencing has resulted in the time series for the three causes of death being stationary. In the next subsection, we will look at the stationarity of the $\kappa(t, j, g)$ terms for England & Wales.

5.4.2 Stationarity of $\kappa(t, j, g)$ for England & Wales

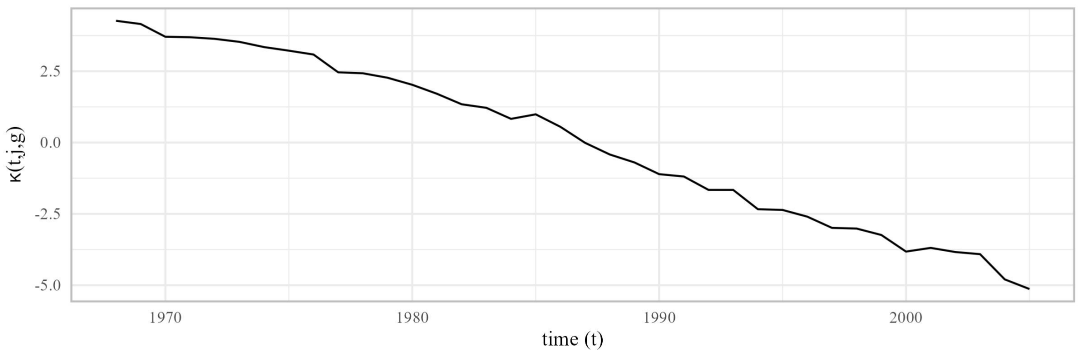
The plots of $\kappa(t, j, g)$ from the MLG-LL model for all six causes in England & Wales are shown in Figure 5.4.



(a) Infectious and Parasitic Diseases

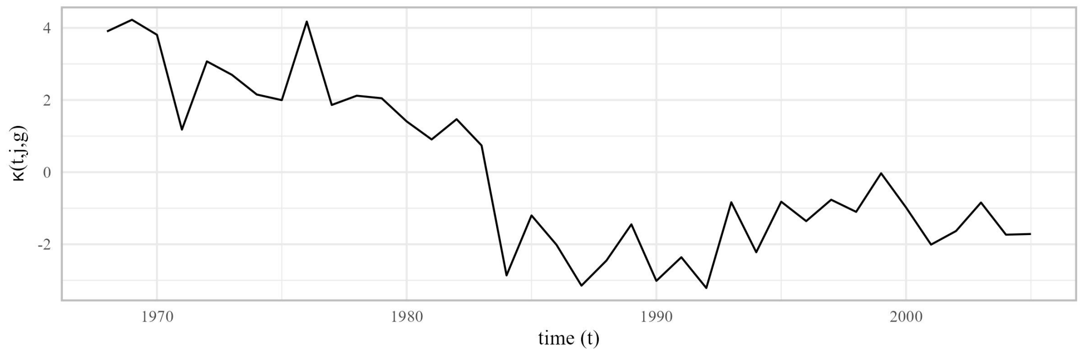


(b) Neoplasms

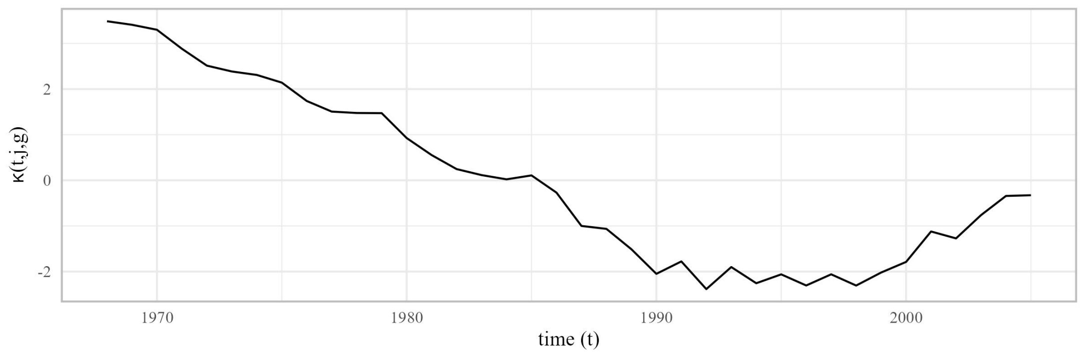


(c) Circulatory Diseases

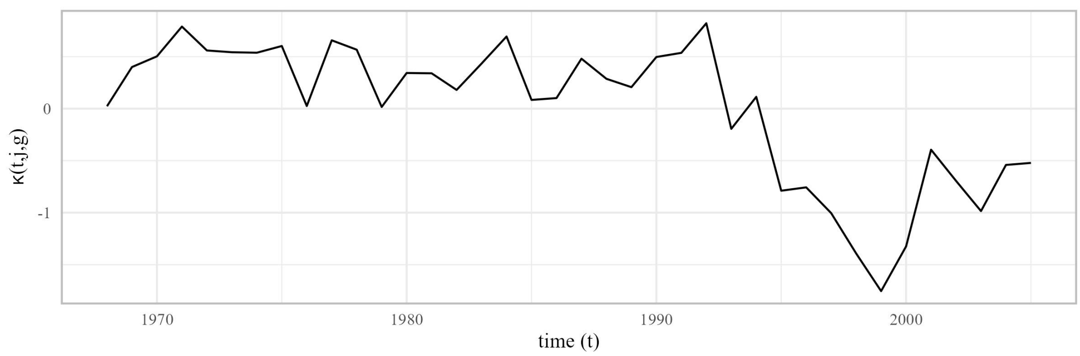
FIGURE 5.4: Time series plots of $\kappa(t, j, g)$
(MLG-LL, England & Wales, Females)



(d) Respiratory Diseases



(e) External Causes



(f) Other Causes

FIGURE 5.4 (CONT.)

The plots of $\kappa(t, j, g)$ (Figure 5.4) suggest that the non-differenced time series are not stationary. As the plots of $\Delta\kappa(t, j, g)$ (see Figure D.7 in the appendix) suggest a non-zero mean, we formally test for stationarity of $\kappa(t, j, g)$ by running an ADF test with drift and present the values of the test statistic τ and the corresponding p-values in Table 5.12.

TABLE 5.12: ADF TEST FOR STATIONARITY OF $\kappa(t, j, g)$ — NO DIFFERENCING (MLG-LL, ENGLAND & WALES, FEMALES)

Cause of Death	Lags	τ	p-value
Infectious	0	2.7960	1.0000
Neoplasm	4	4.4443	1.0000
Circulatory	1	0.4317	0.9808
Respiratory	1	-2.2337	0.1995
External	0	-1.9532	0.3046
Other	0	-1.7850	0.3798

None of the test statistics are significant and we therefore fail to reject the null hypothesis of non-stationarity for all causes. We therefore difference each time series once and repeat the ADF test. The results are presented in Table 5.13.

TABLE 5.13: ADF TEST FOR STATIONARITY OF $\kappa(t, j, g)$ — SINGLE ORDER OF DIFFERENCING (MLG-LL, ENGLAND & WALES, FEMALES)

Cause of Death	Lags	τ	p-value
Infectious	4	0.7018	0.8616
Neoplasm	2	-0.7387	0.3880
Circulatory	1	-1.1355	0.2272
Respiratory	0	-8.9399	0.0000
External	1	-2.0349	0.0419
Other	0	-7.0814	0.0000

The test statistic τ is significant at the 10% level for only three of the six causes. We can therefore only consider the time series for respiratory diseases, external causes, and other causes of death to be stationary after one order of differencing. We fail to reject the null hypothesis of non-stationarity for the other three causes of death: infectious diseases, neoplasms, and circulatory diseases. For these causes we will need to difference the time series a second time and perform the ADF test once more. (See Figure D.8 in the appendix for the plots of $\Delta^2\kappa(t, j, g)$.) We present the results of the ADF test for these causes in Table 5.14.

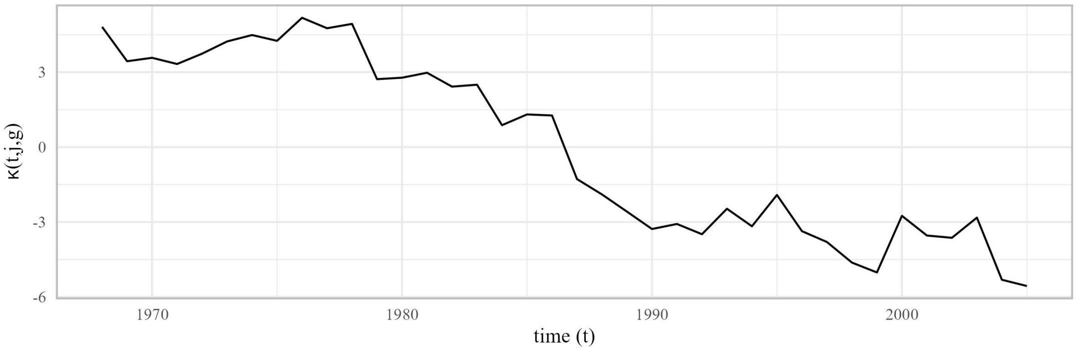
TABLE 5.14: ADF TEST FOR STATIONARITY OF $\kappa(t, j, g)$ — SECOND ORDER OF DIFFERENCING (MLG-LL, ENGLAND & WALES, FEMALES)

Cause of Death	Lags	τ	p-value
Infectious	2	-7.32830	0.00000
Neoplasm	3	-5.60586	0.00000
Circulatory	2	-5.62223	0.00000

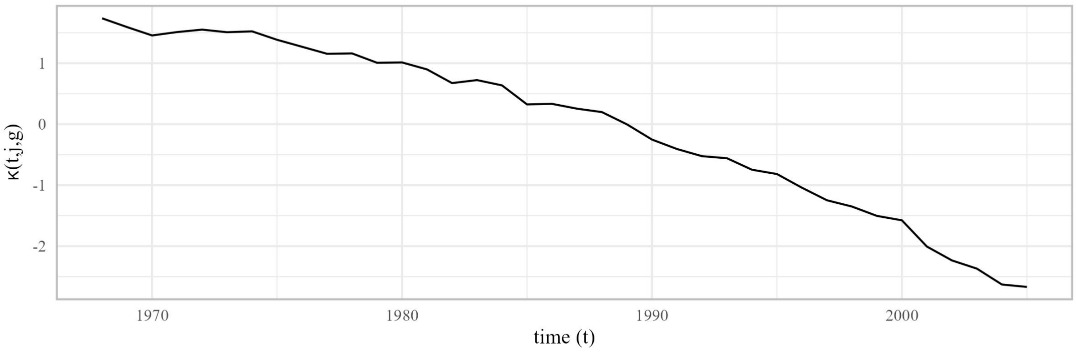
We can see that the second order of differencing has resulted in the time series for infectious diseases, neoplasms, and circulatory diseases being stationary. In the next section, we will look at the stationarity of the $\kappa(t, j, g)$ terms for France.

5.4.3 Stationarity of $\kappa(t, j, g)$ for France

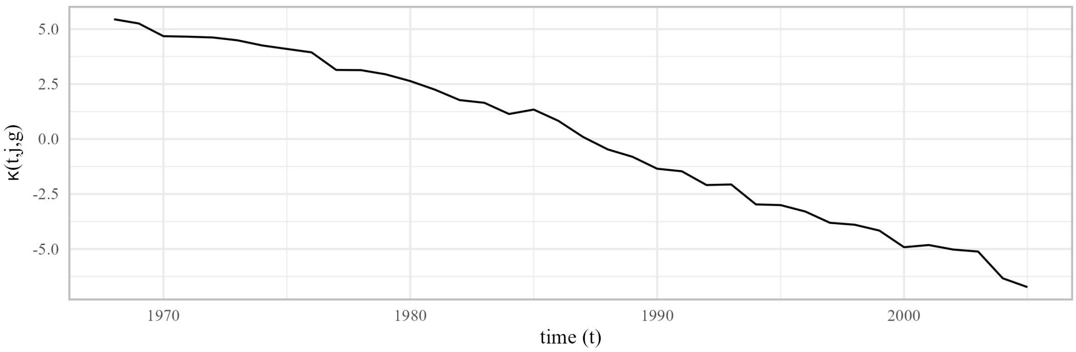
The plots of $\kappa(t, j, g)$ from the MLG-LL model for all six causes in France are shown in Figure 5.5.



(a) Infectious and Parasitic Diseases

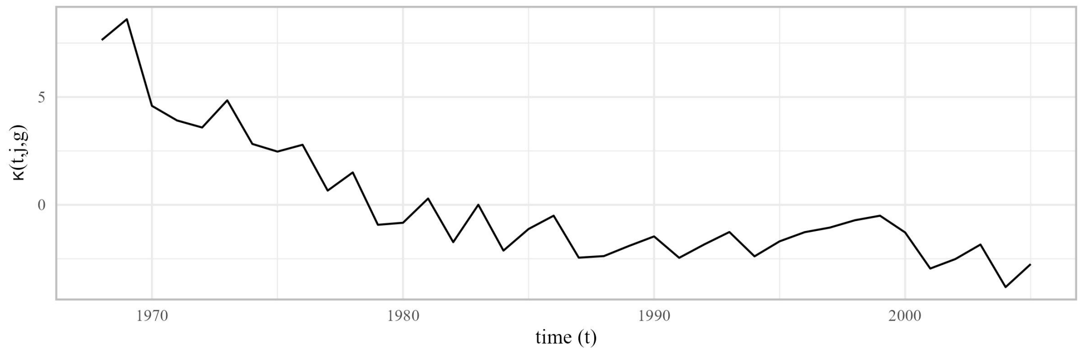


(b) Neoplasms

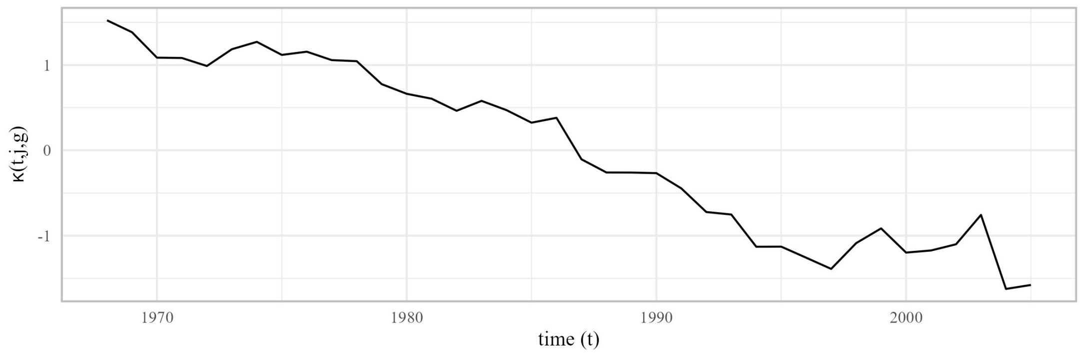


(c) Circulatory Diseases

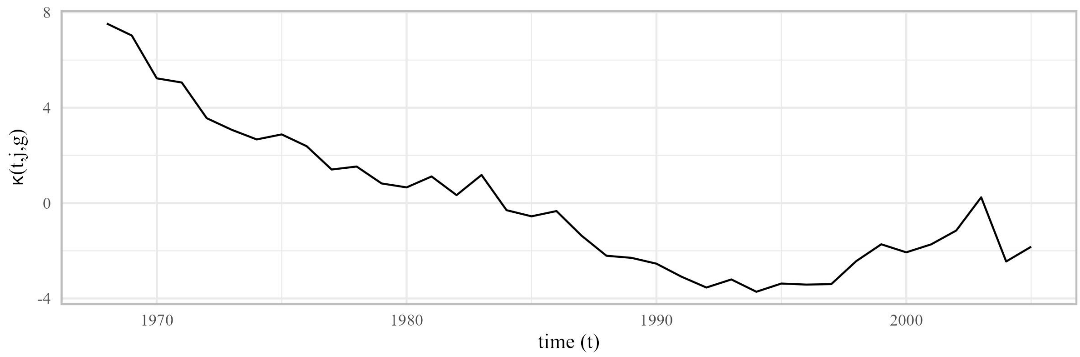
FIGURE 5.5: Time series plots of $\kappa(t, j, g)$
(MLG-LL, France, Females)



(d) Respiratory Diseases



(e) External Causes



(f) Other Causes

FIGURE 5.5 (CONT.)

The plots of $\kappa(t, j, g)$ for France suggest that the non-differenced time series are not stationary. We test this formally by running an ADF test with a drift and present the values of the ADF test statistic τ and the corresponding p-values in Table 5.15.

As the plots of $\Delta\kappa(t, j, g)$ (see Figure D.9 in the appendix) suggest a non-zero mean, we formally test for stationarity of $\kappa(t, j, g)$ by running an ADF test with drift and present the values of the test statistic τ and the corresponding p-values in Table 5.15.

TABLE 5.15: ADF TEST FOR STATIONARITY OF $\kappa(t, j, g)$ — NO DIFFERENCING (MLG-LL, FRANCE, FEMALES)

Cause of Death	Lags	τ	p-value
Infectious	0	-1.47827	0.52964
Neoplasm	4	4.34460	1.00000
Circulatory	1	0.73778	0.99086
Respiratory	2	-2.50552	0.12491
External	0	-1.04985	0.72086
Other	0	-1.83219	0.35794

None of the test statistics are significant and we therefore fail to reject the null hypothesis of non-stationarity for all $\kappa(t, j, g)$ for France. We now difference each time series once and repeat the ADF test. The results of the ADF tests are presented in Table 5.16.

TABLE 5.16: ADF TEST FOR STATIONARITY OF $\kappa(t, j, g)$ — SINGLE ORDER OF DIFFERENCING (MLG-LL, FRANCE, FEMALES)

Cause of Death	Lags	τ	p-value
Infectious	0	-6.44418	0.00000
Neoplasm	6	1.65767	0.97348
Circulatory	1	-1.08714	0.24447
Respiratory	0	-9.28167	0.00000
External	0	-6.05868	0.00000
Other	0	-7.00025	0.00000

We find that the test statistic τ is significant for four out of six causes. The causes for which we fail to reject the null hypothesis of non-stationarity are neoplasms and circulatory diseases. The $\kappa(t, j, g)$ for the same causes in MLG-LC (M0) also needed to be differenced more than once to ensure stationarity. We therefore proceed to difference the $\kappa(t, j, g)$ time series for these causes a second time and perform the ADF test once more. (See Figure D.10 in the appendix for the plots of $\Delta^2\kappa(t, j, g)$.)

TABLE 5.17: ADF TEST FOR STATIONARITY OF $\kappa(t,j,g)$ — SECOND ORDER OF DIFFERENCING (MLG-LL, FRANCE, FEMALES)

Cause of Death	Lags	τ	p-value
Neoplasm	3	-5.64160	0.00000
Circulatory	2	-5.71980	0.00000

The results of the ADF test for stationarity of $\Delta^2\kappa(t,j,g)$ for neoplasms and circulatory diseases are given in Table 5.17. We can see that the second order of differencing has resulted in the time series for these two causes being stationary.

5.4.4 ARIMA Model Selection for MLG-LL Period Effects

We repeat the procedure from Section 5.3.3 to select the non-seasonal ARIMA models for $k(t,j)$ and $\kappa(t,j,g)$ from the MLG-LL model while incorporating our findings from Sections 5.4.1, 5.4.2 & 5.4.3. The ARIMA(p,d,q) specifications for the common cause-specific $k(t,j)$ are presented in Table 5.18.

TABLE 5.18: ARIMA MODEL SPECIFICATIONS FOR $k(t,j)$ SELECTED USING AUTO.ARIMA() FUNCTION IN R — MLG-LL, FEMALES

	ARIMA				
Cause of Death	p	d	q	ℓ	BIC
Infectious	0	1	0	-40.37	87.96
Neoplasm	0	2	1	26.91	-46.65
Circulatory	0	2	2	2.31	6.14
Respiratory	3	1	1	-45.58	105.6
External	0	2	1	-15.96	39.1
Other	0	1	1	-5.67	22.18

As we have seen in the previous subsections, the time series for three of the causes of death required second order of differencing to achieve stationarity. This is one more than for the MLG-LC (M0) where at most two time series needed to be differenced twice. More notable is the value of the autoregressive coefficient $p = 3$ for respiratory diseases. The pooled common term $k(t,j)$ for this cause of death requires more autoregressive terms than $\kappa(t,j,g)$ for either country from MLG-LC (M0).

The ARIMA model specifications for $\kappa(t, j, g)$ for England & Wales and France are presented in Tables 5.19 and 5.20, respectively.

TABLE 5.19: ARIMA MODEL SPECIFICATIONS FOR $\kappa(t, j, g)$ SELECTED USING AUTO.ARIMA() FUNCTION IN R — MLG-LL, ENGLAND & WALES, FEMALES

	ARIMA				
Cause of Death	p	d	q	ℓ	BIC
Infectious	0	2	1	86.02	-164.87
Neoplasm	0	2	1	29.7	-52.24
Circulatory	0	2	2	1.3	8.15
Respiratory	0	1	1	-57.25	121.73
External	0	1	0	-13.07	29.75
Other	0	1	0	-18.94	41.5

We can see that the $\kappa(t, j, g)$ specifications for both countries contain exclusively moving average terms and no autoregressive ones. The time series for circulatory diseases for both

TABLE 5.20: ARIMA MODEL SPECIFICATIONS FOR $\kappa(t, j, g)$ SELECTED USING AUTO.ARIMA() FUNCTION IN R — MLG-LL, FRANCE, FEMALES

	ARIMA				
Cause of Death	p	d	q	ℓ	BIC
Infectious	0	1	0	-53.27	110.14
Neoplasm	0	2	1	29.18	-51.19
Circulatory	0	2	2	-8.47	27.68
Respiratory	0	1	1	-58.11	127.05
External	0	1	0	4.23	-1.25
Other	0	1	0	-46.11	95.83

countries have the highest number of moving average terms with $q = 2$. The ARIMA(p, d, q) specifications of $\kappa(t, j, g)$ for a cause j are the same for both countries with the exception of infectious diseases where $\kappa(t, j, g)$ for England & Wales follows an ARIMA$(0, 2, 1)$ whereas $\kappa(t, j, g)$ for France follows ARIMA$(0, 1, 0)$, i.e. a random walk. Interestingly, the $\kappa(t, j, g)$ ARIMA specifications for respiratory diseases are once again the same for both countries and identical to those for $\kappa(t, j, g)$ from the MLG-LC model.

In the next chapter we will re-estimate the cause-specific mortality probabilities for a selection of models from Chapter 4 using the Bayesian approach – specifically the Markov Chain Monte Carlo and Hamiltonian Monte Carlo algorithms. The ARIMA model specifications presented in this chapter, particularly those presented in Section 5.3, will underpin the prior specifications for the period effects in Section 6.2.

Chapter 6

Bayesian Modelling of Cause-Specific Mortality in England & Wales and France

While the Maximum Likelihood approach provides one method for coefficient estimation, quantification of the uncertainty in our estimates often relies on the limiting distributions of the MLEs — the Bayesian approach, on the other hand, does not require such assumptions. Furthermore, generation of sufficient number of samples using the Parametric Bootstrap method can be time consuming due to the need to find the MLEs for each sample. Bayesian techniques offer an alternative by estimating the entire probability distributions for model coefficients as part of the posterior. The posterior distribution for a parameter θ combines the prior beliefs about the underlying distribution of the parameter with the likelihood of the observed data. This is analogous to the conditional probability formula given by the Bayes theorem

$$p(\theta|y) = \frac{p(\theta, y)}{p(y)} = \frac{p(y|\theta)p(\theta)}{p(y)} \tag{6.1}$$

where y is the data. A natural choice of priors for the mortality problem we are presenting includes uniform, Normal, and multivariate Normal distributions — all of these priors, however, are not conjugate to the multinomial likelihood of the MLG model. The resulting marginal likelihood

$$p(y) = \int p(y,\theta)d\theta = \int p(y|\theta)p(\theta)d\theta \tag{6.2}$$

will therefore be analytically intractable requiring numeric techniques to calculate it (Gelman et al., 2014). Markov Chain Monte Carlo methods are one possible option to allow us to obtain parameter estimates along with their respective posterior distributions when closed-form solutions are not available. The sections that follow will focus on the application of Metropolis(-Hastings) algorithm to the MLG-LL model as well as the simpler MLG-LC model.

6.1 Markov Chain Monte Carlo

6.1.1 Metropolis-Hastings Algorithm

The Metropolis-Hastings algorithm (Hastings, 1970) generates values of parameter estimates $\boldsymbol{\theta}$ from a Markov chain using a proposal distribution based on the estimates from the previous iteration, e.g. $q_t(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{t-1})$. The algorithm then runs as follows

1. Choose $\boldsymbol{\theta}^{\mathbf{0}}$, i.e. the initial parameter estimates, such that $p(\boldsymbol{\theta}^{\mathbf{0}}|\boldsymbol{y}) > 0$, e.g. a crude estimate of the parameters from the data;

2. Generate $\boldsymbol{\theta}^*$ from the proposal distribution $q_t(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{\boldsymbol{t-1}})$;

3. Generate u from a uniform $U(0;1)$;

4. Calculate the ratio of the posterior distributions conditional on previous $\boldsymbol{\theta}^{\boldsymbol{t-1}}$ and proposed $\boldsymbol{\theta}^*$

$$\alpha(\boldsymbol{\theta}^*, \boldsymbol{\theta}^{\boldsymbol{t-1}}, \boldsymbol{y}) = \min\left\{ \frac{p(\boldsymbol{\theta}^*|\boldsymbol{y})q_t(\boldsymbol{\theta}^{\boldsymbol{t-1}}|\boldsymbol{\theta}^*)}{p(\boldsymbol{\theta}^{\boldsymbol{t-1}}|\boldsymbol{y})q_t(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{\boldsymbol{t-1}})}, 1 \right\}$$

5. Accept $\boldsymbol{\theta}^*$ as $\boldsymbol{\theta}^{\boldsymbol{t}}$ if $u < \alpha(\boldsymbol{\theta}^*, \boldsymbol{\theta}^{\boldsymbol{t-1}}, \boldsymbol{y})$, otherwise $\boldsymbol{\theta}^{\boldsymbol{t}} = \boldsymbol{\theta}^{\boldsymbol{t-1}}$;

6. Repeat steps 2-5 for $t \in \{2, \ldots, T\}$.

6.1.1.1 Metropolis Algorithm

The Metropolis algorithm (Metropolis et al., 1953) is a special case of the Metropolis-Hastings algorithm where $q_t(\boldsymbol{\theta^{t-1}}|\boldsymbol{\theta}^*) = q_t(\boldsymbol{\theta}^*|\boldsymbol{\theta^{t-1}})$ i.e. where the proposal distribution centered around the previously accepted values and the newly proposed values are symmetric. The proposal distribution will always be symmetric when a MVN distribution is used. To see why, suppose that $\boldsymbol{\theta}$, a $k \times 1$ column vector of coefficients, follows a k-variate Normal distribution ($\mathcal{N}_k$) such that

$$\boldsymbol{\theta} \sim \mathcal{N}_k(\boldsymbol{\mu}, \boldsymbol{\Lambda})$$

where $\boldsymbol{\mu}$ is a $k \times 1$ mean vector and $\boldsymbol{\Lambda}$ is a $k \times k$ covariance matrix. The probability density function of $\mathcal{N}_k$ would then be given by

$$f(\boldsymbol{\theta}|\boldsymbol{\mu}, \boldsymbol{\Lambda}) = (2\pi)^{-\frac{k}{2}}|\boldsymbol{\Lambda}|^{-\frac{1}{2}} \cdot \exp\big\{ -\frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\mu})'\boldsymbol{\Lambda}^{-1}(\boldsymbol{\theta} - \boldsymbol{\mu})\big\} \tag{6.3}$$

For any pair of $k \times 1$ column vectors, e.g. $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$, such that $\boldsymbol{\phi} \neq \boldsymbol{\psi}$, the ratio of the proposal distributions would be

$$\begin{aligned}
\frac{f(\boldsymbol{\phi}|\ \boldsymbol{\psi}, \boldsymbol{\Lambda})}{f(\boldsymbol{\psi}|\ \boldsymbol{\phi}, \boldsymbol{\Lambda})} &= \frac{(2\pi)^{-\frac{k}{2}}|\boldsymbol{\Lambda}|^{-\frac{1}{2}} \cdot \exp\big\{ -\frac{1}{2}(\boldsymbol{\phi} - \boldsymbol{\psi})^\top\boldsymbol{\Lambda}^{-1}(\boldsymbol{\phi} - \boldsymbol{\psi})\big\}}{(2\pi)^{-\frac{k}{2}}|\boldsymbol{\Lambda}|^{-\frac{1}{2}} \cdot \exp\big\{ -\frac{1}{2}(\boldsymbol{\psi} - \boldsymbol{\phi})^\top\boldsymbol{\Lambda}^{-1}(\boldsymbol{\psi} - \boldsymbol{\phi})\big\}} \\
&= \exp\Big\{-\tfrac{1}{2}(\boldsymbol{\phi} - \boldsymbol{\psi})^\top\boldsymbol{\Lambda}^{-1}(\boldsymbol{\phi} - \boldsymbol{\psi}) + \tfrac{1}{2}(\boldsymbol{\psi} - \boldsymbol{\phi})^\top\boldsymbol{\Lambda}^{-1}(\boldsymbol{\psi} - \boldsymbol{\phi})\Big\} \\
&= \exp\Big\{-\tfrac{1}{2}\Big[(\boldsymbol{\phi} - \boldsymbol{\psi})^\top\boldsymbol{\Lambda}^{-1}(\boldsymbol{\phi} - \boldsymbol{\psi}) - (\boldsymbol{\phi} - \boldsymbol{\psi})^\top\boldsymbol{\Lambda}^{-1}(\boldsymbol{\phi} - \boldsymbol{\psi})\Big]\Big\} \\
&= 1
\end{aligned}$$

The Metropolis-Hastings algorithm therefore reduces to a Metropolis algorithm when a multivariate Normal proposal distribution is used.

6.1.1.2 Metropolis Algorithm Tuning

Roberts, Gelman, and Gilks (1997) have shown that the optimal average acceptance rate for a Metropolis algorithm as the number of coefficients approaches infinity is 23.8%. Given the number of coefficients in the model, manual calibration of variance for every parameter, or even block of parameters, would be extremely time consuming. We therefore used the following algorithm to tune the proposal variance-covariance matrix of the Metropolis algorithm:

1. Calculate

$$\boldsymbol{\omega}^s = \boldsymbol{\xi}^s \circ \boldsymbol{\omega}^{s-1} \tag{6.4}$$

where $\boldsymbol{\omega}^s$ is the scaling vector for calibration iteration s, $\boldsymbol{\xi}^s$ is a vector of calibration scalars and $\circ$ denotes a Hadamard product. The initial scalar vector $\boldsymbol{\omega}^0$ is a vector of ones. The elements ξ_r^s of vector $\boldsymbol{\Xi}^s$ are calculated using the piecewise function (6.5)

$$\xi_r^s = \begin{cases} 0.1 & 0 \leqslant a_r^s < 0.01026334 \\ -\frac{a_r^s(a_r^s-0.40)}{0.04} & 0.01026334 \leqslant a_r^s < 0.20 \\ 1 & 0.20 \leqslant a_r^s \leqslant 0.25 \\ 1+\frac{(a_r^s-0.25)^2}{0.16} & 0.25 < a_r^s < 0.85 \\ 10 & 0.85 \leqslant a_r^s \leqslant 1 \end{cases} \tag{6.5}$$

for $a_r^s, r \in \{1,\ldots,R\}$, where a_r^s is the proportion of accepted proposals for each group of parameters r and R is the total number of blocks in the Fisher information matrix described in Section 4.3.

2. Scale each block of the proposal variance $\boldsymbol{\Lambda}_r$ by ω_r^s, i.e. $\boldsymbol{\Psi}_r^s = \omega_r^s \cdot \boldsymbol{\Lambda}_r$.

3. Estimate the acceptance rate using a random walk with proposal variance $\boldsymbol{\Psi}^s$.

4. Repeat steps 1-3 for $s \in \{2,\ldots,S\}$, i.e. until sufficient convergence.

Figure 6.2 shows the plot of the piecewise function given above. This calibration curve replaced the continuous piecewise function initially proposed for the algorithm (see Figure 6.1). The steep slope of calibration curve 1 for $a > 0.25$ resulted in the acceptance rate decreasing below the desired interval $[0.20; 0.25]$ at the subsequent calibration iteration when the acceptance rate was approaching the interval from above, creating an inefficiency in the algorithm. The visible discontinuity in calibration curve 2 is the result of the applied correction.

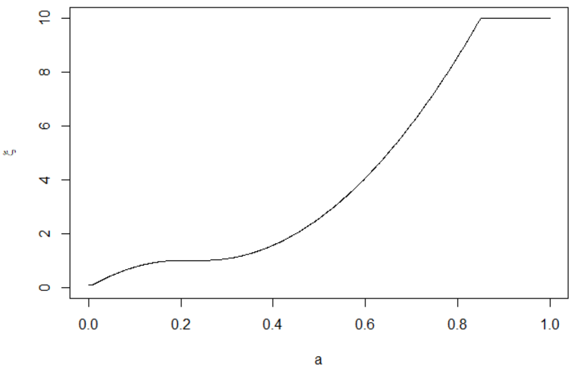


FIGURE 6.1:
Calibration Curve 1

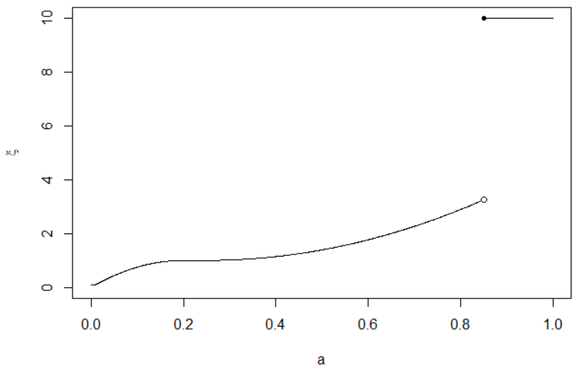


FIGURE 6.2:
Calibration Curve 2

Whilst this approach is not adaptive in the sense of the Adaptive Metropolis (AM) algorithm described by Haario, Saksman, and Tamminen (2001), the tuning process preserves ergodicity of the random walk. The convergence of an adaptive MCMC requires the amount of tuning to diminish as the number of iterations increases (Roberts and Rosenthal, 2007). This condition is clearly satisfied as the $\boldsymbol{\omega}$ is fixed at the end of the tuning stage.

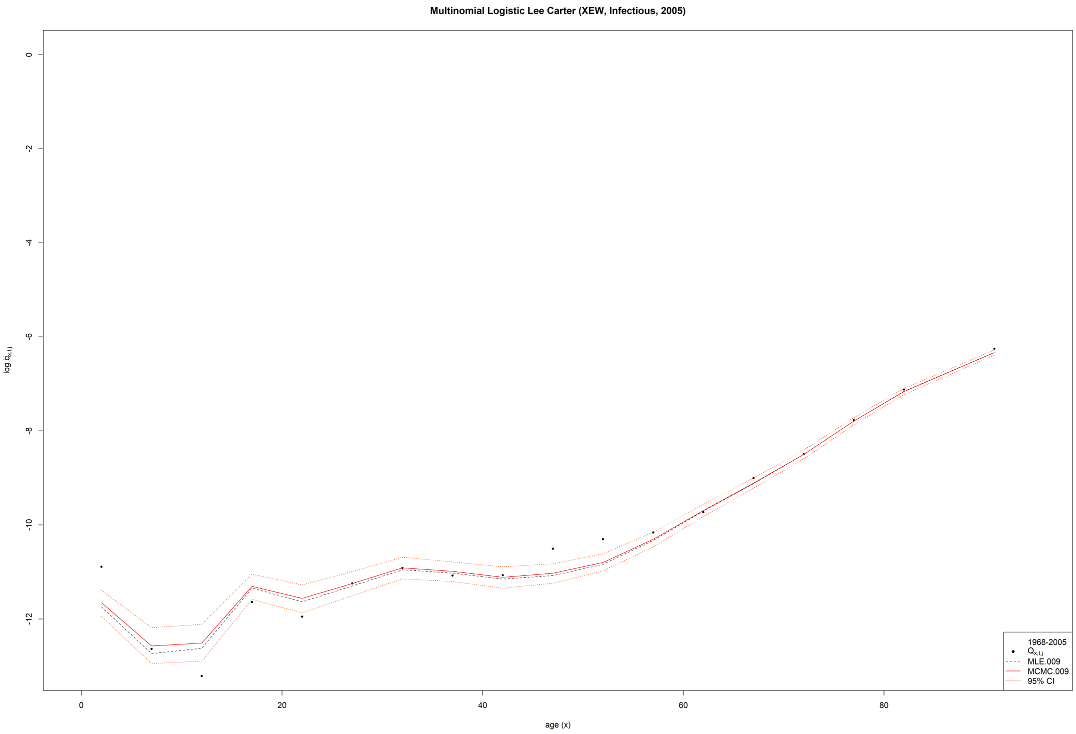
6.1.2 Results

We have estimated the MLG-LC (M0) model using the Metropolis algorithm with the following prior specifications

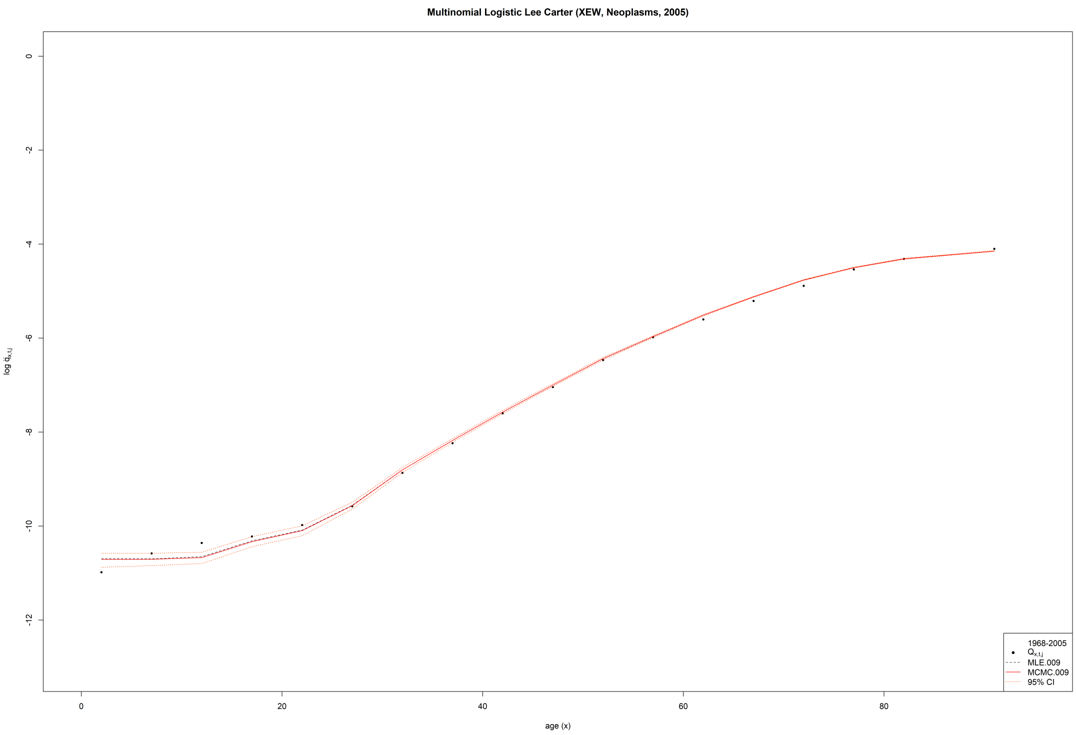
$$\begin{aligned}
\alpha(x,j,g) &\sim \mathrm{N}\left(0, \sigma^2_\alpha(j,g)\right), & \sigma_\alpha(j,g) &= 100 \cdot w^s_r \\
\beta(x,j,g) &\sim \mathrm{N}\left(\tfrac{1}{X}, \sigma^2_\beta(j,g)\right), & \sigma_\beta(j,g) &= 10 \cdot w^s_r \\
\kappa(t,j,g) &\sim \mathrm{N}\left(0, \sigma^2_\kappa(j,g)\right), & \sigma_\kappa(j,g) &= 1 \cdot w^s_r
\end{aligned} \tag{6.6}$$

where X denotes the number of age groups and w^s_r is the scaling factor as described in Section 6.1.1.2. The mean of the priors for $\beta(x,j,g)$ parameters was set to $\frac{1}{X}$ as in Wong, Forster, and Smith (2018). This not only incorporates the sum-to-one constraint but also helps avoid the proposed values from being zero. This is helpful as $\beta(x,j,g)$ forms part of the bilinear term in the predictor function and $\beta(x,j,g) = 0$ would imply $\beta(x,j,g)\kappa(t,j,g) = 0$ making the estimation of $\kappa(t, j, g)$ parameter difficult. Do note that in this section no time series specification is imposed on the priors for the $\kappa(t, j, g)$ parameters.

A comparison of the MLEs for MLG-LC (M0) along with the MCMC estimates and 95% credible intervals is shown in Figure 6.3. The values of the uncertainty estimates under maximum likelihood and Metropolis algorithm for cause group 2 – Neoplasms – are presented in Tables 6.1 and 6.2. The uncertainty estimates for the remainder of the cause groups are presented in Appendix E. We can see that the Bayesian approach yields slightly larger standard errors than ML due to the increased uncertainty imposed by our priors.

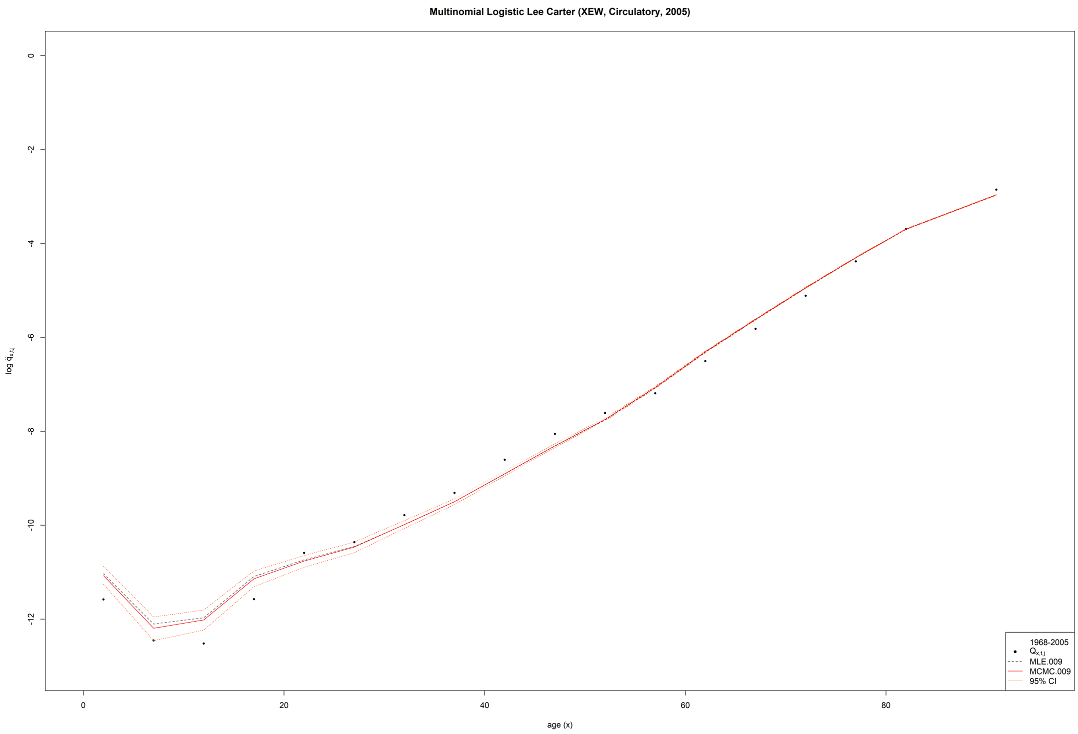


(a) Infectious and Parasitic Diseases

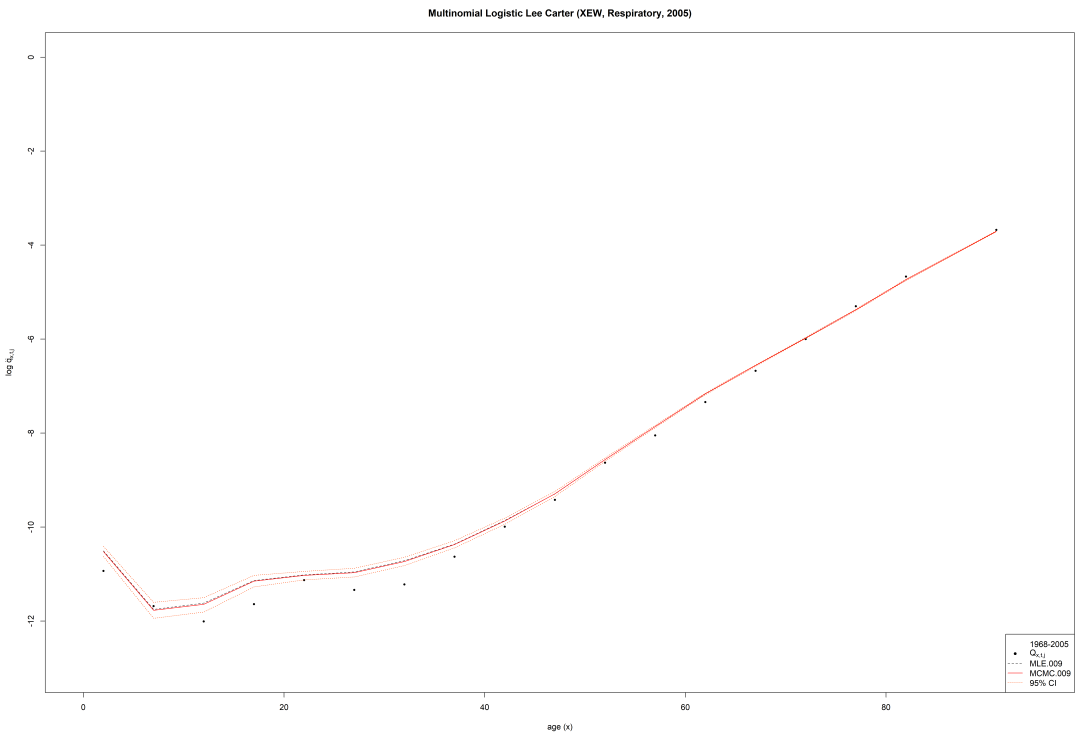


(b) Neoplasms

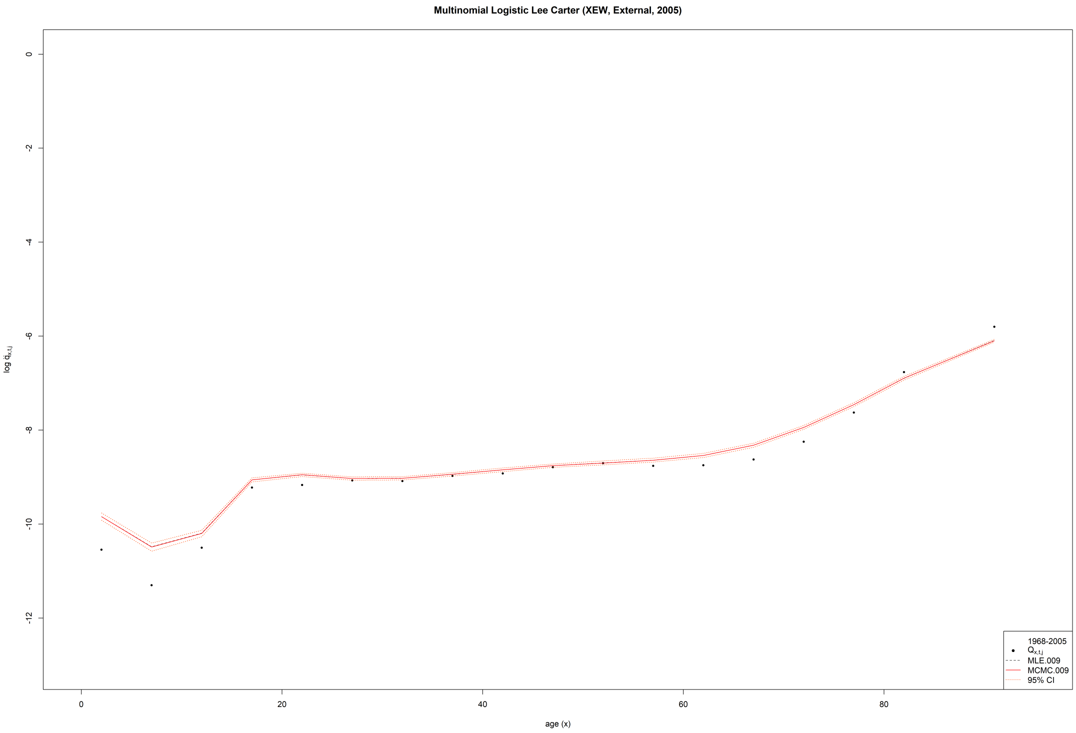
FIGURE 6.3: Log Probability of Death across Age Groups (MLG-LC (M0), England and Wales, 2005)



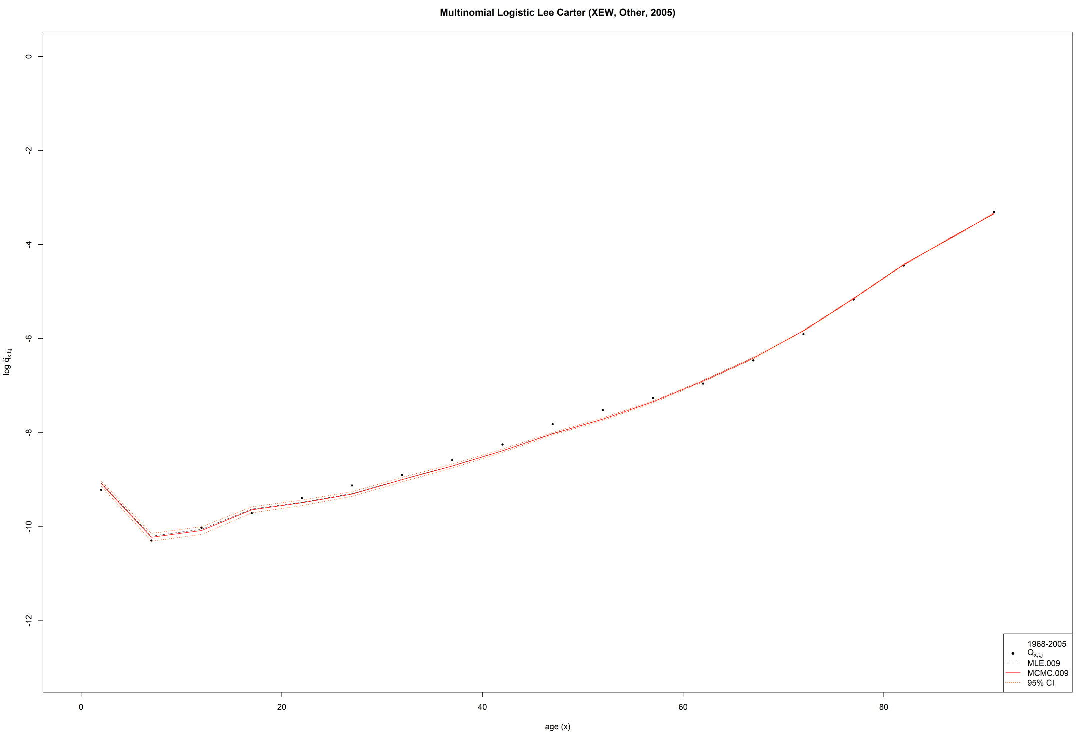
(c) Circulatory Diseases



(d) Respiratory Diseases



(e) External Causes



(f) Other Causes

FIGURE 6.3

TABLE 6.1: BAYESIAN ESTIMATES AND CREDIBLE INTERVALS FOR CAUSE 2 — NEOPLASMS, ENGLAND & WALES, FEMALES

	Cause 2 — Neoplasms				
	Maximum Likelihood		Metropolis Algorithm		
	Estimate	s.e.	Median	s.e.	95 % Credible Interval
$\alpha(x,2)$					
1 –4	-9.9772	0.020049 ***	-9.9805	0.027595 ***	(-10.0358 , -9.9263)
5 –9	-10.1376	0.019368 ***	-10.1414	0.024824 ***	(-10.1909 , -10.0942)
10 –14	-10.2165	0.020233 ***	-10.2194	0.025815 ***	(-10.2699 , -10.1699)
15 –19	-9.9917	0.018135 ***	-9.9932	0.023408 ***	(-10.0390 , -9.9471)
20 –24	-9.7375	0.015657 ***	-9.7376	0.020192 ***	(-9.7792 , -9.7008)
25 –29	-9.2113	0.012030 ***	-9.2123	0.014677 ***	(-9.2407 , -9.1831)
30 –34	-8.5008	0.008558 ***	-8.5009	0.010863 ***	(-8.5228 , -8.4802)
35 –39	-7.8431	0.006297 ***	-7.8425	0.007878 ***	(-7.8581 , -7.8273)
40 –44	-7.2177	0.004723 ***	-7.2172	0.005923 ***	(-7.2286 , -7.2055)
45 –49	-6.6385	0.003582 ***	-6.6382	0.004530 ***	(-6.6469 , -6.6290)
50 –54	-6.1506	0.002859 ***	-6.1505	0.003637 ***	(-6.1577 , -6.1433)
55 –59	-5.7501	0.002385 ***	-5.7502	0.002898 ***	(-5.7559 , -5.7445)
60 –64	-5.3951	0.002055 ***	-5.3957	0.002598 ***	(-5.4008 , -5.3906)
65 –69	-5.0860	0.001821 ***	-5.0864	0.002332 ***	(-5.0911 , -5.0819)
70 –74	-4.8057	0.001678 ***	-4.8062	0.002120 ***	(-4.8102 , -4.8019)
75 –79	-4.5574	0.001646 ***	-4.5580	0.001964 ***	(-4.5618 , -4.5542)
80 –84	-4.3312	0.001764 ***	-4.3321	0.002295 ***	(-4.3365 , -4.3275)
85+	-4.1054	0.001758 ***	-4.1068	0.002351 ***	(-4.1115 , -4.1023)
$\beta(x,2)$					
1 –4	0.1648	0.008414 ***	0.1646	0.011619 ***	(0.1414 , 0.1872)
5 –9	0.1283	0.008070 ***	0.1293	0.010632 ***	(0.1086 , 0.1501)
10 –14	0.1002	0.008312 ***	0.1025	0.011007 ***	(0.0806 , 0.1239)
15 –19	0.0736	0.007685 ***	0.0750	0.009868 ***	(0.0563 , 0.0950)
20 –24	0.0799	0.006889 ***	0.0808	0.008635 ***	(0.0645 , 0.0981)
25 –29	0.0790	0.005424 ***	0.0787	0.006896 ***	(0.0657 , 0.0929)
30 –34	0.0691	0.003811 ***	0.0686	0.004942 ***	(0.0589 , 0.0782)
35 –39	0.0767	0.002798 ***	0.0756	0.003812 ***	(0.0682 , 0.0831)
40 –44	0.0813	0.002111 ***	0.0799	0.003538 ***	(0.0734 , 0.0876)
45 –49	0.0823	0.001605 ***	0.0807	0.003065 ***	(0.0753 , 0.0875)
50 –54	0.0651	0.001259 ***	0.0640	0.002535 ***	(0.0596 , 0.0696)
55 –59	0.0487	0.001042 ***	0.0479	0.001931 ***	(0.0445 , 0.0522)
60 –64	0.0252	0.000915 ***	0.0248	0.001384 ***	(0.0222 , 0.0276)
65 –69	0.0048	0.000810 ***	0.0049	0.001135 ***	(0.0025 , 0.0070)
70 –74	-0.0152	0.000738 ***	-0.0149	0.001098 ***	(-0.0170 , -0.0128)
75 –79	-0.0220	0.000714 ***	-0.0215	0.001134 ***	(-0.0239 , -0.0194)
80 –84	-0.0191	0.000752 ***	-0.0188	0.001210 ***	(-0.0213 , -0.0165)
85+	-0.0227	0.000728 ***	-0.0228	0.001342 ***	(-0.0257 , -0.0205)

TABLE 6.2: BAYESIAN ESTIMATES AND CREDIBLE INTERVALS FOR CAUSE 2 — NEOPLASMS, ENGLAND & WALES, FEMALES (CONTINUED)

	CAUSE 2 — NEOPLASMS				
	Maximum Likelihood		Metropolis Algorithm		
	Estimate	s.e.	Median	s.e.	95 % Credible Interval
$\kappa(t,2)$					
1968	2.4401	0.107936 ***	2.4678	0.154888 ***	(2.1749 , 2.7812)
1969	2.6130	0.107496 ***	2.6492	0.156130 ***	(2.3374 , 2.9443)
1970	2.4249	0.108270 ***	2.4663	0.163217 ***	(2.1561 , 2.8068)
1971	2.5950	0.107946 ***	2.6300	0.156989 ***	(2.3353 , 2.9557)
1972	2.5756	0.108111 ***	2.6058	0.169956 ***	(2.2759 , 2.9445)
1973	2.4781	0.108559 ***	2.5075	0.153137 ***	(2.1974 , 2.8029)
1974	2.3824	0.109066 ***	2.4116	0.158414 ***	(2.1029 , 2.7278)
1975	2.3160	0.109649 ***	2.3578	0.160906 ***	(2.0281 , 2.6682)
1976	2.2496	0.110121 ***	2.2840	0.157894 ***	(1.9748 , 2.6024)
1977	2.1444	0.110563 ***	2.1598	0.158042 ***	(1.8699 , 2.4843)
1978	2.1727	0.110439 ***	2.1998	0.160793 ***	(1.9084 , 2.5312)
1979	1.9286	0.111145 ***	1.9637	0.145893 ***	(1.6723 , 2.2458)
1980	1.8987	0.111525 ***	1.9171	0.151366 ***	(1.6269 , 2.2154)
1981	1.7274	0.111537 ***	1.7567	0.153027 ***	(1.4649 , 2.0682)
1982	1.5831	0.112234 ***	1.6061	0.152747 ***	(1.3148 , 1.9187)
1983	1.4262	0.112410 ***	1.4461	0.147552 ***	(1.1694 , 1.7436)
1984	1.0100	0.113300 ***	1.0168	0.151018 ***	(0.7414 , 1.3292)
1985	0.6530	0.114213 ***	0.6607	0.154463 ***	(0.3575 , 0.9563)
1986	0.6255	0.114175 ***	0.6319	0.147899 ***	(0.3487 , 0.9268)
1987	0.3412	0.114536 **	0.3500	0.152293 *	(0.0579 , 0.6571)
1988	0.2475	0.114331 *	0.2480	0.150532	(-0.0616 , 0.5323)
1989	-0.3156	0.115450 **	-0.3267	0.152381 *	(-0.6322 , -0.0347)
1990	-0.4565	0.115324 ***	-0.4487	0.153004 **	(-0.7468 , -0.1569)
1991	-0.7362	0.115530 ***	-0.7457	0.155274 ***	(-1.0466 , -0.4338)
1992	-0.9894	0.115470 ***	-1.0001	0.153781 ***	(-1.3042 , -0.6995)
1993	-0.8372	0.114369 ***	-0.8440	0.151379 ***	(-1.1445 , -0.5541)
1994	-1.3187	0.115186 ***	-1.3341	0.150233 ***	(-1.6206 , -1.0369)
1995	-1.3792	0.114655 ***	-1.4024	0.152597 ***	(-1.6980 , -1.0990)
1996	-1.7517	0.115078 ***	-1.7719	0.160621 ***	(-2.0928 , -1.4674)
1997	-2.0199	0.115227 ***	-2.0559	0.164308 ***	(-2.3856 , -1.7522)
1998	-2.3692	0.115556 ***	-2.4158	0.169167 ***	(-2.7320 , -2.0741)
1999	-2.6461	0.115728 ***	-2.6810	0.169172 ***	(-3.0029 , -2.3368)
2000	-2.5610	0.114912 ***	-2.5907	0.166592 ***	(-2.9204 , -2.2806)
2001	-3.6302	0.117696 ***	-3.6922	0.188339 ***	(-4.0546 , -3.3141)
2002	-4.0158	0.118166 ***	-4.0669	0.184998 ***	(-4.4336 , -3.7109)
2003	-4.2750	0.118558 ***	-4.3322	0.200498 ***	(-4.7218 , -3.9287)
2004	-4.1715	0.117922 ***	-4.2302	0.196034 ***	(-4.6249 , -3.8475)
2005	-4.3597	0.117874 ***	-4.4215	0.207633 ***	(-4.8387 , -4.0097)

6.2 Hamiltonian Monte Carlo

The already large number of effective parameters seen in Table 4.3 will grow even larger once prior parameters are added, e.g. 1188 for M5. This results in the joint probability density of the model becoming complex and causes issues with the convergence of the MCMC algorithm even after the simulation has been running for a number of days. To make estimation of multi-country models possible, we have used the Hamiltonian Monte Carlo (HMC) algorithm implemented via the R interface to the Stan statistical computation platform (RStan) to fit the Bayesian specifications rather than a Metropolis-Hastings algorithm from Section 6.1 (Stan Development Team, 2023a).

6.2.1 Multinomial Logistic Lee-Carter

We have implemented the Bayesian approach for the MLG-LC model using the prior specifications given below

$$\begin{aligned} \alpha(x,j,g) &\sim \mathrm{N}\left(0,\sigma_\alpha^2(j,g)\right), & \sigma_\alpha(j,g) &\sim \mathrm{N}^+\left(0,100\right) \\ \beta(x,j,g) &\sim \mathrm{N}\left(\tfrac{1}{X},\sigma_\beta^2(j,g)\right), & \sigma_\beta(j,g) &\sim \mathrm{N}^+\left(0,100\right) \end{aligned} \tag{6.7}$$

where X denotes the number of age groups. The rationale for the mean of the prior for $\beta(x,j,g)$ is the same as in (6.6). We have tested a number of specifications for the variance hyperpriors including Uniform$(0,10)$, Beta$(2,2)$, and Inverse-Gamma$(0.001;\ 0.001)$. Gelman (2006) demonstrates estimation issues for prior distributions with heavy tails such as the Inverse-Gamma favoured by Wong, Forster, and Smith (2018). Based on prior expectations of the variance and due to the hierarchical nature of the model, we opt for a weakly informative half-Normal specification similar to that used by Hilton et al. (2018). This specification takes into account plausible values of the variance hyperparameters while providing for optimal performance. The half-Normal priors are implemented in STAN using $\varphi_{[0,100]}(\cdot)$, i.e. a standard Normal density function truncated to $[0,100]$ as the right tail will computationally underflow to 0.

In the existing literature, Vector Autoregressive model (VAR) or Vector Error-Correction Model (VECM) have been used to estimate time series that exhibit co-integration; see for example Arnold and Sherris (2015) or Arnold and Glushko (2022). As discussed earlier, the number of parameters required to be estimated is large and a VAR or VECM specification for the period effects would increase this further. We therefore chose to model the time-dependent $\kappa(t,j,g)$ parameters using a simpler approach, specifically an ARIMA with non-seasonal differencing, a constant term, and autoregressive (p) and moving average (q) parameter specifications based on the analysis from Chapter 5, specifically those listed in Tables 5.7 & 5.8.

6.2.2 Multinomial Logistic Li-Lee

The MLG-LL specification includes not only country-specific period effects $\kappa(t,j,g)$ for each cause j but also a common, or global, period effect $k(t,j)$ for each cause j. One possible model could follow the ARIMA specifications presented in Section 5.4.4. Unfortunately the computational complexity of those specifications has caused convergence issues in RStan when we tried to implement them. In order to provide methodology that can be generalised to other countries, we implemented the approach proposed by Li and Lee (2005) who recommended that the period effect is modelled using a random walk with drift (RWD) for the country/cause specific terms. We therefore imposed the RWD prior on the common cause-specific period effect $k(t,j)$ and ARIMA(1,1,0) prior for the period deviations for each country-cause combination $\kappa(t,j,g)$. These are similar to the baseline time series specifications used by Enchev, Kleinow, and Cairns (2017). The weakly informative prior distributions are then as follows:

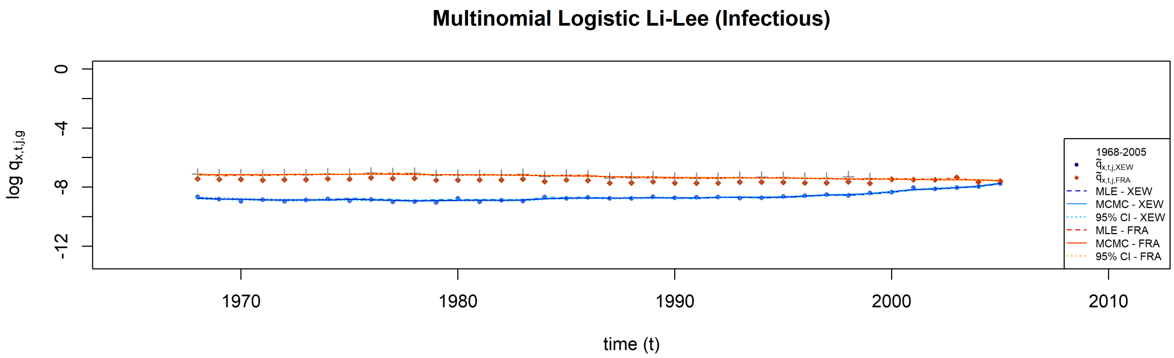
$$
\begin{aligned}
k(t,j) &\sim \mathrm{N}\left(\nabla(j) + k(t-1,j), \sigma_k^2(j)\right), & \sigma_k(j) &\sim \mathrm{N}^+(0,100) \\
\nabla(j) &\sim \mathrm{N}(0,100), & & \\
\Delta\kappa(t,j,g) &\sim \mathrm{N}\left(\rho(j,g)\Delta\kappa(t-1,j,g), \sigma_\kappa^2(j,g)\right), & \sigma_\kappa(j,g) &\sim \mathrm{N}^+(0,100) \\
\rho(j,g) &\sim \mathrm{N}^*(0,100), & \text{s.t. } \rho(j,g) &\in (-1,1) \\
b(x,j) &\sim \mathrm{N}\left(\tfrac{1}{X}, \sigma_b^2(j)\right), & \sigma_b(j) &\sim \mathrm{N}^+(0,100) \\
\beta(x,j,g) &\sim \mathrm{N}\left(\tfrac{1}{X}, \sigma_\beta^2(j,g)\right), & \sigma_\beta(j,g) &\sim \mathrm{N}^+(0,100) \\
\alpha(x,j,g) &\sim \mathrm{N}\left(0, \sigma_\alpha^2(j,g)\right), & \sigma_\alpha(j,g) &\sim \mathrm{N}^+(0,100)
\end{aligned}
\tag{6.8}
$$

where $\nabla(j)$ is the common cause-specific drift term, $\Delta\kappa(t,j,g)$ is the first difference of the period deviations for each country-cause combination, N^* is a truncated Normal distribution, N^+ is a half-Normal distribution, and X again denotes the number of age groups. As $\kappa(t,j,g)$ follows ARIMA(1,1,0), $\Delta\kappa(t,j,g)$ will follow AR(1) process.

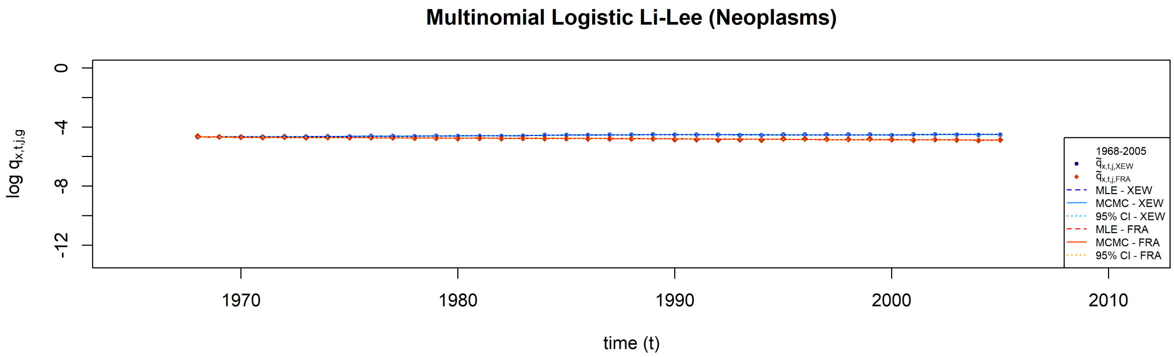
6.2.3 Results

We ran the HMC algorithm via RStan for 4 000 iterations, including 2 000 warm-up iterations, with one half thinning due to RAM constraints. HMC requires significantly lower number of iterations than MCMC for the chains to converge, e.g. HMC took less than 3 days for the MLG-LL model to run with the MLG-LC model converging in only a couple of hours. In comparison the MCMC implementation of the MLG-LC model took 2 days to run whereas the MLG-LL model never converged.

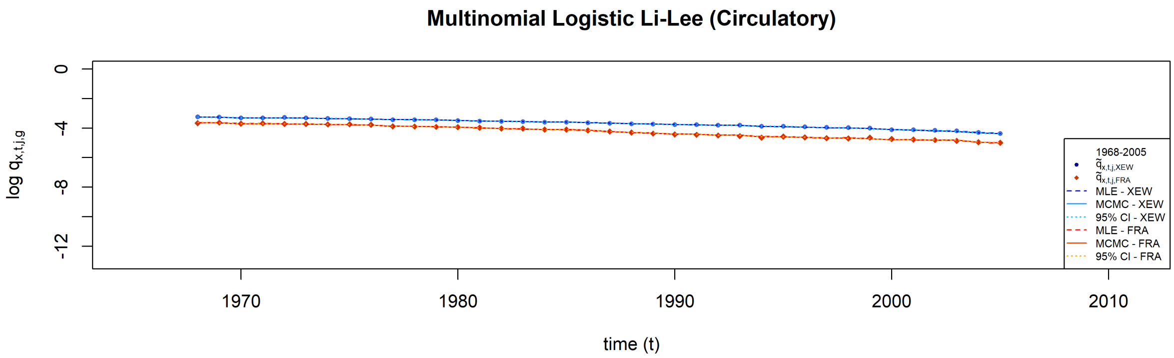
The comparison of the probabilities of death fitted using MLE vs HMC for age group 75-79 is presented in Figure 6.4. We chose this group due to the relatively large number of deaths given its position on the timeline of human mortality. It should be noted that the HMC estimates for the MLG-LL specification follow the approach of Li and Lee (2005) rather than our findings from Section 5.4.4 and are presented here for comparison purposes only.



(a) Infectious and Parasitic Diseases

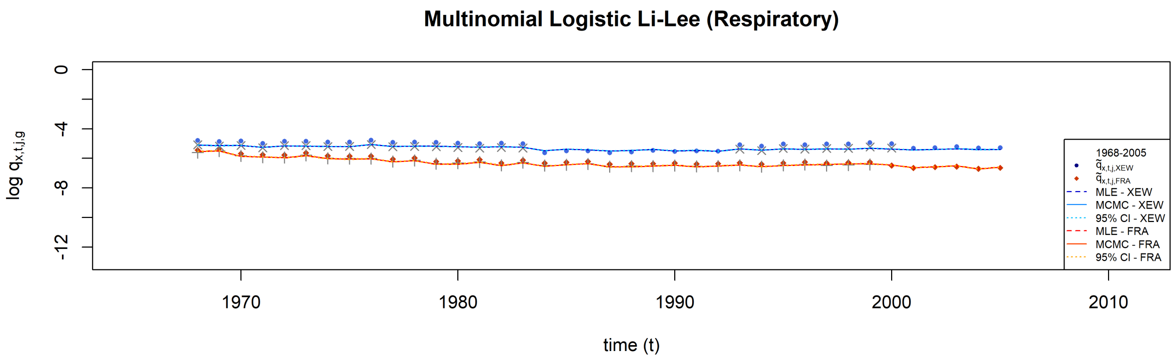


(b) Neoplasms

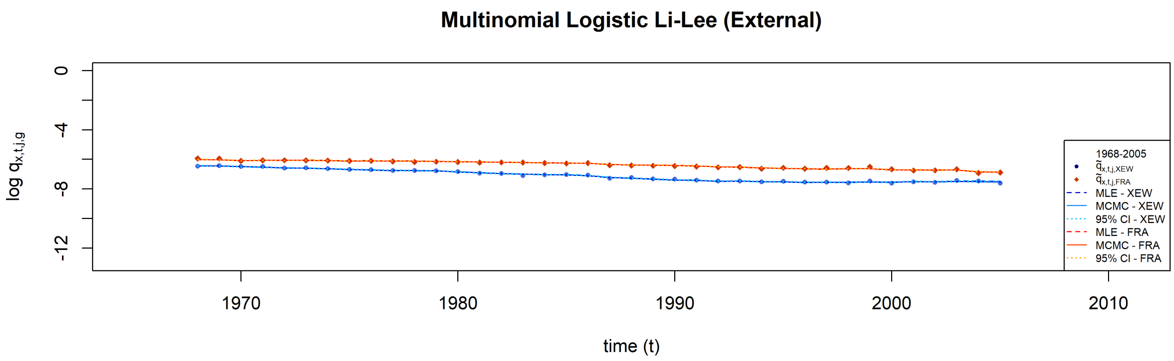


(c) Circulatory Diseases

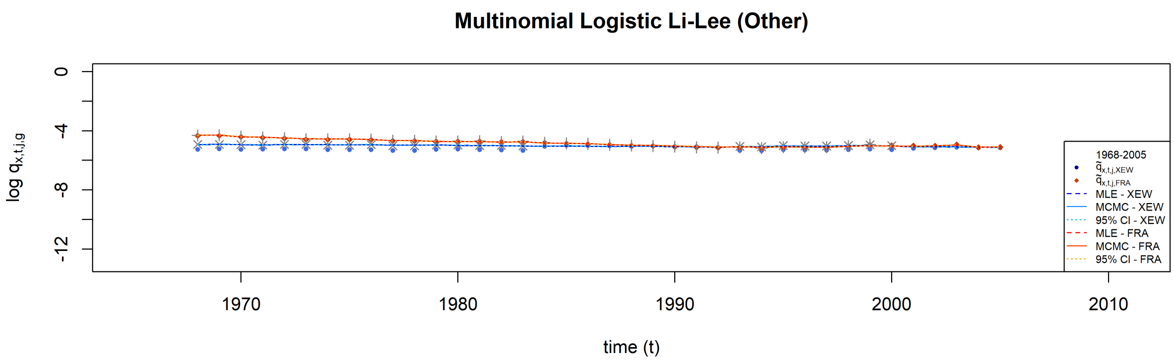
FIGURE 6.4: Log Probability of Death for Age Group 75-79
(MLG-LL (M5), England & Wales and France, 1968-2005)



(d) Respiratory Diseases



(e) External Causes



(f) Other Causes

FIGURE 6.4 (CONT.)

We can see that both approaches yield comparable values of $\hat{q}(x,t,j,g)$ with the benefit of HMC providing a direct method to evaluate the credible intervals for the parameter estimates. The mortality from Circulatory diseases and External causes for England & Wales and France moves largely in line with the French mortality being noticeably lower. There is a sign of convergence of the probabilities of death for the 'Infectious and parasitic diseases' group and especially the 'Other causes' group for the two countries. Meanwhile the probabilities of death due to Neoplasms in France are staying largely constant over time while the estimates for England & Wales are increasing for the past few decades.

Furthermore, we assessed the performance of the above model specifications using statistical criteria, e.g. mean squared error (MSE). The formula for MSE that we used is given in (6.9):

$$MSE_q(j,g) = \frac{1}{X \times T} \sum_{x=1}^{X} \sum_{t=1}^{T} \left[\hat{q}(x,t,j,g) - \tilde{q}(x,t,j,g)\right]^2 \tag{6.9}$$

where X is the number of age groups, T is the number of years, $\tilde{q}(x,t,j,g)$ is the observed probability of death, and $\hat{q}(x,t,j,g)$ is the fitted probability of death. Since we can think of death probabilities as transition probabilities, MSE can also be calculated for survival probabilities as follows:

$$MSE_p(g) = \frac{1}{X \times T} \sum_{x=1}^{X} \sum_{t=1}^{T} \left[\hat{p}(x,t,g) - \tilde{p}(x,t,g)\right]^2 \tag{6.10}$$

where $\tilde{p}(x,t,g)$ is the observed probability of survival and $\hat{p}(x,t,g)$ is the fitted survival probability. All other variables are defined as before.

We have calculated the MSEs of the fitted survival probabilities, probabilities of death for each of the six cause groups, and overall fit for each model specification, MLG-LC and MLG-LL, as well as estimation technique: MLE and HMC. These are presented for England & Wales in Table 6.3 and for France in Table 6.4.

TABLE 6.3: MSES FOR SURVIVAL AND DEATH PROBABILITIES FOR EACH MODEL SPECIFICATION – ENGLAND & WALES

	MLG-LC		**MLG-LL**	
Cause of Death	**MLE**	**HMC**	**MLE**	**HMC**
Alive	1.34×10^{-06}	1.26×10^{-06}	3.77×10^{-07}	4.22×10^{-07}
Infectious	1.72×10^{-10}	1.50×10^{-10}	1.13×10^{-10}	1.15×10^{-10}
Neoplasms	1.01×10^{-07}	1.07×10^{-07}	3.89×10^{-08}	3.82×10^{-08}
Circulatory	8.64×10^{-07}	8.05×10^{-07}	1.71×10^{-07}	1.65×10^{-07}
Respiratory	3.04×10^{-07}	2.89×10^{-07}	4.31×10^{-08}	4.10×10^{-08}
External	8.68×10^{-09}	8.67×10^{-09}	1.11×10^{-09}	8.47×10^{-10}
Other	7.92×10^{-08}	8.07×10^{-08}	2.02×10^{-08}	2.56×10^{-08}
Overall	3.85×10^{-07}	3.64×10^{-07}	9.31×10^{-08}	9.90×10^{-08}

The ratios of the MSEs for the MLE vs the HMC estimates for England & Wales are generally close to one. The notable exception being the HMC estimates of the MLG-LL model for External causes, which are 0.76 times those of the MLEs.

TABLE 6.4: MSES FOR SURVIVAL AND DEATH PROBABILITIES FOR EACH MODEL SPECIFICATION – FRANCE

	MLG-LC		**MLG-LL**	
Cause of Death	**MLE**	**HMC**	**MLE**	**HMC**
Alive	2.03×10^{-06}	2.01×10^{-06}	5.48×10^{-07}	4.00×10^{-07}
Infectious	3.24×10^{-09}	3.36×10^{-09}	9.25×10^{-10}	1.45×10^{-09}
Neoplasms	2.74×10^{-08}	2.82×10^{-08}	2.44×10^{-08}	2.79×10^{-08}
Circulatory	3.57×10^{-07}	3.67×10^{-07}	1.92×10^{-07}	6.93×10^{-08}
Respiratory	6.23×10^{-08}	6.23×10^{-08}	2.04×10^{-08}	1.68×10^{-08}
External	1.16×10^{-08}	1.01×10^{-08}	2.57×10^{-09}	2.62×10^{-09}
Other	6.55×10^{-07}	6.51×10^{-07}	4.76×10^{-08}	4.43×10^{-08}
Overall	4.49×10^{-07}	4.47×10^{-07}	1.19×10^{-07}	8.04×10^{-08}

The MSE ratios for the MLG-LC model for France also tend to be close to one as is the case for England & Wales. The MSEs for the French HMC mortality estimates from the MLG-LL model for Infectious diseases and Neoplasms, however, are 1.57 and 1.14 times those obtained via MLE, respectively. The MSE for the French HMC estimates for the same model for Circulatory diseases is 0.36 times that of the MLE and the overall HMC MSE is 0.67 times that of the MLE. This is likely due to the restrictions imposed on the priors in order to ensure model convergence.

Overall we find that the MLG-LL model outperforms the MLG-LC model when applied to mortality data for both England & Wales and for France. The MLG-LL model performs much better particularly for Circulatory diseases group in both countries, for External causes of death in England & Wales, and for Other causes of death in France. We do note that the confidence intervals for the HMC estimates are extremely small and further work to incorporate additional parameter uncertainty through the prior distributions will be needed.

Chapter 7

Projections of Cause-Specific Mortality in England & Wales and France

Both the MLG-LC and MLG-LL models provide for a natural extension to future projections as only the period effects need to be predicted; the age effects are assumed to remain unchanged in the short term. The Bayesian approach makes this straightforward as the distributions of the time series parameters, e.g. variance and drift terms, can be estimated by the model rather than be imposed by assumption. Furthermore, a time series is available for each cause of death at each MCMC/HMC iteration – a fact that we can exploit to quantify forecast uncertainty.

The data we have used to fit the models in previous chapters cover the period from 1968 to 2005. We held out the mortality data for years 2006 to 2014 to allow us to assess the performance of the forecasts by calculating the coverage of the prediction intervals (PIs) of the projections. The models were implemented via the RStan interface using largely the methodology described in Section 6.2.

In Section 7.1 we explain the methodology for projecting the period-effect $\kappa(t, j, g)$ within the MLG-LC specification and present the parameter estimates as well as projected probabilities of death for females in both England & Wales and France. The projections of the common cause-specific period-effect $k(t, j)$ as well as the country-specific $\kappa(t, j, g)$ for each cause are presented in Section 7.2. The resulting projected probabilities of death are then shown in Subsection 7.2.2.

7.1 Projections Using the Multinomial Logistic Lee-Carter Model

We propose to forecast the $\kappa(t,j,g)$ period effects for the LC specification of the MLG model using the same ARIMA specifications as in Section 6.2. (These are the specifications selected in Section 5.3.3, which can be found in Tables 5.7 & 5.8.) For example, where the selected specification is ARIMA(0,1,0) this can be modelled using RWD as follows:

$$\kappa(t,j,g) = \nabla(j,g) + \kappa(t-1,j,g) + \epsilon(t,j,g), \qquad \epsilon(t,j,g) \sim \mathrm{N}\left(0, \sigma^2_\kappa(j,g)\right) \tag{7.1}$$

where $\nabla(j,g)$ is the cause-specific constant, i.e. drift term, and $\epsilon(t,j,g)$ is the cause-specific random noise or error term at time t. Similarly, for causes where the specification is ARIMA(1,1,0) the time series is forecast using the iterated process

$$\kappa(t,j,g) = \nabla(j,g) + \kappa(t-1,j,g) + \psi(j,g)[\kappa(t-1,j,g) - \kappa(t-2,j,g)] + \epsilon(t,j,g), \qquad \epsilon(t,j,g) \sim \mathrm{N}\left(0, \sigma^2_\kappa(j,g)\right) \tag{7.2}$$

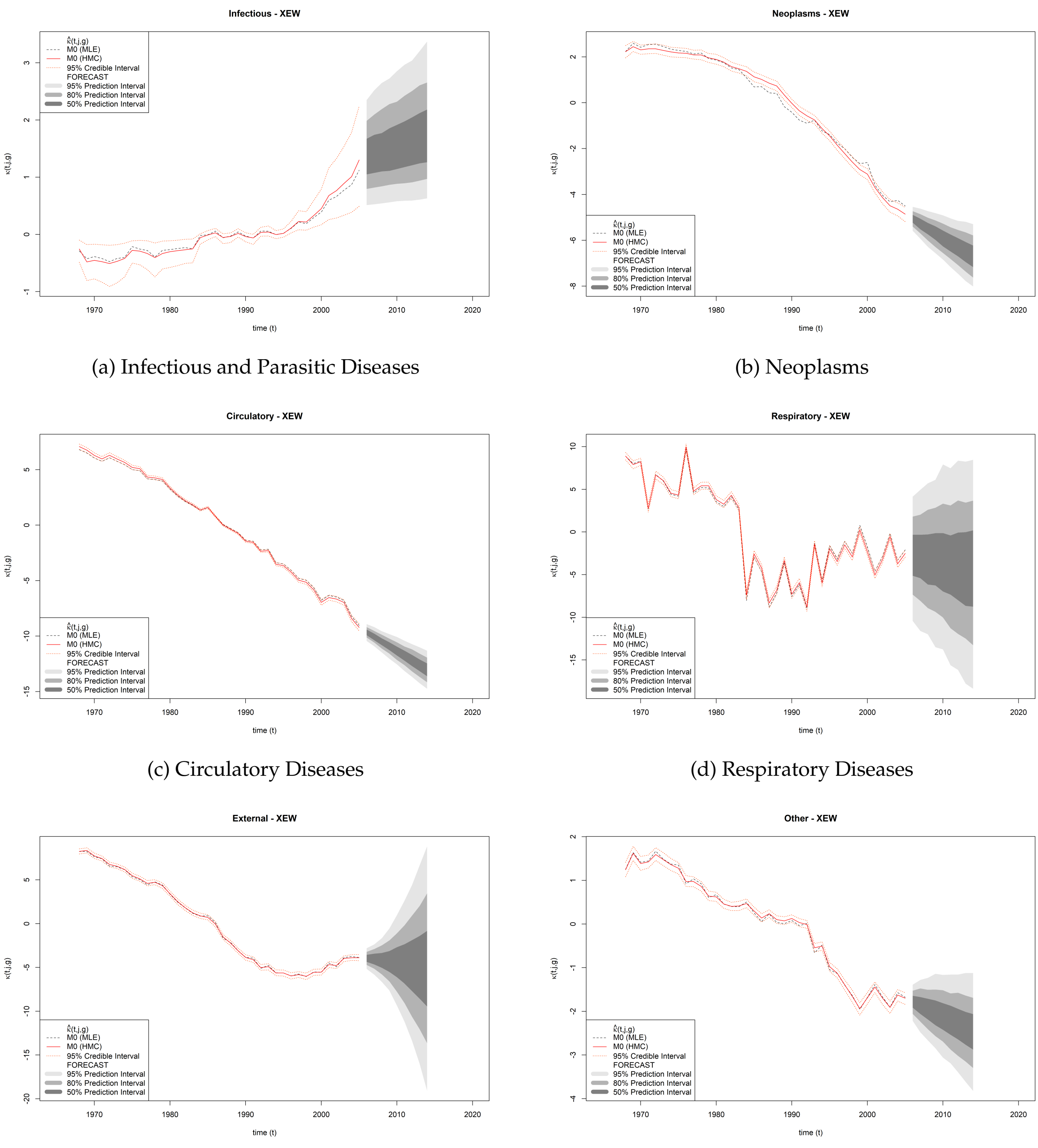
where $\nabla(j,g)$ is the cause-specific drift term, $\psi(j,g)$ is the cause-specific autoregressive term, and $\epsilon(t,j,g)$ is the cause-specific random noise or error term at time t. Lastly, the ARIMA(0,1,1) time series specification can be forecast using

$$\kappa(t,j,g) = \nabla(j,g) + \kappa(t-1,j,g) + \theta(j,g)\epsilon(t-1,j,g) + \epsilon(t,j,g), \qquad \epsilon(t,j,g) \sim \mathrm{N}\left(0, \sigma^2_\kappa(j,g)\right) \tag{7.3}$$

where $\nabla(j,g)$ is the cause-specific drift term, $\theta(j,g)$ is the cause-specific moving average term, and $\epsilon(t,j,g)$ is the cause-specific random noise or error term at time t.

7.1.1 Period-Effect Parameter Projections for England & Wales using MLG-LC

The fitted values of the time-specific $\kappa(t,j,g)$ parameters for England & Wales are shown in Figure 7.1. The dark grey area indicates the 50% prediction interval, the lighter grey area contains the 80% prediction interval, and the light grey area contains the 95% prediction interval.



(a) Infectious and Parasitic Diseases

(b) Neoplasms

(c) Circulatory Diseases

(d) Respiratory Diseases

(e) External Causes

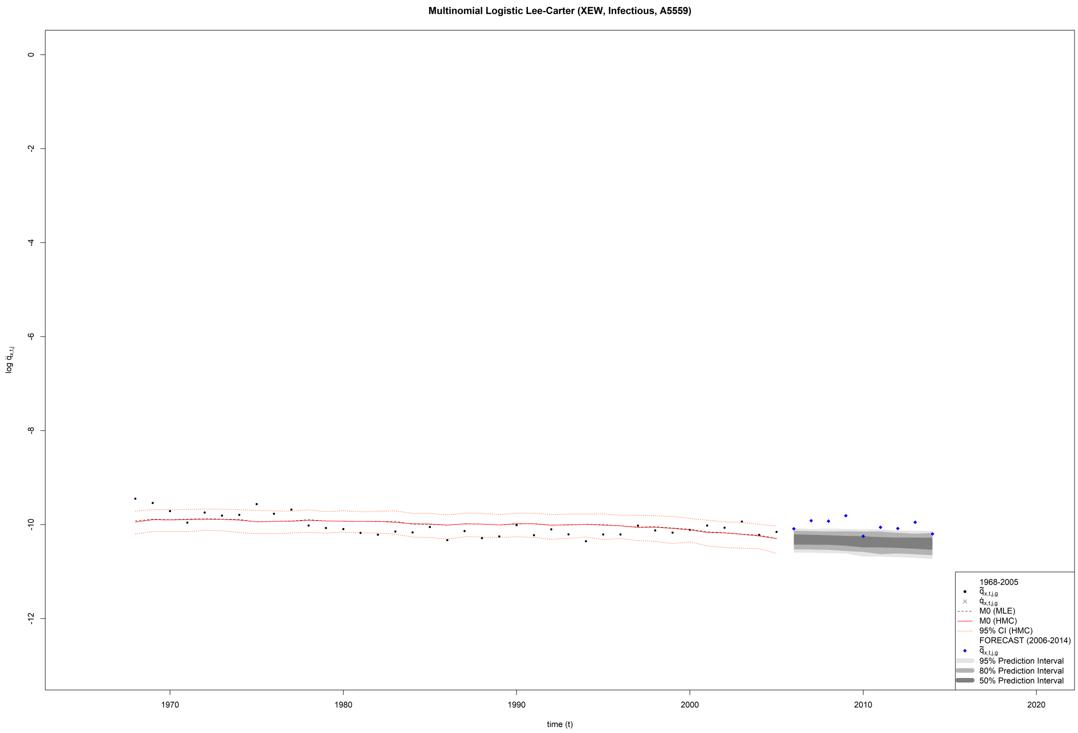
(f) Other Causes

FIGURE 7.1: $\kappa(t, j, g)$ coefficients fitted using MLG-LC for years 1968-2005 and their prediction intervals for years 2006-2014 (England & Wales)

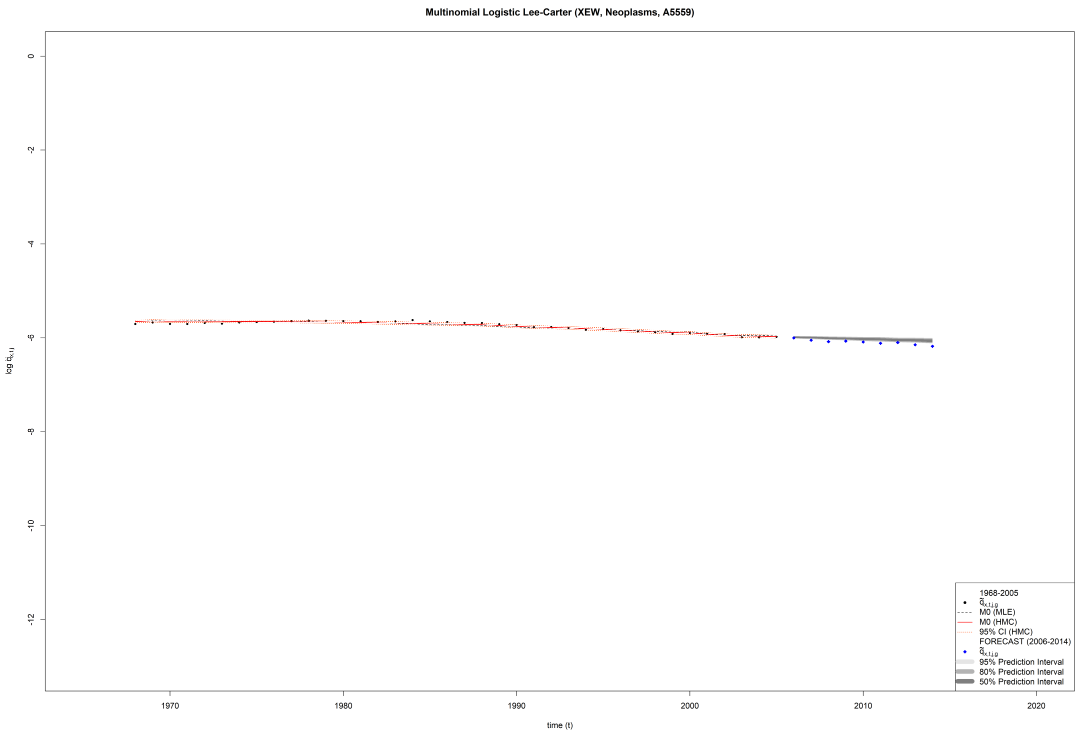
The overall trend is for $\kappa(t, j, g)$ to decrease over time with one notable exception being infectious and parasitic diseases in England & Wales. One of the contributing factors to the sudden rise in mortality due to this group was the outbreak of Clostridium difficile in NHS England that started in 2005 (Duerden, 2011). As this is the last year in the data used to fit the model, the resulting forecast for England & Wales is much steeper than would have been the case otherwise. While a comparability ratio was applied to respiratory deaths for age groups 50-54 and older, a noticeable dip remains during the years 1984 until 1992. This in turn results in a slight spike in the values of $\kappa(t, j, g)$ for other causes, which manifests itself as a drop in 1993.

7.1.2 Projected Probabilities of Death for England & Wales using MLG-LC

The projections of the probabilities of death due to the various causes in England & Wales for ages 55-59 and 80-84 are presented in Figures 7.2 and 7.3, respectively. Additional plots for age groups 20-24, 40-44, 60-64, and 85+ are presented in Appendix F.1.

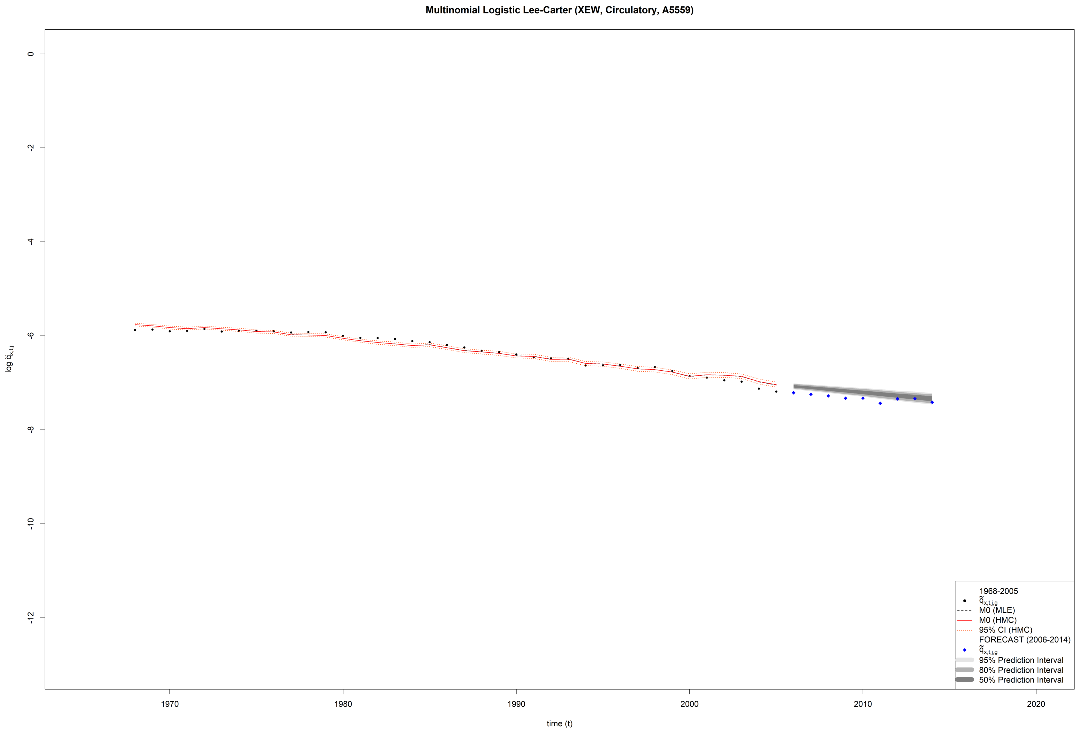


(a) Infectious and Parasitic Diseases

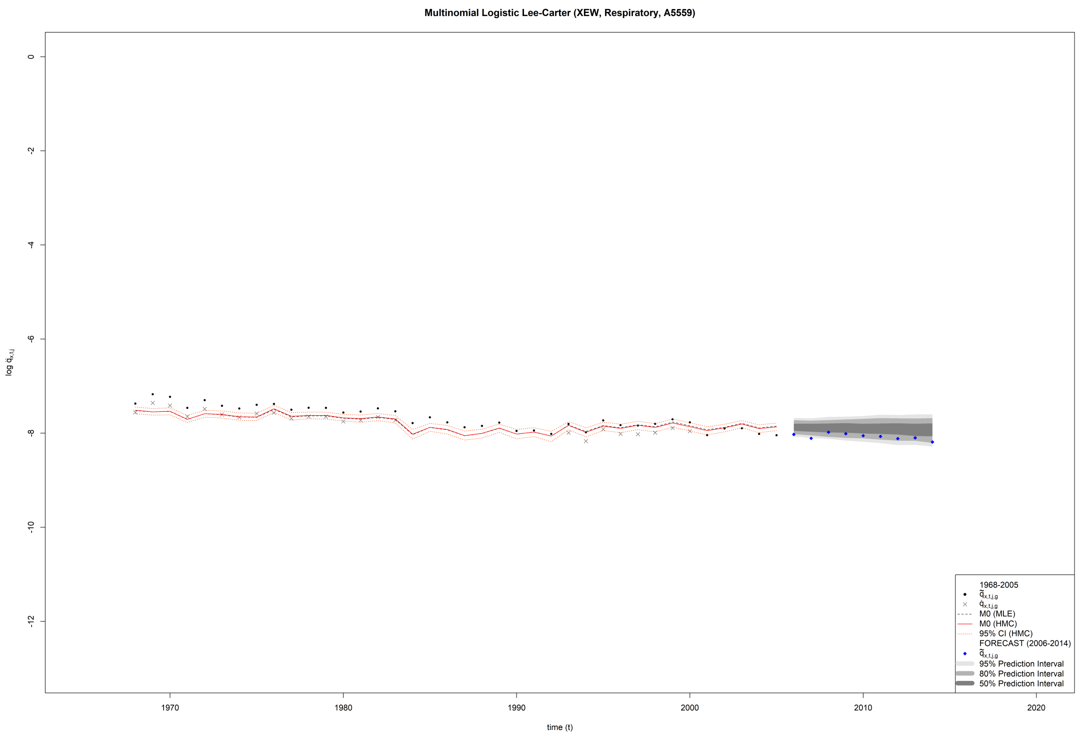


(b) Neoplasms

FIGURE 7.2: Probability of Death Forecasts for Age Group 55-59 (MLG-LC (M0), England and Wales, 2006-2014)

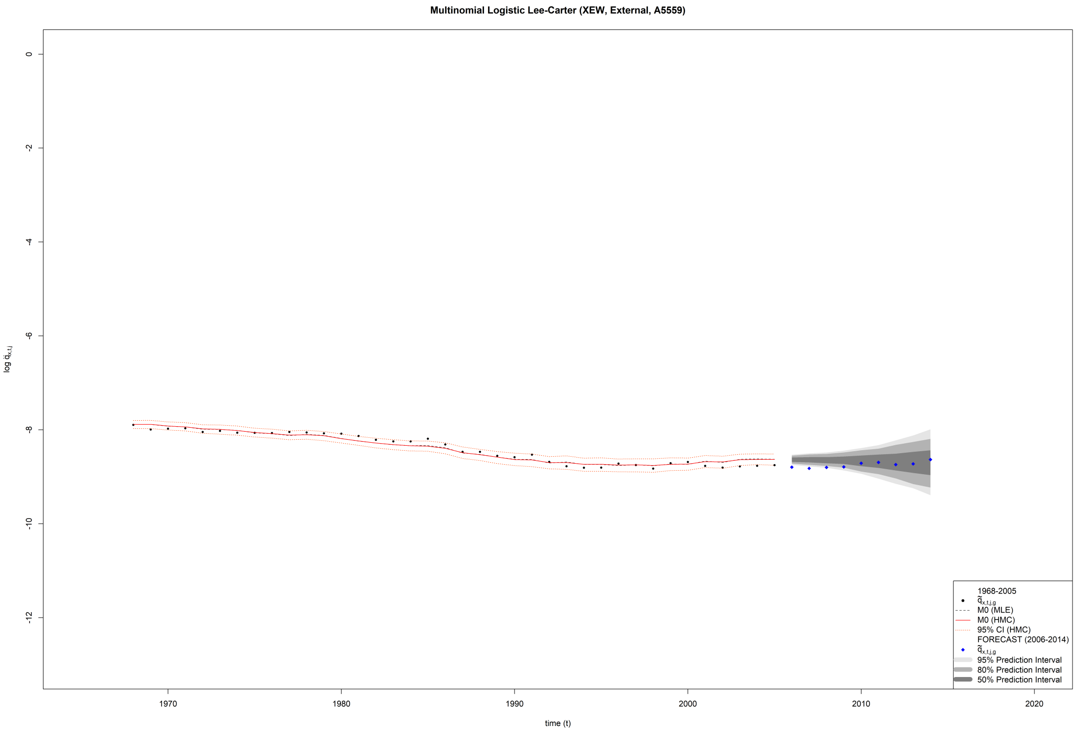


(c) Circulatory Diseases

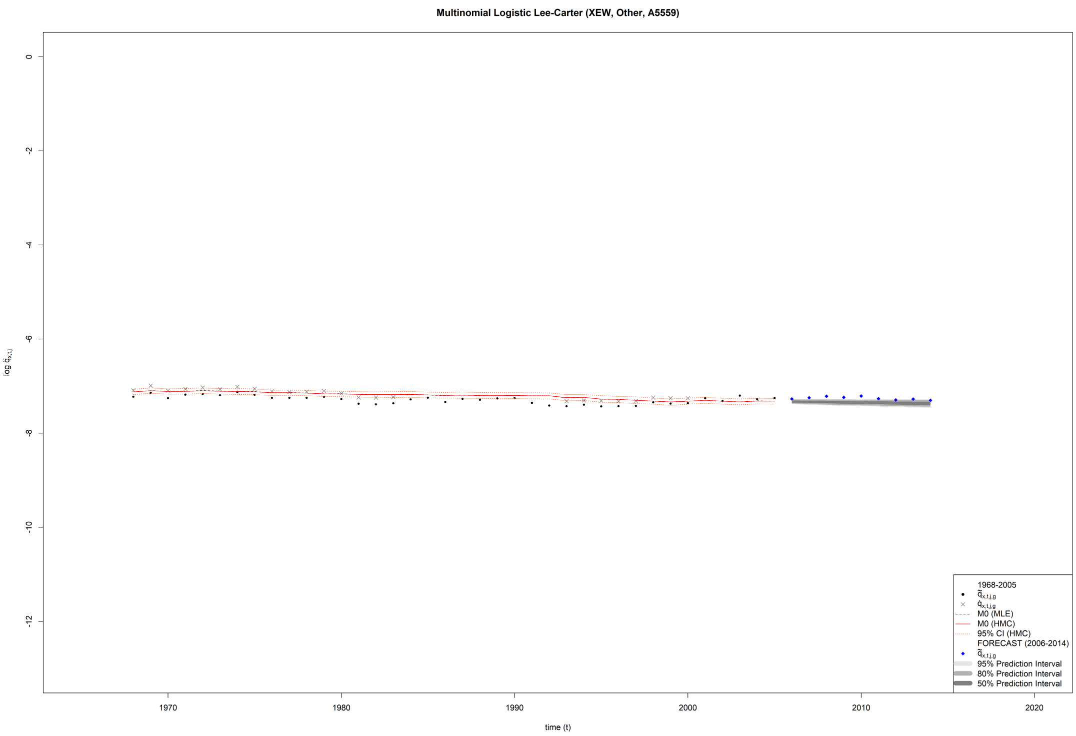


(d) Respiratory Diseases

FIGURE 7.2 (CONT.)

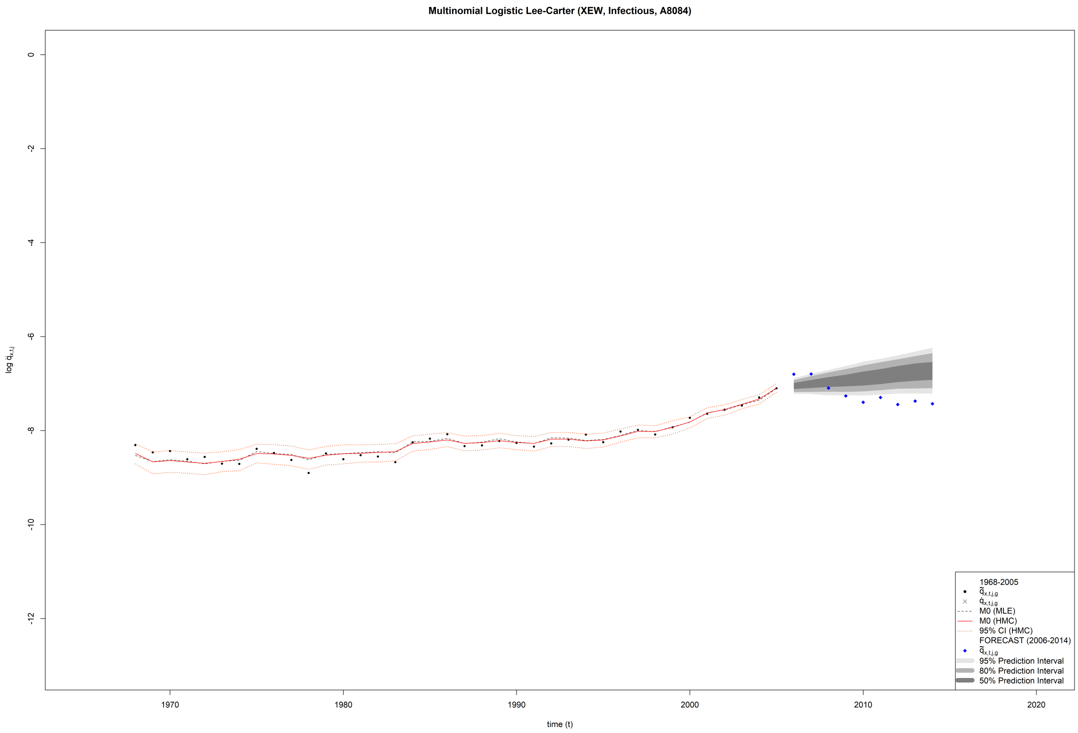


(e) External Causes

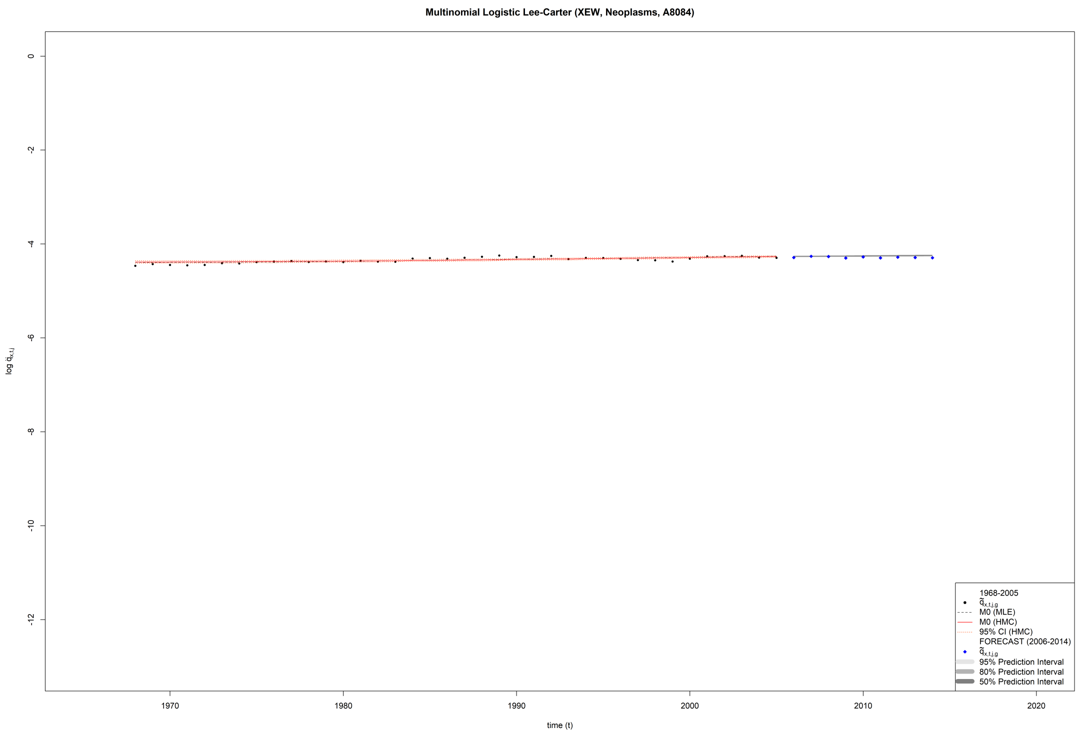


(f) Other Causes

FIGURE 7.2 (CONT.)

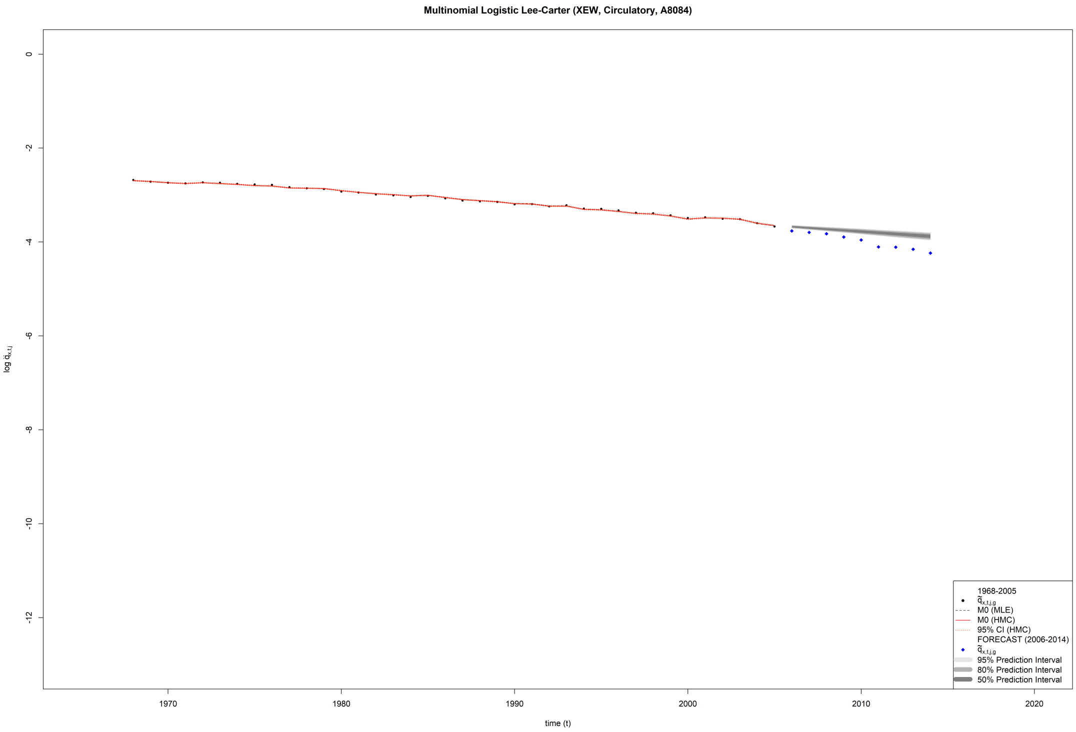


(a) Infectious and Parasitic Diseases

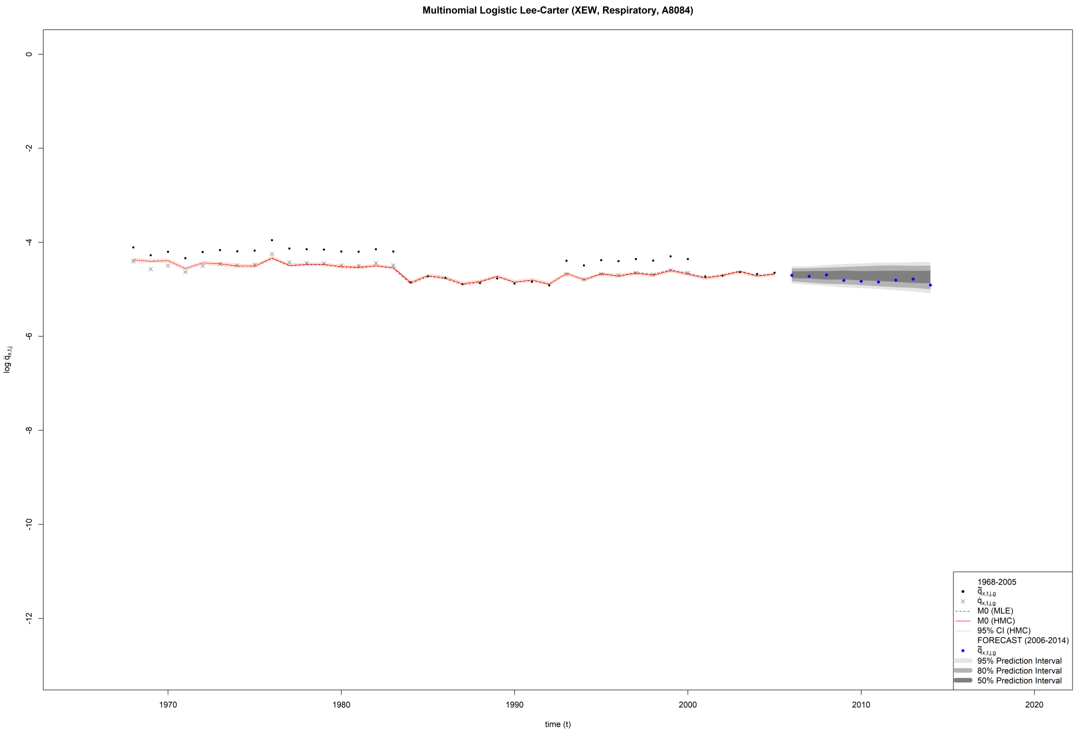


(b) Neoplasms

FIGURE 7.3: Probability of Death Forecasts for Age Group 80-84
(MLG-LC (M0), England and Wales, 2006-2014)

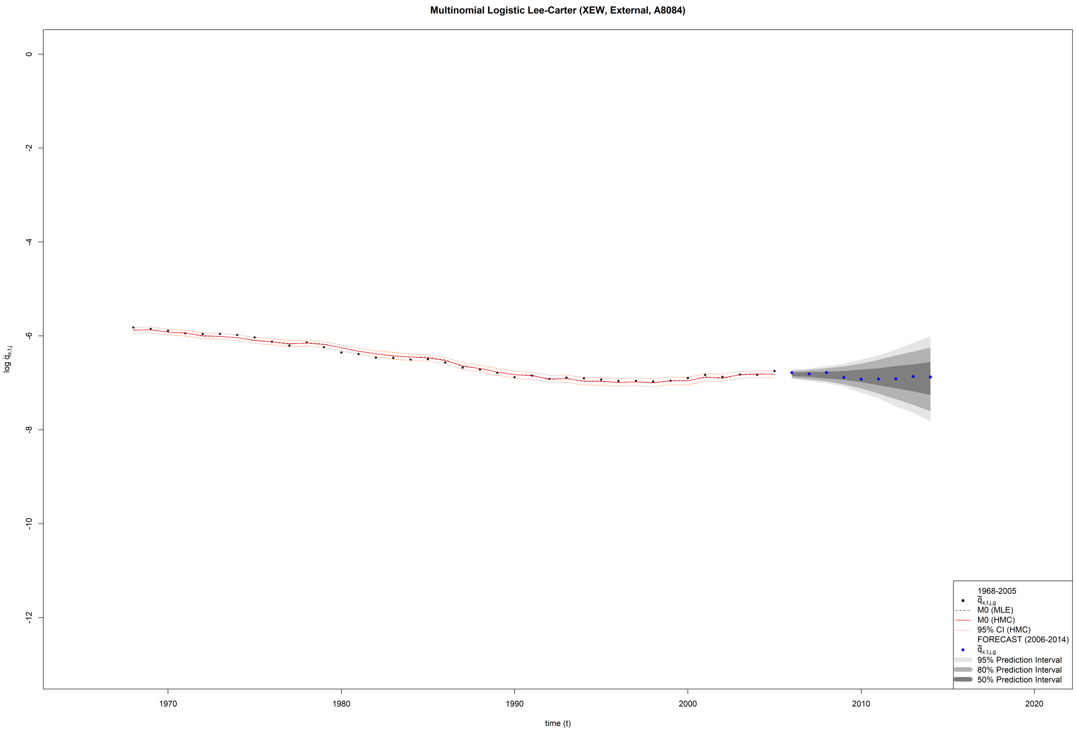


(c) Circulatory Diseases

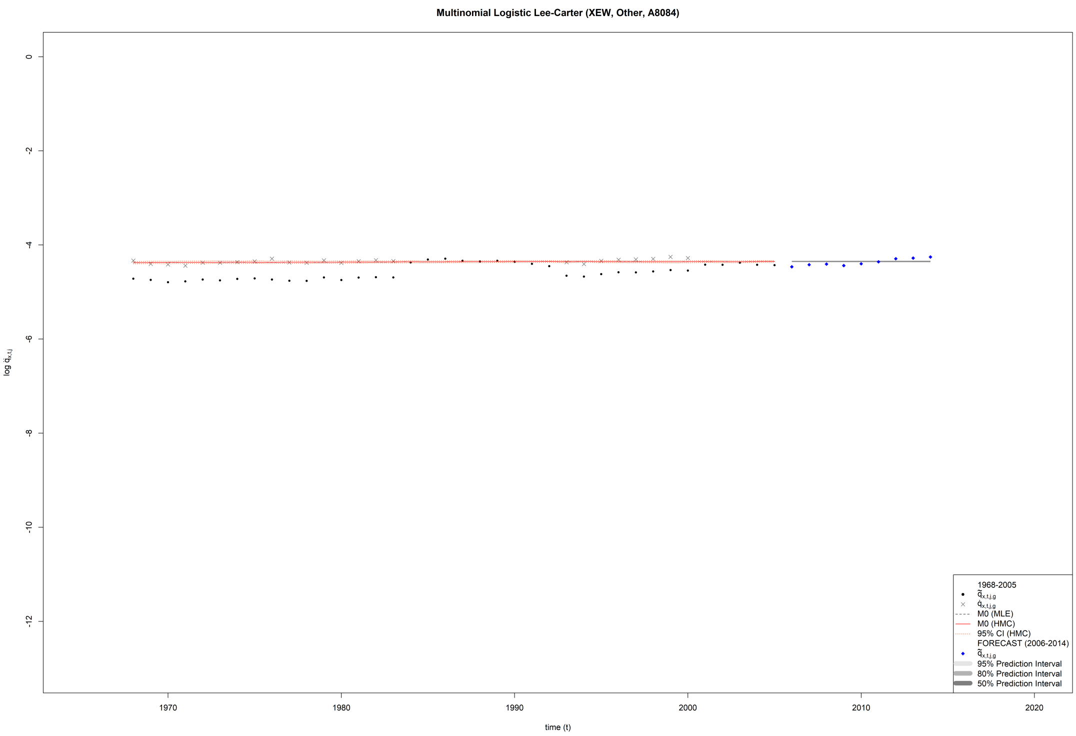


(d) Respiratory Diseases

FIGURE 7.3 (CONT.)



(e) External Causes



(f) Other Causes

FIGURE 7.3 (CONT.)

Our forecasts capture most of the crude probabilities for four of the causes of death for higher ages. Even the projected mortality probabilities for respiratory diseases fit the data well for age groups 20-24 and older contrary to the experience of Girosi and King (2007). The forecasts for circulatory deaths follows the long-term trend and therefore fails to take into account the fall in the incidence of coronary heart disease leading to a notable reduction in deaths in the 2000s (Mayor, 2015; Conrad et al., 2024). Furthermore, the forecasts for group of causes 1 – infectious and parasitic diseases – underestimate the mortality due to this cause. At the peak of the 'superbug' epidemic in 2007, 49% of deaths classified under infectious and parasitic diseases (0.8% of total deaths) were attributed to enterocolitis due to C. difficile (ICD-10 A04.7). Furthermore, the severity of the spread of C. difficile did not change following the introduction of additional infection control procedures, e.g. increased hand washing and cleaning (Valiquette et al., 2007; Dingle et al., 2017). Research by Dingle et al. (2017) has shown that the rise of C. difficile was the result of overprescribing of fluoroquinolones, a class of antibiotics, rather than poor hygiene in hospitals or other causes. On the other hand, 'superbugs' such as meticillin-resistant Staphylococcus aureus (MRSA) and norovirus appear to be susceptible to copper surfaces and their increased use in hospital may lead to a decrease in incidence (Noyce, Michels, and Keevil, 2006; Warnes and Keevil, 2013). The unpredictability of outbreaks of newly identified diseases makes the forecasting of this category difficult.

In addition to 'superbugs', there are other shocks that are likely to influence infectious mortality. While the US declared measles to be eradicated in 2000, there were 16 outbreaks of the disease during 2024 (CDC, 2024; 2025). Meanwhile Trentini et al. (2019) predict a 50% increase in the proportion of population at risk of a measles infection in the UK by 2050 under the current vaccination programmes. While only a single death was attributed to measles (ICD-10 B05) in 2017, the future trend will depend on government action in regards to vaccination policies (ONS, 2018a). The same can be said about other viruses such as SARS-Cov-2 that started the COVID-19 pandemic at the end of 2019 and the monkeypox virus that caused the Mpox outbreak in 2022 (WHO, 2024).

7.1.3 Coverage of MLG-LC Projections for England & Wales

We have calculated the coverage of the prediction intervals for the ARIMA model specifications from Chapter 5 (see Table 5.7) and present them in Table 7.1. Coverage here denotes the percentage of observations from the hold out sample (years 2006 to 2014) that fall within the given prediction interval.

TABLE 7.1: COVERAGE OF PREDICTION INTERVALS FROM ARIMA, ENGLAND & WALES

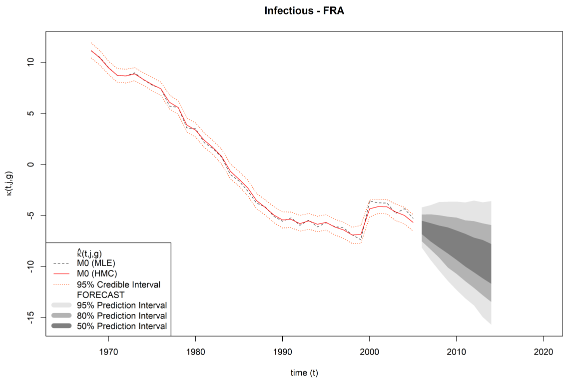
	Prediction Interval			
Cause of Death	50%	80%	90%	95%
Infectious	18.33%	40.00%	49.44%	57.22%
Neoplasms	21.67%	41.11%	54.44%	58.89%
Circulatory	12.78%	28.89%	35.00%	41.11%
Respiratory	36.67%	67.78%	73.33%	80.56%
External	32.22%	48.33%	55.00%	60.00%
Other	23.89%	40.56%	49.44%	57.78%
Alive	19.44%	30.56%	36.67%	42.78%
Overall	23.57%	42.46%	50.48%	56.90%

We note that the overall coverage of our prediction intervals is low. In order to ensure convergence of the model specifications with autoregressive terms, we had to constrain the range of possible values for the autoregressive parameters to the interval (-1,1). This has likely limited the posterior distribution that the HMC algorithm could explore. The forecast coverage is highest for respiratory diseases owing to the much wider PIs. On the other hand, the coverage of the forecasts for circulatory diseases is lowest among the six groups of causes of death. This is not surprising given the changes in mortality discussed in Section 7.1.2. The 'Circulatory diseases' group does, however, have better coverage at younger ages due to wider PIs. Although the group 'External causes' has second highest coverage of the six causes of death, it has worse coverage for younger ages – likely due to the reduction of the accident hump in future years.

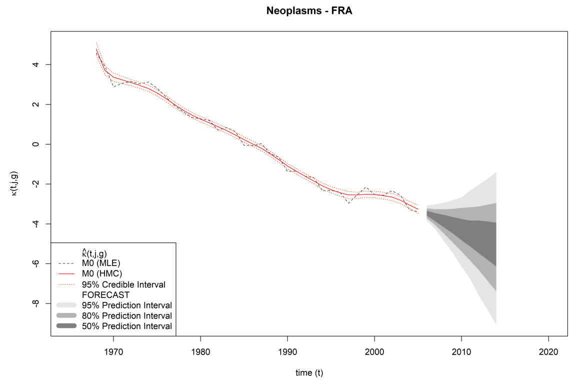
The coverage of our prediction intervals could be improved by increasing the uncertainty through the prior specifications listed in Section 6.2.1; however, this would likely cause problems with the convergence of the HMC algorithm. A further possibility would be to include variability of the comparability ratios into our model as this would introduce additional uncertainty into the model. We can already see the impact of this on the prediction intervals for respiratory deaths in England & Wales where the coverage is much higher than for the other groups.

7.1.4 Period-Effect Parameter Projections for France using MLG-LC

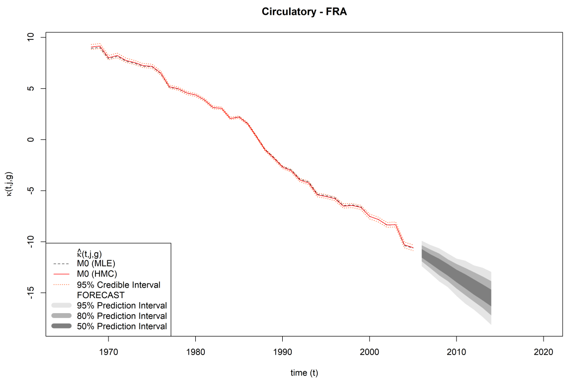
The ARIMA specifications for France follow those listed in Table 5.8. Just like the $\kappa(t, j, g)$ time series for England & Wales presented in Section 7.1.1, the ARIMA specifications for all French causes incorporate at least one order of differencing. The fitted values of the time-specific $\kappa(t, j, g)$ parameters for France are shown in Figure 7.4. The dark grey area indicates the 50% prediction interval, the lighter grey area contains the 80% prediction interval, and the light grey area contains the 95% prediction interval.



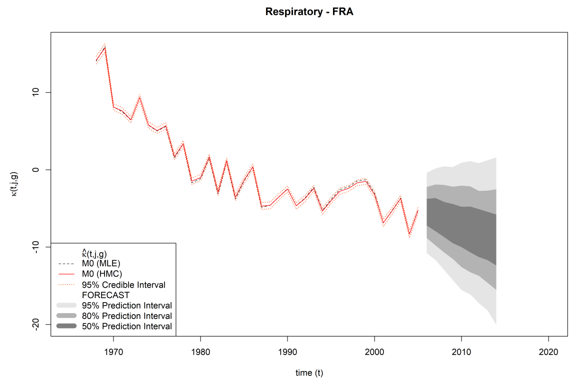
(a) Infectious and Parasitic Diseases



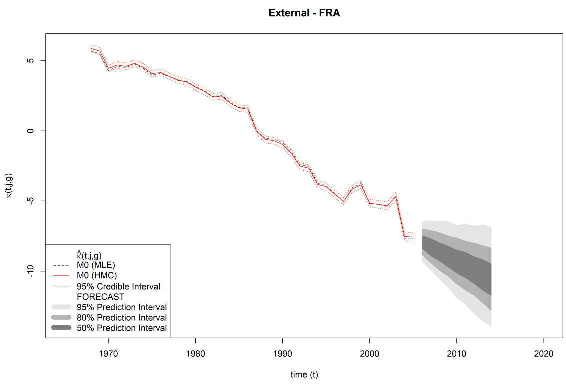
(b) Neoplasms



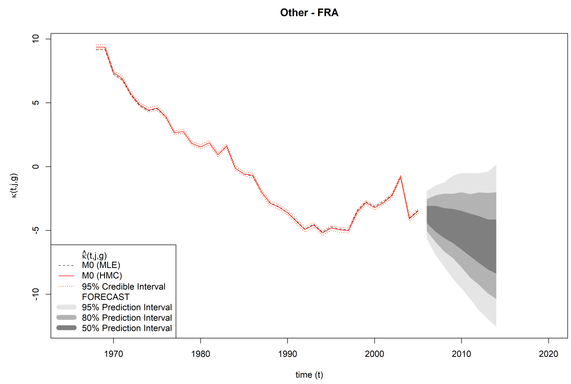
(c) Circulatory Diseases



(d) Respiratory Diseases



(e) External Causes



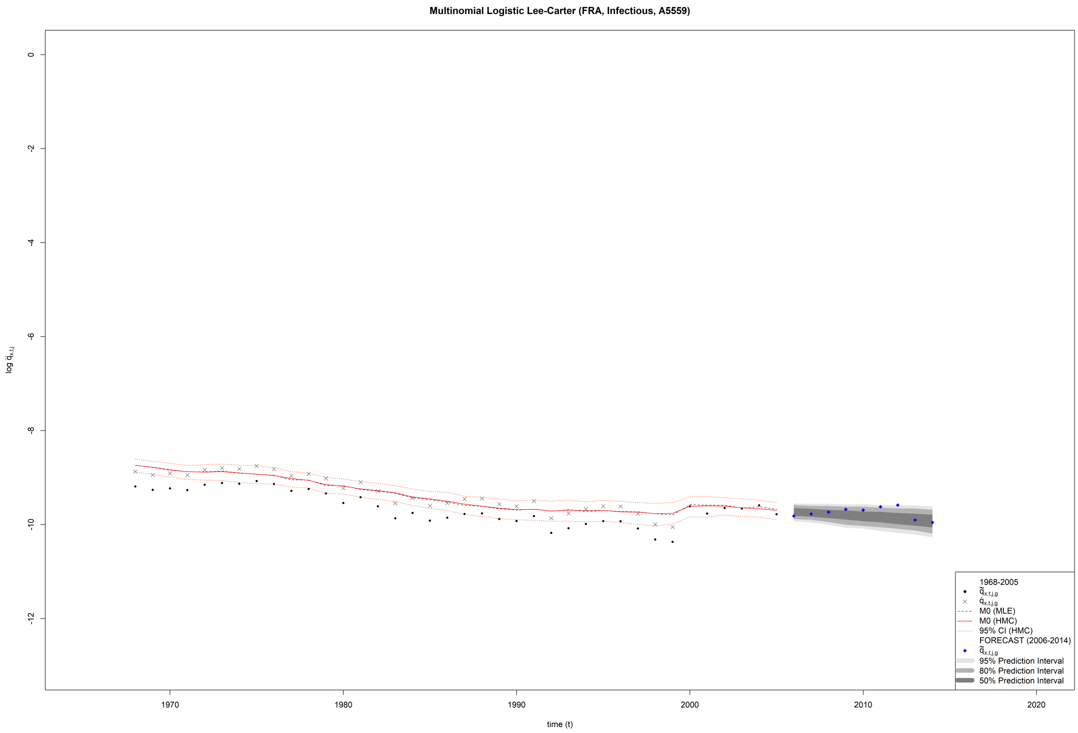
(f) Other Causes

FIGURE 7.4: $\kappa(t, j, g)$ coefficients fitted using MLG-LC for years 1968-2005 and their prediction intervals for years 2006-2014 (France)

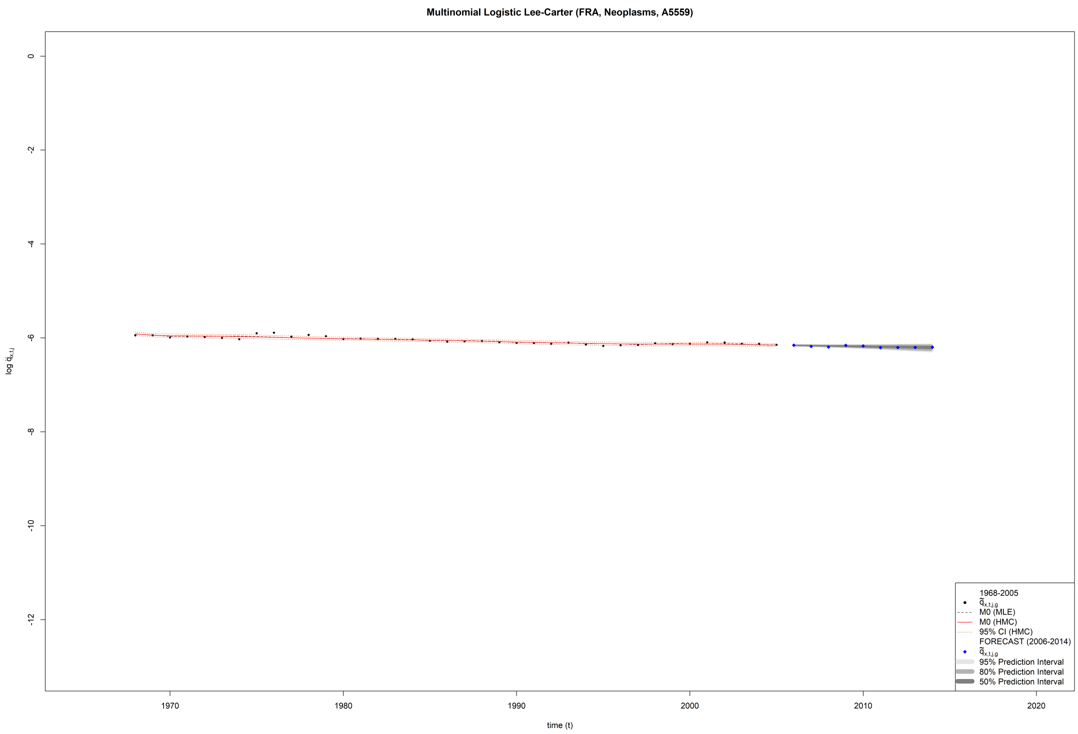
All six $\kappa(t, j, g)$ parameters for France exhibit a downward trend and this is reflected in their projections. Unlike in England & Wales, the French data does not experience an increase in the period-effect parameter for deaths due to infectious diseases although there is a noticeable jump in the year 2000 due to the change from ICD-9 to ICD-10. Another notable difference in the trends between the two countries is the jagged pattern of the $\kappa(t, j, g)$ parameter for respiratory diseases. This trend in the period-effect for France is a result of fluctuations in the prevalence of respiratory viruses during winter rather than any particular coding change as was the case for England & Wales.

7.1.5 Projected Probabilities of Death for France using MLG-LC

The forecasts for the probability of death due to the various causes in France for ages 55-59 and 80-84 are presented in Figures 7.5 and 7.6, respectively. Additional plots for age groups 20-24, 40-44, 60-44, and 85+ are presented in Appendix F.2.

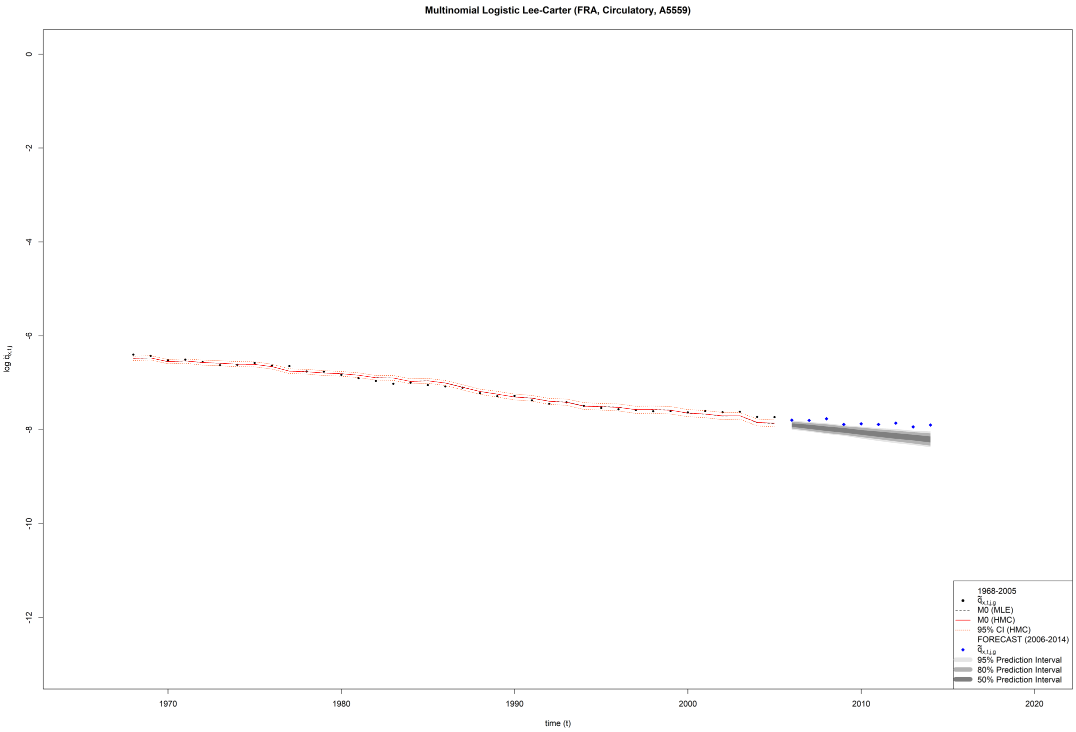


(a) Infectious and Parasitic Diseases

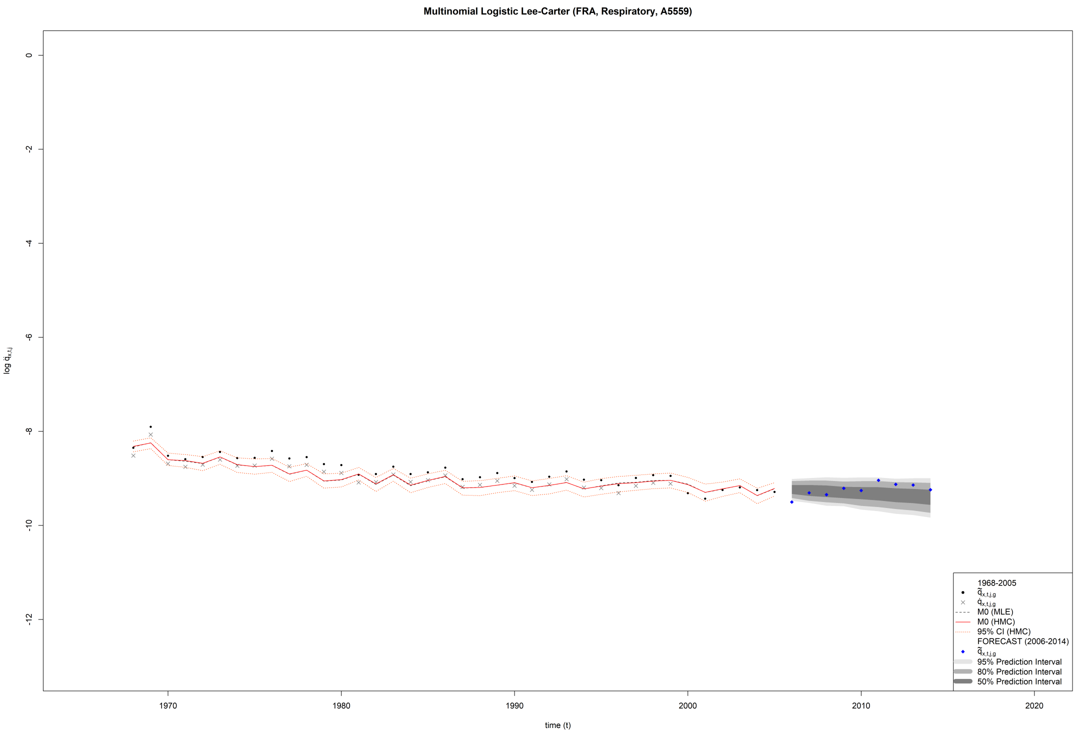


(b) Neoplasms

FIGURE 7.5: Probability of Death Forecasts for Age Group 55-59
(MLG-LC (M0), France, 2006-2014)

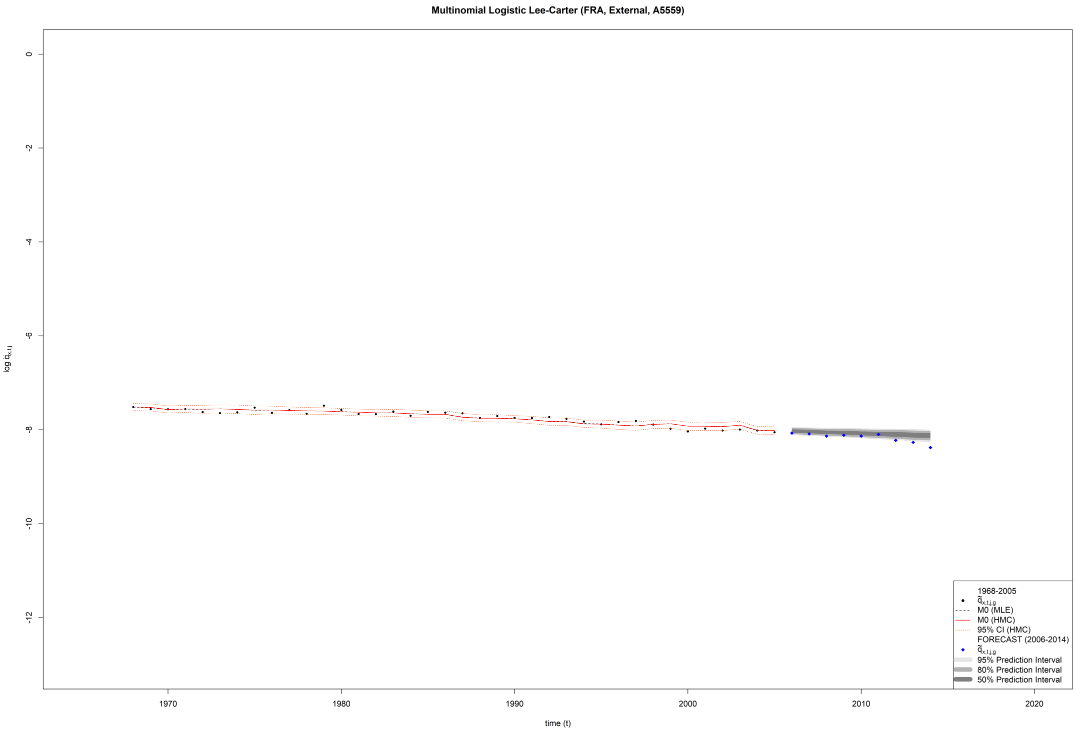


(c) Circulatory Diseases

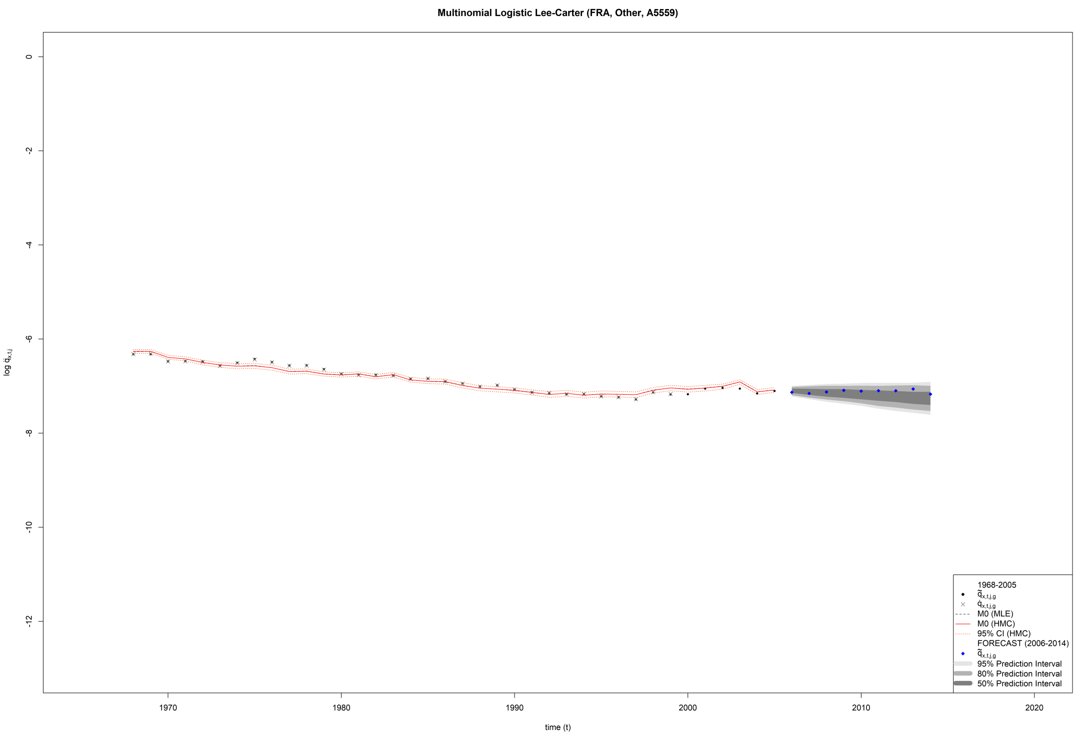


(d) Respiratory Diseases

FIGURE 7.5 (CONT.)

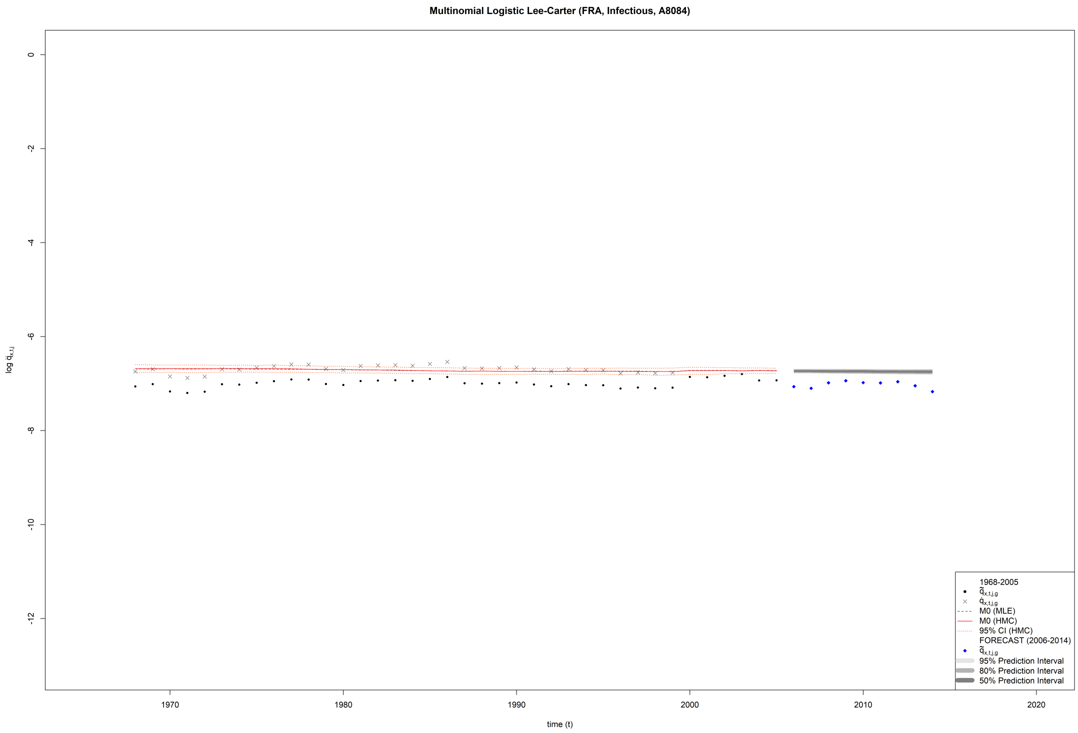


(e) External Causes

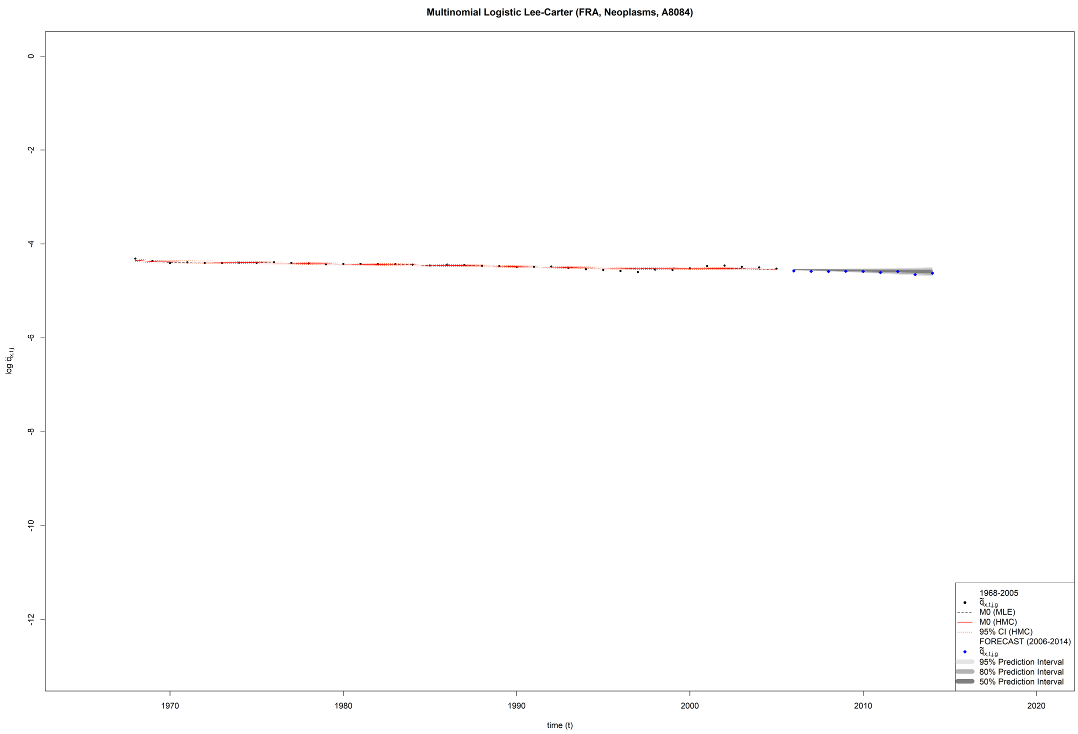


(f) Other Causes

FIGURE 7.5 (CONT.)

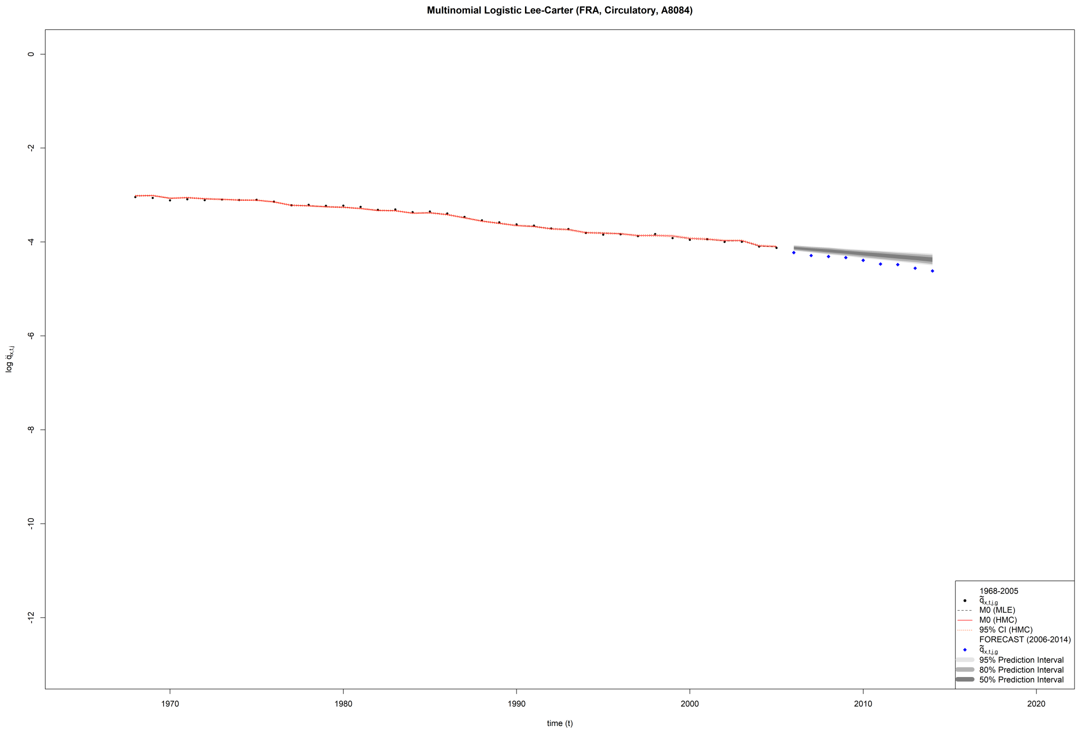


(a) Infectious and Parasitic Diseases

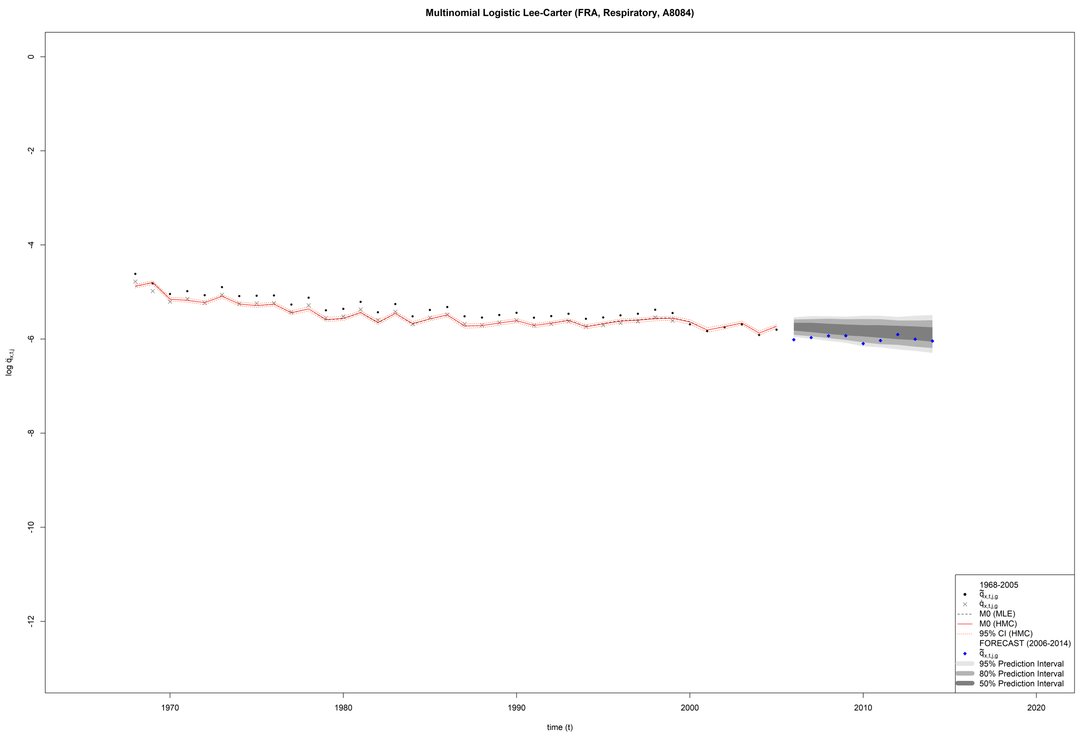


(b) Neoplasms

FIGURE 7.6: Probability of Death Forecasts for Age Group 80-84 (MLG-LC (M0), France, 2006-2014)

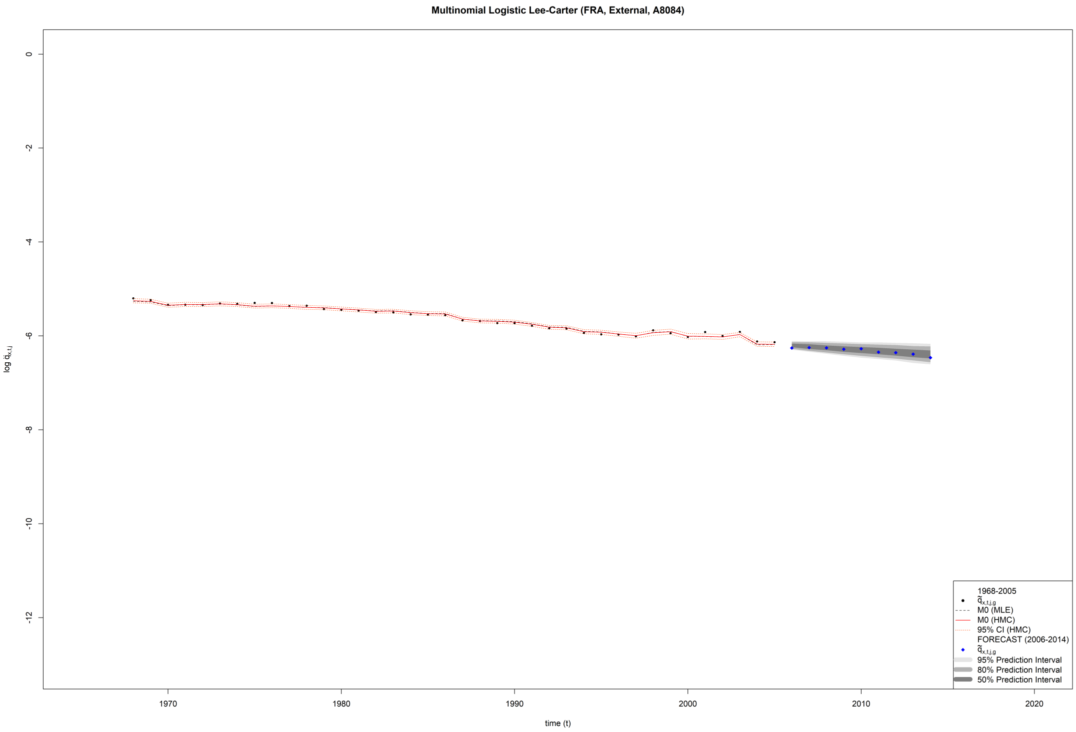


(c) Circulatory Diseases

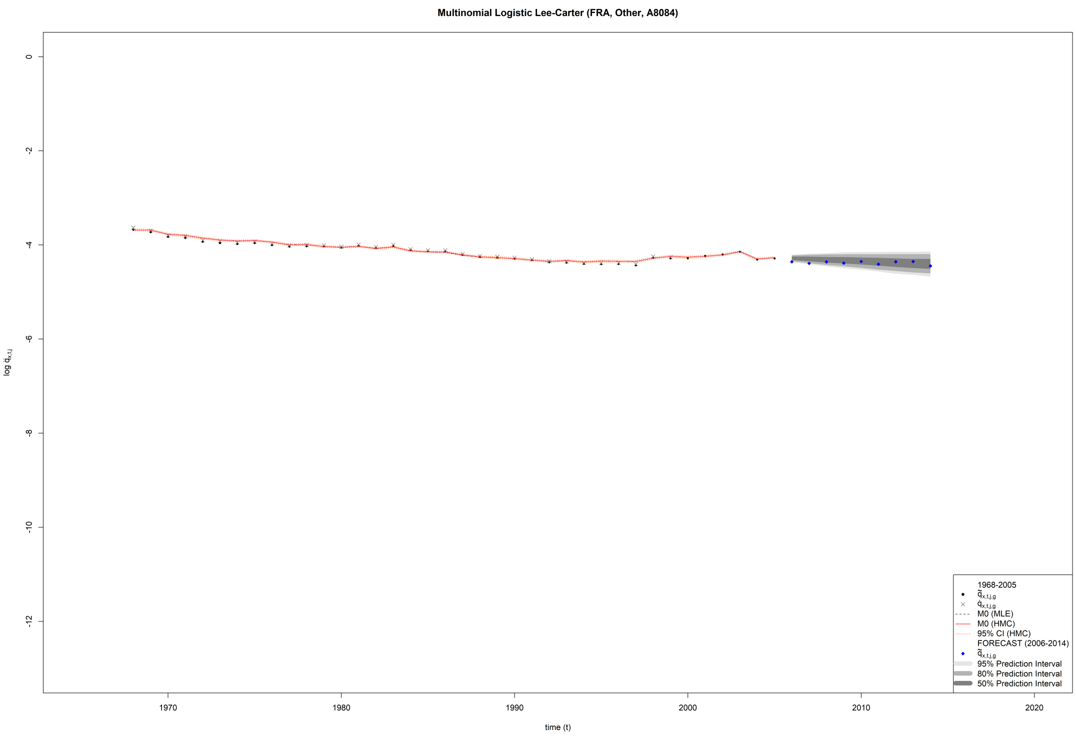


(d) Respiratory Diseases

FIGURE 7.6 (CONT.)



(e) External Causes



(f) Other Causes

FIGURE 7.6 (CONT.)

Similar to England & Wales, the forecasts for France capture most of the crude probabilities for four of the causes of death. The fact that the bilinear term $\beta(x, j, g)\kappa(t, j, g)$ can be difficult to estimate can sometimes result in a poor fit, especially if there is a sudden spike or drop in deaths. This is notable in Figure 7.6.a where there is a sudden spike in the number of deaths starting in year 2000 – the first year when ICD-10 was used for mortality coding in France. This spike is present despite the relevant comparability ratio for infectious and parasitic diseases being applied. The forecasts for deaths due to circulatory diseases in Figures 7.5.c & 7.6.c also follow the long-term trend and therefore fail to take into account changes in the incidence of ischaemic heart disease in the 2000s. These were likely caused by changes to a number of behavioural risk factors, notably the rise in smoking among the female population (Grave et al., 2024; Olié et al., 2019).

7.1.6 Coverage of MLG-LC Projections for France

We present the calculated coverage of the prediction intervals for the ARIMA specifications from Chapter 5 (see Table 5.8) for the hold out sample (years 2006 to 2014) for France in Table 7.2:

TABLE 7.2: COVERAGE OF PREDICTION INTERVALS FROM ARIMA, FRANCE

	Prediction Interval			
Cause of Death	50%	80%	90%	95%
Infectious	26.67%	45.56%	56.11%	62.22%
Neoplasms	34.44%	52.22%	61.11%	69.44%
Circulatory	28.89%	49.44%	60.56%	67.22%
Respiratory	34.44%	57.22%	68.89%	77.22%
External	10.56%	32.78%	45.00%	54.44%
Other	33.33%	56.11%	65.00%	70.00%
Alive	12.22%	23.33%	28.33%	31.11%
Overall	25.79%	45.24%	55.00%	61.67%

As was the case for the English & Welsh MLG-LC projections, those for France also suffer from a low coverage especially for external causes of death although coverage is closest to the values of the prediction intervals for respiratory diseases followed by other causes of death. This can be attributed to the additional uncertainty arising from the use of comparability ratios. The second group of causes where comparability ratios were applied – infectious and parasitic diseases – does not exhibit the same increase in coverage. The values are also low for the relatively stable causes of death, i.e. neoplasms and circulatory diseases. We note that the coverage of the projections for the 'Alive' group is especially poor and even worse than for England & Wales. In addition to the possible remedies suggested in Section 7.1.3, additional uncertainty may need to be applied to external causes of death using a different comparability ratio.

7.2 Projections Using the Multinomial Logistic Li-Lee Model

Compared to the MLG-LC model, the MLG-LL specification requires that not only country-specific period effects be forecast for each cause but also common, or global, period effect. Li and Lee (2005) recommend that the period effect is modelled using RWD for the country/cause specific terms. This follows on from our approach presented in Section 6.2.2. We therefore forecast the common cause-specific period effect $k(t,j)$ and the country-specific cause effect $\kappa(t,j,g)$ for M5 using RWD and ARIMA(1,1,0) model, respectively. The processes are then given as follows:

$$\begin{aligned} k(t,j) &\sim \mathrm{N}\left(\nabla(j) + k(t-1,j), \sigma_k^2(j)\right) \\ \Delta\kappa(t,j,g) &\sim \mathrm{N}\left(\rho(j,g)\Delta\kappa(t-1,j,g), \sigma_\kappa^2(j,g)\right) \end{aligned} \tag{7.4}$$

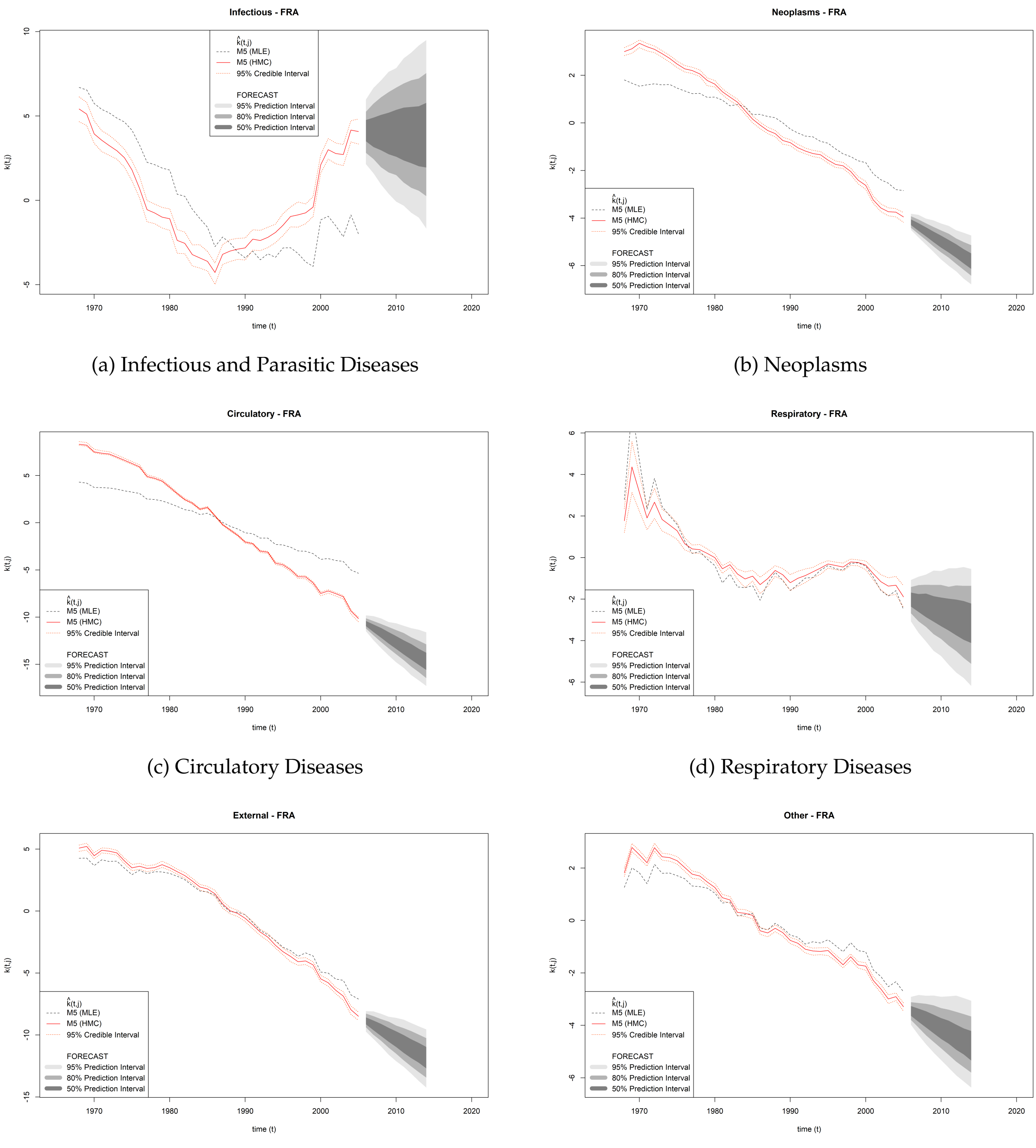
where $\nabla(j)$ is again the cause-specific drift term and $\Delta\kappa(t,j,g)$ is the first difference of the period deviations for each country-cause combination, and $\rho(j,g)$ is the cause-specific autoregressive term for country g. We forecast $\kappa(t,j,g)$ using the identity

$$\begin{aligned} \Delta\kappa(t,j,g) &= \rho(j,g)\Delta\kappa(t-1,j,g) + \epsilon(t,j,g) \\ \implies \kappa(t,j,g) - \kappa(t-1,j,g) &= \rho(j,g)\left[\kappa(t-1,j,g) - \kappa(t-2,j,g)\right] + \epsilon(t,j,g) \\ \implies \kappa(t,j,g) &= \left[1 + \rho(j,g)\right]\kappa(t-1,j,g) - \rho(j,g)\kappa(t-2,j,g) + \epsilon(t,j,g). \end{aligned} \tag{7.5}$$

Here the error term $\epsilon(t,j,g)$ follows $\mathrm{N}\left(0, \sigma_\kappa^2(j,g)\right)$.

7.2.1 Period-Effect Parameter Projections for MLG-LL

The fitted values and projections for the common, or global, cause-specific period effect $k(t,j)$ are shown in Figure 7.7. The dark grey area indicates the 50% prediction interval, the lighter grey area contains the 80% prediction interval, and the light grey area contains the 95% prediction interval.



(a) Infectious and Parasitic Diseases

(b) Neoplasms

(c) Circulatory Diseases

(d) Respiratory Diseases

(e) External Causes

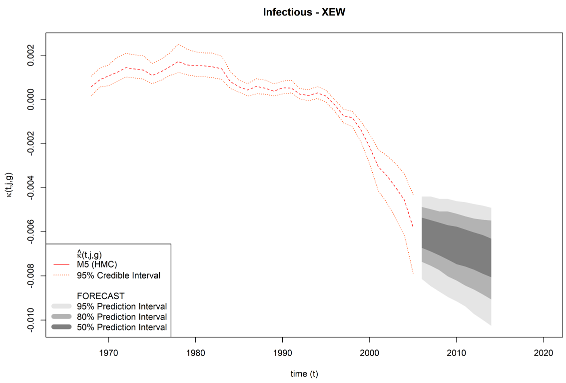
(f) Other Causes

FIGURE 7.7: $k(t, j)$ coefficients fitted using MLG-LL for years 1968-2005 and their prediction intervals for years 2006-2014

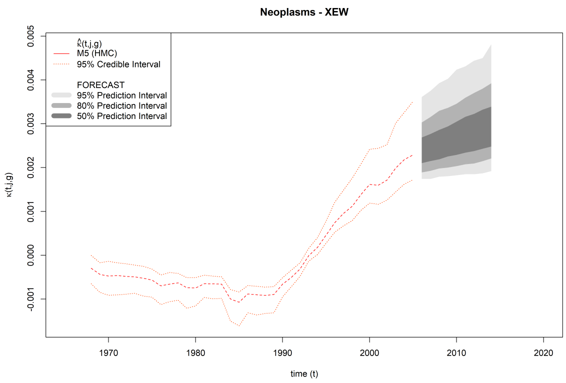
We can see in Figure 7.7 that the trend of the common period effect is largely comparable to the $\kappa(t, j, g)$ terms for England & Wales presented previously in Figure 7.1. It is notable that the increase in deaths due to infectious diseases in England & Wales dominates over the decreasing trend for the same cause of death group in France. The main exception is the common trend for external causes, which resembles more closely $\kappa(t, j, g)$ for France shown in

Figure 7.4. We also note that the magnitude of the country-specific period effect $\kappa(t, j = 4, g)$, i.e. for respiratory diseases, is higher for both countries than the magnitude of the common trend $k(t, j = 4, g)$. This is likely a combination of the effect of two bilinear terms being used in this model and the shocks to the time series of respiratory deaths due to ICD changes. It can also be seen that the common trend for Respiratory diseases is much smoother than the individual parameters for either England & Wales or France.

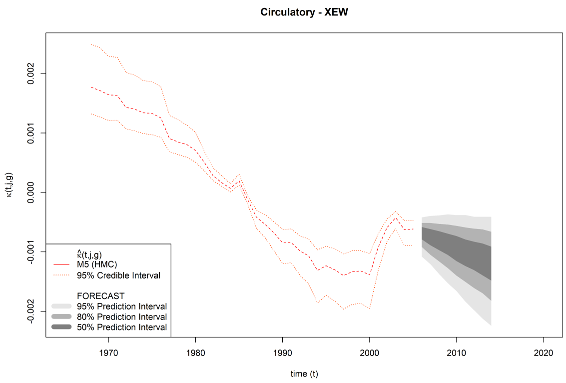
The fitted values and projections of the time-specific $\kappa(t, j, g)$ parameters for England & Wales and France are presented in Figures 7.8 and 7.9, respectively. The dark grey area indicates the 50% prediction interval, the lighter grey area contains the 80% prediction interval, and the light grey area contains the 95% prediction interval.



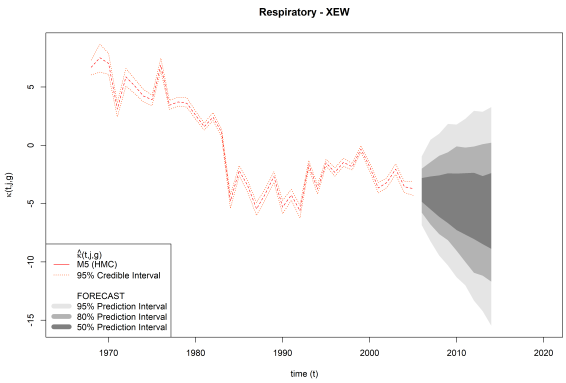
(a) Infectious and Parasitic Diseases



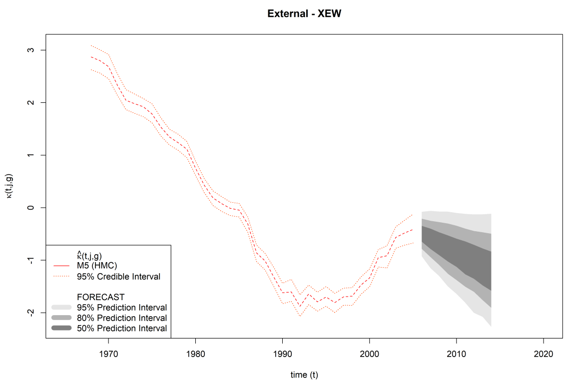
(b) Neoplasms



(c) Circulatory Diseases



(d) Respiratory Diseases

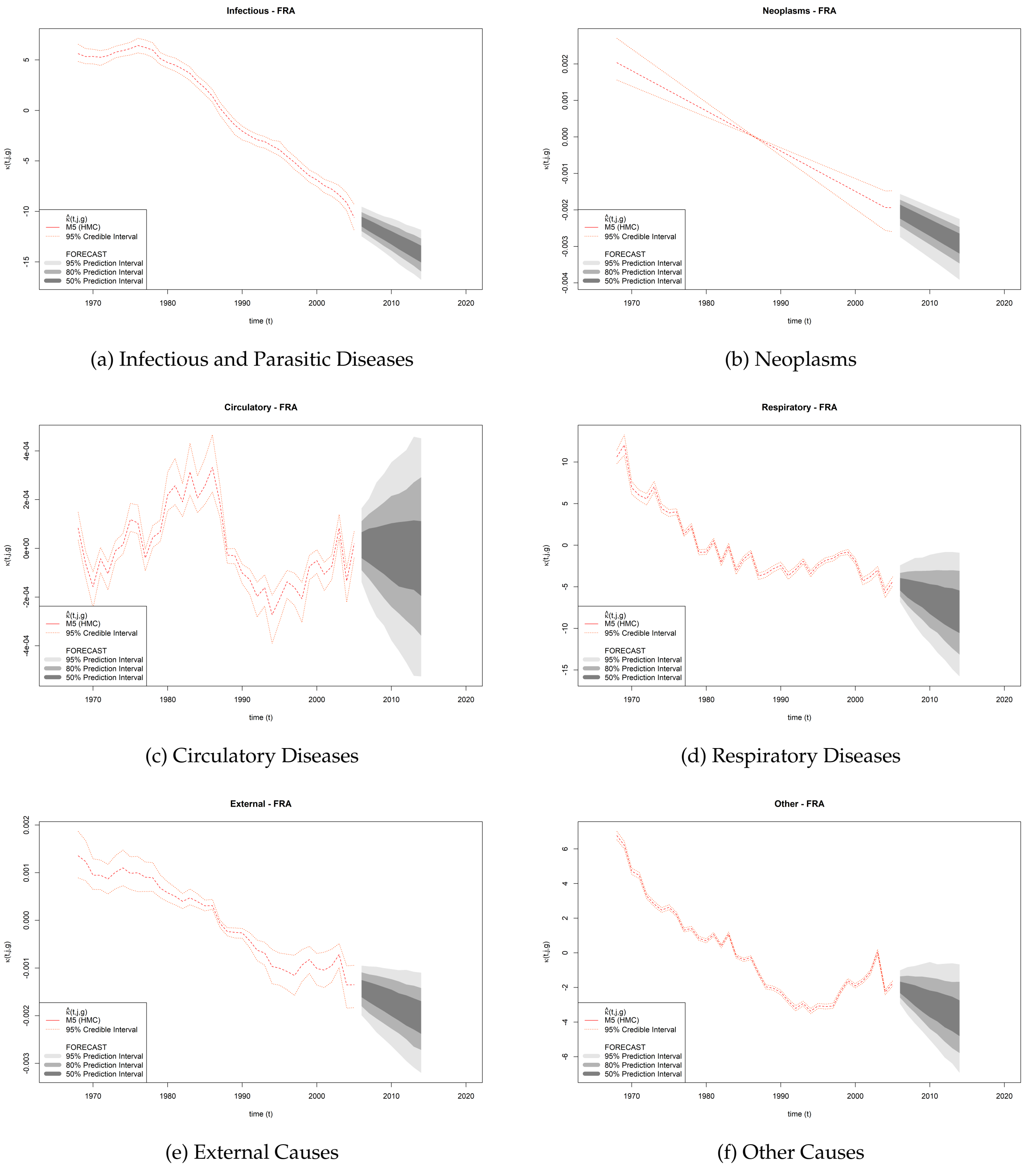


(e) External Causes

(f) Other Causes

FIGURE 7.8: $\kappa(t, j, g)$ coefficients fitted using MLG-LL for years 1968-2005 and their prediction intervals for years 2006-2014 (England & Wales)

Two of the cause-specific period effects $\kappa(t, j, g)$ for England & Wales – Infectious & parasitic diseases and Neoplasms – changed direction of the trend compared to $\kappa(t, j)$ from the MLG-LC specification. This can be explained by considering $\kappa(t, j, g)$ to be the country-specific deviation for country g of the period effect for a particular cause of death j from the common trend given by $k(t, j)$. This becomes even more clear when the magnitude of the $\kappa(t, j, g)$ parameters is compared to that of $k(t, j)$.



(a) Infectious and Parasitic Diseases

(b) Neoplasms

(c) Circulatory Diseases

(d) Respiratory Diseases

(e) External Causes

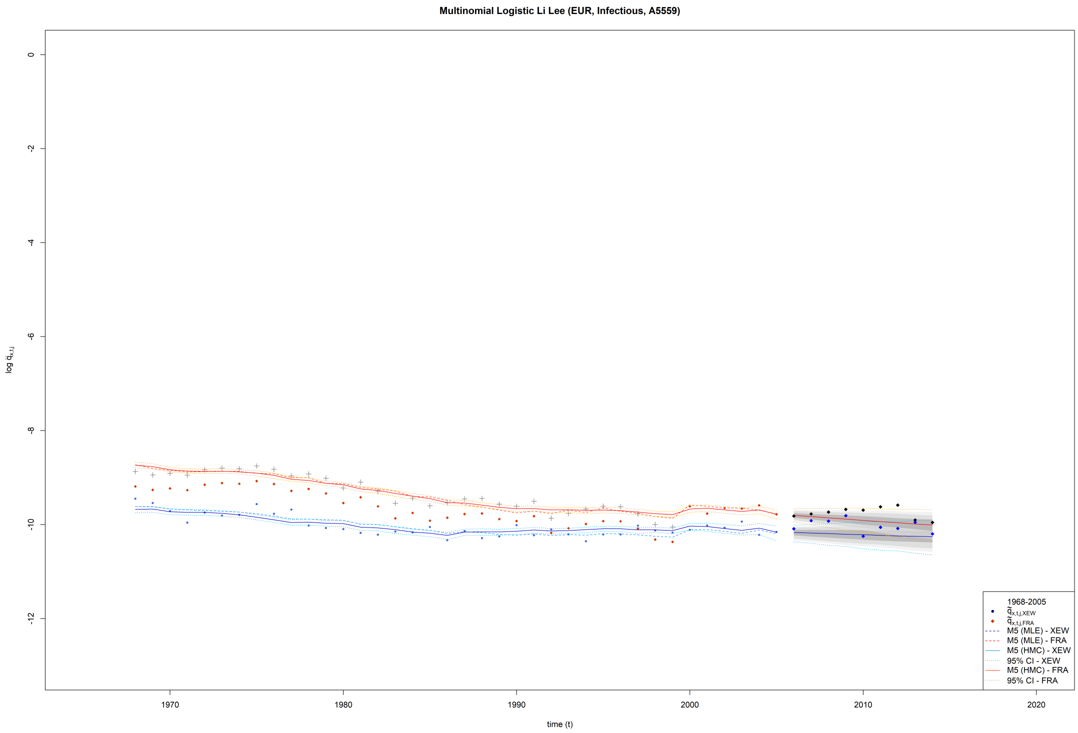
(f) Other Causes

FIGURE 7.9: $\kappa(t, j, g)$ coefficients fitted using MLG-LL for years 1968-2005 and their prediction intervals for years 2006-2014 (France)

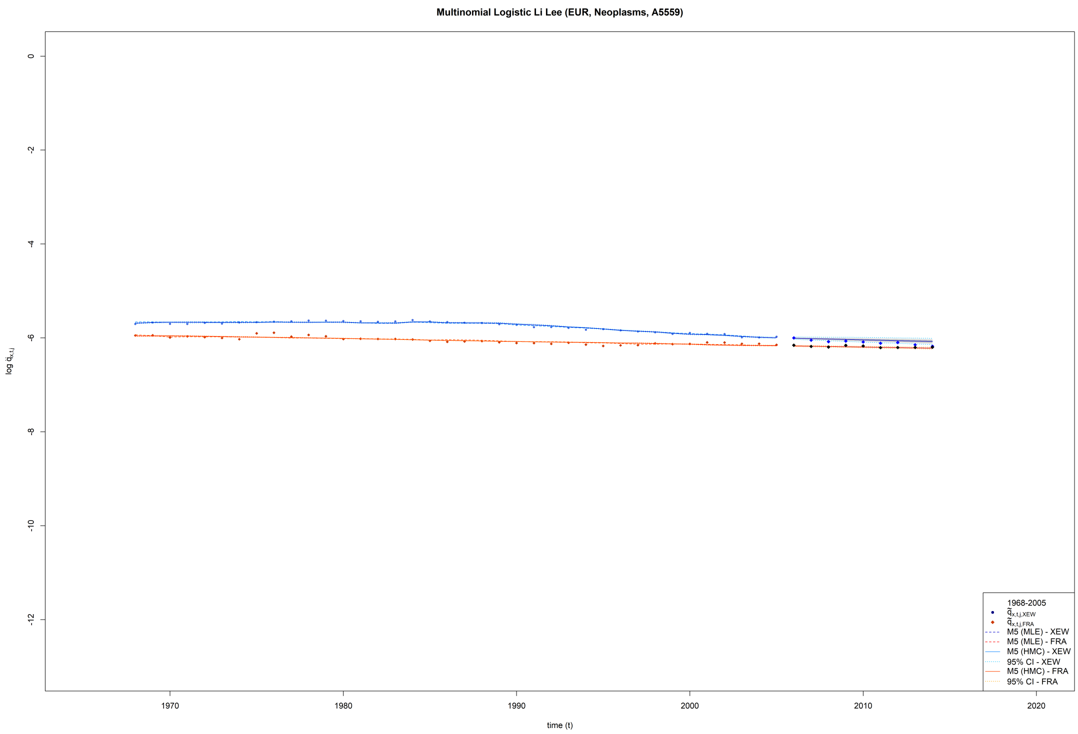
A pattern similar to that of England & Wales emerges for the $\kappa(t, j, g)$ parameters for France. Three of the parameters for cause of death groups that have comparable experience in the two countries – neoplasms, circulatory diseases, and external causes – have a $\kappa(t, j, g)$ term with a much smaller order of magnitude than the common period effect $k(t, j)$. The biggest exception is the $\kappa(t, j, g)$ term for infectious and parasitic diseases in France that has to compensate for the spike in mortality due to this cause in England & Wales picked up by the common term $k(t, j)$. The period effect $\kappa(t, j, g)$ for respiratory diseases in France is also of a magnitude comparable to the corresponding $k(t, j)$ in order to offset the much higher mortality experience due to this cause of death in England & Wales.

7.2.2 Projected Probabilities of Death for England & Wales and France using MLG-LL

The projections of the probabilities of death due to the various causes in England & Wales and France for ages 55-59 and 80-84 using MLG-LL are presented in Figures 7.10 and 7.11, respectively. Additional plots for age groups 20-24, 40-44, 60-44, and 85+ are presented in Appendix F.3. Please note that the cause-specific death probabilities for England & Wales and France denoted with a cross and a plus symbol, respectively, indicate the adjusted probabilities calculated by applying a comparability ratio as per Section 2.4.

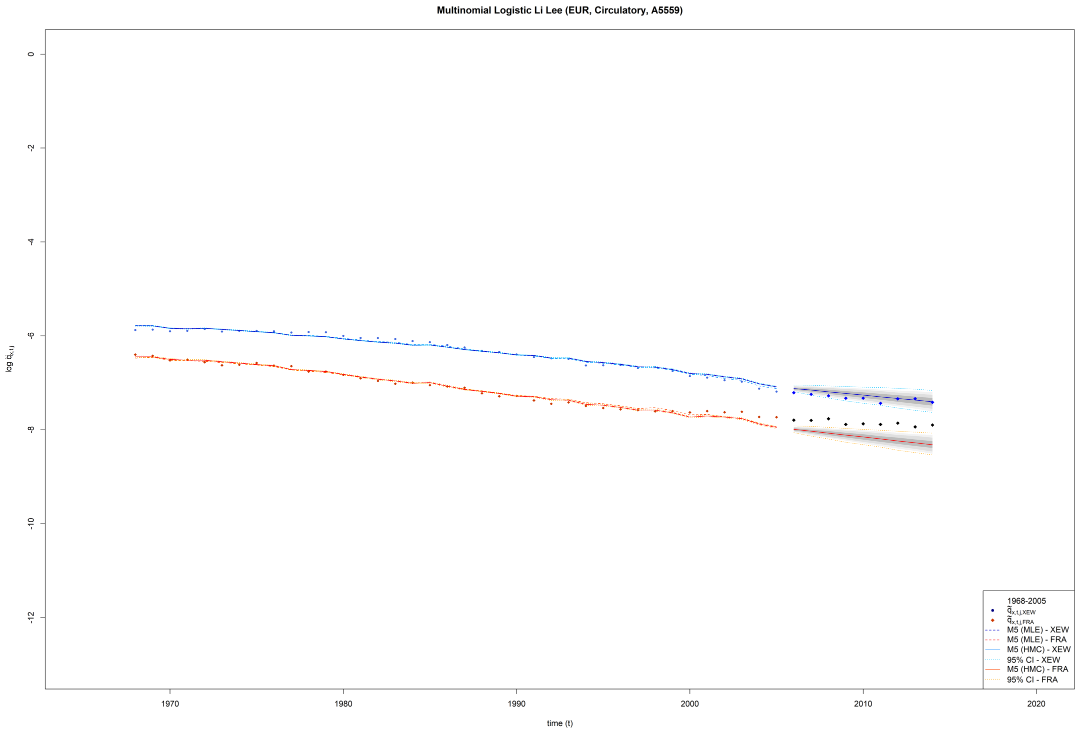


(a) Infectious and Parasitic Diseases

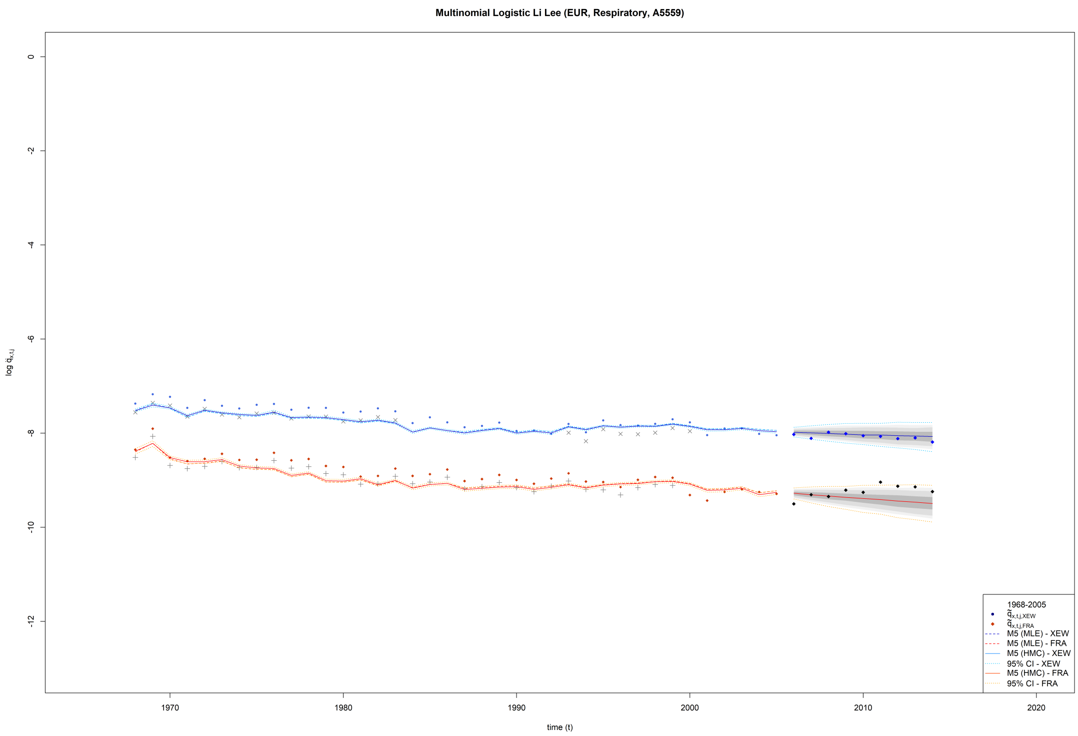


(b) Neoplasms

FIGURE 7.10: Probability of Death Forecasts for Age Group 55-59 (MLG-LL (M5), England & Wales + France, 2006-2014)

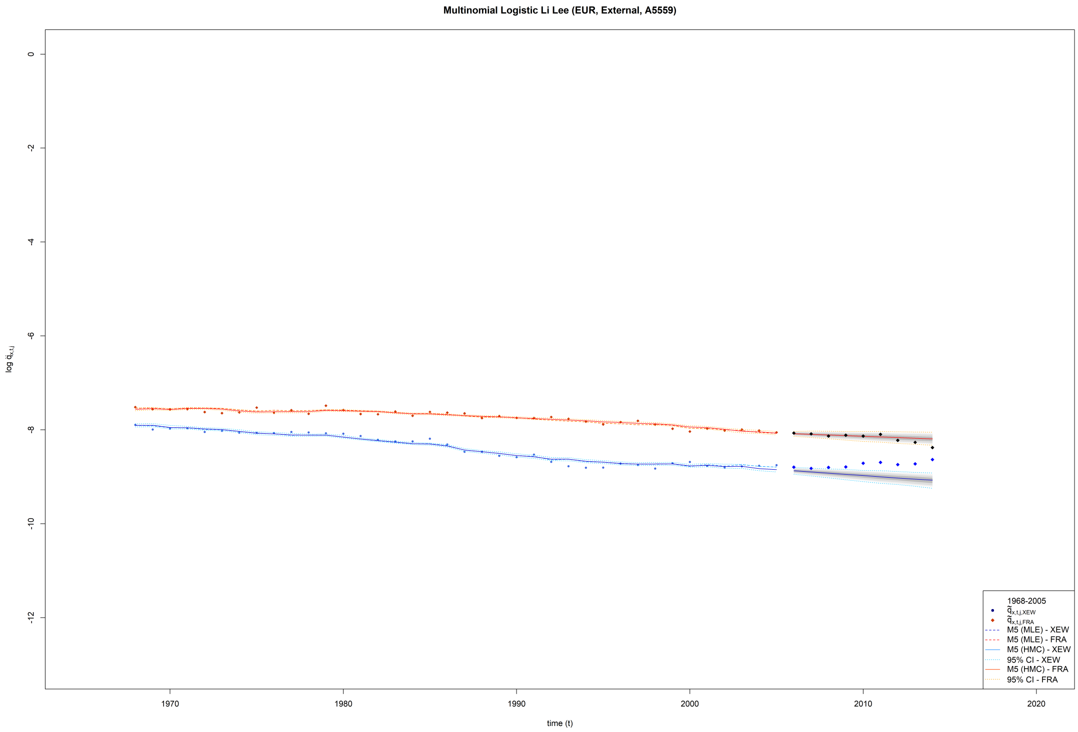


(c) Circulatory Diseases

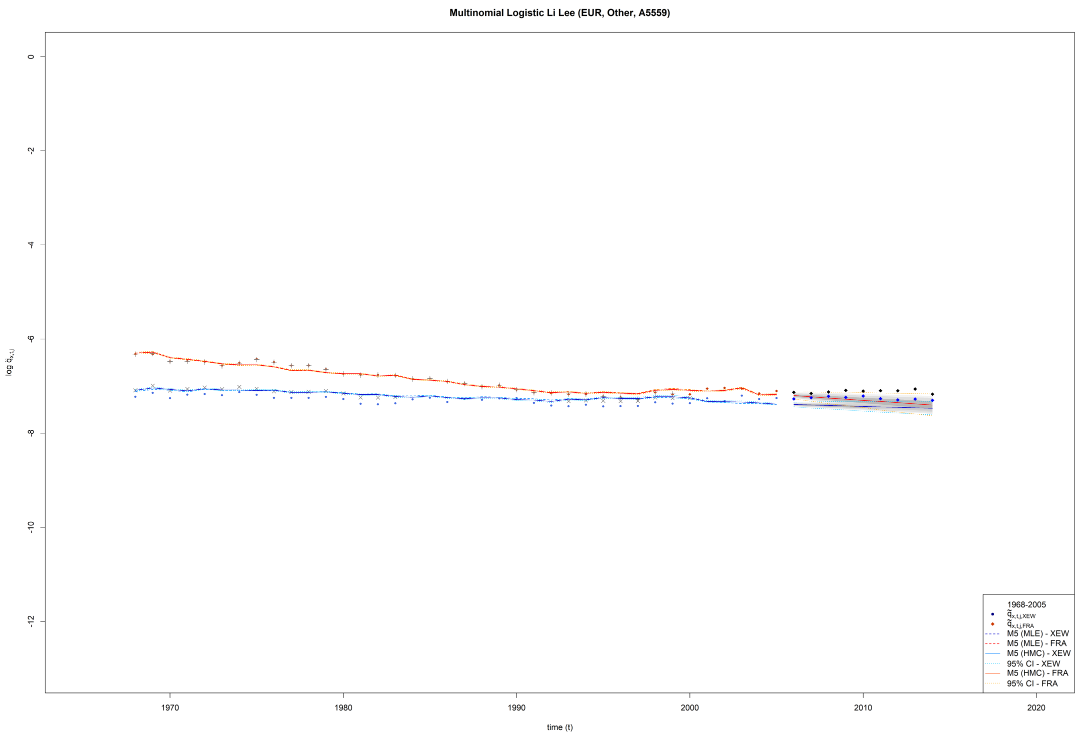


(d) Respiratory Diseases

FIGURE 7.10 (CONT.)

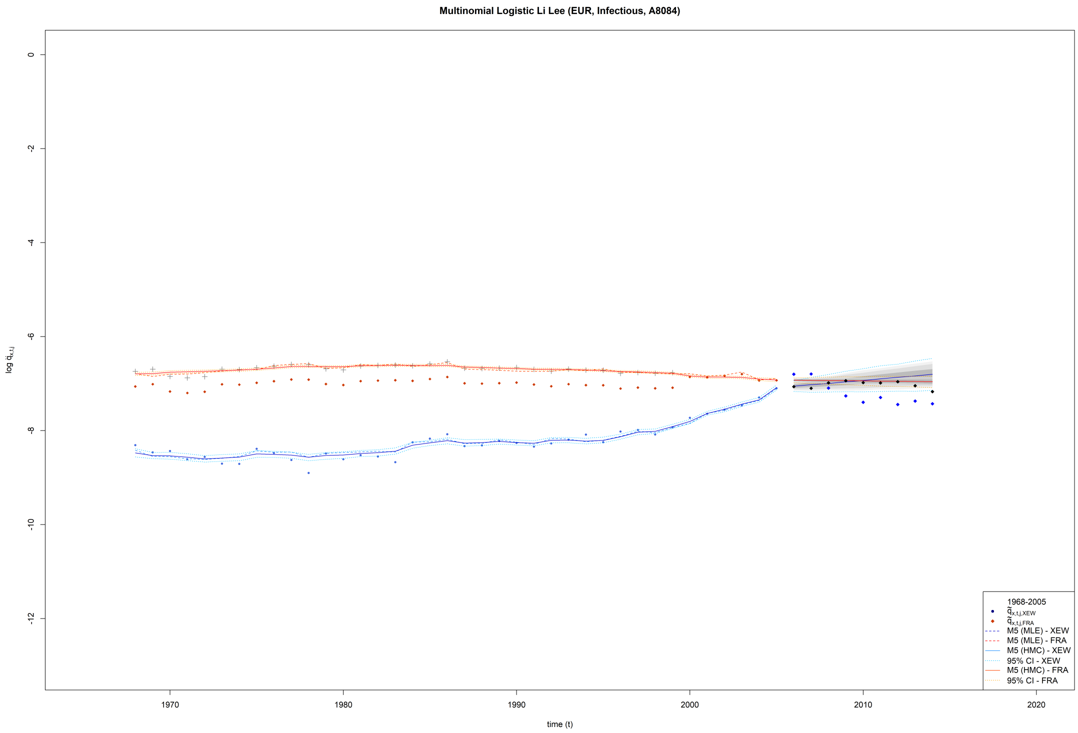


(e) External Causes

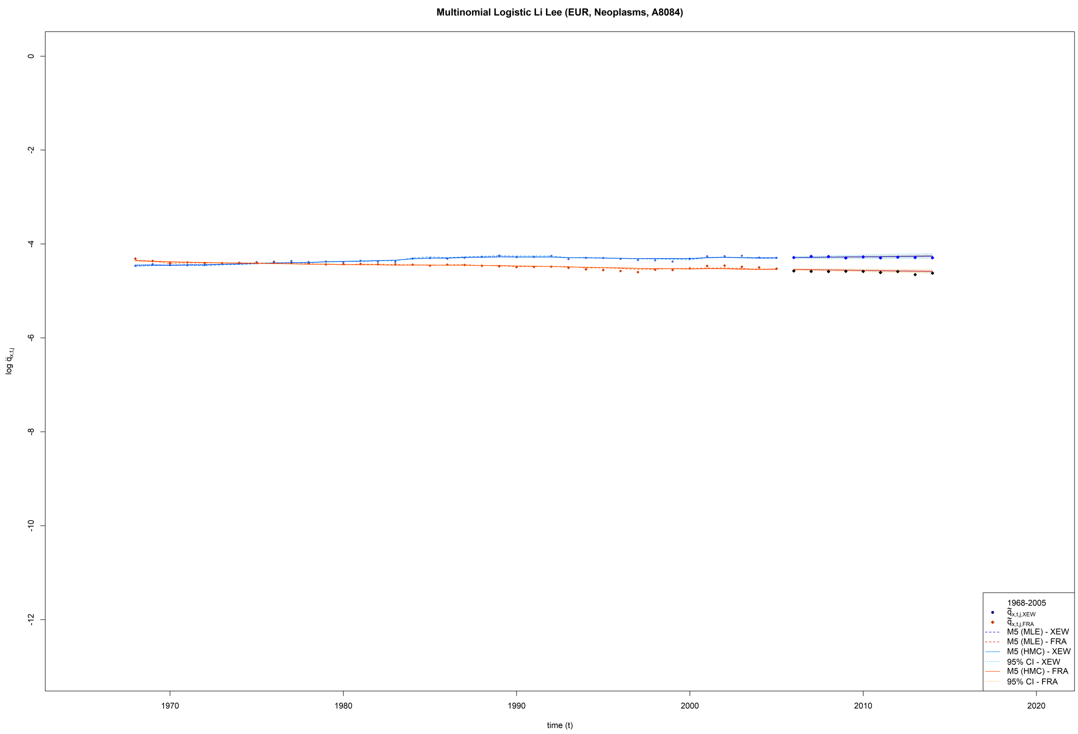


(f) Other Causes

FIGURE 7.10 (CONT.)

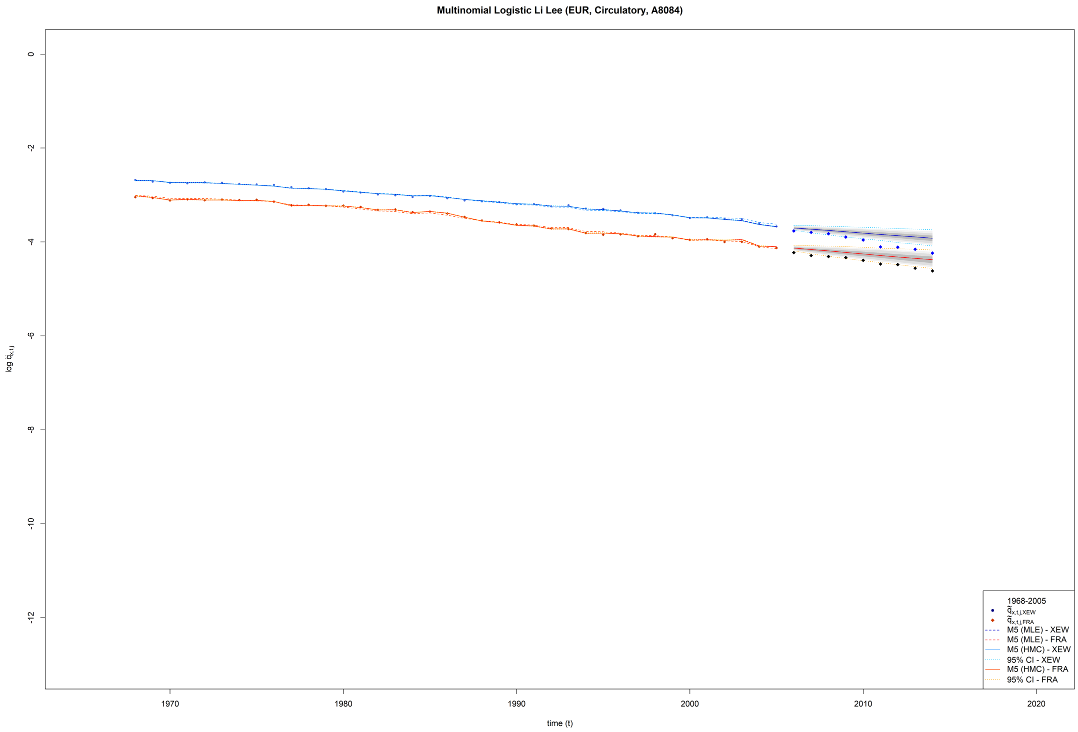


(a) Infectious and Parasitic Diseases

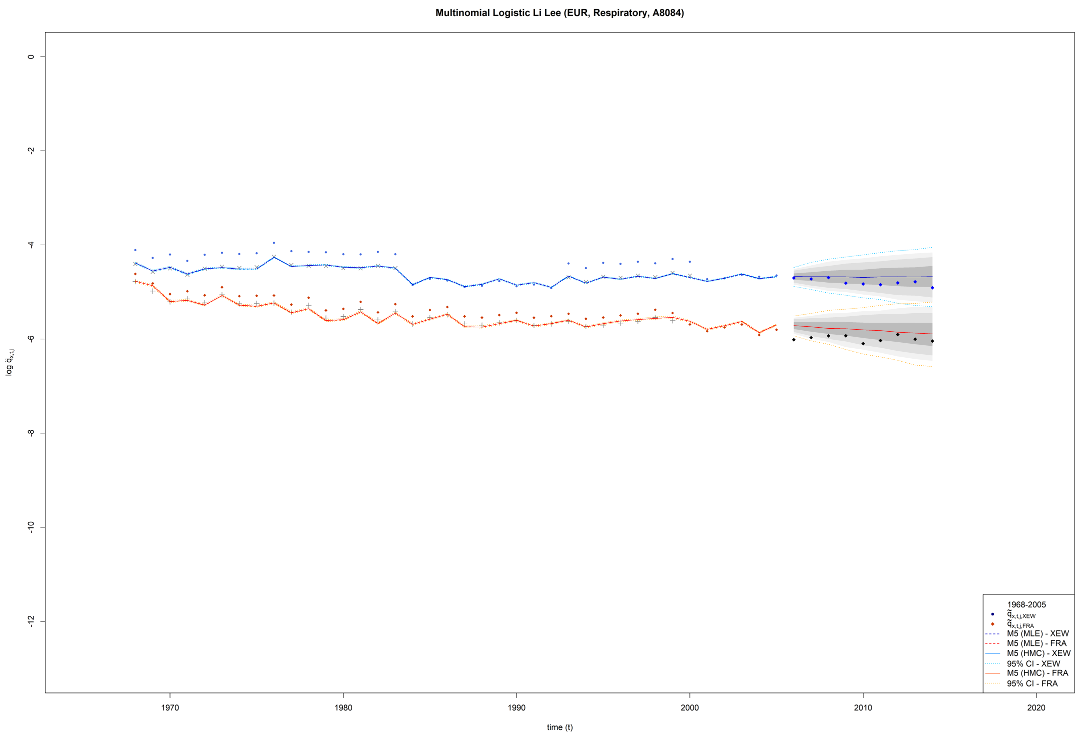


(b) Neoplasms

FIGURE 7.11: Probability of Death Forecasts for Age Group 80-84 (MLG-LL (M5), England & Wales + France, 2006-2014)

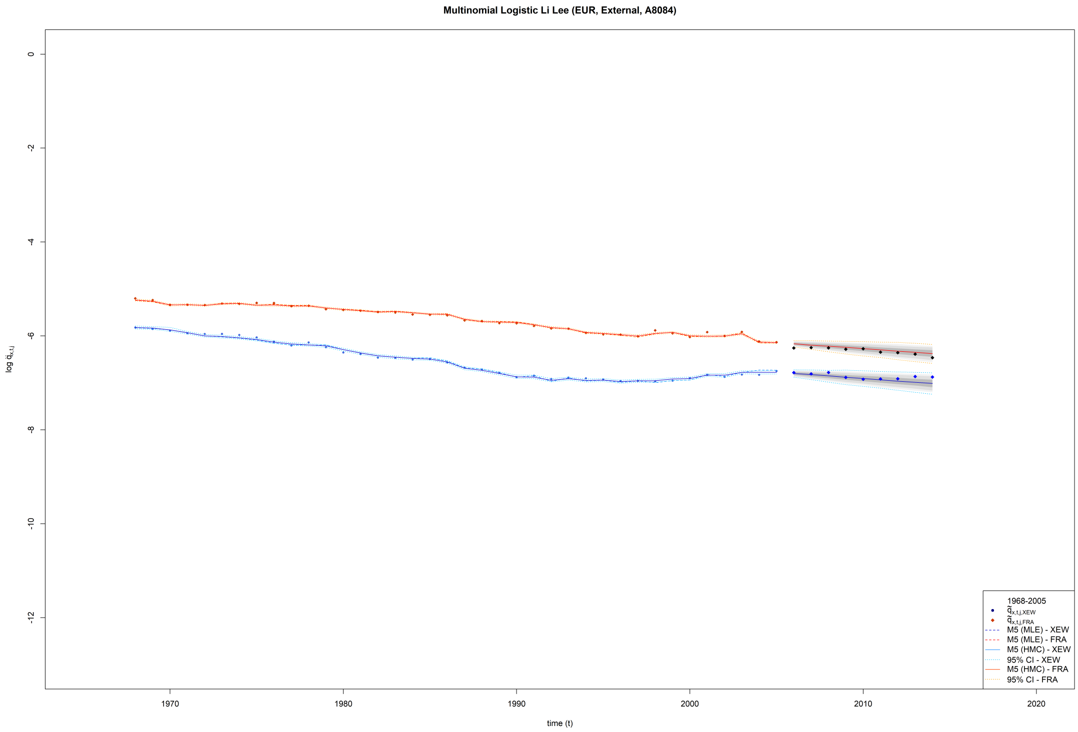


(c) Circulatory Diseases

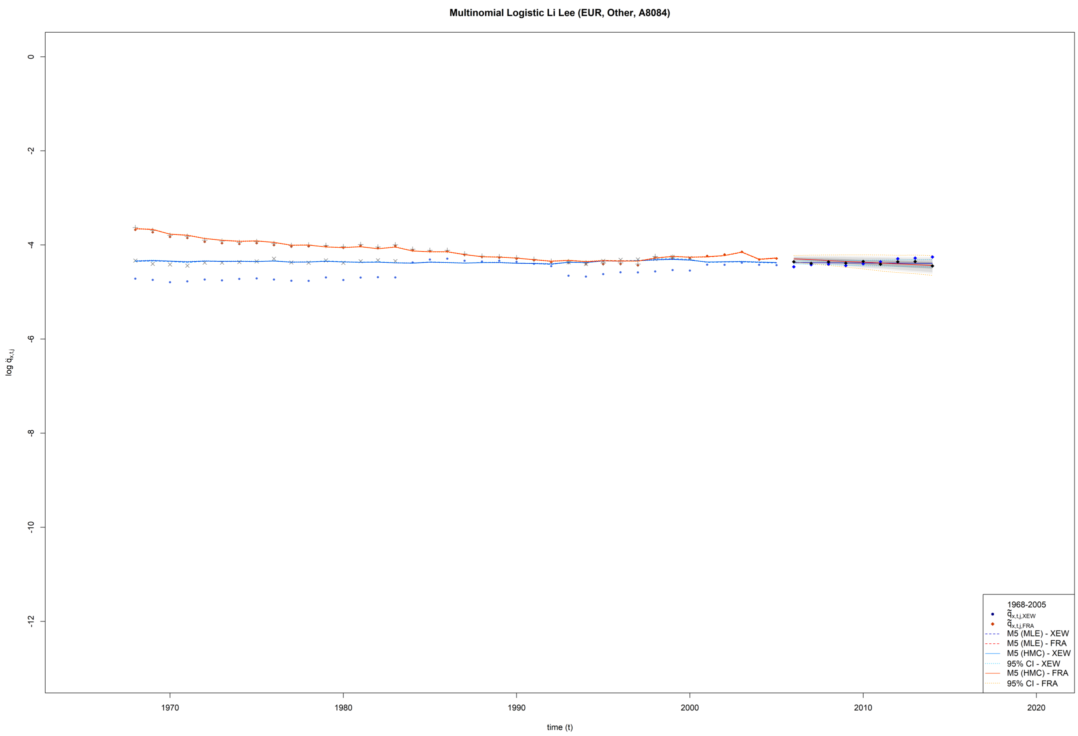


(d) Respiratory Diseases

FIGURE 7.11 (CONT.)



(e) External Causes



(f) Other Causes

FIGURE 7.11 (CONT.)

The direction of the trend for age group 55-59 is the same for both countries with France experiencing lower mortality for neoplasms, circulatory diseases, and respiratory diseases. The apparent convergence of the probabilities of death for 'Other causes' in the two countries should be noted as there is not a corresponding divergence in the probabilities for the 'Respiratory diseases' group in Figure 7.10. The convergence of probabilities for 'Other causes' is also present in the 80-84 age group and shown in Figure 7.11. The impact of the ICD Rule 3 on respiratory deaths in England & Wales is most apparent in Figure 7.11 as there is no obvious dip in the raw probabilities of death for France.

Chapter 8

Conclusions and Further Research

8.1 Conclusions

In this thesis, we develop a methodology to estimate cause specific mortality using the MLG-LC and MLG-LL models. This is the first use of the MLG-LL model and also the first application of a MLG-LC model to cause-specific mortality as far as we are aware. While Alai, Arnold, and Sherris (2015) did fit a multinomial logistic model for six groups of cause of death, the predictor function incorporated a continuous linear time effect t rather than a categorical period effect $\kappa(t)$. This approach oversimplifies the time series for respiratory deaths. Although a binomial logistic Lee-Carter model has been estimated before by Zhao (2012), the application was for all-cause mortality. We extend this modification of the Lee-Carter model to the multi-cause case as well as the multi-country version, i.e. the Li-Lee model. Furthermore, existing literature on cause-specific mortality obtains estimates using the classical, i.e. frequentist, approach and in many cases lacks uncertainty estimates. We explore possible approaches to quantify uncertainty including Bootstrap sampling for MLEs in Section 4.3 and Bayesian methods in Chapter 6. One of the benefits of the Bayesian approach is that it allows us to incorporate expert opinion in our model in the form of prior information.

In Chapter 4 we fit the MLG-LC model using maximum likelihood as well as four specifications of the MLG-LL model. The models fit the data comparably well for adult ages for neoplasms, circulatory diseases, and external causes of death, i.e. causes with a high number of annual deaths. The periodic fluctuations in respiratory deaths prove to be problematic for the models studied irrespective of the period-specific terms used. The performance of the models for young ages is mixed. Overall, the global age and period parameters prove to not significantly improve the regression fit and MLG-LL specification M5, with only a cause-specific common term, is chosen for further study based on the low value of its information criteria.

Section 4.3 explores alternative methods for quantifying uncertainty of our coefficient estimates. The two approaches we look at are a Multivariate Normal sampler and Parametric Bootstrap. While the MVN sampler requires the imposition of the Normal assumption onto all of the fitted coefficients, the Parametric Bootstrap instead relies on the assumption of a multinomial distribution for deaths and lives – a property that is consistent with the Multinomial Logistic model. Unsurprisingly the standard errors from the MVN sampler are very close to the standard errors of the Maximum Likelihood estimates. On the other hand standard error estimates from Parametric Bootstrap are much larger owing to the fact that the Bootstrap approach samples deaths data prior to the estimation of the model, which introduces additional variability.

We provide an overview of classical time series models in Chapter 5. Specifications of ARIMA models for England & Wales and France are selected for each cause of death using a combination of ADF tests and the `auto.arima()` function from the forecast package in R. The resulting selections yield a different specification for various causes as well as variation in the model specification between countries. All causes require at least one order of differencing to achieve stationarity.

Chapter 6 focused on the application of Bayesian methods to mortality models presented in previous chapters. In Section 6.1 we estimated the MLG-LC model using MCMC methods, more specifically the Metropolis algorithm. The fitted coefficients were comparable with those estimated using ML; the credible intervals, however, were wider due to the increased uncertainty imposed by the prior distribution. The large number of model parameters, e.g. 864 parameters for the single-country MLG-LC model, posed computational difficulties when it came to mixing and convergence of the Markov chains. Subsection 6.1.1.2 contains a description of an approach to remedy this issue by tuning the variance-covariance matrix of the Metropolis algorithm; nonetheless, this algorithm was slow to run and to converge.

Due to the complexities of the MLG-LL model as a result of the large number of effective parameters, e.g. 1188 for M5, and the associated computational difficulties, we switched from Metropolis to the HMC algorithm and coded our models via the R interface for the Stan programming language in Section 6.2. In Section 6.2.1 we built on the time series work from Chapter 5 and incorporated a time series prior for the $\kappa(t, j, g)$ terms of the MLG-LC model. The MLG-LL model presented in Section 6.2.2 incorporated a time series prior for the cause-specific common period effect $k(t, j)$ in addition to the $\kappa(t, j, g)$ term also present in MLG-LC. We found that the MLG-LL model outperforms the MLG-LC model when applied to mortality data for both England & Wales and France.

Finally in Chapter 7 we created mortality projections using the time-specific term(s) of the Multinomial Logistic Lee-Carter and Li-Lee models. While others, e.g. Arnold and Sherris (2013), fitted VECM to forecast cause-specific mortality rates, our approach wraps cause-specific mortality time series inside a multinomial logistic transformation to restrict the projected probabilities between 0 and 1. While this approach allows us to forecast the probability of death in the future, it does not provide us with a framework to extrapolate the exposures to later years and in turn the number of future deaths. We assess the performance of forecasts using out-of-sample data for years 2006 to 2014. We note the low coverage of our prediction intervals following the logit transformation. This is especially apparent for the higher age groups, which have a smaller underlying model uncertainty due to a higher number of deaths relative to the younger age groups. While the logit transformation ensures that the future transition probabilities still sum to one, the projections for total number of deaths are not constrained in any way to follow the all-cause mortality trend and therefore distort survival projections.

Overall, we have seen that the MLG-LL model outperforms the MLG-LC model for all groups of causes of death. The MLG-LL specification should therefore be used to model mortality for multiple countries provided that they are similar economically, socially, and in terms of population size and are experiencing converging trends in mortality. In contrast, the MLG-LC model should be used in situations where the countries studied are not similar enough, especially if the population sizes are different orders of magnitude. The MLE approach for the MLG-LL can be used in instances where the number of countries is small, say no more than three, or where the need for uncertainty estimates is limited. The Parametric Bootstrap approach can be useful in estimating uncertainty for the MLG-LC model or simplified version of MLG-LL where the number of time periods or the number of causes, or both, is reduced. This will decrease the number of parameters needed to be estimated and thereby reduce the computational expense. The Bayesian approach provides a possible way to not only estimate model coefficients and their uncertainty but also allows for the incorporation of prior information into the model. The HMC algorithm was able to handle a simple two-country model although further modelling is needed to assess its usability for much larger countries or regions.

8.2 Further Research

The current MLG specification of the model imposes a constant force of mortality assumption for each five-year age group, i.e. for age intervals $[x, x+5)$ for $x \geq 5$. This simplification does not capture the mortality experience well for ages less than 25. The decision to model age in aggregate groups was not by choice but rather due to the availability of mortality data from WHO as well as the relevant national statistics agencies. It would have been preferable if single age data was available to provide a more granular and therefore smoother fit for the age effects. Although we have not modelled single-year age effects in this thesis, we would expect the age-specific terms $\alpha(x, j, g)$ and $\beta(x, j, g)$ to be smooth. One possible approach to achieve this would be by using penalised splines as demonstrated by Hilton et al. (2018).

Furthermore, the RWD and ARIMA models used to forecast the period effects could be replaced with a VAR or VECM models. While the probabilities of death and survival are constrained via the multinomial logit transformation in the models that we have used, the period effects for each cause of death are allowed to move independently of each other. The VECM would account for any time series that exhibit co-integration and allow for long-run common stochastic trends similarly to the approach used by Arnold and Sherris (2015; 2016) and Arnold and Glushko (2021; 2022), for example.

Another area that could be explored further is the use of comparability ratios. At present our models treat comparability ratios as fixed values and do not take into account the confidence intervals that are available for them. While we looked at the impact of the Parametric Bootstrap on the confidence intervals in Section 4.3, deaths for all groups of causes were allowed to vary subject to a constraint. An interesting exploration could be performed by allowing the comparability ratio for respiratory diseases to vary. Sampling of different ratios in the Bayesian setting would not only provide us with a distribution for each year where the ratio was applied but also quantify the uncertainty in the underlying data.

The COVID-19 pandemic provides an opportunity to apply the models presented in this thesis to recent years and beyond. While the spike in respiratory deaths in 2020 due to the SARS-CoV-2 infection is likely to cause the forecasts to overestimate future mortality, similarly to the forecasts for infectious diseases, we would expect the time-specific ARIMA specification of the kappa series to become less sensitive to this over time as the effects of the excess deaths dissipate. This highlights the importance of quality historic data to produce a long time series so that a small number of outliers is less likely to significantly impact the long-term projections. Another possible remedy would be to follow the methodology of the 2022 CMI Mortality Projections Model and apply specific weights to the years impacted (CMI, 2023).

Lastly, the start of mortality coding using ICD-11 opens an opportunity to revise the aggregate grouping of causes of death to reflect current epidemiology and aetiology. Nonetheless, statistical estimation of cause-specific mortality will continue to be impacted by the periodic changes to ICD codes and the resulting inconsistencies in long-term time series. One possible area to revisit would be the ICD codes that make up infectious and parasitic diseases to ensure consistency across time as the mortality rate is sensitive to minor changes in classification due to the small number of annual deaths (less than 1%) attributable to this category. With that being said, the reclassification of influenza from diseases of the respiratory system under ICD-10 to infectious and parasitic diseases under ICD-11 will undoubtedly lead to a need for a correction mechanism given the high number of deaths as a result of this virus. Obviously each additional category of causes of death will increase the computational cost of the model due to the increase of coefficients that need to be estimated. While an additional group or two is unlikely to exceed a computer's available memory during the estimation of a single-country MLG-LC model, this issue will become ever pressing for the multi-country MLG-LL model. The added benefit of an additional group will then need to be weighted against the associated decrease in computational efficiency.

Appendix A

Local Mortality Database Specifications

The mapping of chapter, subchapter, superhigh, and high CoDCM codes is presented in Table A.1.

TABLE A.1: CAUSE OF DEATH CODE MAPPING (CODCM) RELATIONSHIPS

Subchapter Code	Chapter Code	High Group	Superhigh Group
CH01	CH01	Infectious and Parasitic	Infectious and Parasitic
CH02-1	CH02	Neoplasms	Neoplasms
CH02-2	CH02	Neoplasms	Neoplasms
CH02-3	CH02	Neoplasms	Neoplasms
CH03	CH03	Endocrine and Blood	Other
CH04	CH04	Endocrine and Blood	Other
CH05	CH05	Mental	Mental
CH06-1	CH06	Nervous	Nervous and Sensory
CH06-2	CH06	Sensory	Nervous and Sensory
CH06-3	CH06	Sensory	Nervous and Sensory
CH06-9	CH06	Nervous	Nervous and Sensory
CH07	CH07	Circulatory	Circulatory
CH08	CH08	Respiratory	Respiratory
CH09	CH09	Digestive	Digestive
CH10	CH10	Genitourinary and Obstetric	Other
CH11	CH11	Genitourinary and Obstetric	Other
CH12	CH12	Skin and Musculoskeletal	Other
CH13	CH13	Skin and Musculoskeletal	Other
CH13-9	CH13	Skin and Musculoskeletal	Other
CH14	CH14	Congenital	Other
CH15	CH15	Perinatal	Other
CH16	CH16	Illdefined	Other
CH17-1	CH17	External	External
CH17-3	CH17	External	External
CH17-5	CH17	External	External
CH17-6	CH17	External	External
CH17-7	CH17	External	External
CH17-9	CH17	External	External
CH17-0	CH17	External	External
CH18	CH18	Other	Other

The relationships between ICD-7, ICD-8, ICD-9, ICD-10, and CoDCM codes in the local mortality database are presented in Figure A.1 on the next page where different colours represent different versions of ICD. This mapping via SQL allowed for the aggregation of death counts coded using different versions of ICD into our CoDCM subchapters. These subchapters were then aggregated further into chapters, high groups, and superhigh groups to create time series that are comparable, or mostly comparable, over time. Finally, the superhigh groups were aggregated into the six cause of death groups shown in Table 2.1 and used in the analysis throughout this thesis.

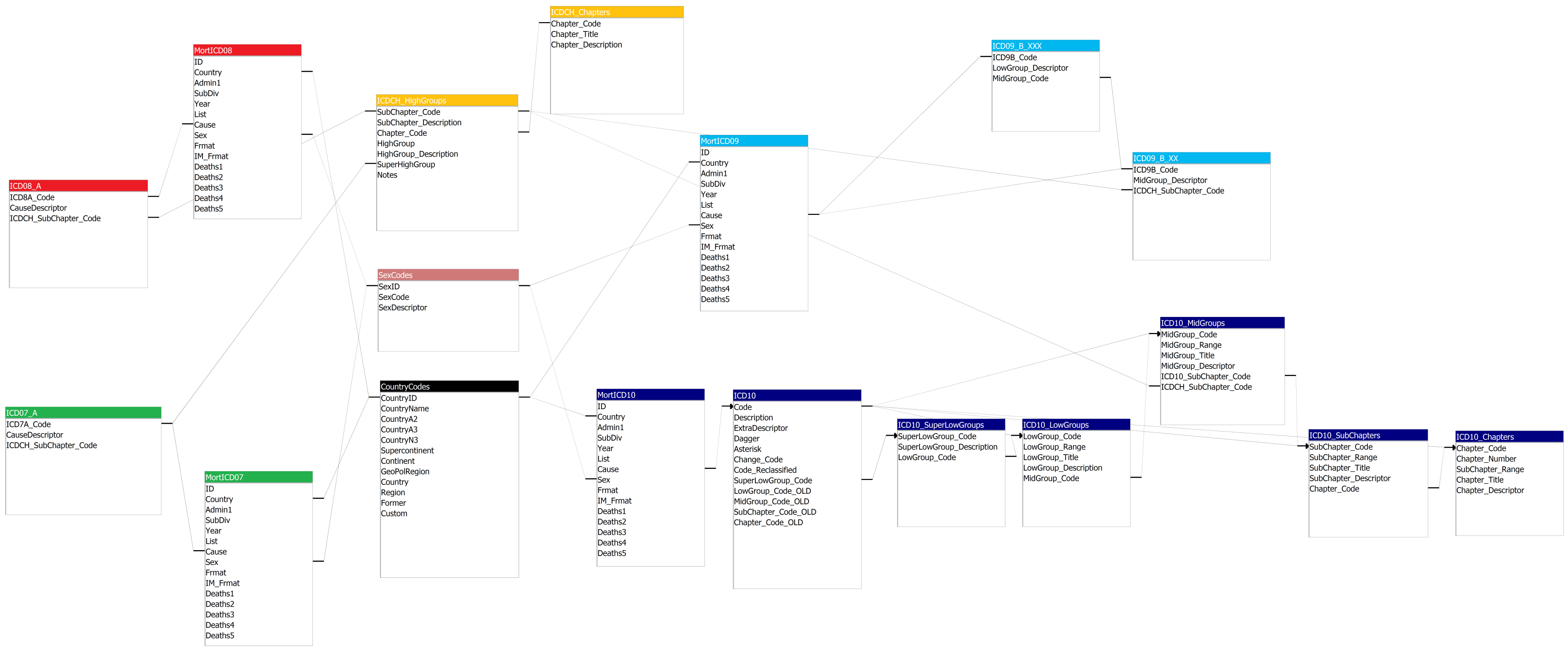


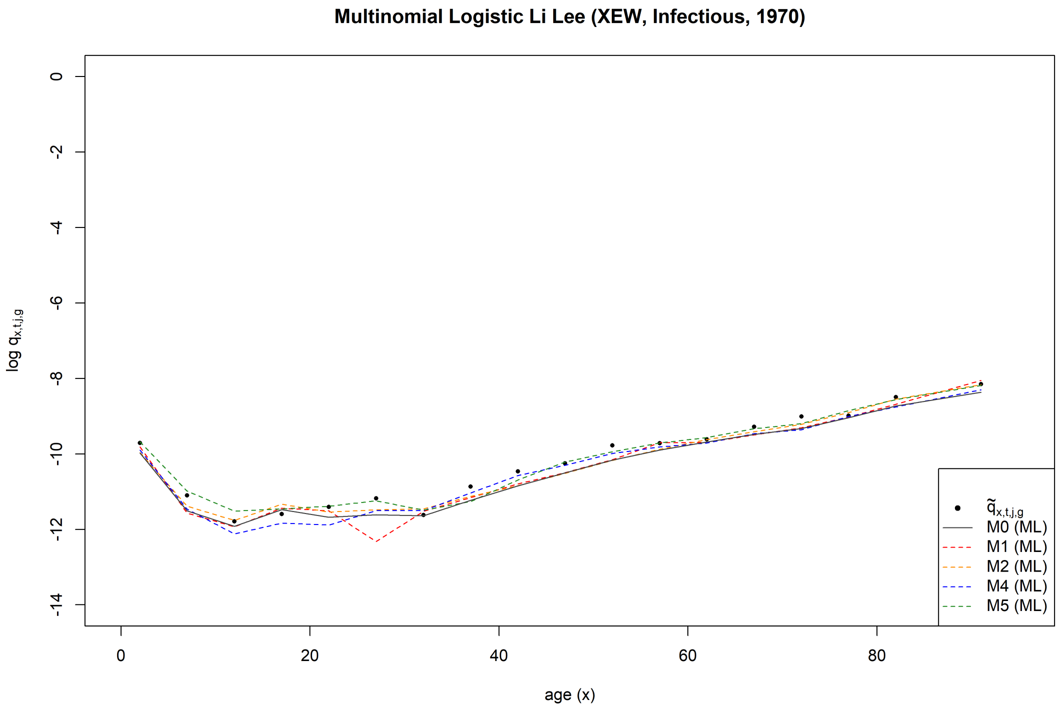
FIGURE A.1: Local Mortality Database Relationships between Versions of ICD and CoDCM

Appendix B

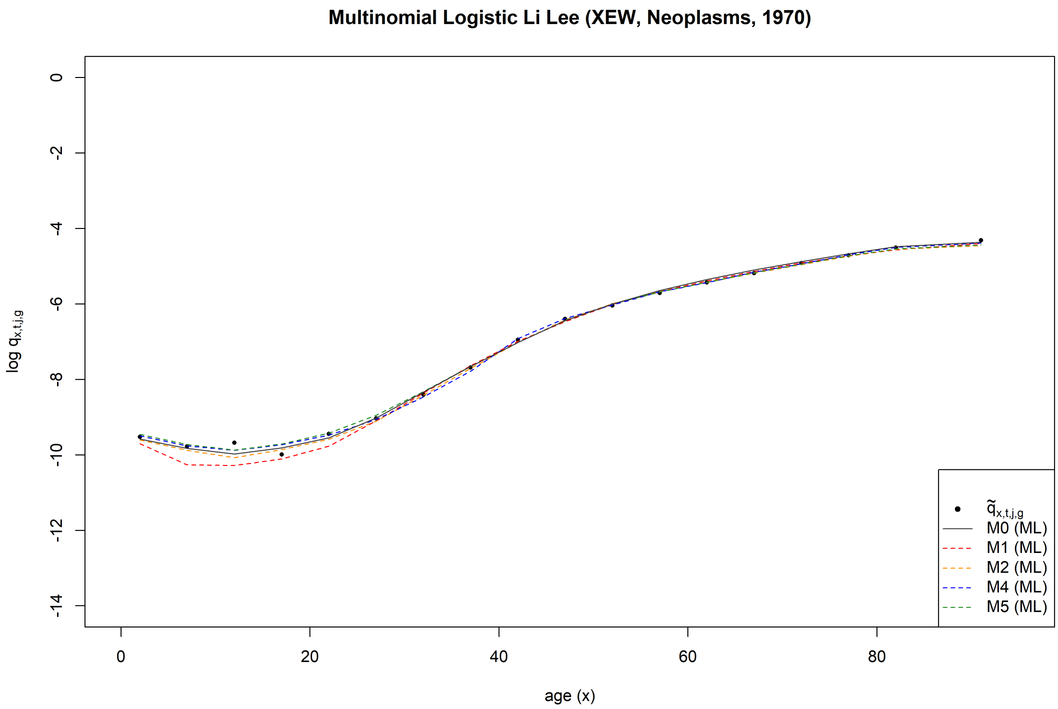
Model Comparison: Additional Plots

B.1 Maximum Likelihood Estimates

The plots of maximum likelihood estimates of probabilities of death $\log \hat{q}(x, t, j, g)$ for models M0, M1, M2, M4, and M5 for years 1970, 1980, 1990, and 2000 are presented on the pages that follow.

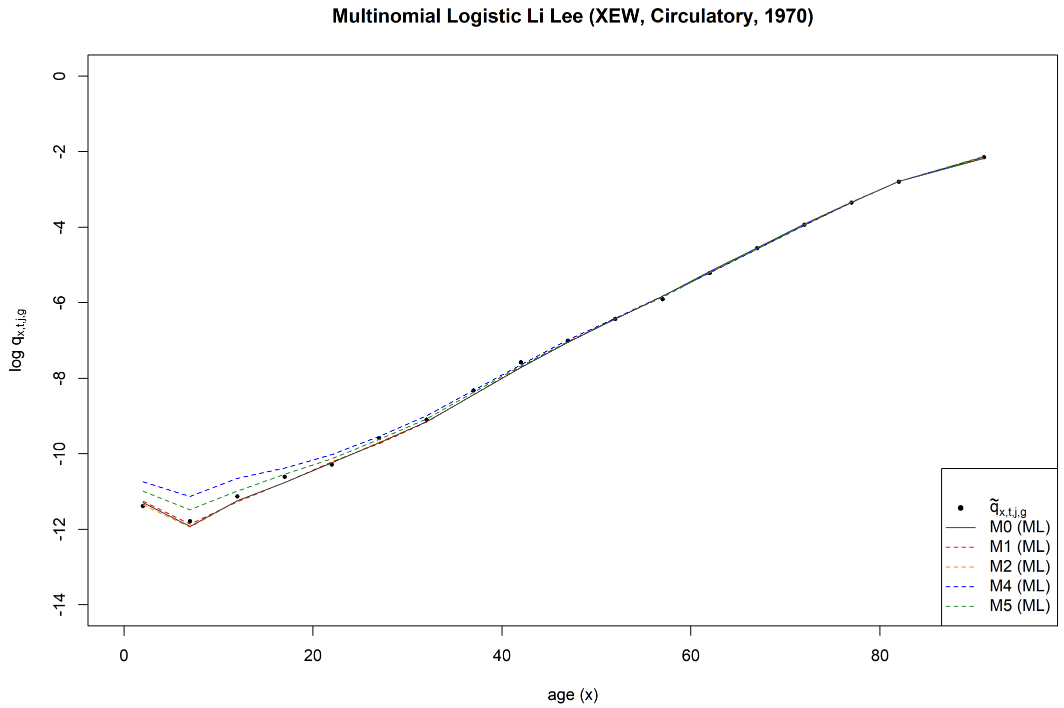


(a) Infectious and Parasitic Diseases

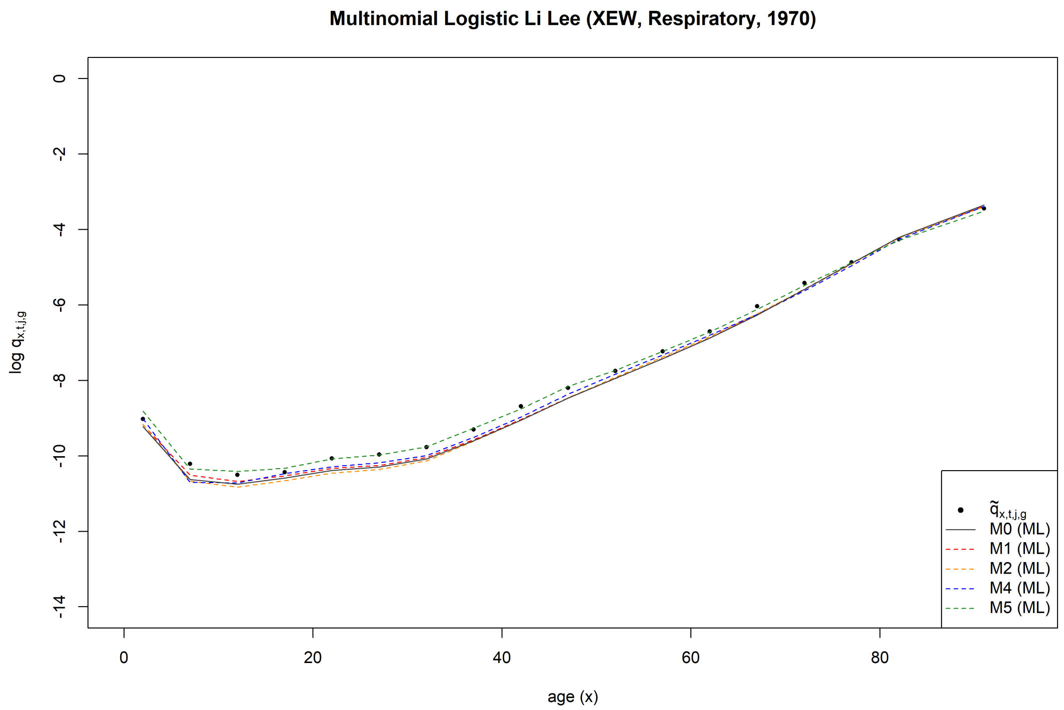


(b) Neoplasms

FIGURE B.1: Probabilities of Death in Year 1970
(England & Wales)

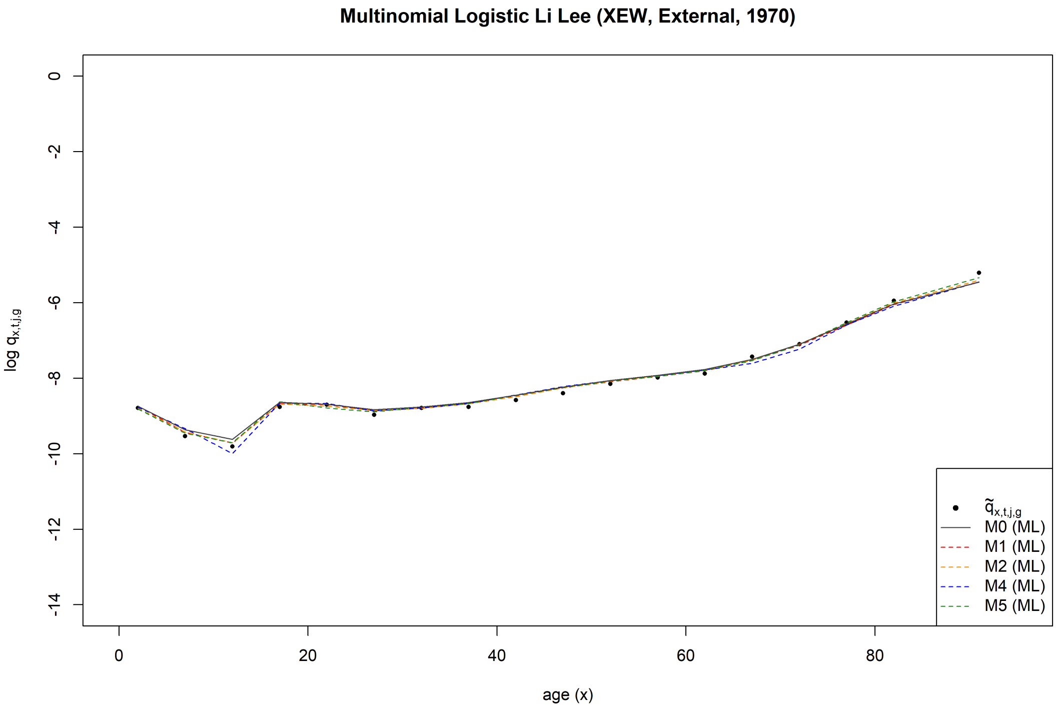


(c) Circulatory Diseases

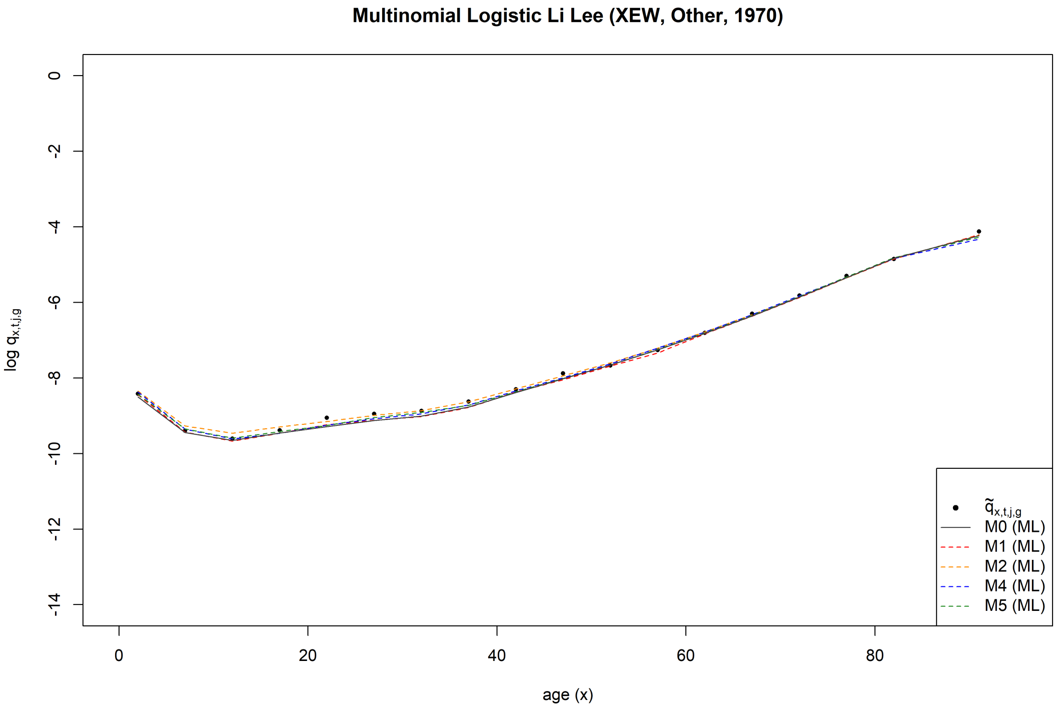


(d) Respiratory Diseases

FIGURE B.1 (cont.)

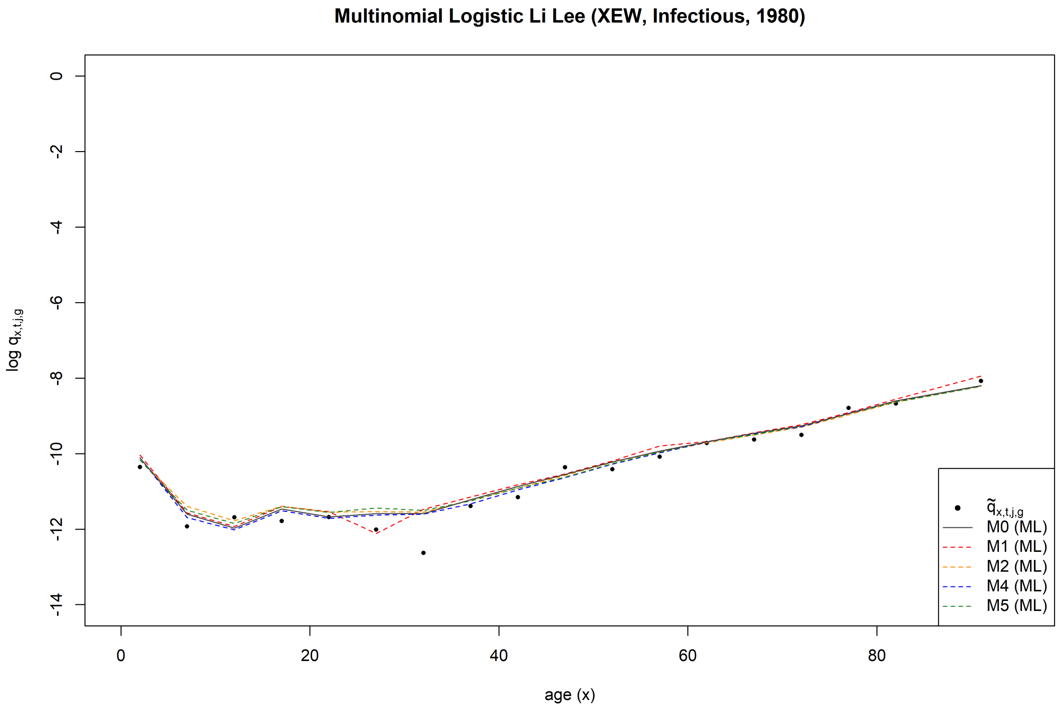


(e) External Causes

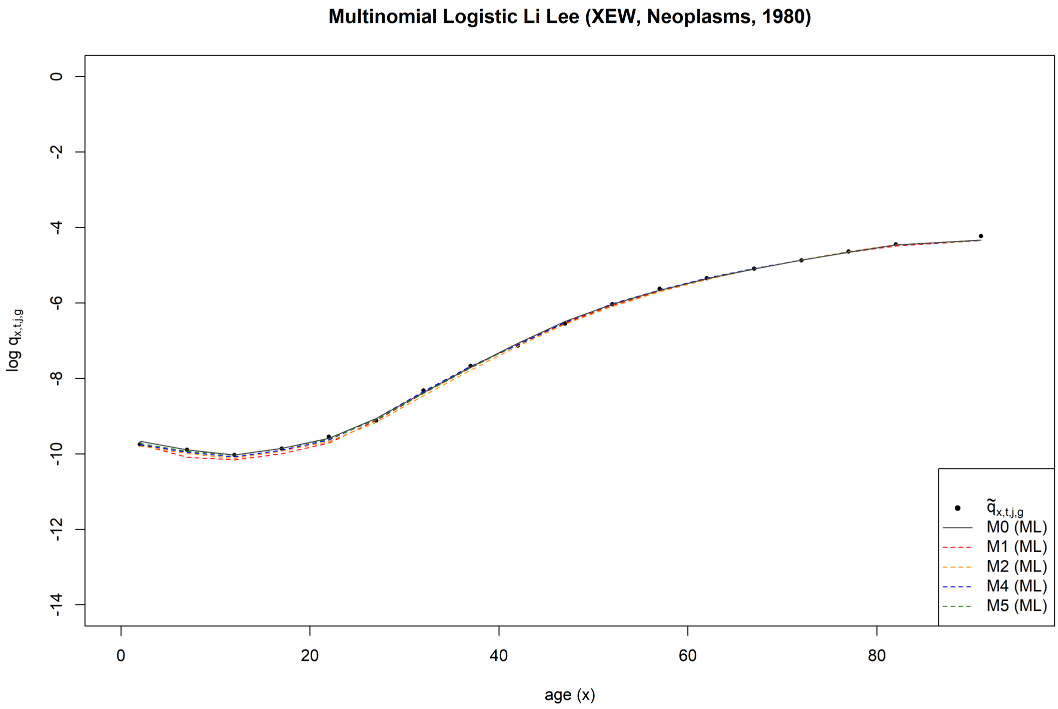


(f) Other Causes

FIGURE B.1 (cont.)

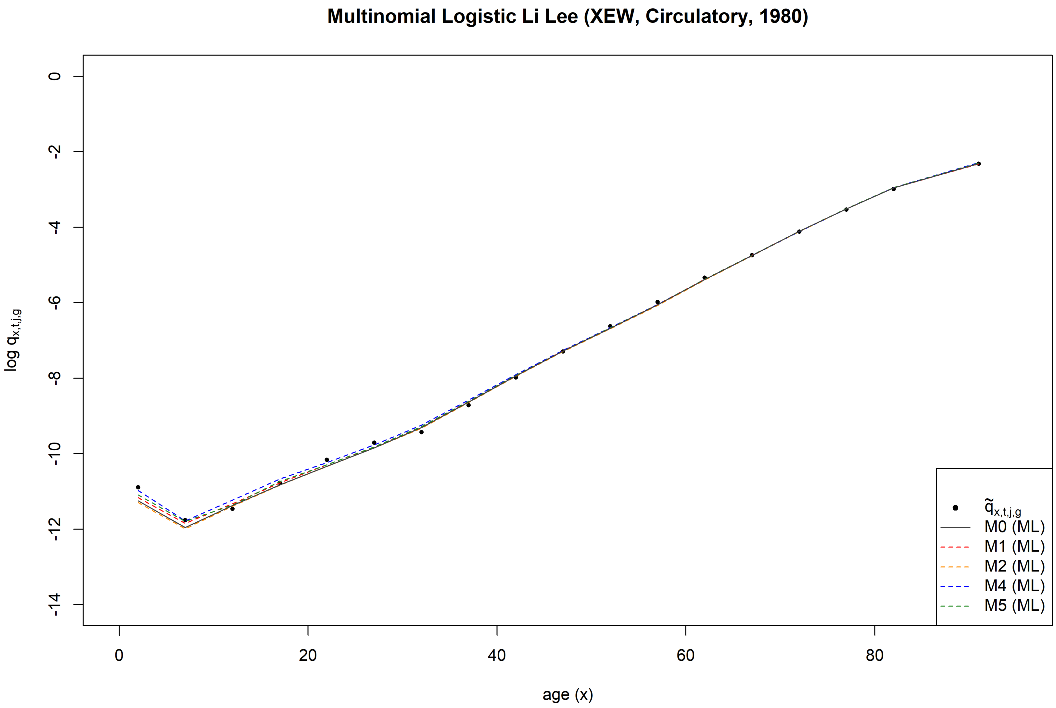


(a) Infectious and Parasitic Diseases

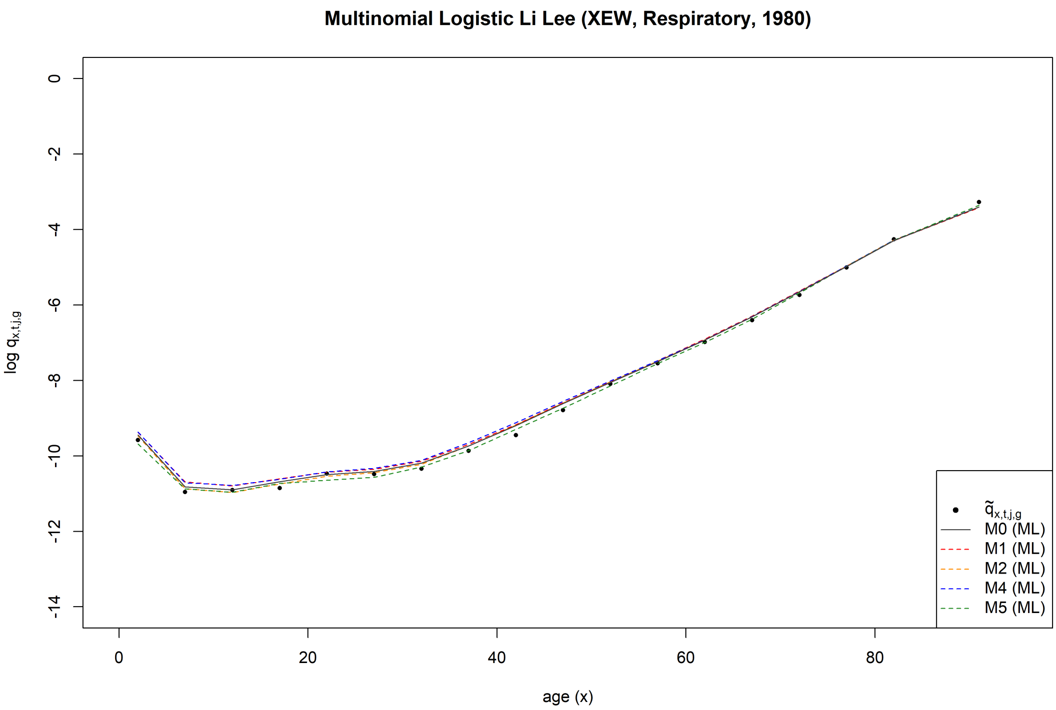


(b) Neoplasms

FIGURE B.2: Probabilities of Death in Year 1980
(England & Wales)

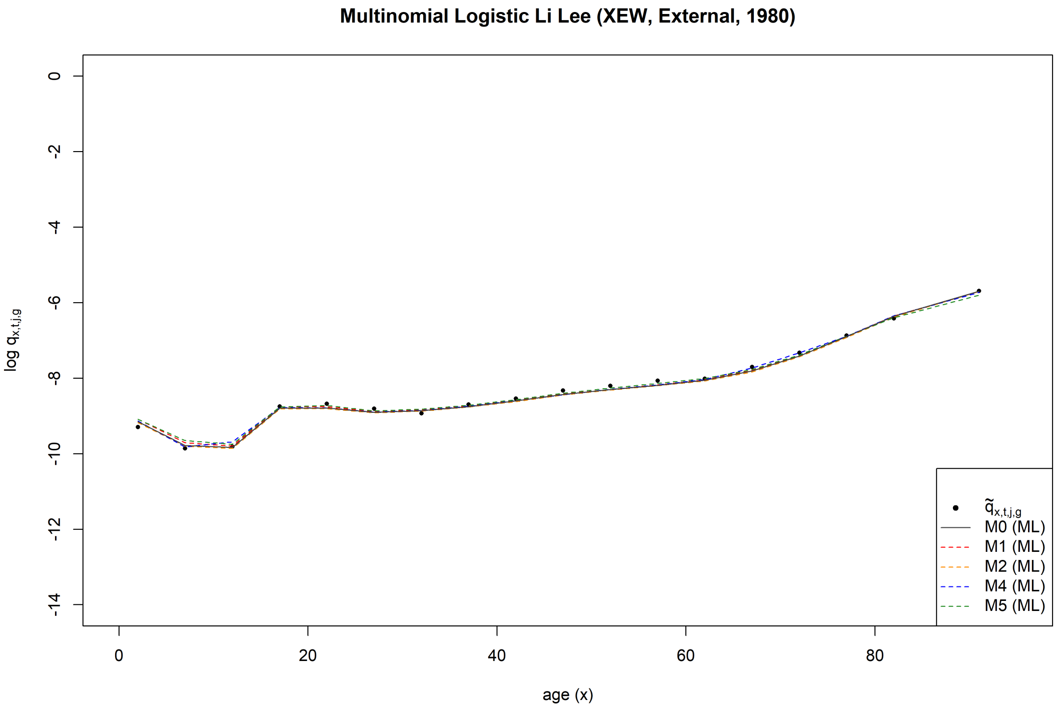


(c) Circulatory Diseases

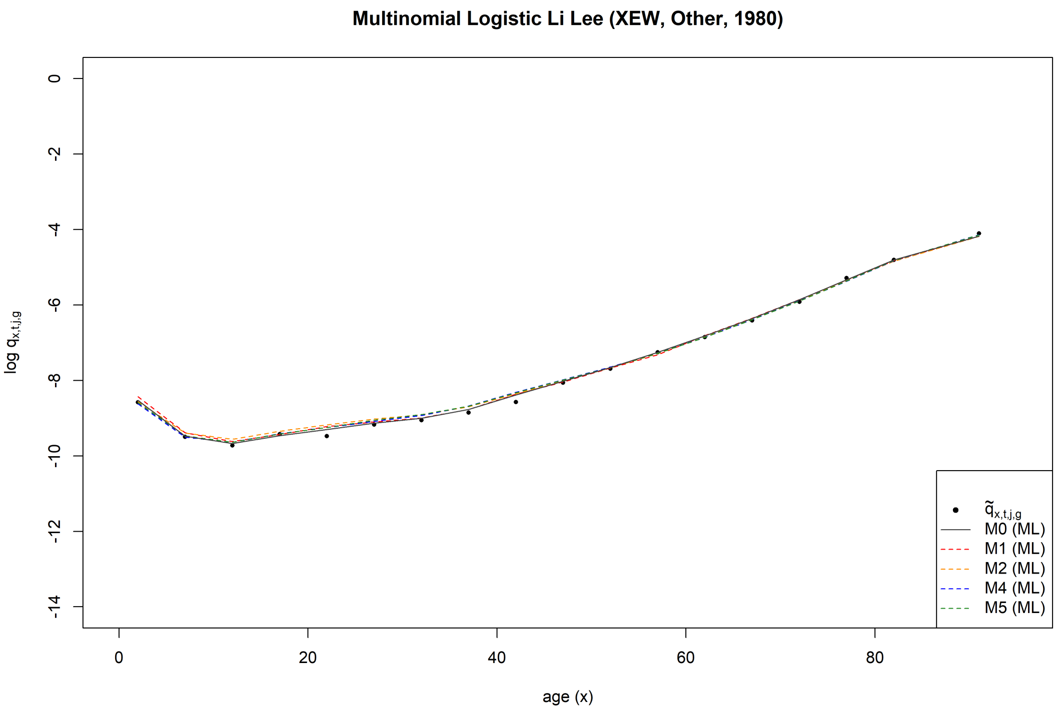


(d) Respiratory Diseases

FIGURE B.2 (cont.)

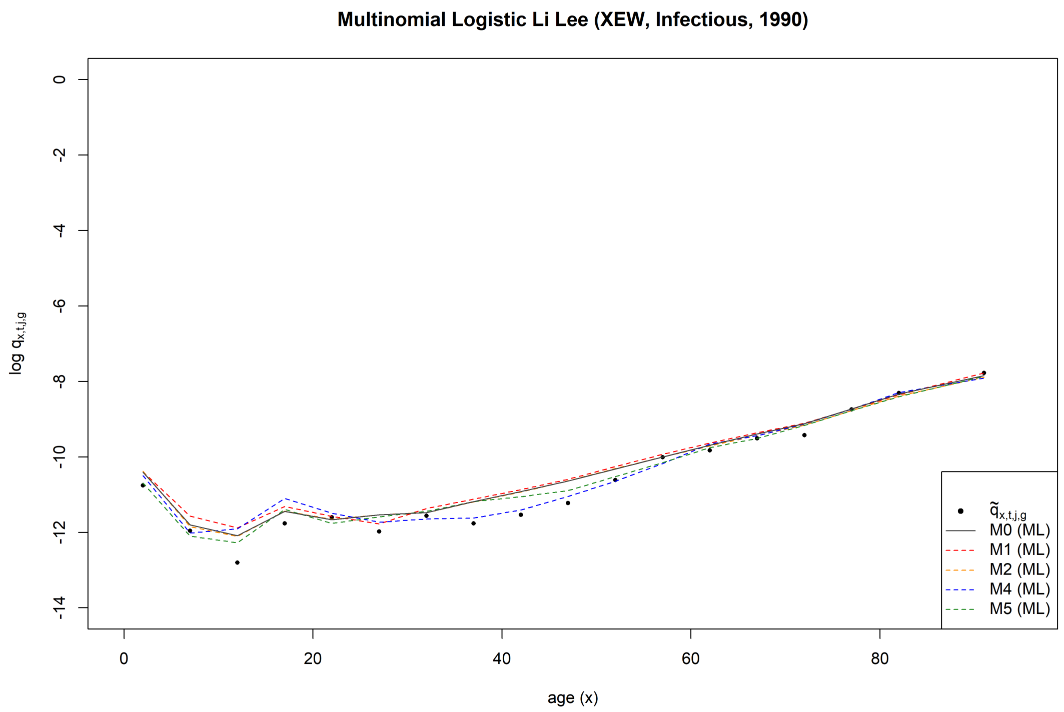


(e) External Causes

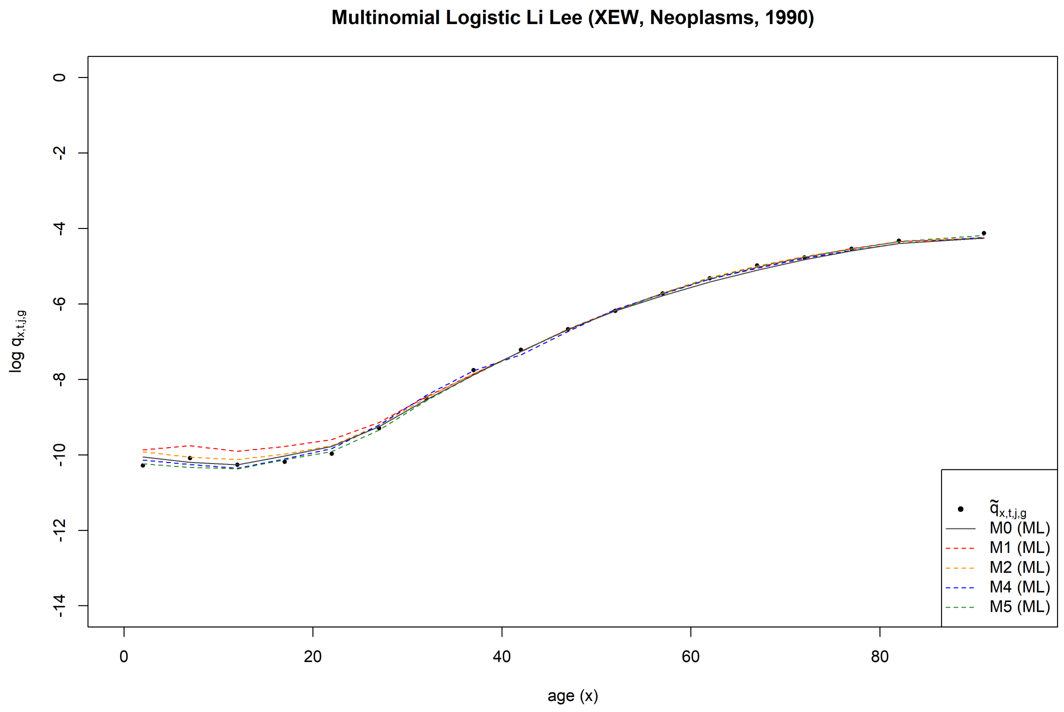


(f) Other Causes

FIGURE B.2 (cont.)

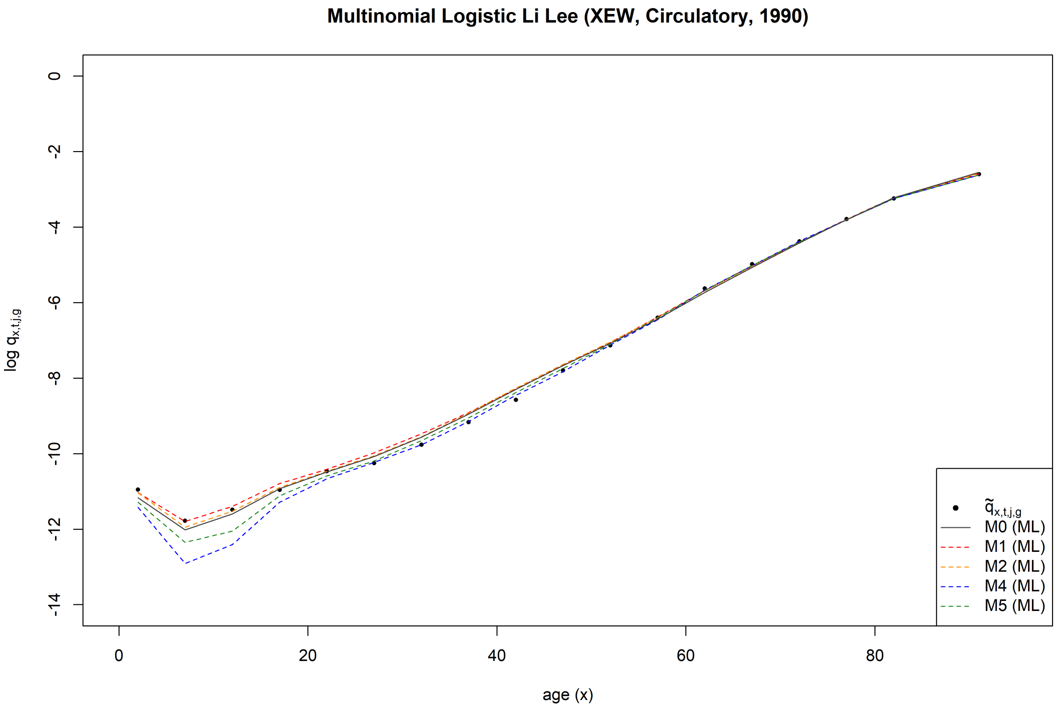


(a) Infectious and Parasitic Diseases

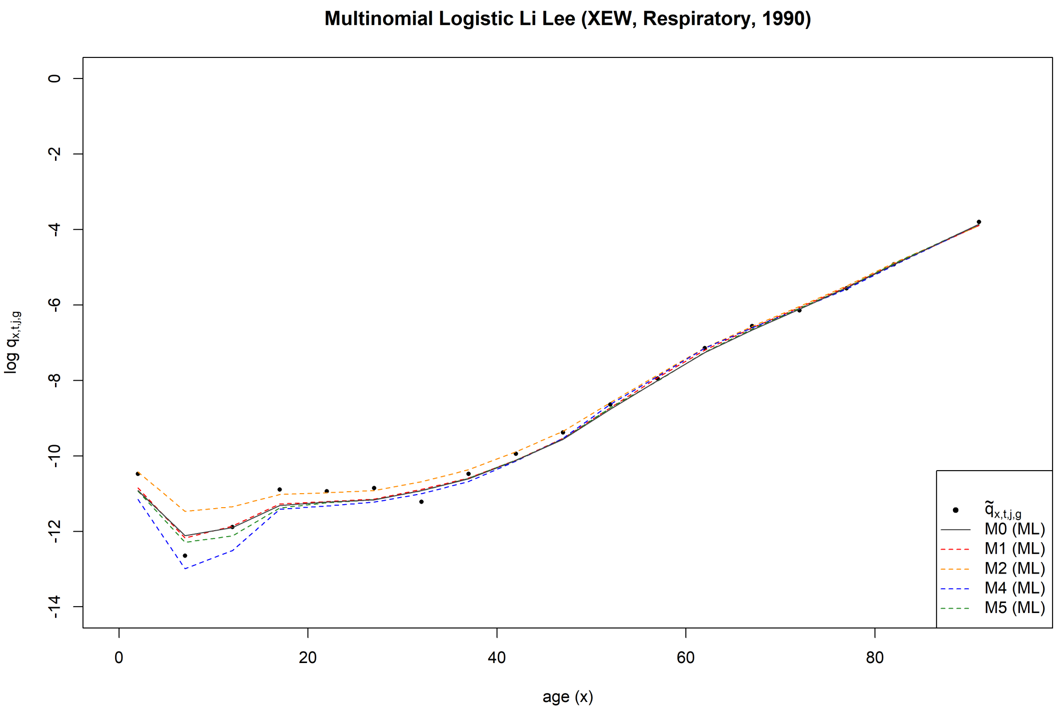


(b) Neoplasms

FIGURE B.3: Probabilities of Death in Year 1990
(England & Wales)

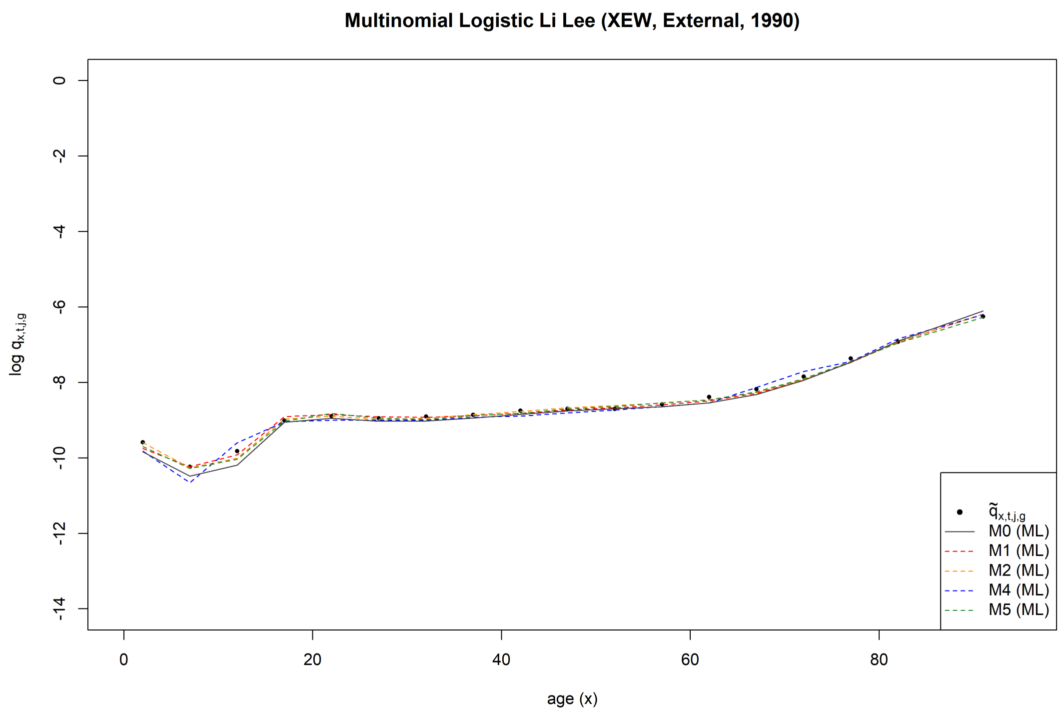


(c) Circulatory Diseases

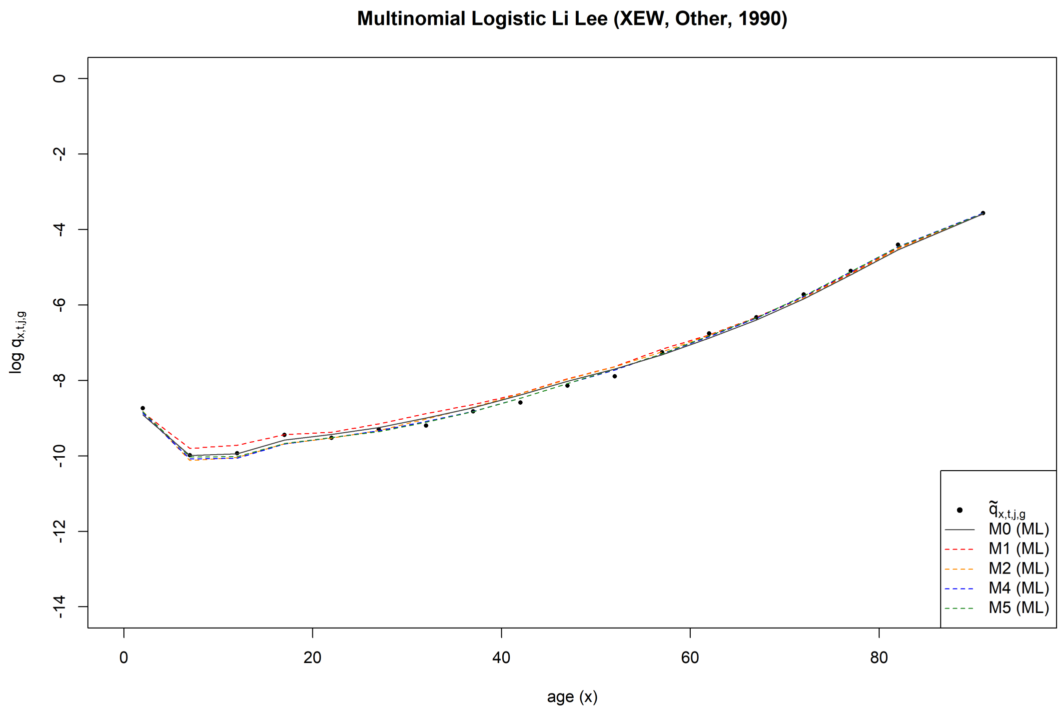


(d) Respiratory Diseases

FIGURE B.3 (cont.)

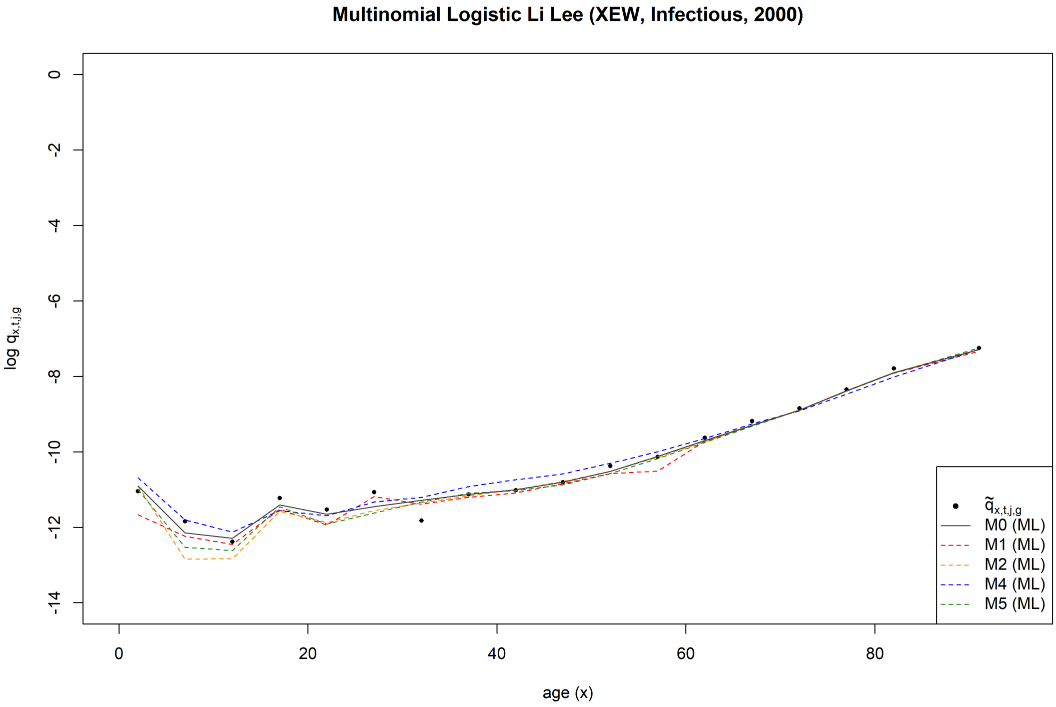


(e) External Causes

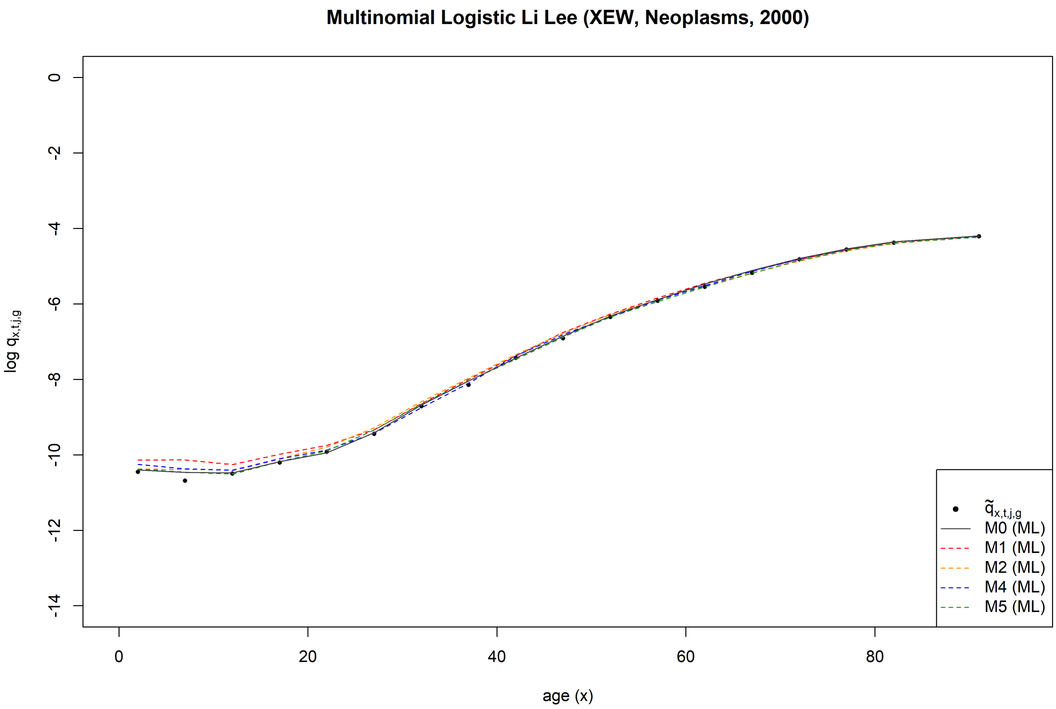


(f) Other Causes

FIGURE B.3 (cont.)

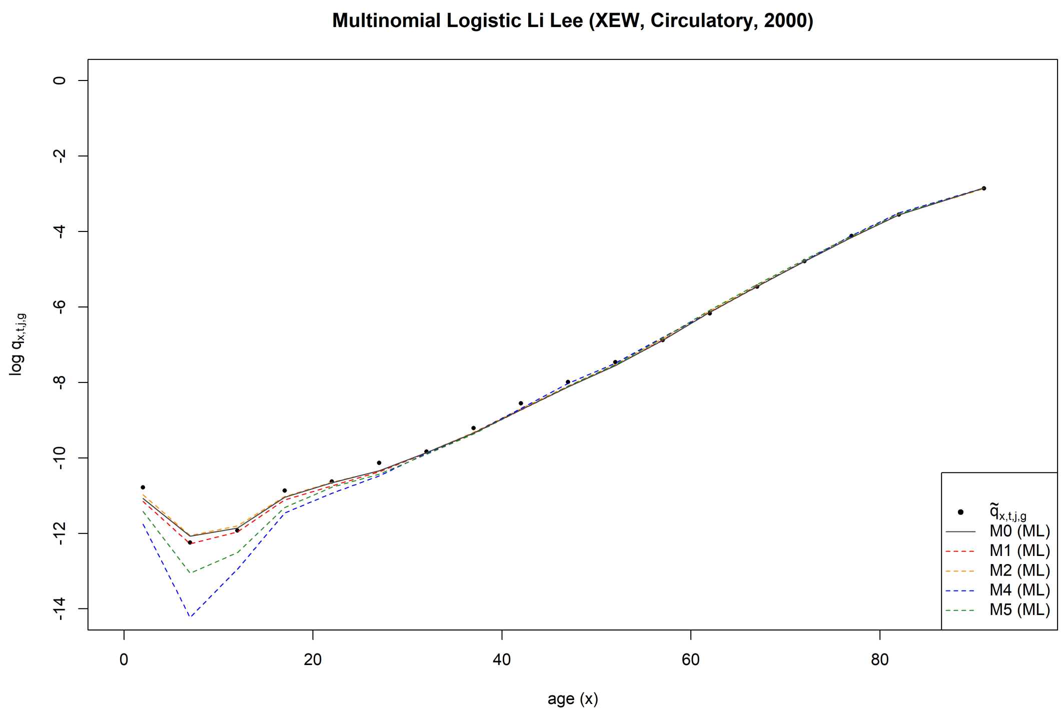


(a) Infectious and Parasitic Diseases

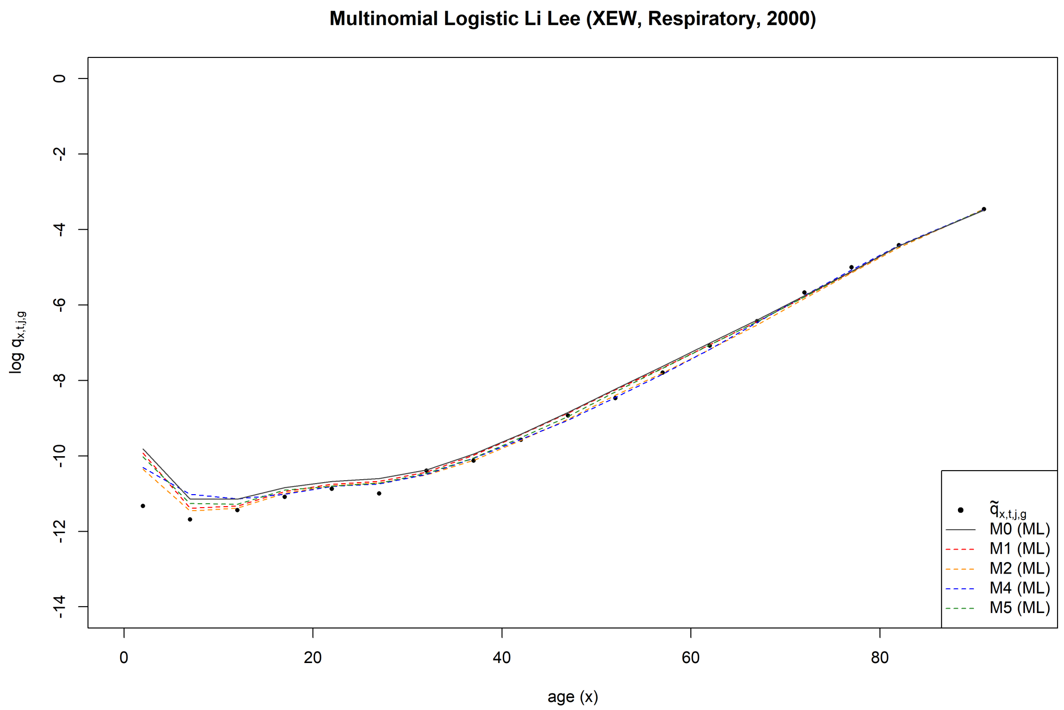


(b) Neoplasms

FIGURE B.4: Probabilities of Death in Year 2000
(England & Wales)

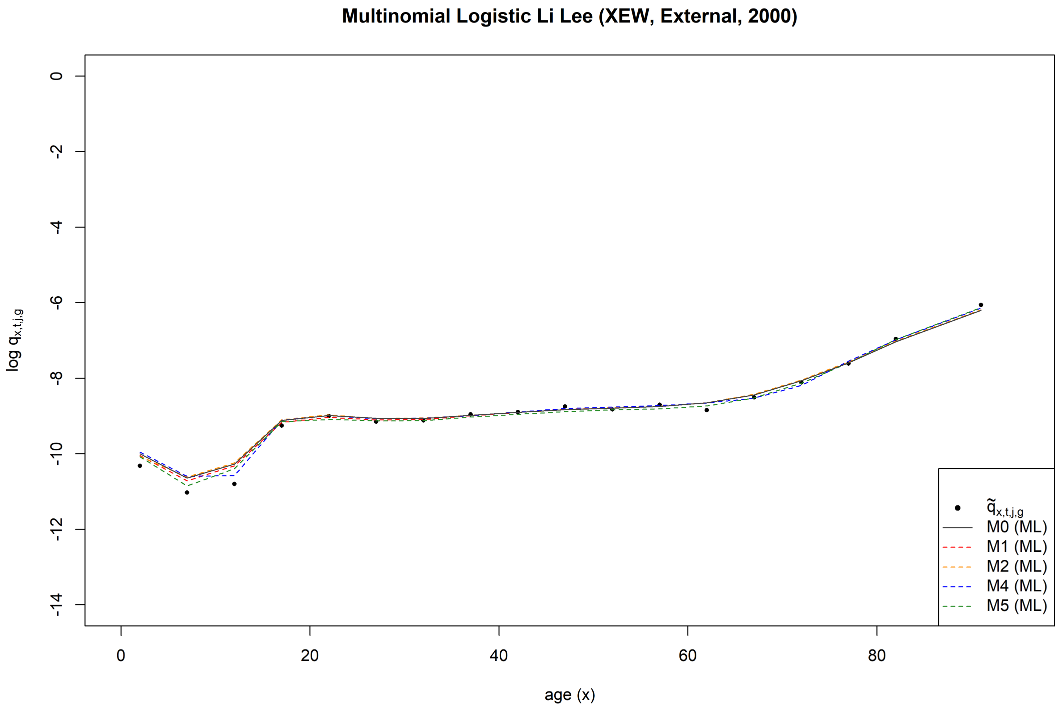


(c) Circulatory Diseases

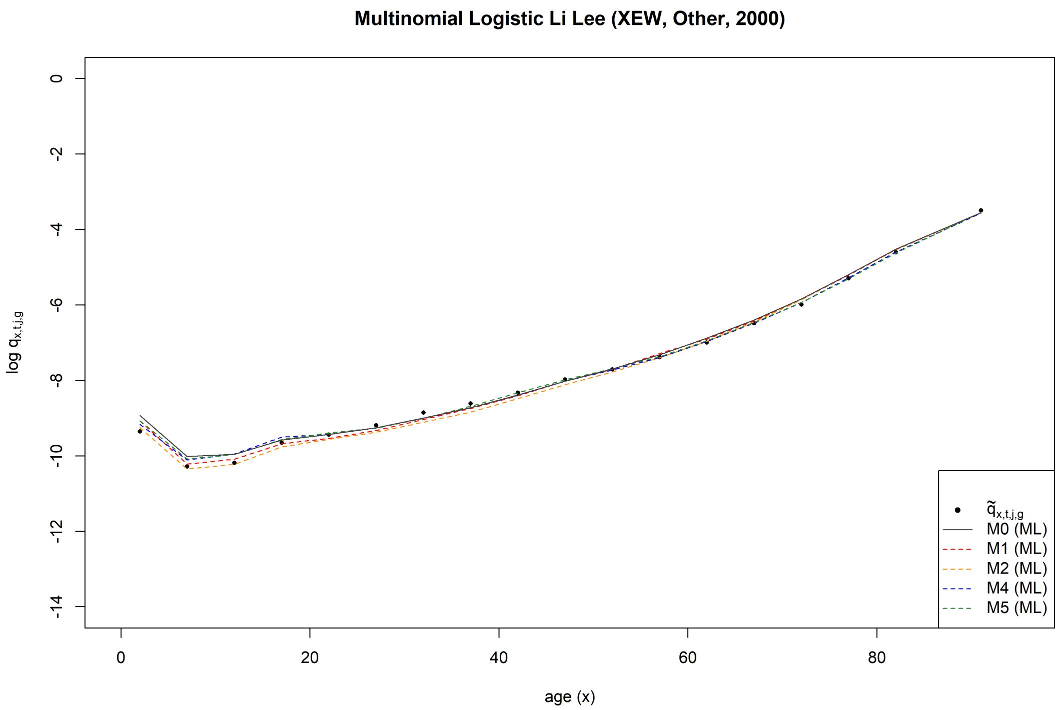


(d) Respiratory Diseases

FIGURE B.4 (cont.)



(e) External Causes



(f) Other Causes

FIGURE B.4 (cont.)

Appendix C

Frequentist Parameter Estimates

The remainder of the MLG-LC (M0) parameter estimates for deaths due to infectious and parasitic diseases (Cause 1), circulatory diseases (Cause 3), diseases of the respiratory system (Cause 4), external causes (Cause 5), and other causes (Cause 6) can be found in tables that follow.

TABLE C.1: PARAMETER ESTIMATES AND STANDARD ERRORS FOR CAUSE 1 — INFECTIOUS AND PARASITIC DISEASES, ENGLAND & WALES, FEMALES

	CAUSE 1 — INFECTIOUS AND PARASITIC DISEASES					
	Maximum Likelihood		Multivariate Normal Sampler		Parametric Bootstrap	
	Estimate	s.e.	Median	s.e.	Median	s.e.
$\alpha(x,1)$						
1 –4	-10.4338	0.024884 ***	-10.4338	0.024695 ***	-10.4338	0.030063 ***
5 –9	-11.8236	0.044901 ***	-11.8242	0.045774 ***	-11.8226	0.047107 ***
10 –14	-12.1008	0.052271 ***	-12.1014	0.052385 ***	-12.1018	0.049752 ***
15 –19	-11.4402	0.037824 ***	-11.4392	0.037683 ***	-11.4420	0.038070 ***
20 –24	-11.6645	0.041487 ***	-11.6644	0.041979 ***	-11.6666	0.040609 ***
25 –29	-11.5317	0.038519 ***	-11.5330	0.038127 ***	-11.5315	0.039973 ***
30 –34	-11.4623	0.036962 ***	-11.4625	0.036760 ***	-11.4607	0.038807 ***
35 –39	-11.1779	0.033078 ***	-11.1773	0.032952 ***	-11.1795	0.033741 ***
40 –44	-10.9219	0.030097 ***	-10.9221	0.030585 ***	-10.9237	0.030061 ***
45 –49	-10.6475	0.026585 ***	-10.6479	0.026683 ***	-10.6502	0.028097 ***
50 –54	-10.3315	0.023046 ***	-10.3316	0.023075 ***	-10.3327	0.023793 ***
55 –59	-10.0021	0.019945 ***	-10.0022	0.019952 ***	-10.0028	0.020147 ***
60 –64	-9.6790	0.017459 ***	-9.6788	0.017387 ***	-9.6793	0.017785 ***
65 –69	-9.3721	0.015430 ***	-9.3721	0.015414 ***	-9.3718	0.015088 ***
70 –74	-9.0810	0.013972 ***	-9.0810	0.013853 ***	-9.0814	0.013800 ***
75 –79	-8.6619	0.012256 ***	-8.6617	0.012345 ***	-8.6624	0.012587 ***
80 –84	-8.2328	0.011419 ***	-8.2330	0.011503 ***	-8.2352	0.012369 ***
85+	-7.6565	0.008907 ***	-7.6567	0.008929 ***	-7.6575	0.011539 ***
$\beta(x,1)$						
1 –4	-1.1391	0.072187 ***	-1.1394	0.073325 ***	-1.1375	7.160024
5 –9	-0.7920	0.118045 ***	-0.7917	0.117932 ***	-0.8183	5.252635
10 –14	-0.4539	0.128381 ***	-0.4527	0.128696 ***	-0.4433	3.422758
15 –19	0.0850	0.090141	0.0846	0.089602	0.0840	0.688844
20 –24	0.0265	0.100761	0.0268	0.099911	0.0278	1.051194
25 –29	0.2013	0.091377 *	0.2015	0.091090 *	0.2061	0.952015
30 –34	0.4417	0.079525 ***	0.4413	0.078771 ***	0.4350	2.175401
35 –39	0.1334	0.074726	0.1342	0.074919	0.1383	0.422075
40 –44	-0.2001	0.073564 **	-0.2014	0.073382 **	-0.2135	1.708381
45 –49	-0.3708	0.068688 ***	-0.3706	0.068883 ***	-0.3758	2.514457
50 –54	-0.4360	0.060099 ***	-0.4364	0.060091 ***	-0.4299	2.964968
55 –59	-0.2823	0.050102 ***	-0.2819	0.050149 ***	-0.2893	1.734129
60 –64	-0.0160	0.043587	-0.0162	0.043294	-0.0138	0.344060
65 –69	0.2296	0.037036 ***	0.2290	0.036575 ***	0.2346	1.235710
70 –74	0.5220	0.031172 ***	0.5220	0.031378 ***	0.5242	3.118747
75 –79	0.7886	0.024905 ***	0.7885	0.025035 ***	0.7941	4.728305
80 –84	0.9883	0.020927 ***	0.9885	0.020801 ***	0.9862	5.851026
85+	1.2738	0.014777 ***	1.2739	0.014725 ***	1.2924	7.601697

Table C.2: Parameter Estimates and Standard Errors for Cause 1 — Infectious and Parasitic Diseases, England & Wales, Females (continued)

	Cause 1 — Infectious and Parasitic Diseases					
	Maximum Likelihood		Multivariate Normal Sampler		Parametric Bootstrap	
	Estimate	s.e.	Median	s.e.	Median	s.e.
$\kappa(t,1)$						
1968	-0.2543	0.045212 ***	-0.2537	0.045007 ***	-0.2281	0.116311 *
1969	-0.4412	0.046734 ***	-0.4415	0.047016 ***	-0.4193	0.199636 *
1970	-0.4002	0.046130 ***	-0.4005	0.046275 ***	-0.3826	0.188356 *
1971	-0.4430	0.045974 ***	-0.4439	0.046118 ***	-0.4197	0.214839
1972	-0.4871	0.046122 ***	-0.4877	0.045940 ***	-0.4681	0.240758
1973	-0.4338	0.045434 ***	-0.4345	0.045422 ***	-0.4123	0.208904 *
1974	-0.4162	0.045095 ***	-0.4153	0.044825 ***	-0.3848	0.199512
1975	-0.2300	0.042720 ***	-0.2293	0.042582 ***	-0.2160	0.112589
1976	-0.2739	0.043194 ***	-0.2743	0.043721 ***	-0.2605	0.143425
1977	-0.2947	0.043337 ***	-0.2951	0.043070 ***	-0.2765	0.155308
1978	-0.4089	0.044651 ***	-0.4088	0.044624 ***	-0.3960	0.210446
1979	-0.2905	0.042858 ***	-0.2909	0.042230 ***	-0.2754	0.153723
1980	-0.2806	0.042313 ***	-0.2811	0.042312 ***	-0.2666	0.152003
1981	-0.2637	0.041310 ***	-0.2636	0.040684 ***	-0.2420	0.146311
1982	-0.2412	0.040495 ***	-0.2414	0.040446 ***	-0.2234	0.135420
1983	-0.2592	0.040345 ***	-0.2594	0.039968 ***	-0.2509	0.144484
1984	-0.0275	0.036465	-0.0277	0.036196	-0.0204	0.042404
1985	-0.0175	0.035887	-0.0178	0.035932	-0.0116	0.040707
1986	0.0506	0.034479	0.0495	0.034771	0.0403	0.043364
1987	-0.0667	0.035662	-0.0661	0.035941	-0.0581	0.055093
1988	-0.0324	0.034740	-0.0326	0.034871	-0.0241	0.042731
1989	0.0394	0.033216	0.0395	0.033262	0.0316	0.038001
1990	-0.0281	0.033873	-0.0279	0.034116	-0.0202	0.041586
1991	-0.0702	0.033938 *	-0.0708	0.033869 *	-0.0605	0.051006
1992	0.0598	0.031671	0.0599	0.031726	0.0523	0.045032
1993	0.0558	0.031401	0.0562	0.031274	0.0472	0.039857
1994	0.0038	0.031985	0.0037	0.032166	0.0027	0.035526
1995	0.0224	0.031402	0.0226	0.031282	0.0183	0.036710
1996	0.1129	0.029925 ***	0.1133	0.029727 ***	0.1048	0.065946
1997	0.2297	0.028169 ***	0.2297	0.028130 ***	0.2188	0.116264
1998	0.1972	0.028517 ***	0.1972	0.028583 ***	0.1848	0.099988
1999	0.3074	0.026936 ***	0.3080	0.027027 ***	0.2963	0.150936 *
2000	0.4072	0.025391 ***	0.4074	0.025339 ***	0.3952	0.199322 *
2001	0.6327	0.022762 ***	0.6326	0.022684 ***	0.6129	0.309509 *
2002	0.6846	0.022088 ***	0.6847	0.022047 ***	0.6602	0.333695 *
2003	0.8023	0.020803 ***	0.8025	0.020918 ***	0.7808	0.391966 *
2004	0.9038	0.019656 ***	0.9038	0.019540 ***	0.8694	0.441583 *
2005	1.1516	0.016887 ***	1.1519	0.016817 ***	1.1125	0.560077 *

TABLE C.3: PARAMETER ESTIMATES AND STANDARD ERRORS FOR CAUSE 3 — DISEASES OF THE CIRCULATORY SYSTEM, ENGLAND & WALES, FEMALES

	CAUSE 3 — DISEASES OF THE CIRCULATORY SYSTEM					
	Maximum Likelihood		Multivariate Normal Sampler		Parametric Bootstrap	
	Estimate	s.e.	Median	s.e.	Median	s.e.
$\alpha(x,3)$						
1 –4	-11.1901	0.038287 ***	-11.1902	0.038492 ***	-11.1920	0.037955 ***
5 –9	-11.9976	0.050346 ***	-11.9985	0.051026 ***	-11.9979	0.046347 ***
10 –14	-11.5296	0.039005 ***	-11.5295	0.038486 ***	-11.5305	0.041627 ***
15 –19	-10.8942	0.028647 ***	-10.8937	0.028604 ***	-10.8927	0.029491 ***
20 –24	-10.4358	0.022258 ***	-10.4361	0.022698 ***	-10.4338	0.023342 ***
25 –29	-10.0050	0.017824 ***	-10.0050	0.017893 ***	-10.0060	0.018791 ***
30 –34	-9.4840	0.013931 ***	-9.4841	0.013866 ***	-9.4844	0.013793 ***
35 –39	-8.8585	0.010372 ***	-8.8584	0.010343 ***	-8.8585	0.010681 ***
40 –44	-8.1870	0.007547 ***	-8.1871	0.007557 ***	-8.1876	0.007647 ***
45 –49	-7.5484	0.005507 ***	-7.5485	0.005458 ***	-7.5482	0.005887 ***
50 –54	-6.9512	0.004127 ***	-6.9513	0.004105 ***	-6.9514	0.004230 ***
55 –59	-6.3139	0.003051 ***	-6.3139	0.003004 ***	-6.3140	0.003280 ***
60 –64	-5.6143	0.002210 ***	-5.6143	0.002214 ***	-5.6146	0.002430 ***
65 –69	-4.9560	0.001655 ***	-4.9560	0.001653 ***	-4.9565	0.001862 ***
70 –74	-4.2965	0.001279 ***	-4.2965	0.001291 ***	-4.2964	0.001348 ***
75 –79	-3.6696	0.001064 ***	-3.6696	0.001067 ***	-3.6694	0.001048 ***
80 –84	-3.0628	0.000976 ***	-3.0628	0.000981 ***	-3.0629	0.001065 ***
85+	-2.3136	0.000802 ***	-2.3136	0.000792 ***	-2.3135	0.000918 ***
$\beta(x,3)$						
1 –4	-0.0174	0.007183 *	-0.0176	0.007136 *	-0.0179	0.007644 *
5 –9	0.0116	0.008638	0.0115	0.008615	0.0125	0.010329
10 –14	0.0478	0.007430 ***	0.0481	0.007477 ***	0.0473	0.008191 ***
15 –19	0.0214	0.005849 ***	0.0213	0.005811 ***	0.0214	0.006072 ***
20 –24	0.0327	0.004707 ***	0.0327	0.004700 ***	0.0330	0.004709 ***
25 –29	0.0489	0.003829 ***	0.0489	0.003823 ***	0.0489	0.004087 ***
30 –34	0.0546	0.002987 ***	0.0546	0.002980 ***	0.0545	0.003112 ***
35 –39	0.0697	0.002223 ***	0.0697	0.002224 ***	0.0696	0.002560 ***
40 –44	0.0782	0.001610 ***	0.0782	0.001616 ***	0.0781	0.002192 ***
45 –49	0.0830	0.001160 ***	0.0830	0.001150 ***	0.0829	0.001891 ***
50 –54	0.0875	0.000868 ***	0.0875	0.000862 ***	0.0874	0.001770 ***
55 –59	0.0823	0.000643 ***	0.0823	0.000642 ***	0.0824	0.001580 ***
60 –64	0.0754	0.000473 ***	0.0754	0.000480 ***	0.0755	0.001411 ***
65 –69	0.0712	0.000358 ***	0.0712	0.000357 ***	0.0711	0.001347 ***
70 –74	0.0689	0.000280 ***	0.0689	0.000281 ***	0.0688	0.001243 ***
75 –79	0.0654	0.000234 ***	0.0654	0.000232 ***	0.0654	0.001196 ***
80 –84	0.0626	0.000214 ***	0.0626	0.000215 ***	0.0625	0.001088 ***
85+	0.0562	0.000173 ***	0.0562	0.000172 ***	0.0560	0.001081 ***

TABLE C.4: PARAMETER ESTIMATES AND STANDARD ERRORS FOR CAUSE 3 — DISEASES OF THE CIRCULATORY SYSTEM, ENGLAND & WALES, FEMALES (CONTINUED)

	CAUSE 3 — DISEASES OF THE CIRCULATORY SYSTEM					
	Maximum Likelihood		Multivariate Normal Sampler		Parametric Bootstrap	
	Estimate	s.e.	Median	s.e.	Median	s.e.
$\kappa(t,3)$						
1968	6.8472	0.039519 ***	6.8478	0.039017 ***	6.8463	0.125306 ***
1969	6.4954	0.039638 ***	6.4955	0.039933 ***	6.4937	0.120385 ***
1970	6.0562	0.039895 ***	6.0557	0.039802 ***	6.0545	0.116610 ***
1971	5.6690	0.039873 ***	5.6692	0.040143 ***	5.6693	0.109332 ***
1972	6.0999	0.039109 ***	6.0992	0.039074 ***	6.1064	0.116502 ***
1973	5.7883	0.039266 ***	5.7886	0.039575 ***	5.7937	0.107005 ***
1974	5.4814	0.039395 ***	5.4819	0.039265 ***	5.4862	0.105668 ***
1975	5.0386	0.039665 ***	5.0386	0.039786 ***	5.0408	0.097973 ***
1976	4.9360	0.039574 ***	4.9368	0.039450 ***	4.9329	0.093607 ***
1977	4.2172	0.040198 ***	4.2172	0.040343 ***	4.2153	0.084818 ***
1978	4.1459	0.040047 ***	4.1460	0.039917 ***	4.1506	0.085494 ***
1979	4.0151	0.039980 ***	4.0154	0.039729 ***	4.0140	0.079673 ***
1980	3.2346	0.040602 ***	3.2343	0.040420 ***	3.2360	0.069381 ***
1981	2.5163	0.040892 ***	2.5161	0.040466 ***	2.5125	0.063197 ***
1982	2.1727	0.041105 ***	2.1726	0.041404 ***	2.1689	0.055742 ***
1983	1.8216	0.041291 ***	1.8220	0.041705 ***	1.8230	0.054670 ***
1984	1.3733	0.041572 ***	1.3730	0.041763 ***	1.3755	0.048883 ***
1985	1.5769	0.041022 ***	1.5766	0.041506 ***	1.5724	0.050019 ***
1986	0.8271	0.041712 ***	0.8270	0.041555 ***	0.8287	0.045076 ***
1987	0.0363	0.042336	0.0370	0.042903	0.0359	0.044437
1988	-0.2700	0.042503 ***	-0.2694	0.042616 ***	-0.2704	0.042604 ***
1989	-0.6702	0.042748 ***	-0.6707	0.042723 ***	-0.6686	0.044698 ***
1990	-1.3296	0.043330 ***	-1.3297	0.043701 ***	-1.3271	0.048714 ***
1991	-1.5293	0.043103 ***	-1.5292	0.042744 ***	-1.5346	0.048647 ***
1992	-2.2063	0.043743 ***	-2.2066	0.043732 ***	-2.2105	0.061512 ***
1993	-2.1592	0.043493 ***	-2.1594	0.043514 ***	-2.1615	0.060471 ***
1994	-3.3134	0.044885 ***	-3.3140	0.044788 ***	-3.3184	0.073712 ***
1995	-3.5277	0.044903 ***	-3.5282	0.044764 ***	-3.5302	0.078455 ***
1996	-4.0466	0.045449 ***	-4.0477	0.045896 ***	-4.0474	0.085203 ***
1997	-4.7675	0.046334 ***	-4.7668	0.046281 ***	-4.7738	0.097927 ***
1998	-4.9566	0.046499 ***	-4.9571	0.046904 ***	-4.9531	0.099319 ***
1999	-5.6290	0.047402 ***	-5.6286	0.047633 ***	-5.6290	0.108075 ***
2000	-6.8495	0.048885 ***	-6.8485	0.048908 ***	-6.8500	0.130964 ***
2001	-6.2404	0.048212 ***	-6.2406	0.048756 ***	-6.2353	0.119114 ***
2002	-6.5498	0.048477 ***	-6.5499	0.048760 ***	-6.5475	0.123992 ***
2003	-6.8258	0.048841 ***	-6.8257	0.048628 ***	-6.8246	0.128359 ***
2004	-8.2874	0.050872 ***	-8.2860	0.050146 ***	-8.2848	0.152445 ***
2005	-9.1910	0.051957 ***	-9.1911	0.051890 ***	-9.2076	0.172710 ***

TABLE C.5: PARAMETER ESTIMATES AND STANDARD ERRORS FOR CAUSE 4 — DISEASES OF THE RESPIRATORY SYSTEM, ENGLAND & WALES, FEMALES

	CAUSE 4 — DISEASES OF THE RESPIRATORY SYSTEM					
	Maximum Likelihood		Multivariate Normal Sampler		Parametric Bootstrap	
	Estimate	s.e.	Median	s.e.	Median	s.e.
$\alpha(x,4)$						
1 –4	-9.9645	0.018761 ***	-9.9644	0.018726 ***	-9.9652	0.022102 ***
5 –9	-11.2777	0.032241 ***	-11.2784	0.032772 ***	-11.2806	0.039025 ***
10 –14	-11.2502	0.032648 ***	-11.2508	0.032709 ***	-11.2468	0.037596 ***
15 –19	-10.9056	0.028394 ***	-10.9056	0.028439 ***	-10.9087	0.031401 ***
20 –24	-10.7504	0.025783 ***	-10.7501	0.025922 ***	-10.7505	0.026781 ***
25 –29	-10.6763	0.024684 ***	-10.6762	0.024625 ***	-10.6768	0.026979 ***
30 –34	-10.4462	0.022216 ***	-10.4462	0.022411 ***	-10.4475	0.023977 ***
35 –39	-10.0406	0.018487 ***	-10.0404	0.018590 ***	-10.0427	0.018407 ***
40 –44	-9.5191	0.014582 ***	-9.5190	0.014461 ***	-9.5192	0.015102 ***
45 –49	-8.9440	0.010952 ***	-8.9440	0.010951 ***	-8.9439	0.012513 ***
50 –54	-8.2981	0.008163 ***	-8.2982	0.008058 ***	-8.2984	0.008681 ***
55 –59	-7.6694	0.006143 ***	-7.6694	0.006126 ***	-7.6694	0.005995 ***
60 –64	-7.0335	0.004625 ***	-7.0334	0.004636 ***	-7.0336	0.004469 ***
65 –69	-6.4183	0.003506 ***	-6.4182	0.003484 ***	-6.4184	0.003636 ***
70 –74	-5.7807	0.002693 ***	-5.7806	0.002664 ***	-5.7813	0.002902 ***
75 –79	-5.1279	0.002169 ***	-5.1279	0.002168 ***	-5.1284	0.002217 ***
80 –84	-4.4336	0.001861 ***	-4.4337	0.001860 ***	-4.4346	0.001984 ***
85+	-3.3908	0.001272 ***	-3.3908	0.001275 ***	-3.3907	0.001357 ***
$\beta(x,4)$						
1 –4	0.1097	0.003270 ***	0.1096	0.003291 ***	0.1095	0.004079 ***
5 –9	0.0962	0.005269 ***	0.0962	0.005219 ***	0.0963	0.006610 ***
10 –14	0.0743	0.005323 ***	0.0743	0.005345 ***	0.0742	0.006437 ***
15 –19	0.0470	0.004672 ***	0.0469	0.004671 ***	0.0476	0.005405 ***
20 –24	0.0541	0.004296 ***	0.0540	0.004240 ***	0.0540	0.004548 ***
25 –29	0.0562	0.004201 ***	0.0563	0.004211 ***	0.0560	0.004385 ***
30 –34	0.0544	0.003893 ***	0.0543	0.003887 ***	0.0544	0.004208 ***
35 –39	0.0650	0.003288 ***	0.0650	0.003265 ***	0.0654	0.003780 ***
40 –44	0.0689	0.002599 ***	0.0690	0.002603 ***	0.0689	0.002921 ***
45 –49	0.0706	0.001985 ***	0.0706	0.001979 ***	0.0705	0.002435 ***
50 –54	0.0531	0.001476 ***	0.0531	0.001474 ***	0.0532	0.001696 ***
55 –59	0.0376	0.001089 ***	0.0375	0.001085 ***	0.0374	0.001232 ***
60 –64	0.0250	0.000807 ***	0.0250	0.000799 ***	0.0250	0.000927 ***
65 –69	0.0260	0.000614 ***	0.0260	0.000610 ***	0.0260	0.000732 ***
70 –74	0.0347	0.000480 ***	0.0347	0.000478 ***	0.0348	0.000764 ***
75 –79	0.0430	0.000388 ***	0.0430	0.000388 ***	0.0431	0.000838 ***
80 –84	0.0486	0.000337 ***	0.0486	0.000335 ***	0.0487	0.000923 ***
85+	0.0356	0.000236 ***	0.0356	0.000237 ***	0.0353	0.000773 ***

TABLE C.6: PARAMETER ESTIMATES AND STANDARD ERRORS FOR CAUSE 4 — DISEASES OF THE RESPIRATORY SYSTEM, ENGLAND & WALES, FEMALES (CONTINUED)

	CAUSE 4 — DISEASES OF THE RESPIRATORY SYSTEM					
	Maximum Likelihood		Multivariate Normal Sampler		Parametric Bootstrap	
	Estimate	s.e.	Median	s.e.	Median	s.e.
$\kappa(t,4)$						
1968	7.532220	0.126803 ***	7.5330	0.126057 ***	7.5404	0.185344 ***
1969	6.030353	0.129849 ***	6.0279	0.130176 ***	6.0288	0.175340 ***
1970	6.764204	0.126894 ***	6.7643	0.126478 ***	6.7564	0.176219 ***
1971	2.336198	0.137316 ***	2.3362	0.137341 ***	2.3289	0.159752 ***
1972	5.483760	0.127623 ***	5.4851	0.127552 ***	5.4827	0.168185 ***
1973	5.476098	0.126896 ***	5.4763	0.127318 ***	5.4716	0.166242 ***
1974	4.539837	0.128587 ***	4.5394	0.127165 ***	4.5348	0.156472 ***
1975	4.542741	0.127718 ***	4.5406	0.127930 ***	4.5517	0.152142 ***
1976	9.253285	0.114960 ***	9.2514	0.115090 ***	9.2502	0.182075 ***
1977	5.228648	0.124252 ***	5.2274	0.124857 ***	5.2327	0.140954 ***
1978	5.670809	0.122150 ***	5.6720	0.122212 ***	5.6638	0.153360 ***
1979	5.806135	0.120833 ***	5.8045	0.121289 ***	5.8078	0.147591 ***
1980	4.747999	0.122284 ***	4.7495	0.122504 ***	4.7554	0.145725 ***
1981	4.280053	0.121400 ***	4.2766	0.120787 ***	4.2863	0.136843 ***
1982	5.332206	0.117597 ***	5.3324	0.118112 ***	5.3218	0.137276 ***
1983	4.484355	0.118501 ***	4.4846	0.117205 ***	4.4819	0.139903 ***
1984	-9.146768	0.154527 ***	-9.1475	0.153511 ***	-9.1388	0.226001 ***
1985	-5.824803	0.143186 ***	-5.8274	0.143254 ***	-5.8336	0.169075 ***
1986	-6.704772	0.144538 ***	-6.7074	0.142095 ***	-6.7111	0.190677 ***
1987	-9.767162	0.151606 ***	-9.7684	0.151843 ***	-9.7720	0.223411 ***
1988	-8.533805	0.146679 ***	-8.5310	0.147230 ***	-8.5316	0.205045 ***
1989	-6.034001	0.138212 ***	-6.0322	0.137425 ***	-6.0260	0.174624 ***
1990	-8.693259	0.144521 ***	-8.6933	0.144774 ***	-8.7006	0.211445 ***
1991	-7.814359	0.140053 ***	-7.8160	0.139646 ***	-7.8036	0.199182 ***
1992	-9.533724	0.143849 ***	-9.5333	0.144563 ***	-9.5208	0.213261 ***
1993	1.949407	0.113643 ***	1.9530	0.114064 ***	1.9557	0.115823 ***
1994	-1.062066	0.120388 ***	-1.0625	0.118161 ***	-1.0568	0.127294 ***
1995	1.545725	0.113291 ***	1.5457	0.113524 ***	1.5369	0.116519 ***
1996	0.728256	0.114729 ***	0.7295	0.114739 ***	0.7276	0.109563 ***
1997	2.029130	0.111620 ***	2.0302	0.111865 ***	2.0302	0.115936 ***
1998	1.093103	0.113656 ***	1.0910	0.114746 ***	1.0993	0.116780 ***
1999	3.310390	0.108826 ***	3.3114	0.109860 ***	3.3229	0.111916 ***
2000	1.386583	0.112297 ***	1.3849	0.111791 ***	1.3867	0.110405 ***
2001	-6.634404	0.132221 ***	-6.6336	0.132335 ***	-6.6365	0.176906 ***
2002	-5.546280	0.128848 ***	-5.5446	0.129871 ***	-5.5466	0.162490 ***
2003	-3.557503	0.123787 ***	-3.5585	0.124566 ***	-3.5519	0.143613 ***
2004	-5.765025	0.129136 ***	-5.7642	0.128814 ***	-5.7650	0.160717 ***
2005	-4.933564	0.126324 ***	-4.9320	0.125239 ***	-4.9308	0.155895 ***

TABLE C.7: PARAMETER ESTIMATES AND STANDARD ERRORS FOR CAUSE 5 — EXTERNAL CAUSES, ENGLAND & WALES, FEMALES

	CAUSE 5 — EXTERNAL CAUSES					
	Maximum Likelihood		Multivariate Normal Sampler		Parametric Bootstrap	
	Estimate	s.e.	Median	s.e.	Median	s.e.
$\alpha(x,5)$						
1 –4	-9.4697	0.015098 ***	-9.4698	0.015133 ***	-9.4692	0.017254 ***
5 –9	-10.1070	0.018172 ***	-10.1070	0.017970 ***	-10.1074	0.021111 ***
10 –14	-9.9996	0.018070 ***	-9.9997	0.018102 ***	-10.0027	0.018731 ***
15 –19	-8.9152	0.010575 ***	-8.9153	0.010476 ***	-8.9157	0.010313 ***
20 –24	-8.8636	0.010184 ***	-8.8634	0.010140 ***	-8.8634	0.010228 ***
25 –29	-8.9644	0.010697 ***	-8.9645	0.010736 ***	-8.9650	0.011020 ***
30 –34	-8.9345	0.010643 ***	-8.9343	0.010669 ***	-8.9341	0.010707 ***
35 –39	-8.8391	0.010358 ***	-8.8391	0.010461 ***	-8.8387	0.010276 ***
40 –44	-8.7112	0.009955 ***	-8.7112	0.009964 ***	-8.7106	0.010077 ***
45 –49	-8.5790	0.009396 ***	-8.5789	0.009411 ***	-8.5790	0.009369 ***
50 –54	-8.4802	0.009015 ***	-8.4802	0.008980 ***	-8.4795	0.008892 ***
55 –59	-8.3933	0.008688 ***	-8.3934	0.008734 ***	-8.3937	0.009612 ***
60 –64	-8.2702	0.008323 ***	-8.2702	0.008309 ***	-8.2710	0.008893 ***
65 –69	-8.0288	0.007600 ***	-8.0288	0.007515 ***	-8.0292	0.008475 ***
70 –74	-7.6305	0.006628 ***	-7.6306	0.006577 ***	-7.6300	0.006971 ***
75 –79	-7.1122	0.005735 ***	-7.1122	0.005687 ***	-7.1123	0.005920 ***
80 –84	-6.5285	0.005229 ***	-6.5286	0.005172 ***	-6.5285	0.005397 ***
85+	-5.7168	0.004020 ***	-5.7168	0.003977 ***	-5.7168	0.004074 ***
$\beta(x,5)$						
1 –4	0.0956	0.002581 ***	0.0955	0.002595 ***	0.0956	0.002865 ***
5 –9	0.0971	0.003038 ***	0.0972	0.003052 ***	0.0971	0.003332 ***
10 –14	0.0496	0.003217 ***	0.0496	0.003198 ***	0.0497	0.003419 ***
15 –19	0.0367	0.002067 ***	0.0367	0.002066 ***	0.0365	0.002142 ***
20 –24	0.0229	0.001999 ***	0.0230	0.001966 ***	0.0229	0.002020 ***
25 –29	0.0171	0.002100 ***	0.0171	0.002091 ***	0.0170	0.002091 ***
30 –34	0.0231	0.002096 ***	0.0231	0.002108 ***	0.0231	0.002100 ***
35 –39	0.0260	0.002052 ***	0.0261	0.002057 ***	0.0261	0.002036 ***
40 –44	0.0343	0.001935 ***	0.0343	0.001925 ***	0.0341	0.001990 ***
45 –49	0.0456	0.001770 ***	0.0456	0.001758 ***	0.0458	0.001761 ***
50 –54	0.0558	0.001699 ***	0.0558	0.001705 ***	0.0556	0.001760 ***
55 –59	0.0630	0.001632 ***	0.0630	0.001644 ***	0.0628	0.001708 ***
60 –64	0.0675	0.001559 ***	0.0675	0.001583 ***	0.0674	0.001733 ***
65 –69	0.0717	0.001431 ***	0.0717	0.001436 ***	0.0716	0.001566 ***
70 –74	0.0746	0.001269 ***	0.0746	0.001264 ***	0.0746	0.001398 ***
75 –79	0.0794	0.001116 ***	0.0794	0.001117 ***	0.0793	0.001327 ***
80 –84	0.0789	0.001031 ***	0.0789	0.001042 ***	0.0789	0.001169 ***
85+	0.0609	0.000815 ***	0.0609	0.000820 ***	0.0620	0.001150 ***

Table C.8: Parameter Estimates and Standard Errors for Cause 5 — External Causes, England & Wales, Females (continued)

	Cause 5 — External Causes					
	Maximum Likelihood		Multivariate Normal Sampler		Parametric Bootstrap	
	Estimate	s.e.	Median	s.e.	Median	s.e.
$\kappa(t,5)$						
1968	8.1063	0.150840 ***	8.1036	0.150384 ***	8.1183	0.158692 ***
1969	8.2147	0.149252 ***	8.2125	0.149317 ***	8.2122	0.158277 ***
1970	7.6203	0.151548 ***	7.6186	0.153323 ***	7.6202	0.158861 ***
1971	7.3378	0.151686 ***	7.3379	0.151894 ***	7.3195	0.167106 ***
1972	6.5855	0.154957 ***	6.5856	0.155579 ***	6.5926	0.161225 ***
1973	6.5051	0.154770 ***	6.5053	0.154673 ***	6.4944	0.161492 ***
1974	6.1748	0.155949 ***	6.1736	0.156357 ***	6.1743	0.161121 ***
1975	5.3723	0.159673 ***	5.3702	0.162188 ***	5.3769	0.163587 ***
1976	5.0647	0.160837 ***	5.0645	0.161815 ***	5.0661	0.167749 ***
1977	4.4365	0.163709 ***	4.4369	0.163519 ***	4.4346	0.166240 ***
1978	4.7862	0.161020 ***	4.7862	0.159727 ***	4.7946	0.161133 ***
1979	4.4416	0.162262 ***	4.4391	0.163090 ***	4.4458	0.167274 ***
1980	3.3544	0.167416 ***	3.3555	0.166572 ***	3.3563	0.173886 ***
1981	2.3572	0.171179 ***	2.3566	0.172681 ***	2.3545	0.176992 ***
1982	1.8138	0.173645 ***	1.8111	0.173650 ***	1.7964	0.179626 ***
1983	1.1840	0.176485 ***	1.1844	0.175721 ***	1.1927	0.175486 ***
1984	0.8544	0.177429 ***	0.8550	0.177234 ***	0.8574	0.186278 ***
1985	0.8771	0.176186 ***	0.8757	0.176441 ***	0.8718	0.187563 ***
1986	0.1249	0.179898	0.1287	0.180452	0.1249	0.177929
1987	-1.6963	0.189953 ***	-1.6960	0.190011 ***	-1.7055	0.198098 ***
1988	-2.1474	0.191907 ***	-2.1498	0.193919 ***	-2.1607	0.191872 ***
1989	-3.1085	0.197088 ***	-3.1095	0.197035 ***	-3.0958	0.198247 ***
1990	-3.8409	0.201029 ***	-3.8377	0.199931 ***	-3.8602	0.199031 ***
1991	-4.0193	0.200325 ***	-4.0181	0.200307 ***	-4.0054	0.203520 ***
1992	-5.1114	0.206866 ***	-5.1082	0.205335 ***	-5.1160	0.213754 ***
1993	-4.7726	0.203726 ***	-4.7733	0.201770 ***	-4.7738	0.207221 ***
1994	-5.5887	0.208913 ***	-5.5894	0.208954 ***	-5.5925	0.207923 ***
1995	-5.6522	0.208188 ***	-5.6502	0.208432 ***	-5.6367	0.206928 ***
1996	-5.9285	0.209511 ***	-5.9310	0.210765 ***	-5.9431	0.214825 ***
1997	-5.7012	0.207589 ***	-5.6983	0.207725 ***	-5.7084	0.208722 ***
1998	-6.0052	0.209297 ***	-6.0027	0.212273 ***	-5.9916	0.217029 ***
1999	-5.4606	0.205478 ***	-5.4607	0.205647 ***	-5.4525	0.206309 ***
2000	-5.5624	0.205238 ***	-5.5621	0.205950 ***	-5.5627	0.214305 ***
2001	-4.3391	0.197612 ***	-4.3403	0.197742 ***	-4.3308	0.211161 ***
2002	-4.8992	0.200570 ***	-4.9000	0.198768 ***	-4.9062	0.216768 ***
2003	-3.7987	0.193097 ***	-3.7953	0.192461 ***	-3.7892	0.201545 ***
2004	-3.7020	0.192080 ***	-3.6995	0.193648 ***	-3.6873	0.199221 ***
2005	-3.8774	0.192385 ***	-3.8779	0.193649 ***	-3.8714	0.203477 ***

TABLE C.9: PARAMETER ESTIMATES AND STANDARD ERRORS FOR CAUSE 6 — OTHER CAUSES, ENGLAND & WALES, FEMALES

	CAUSE 6 — OTHER CAUSES					
	Maximum Likelihood		Multivariate Normal Sampler		Parametric Bootstrap	
	Estimate	s.e.	Median	s.e.	Median	s.e.
$\alpha(x,6)$						
1 –4	-8.7523	0.011124 ***	-8.7522	0.011054 ***	-8.7517	0.012119 ***
5 –9	-9.7787	0.016120 ***	-9.7785	0.016227 ***	-9.7784	0.016994 ***
10 –14	-9.8298	0.016786 ***	-9.8299	0.016790 ***	-9.8308	0.017167 ***
15 –19	-9.5295	0.014516 ***	-9.5291	0.014674 ***	-9.5302	0.014592 ***
20 –24	-9.3736	0.013176 ***	-9.3735	0.013079 ***	-9.3741	0.012752 ***
25 –29	-9.2006	0.012031 ***	-9.2007	0.012086 ***	-9.2012	0.011854 ***
30 –34	-8.9992	0.010955 ***	-8.9991	0.010925 ***	-8.9987	0.010958 ***
35 –39	-8.7395	0.009798 ***	-8.7394	0.009795 ***	-8.7391	0.009455 ***
40 –44	-8.3818	0.008430 ***	-8.3818	0.008387 ***	-8.3821	0.008827 ***
45 –49	-8.0181	0.007151 ***	-8.0182	0.007198 ***	-8.0186	0.007090 ***
50 –54	-7.6778	0.006138 ***	-7.6777	0.006108 ***	-7.6782	0.005844 ***
55 –59	-7.2861	0.005140 ***	-7.2862	0.005151 ***	-7.2860	0.005014 ***
60 –64	-6.8437	0.004231 ***	-6.8437	0.004258 ***	-6.8435	0.004153 ***
65 –69	-6.3628	0.003432 ***	-6.3628	0.003432 ***	-6.3629	0.003625 ***
70 –74	-5.8171	0.002769 ***	-5.8170	0.002783 ***	-5.8176	0.002835 ***
75 –79	-5.2069	0.002263 ***	-5.2070	0.002256 ***	-5.2077	0.002335 ***
80 –84	-4.5654	0.001955 ***	-4.5654	0.001959 ***	-4.5663	0.002074 ***
85+	-3.6619	0.001347 ***	-3.6619	0.001341 ***	-3.6628	0.001695 ***
$\beta(x,6)$						
1 –4	0.5221	0.024951 ***	0.5220	0.024756 ***	0.5209	0.051406 ***
5 –9	0.6948	0.034031 ***	0.6952	0.034049 ***	0.6938	0.067148 ***
10 –14	0.3701	0.035315 ***	0.3708	0.035336 ***	0.3706	0.045128 ***
15 –19	0.1560	0.031698 ***	0.1555	0.031584 ***	0.1531	0.033818 ***
20 –24	0.1747	0.029508 ***	0.1747	0.029435 ***	0.1754	0.032092 ***
25 –29	0.1529	0.027386 ***	0.1528	0.027178 ***	0.1521	0.029288 ***
30 –34	-0.0112	0.024964	-0.0117	0.024938	-0.0093	0.027225
35 –39	-0.0603	0.022291 **	-0.0600	0.022361 **	-0.0599	0.027606 *
40 –44	-0.0015	0.019298	-0.0011	0.019343	-0.0003	0.020865
45 –49	-0.0011	0.016525	-0.0014	0.016601	-0.0031	0.018251
50 –54	0.0524	0.014034 ***	0.0523	0.013903 ***	0.0538	0.014193 ***
55 –59	0.0837	0.011718 ***	0.0837	0.011879 ***	0.0844	0.013623 ***
60 –64	0.0800	0.009814 ***	0.0800	0.009660 ***	0.0801	0.011623 ***
65 –69	0.0604	0.008019 ***	0.0604	0.008099 ***	0.0607	0.009976 ***
70 –74	-0.0125	0.006536	-0.0125	0.006476	-0.0124	0.007107
75 –79	-0.1600	0.005346 ***	-0.1599	0.005334 ***	-0.1598	0.018159 ***
80 –84	-0.3364	0.004600 ***	-0.3363	0.004583 ***	-0.3369	0.035241 ***
85+	-0.7641	0.003128 ***	-0.7642	0.003133 ***	-0.7653	0.078760 ***

TABLE C.10: PARAMETER ESTIMATES AND STANDARD ERRORS FOR CAUSE 6 — OTHER CAUSES, ENGLAND & WALES, FEMALES (CONTINUED)

	CAUSE 6 — OTHER CAUSES					
	Maximum Likelihood		Multivariate Normal Sampler		Parametric Bootstrap	
	Estimate	s.e.	Median	s.e.	Median	s.e.
$\kappa(t,6)$						
1968	0.409876	0.016061 ***	0.4098	0.016081 ***	0.4092	0.046431 ***
1969	0.446649	0.016026 ***	0.4467	0.016110 ***	0.4454	0.049810 ***
1970	0.484493	0.016007 ***	0.4845	0.015934 ***	0.4841	0.052056 ***
1971	0.533607	0.015671 ***	0.5332	0.015604 ***	0.5299	0.055864 ***
1972	0.494493	0.015328 ***	0.4947	0.015188 ***	0.4946	0.054171 ***
1973	0.517672	0.015290 ***	0.5175	0.015235 ***	0.5163	0.055234 ***
1974	0.504196	0.015065 ***	0.5043	0.015059 ***	0.5047	0.054686 ***
1975	0.512277	0.015001 ***	0.5123	0.014975 ***	0.5097	0.053304 ***
1976	0.534507	0.014986 ***	0.5345	0.014856 ***	0.5357	0.056676 ***
1977	0.546233	0.014943 ***	0.5467	0.014966 ***	0.5460	0.056650 ***
1978	0.528150	0.014746 ***	0.5282	0.014687 ***	0.5269	0.053931 ***
1979	0.370913	0.013971 ***	0.3707	0.014006 ***	0.3669	0.039225 ***
1980	0.437682	0.014058 ***	0.4375	0.013962 ***	0.4356	0.045599 ***
1981	0.409771	0.013559 ***	0.4098	0.013468 ***	0.4083	0.042080 ***
1982	0.385759	0.013219 ***	0.3856	0.013237 ***	0.3853	0.039714 ***
1983	0.389774	0.013069 ***	0.3902	0.013242 ***	0.3928	0.041047 ***
1984	-0.207119	0.010662 ***	-0.2069	0.010755 ***	-0.2066	0.023023 ***
1985	-0.324775	0.010086 ***	-0.3249	0.010111 ***	-0.3248	0.034339 ***
1986	-0.352025	0.009837 ***	-0.3518	0.009770 ***	-0.3516	0.037238 ***
1987	-0.275021	0.009809 ***	-0.2748	0.009862 ***	-0.2745	0.029068 ***
1988	-0.318836	0.009495 ***	-0.3190	0.009491 ***	-0.3163	0.033569 ***
1989	-0.339446	0.009245 ***	-0.3395	0.009203 ***	-0.3368	0.035596 ***
1990	-0.306612	0.009204 ***	-0.3066	0.009201 ***	-0.3044	0.033070 ***
1991	-0.292234	0.009030 ***	-0.2924	0.009063 ***	-0.2914	0.030446 ***
1992	-0.263915	0.008949 ***	-0.2640	0.008908 ***	-0.2620	0.027314 ***
1993	0.002063	0.009580	0.0022	0.009528	0.0029	0.009309
1994	-0.009555	0.009437	-0.0096	0.009327	-0.0104	0.009701
1995	-0.153232	0.008884 ***	-0.1532	0.008899 ***	-0.1543	0.018347 ***
1996	-0.192277	0.008703 ***	-0.1924	0.008834 ***	-0.1913	0.022093 ***
1997	-0.249269	0.008492 ***	-0.2492	0.008537 ***	-0.2483	0.027442 ***
1998	-0.337925	0.008174 ***	-0.3379	0.008160 ***	-0.3373	0.034470 ***
1999	-0.402912	0.007960 ***	-0.4030	0.007990 ***	-0.4023	0.041309 ***
2000	-0.339592	0.008066 ***	-0.3394	0.008159 ***	-0.3394	0.035231 ***
2001	-0.593225	0.007560 ***	-0.5932	0.007470 ***	-0.5925	0.060212 ***
2002	-0.629014	0.007480 ***	-0.6291	0.007438 ***	-0.6290	0.063013 ***
2003	-0.702676	0.007374 ***	-0.7027	0.007375 ***	-0.7019	0.070462 ***
2004	-0.611563	0.007602 ***	-0.6114	0.007648 ***	-0.6114	0.062029 ***
2005	-0.606892	0.007472 ***	-0.6068	0.007555 ***	-0.6074	0.062246 ***

Appendix D

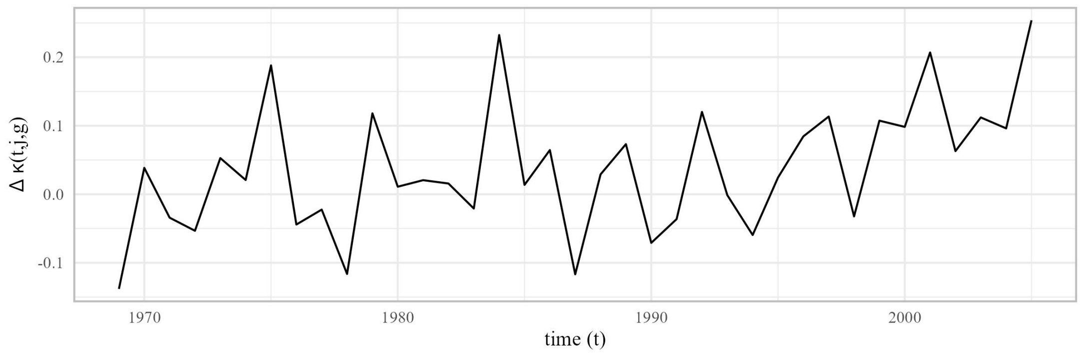
Additional Time Series Plots

The plots of the single and double-differenced time series of the parameters $\kappa(t, j, g)$ for MLG-LC (M0) and MLG-LL (M5) as well as the single and double-differenced time series of the parameters $k(t, j)$ for the MLG-LL (M5) are presented on the pages that follow.

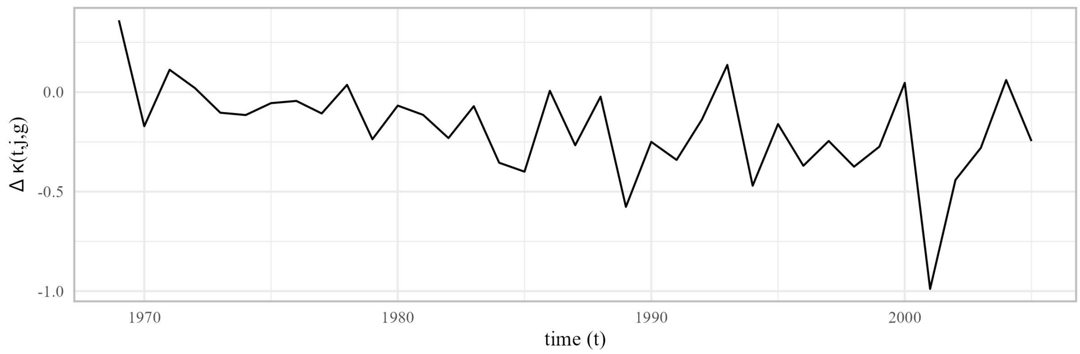
D.1 Multinomial Logistic Lee-Carter (M0)

D.1.1 England & Wales

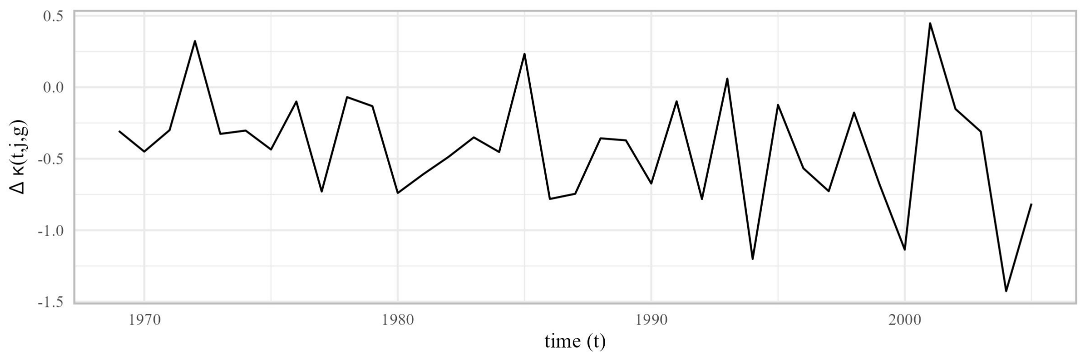
The time series of $\Delta\kappa(t, j, g)$ for all six causes in England & Wales are presented in Figure D.1. The plots suggest that the means of the time series $\Delta\kappa(t, j, g)$ are non-zero except for perhaps respiratory diseases. (This observation is formally tested in Section 5.3.1.) The mean of



(a) Infectious and Parasitic Diseases

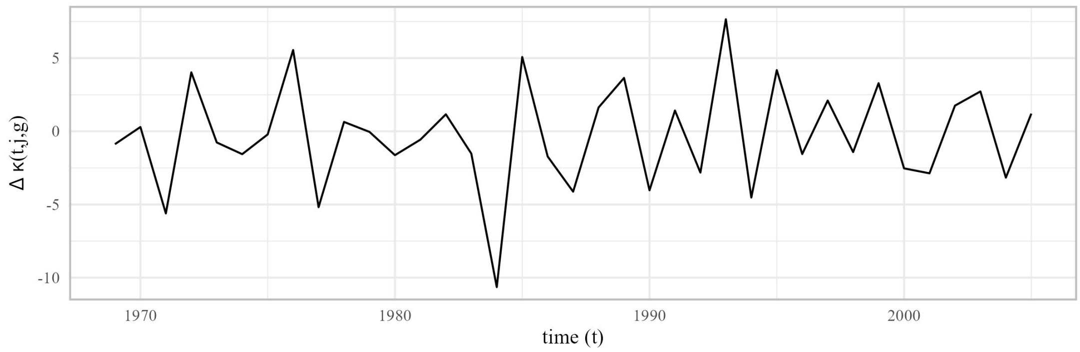


(b) Neoplasms

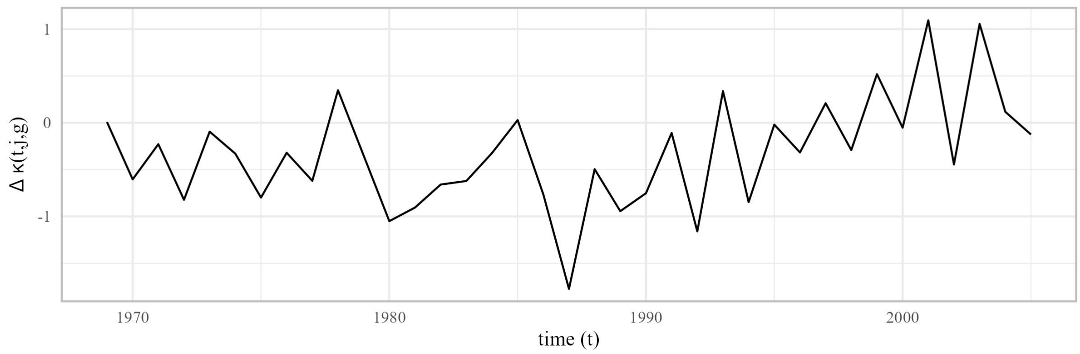


(c) Circulatory Diseases

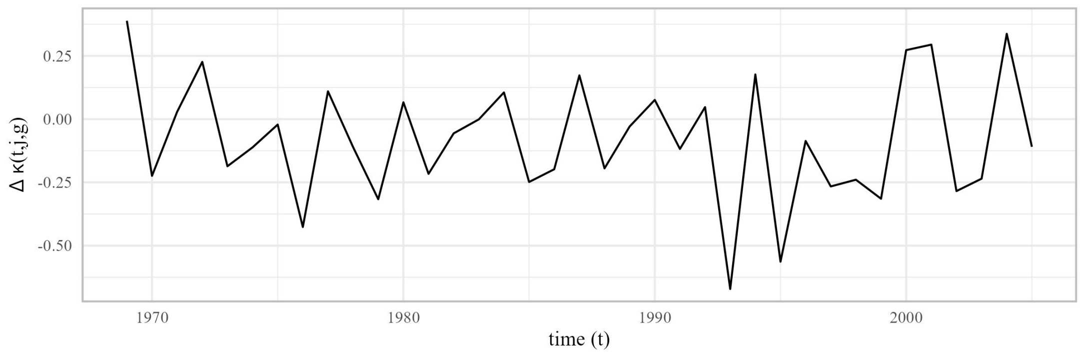
FIGURE D.1: Time series plots of single-differenced $\kappa(t, j, g)$
(England & Wales, Females)



(d) Respiratory Diseases



(e) External Causes

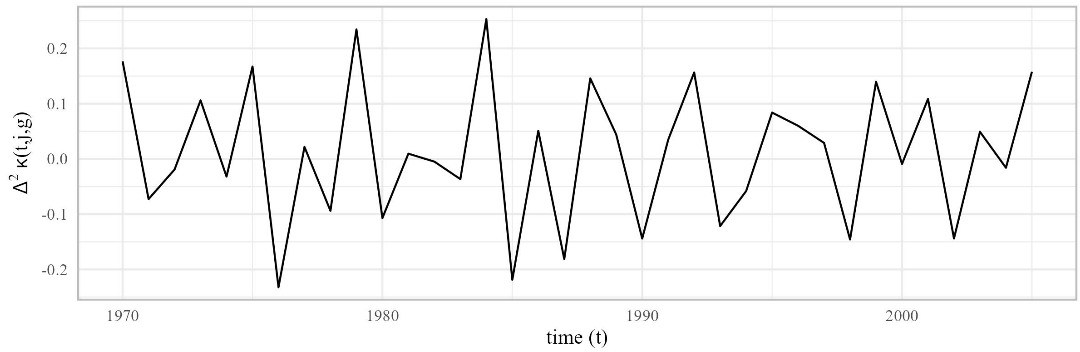


(f) Other Causes

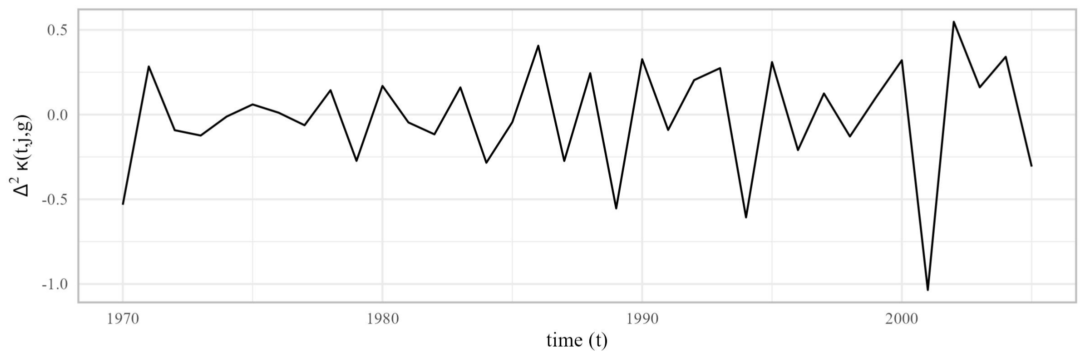
FIGURE D.1 (CONT.)

$\Delta\kappa(t, j, g)$ for infectious diseases appears to be positive, which is consistent with the upward drift shown in Figure 5.1.a. On the other hand, the time series plots for the remaining causes of death suggest a negative mean consistent with a downward drift shown in Figures 5.1.b-5.1.c & 5.1.e-5.1.f. We note that there is a visible dip around the year 2001 in the single-differenced time series for neoplasms – this corresponds to the year when ONS in England & Wales switched from ICD-9 to ICD-10.

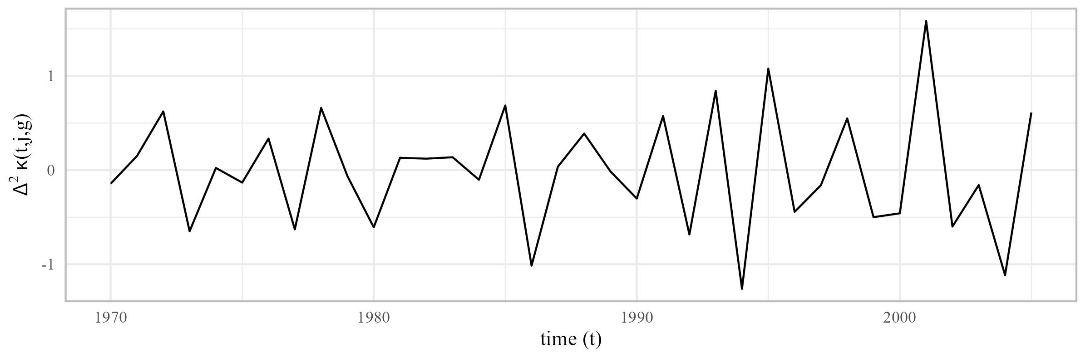
The plots of $\Delta^2\kappa(t,j,g)$ for all causes in England & Wales are presented in Figure D.2. The double-differenced time series for all causes now resemble a random walk with a mean around zero.



(a) Infectious and Parasitic Diseases

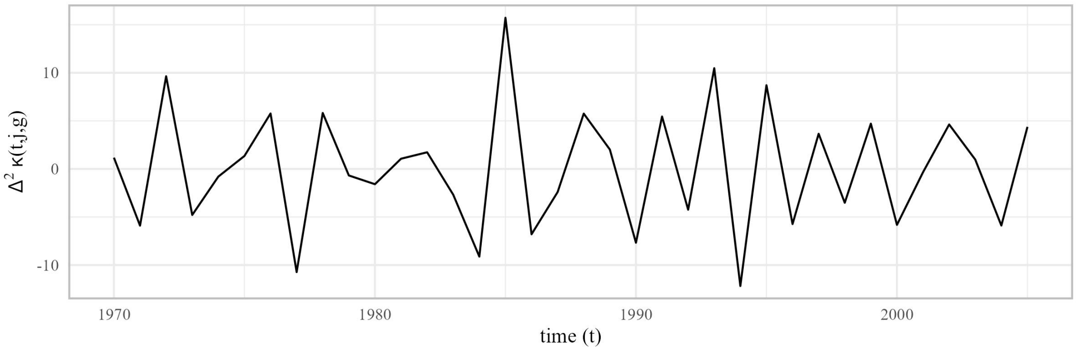


(b) Neoplasms

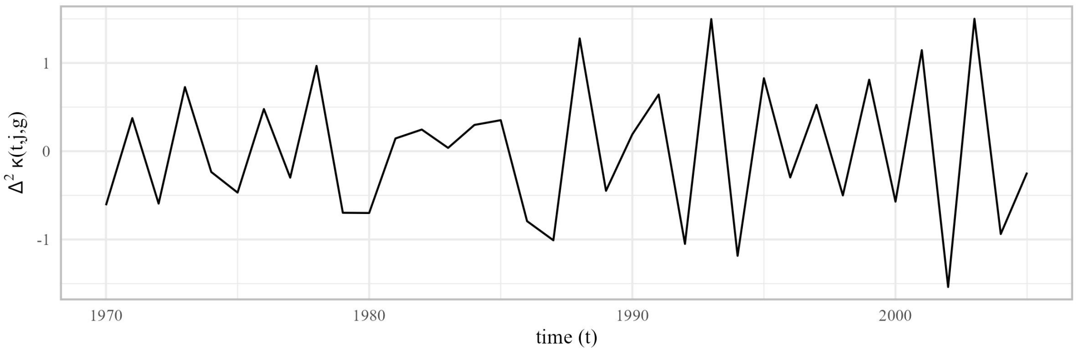


(c) Circulatory Diseases

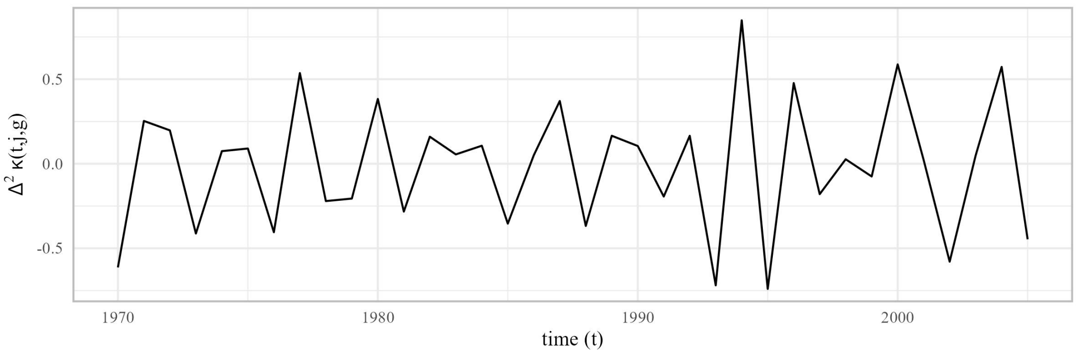
FIGURE D.2: Time series plots of double-differenced $\kappa(t,j,g)$ (England & Wales, Females)



(d) Respiratory Diseases



(e) External Causes

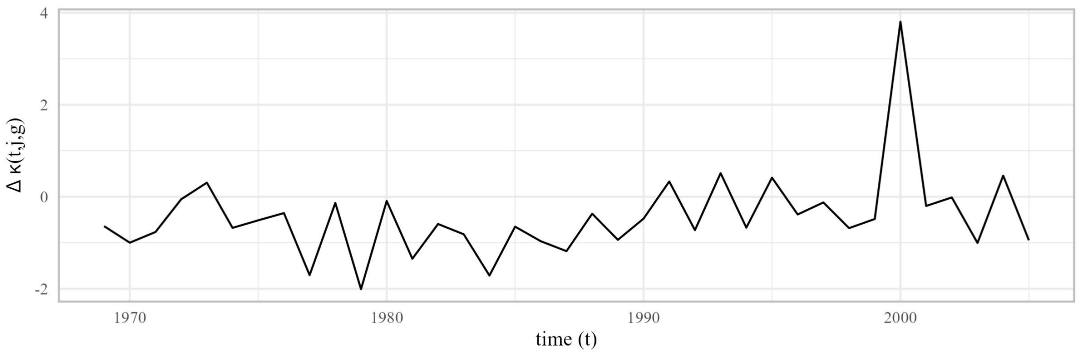


(f) Other Causes

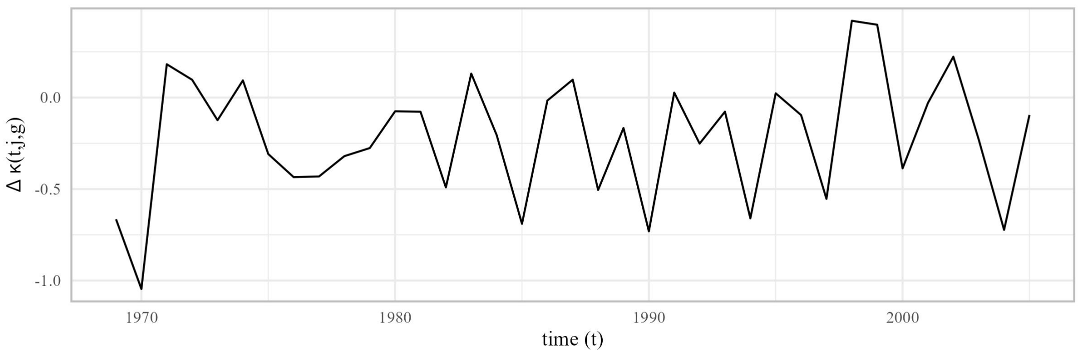
FIGURE D.2 (CONT.)

D.1.2 France

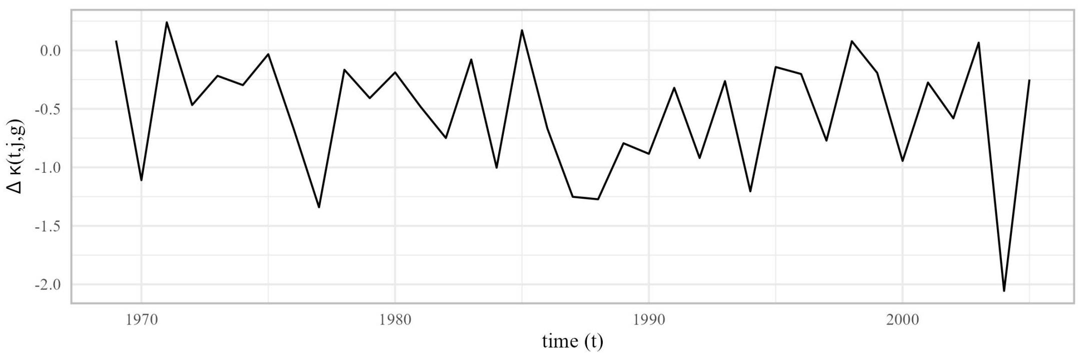
The plots of $\Delta\kappa(t, j, g)$ for all six causes in France are presented in Figure D.3. The plots suggest that the means of the time series $\Delta\kappa(t, j, g)$ are non-zero. (This observation is formally tested in Section 5.3.2.) In contrast to England & Wales, the means of $\Delta\kappa(t, j, g)$ for all causes in France appear to be negative, which is consistent with the downward drifts shown in Figure 5.2. This is true even for infectious diseases where a positive spike remains around the year



(a) Infectious and Parasitic Diseases

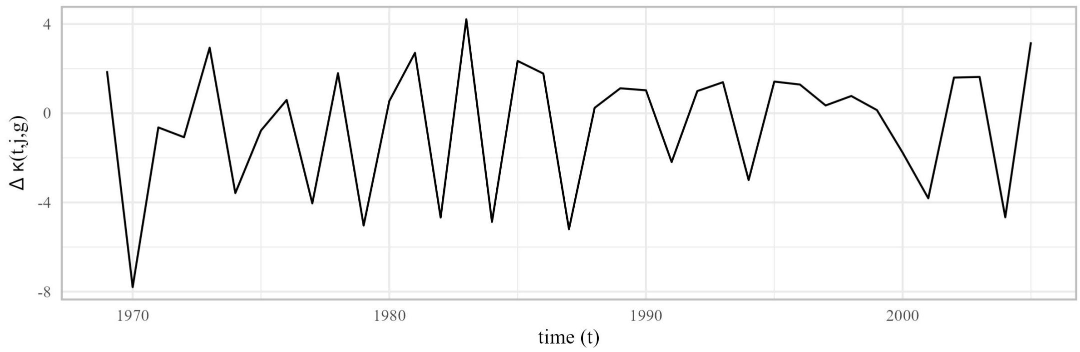


(b) Neoplasms

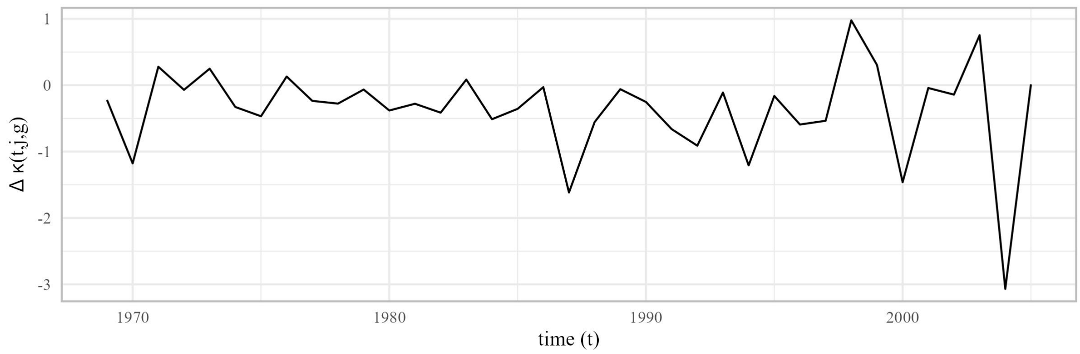


(c) Circulatory Diseases

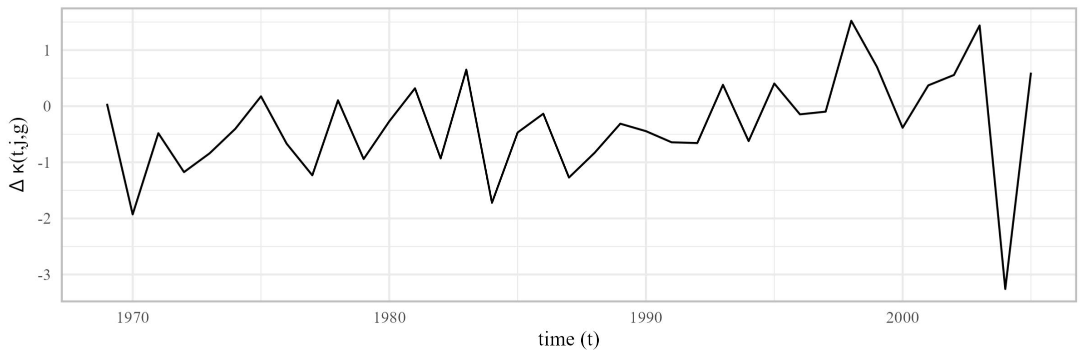
FIGURE D.3: Time series plots of single-differenced $\kappa(t, j, g)$ (France, Females)



(d) Respiratory Diseases



(e) External Causes

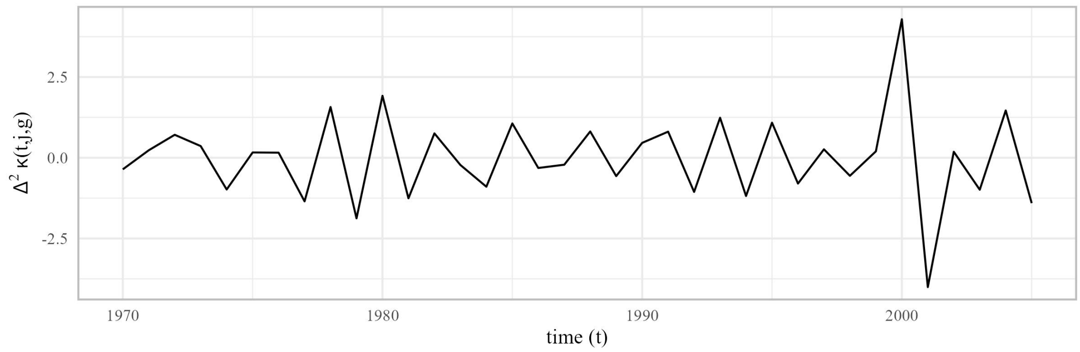


(f) Other Causes

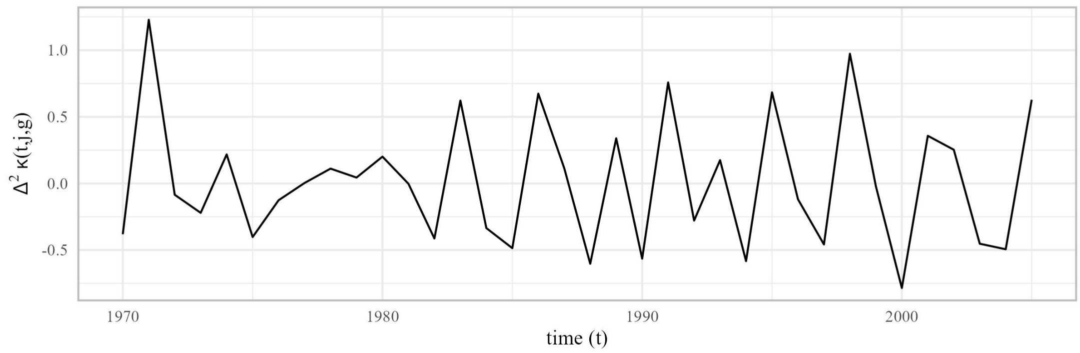
FIGURE D.3 (CONT.)

2000, i.e. the year when France switched from ICD-9 to ICD-10.

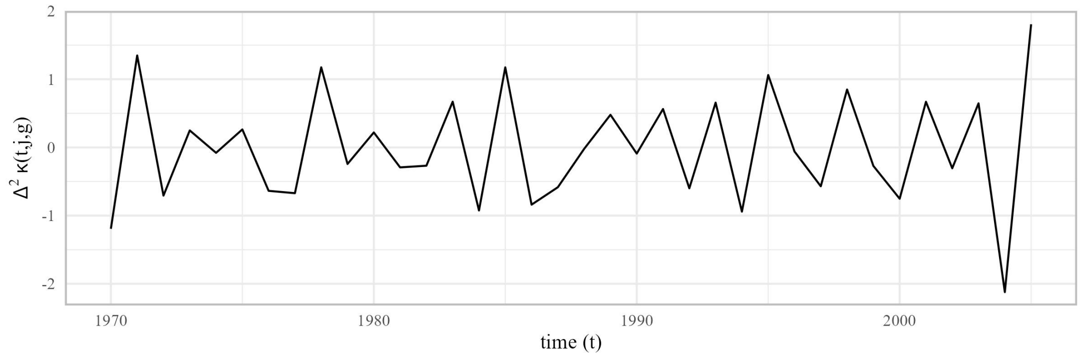
The plots of $\Delta^2\kappa(t, j, g)$ for all six causes in France are presented in Figure D.4. The double-differenced time series for all causes now resemble a random walk with a mean around zero.



(a) Infectious and Parasitic Diseases

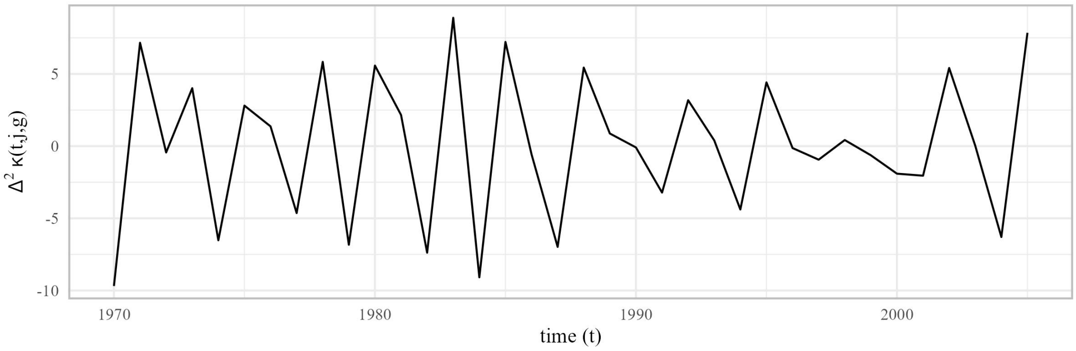


(b) Neoplasms

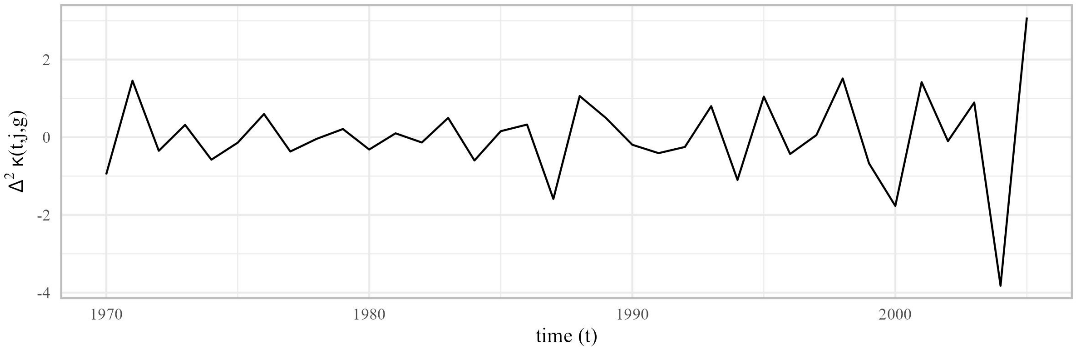


(c) Circulatory Diseases

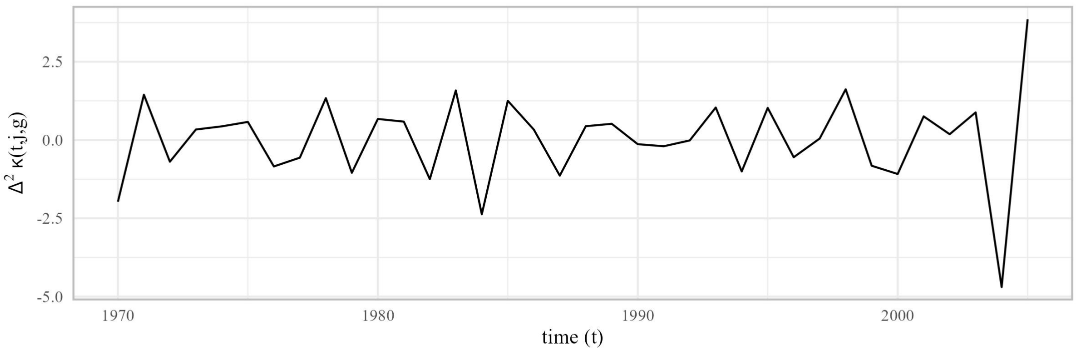
FIGURE D.4: Time series plots of double-differenced $\kappa(t, j, g)$ (France, Females)



(d) Respiratory Diseases



(e) External Causes

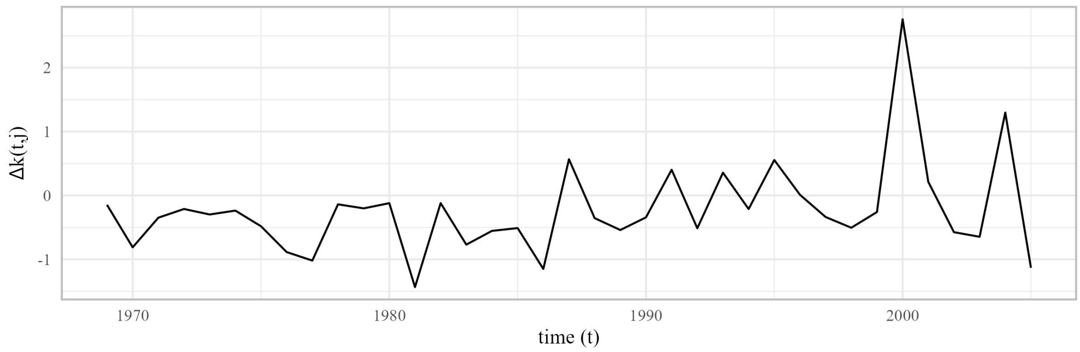


(f) Other Causes

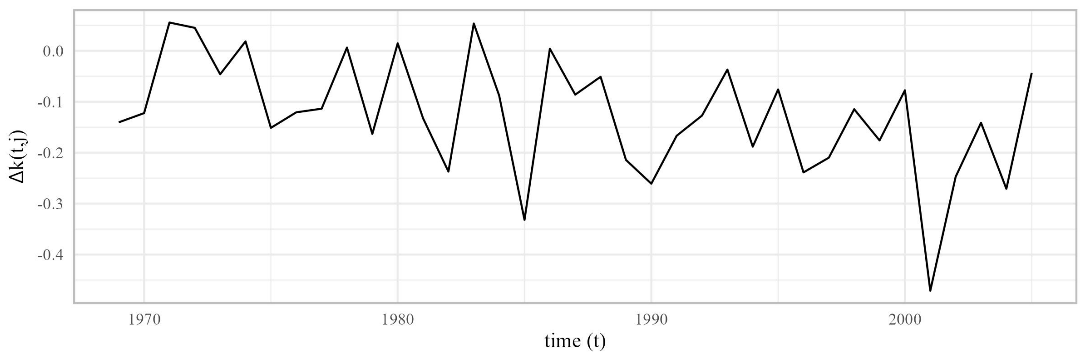
FIGURE D.4 (CONT.)

D.2 Multinomial Logistic Li-Lee (M5)

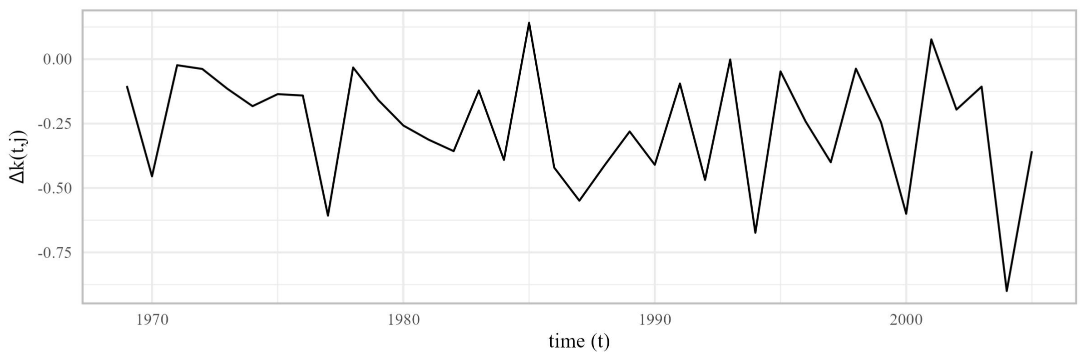
In addition to $\kappa(t, j, g)$, the MLG-LL (M5) model contains a common cause-specific time series $k(t, j)$. We will begin by looking at the differenced time series for $k(t, j)$ before looking at differenced time series for $\kappa(t, j, g)$ for England & Wales and for France in Sections D.2.1 and D.2.2, respectively.



(a) Infectious and Parasitic Diseases

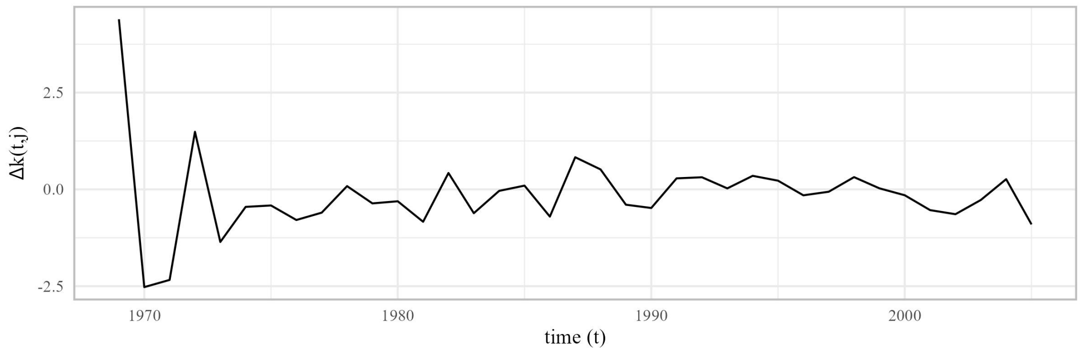


(b) Neoplasms

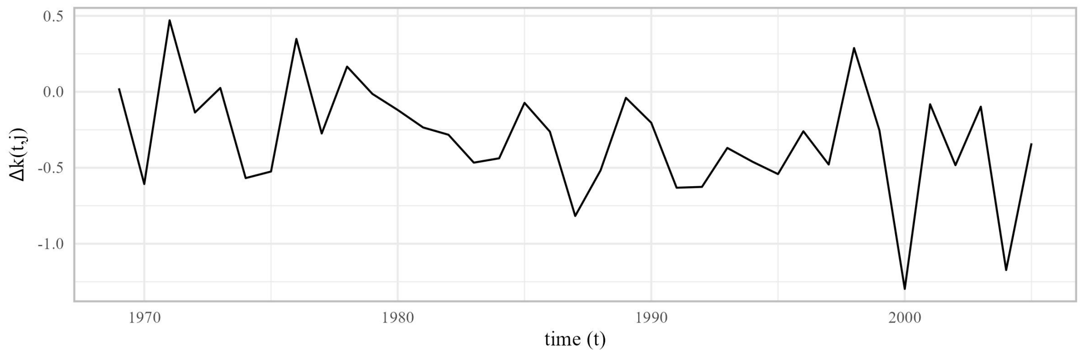


(c) Circulatory Diseases

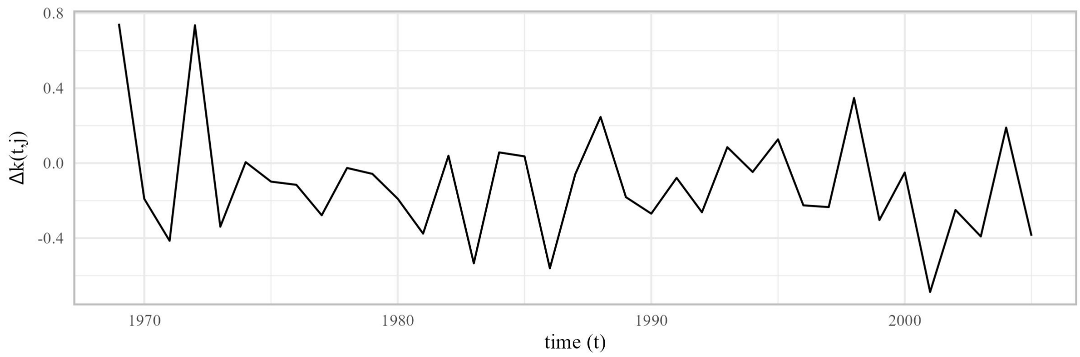
FIGURE D.5: Time series plots of single-differenced common $k(t, j)$ (MLG-LL, Females)



(d) Respiratory Diseases



(e) External Causes



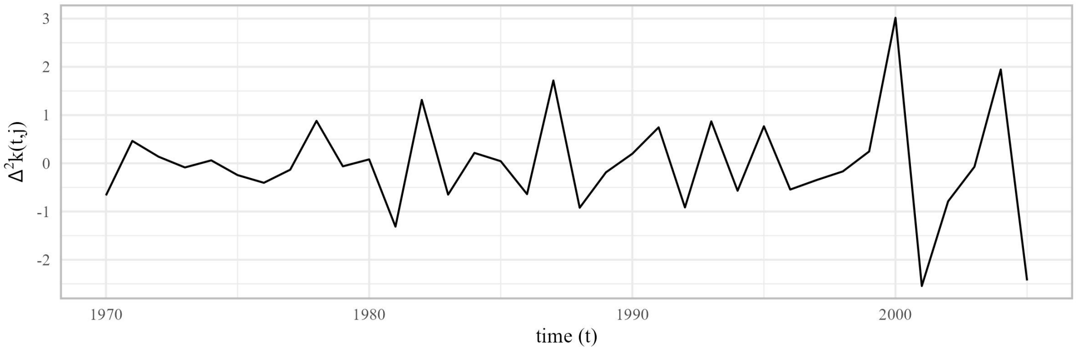
(f) Other Causes

FIGURE D.5 (CONT.)

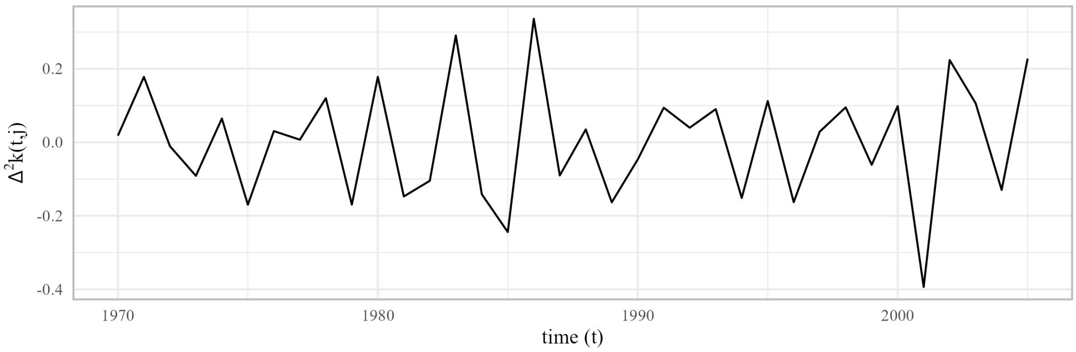
The time series of common cause-specific $\Delta k(t, j)$ for all six causes are presented in Figure D.5. The plots suggest that the means of the time series $\Delta\kappa(t, j, g)$ are non-zero except for perhaps respiratory diseases. (This observation is formally tested in Section 5.4.) The mean of $\Delta\kappa(t, j, g)$ for infectious diseases appears to be negative, which is consistent with the downward drift shown in Figure 5.3.a. There is a visible spike around the year 2000 similar to the one we have seen previously in the plot for the French $\Delta\kappa(t, j, g)$ from MLG-LC (M0). There is also a visible dip around the year 2001 in the single-differenced time series for neoplasms similar to the one we have seen previously in the $\Delta\kappa(t, j, g)$ from MLG-LC (M0) for

England & Wales. The time series plots for the remaining causes of death suggest a negative mean consistent with the downward drifts shown in Figure 5.1.

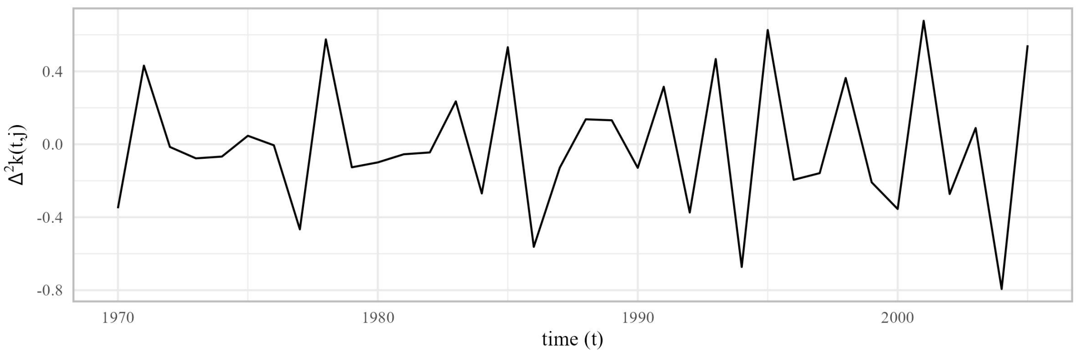
The plots of common cause-specific $\Delta^2 k(t,j)$ for all six causes are presented in Figure D.6. The double-differenced time series for all causes now resemble a random walk with a mean around zero.



(a) Infectious and Parasitic Diseases

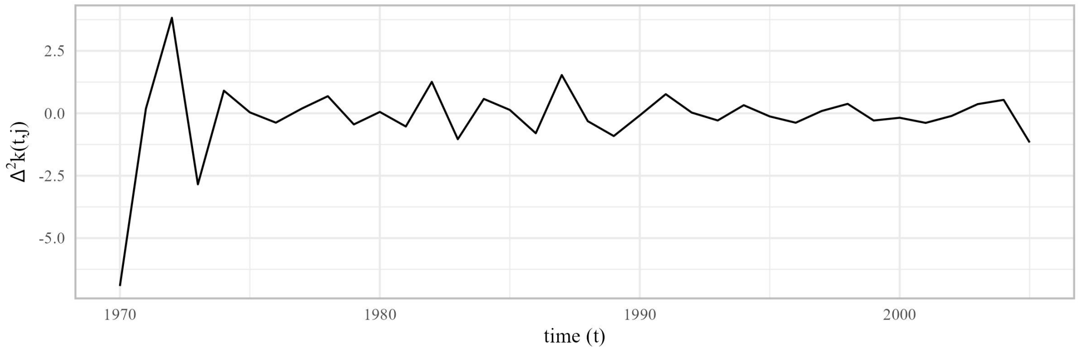


(b) Neoplasms

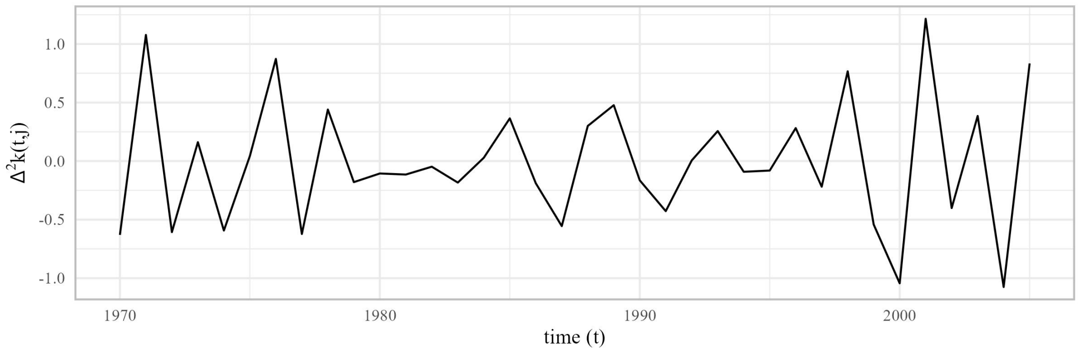


(c) Circulatory Diseases

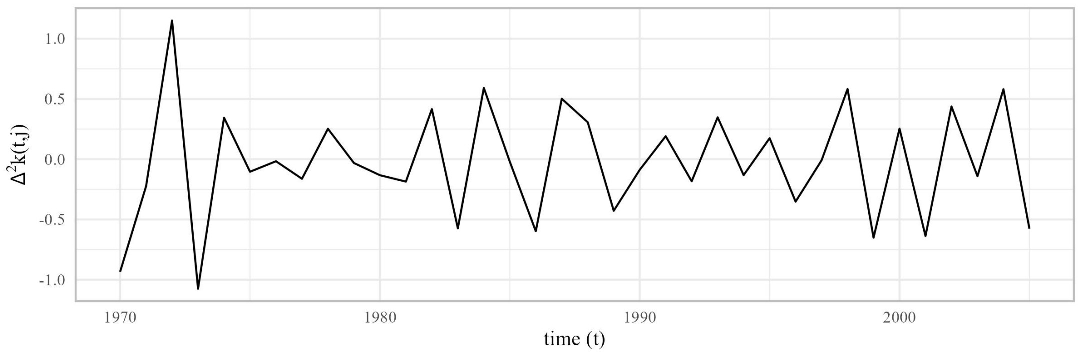
FIGURE D.6: Time series plots of double-differenced common $k(t,j)$ (MLG-LL, Females)



(d) Respiratory Diseases



(e) External Causes

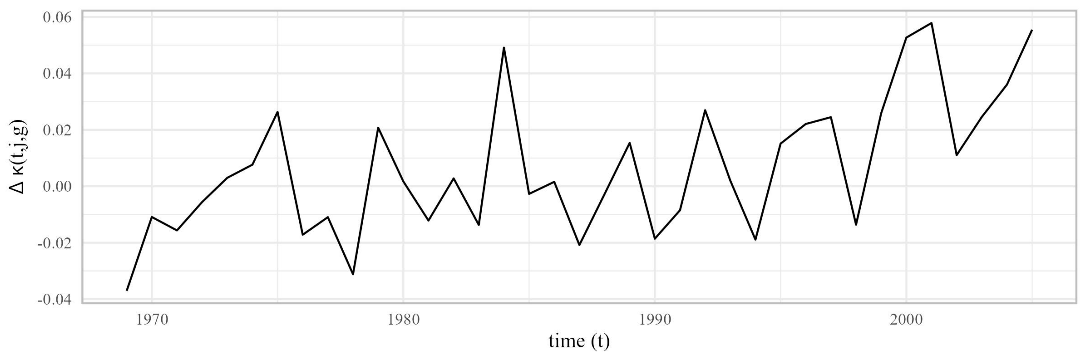


(f) Other Causes

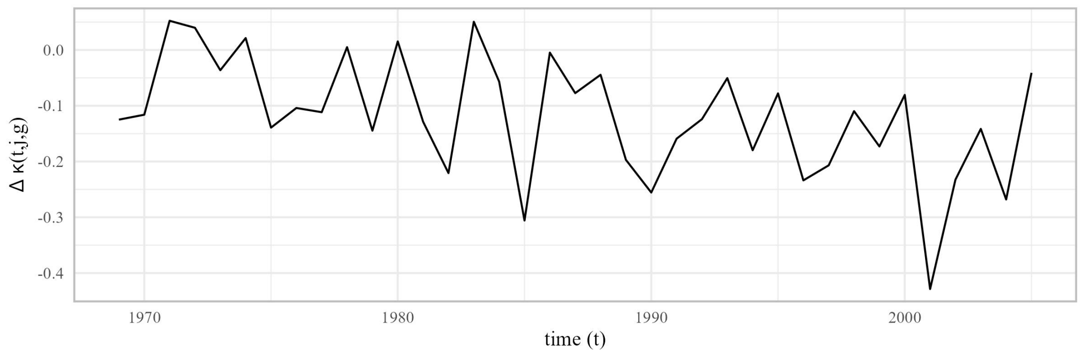
FIGURE D.6 (CONT.)

D.2.1 England & Wales

The time series of $\Delta\kappa(t, j, g)$ for all six causes in England & Wales are presented in Figure D.7. The plots suggest that the means of the time series $\Delta\kappa(t, j, g)$ are clearly non-zero except for perhaps respiratory and other diseases. (This observation is formally tested in Section 5.4.2.) The mean of $\Delta\kappa(t, j, g)$ for infectious diseases appears to be positive, which is consistent with the upward drift in the later years shown in Figure 5.4.a. On the other hand, the time series



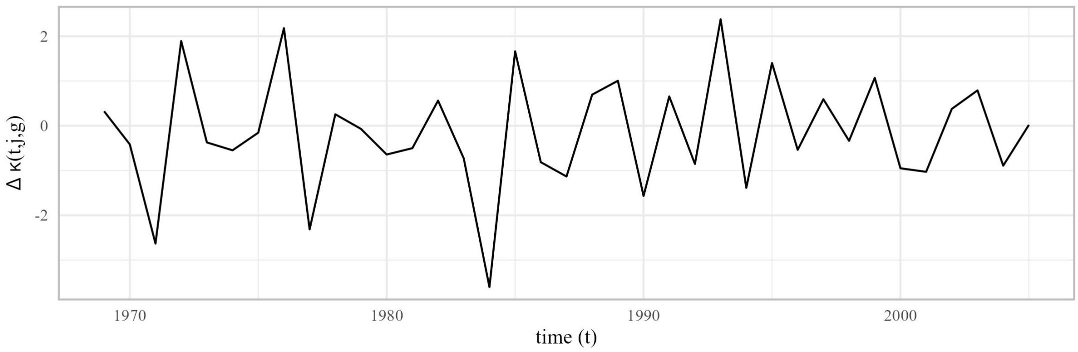
(a) Infectious and Parasitic Diseases



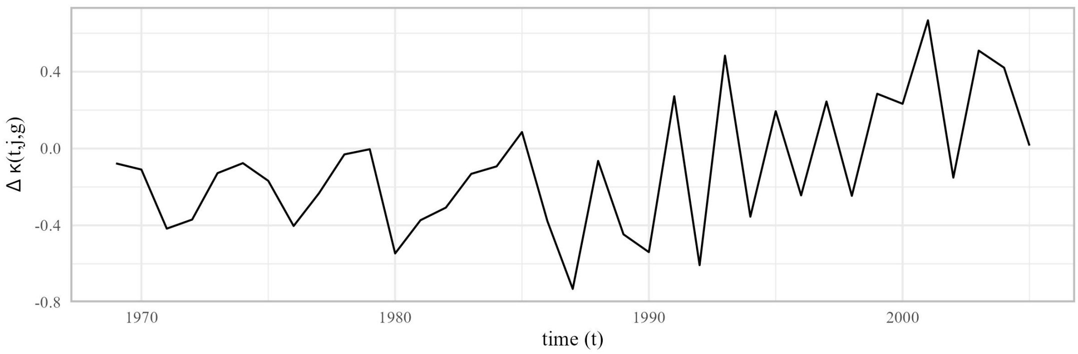
(b) Neoplasms

(c) Circulatory Diseases

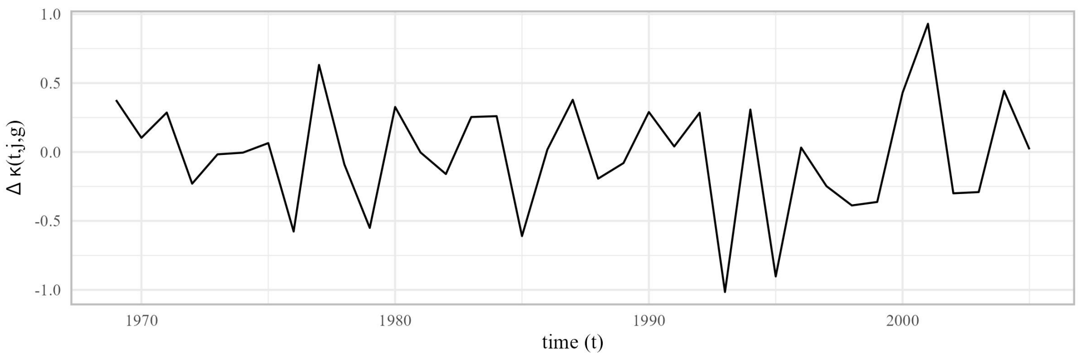
FIGURE D.7: Time series plots of single-differenced $\kappa(t, j, g)$ (MLG-LL, England & Wales, Females)



(d) Respiratory Diseases



(e) External Causes

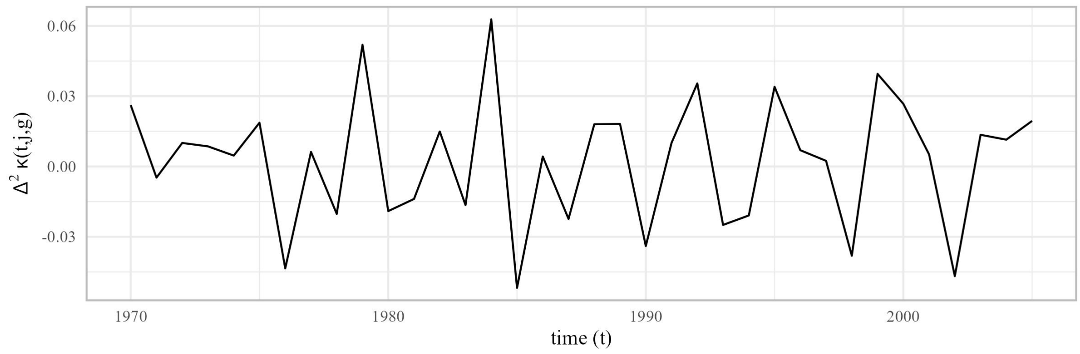


(f) Other Causes

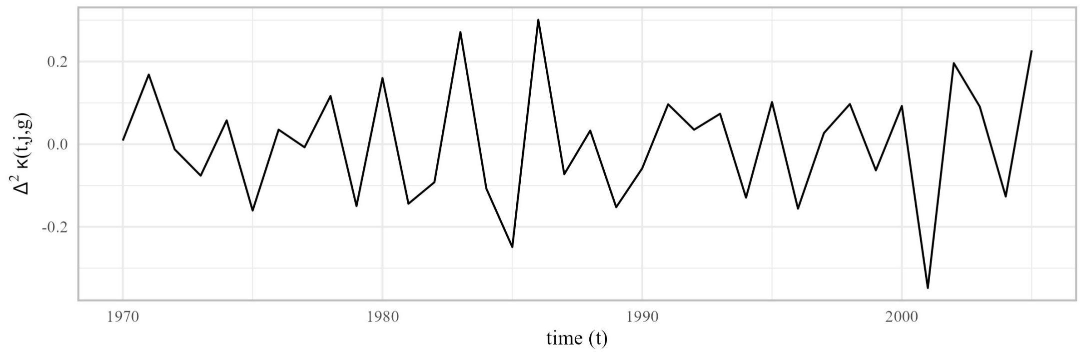
FIGURE D.7 (CONT.)

plots for the remaining causes of death suggest a negative mean consistent with the downward drifts shown in Figures 5.4.b, 5.4.c, & 5.4.e and to an extent in 5.4.d & 5.4.f.

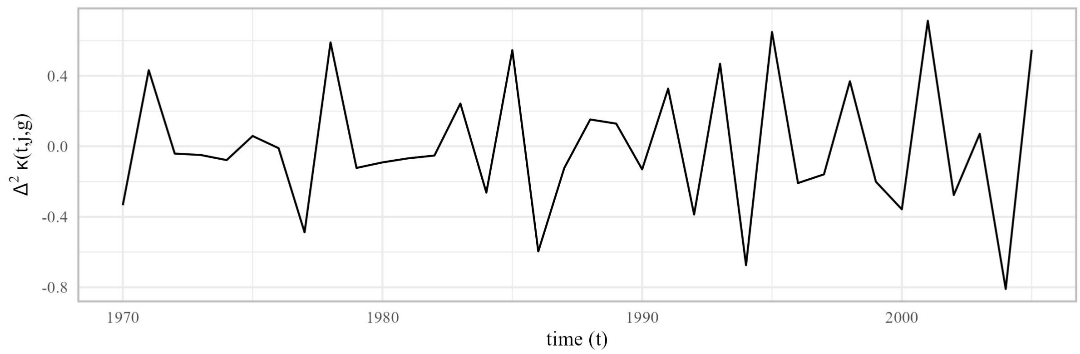
The plots of $\Delta^2\kappa(t, j, g)$ for all six causes in England & Wales are presented in Figure D.8. The double-differenced time series for all causes now resemble a random walk with a mean around zero.



(a) Infectious and Parasitic Diseases

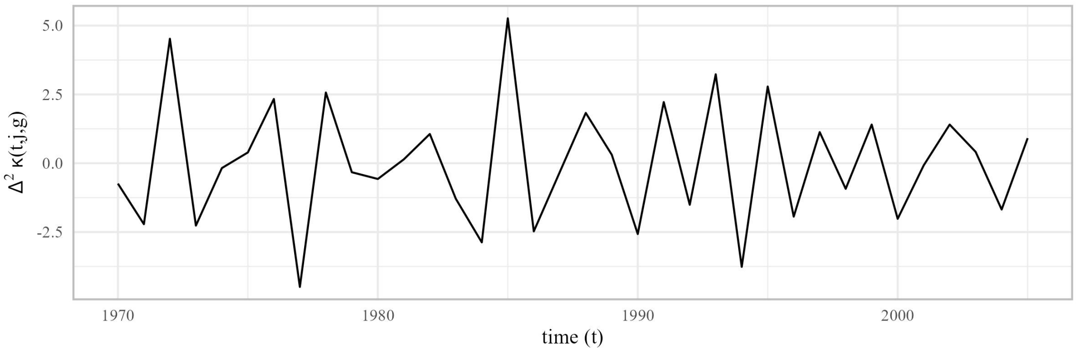


(b) Neoplasms

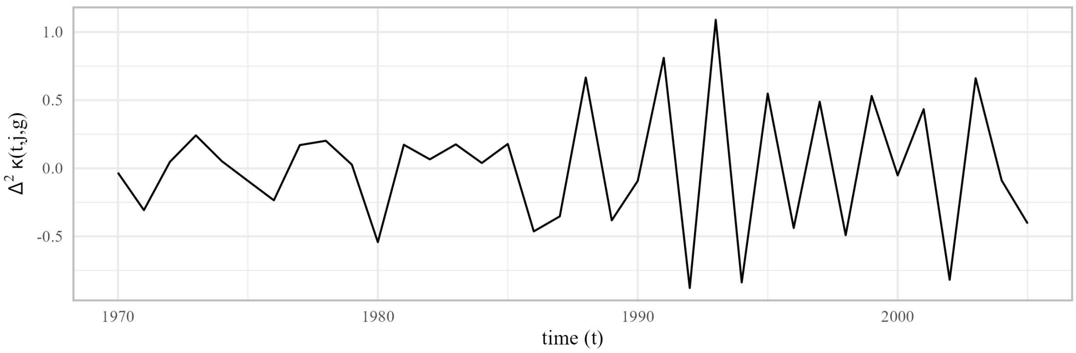


(c) Circulatory Diseases

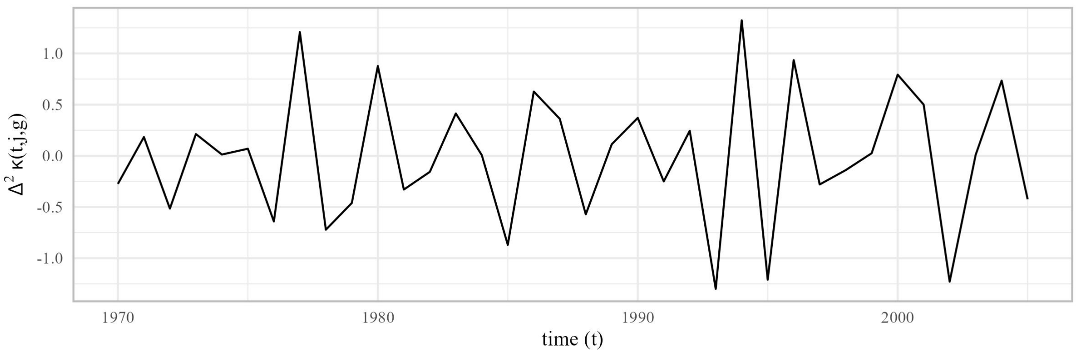
FIGURE D.8: Time series plots of double-differenced $\kappa(t, j, g)$ (MLG-LL, England & Wales, Females)



(d) Respiratory Diseases



(e) External Causes

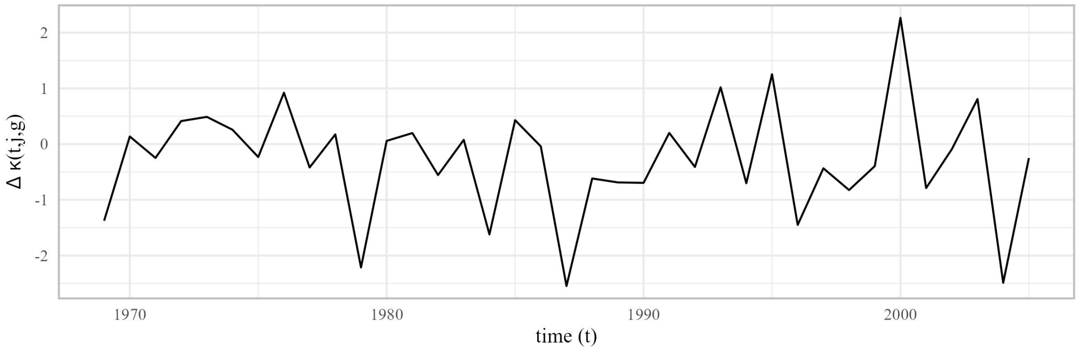


(f) Other Causes

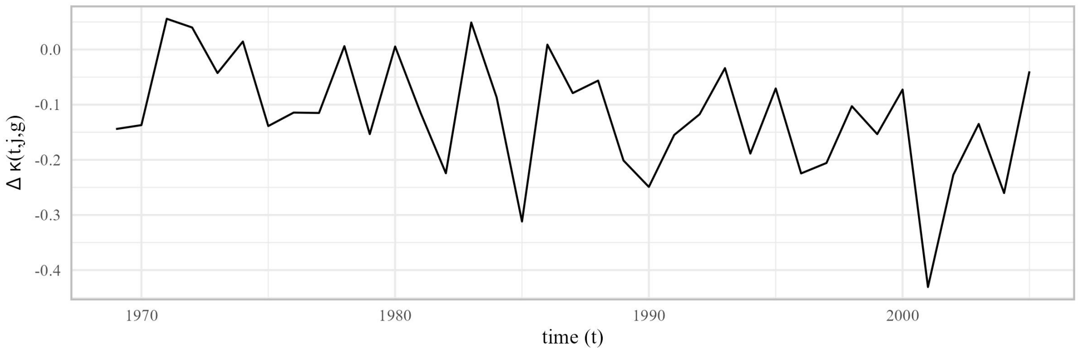
FIGURE D.8 (CONT.)

D.2.2 France

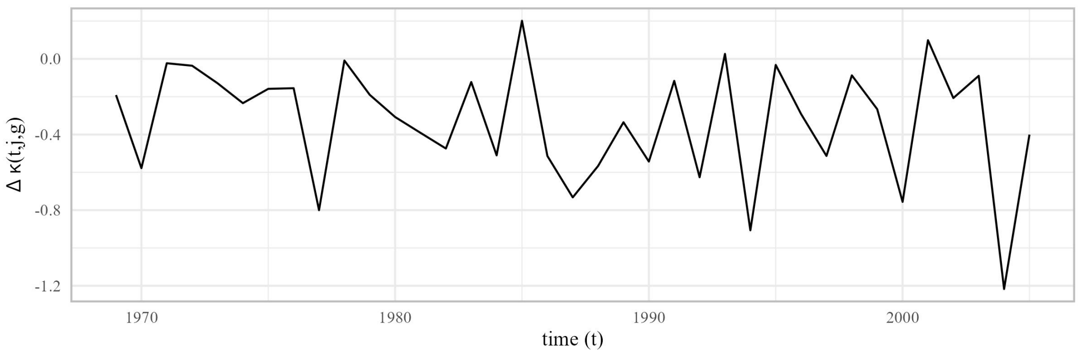
The plots of $\Delta\kappa(t, j, g)$ for all six causes in France are presented in Figure D.9. The plots suggest that the means of the time series $\Delta\kappa(t, j, g)$ are non-zero. (This observation is formally tested in Section 5.4.3.)



(a) Infectious and Parasitic Diseases



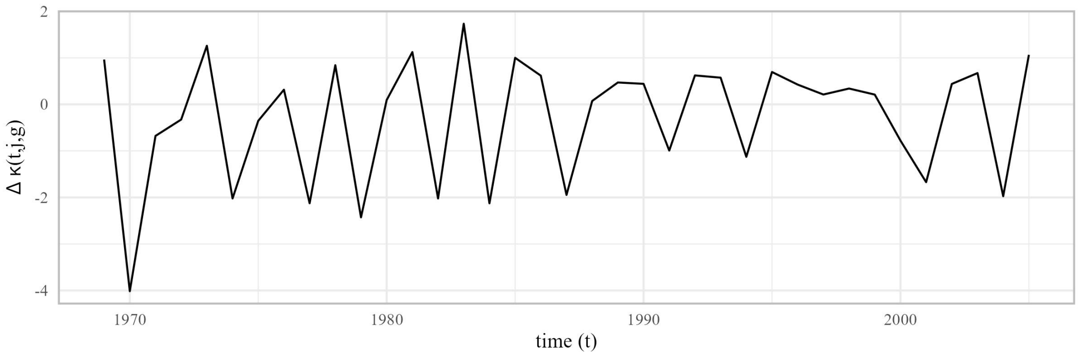
(b) Neoplasms



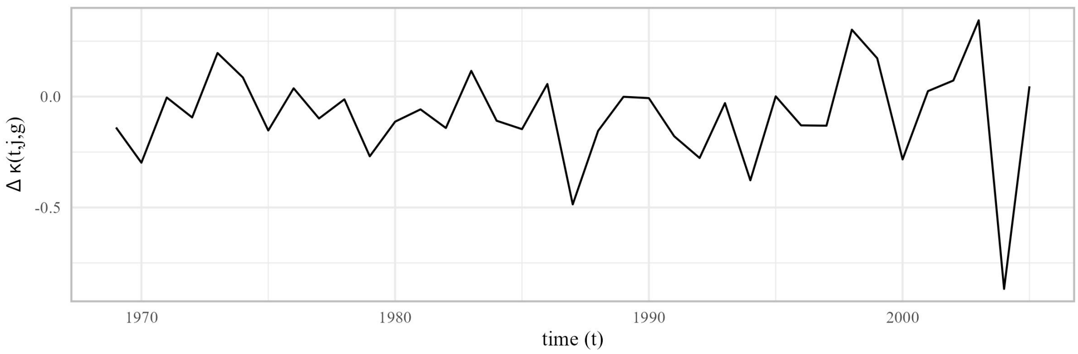
(c) Circulatory Diseases

FIGURE D.9: Time series plots of single-differenced $\kappa(t, j, g)$ (MLG-LL, France, Females)

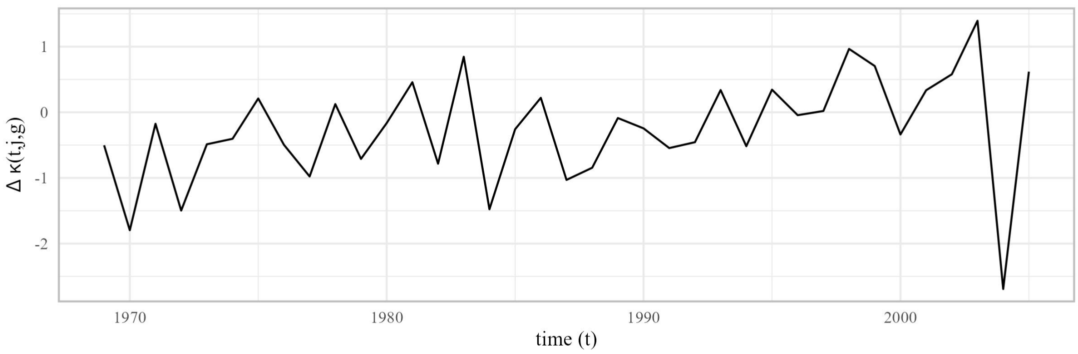
Similarly to the time series for MLG-LC (M0), the means of $\Delta\kappa(t, j, g)$ for MLG-LL (M5) for all French causes also appear to be negative, which is consistent with the downward drifts shown



(d) Respiratory Diseases



(e) External Causes

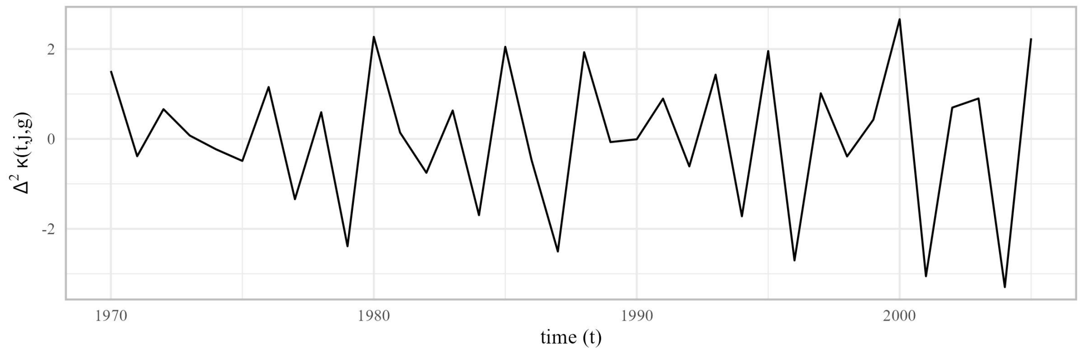


(f) Other Causes

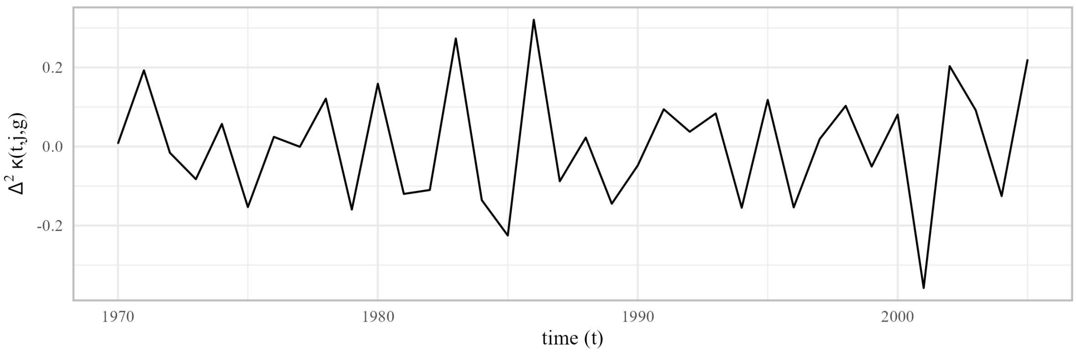
FIGURE D.9 (CONT.)

in Figure 5.5. This is true even for infectious diseases where there is once again a positive spike around the year 2000.

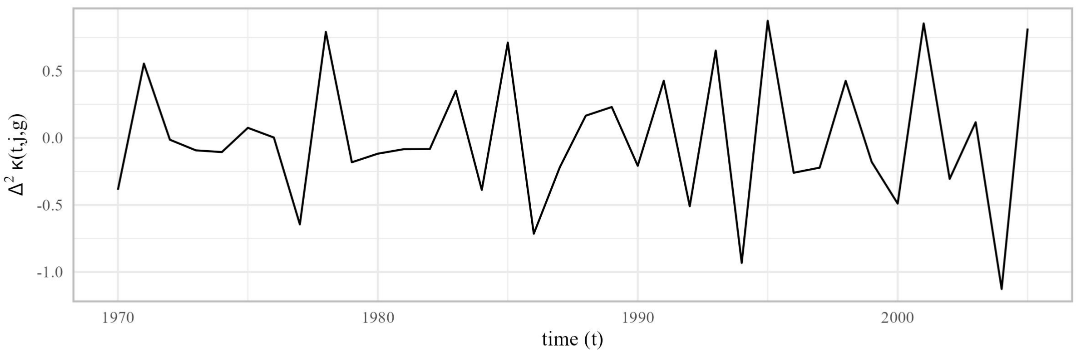
The plots of $\Delta^2\kappa(t, j, g)$ for all six causes in France are presented in Figure D.10. The double-differenced time series for all causes now resemble a random walk with a mean around zero.



(a) Infectious and Parasitic Diseases

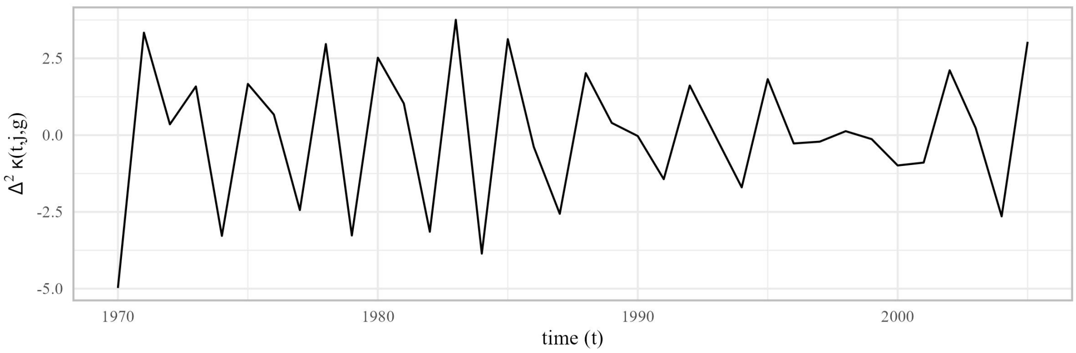


(b) Neoplasms

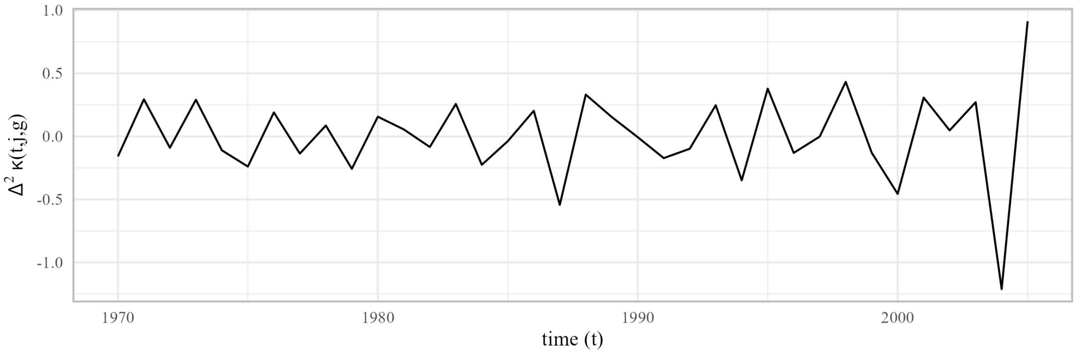


(c) Circulatory Diseases

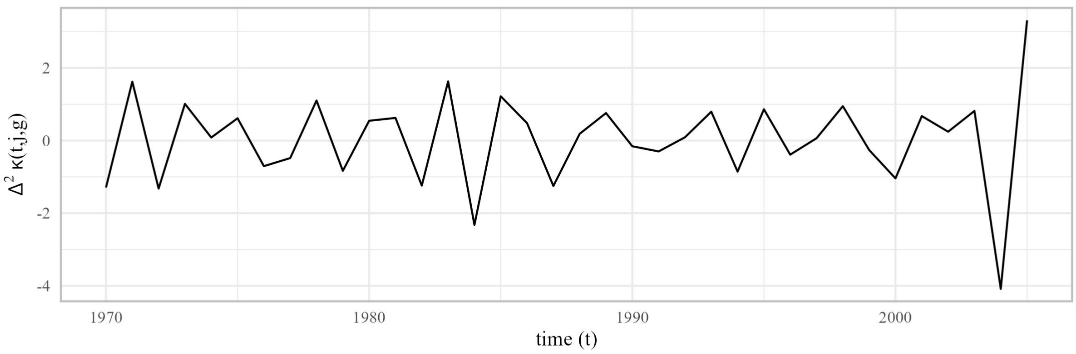
FIGURE D.10: Time series plots of double-differenced $\kappa(t, j, g)$ (MLG-LL, France, Females)



(d) Respiratory Diseases



(e) External Causes



(f) Other Causes

FIGURE D.10 (CONT.)

Appendix E

MCMC Parameter Estimates

The remainder of the MLG-LC (M0) parameter estimates for deaths due to infectious and parasitic diseases (Cause 1), circulatory diseases (Cause 3), diseases of the respiratory system (Cause 4), external causes (Cause 5), and other causes (Cause 6) can be found in the tables that follow.

TABLE E.1: BAYESIAN ESTIMATES AND CREDIBLE INTERVALS FOR CAUSE 1 — INFECTIOUS AND PARASITIC DISEASES, ENGLAND & WALES, FEMALES

	CAUSE 1 — INFECTIOUS AND PARASITIC DISEASES				
	Maximum Likelihood		Metropolis Algorithm		
	Estimate	s.e.	Median	s.e.	95 % Credible Interval
$\alpha(x,1)$					
1 –4	-10.4338	0.024884 ***	-10.4225	0.035127 ***	(-10.4894 , -10.3526)
5 –9	-11.8236	0.044901 ***	-11.8129	0.059449 ***	(-11.9330 , -11.6999)
10 –14	-12.1008	0.052271 ***	-12.0998	0.066827 ***	(-12.2337 , -11.9739)
15 –19	-11.4402	0.037824 ***	-11.4424	0.047294 ***	(-11.5384 , -11.3522)
20 –24	-11.6645	0.041487 ***	-11.6627	0.051487 ***	(-11.7647 , -11.5625)
25 –29	-11.5317	0.038519 ***	-11.5393	0.047116 ***	(-11.6311 , -11.4467)
30 –34	-11.4623	0.036962 ***	-11.4679	0.048525 ***	(-11.5666 , -11.3770)
35 –39	-11.1779	0.033078 ***	-11.1799	0.042290 ***	(-11.2635 , -11.0971)
40 –44	-10.9219	0.030097 ***	-10.9222	0.037007 ***	(-10.9946 , -10.8525)
45 –49	-10.6475	0.026585 ***	-10.6474	0.032914 ***	(-10.7119 , -10.5836)
50 –54	-10.3315	0.023046 ***	-10.3312	0.027784 ***	(-10.3846 , -10.2772)
55 –59	-10.0021	0.019945 ***	-10.0013	0.025057 ***	(-10.0512 , -9.9525)
60 –64	-9.6790	0.017459 ***	-9.6802	0.021068 ***	(-9.7218 , -9.6387)
65 –69	-9.3721	0.015430 ***	-9.3720	0.019027 ***	(-9.4107 , -9.3354)
70 –74	-9.0810	0.013972 ***	-9.0827	0.017834 ***	(-9.1188 , -9.0484)
75 –79	-8.6619	0.012256 ***	-8.6622	0.016125 ***	(-8.6942 , -8.6309)
80 –84	-8.2328	0.011419 ***	-8.2320	0.016628 ***	(-8.2640 , -8.1993)
85+	-7.6565	0.008907 ***	-7.6560	0.014349 ***	(-7.6838 , -7.6277)
$\beta(x,1)$					
1 –4	-1.1391	0.072187 ***	-0.6120	0.113114 ***	(-0.9476 , -0.4686)
5 –9	-0.7920	0.118045 ***	-0.3887	0.123722 **	(-0.7070 , -0.2057)
10 –14	-0.4539	0.128381 ***	-0.2003	0.112229	(-0.4603 , -0.0141)
15 –19	0.0850	0.090141	0.0767	0.068559	(-0.0667 , 0.2065)
20 –24	0.0265	0.100761	0.0503	0.075807	(-0.1029 , 0.1939)
25 –29	0.2013	0.091377 *	0.1457	0.069943 *	(0.0129 , 0.2882)
30 –34	0.4417	0.079525 ***	0.2829	0.067494 ***	(0.1626 , 0.4315)
35 –39	0.1334	0.074726	0.0967	0.056908	(-0.0139 , 0.2098)
40 –44	-0.2001	0.073564 **	-0.0891	0.058701	(-0.2165 , 0.0108)
45 –49	-0.3708	0.068688 ***	-0.1916	0.061427 **	(-0.3369 , -0.0914)
50 –54	-0.4360	0.060099 ***	-0.2326	0.061470 ***	(-0.3843 , -0.1394)
55 –59	-0.2823	0.050102 ***	-0.1520	0.045037 ***	(-0.2565 , -0.0765)
60 –64	-0.0160	0.043587	-0.0007	0.032799	(-0.0679 , 0.0607)
65 –69	0.2296	0.037036 ***	0.1405	0.031688 ***	(0.0879 , 0.2139)
70 –74	0.5220	0.031172 ***	0.3057	0.048034 ***	(0.2455 , 0.4500)
75 –79	0.7886	0.024905 ***	0.4598	0.067416 ***	(0.3837 , 0.6767)
80 –84	0.9883	0.020927 ***	0.5753	0.084274 ***	(0.4875 , 0.8526)
85+	1.2738	0.014777 ***	0.7463	0.107368 ***	(0.6342 , 1.1214)

TABLE E.2: BAYESIAN ESTIMATES AND CREDIBLE INTERVALS FOR CAUSE 1 — INFECTIOUS AND PARASITIC DISEASES, ENGLAND & WALES, FEMALES (CONTINUED)

	CAUSE 1 — INFECTIOUS AND PARASITIC DISEASES				
	Maximum Likelihood		Metropolis Algorithm		
	Estimate	s.e.	Median	s.e.	95 % Credible Interval
$\kappa(t,1)$					
1968	-0.2543	0.045212 ***	-0.3819	0.109206 ***	(-0.6105 , -0.1865)
1969	-0.4412	0.046734 ***	-0.7108	0.130676 ***	(-0.9702 , -0.4456)
1970	-0.4002	0.046130 ***	-0.6608	0.138655 ***	(-0.9669 , -0.3984)
1971	-0.4430	0.045974 ***	-0.7309	0.132374 ***	(-0.9930 , -0.4742)
1972	-0.4871	0.046122 ***	-0.8343	0.151269 ***	(-1.1104 , -0.5208)
1973	-0.4338	0.045434 ***	-0.7284	0.128322 ***	(-0.9820 , -0.4842)
1974	-0.4162	0.045095 ***	-0.7090	0.129880 ***	(-0.9581 , -0.4397)
1975	-0.2300	0.042720 ***	-0.3727	0.109734 ***	(-0.6208 , -0.1874)
1976	-0.2739	0.043194 ***	-0.4506	0.108189 ***	(-0.6833 , -0.2584)
1977	-0.2947	0.043337 ***	-0.5175	0.120483 ***	(-0.7553 , -0.2918)
1978	-0.4089	0.044651 ***	-0.7001	0.135010 ***	(-0.9727 , -0.4511)
1979	-0.2905	0.042858 ***	-0.4993	0.113626 ***	(-0.7306 , -0.2961)
1980	-0.2806	0.042313 ***	-0.4885	0.109353 ***	(-0.7065 , -0.2782)
1981	-0.2637	0.041310 ***	-0.4590	0.107694 ***	(-0.6799 , -0.2562)
1982	-0.2412	0.040495 ***	-0.4223	0.103627 ***	(-0.6282 , -0.2196)
1983	-0.2592	0.040345 ***	-0.4684	0.108168 ***	(-0.6666 , -0.2470)
1984	-0.0275	0.036465	-0.0670	0.082040	(-0.2352 , 0.0837)
1985	-0.0175	0.035887	-0.0275	0.076543	(-0.1797 , 0.1177)
1986	0.0506	0.034479	0.0738	0.069223	(-0.0558 , 0.2167)
1987	-0.0667	0.035662	-0.1190	0.083855	(-0.2998 , 0.0293)
1988	-0.0324	0.034740	-0.0558	0.076760	(-0.2155 , 0.0868)
1989	0.0394	0.033216	0.0683	0.072731	(-0.0816 , 0.2048)
1990	-0.0281	0.033873	-0.0588	0.070901	(-0.2029 , 0.0760)
1991	-0.0702	0.033938 *	-0.1265	0.073905	(-0.2757 , 0.0097)
1992	0.0598	0.031671	0.0952	0.070854	(-0.0472 , 0.2304)
1993	0.0558	0.031401	0.0883	0.071954	(-0.0518 , 0.2291)
1994	0.0038	0.031985	0.0077	0.067758	(-0.1288 , 0.1361)
1995	0.0224	0.031402	0.0402	0.068027	(-0.0931 , 0.1801)
1996	0.1129	0.029925 ***	0.1997	0.068283 **	(0.0710 , 0.3298)
1997	0.2297	0.028169 ***	0.4016	0.080410 ***	(0.2257 , 0.5430)
1998	0.1972	0.028517 ***	0.3346	0.073577 ***	(0.1867 , 0.4722)
1999	0.3074	0.026936 ***	0.5228	0.084818 ***	(0.3384 , 0.6813)
2000	0.4072	0.025391 ***	0.6923	0.102526 ***	(0.4674 , 0.8879)
2001	0.6327	0.022762 ***	1.0880	0.137851 ***	(0.7192 , 1.2967)
2002	0.6846	0.022088 ***	1.1731	0.150909 ***	(0.7744 , 1.4111)
2003	0.8023	0.020803 ***	1.3696	0.170472 ***	(0.9146 , 1.6490)
2004	0.9038	0.019656 ***	1.5418	0.185916 ***	(1.0523 , 1.8206)
2005	1.1516	0.016887 ***	1.9648	0.231583 ***	(1.3076 , 2.2797)

TABLE E.3: BAYESIAN ESTIMATES AND CREDIBLE INTERVALS FOR CAUSE 3 — DISEASES OF THE CIRCULATORY SYSTEM, ENGLAND & WALES, FEMALES

	CAUSE 3 — DISEASES OF THE CIRCULATORY SYSTEM				
	Maximum Likelihood		Metropolis Algorithm		
	Estimate	s.e.	Median	s.e.	95 % Credible Interval
$\alpha(x,3)$					
1 –4	-11.1901	0.038287 ***	-11.1907	0.046073 ***	(-11.2817 , -11.0990)
5 –9	-11.9976	0.050346 ***	-11.9998	0.062525 ***	(-12.1237 , -11.8770)
10 –14	-11.5296	0.039005 ***	-11.5351	0.051468 ***	(-11.6426 , -11.4388)
15 –19	-10.8942	0.028647 ***	-10.8977	0.036069 ***	(-10.9695 , -10.8269)
20 –24	-10.4358	0.022258 ***	-10.4351	0.028444 ***	(-10.4902 , -10.3801)
25 –29	-10.0050	0.017824 ***	-10.0059	0.022179 ***	(-10.0497 , -9.9629)
30 –34	-9.4840	0.013931 ***	-9.4841	0.017192 ***	(-9.5182 , -9.4511)
35 –39	-8.8585	0.010372 ***	-8.8587	0.013146 ***	(-8.8842 , -8.8326)
40 –44	-8.1870	0.007547 ***	-8.1867	0.009784 ***	(-8.2060 , -8.1678)
45 –49	-7.5484	0.005507 ***	-7.5482	0.006931 ***	(-7.5614 , -7.5346)
50 –54	-6.9512	0.004127 ***	-6.9510	0.005563 ***	(-6.9619 , -6.9396)
55 –59	-6.3139	0.003051 ***	-6.3132	0.004112 ***	(-6.3215 , -6.3053)
60 –64	-5.6143	0.002210 ***	-5.6143	0.002974 ***	(-5.6201 , -5.6085)
65 –69	-4.9560	0.001655 ***	-4.9563	0.002154 ***	(-4.9604 , -4.9520)
70 –74	-4.2965	0.001279 ***	-4.2960	0.001645 ***	(-4.2993 , -4.2928)
75 –79	-3.6696	0.001064 ***	-3.6693	0.001314 ***	(-3.6718 , -3.6667)
80 –84	-3.0628	0.000976 ***	-3.0627	0.001268 ***	(-3.0653 , -3.0603)
85+	-2.3136	0.000802 ***	-2.3130	0.001016 ***	(-2.3150 , -2.3110)
$\beta(x,3)$					
1 –4	-0.0174	0.007183 *	-0.0133	0.009399	(-0.0318 , 0.0052)
5 –9	0.0116	0.008638	0.0157	0.011452	(-0.0069 , 0.0379)
10 –14	0.0478	0.007430 ***	0.0510	0.009738 ***	(0.0318 , 0.0698)
15 –19	0.0214	0.005849 ***	0.0237	0.007221 **	(0.0093 , 0.0375)
20 –24	0.0327	0.004707 ***	0.0339	0.006168 ***	(0.0218 , 0.0462)
25 –29	0.0489	0.003829 ***	0.0492	0.004712 ***	(0.0402 , 0.0586)
30 –34	0.0546	0.002987 ***	0.0546	0.003644 ***	(0.0476 , 0.0618)
35 –39	0.0697	0.002223 ***	0.0689	0.002853 ***	(0.0636 , 0.0748)
40 –44	0.0782	0.001610 ***	0.0770	0.002180 ***	(0.0728 , 0.0813)
45 –49	0.0830	0.001160 ***	0.0815	0.001707 ***	(0.0782 , 0.0849)
50 –54	0.0875	0.000868 ***	0.0858	0.001482 ***	(0.0831 , 0.0888)
55 –59	0.0823	0.000643 ***	0.0805	0.001171 ***	(0.0783 , 0.0828)
60 –64	0.0754	0.000473 ***	0.0739	0.001026 ***	(0.0721 , 0.0762)
65 –69	0.0712	0.000358 ***	0.0696	0.000844 ***	(0.0682 , 0.0714)
70 –74	0.0689	0.000280 ***	0.0674	0.000820 ***	(0.0659 , 0.0691)
75 –79	0.0654	0.000234 ***	0.0641	0.000744 ***	(0.0628 , 0.0656)
80 –84	0.0626	0.000214 ***	0.0613	0.000709 ***	(0.0601 , 0.0627)
85+	0.0562	0.000173 ***	0.0552	0.000632 ***	(0.0541 , 0.0565)

TABLE E.4: BAYESIAN ESTIMATES AND CREDIBLE INTERVALS FOR CAUSE 3 — DISEASES OF THE CIRCULATORY SYSTEM, ENGLAND & WALES, FEMALES (CONTINUED)

	CAUSE 3 — DISEASES OF THE CIRCULATORY SYSTEM				
	Maximum Likelihood		Metropolis Algorithm		
	Estimate	s.e.	Median	s.e.	95 % Credible Interval
$\kappa(t,3)$					
1968	6.8472	0.039519 ***	7.0219	0.089575 ***	(6.8408 , 7.1909)
1969	6.4954	0.039638 ***	6.6580	0.087369 ***	(6.4781 , 6.8198)
1970	6.0562	0.039895 ***	6.2083	0.083293 ***	(6.0266 , 6.3583)
1971	5.6690	0.039873 ***	5.8031	0.080924 ***	(5.6424 , 5.9512)
1972	6.0999	0.039109 ***	6.2560	0.087205 ***	(6.0788 , 6.4158)
1973	5.7883	0.039266 ***	5.9309	0.080616 ***	(5.7687 , 6.0799)
1974	5.4814	0.039395 ***	5.6075	0.079830 ***	(5.4517 , 5.7568)
1975	5.0386	0.039665 ***	5.1572	0.074161 ***	(5.0134 , 5.2997)
1976	4.9360	0.039574 ***	5.0550	0.074774 ***	(4.9003 , 5.1923)
1977	4.2172	0.040198 ***	4.2989	0.068736 ***	(4.1685 , 4.4305)
1978	4.1459	0.040047 ***	4.2267	0.068298 ***	(4.0887 , 4.3619)
1979	4.0151	0.039980 ***	4.0959	0.068631 ***	(3.9420 , 4.2177)
1980	3.2346	0.040602 ***	3.2913	0.065536 ***	(3.1620 , 3.4121)
1981	2.5163	0.040892 ***	2.5409	0.059250 ***	(2.4223 , 2.6571)
1982	2.1727	0.041105 ***	2.1818	0.059067 ***	(2.0639 , 2.2984)
1983	1.8216	0.041291 ***	1.8172	0.057396 ***	(1.7051 , 1.9287)
1984	1.3733	0.041572 ***	1.3688	0.056183 ***	(1.2617 , 1.4781)
1985	1.5769	0.041022 ***	1.5891	0.053680 ***	(1.4818 , 1.6927)
1986	0.8271	0.041712 ***	0.8076	0.056602 ***	(0.6949 , 0.9137)
1987	0.0363	0.042336	-0.0126	0.054063	(-0.1199 , 0.0953)
1988	-0.2700	0.042503 ***	-0.3222	0.055258 ***	(-0.4329 , -0.2151)
1989	-0.6702	0.042748 ***	-0.7295	0.057365 ***	(-0.8445 , -0.6191)
1990	-1.3296	0.043330 ***	-1.4064	0.058564 ***	(-1.5216 , -1.2903)
1991	-1.5293	0.043103 ***	-1.6047	0.058461 ***	(-1.7178 , -1.4865)
1992	-2.2063	0.043743 ***	-2.2942	0.064239 ***	(-2.4294 , -2.1727)
1993	-2.1592	0.043493 ***	-2.2469	0.058800 ***	(-2.3659 , -2.1365)
1994	-3.3134	0.044885 ***	-3.4260	0.070486 ***	(-3.5681 , -3.2904)
1995	-3.5277	0.044903 ***	-3.6377	0.072206 ***	(-3.7680 , -3.4878)
1996	-4.0466	0.045449 ***	-4.1584	0.074193 ***	(-4.3024 , -4.0103)
1997	-4.7675	0.046334 ***	-4.8880	0.081726 ***	(-5.0422 , -4.7265)
1998	-4.9566	0.046499 ***	-5.0739	0.080238 ***	(-5.2288 , -4.9206)
1999	-5.6290	0.047402 ***	-5.7464	0.087211 ***	(-5.9046 , -5.5654)
2000	-6.8495	0.048885 ***	-6.9708	0.097837 ***	(-7.1600 , -6.7839)
2001	-6.2404	0.048212 ***	-6.3164	0.090535 ***	(-6.4771 , -6.1317)
2002	-6.5498	0.048477 ***	-6.6053	0.094643 ***	(-6.7904 , -6.4216)
2003	-6.8258	0.048841 ***	-6.8791	0.094030 ***	(-7.0490 , -6.6835)
2004	-8.2874	0.050872 ***	-8.3428	0.110103 ***	(-8.5446 , -8.1153)
2005	-9.1910	0.051957 ***	-9.2487	0.121633 ***	(-9.4744 , -9.0013)

TABLE E.5: BAYESIAN ESTIMATES AND CREDIBLE INTERVALS FOR CAUSE 4 — DISEASES OF THE RESPIRATORY SYSTEM, ENGLAND & WALES, FEMALES

	CAUSE 4 — DISEASES OF THE RESPIRATORY SYSTEM				
	Maximum Likelihood		Metropolis Algorithm		
	Estimate	s.e.	Median	s.e.	95 % Credible Interval
$\alpha(x,4)$					
1 –4	-9.9645	0.018761 ***	-9.9644	0.027634 ***	(-10.0209 , -9.9117)
5 –9	-11.2777	0.032241 ***	-11.2856	0.048115 ***	(-11.3838 , -11.1958)
10 –14	-11.2502	0.032648 ***	-11.2515	0.045358 ***	(-11.3415 , -11.1633)
15 –19	-10.9056	0.028394 ***	-10.9091	0.037002 ***	(-10.9825 , -10.8388)
20 –24	-10.7504	0.025783 ***	-10.7513	0.032210 ***	(-10.8151 , -10.6902)
25 –29	-10.6763	0.024684 ***	-10.6772	0.031912 ***	(-10.7389 , -10.6151)
30 –34	-10.4462	0.022216 ***	-10.4454	0.027779 ***	(-10.4988 , -10.3897)
35 –39	-10.0406	0.018487 ***	-10.0420	0.023288 ***	(-10.0865 , -9.9959)
40 –44	-9.5191	0.014582 ***	-9.5187	0.019225 ***	(-9.5567 , -9.4813)
45 –49	-8.9440	0.010952 ***	-8.9434	0.015108 ***	(-8.9728 , -8.9137)
50 –54	-8.2981	0.008163 ***	-8.2980	0.010583 ***	(-8.3180 , -8.2769)
55 –59	-7.6694	0.006143 ***	-7.6692	0.008108 ***	(-7.6847 , -7.6528)
60 –64	-7.0335	0.004625 ***	-7.0326	0.005921 ***	(-7.0445 , -7.0212)
65 –69	-6.4183	0.003506 ***	-6.4179	0.004386 ***	(-6.4266 , -6.4095)
70 –74	-5.7807	0.002693 ***	-5.7806	0.003454 ***	(-5.7873 , -5.7738)
75 –79	-5.1279	0.002169 ***	-5.1284	0.002746 ***	(-5.1337 , -5.1229)
80 –84	-4.4336	0.001861 ***	-4.4347	0.002432 ***	(-4.4395 , -4.4300)
85+	-3.3908	0.001272 ***	-3.3901	0.001576 ***	(-3.3933 , -3.3871)
$\beta(x,4)$					
1 –4	0.1097	0.003270 ***	0.1091	0.004994 ***	(0.0993 , 0.1189)
5 –9	0.0962	0.005269 ***	0.0978	0.008108 ***	(0.0820 , 0.1137)
10 –14	0.0743	0.005323 ***	0.0751	0.007600 ***	(0.0598 , 0.0898)
15 –19	0.0470	0.004672 ***	0.0479	0.006123 ***	(0.0356 , 0.0595)
20 –24	0.0541	0.004296 ***	0.0544	0.005626 ***	(0.0433 , 0.0651)
25 –29	0.0562	0.004201 ***	0.0568	0.005628 ***	(0.0456 , 0.0681)
30 –34	0.0544	0.003893 ***	0.0544	0.005127 ***	(0.0445 , 0.0646)
35 –39	0.0650	0.003288 ***	0.0649	0.004276 ***	(0.0565 , 0.0733)
40 –44	0.0689	0.002599 ***	0.0686	0.003499 ***	(0.0618 , 0.0753)
45 –49	0.0706	0.001985 ***	0.0700	0.002766 ***	(0.0648 , 0.0755)
50 –54	0.0531	0.001476 ***	0.0527	0.002130 ***	(0.0484 , 0.0569)
55 –59	0.0376	0.001089 ***	0.0373	0.001571 ***	(0.0343 , 0.0405)
60 –64	0.0250	0.000807 ***	0.0248	0.001141 ***	(0.0226 , 0.0270)
65 –69	0.0260	0.000614 ***	0.0258	0.000899 ***	(0.0241 , 0.0276)
70 –74	0.0347	0.000480 ***	0.0345	0.000917 ***	(0.0326 , 0.0364)
75 –79	0.0430	0.000388 ***	0.0427	0.000983 ***	(0.0405 , 0.0445)
80 –84	0.0486	0.000337 ***	0.0484	0.001029 ***	(0.0461 , 0.0504)
85+	0.0356	0.000236 ***	0.0351	0.000752 ***	(0.0333 , 0.0365)

TABLE E.6: BAYESIAN ESTIMATES AND CREDIBLE INTERVALS FOR CAUSE 4 — DISEASES OF THE RESPIRATORY SYSTEM, ENGLAND & WALES, FEMALES (CONTINUED)

	CAUSE 4 — DISEASES OF THE RESPIRATORY SYSTEM				
	Maximum Likelihood		Metropolis Algorithm		
	Estimate	s.e.	Median	s.e.	95 % Credible Interval
$\kappa(t,4)$					
1968	7.5322	0.126803 ***	7.5667	0.214460 ***	(7.1678 , 8.0466)
1969	6.0304	0.129849 ***	6.0287	0.210273 ***	(5.6205 , 6.4656)
1970	6.7642	0.126894 ***	6.7737	0.211724 ***	(6.3862 , 7.2272)
1971	2.3362	0.137316 ***	2.3349	0.182927 ***	(1.9654 , 2.6770)
1972	5.4838	0.127623 ***	5.4796	0.198942 ***	(5.1032 , 5.8896)
1973	5.4761	0.126896 ***	5.4895	0.200898 ***	(5.1108 , 5.9119)
1974	4.5398	0.128587 ***	4.5524	0.197901 ***	(4.1893 , 4.9727)
1975	4.5427	0.127718 ***	4.5561	0.186373 ***	(4.1902 , 4.9225)
1976	9.2533	0.114960 ***	9.3516	0.226012 ***	(8.9026 , 9.8289)
1977	5.2286	0.124252 ***	5.2720	0.181794 ***	(4.9235 , 5.6293)
1978	5.6708	0.122150 ***	5.7180	0.192007 ***	(5.3631 , 6.1315)
1979	5.8061	0.120833 ***	5.8599	0.184275 ***	(5.5210 , 6.2494)
1980	4.7480	0.122284 ***	4.7895	0.185490 ***	(4.4550 , 5.2048)
1981	4.2801	0.121400 ***	4.3315	0.179994 ***	(3.9869 , 4.7033)
1982	5.3322	0.117597 ***	5.3838	0.181512 ***	(5.0534 , 5.7577)
1983	4.4844	0.118501 ***	4.5466	0.170816 ***	(4.2360 , 4.9071)
1984	-9.1468	0.154527 ***	-9.1706	0.263803 ***	(-9.7615 , -8.7153)
1985	-5.8248	0.143186 ***	-5.8493	0.206131 ***	(-6.2615 , -5.4546)
1986	-6.7048	0.144538 ***	-6.7404	0.239432 ***	(-7.2495 , -6.2991)
1987	-9.7672	0.151606 ***	-9.8206	0.259174 ***	(-10.3452 , -9.3055)
1988	-8.5338	0.146679 ***	-8.5844	0.252143 ***	(-9.1071 , -8.1300)
1989	-6.0340	0.138212 ***	-6.0781	0.215089 ***	(-6.5225 , -5.6668)
1990	-8.6933	0.144521 ***	-8.7371	0.252310 ***	(-9.3136 , -8.2729)
1991	-7.8144	0.140053 ***	-7.8557	0.228284 ***	(-8.3404 , -7.4202)
1992	-9.5337	0.143849 ***	-9.5936	0.264488 ***	(-10.1596 , -9.1106)
1993	1.9494	0.113643 ***	1.9616	0.144931 ***	(1.6789 , 2.2506)
1994	-1.0621	0.120388 ***	-1.0739	0.154951 ***	(-1.3750 , -0.7657)
1995	1.5457	0.113291 ***	1.5496	0.147196 ***	(1.2624 , 1.8371)
1996	0.7283	0.114729 ***	0.7339	0.150097 ***	(0.4456 , 1.0282)
1997	2.0291	0.111620 ***	2.0491	0.150628 ***	(1.7502 , 2.3432)
1998	1.0931	0.113656 ***	1.0951	0.148699 ***	(0.8051 , 1.3850)
1999	3.3104	0.108826 ***	3.3350	0.152850 ***	(3.0428 , 3.6437)
2000	1.3866	0.112297 ***	1.3927	0.148694 ***	(1.0999 , 1.6939)
2001	-6.6344	0.132221 ***	-6.6962	0.210366 ***	(-7.1286 , -6.2802)
2002	-5.5463	0.128848 ***	-5.5789	0.198578 ***	(-5.9787 , -5.2021)
2003	-3.5575	0.123787 ***	-3.5782	0.182993 ***	(-3.9478 , -3.2280)
2004	-5.7650	0.129136 ***	-5.8078	0.205134 ***	(-6.2199 , -5.4099)
2005	-4.9336	0.126324 ***	-4.9784	0.194793 ***	(-5.3801 , -4.6030)

TABLE E.7: BAYESIAN ESTIMATES AND CREDIBLE INTERVALS FOR CAUSE 5 — EXTERNAL CAUSES, ENGLAND & WALES, FEMALES

	CAUSE 5 — EXTERNAL CAUSES				
	Maximum Likelihood		Metropolis Algorithm		
	Estimate	s.e.	Median	s.e.	95 % Credible Interval
$\alpha(x,5)$					
1 –4	-9.4697	0.015098 ***	-9.4719	0.020873 ***	(-9.5131 , -9.4321)
5 –9	-10.1070	0.018172 ***	-10.1103	0.026276 ***	(-10.1601 , -10.0570)
10 –14	-9.9996	0.018070 ***	-10.0010	0.023757 ***	(-10.0476 , -9.9543)
15 –19	-8.9152	0.010575 ***	-8.9151	0.013597 ***	(-8.9415 , -8.8890)
20 –24	-8.8636	0.010184 ***	-8.8638	0.012517 ***	(-8.8884 , -8.8393)
25 –29	-8.9644	0.010697 ***	-8.9656	0.013313 ***	(-8.9908 , -8.9389)
30 –34	-8.9345	0.010643 ***	-8.9347	0.013472 ***	(-8.9610 , -8.9083)
35 –39	-8.8391	0.010358 ***	-8.8392	0.013270 ***	(-8.8648 , -8.8127)
40 –44	-8.7112	0.009955 ***	-8.7111	0.012292 ***	(-8.7347 , -8.6871)
45 –49	-8.5790	0.009396 ***	-8.5787	0.011805 ***	(-8.6019 , -8.5560)
50 –54	-8.4802	0.009015 ***	-8.4805	0.011908 ***	(-8.5046 , -8.4579)
55 –59	-8.3933	0.008688 ***	-8.3925	0.011564 ***	(-8.4166 , -8.3711)
60 –64	-8.2702	0.008323 ***	-8.2708	0.011369 ***	(-8.2940 , -8.2489)
65 –69	-8.0288	0.007600 ***	-8.0282	0.009832 ***	(-8.0473 , -8.0091)
70 –74	-7.6305	0.006628 ***	-7.6303	0.008722 ***	(-7.6471 , -7.6129)
75 –79	-7.1122	0.005735 ***	-7.1112	0.007527 ***	(-7.1257 , -7.0959)
80 –84	-6.5285	0.005229 ***	-6.5277	0.006583 ***	(-6.5407 , -6.5149)
85+	-5.7168	0.004020 ***	-5.7155	0.004974 ***	(-5.7251 , -5.7057)
$\beta(x,5)$					
1 –4	0.0956	0.002581 ***	0.0958	0.003671 ***	(0.0885 , 0.1030)
5 –9	0.0971	0.003038 ***	0.0976	0.004471 ***	(0.0886 , 0.1060)
10 –14	0.0496	0.003217 ***	0.0502	0.004439 ***	(0.0413 , 0.0587)
15 –19	0.0367	0.002067 ***	0.0367	0.002717 ***	(0.0314 , 0.0422)
20 –24	0.0229	0.001999 ***	0.0231	0.002602 ***	(0.0179 , 0.0281)
25 –29	0.0171	0.002100 ***	0.0173	0.002814 ***	(0.0119 , 0.0227)
30 –34	0.0231	0.002096 ***	0.0231	0.002703 ***	(0.0177 , 0.0283)
35 –39	0.0260	0.002052 ***	0.0262	0.002607 ***	(0.0210 , 0.0311)
40 –44	0.0343	0.001935 ***	0.0343	0.002437 ***	(0.0296 , 0.0392)
45 –49	0.0456	0.001770 ***	0.0454	0.002247 ***	(0.0411 , 0.0500)
50 –54	0.0558	0.001699 ***	0.0556	0.002214 ***	(0.0513 , 0.0600)
55 –59	0.0630	0.001632 ***	0.0626	0.002265 ***	(0.0583 , 0.0671)
60 –64	0.0675	0.001559 ***	0.0673	0.002204 ***	(0.0630 , 0.0715)
65 –69	0.0717	0.001431 ***	0.0714	0.001948 ***	(0.0677 , 0.0753)
70 –74	0.0746	0.001269 ***	0.0744	0.001782 ***	(0.0708 , 0.0778)
75 –79	0.0794	0.001116 ***	0.0789	0.001686 ***	(0.0756 , 0.0821)
80 –84	0.0789	0.001031 ***	0.0786	0.001579 ***	(0.0755 , 0.0818)
85+	0.0609	0.000815 ***	0.0617	0.001274 ***	(0.0591 , 0.0642)

TABLE E.8: BAYESIAN ESTIMATES AND CREDIBLE INTERVALS FOR CAUSE 5 — EXTERNAL CAUSES, ENGLAND & WALES, FEMALES (CONTINUED)

	CAUSE 5 — EXTERNAL CAUSES				
	Maximum Likelihood		Metropolis Algorithm		
	Estimate	s.e.	Median	s.e.	95 % Credible Interval
$\kappa(t,5)$					
1968	8.1063	0.150840 ***	8.1240	0.203551 ***	(7.7299 , 8.5343)
1969	8.2147	0.149252 ***	8.2266	0.202864 ***	(7.8367 , 8.6223)
1970	7.6203	0.151548 ***	7.6406	0.208300 ***	(7.2459 , 8.0650)
1971	7.3378	0.151686 ***	7.3566	0.210560 ***	(6.9632 , 7.7729)
1972	6.5855	0.154957 ***	6.6040	0.207963 ***	(6.1900 , 7.0053)
1973	6.5051	0.154770 ***	6.5225	0.212315 ***	(6.1051 , 6.9258)
1974	6.1748	0.155949 ***	6.1968	0.203914 ***	(5.8103 , 6.6098)
1975	5.3723	0.159673 ***	5.3845	0.218364 ***	(4.9527 , 5.8137)
1976	5.0647	0.160837 ***	5.0828	0.211821 ***	(4.6664 , 5.4834)
1977	4.4365	0.163709 ***	4.4421	0.214114 ***	(4.0315 , 4.8700)
1978	4.7862	0.161020 ***	4.7928	0.217826 ***	(4.3793 , 5.2288)
1979	4.4416	0.162262 ***	4.4701	0.217888 ***	(4.0506 , 4.8891)
1980	3.3544	0.167416 ***	3.3593	0.212898 ***	(2.9477 , 3.7829)
1981	2.3572	0.171179 ***	2.3423	0.232152 ***	(1.8854 , 2.7992)
1982	1.8138	0.173645 ***	1.8215	0.230294 ***	(1.3483 , 2.2501)
1983	1.1840	0.176485 ***	1.1998	0.224244 ***	(0.7524 , 1.6331)
1984	0.8544	0.177429 ***	0.8561	0.231766 ***	(0.4082 , 1.3121)
1985	0.8771	0.176186 ***	0.8881	0.227554 ***	(0.4337 , 1.3211)
1986	0.1249	0.179898	0.1288	0.230163	(-0.3237 , 0.5709)
1987	-1.6963	0.189953 ***	-1.6753	0.235377 ***	(-2.1227 , -1.2227)
1988	-2.1474	0.191907 ***	-2.1333	0.237521 ***	(-2.6219 , -1.6755)
1989	-3.1085	0.197088 ***	-3.1204	0.255779 ***	(-3.6155 , -2.6040)
1990	-3.8409	0.201029 ***	-3.8377	0.247347 ***	(-4.3569 , -3.3921)
1991	-4.0193	0.200325 ***	-4.0347	0.252101 ***	(-4.5436 , -3.5545)
1992	-5.1114	0.206866 ***	-5.1256	0.268404 ***	(-5.6714 , -4.6243)
1993	-4.7726	0.203726 ***	-4.7921	0.266333 ***	(-5.3163 , -4.2680)
1994	-5.5887	0.208913 ***	-5.6093	0.272958 ***	(-6.1528 , -5.1017)
1995	-5.6522	0.208188 ***	-5.6761	0.274390 ***	(-6.2165 , -5.1329)
1996	-5.9285	0.209511 ***	-5.9342	0.280030 ***	(-6.4826 , -5.3958)
1997	-5.7012	0.207589 ***	-5.7207	0.271518 ***	(-6.2436 , -5.1729)
1998	-6.0052	0.209297 ***	-6.0127	0.271405 ***	(-6.5721 , -5.5120)
1999	-5.4606	0.205478 ***	-5.4644	0.265900 ***	(-5.9833 , -4.9452)
2000	-5.5624	0.205238 ***	-5.5772	0.276088 ***	(-6.1450 , -5.0607)
2001	-4.3391	0.197612 ***	-4.3459	0.254976 ***	(-4.8504 , -3.8634)
2002	-4.8992	0.200570 ***	-4.9251	0.264467 ***	(-5.4490 , -4.4107)
2003	-3.7987	0.193097 ***	-3.8110	0.251229 ***	(-4.3008 , -3.3219)
2004	-3.7020	0.192080 ***	-3.7173	0.245055 ***	(-4.1851 , -3.2356)
2005	-3.8774	0.192385 ***	-3.8939	0.245367 ***	(-4.3862 , -3.4271)

TABLE E.9: BAYESIAN ESTIMATES AND CREDIBLE INTERVALS FOR CAUSE 6 — EXTERNAL CAUSES, ENGLAND & WALES, FEMALES

	CAUSE 6 — OTHER CAUSES				
	Maximum Likelihood		Metropolis Algorithm		
	Estimate	s.e.	Median	s.e.	95 % Credible Interval
$\alpha(x,6)$					
1 –4	-8.7523	0.011124 ***	-8.7522	0.014254 ***	(-8.7789 ; -8.7230)
5 –9	-9.7787	0.016120 ***	-9.7782	0.021556 ***	(-9.8208 ; -9.7355)
10 –14	-9.8298	0.016786 ***	-9.8305	0.021664 ***	(-9.8749 ; -9.7886)
15 –19	-9.5295	0.014516 ***	-9.5293	0.018075 ***	(-9.5647 ; -9.4937)
20 –24	-9.3736	0.013176 ***	-9.3745	0.016422 ***	(-9.4065 ; -9.3414)
25 –29	-9.2006	0.012031 ***	-9.2012	0.014916 ***	(-9.2313 ; -9.1723)
30 –34	-8.9992	0.010955 ***	-8.9996	0.013993 ***	(-9.0264 ; -8.9716)
35 –39	-8.7395	0.009798 ***	-8.7389	0.012381 ***	(-8.7635 ; -8.7147)
40 –44	-8.3818	0.008430 ***	-8.3818	0.010342 ***	(-8.4024 ; -8.3615)
45 –49	-8.0181	0.007151 ***	-8.0172	0.008683 ***	(-8.0351 ; -8.0007)
50 –54	-7.6778	0.006138 ***	-7.6775	0.007519 ***	(-7.6923 ; -7.6629)
55 –59	-7.2861	0.005140 ***	-7.2857	0.006307 ***	(-7.2982 ; -7.2735)
60 –64	-6.8437	0.004231 ***	-6.8432	0.005091 ***	(-6.8530 ; -6.8332)
65 –69	-6.3628	0.003432 ***	-6.3626	0.004332 ***	(-6.3709 ; -6.3539)
70 –74	-5.8171	0.002769 ***	-5.8171	0.003371 ***	(-5.8235 ; -5.8104)
75 –79	-5.2069	0.002263 ***	-5.2074	0.002886 ***	(-5.2131 ; -5.2018)
80 –84	-4.5654	0.001955 ***	-4.5667	0.002416 ***	(-4.5715 ; -4.5619)
85+	-3.6619	0.001347 ***	-3.6627	0.002006 ***	(-3.6666 ; -3.6587)
$\beta(x,6)$					
1 –4	0.5221	0.024951 ***	0.5179	0.040144 ***	(0.4427 ; 0.5967)
5 –9	0.6948	0.034031 ***	0.6863	0.051695 ***	(0.5915 ; 0.7947)
10 –14	0.3701	0.035315 ***	0.3695	0.046625 ***	(0.2820 ; 0.4659)
15 –19	0.1560	0.031698 ***	0.1580	0.038936 ***	(0.0831 ; 0.2372)
20 –24	0.1747	0.029508 ***	0.1748	0.036866 ***	(0.1018 ; 0.2484)
25 –29	0.1529	0.027386 ***	0.1553	0.035348 ***	(0.0851 ; 0.2226)
30 –34	-0.0112	0.024964	-0.0099	0.030775	(-0.0741 ; 0.0487)
35 –39	-0.0603	0.022291 **	-0.0589	0.029243 *	(-0.1150 ; -0.0022)
40 –44	-0.0015	0.019298	-0.0028	0.024504	(-0.0527 ; 0.0445)
45 –49	-0.0011	0.016525	-0.0017	0.020636	(-0.0426 ; 0.0390)
50 –54	0.0524	0.014034 ***	0.0519	0.017951 **	(0.0175 ; 0.0880)
55 –59	0.0837	0.011718 ***	0.0822	0.014737 ***	(0.0547 ; 0.1117)
60 –64	0.0800	0.009814 ***	0.0768	0.012974 ***	(0.0536 ; 0.1050)
65 –69	0.0604	0.008019 ***	0.0589	0.010411 ***	(0.0393 ; 0.0803)
70 –74	-0.0125	0.006536	-0.0129	0.007977	(-0.0288 ; 0.0024)
75 –79	-0.1600	0.005346 ***	-0.1586	0.011645 ***	(-0.1820 ; -0.1381)
80 –84	-0.3364	0.004600 ***	-0.3298	0.020057 ***	(-0.3685 ; -0.2993)
85+	-0.7641	0.003128 ***	-0.7474	0.042954 ***	(-0.8273 ; -0.6874)

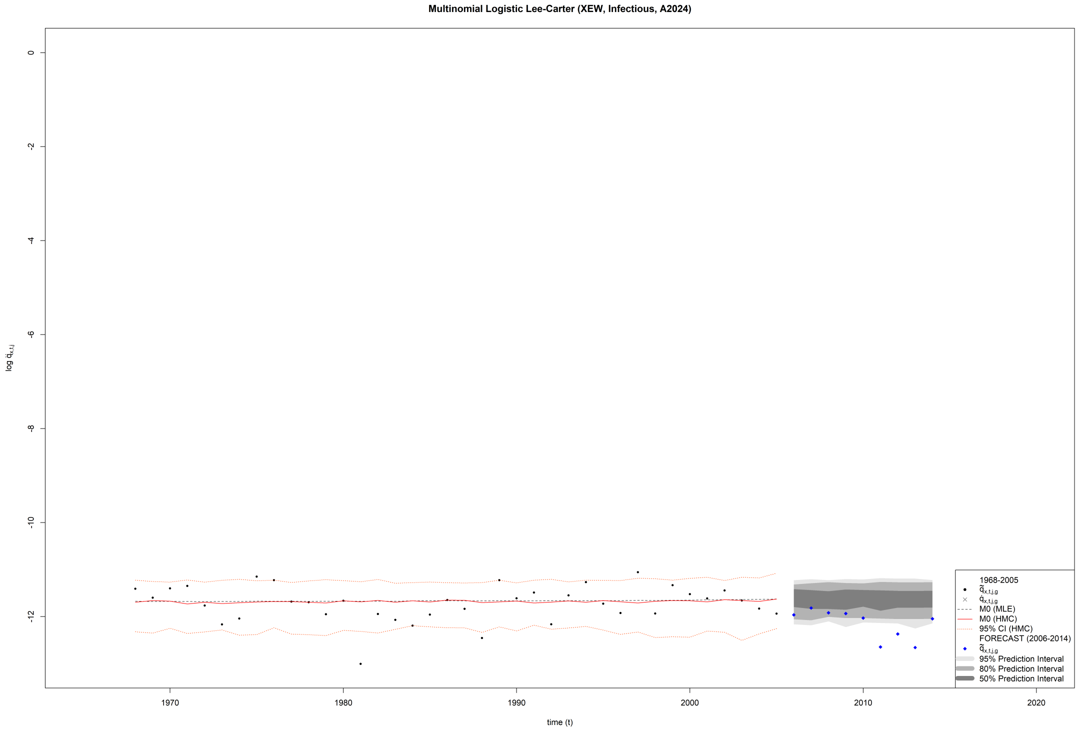
TABLE E.10: BAYESIAN ESTIMATES AND CREDIBLE INTERVALS FOR CAUSE 6 — EXTERNAL CAUSES, ENGLAND & WALES, FEMALES (CONTINUED)

	CAUSE 6 — OTHER CAUSES				
	Maximum Likelihood		Metropolis Algorithm		
	Estimate	s.e.	Median	s.e.	95 % Credible Interval
$\kappa(t,6)$					
1968	0.409876	0.016061 ***	0.4141	0.031269 ***	(0.356849 , 0.4764)
1969	0.446649	0.016026 ***	0.4522	0.033851 ***	(0.390221 , 0.5178)
1970	0.484493	0.016007 ***	0.4903	0.036170 ***	(0.426802 , 0.5619)
1971	0.533607	0.015671 ***	0.5405	0.036719 ***	(0.473135 , 0.6084)
1972	0.494493	0.015328 ***	0.5020	0.036028 ***	(0.438590 , 0.5733)
1973	0.517672	0.015290 ***	0.5251	0.034783 ***	(0.463446 , 0.5874)
1974	0.504196	0.015065 ***	0.5098	0.034096 ***	(0.447875 , 0.5730)
1975	0.512277	0.015001 ***	0.5198	0.033818 ***	(0.457242 , 0.5822)
1976	0.534507	0.014986 ***	0.5429	0.035827 ***	(0.476189 , 0.6125)
1977	0.546233	0.014943 ***	0.5532	0.036932 ***	(0.488533 , 0.6249)
1978	0.528150	0.014746 ***	0.5360	0.036065 ***	(0.469680 , 0.6008)
1979	0.370913	0.013971 ***	0.3759	0.026079 ***	(0.326654 , 0.4257)
1980	0.437682	0.014058 ***	0.4416	0.030409 ***	(0.386573 , 0.5027)
1981	0.409771	0.013559 ***	0.4154	0.029249 ***	(0.363006 , 0.4711)
1982	0.385759	0.013219 ***	0.3902	0.028127 ***	(0.341911 , 0.4483)
1983	0.389774	0.013069 ***	0.3953	0.028211 ***	(0.347512 , 0.4512)
1984	-0.207119	0.010662 ***	-0.2069	0.017281 ***	(-0.241966 , -0.1762)
1985	-0.324775	0.010086 ***	-0.3290	0.023071 ***	(-0.370601 , -0.2854)
1986	-0.352025	0.009837 ***	-0.3576	0.023426 ***	(-0.401453 , -0.3141)
1987	-0.275021	0.009809 ***	-0.2776	0.020287 ***	(-0.316412 , -0.2412)
1988	-0.318836	0.009495 ***	-0.3215	0.022162 ***	(-0.365559 , -0.2846)
1989	-0.339446	0.009245 ***	-0.3429	0.022413 ***	(-0.384146 , -0.3017)
1990	-0.306612	0.009204 ***	-0.3104	0.021240 ***	(-0.350461 , -0.2720)
1991	-0.292234	0.009030 ***	-0.2956	0.019768 ***	(-0.332291 , -0.2589)
1992	-0.263915	0.008949 ***	-0.2658	0.018771 ***	(-0.301715 , -0.2313)
1993	0.002063	0.009580	0.0039	0.012856	(-0.021553 , 0.0290)
1994	-0.009555	0.009437	-0.0083	0.012083	(-0.031837 , 0.0155)
1995	-0.153232	0.008884 ***	-0.1532	0.014326 ***	(-0.181730 , -0.1267)
1996	-0.192277	0.008703 ***	-0.1934	0.016167 ***	(-0.226848 , -0.1644)
1997	-0.249269	0.008492 ***	-0.2524	0.017874 ***	(-0.286274 , -0.2199)
1998	-0.337925	0.008174 ***	-0.3434	0.022583 ***	(-0.385488 , -0.3026)
1999	-0.402912	0.007960 ***	-0.4114	0.025471 ***	(-0.454693 , -0.3654)
2000	-0.339592	0.008066 ***	-0.3442	0.021429 ***	(-0.381840 , -0.3036)
2001	-0.593225	0.007560 ***	-0.6075	0.035465 ***	(-0.667756 , -0.5451)
2002	-0.629014	0.007480 ***	-0.6452	0.037276 ***	(-0.706568 , -0.5797)
2003	-0.702676	0.007374 ***	-0.7197	0.042099 ***	(-0.790895 , -0.6460)
2004	-0.611563	0.007602 ***	-0.6258	0.035923 ***	(-0.685727 , -0.5630)
2005	-0.606892	0.007472 ***	-0.6215	0.036282 ***	(-0.681633 , -0.5572)

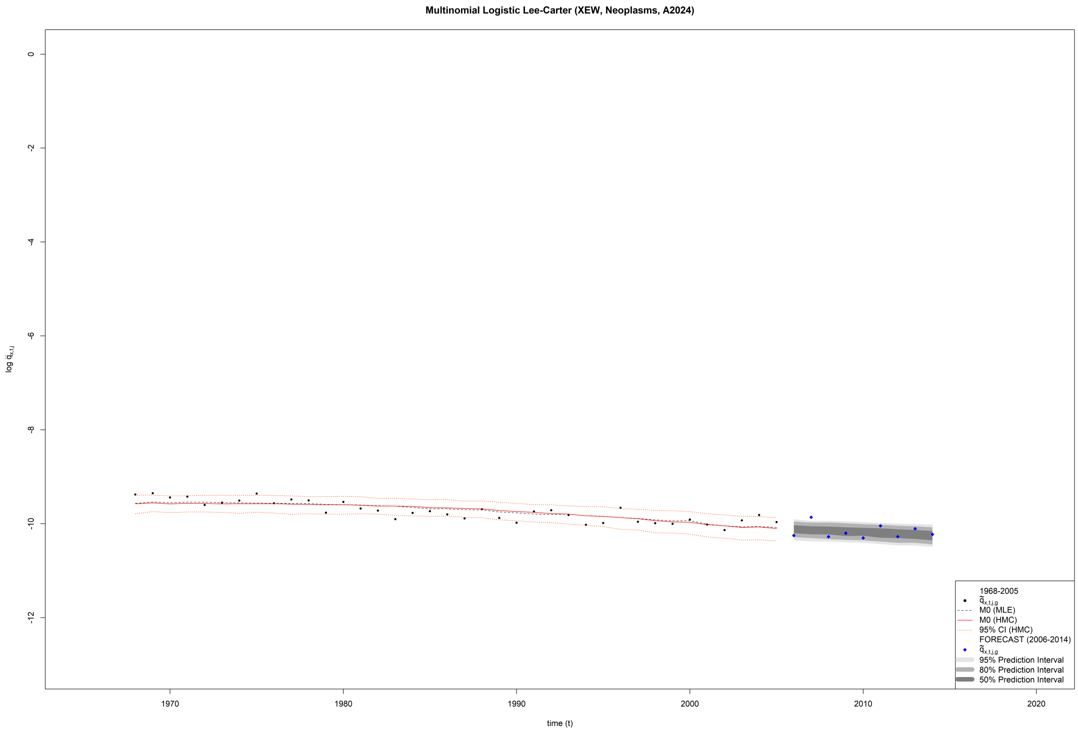
Appendix F

Additional Projection Plots

F.1 Additional Multinomial Logistic Lee-Carter Projections for England & Wales

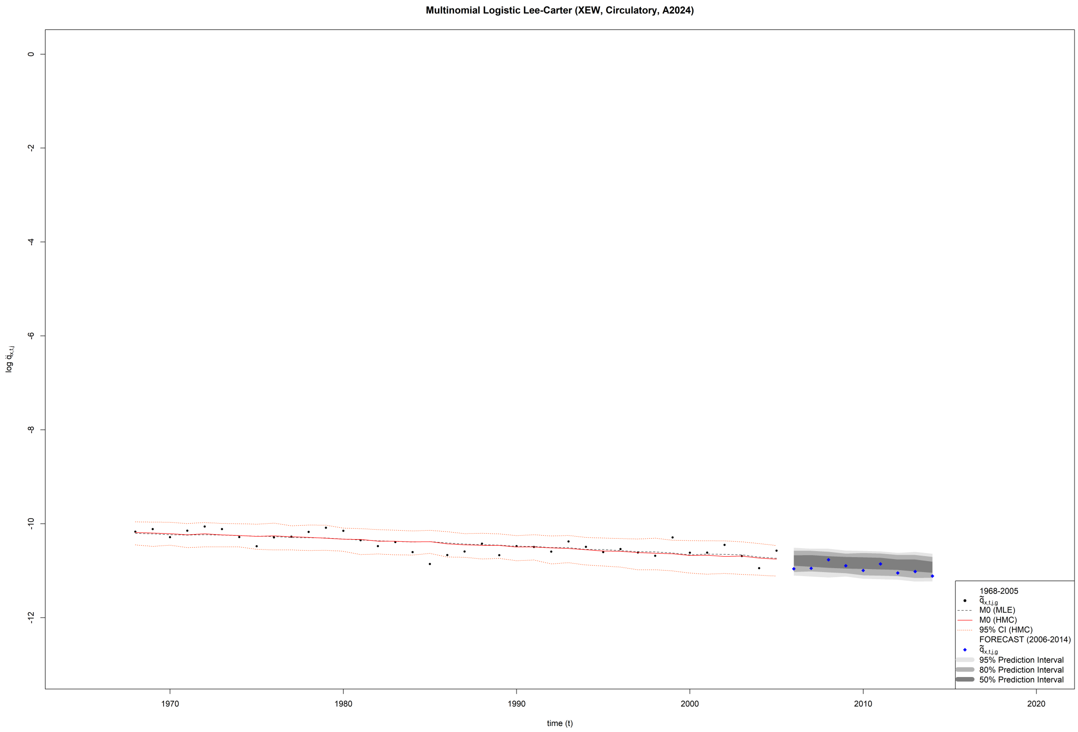


(a) Infectious and Parasitic Diseases

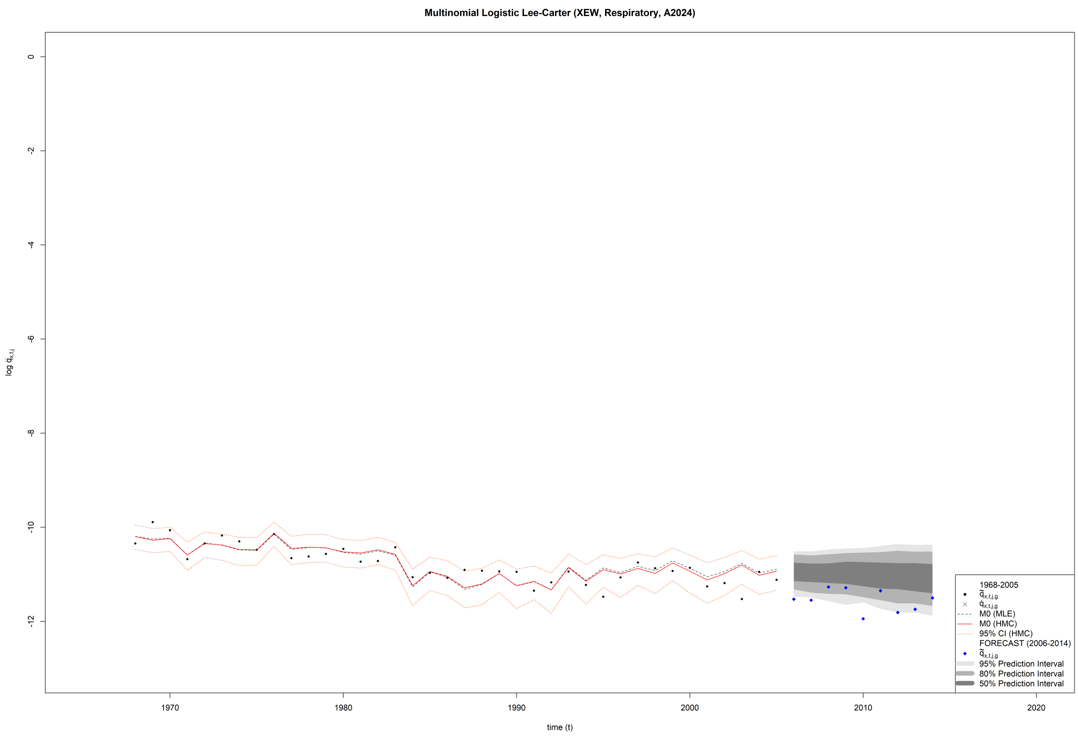


(b) Neoplasms

FIGURE F.1: Probability of Death Forecasts for Age Group 20-24 (MLG-LC (M0), England and Wales, 2006-2014)

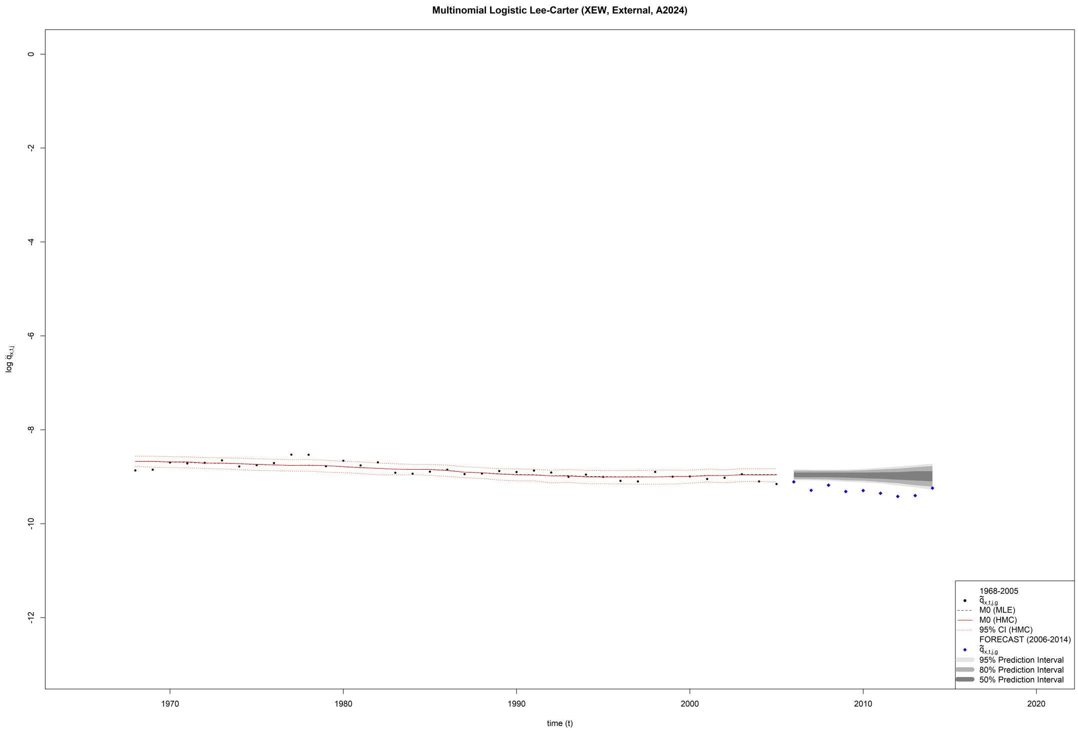


(c) Circulatory Diseases

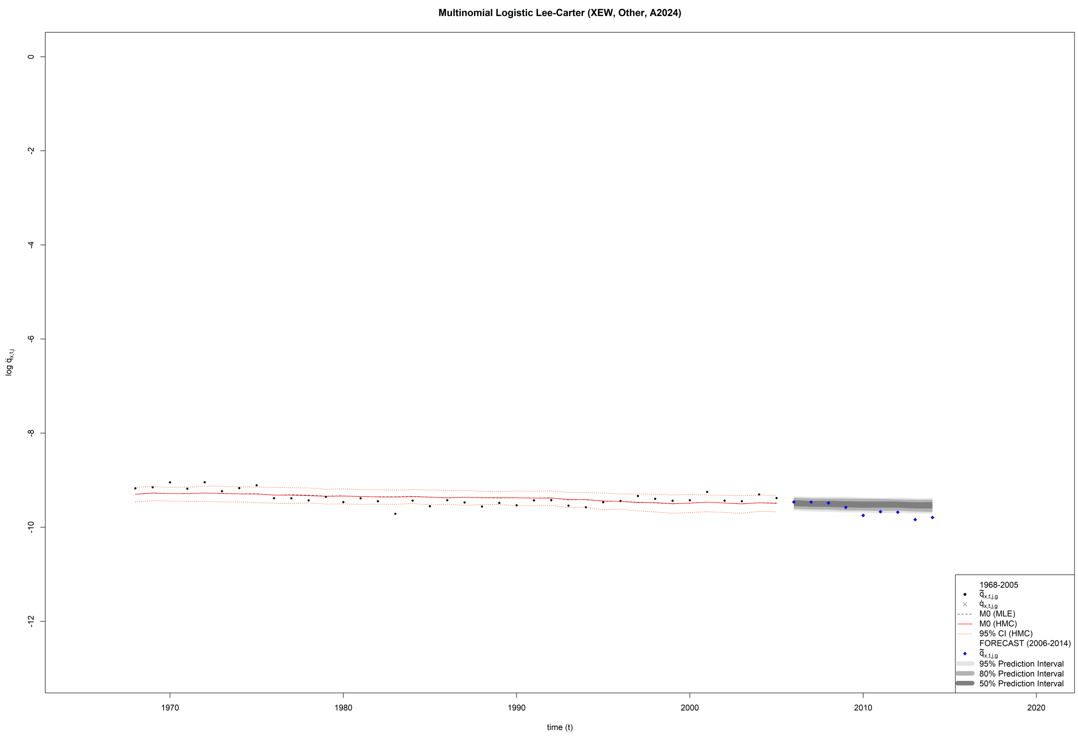


(d) Respiratory Diseases

FIGURE F.1 (CONT.)

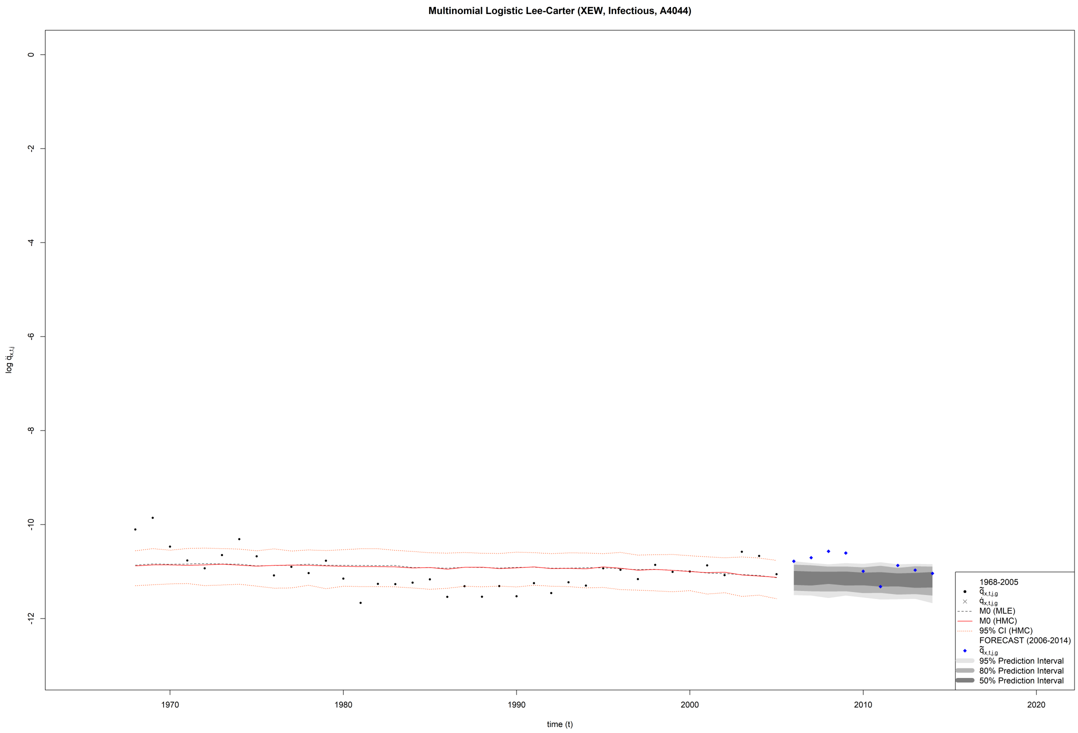


(e) External Causes

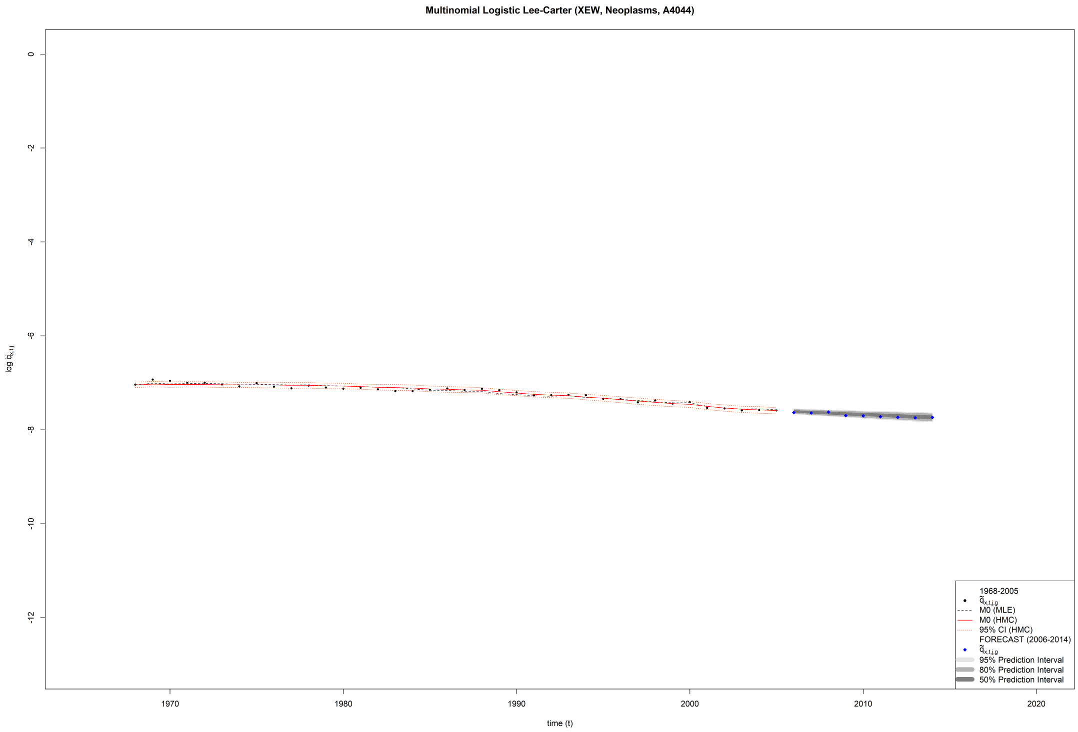


(f) Other Causes

FIGURE F.1 (CONT.)

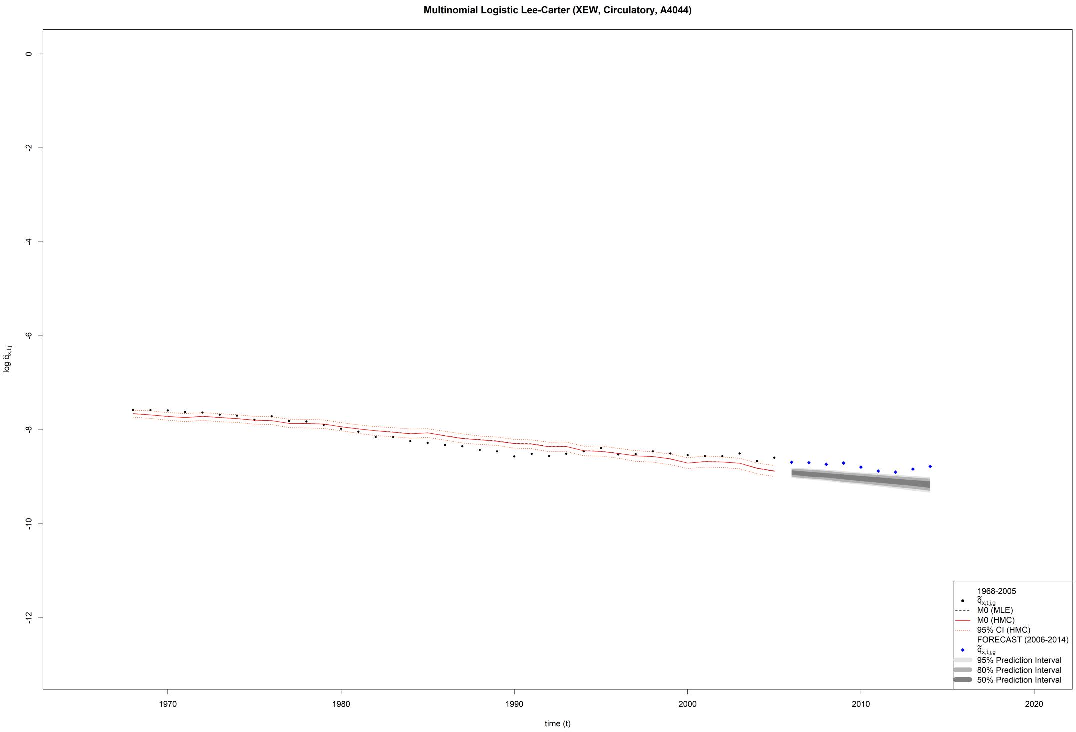


(a) Infectious and Parasitic Diseases

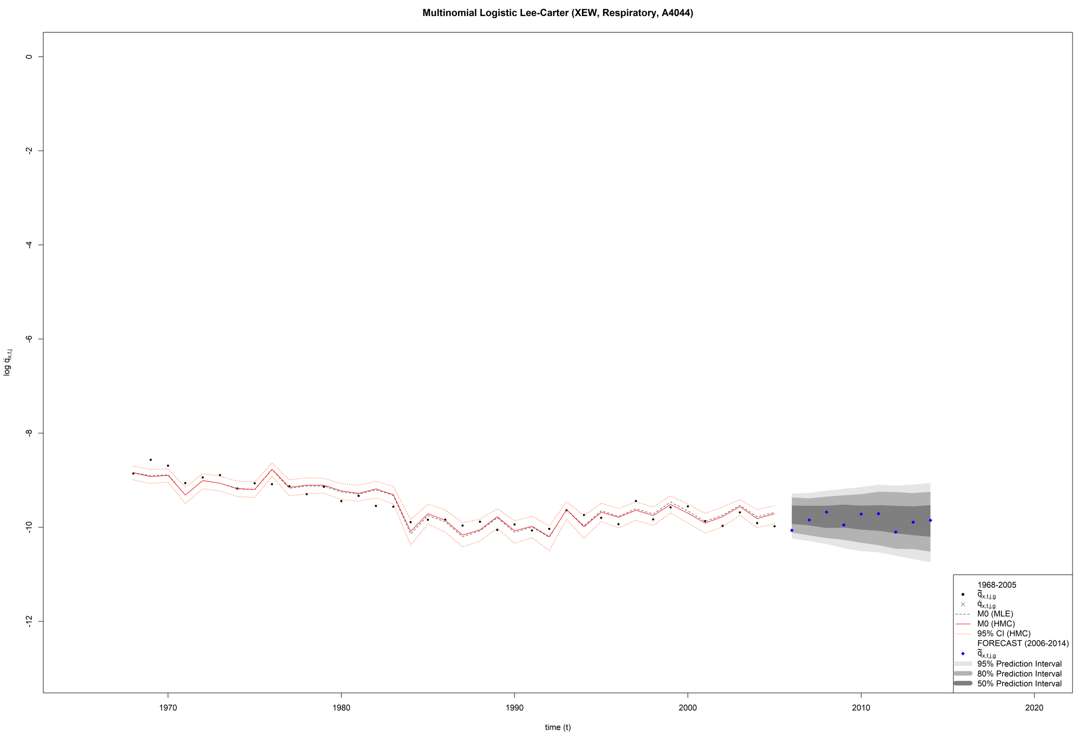


(b) Neoplasms

FIGURE F.2: Probability of Death Forecasts for Age Group 40-44 (MLG-LC (M0), England and Wales, 2006-2014)

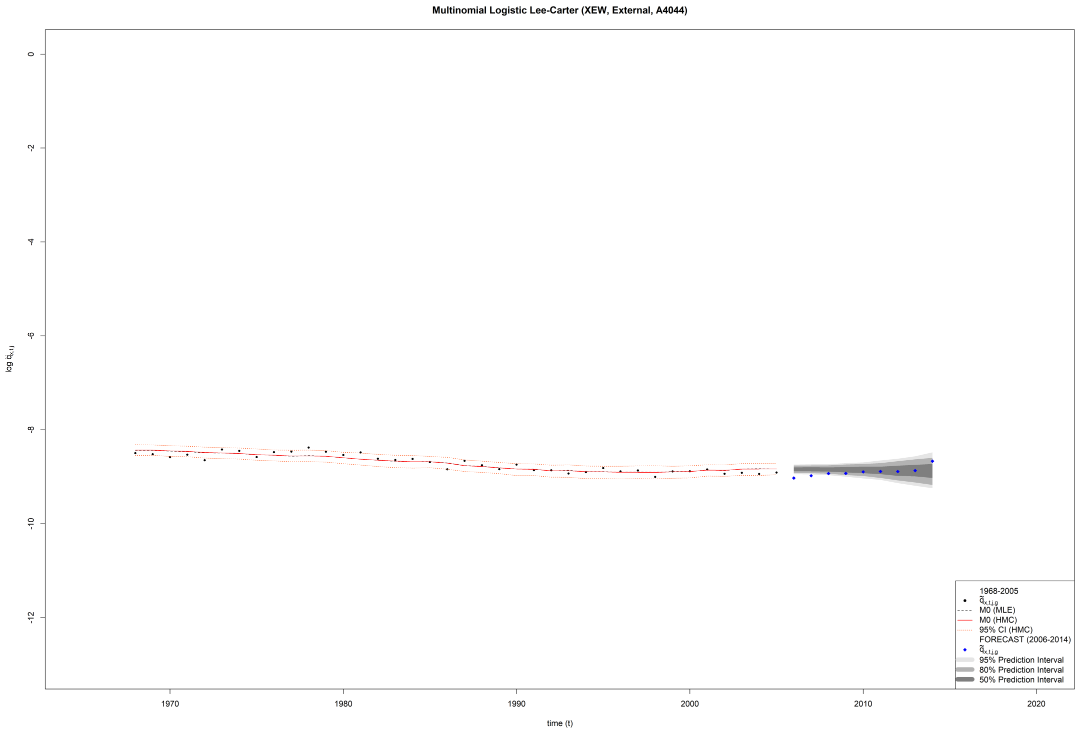


(c) Circulatory Diseases

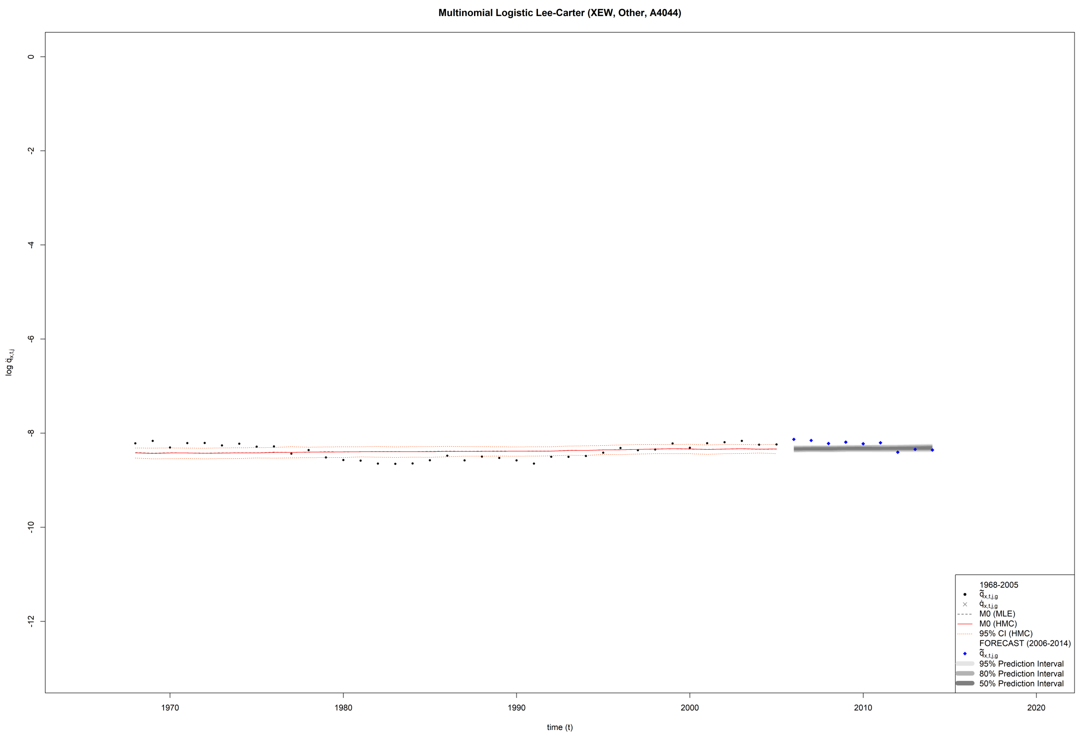


(d) Respiratory Diseases

FIGURE F.2 (CONT.)

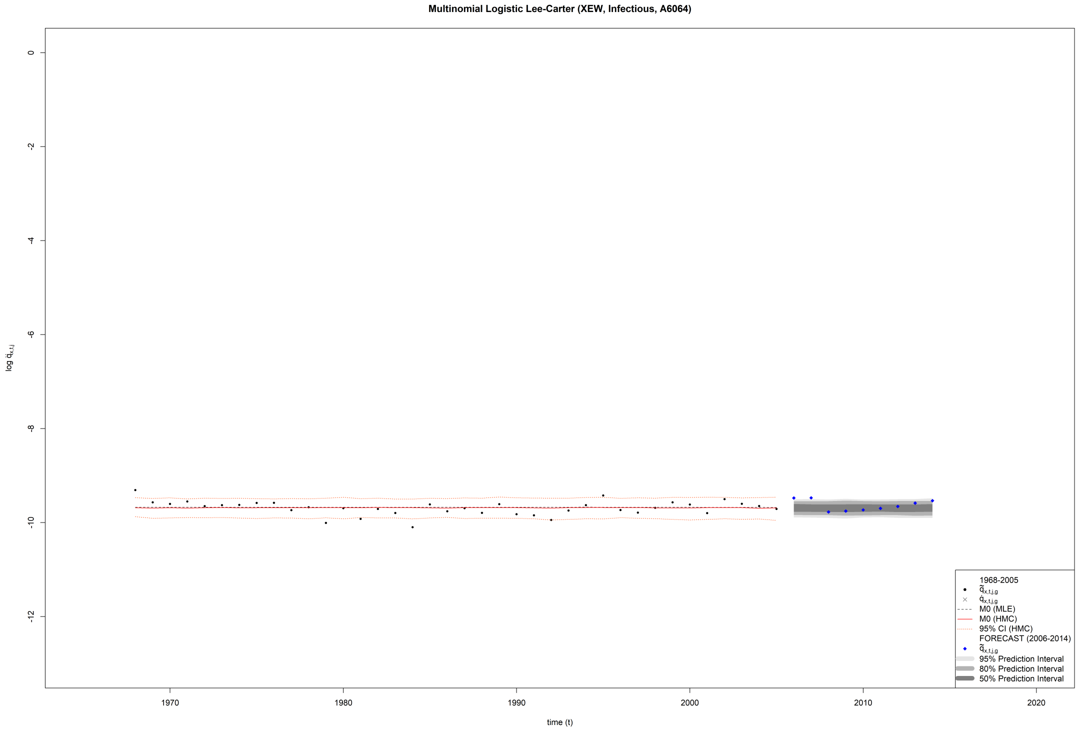


(e) External Causes

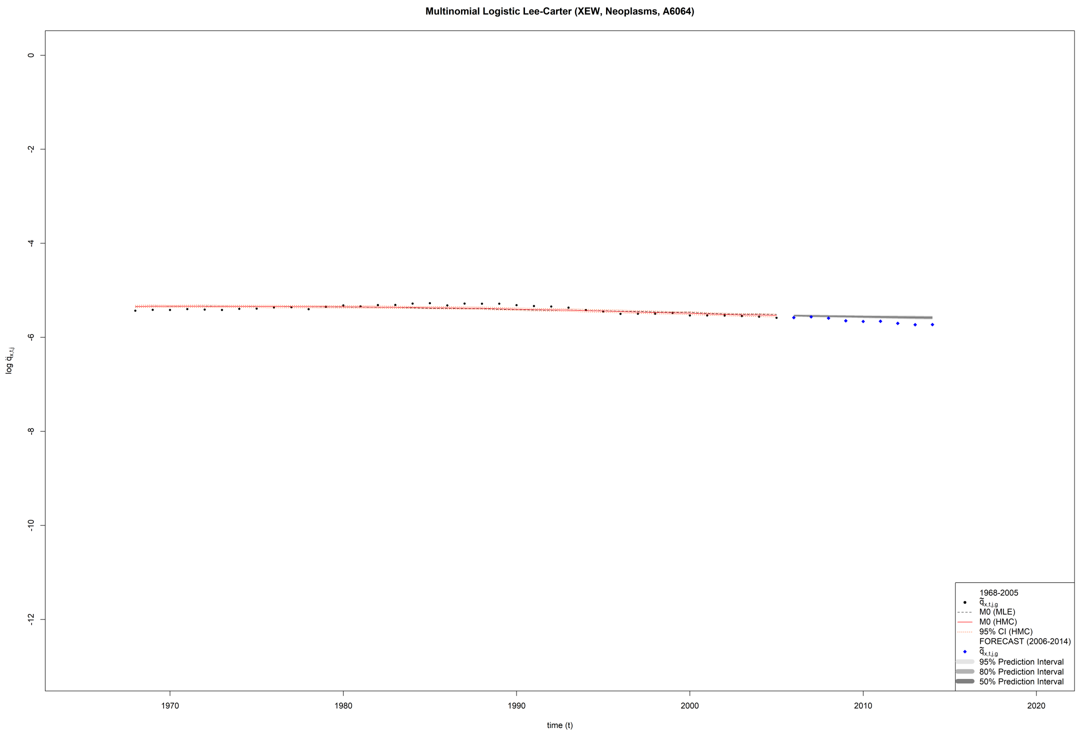


(f) Other Causes

FIGURE F.2 (CONT.)

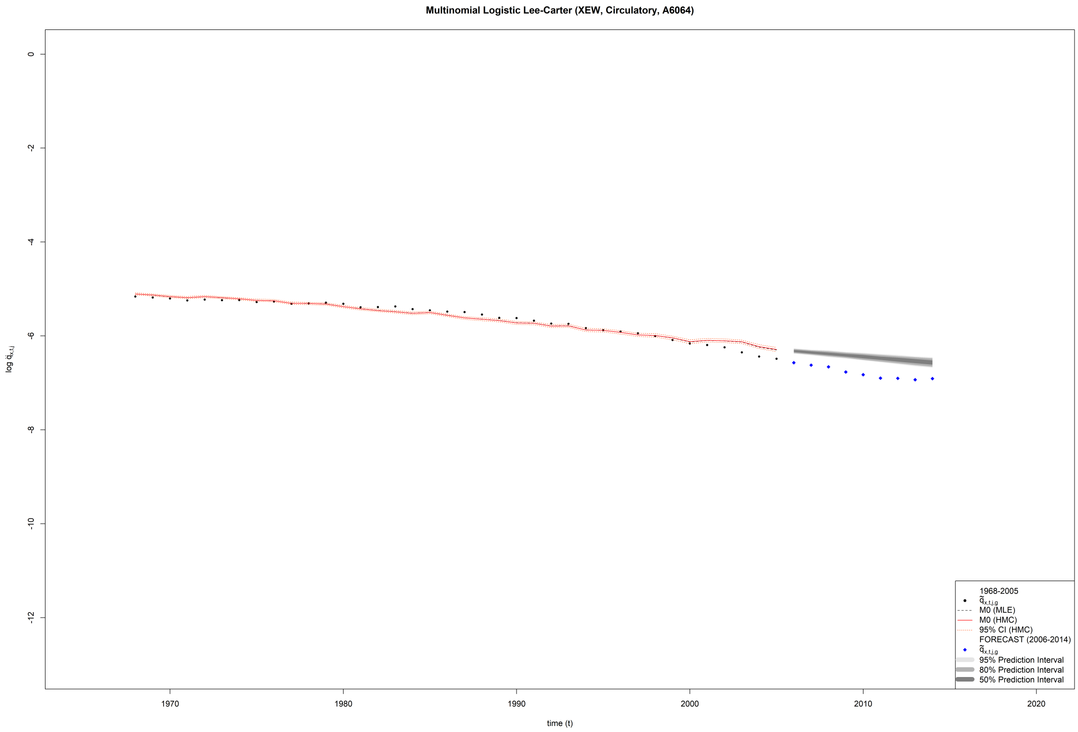


(a) Infectious and Parasitic Diseases

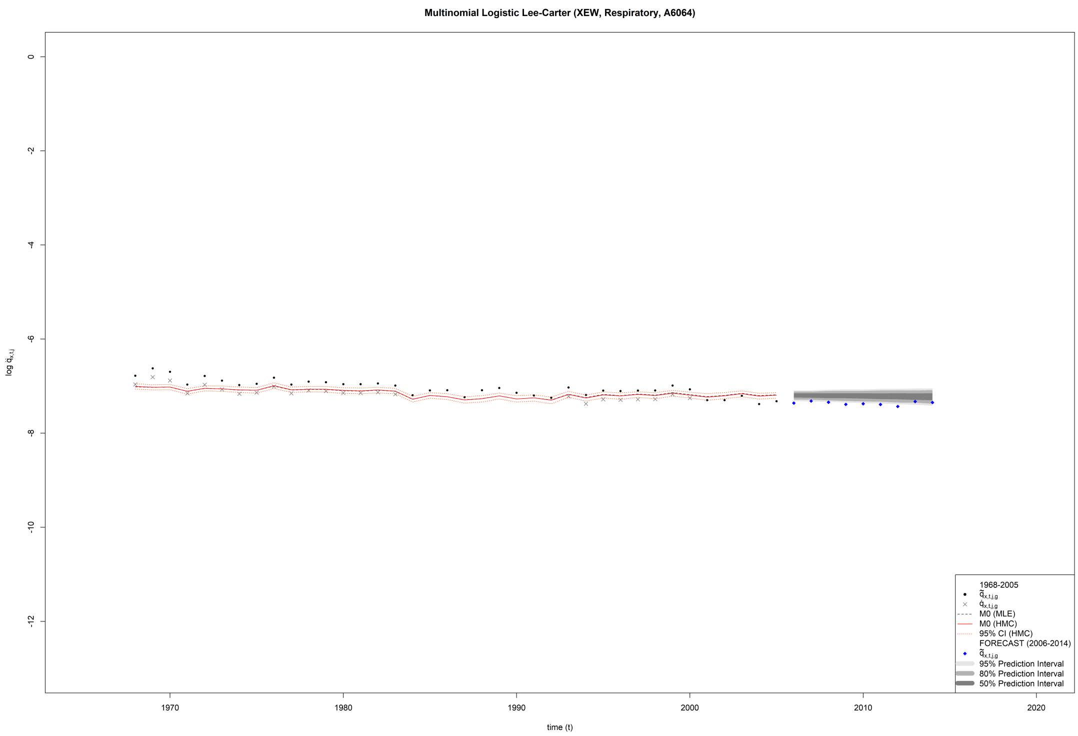


(b) Neoplasms

FIGURE F.3: Probability of Death Forecasts for Age Group 60-64 (MLG-LC (M0), England and Wales, 2006-2014)

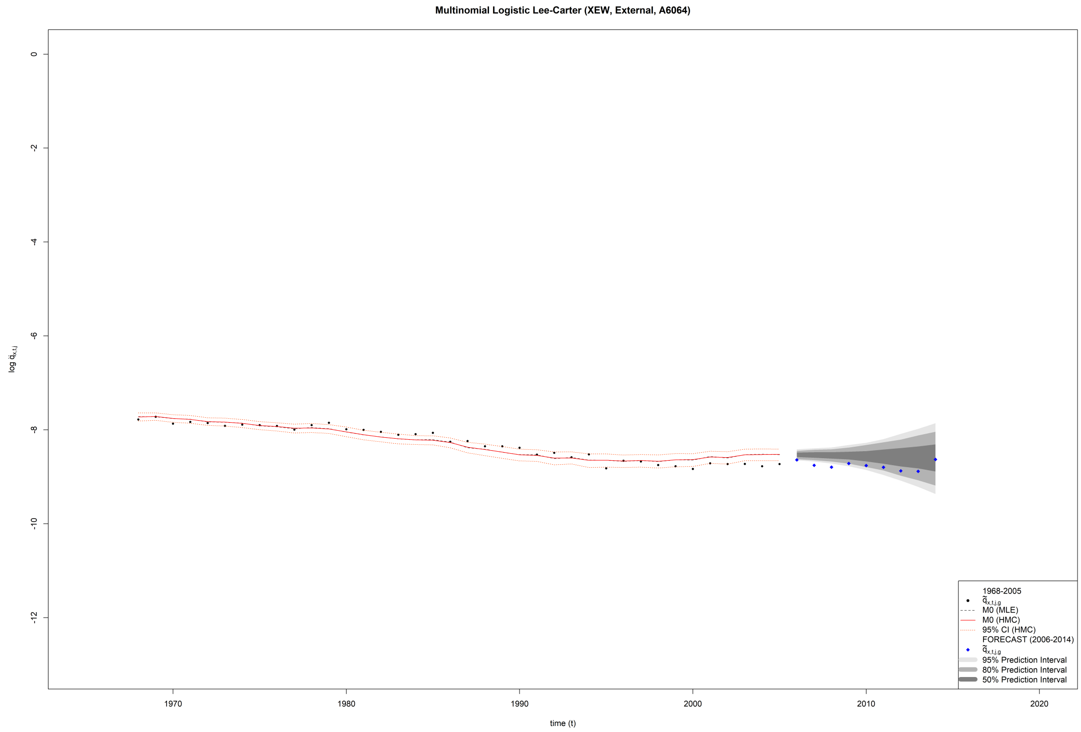


(c) Circulatory Diseases

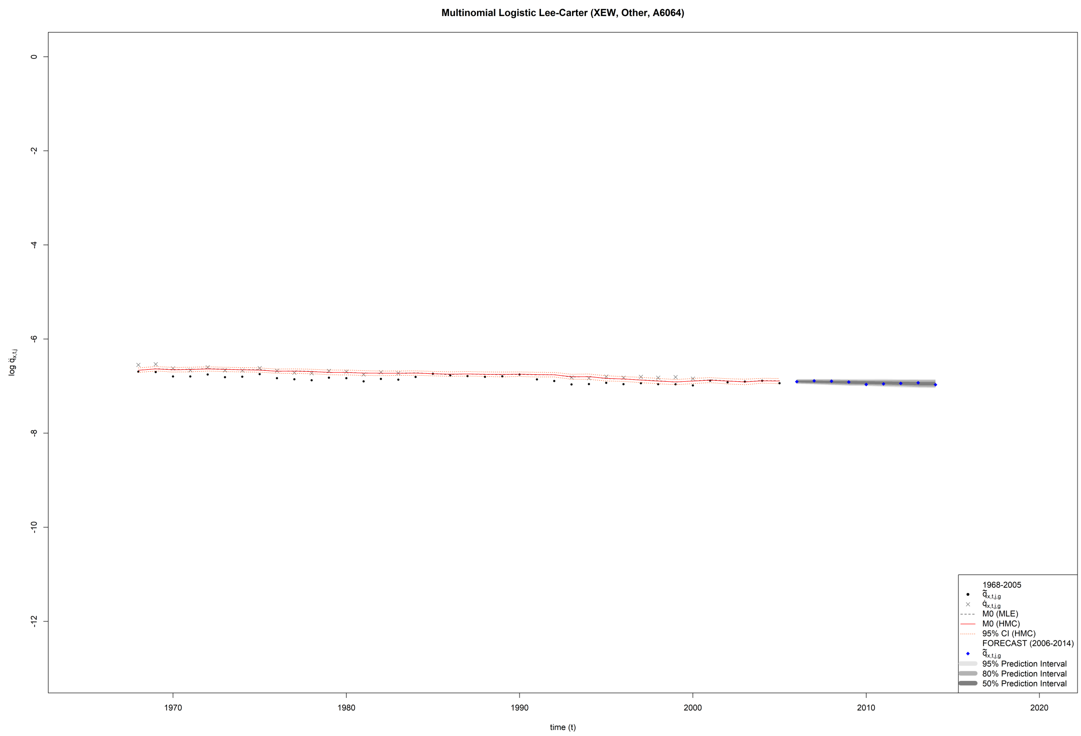


(d) Respiratory Diseases

FIGURE F.3 (CONT.)

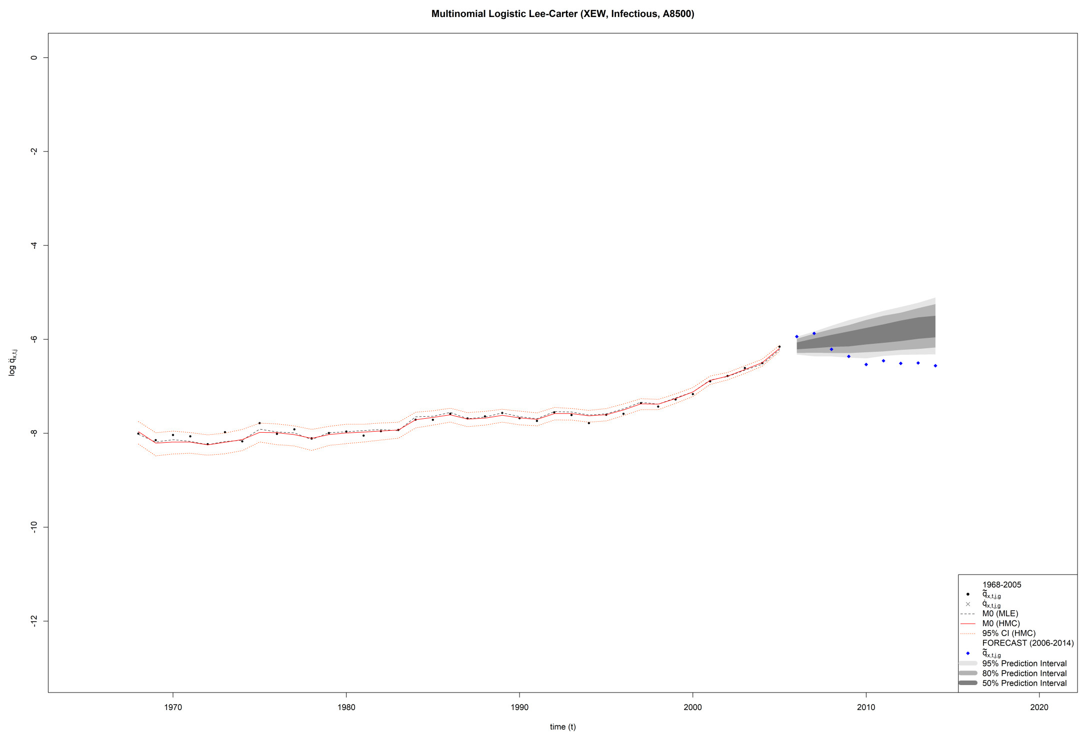


(e) External Causes

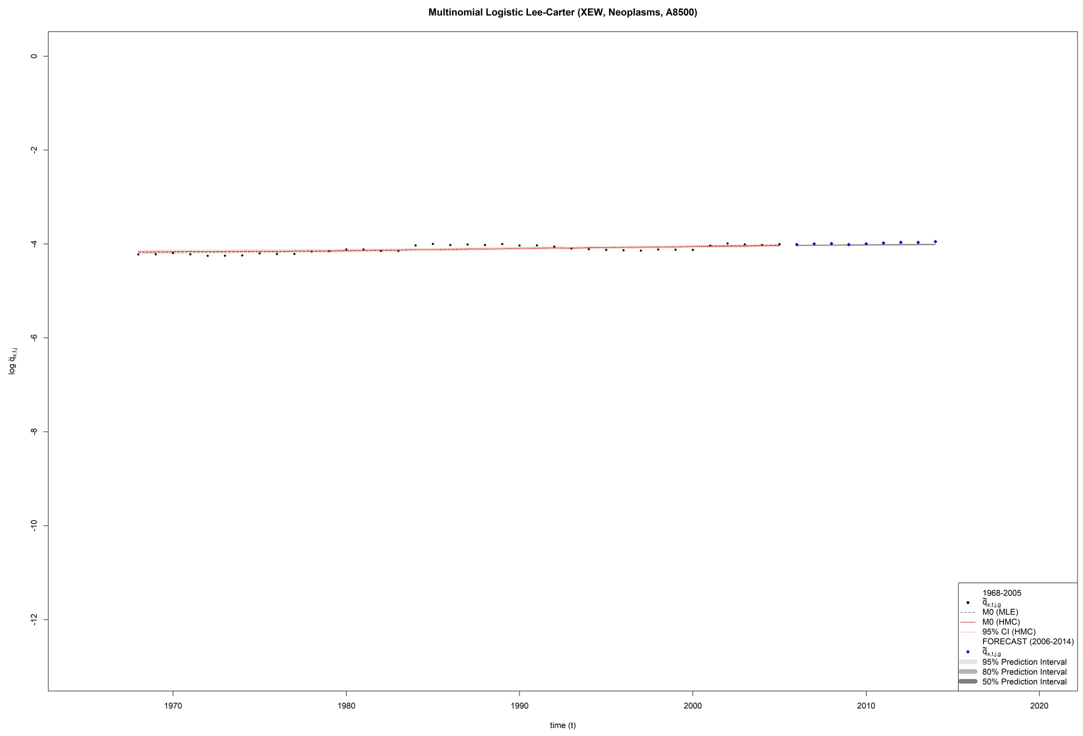


(f) Other Causes

FIGURE F.3 (CONT.)

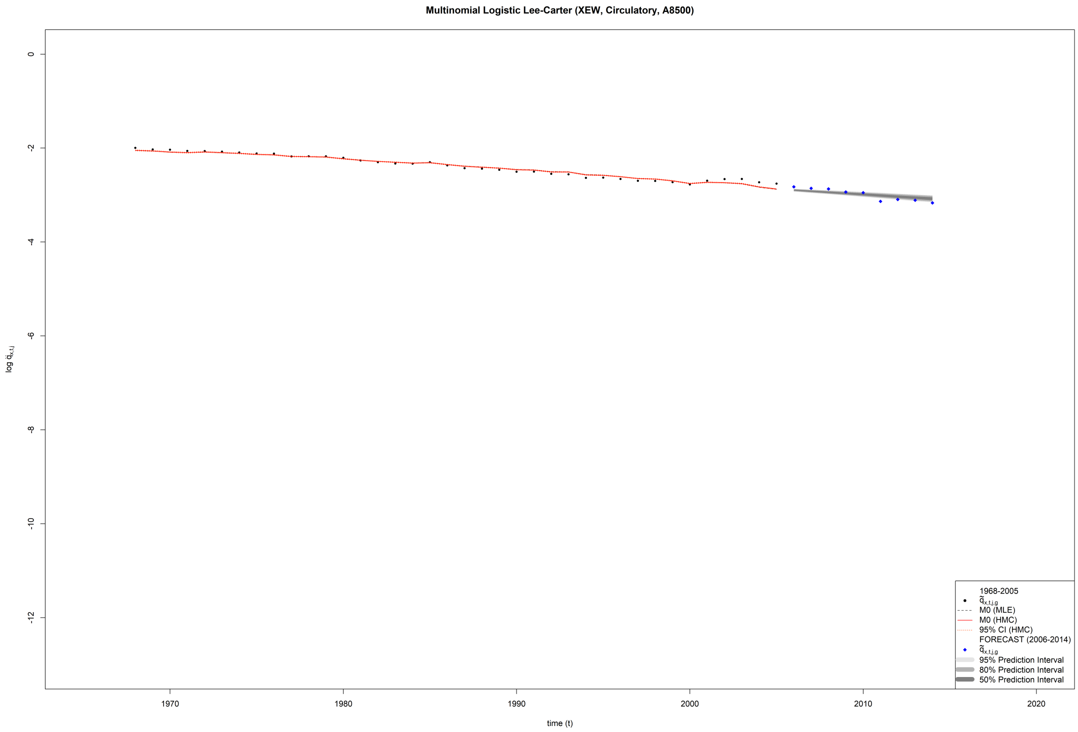


(a) Infectious and Parasitic Diseases

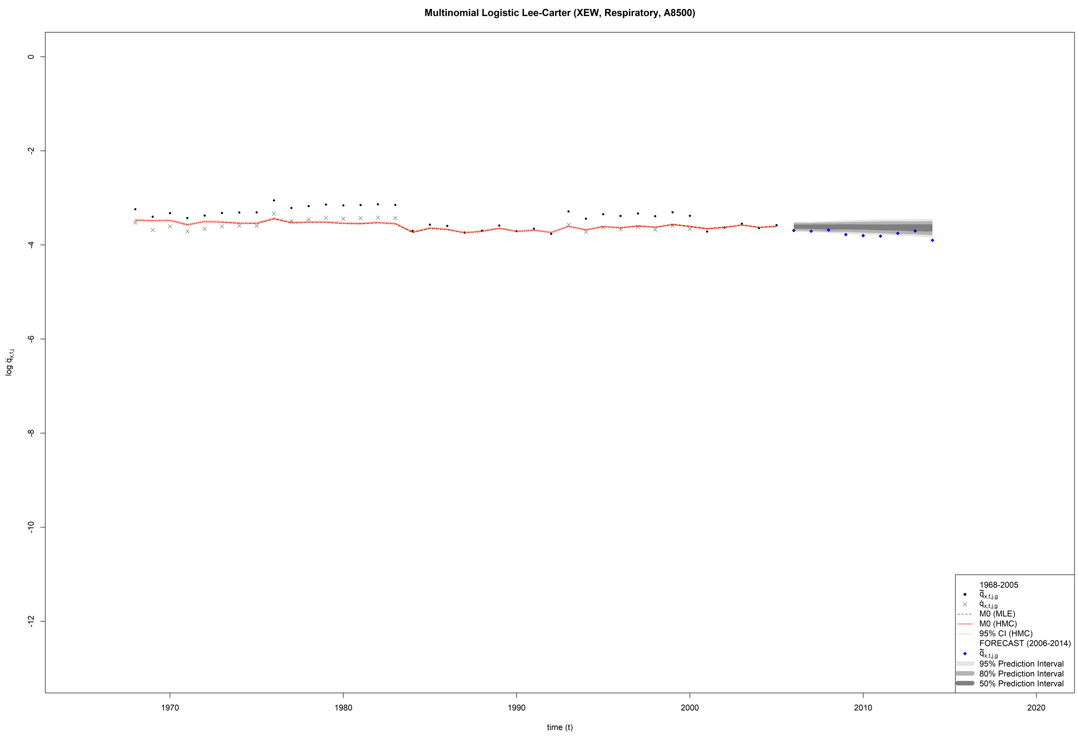


(b) Neoplasms

FIGURE F.4: Probability of Death Forecasts for Age Group 85+
(MLG-LC (M0), England and Wales, 2006-2014)

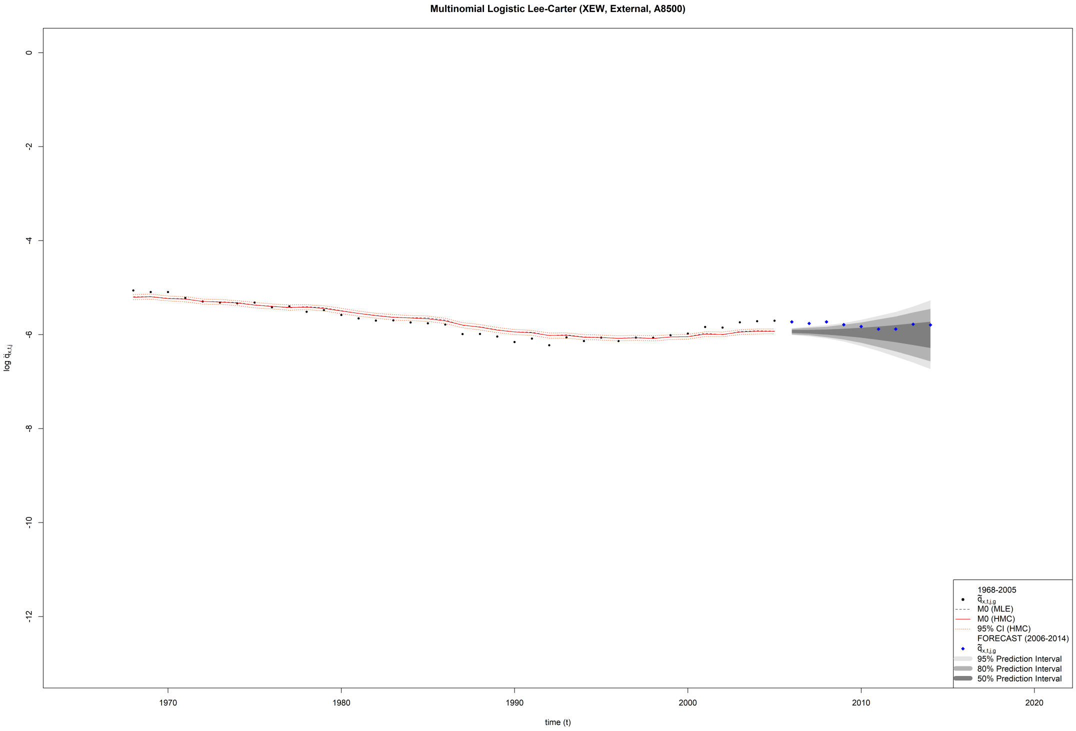


(c) Circulatory Diseases

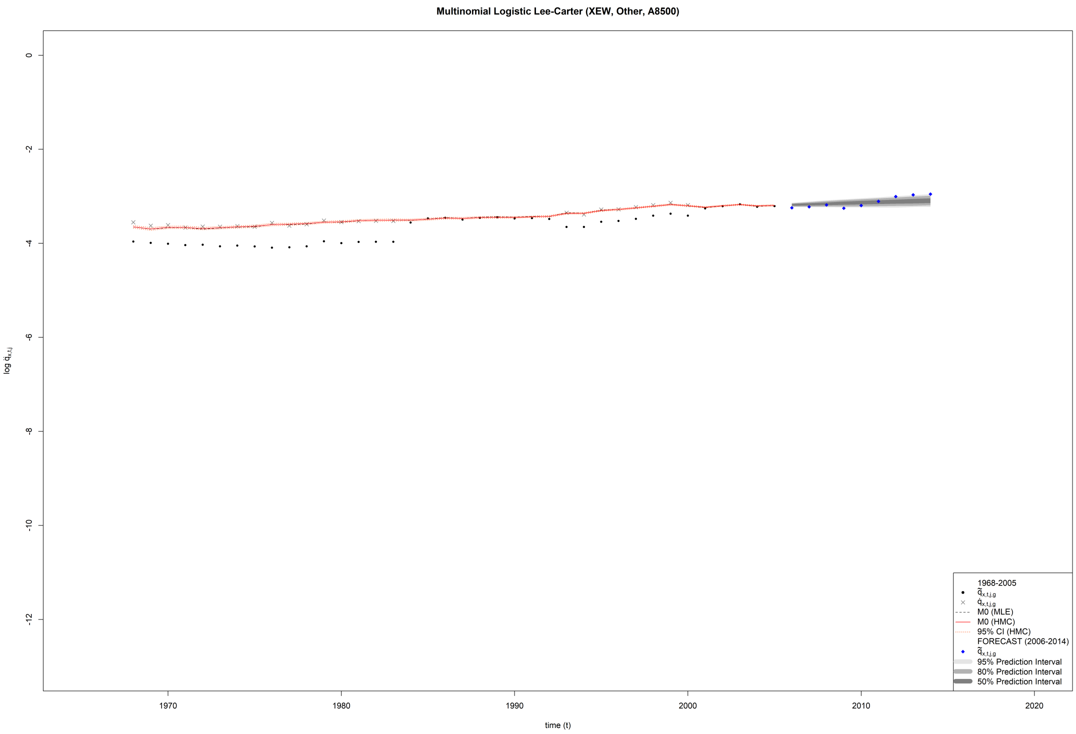


(d) Respiratory Diseases

FIGURE F.4 (CONT.)



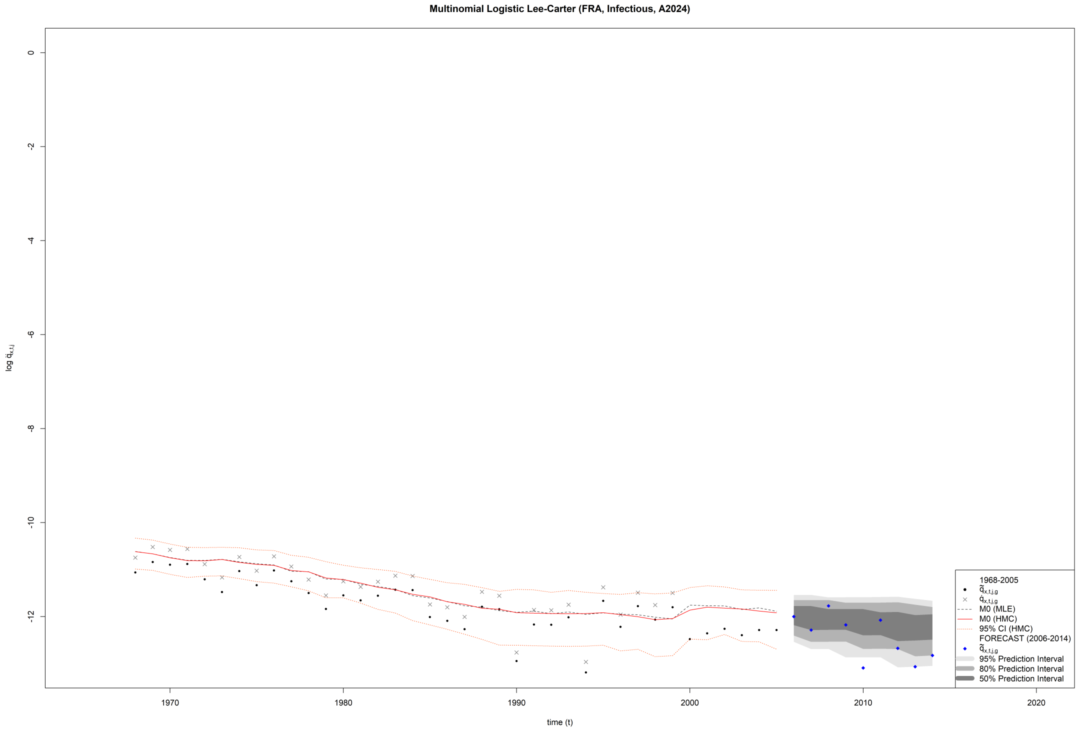
(e) External Causes



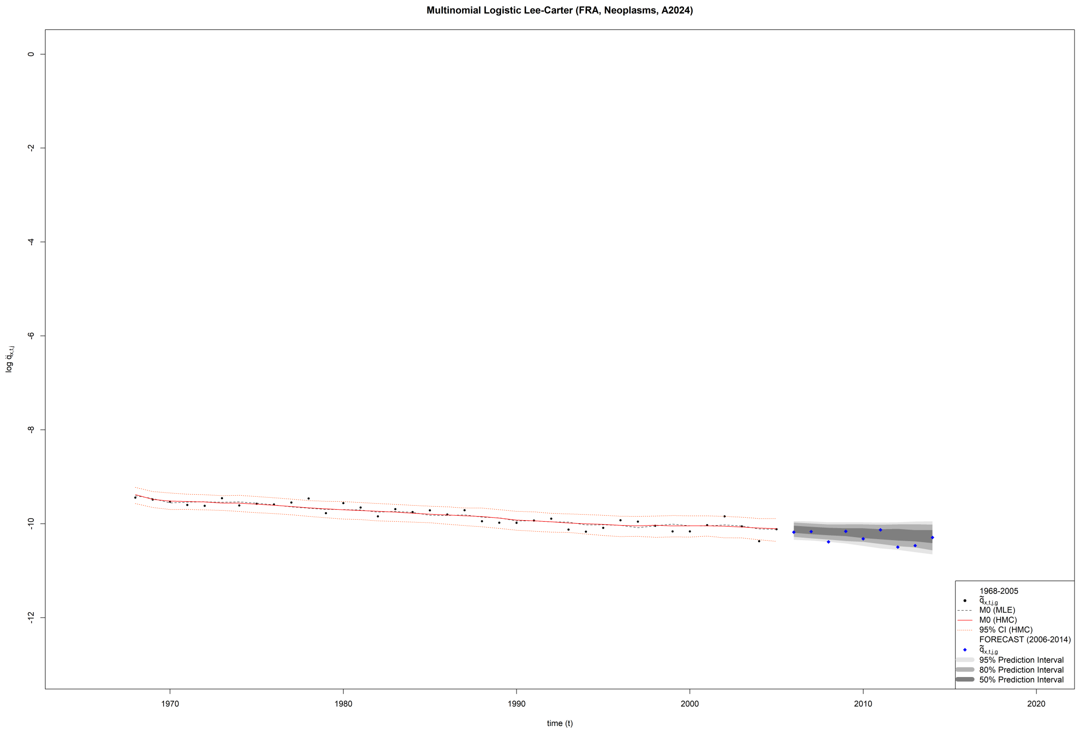
(f) Other Causes

FIGURE F.4 (CONT.)

F.2 Additional Multinomial Logistic Lee-Carter Projections for France

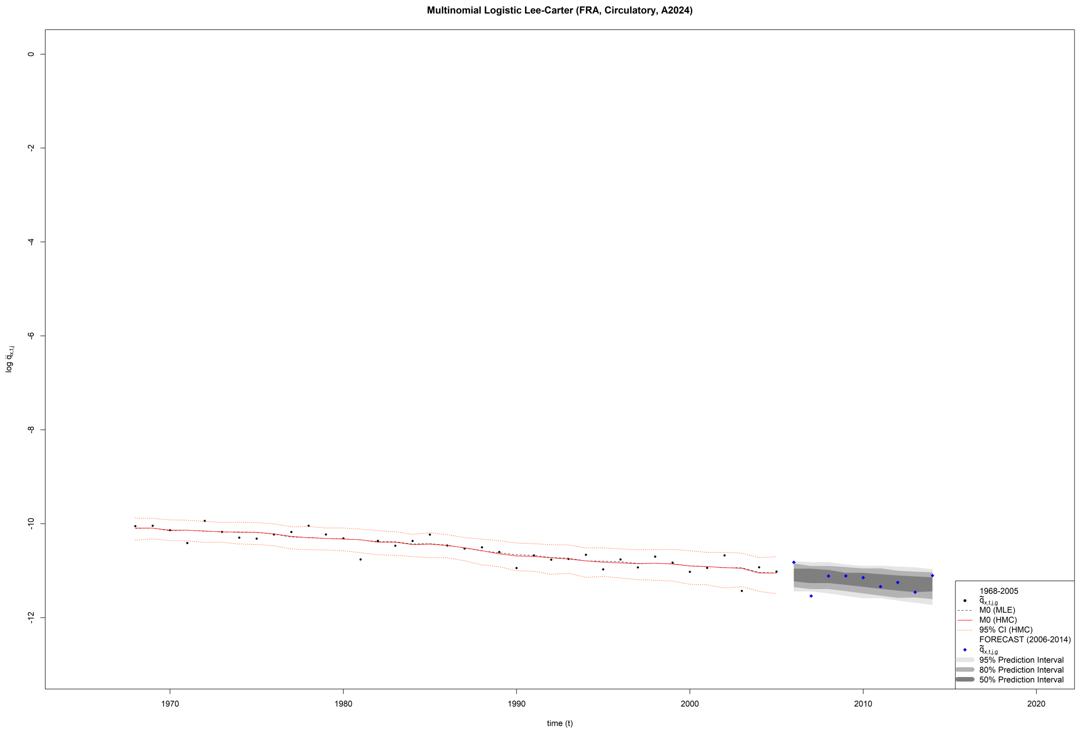


(a) Infectious and Parasitic Diseases

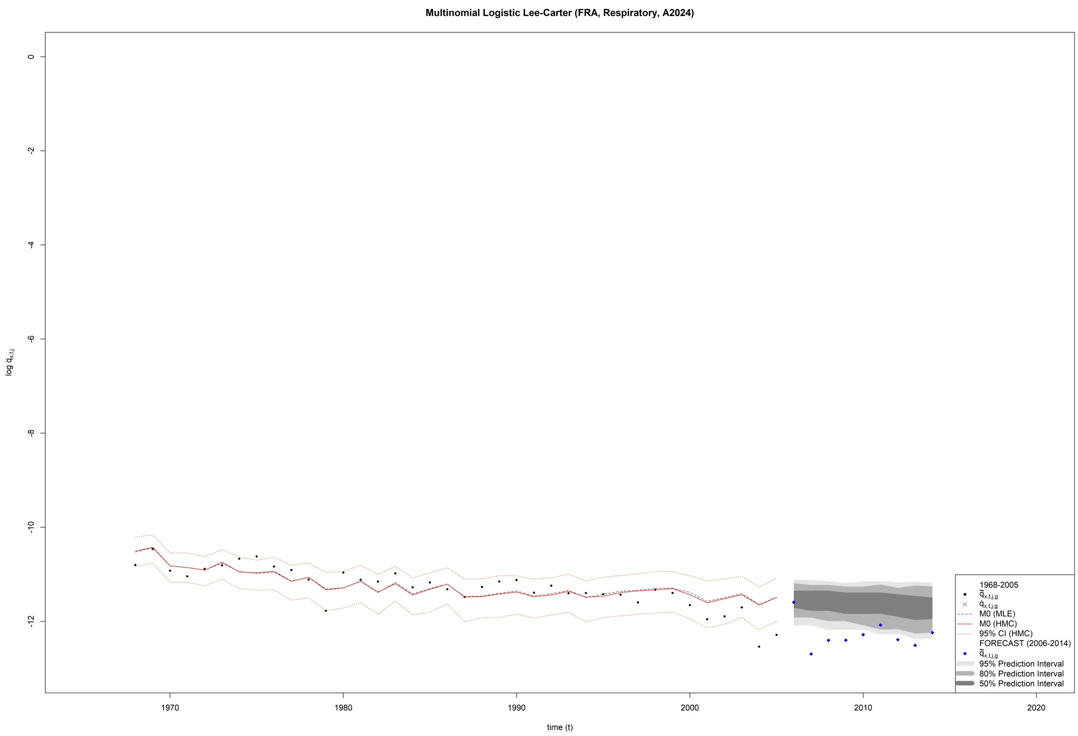


(b) Neoplasms

FIGURE F.5: Probability of Death Forecasts for Age Group 20-24 (MLG-LC (M0), England and Wales, 2006-2014)

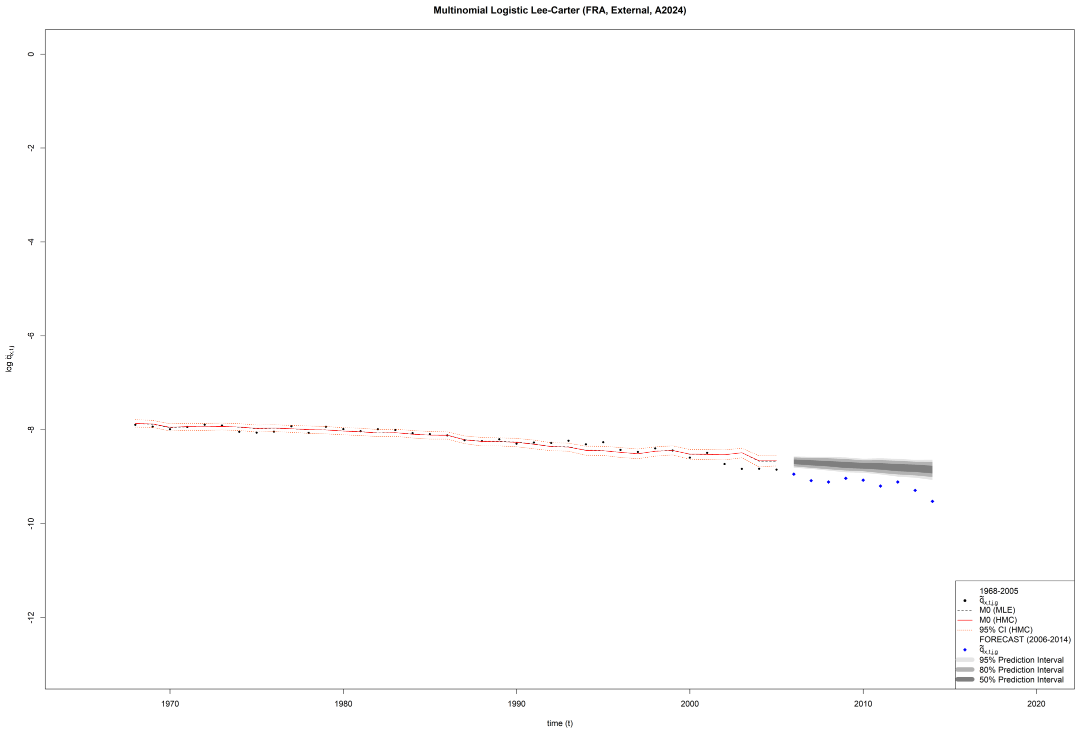


(c) Circulatory Diseases

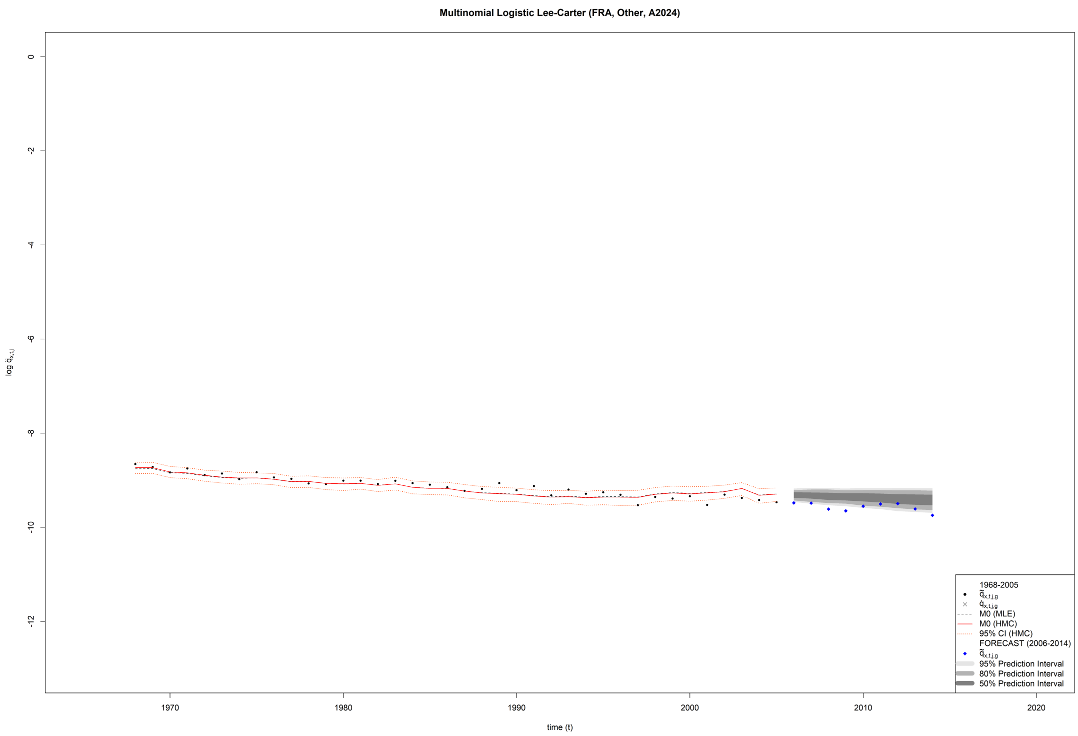


(d) Respiratory Diseases

FIGURE F.5 (CONT.)

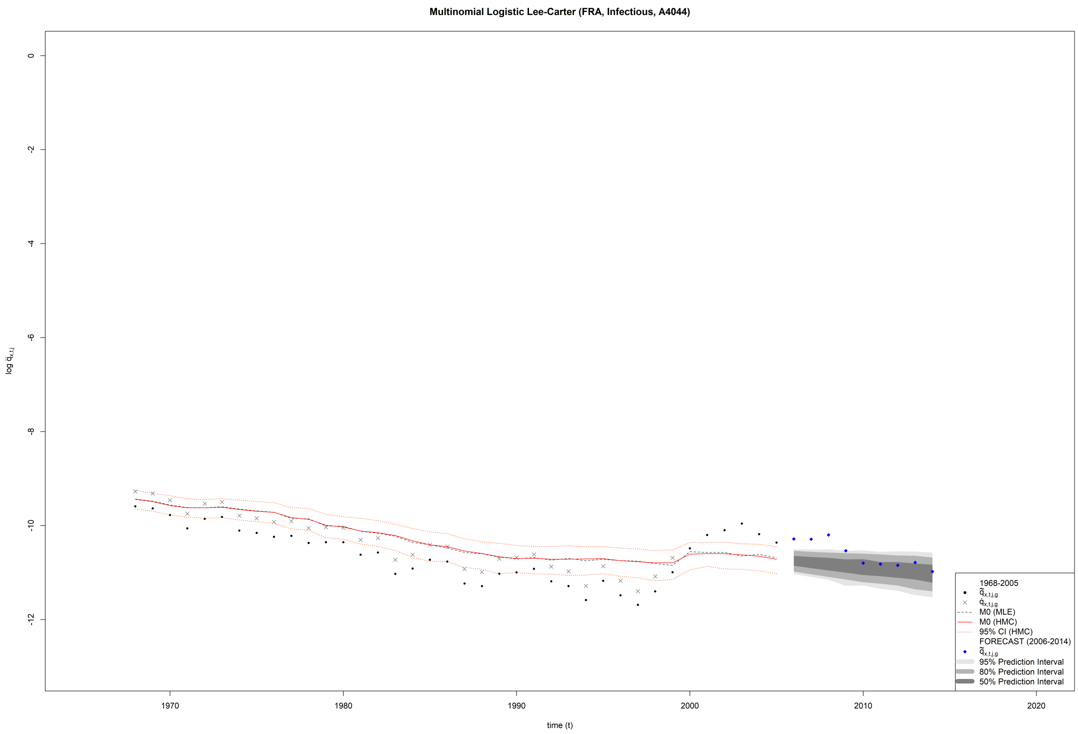


(e) External Causes

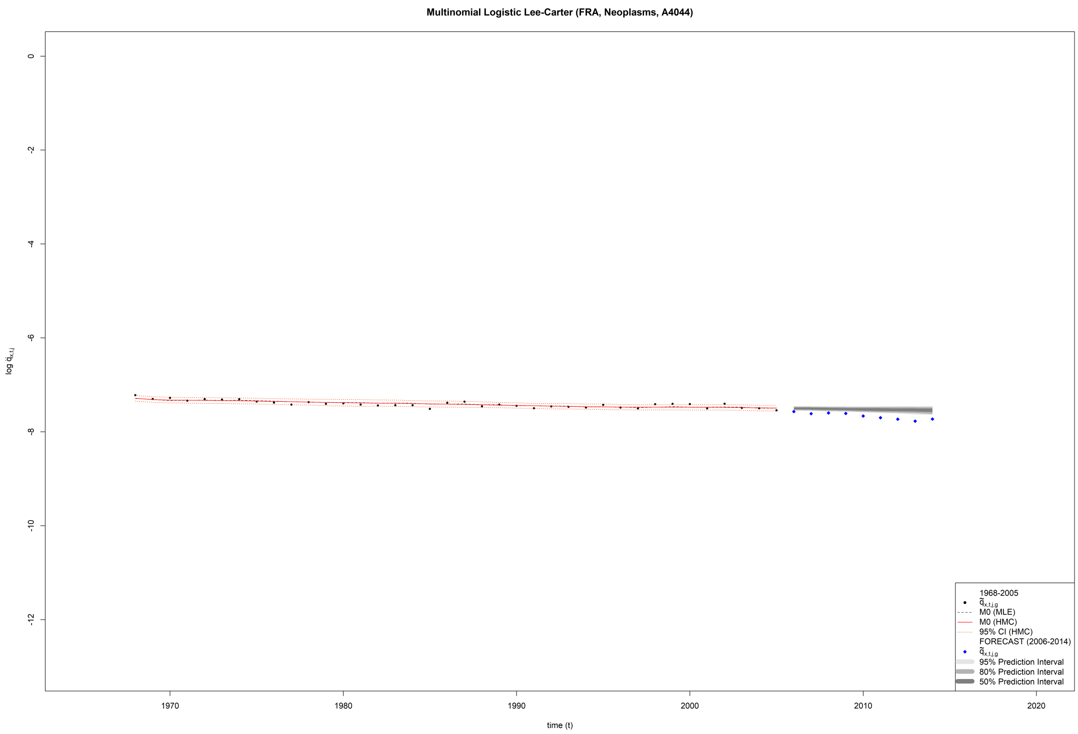


(f) Other Causes

FIGURE F.5 (CONT.)

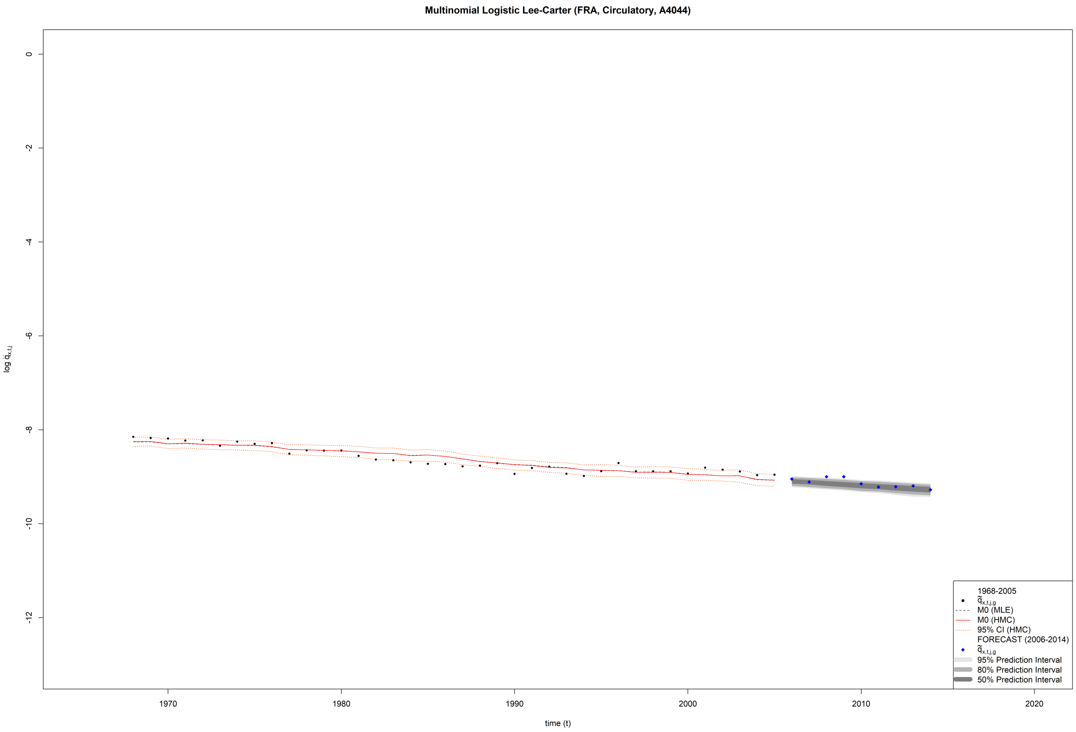


(a) Infectious and Parasitic Diseases

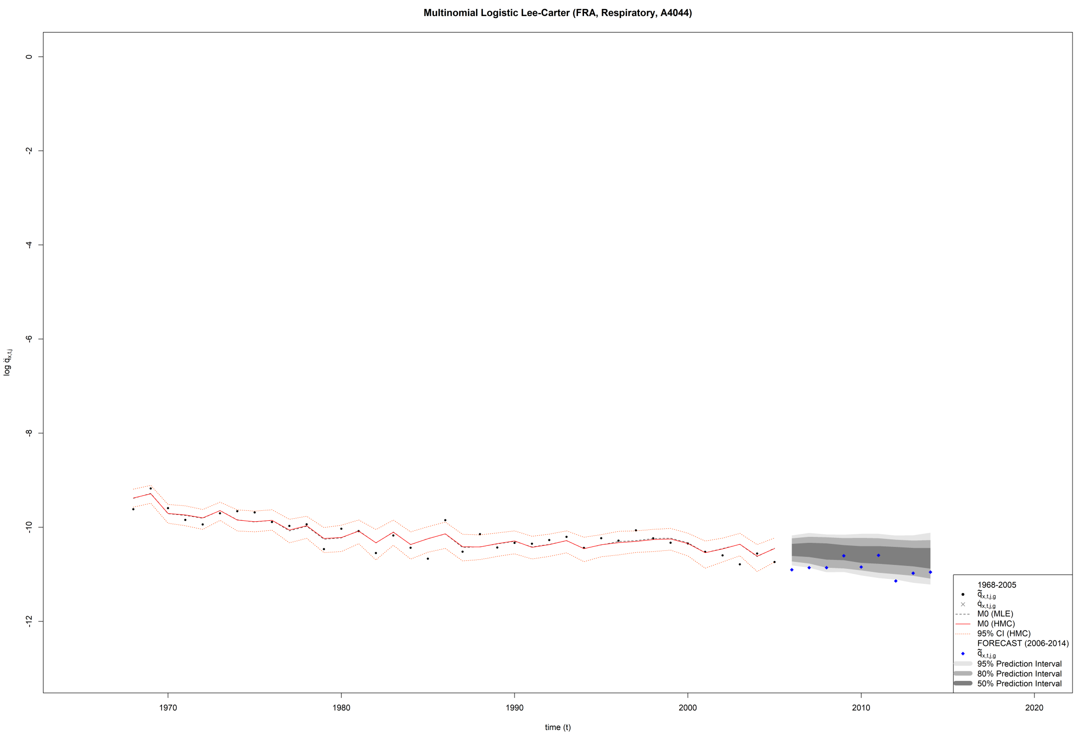


(b) Neoplasms

FIGURE F.6: Probability of Death Forecasts for Age Group 40-44
(MLG-LC (M0), England and Wales, 2006-2014)

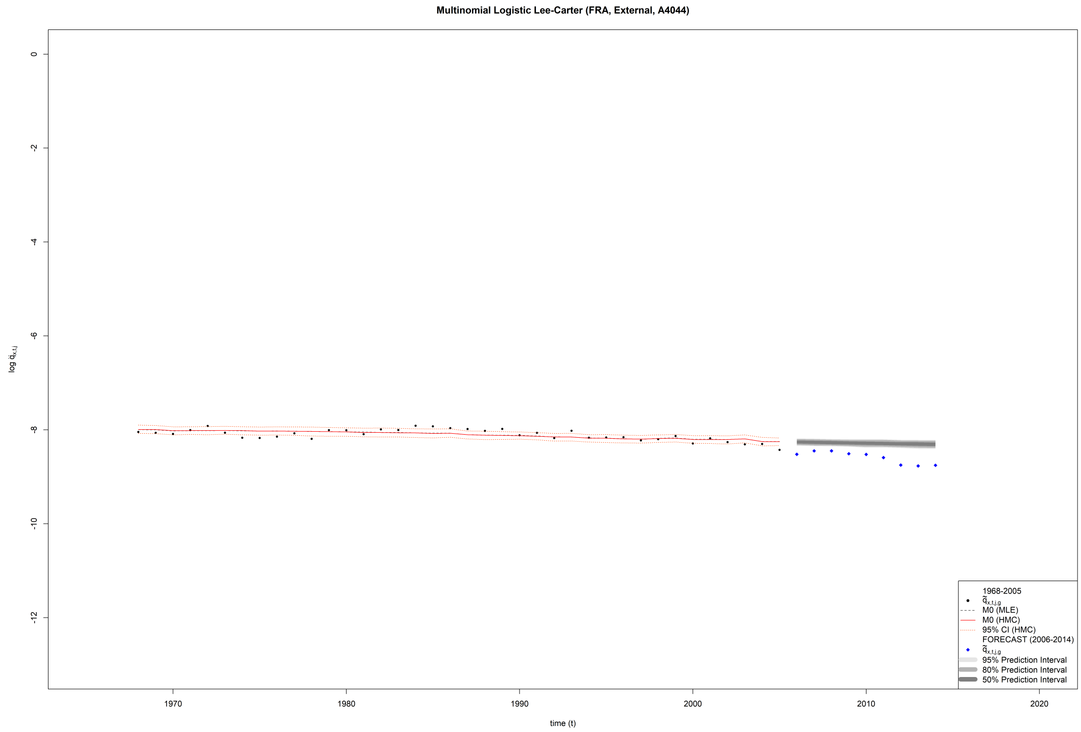


(c) Circulatory Diseases

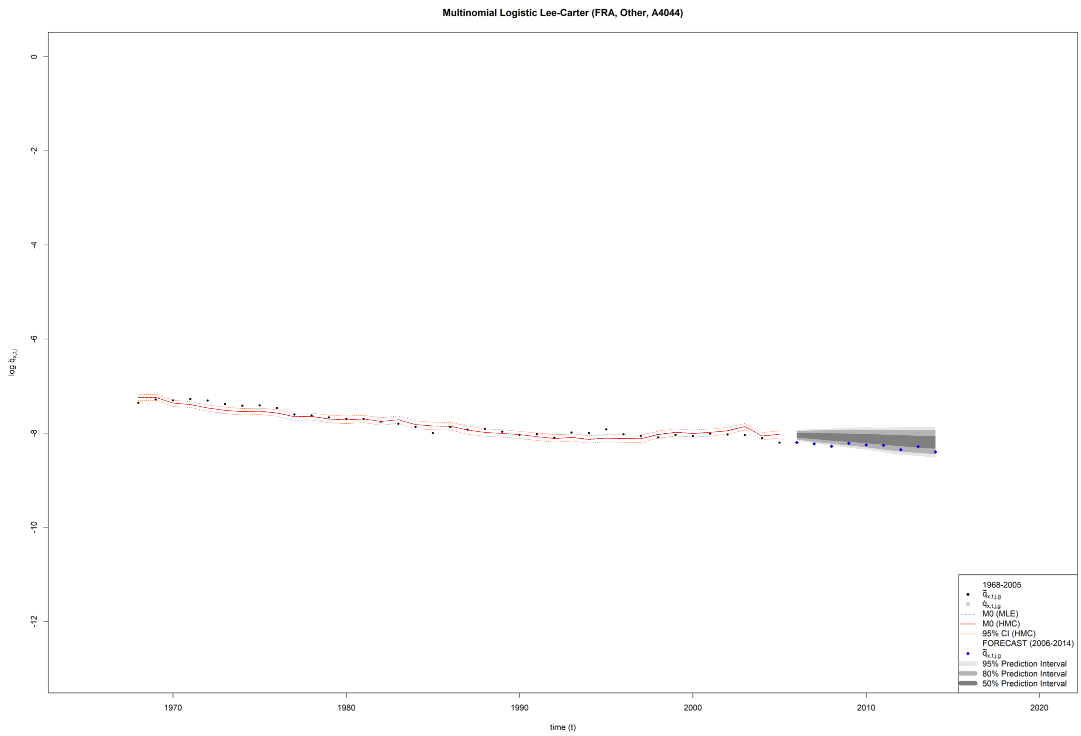


(d) Respiratory Diseases

FIGURE F.6 (CONT.)

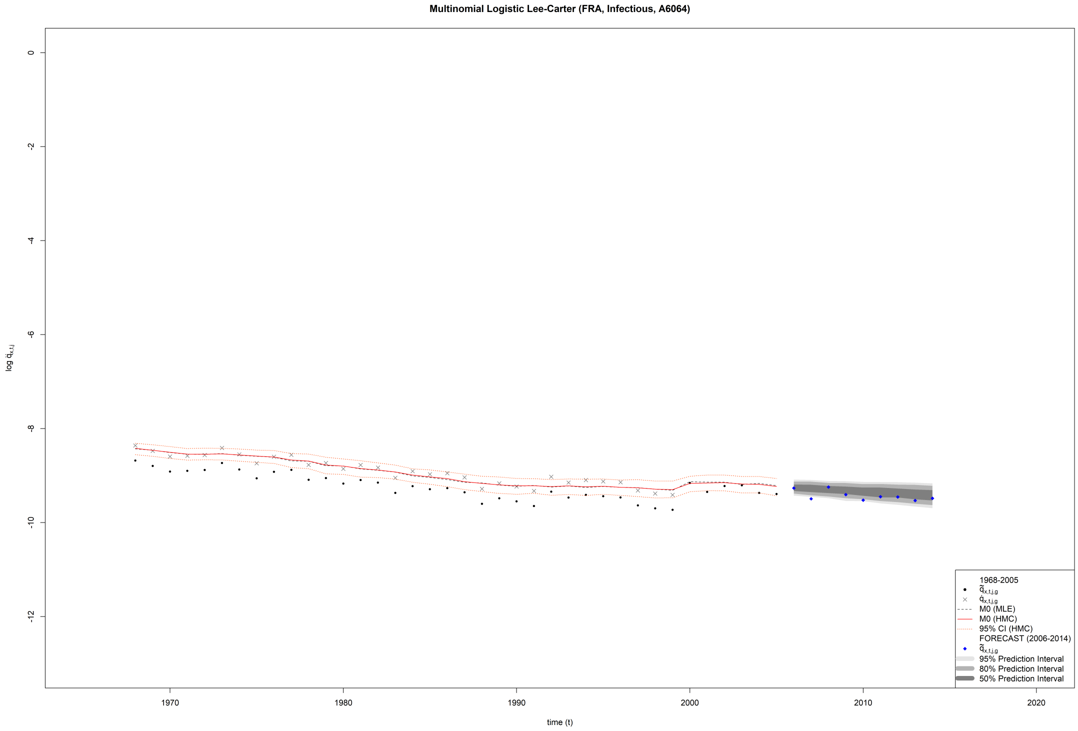


(e) External Causes

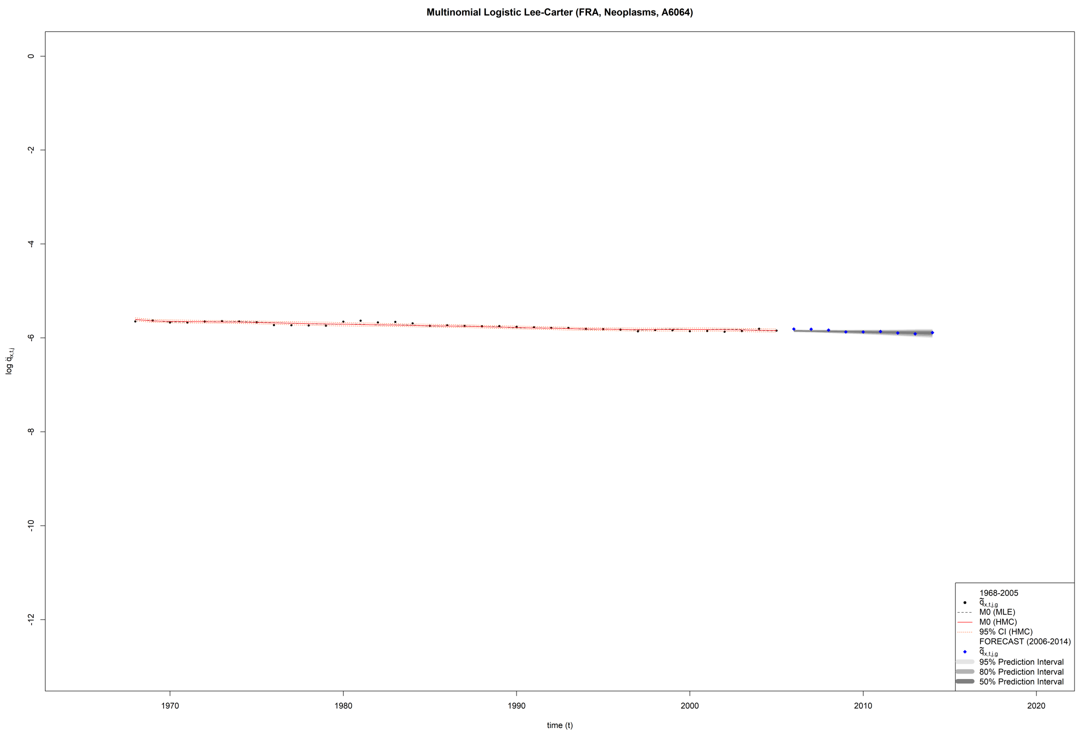


(f) Other Causes

FIGURE F.6 (CONT.)

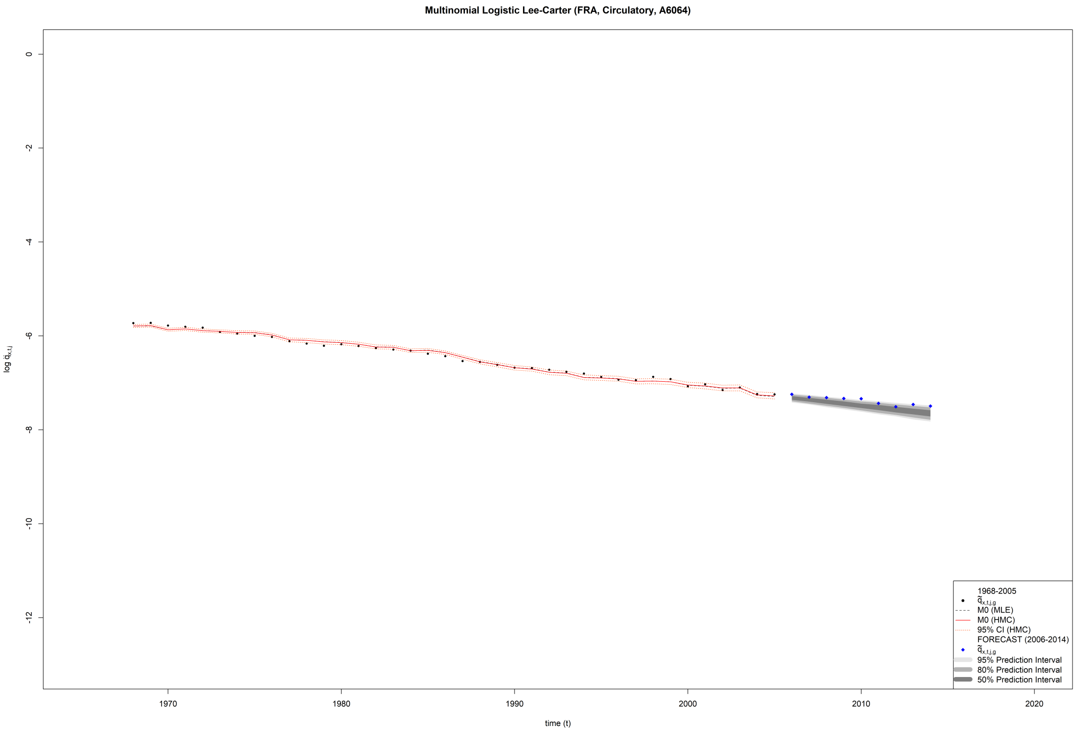


(a) Infectious and Parasitic Diseases

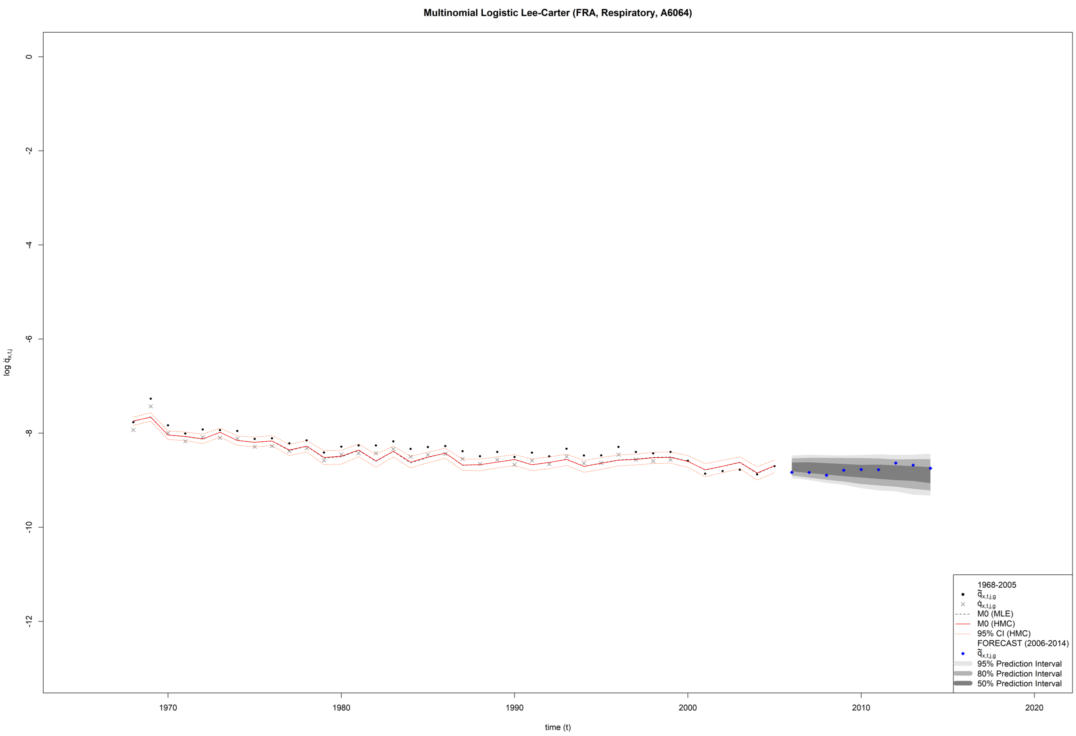


(b) Neoplasms

FIGURE F.7: Probability of Death Forecasts for Age Group 60-64 (MLG-LC (M0), England and Wales, 2006-2014)

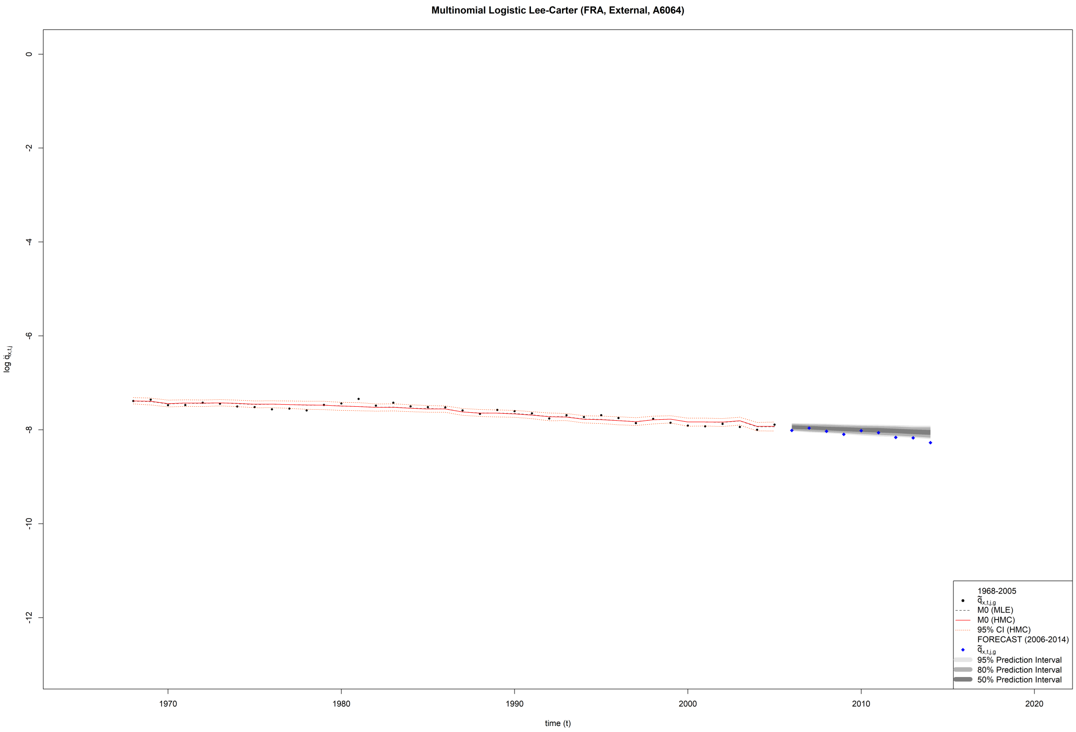


(c) Circulatory Diseases

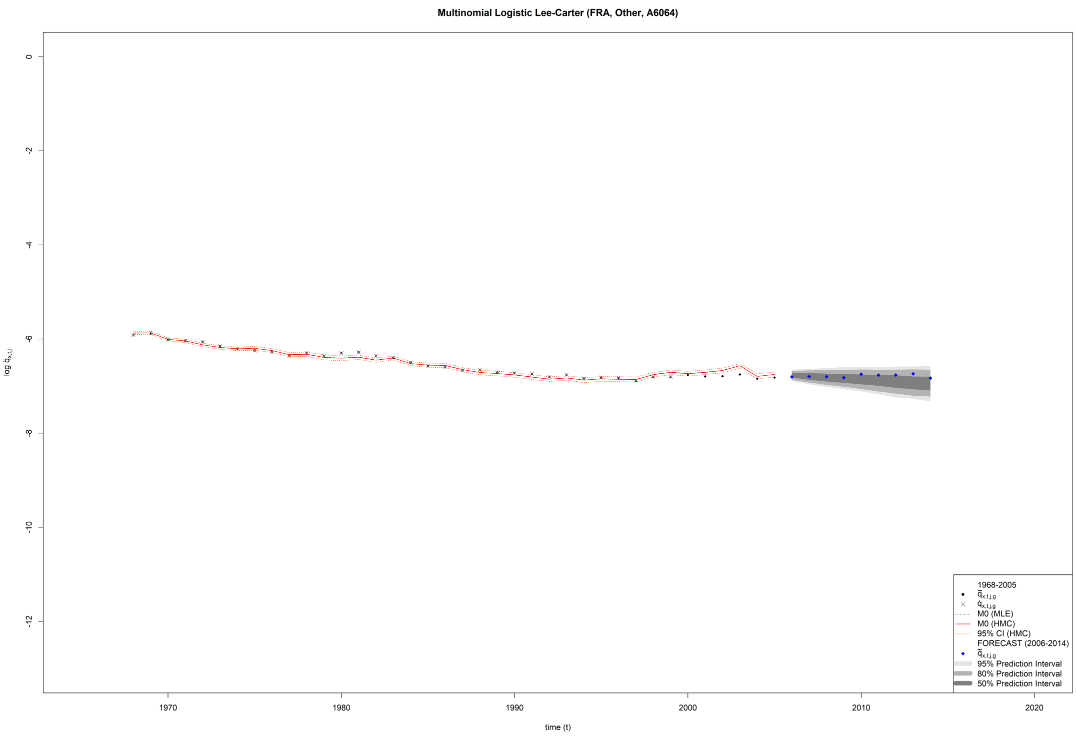


(d) Respiratory Diseases

FIGURE F.7 (CONT.)

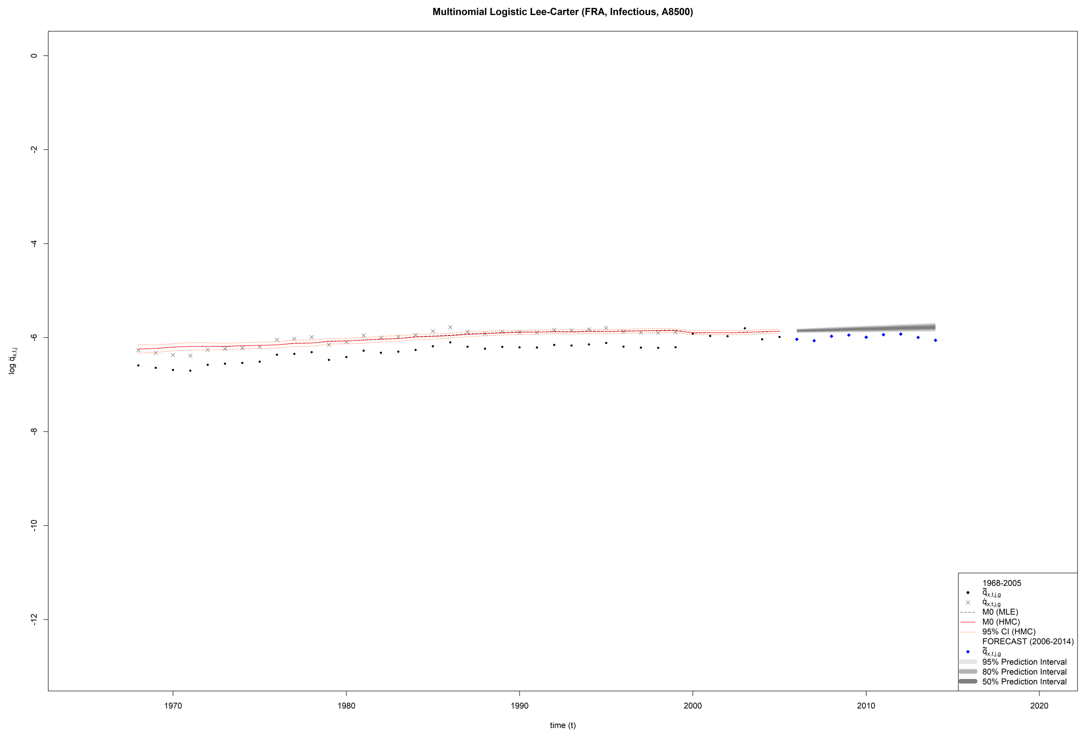


(e) External Causes

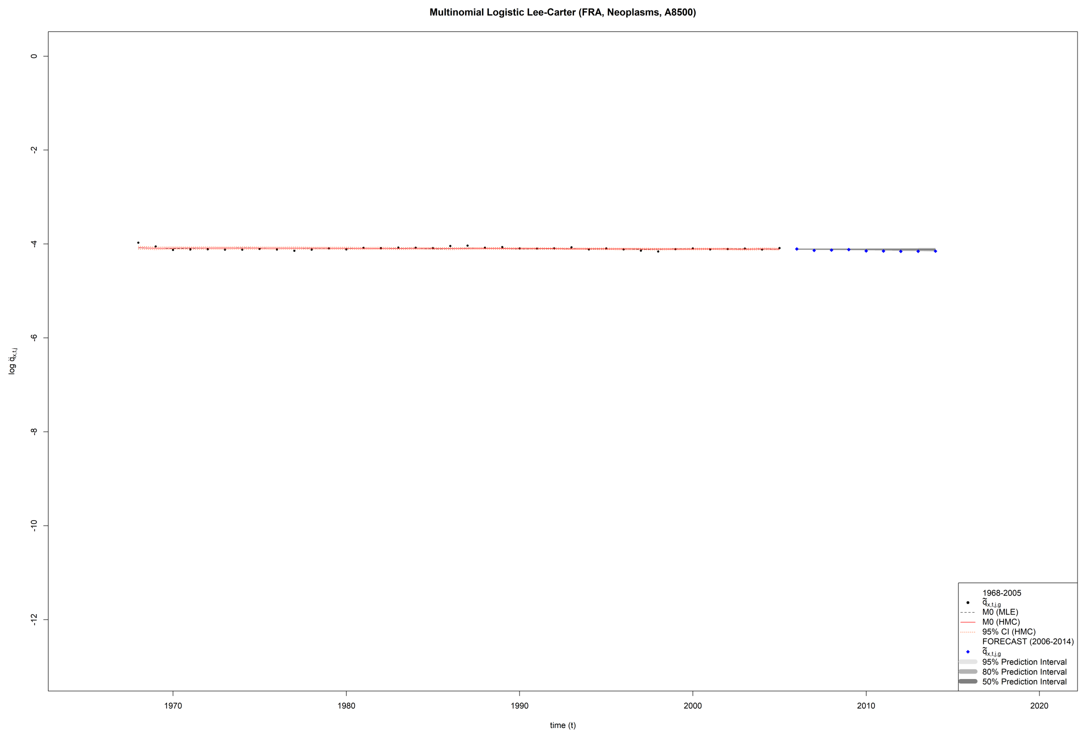


(f) Other Causes

FIGURE F.7 (CONT.)

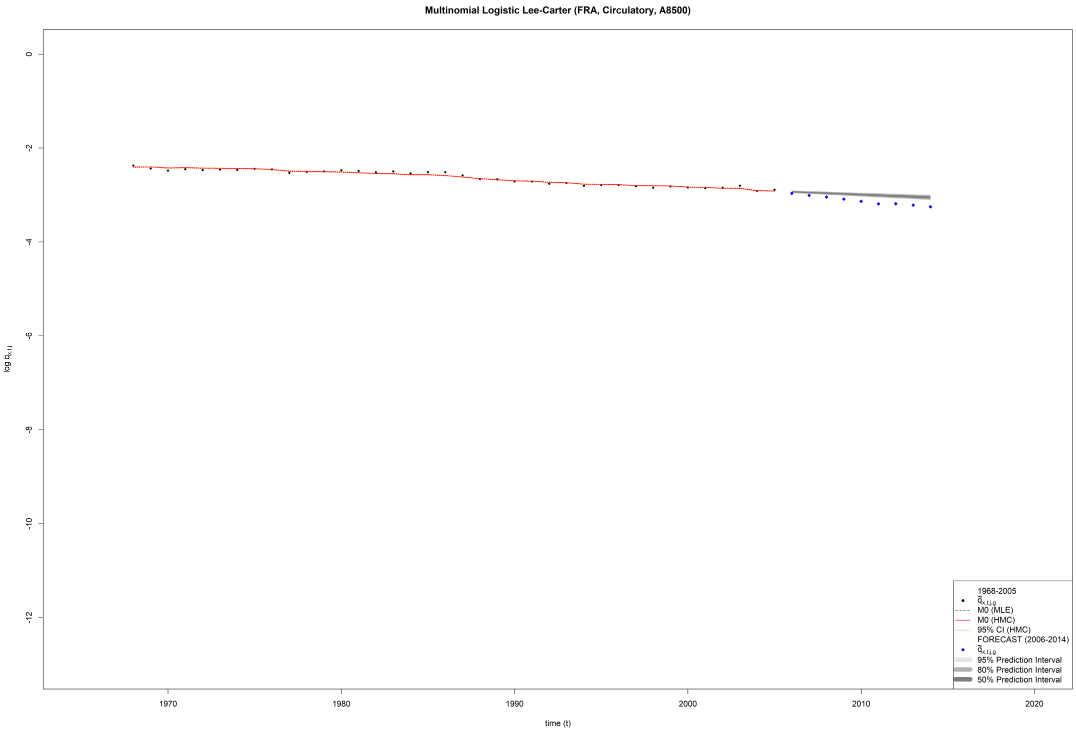


(a) Infectious and Parasitic Diseases

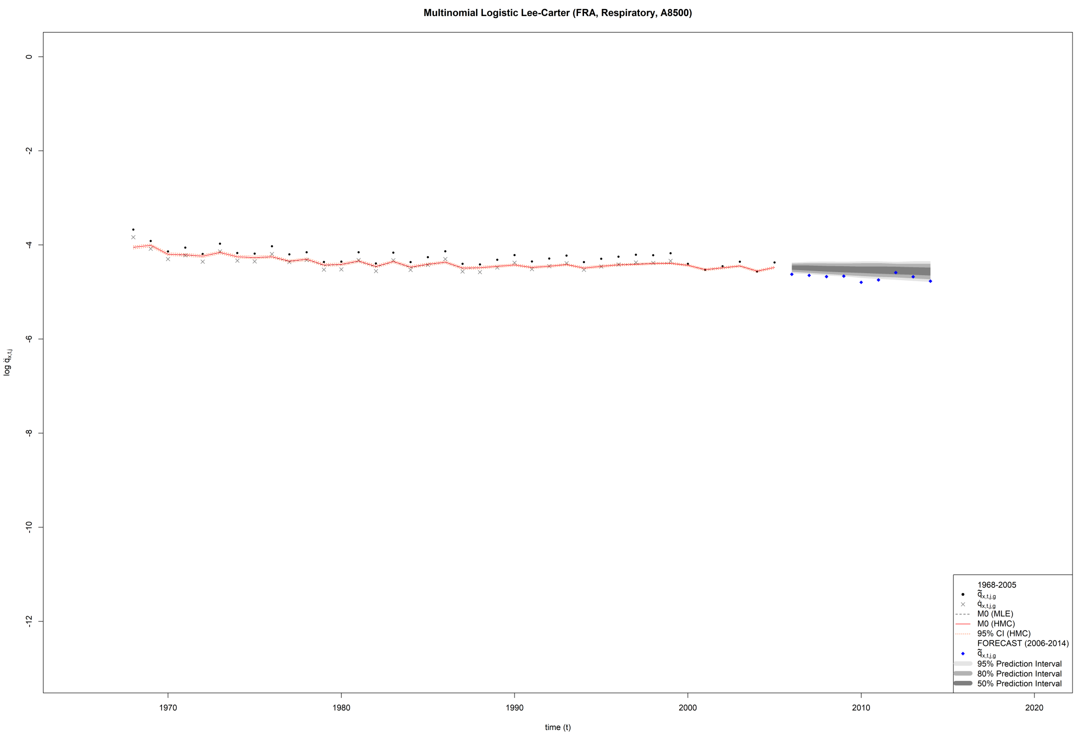


(b) Neoplasms

FIGURE F.8: Probability of Death Forecasts for Age Group 85+ (MLG-LC (M0), England and Wales, 2006-2014)

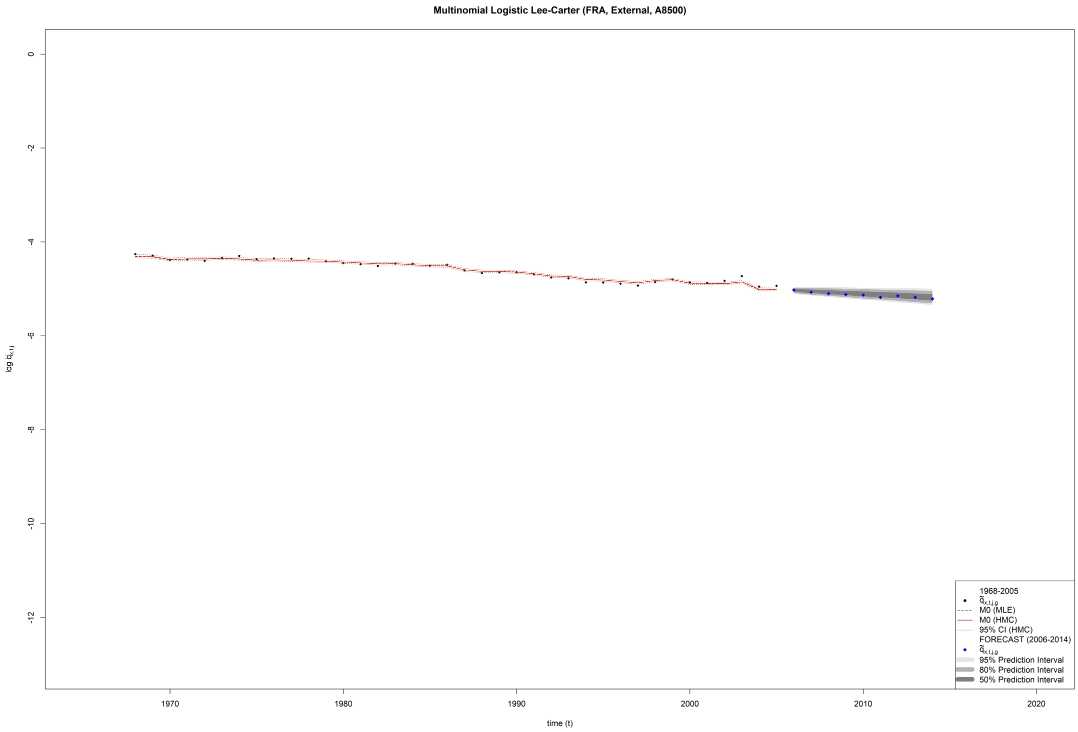


(c) Circulatory Diseases

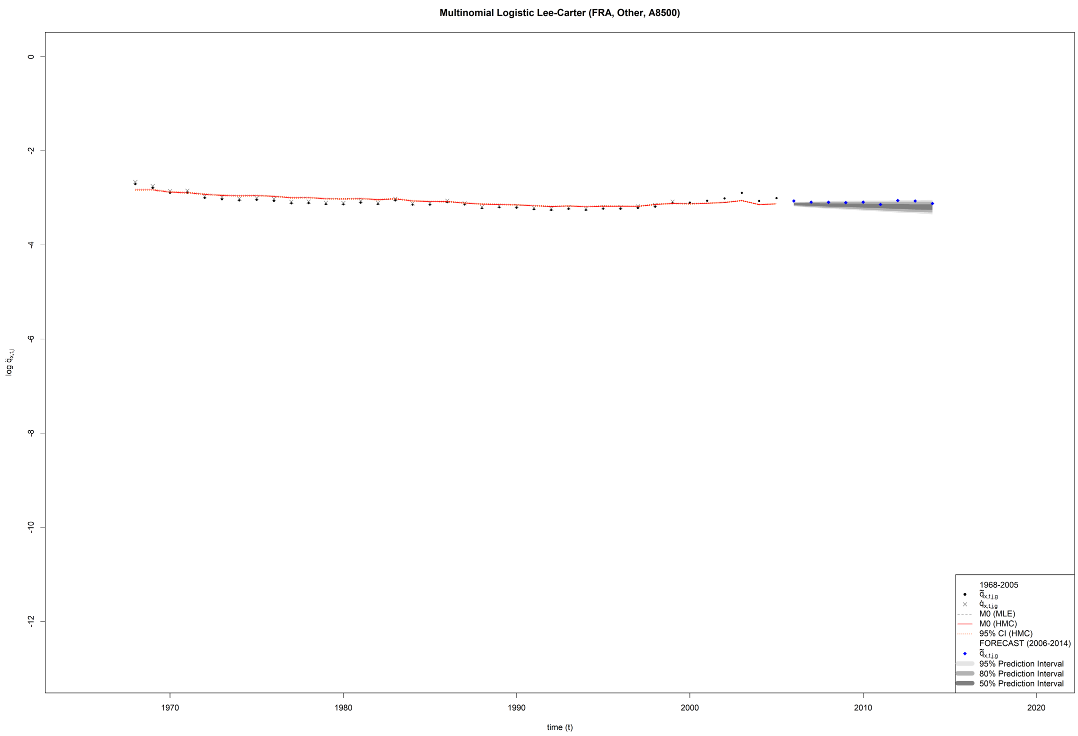


(d) Respiratory Diseases

FIGURE F.8 (CONT.)



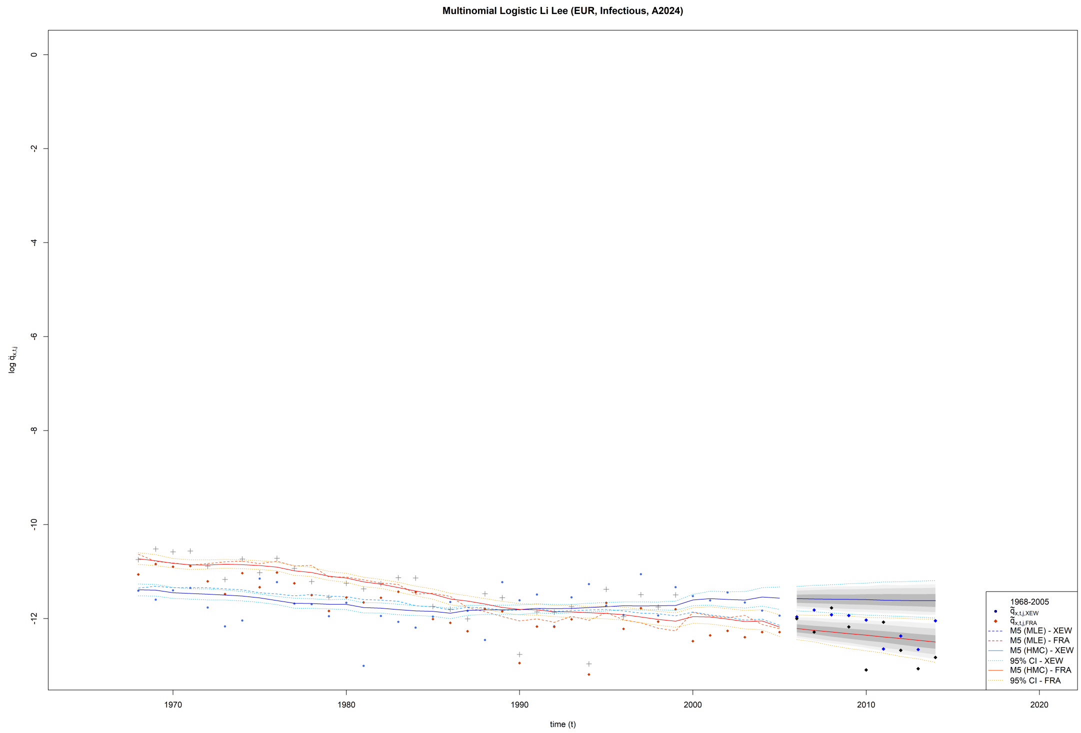
(e) External Causes



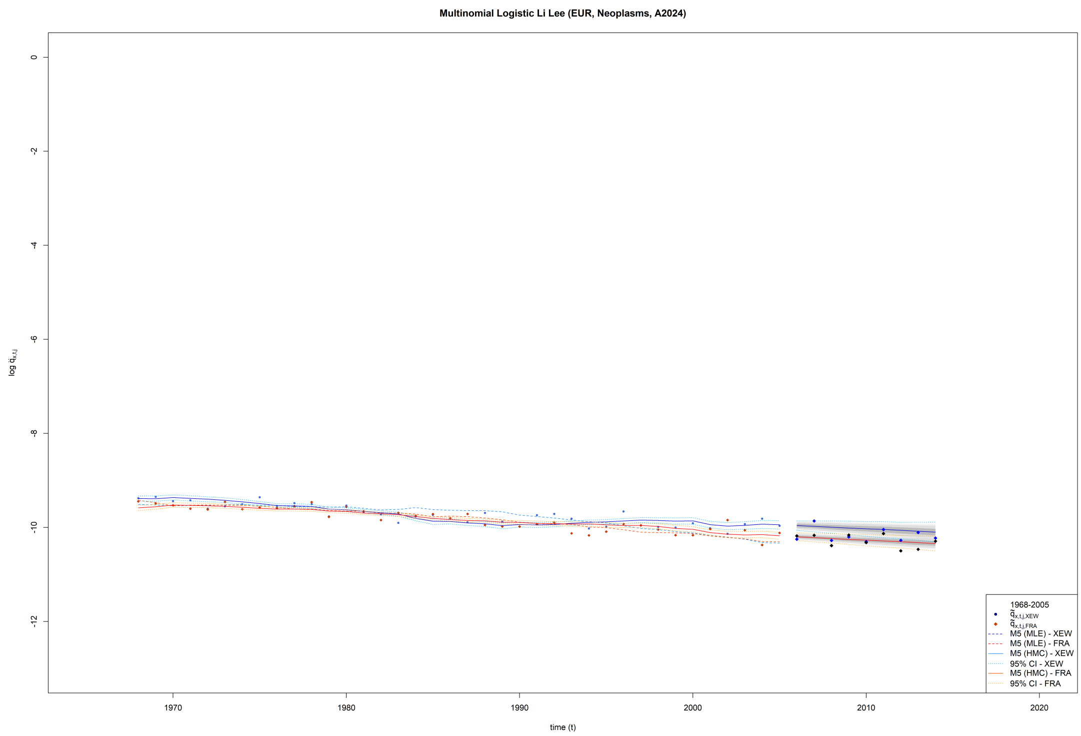
(f) Other Causes

FIGURE F.8 (CONT.)

F.3 Additional Multinomial Logistic Li-Lee Projections for England & Wales and France

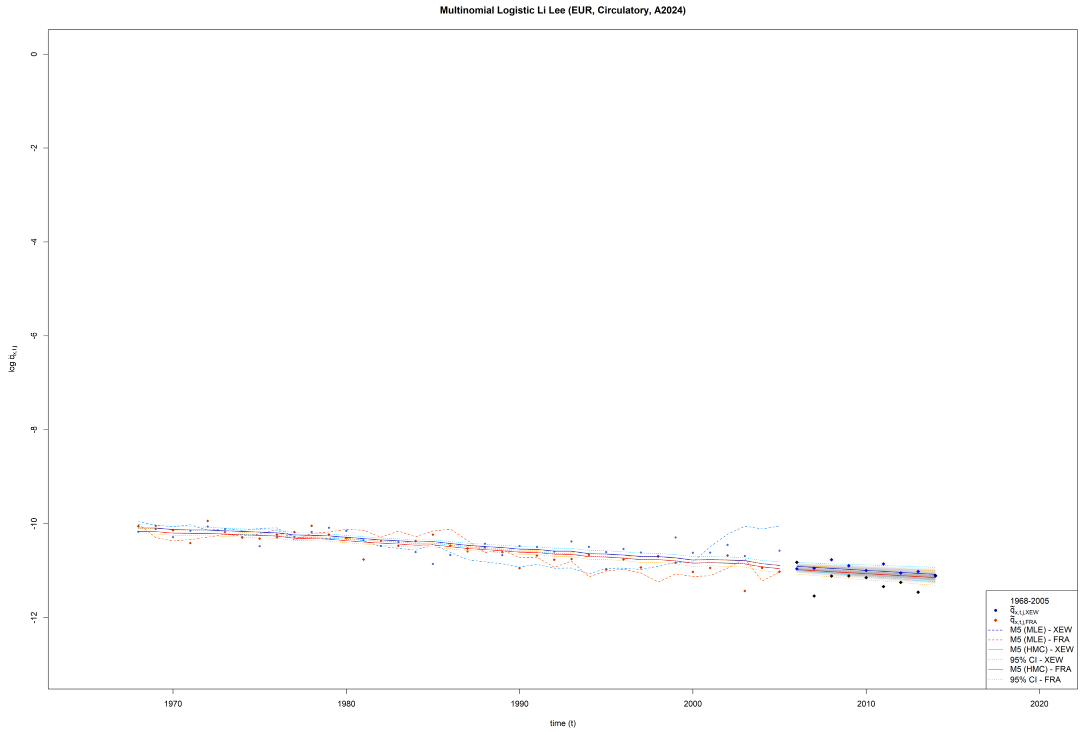


(a) Infectious and Parasitic Diseases

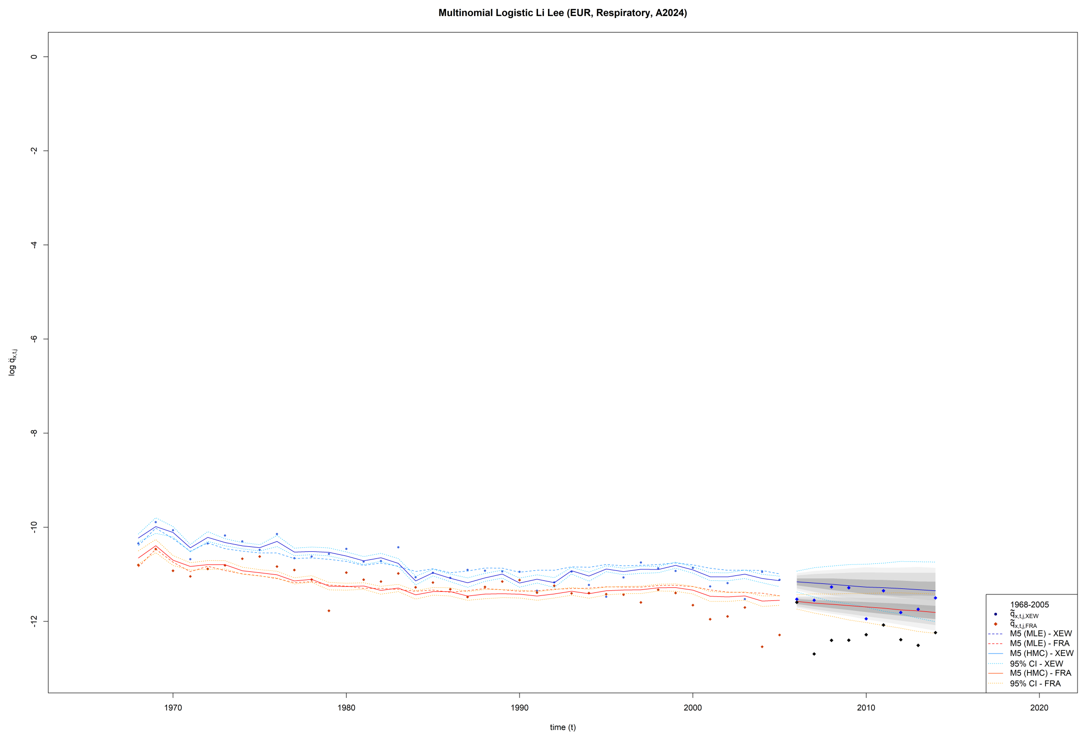


(b) Neoplasms

FIGURE F.9: Probability of Death Forecasts for Age Group 20-24 (MLG-LL (M5), England & Wales + France, 2006-2014)

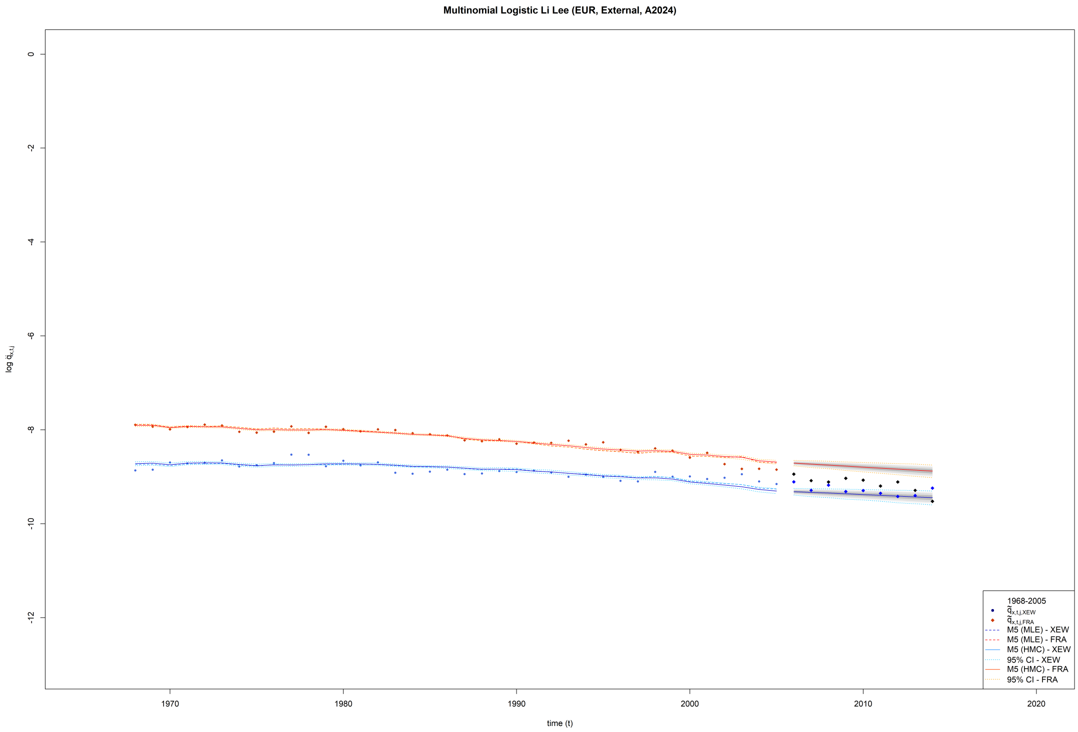


(c) Circulatory Diseases

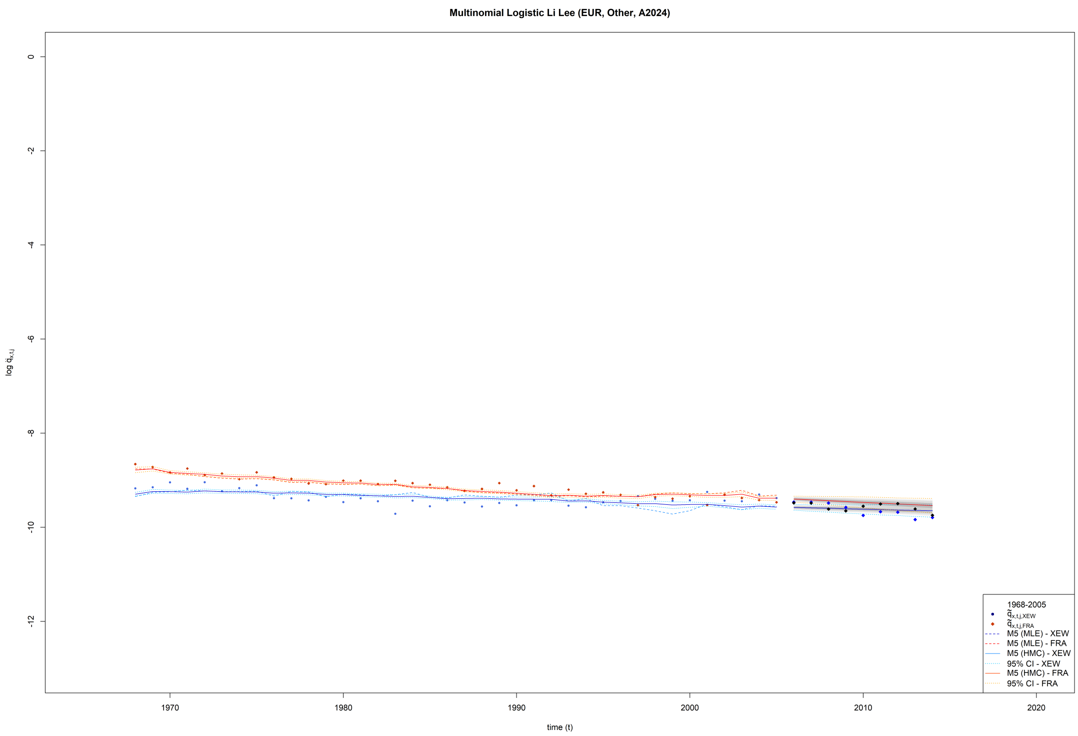


(d) Respiratory Diseases

FIGURE F.9 (CONT.)

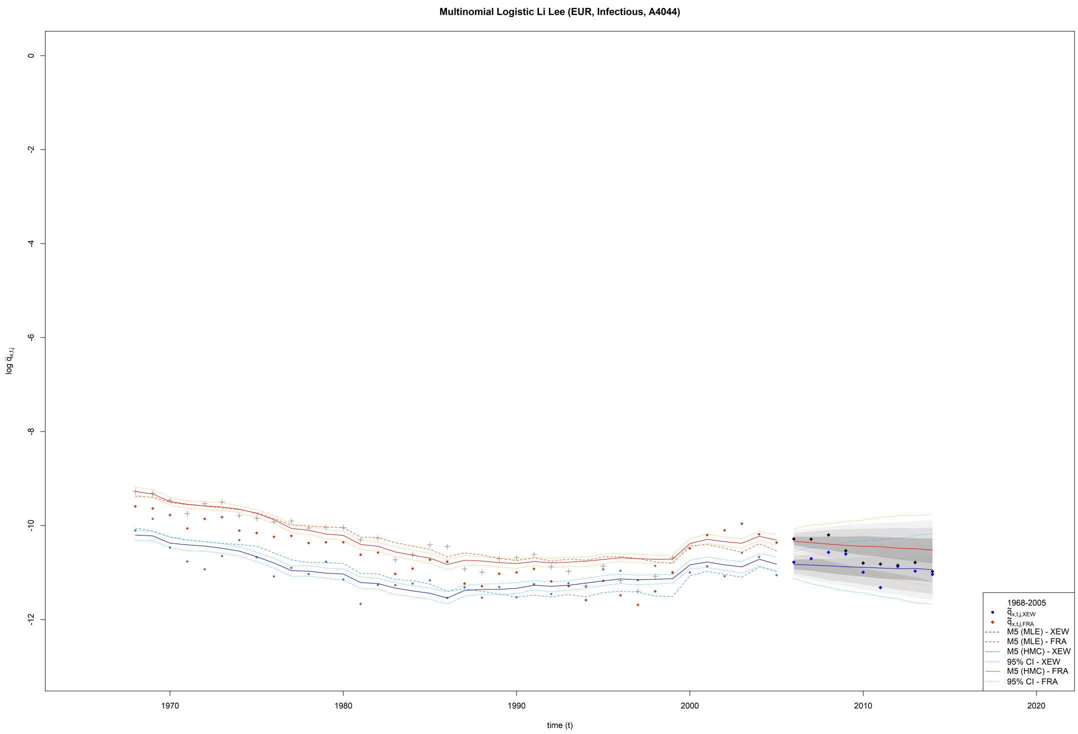


(e) External Causes

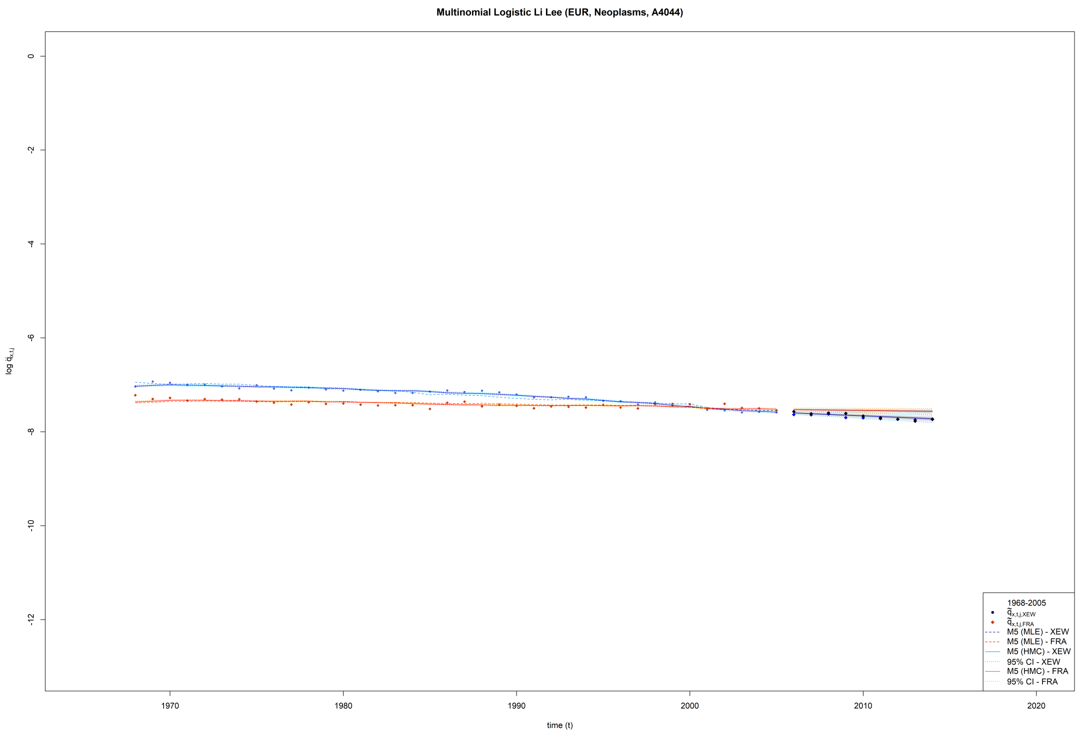


(f) Other Causes

FIGURE F.9 (CONT.)

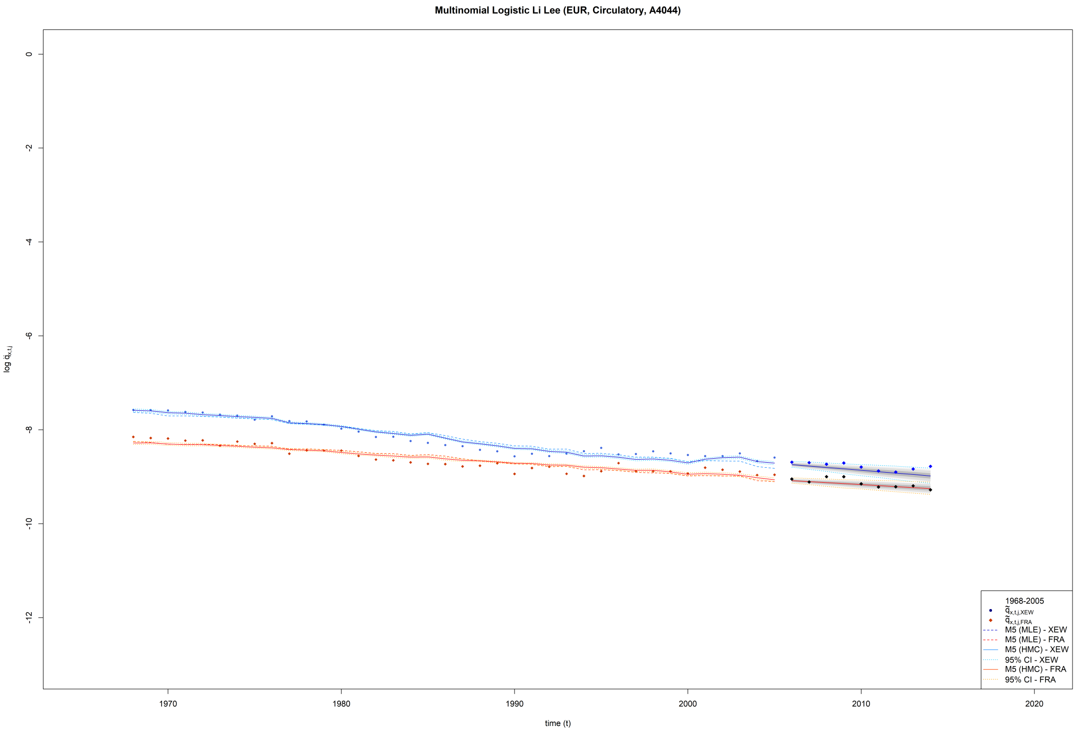


(a) Infectious and Parasitic Diseases

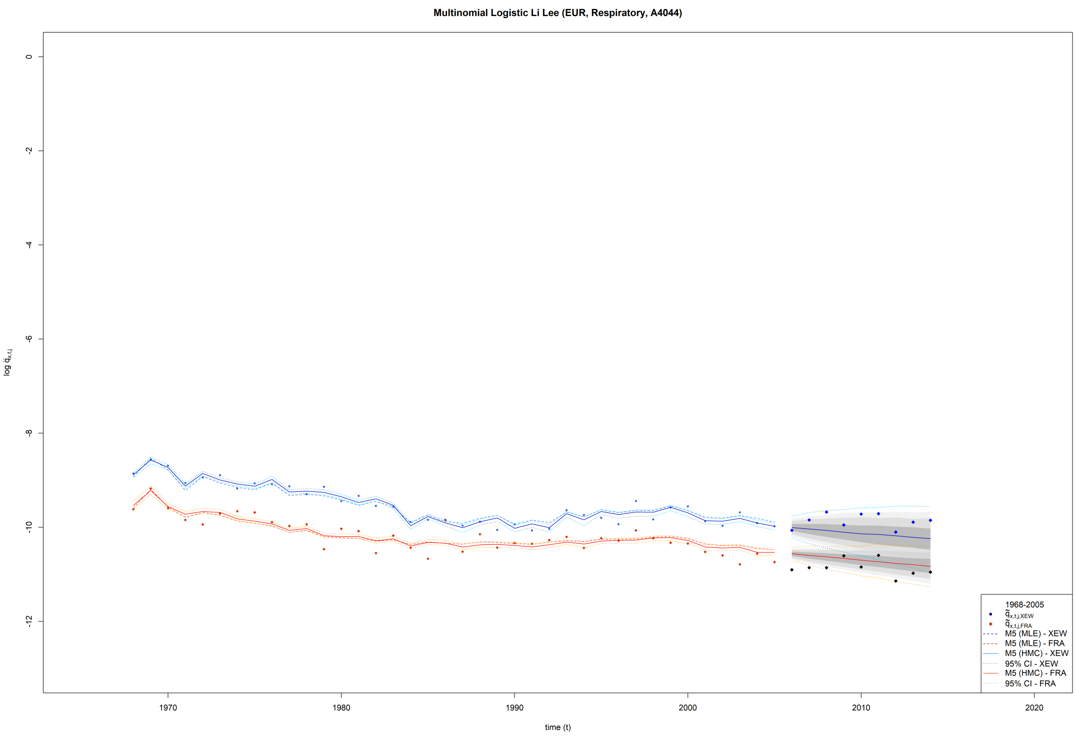


(b) Neoplasms

FIGURE F.10: Probability of Death Forecasts for Age Group 40-44 (MLG-LL (M5), England & Wales + France, 2006-2014)

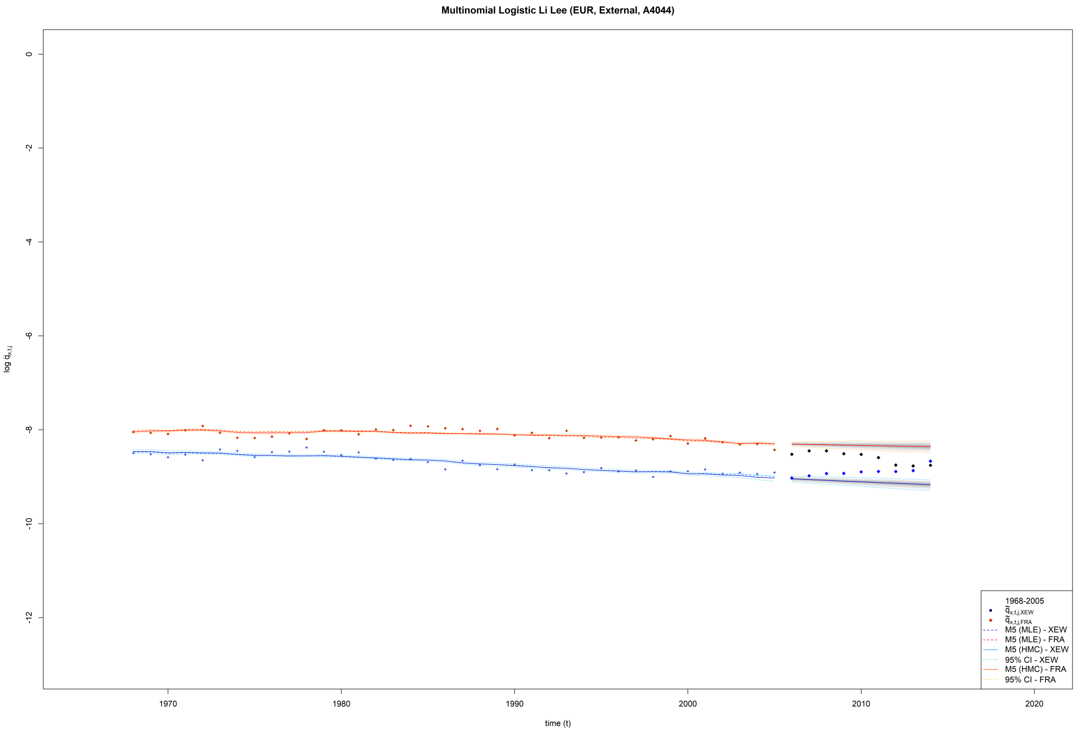


(c) Circulatory Diseases

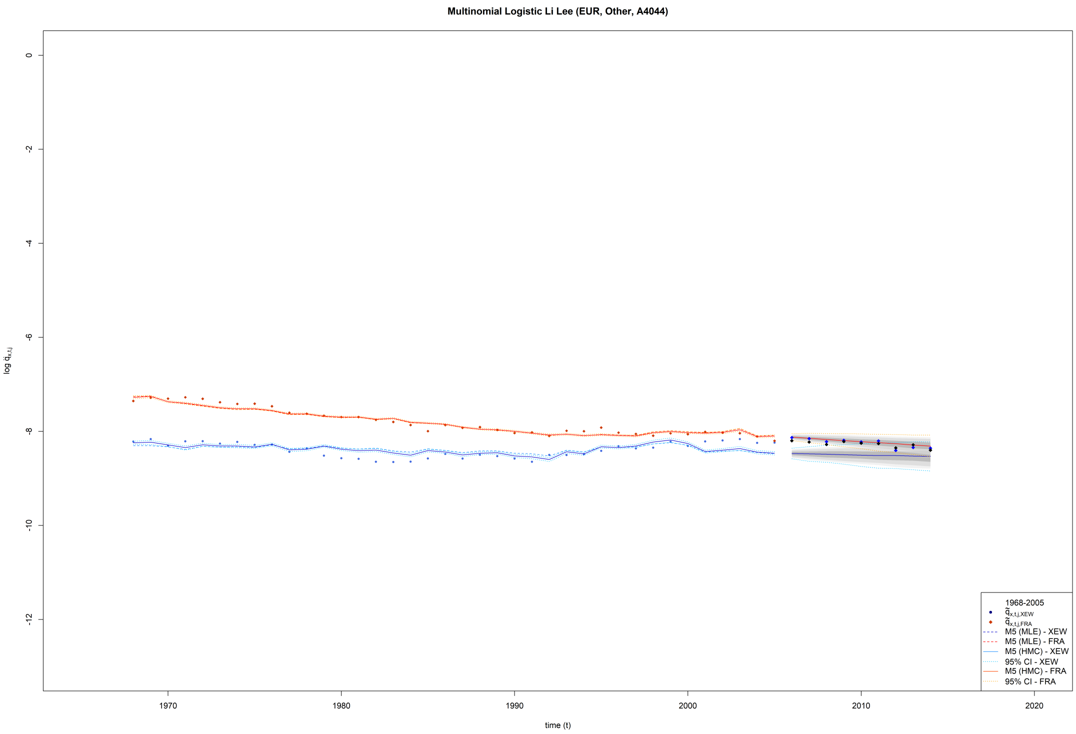


(d) Respiratory Diseases

FIGURE F.10 (CONT.)

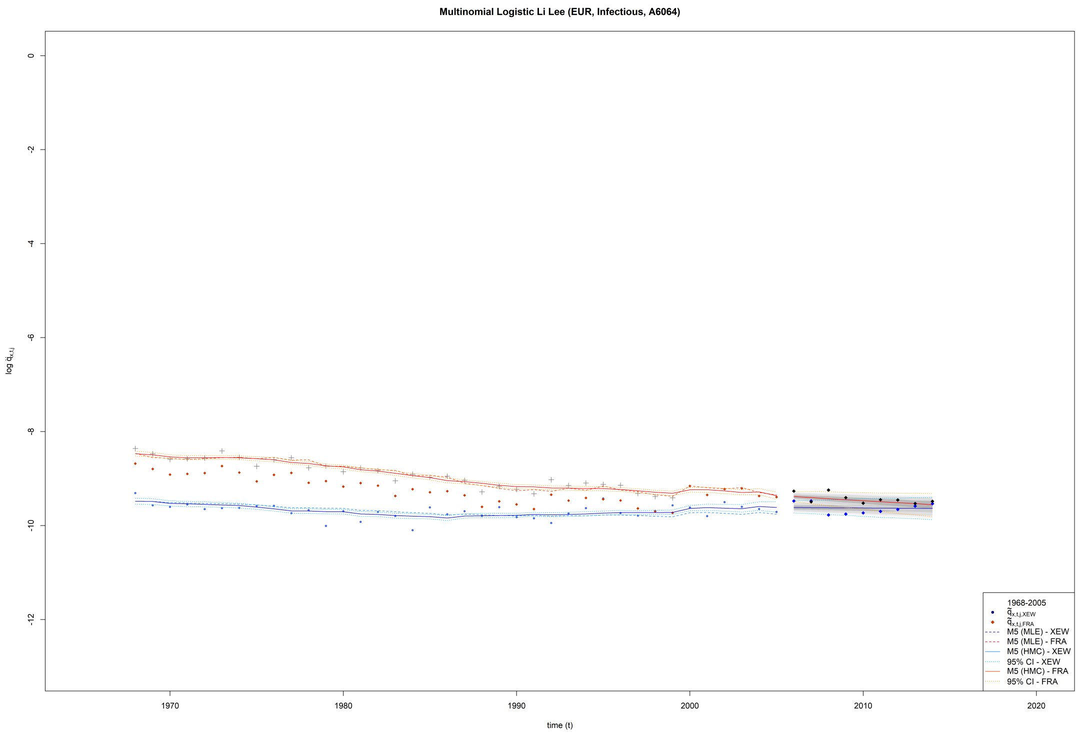


(e) External Causes

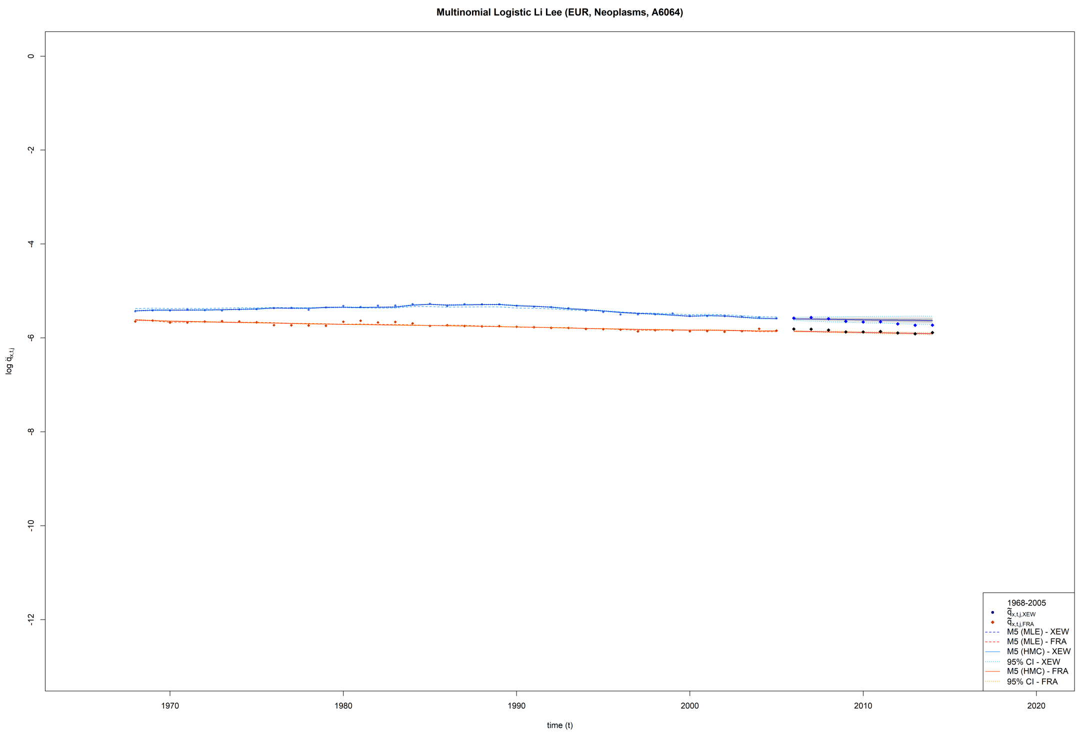


(f) Other Causes

FIGURE F.10 (CONT.)

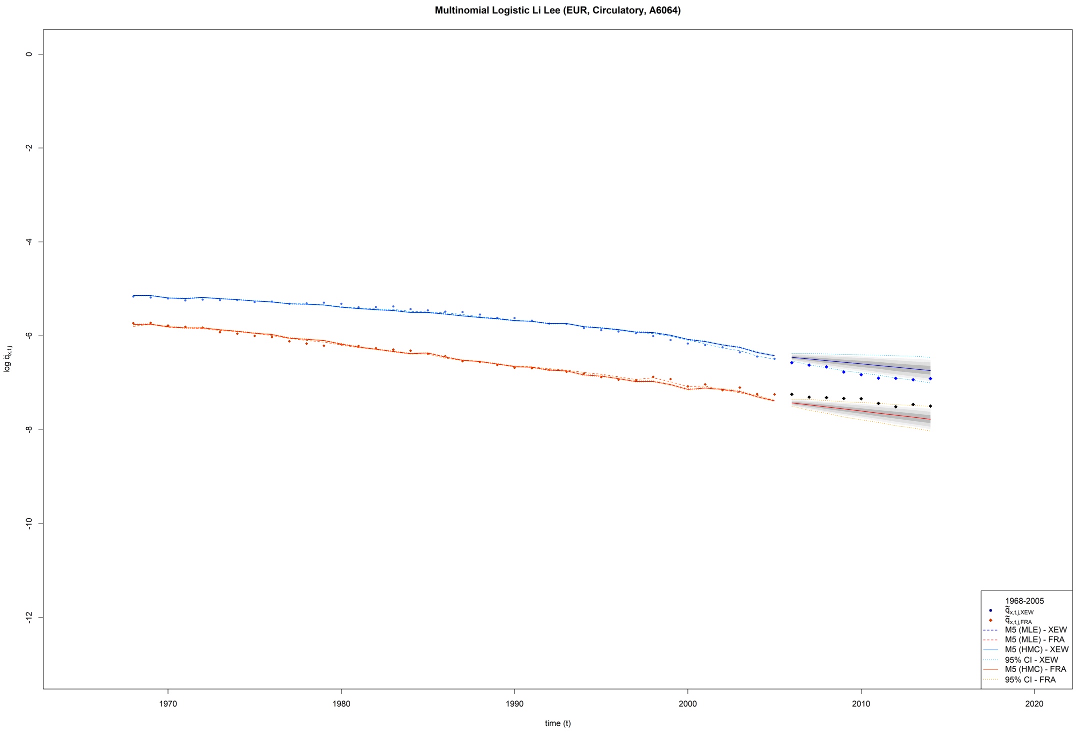


(a) Infectious and Parasitic Diseases

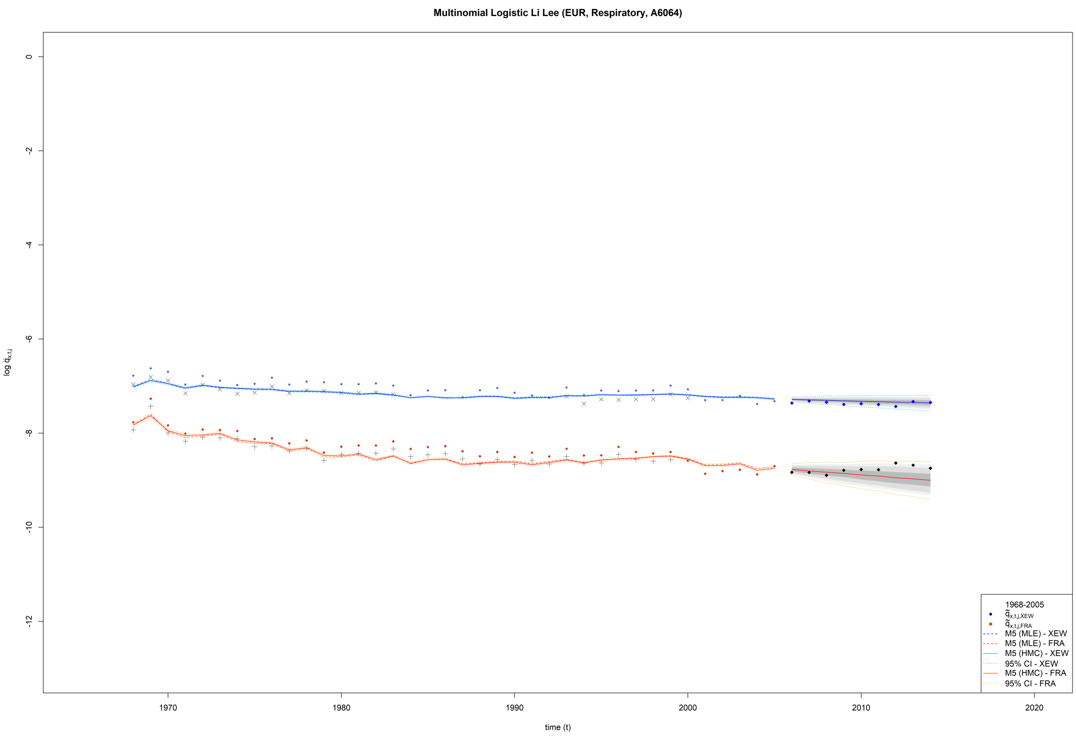


(b) Neoplasms

FIGURE F.11: Probability of Death Forecasts for Age Group 60-64 (MLG-LL (M5), England & Wales + France, 2006-2014)

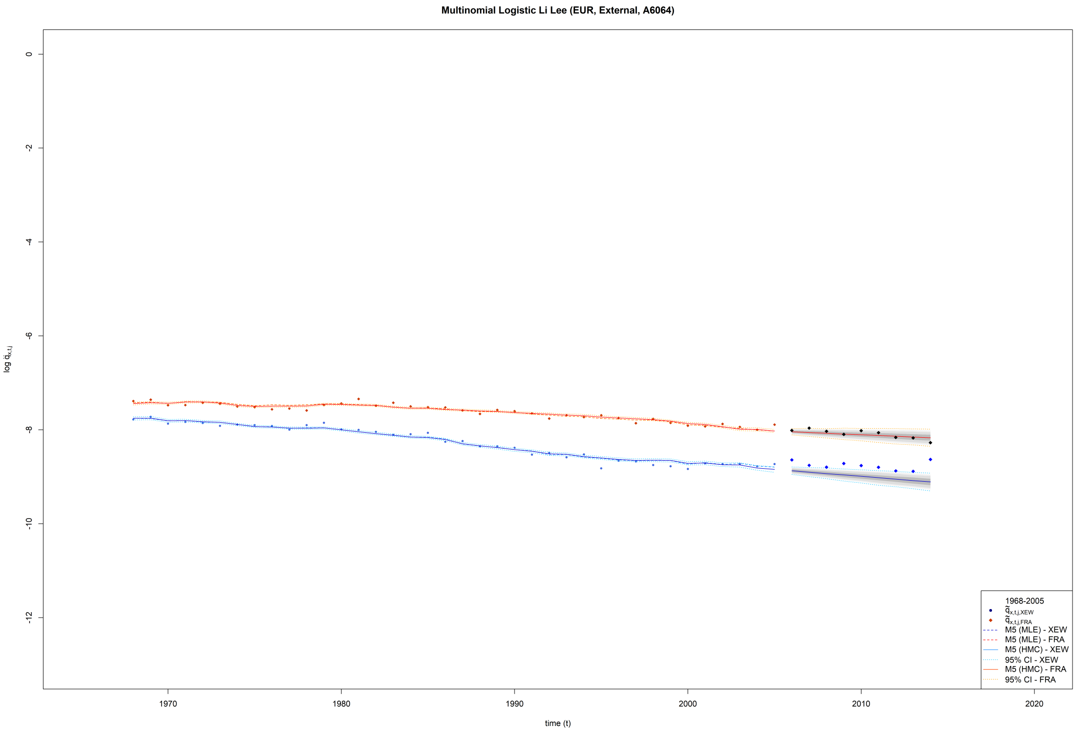


(c) Circulatory Diseases

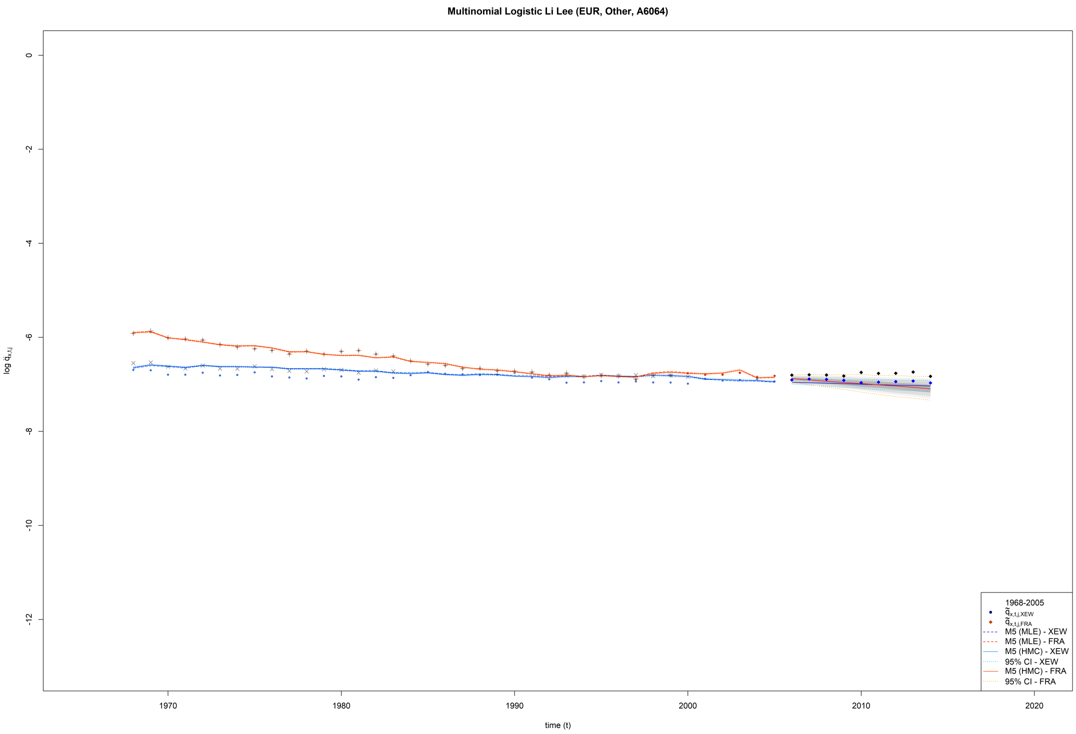


(d) Respiratory Diseases

FIGURE F.11 (CONT.)

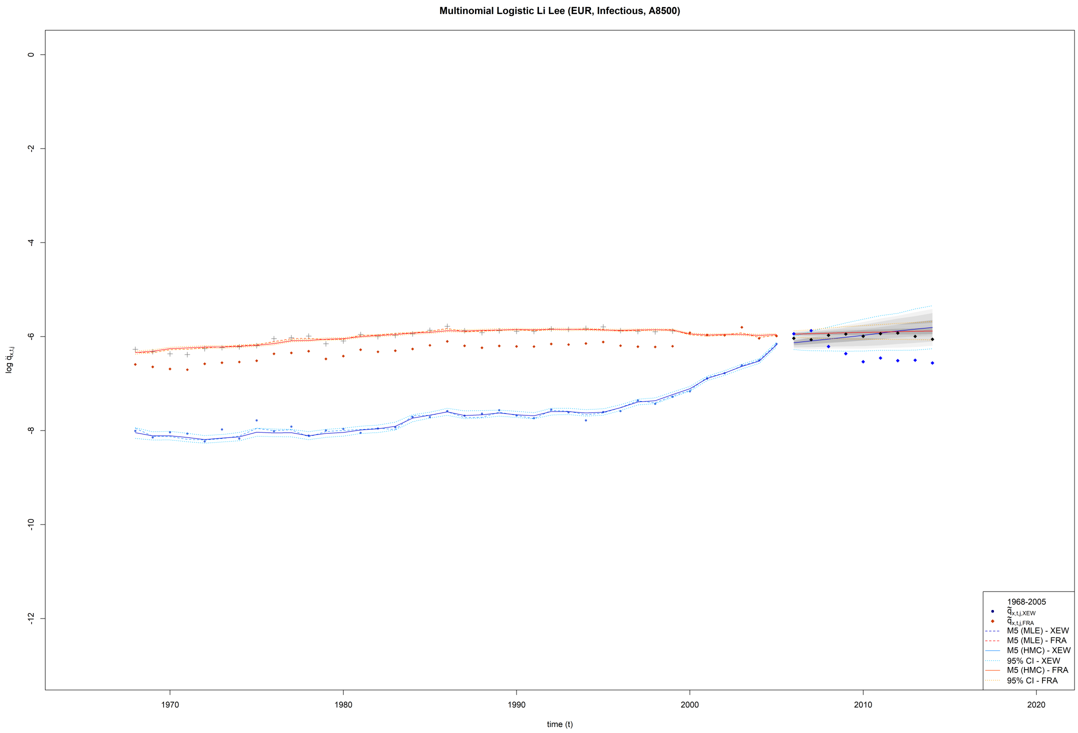


(e) External Causes

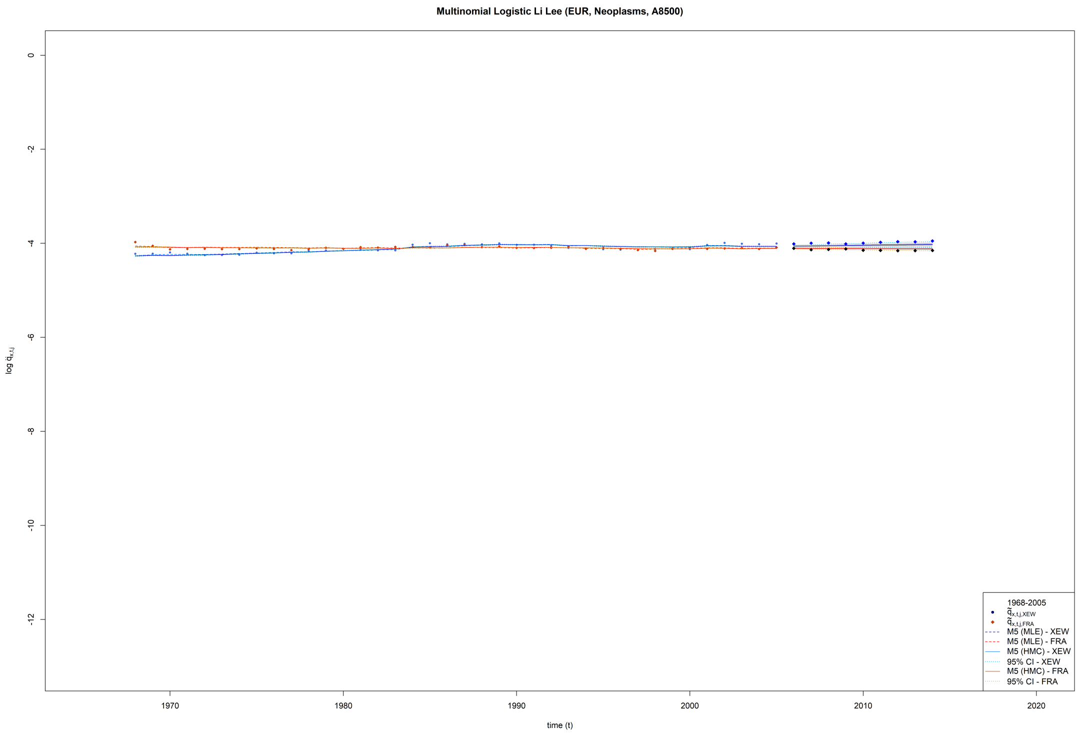


(f) Other Causes

FIGURE F.11 (CONT.)

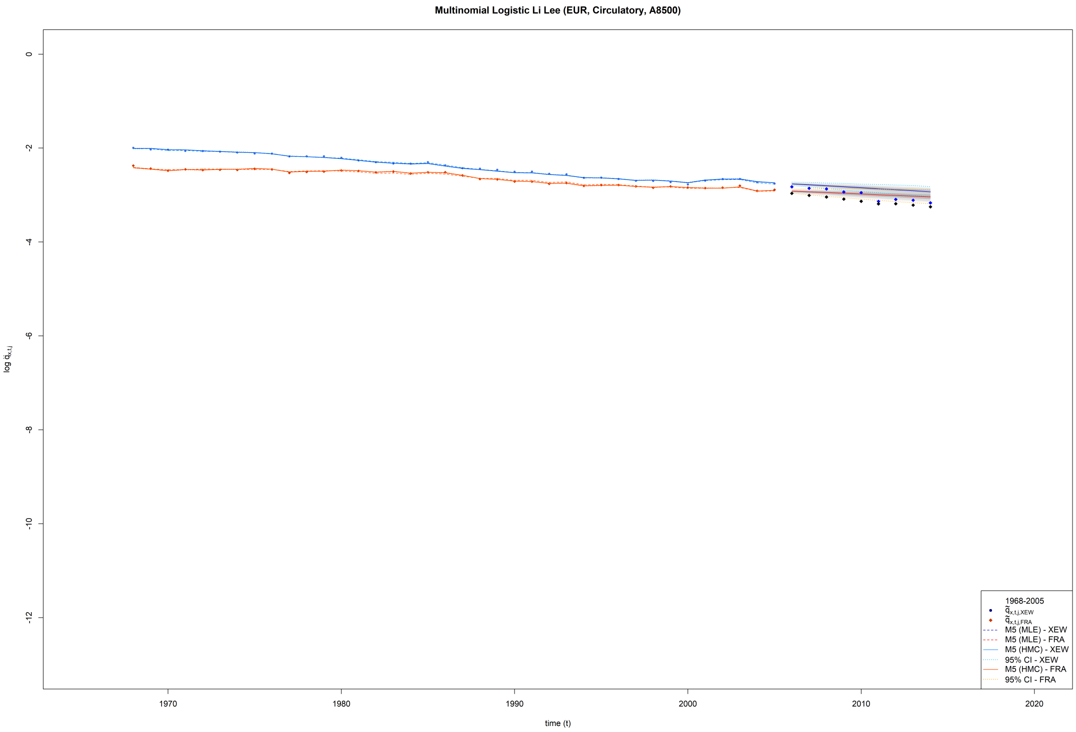


(a) Infectious and Parasitic Diseases

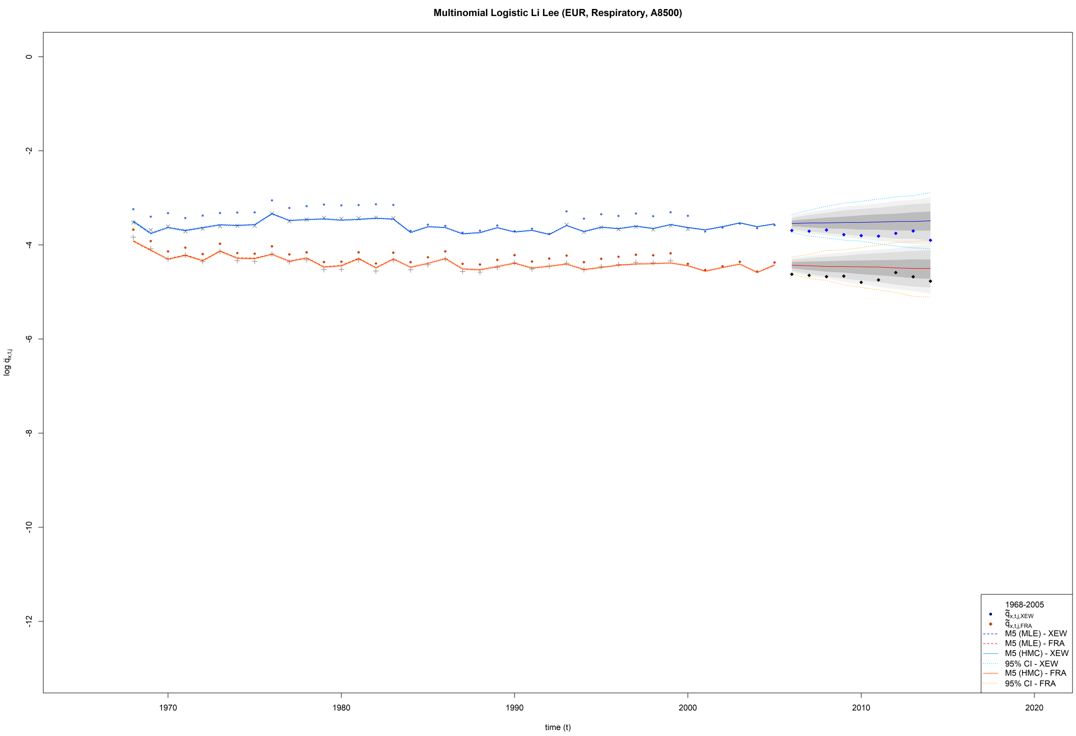


(b) Neoplasms

FIGURE F.12: Probability of Death Forecasts for Age Group 85+ (MLG-LL (M5), England & Wales + France, 2006-2014)

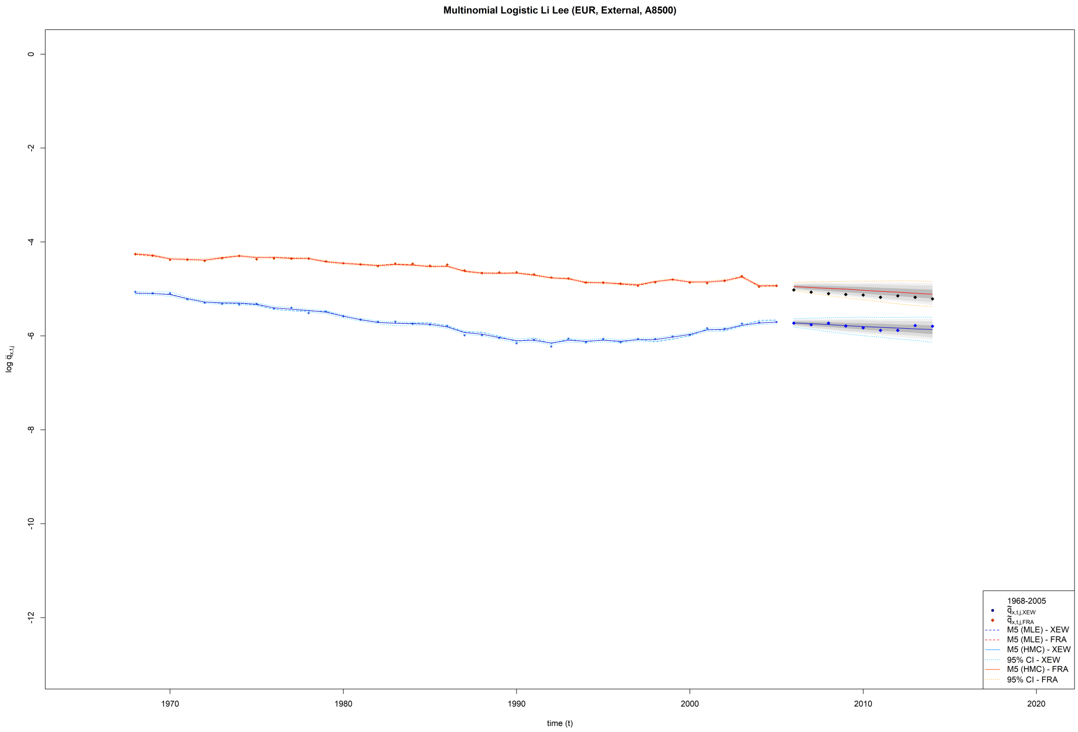


(c) Circulatory Diseases

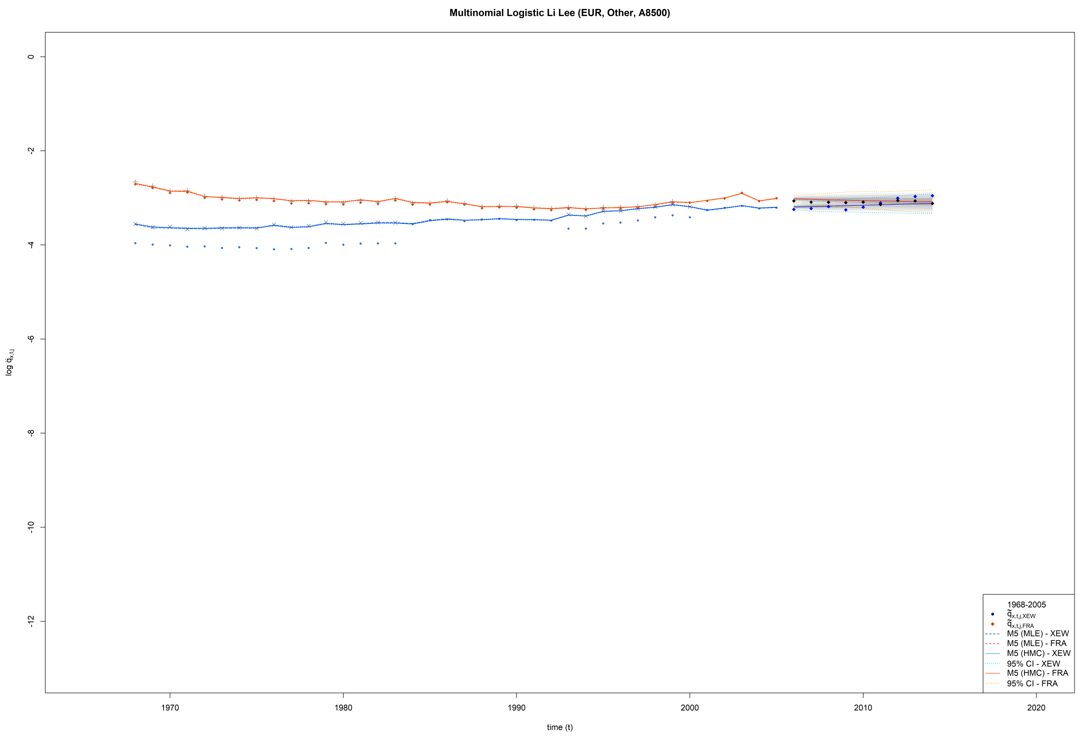


(d) Respiratory Diseases

FIGURE F.12 (CONT.)



(e) External Causes



(f) Other Causes

FIGURE F.12 (CONT.)

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