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SHORT-RUN DEVIATIONS AND TIME-VARYING HEDGE RATIOS: EVIDENCE FROM AGRICULTURAL FUTURES MARKETS¹

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ABSTRACT

This paper investigates the hedging effectiveness of time-varying hedge ratios in the agricultural commodities futures markets based on four different versions of the GARCH models. The GARCH models applied are the standard bivariate GARCH, the bivariate BEKK GARCH, the bivariate GARCH-X and the bivariate BEKK GARCH-X. The GARCH-X and the BEKK GARCH-X models are uniquely different from the other two models because they take into consideration the effect of the short-run deviations from the long-run relationship between the cash and futures prices on the second conditional moments of the bivariate distribution of the variable. For comparison, a constant minimum variance hedge ratio estimated by means of OLS is also applied. Futures data for corn, coffee, wheat, sugar and soybean are applied. Comparison of the hedging effectiveness is done for the within sample period (1980-2004), and two out-of-sample periods (2002-2004 and 2003-2004) performance. Results indicate superior performance of the portfolios based on the GARCH-X model estimated hedge ratio during most periods.

JEL Classification: G1, G13, G15

Key Words: Hedge Ratio, GARCH, BEKK GARCH, GARCH-X, BEKK GARCH-X and Minimum Variance

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1. Introduction

The rapid expansion of derivatives markets over the last twenty-five years has led to a corresponding increase in interest in the theory and practice of hedging. Numerous empirical and statistical methods are applied to estimate hedge ratios in the futures markets. The traditional constant hedge ratio obtained by means of the ordinary least square (OLS) has been discarded as being inappropriate, because it ignores the heteroskedasticity often encountered in price series. Baillie and Myers (1991) further claim that if the joint distribution of cash price and futures prices is changing over time, estimating a constant hedge ratio may not be appropriate. Recently, autoregressive conditional heteroskedastic (ARCH) and the generalized ARCH (GARCH) have been applied to estimate time-varying hedge ratios in the futures markets (see Choudhry, 2004; Moschini and Myers, 2002; Gagnon et al., 1998; Baillie and Myers, 1991; Myers, 1991; Kroner and Sultan, 1993; Gagnon and Lypny, 1995; Park and Switzer, 1995; and Tong, 1995). The optimal hedge ratios estimated by means of the GARCH models is time-varying, because these models take into consideration the time-varying distribution of the cash and futures price changes.

This paper investigates and compares the risk-reducing ability of different optimal time-varying hedge ratios (and constant hedge ratio) for the futures of five agricultural commodities: corn, coffee, wheat, sugar and soybean. An optimal hedge ratio is defined as the proportion of a cash position that should be covered with an opposite position on a futures market. When using a futures contract in order to hedge a portfolio of risky assets, the primary objective is to estimate the size of the short position that must be held in the futures market, as a proportion of the long position held in the spot market, that maximises the agent's expected utility, defined over the risk and expected return of the hedged portfolio.

In this paper, the (time-varying) optimal hedge ratios are estimated using four different types of the generalized autoregressive conditional heteroscedasticity (GARCH) models: the standard bivariate GARCH, bivariate BEKK GARCH, the bivariate GARCH-X, and the bivariate BEKK GARCH-X. The GARCH-X and the BEKK GARCH-X models are different from the other two GARCH models because they take into

consideration the effects of the short-run deviations from the long-run equilibrium relationship between the cash and futures prices on the conditional variance and covariance (second conditional moments of the bivariate distribution) of log difference of the cash and the futures prices. The short-run deviations are represented by the error correction term from a cointegration relationship between the cash and the futures prices.² The BEKK GARCH and the BEKK GARCH-X models are also unique because they allow time variation in the conditional correlations as well as the conditional variance. All GARCH methods applied take into consideration the effects of the short-run deviations on the first moment (mean) of the bivariate distributions of the variables. The long-run relationship between the commodities cash price and the futures price is determined by means of the Engle and Granger (1987) cointegration test. Long-run stationary relationship (cointegration) between the cash price and the futures price has been extensively investigated.³ Yang et al. (2001) further claim that prevalent cointegration between cash and futures prices on commodity markets suggest that cointegration should be incorporated into commodity hedging decisions.⁴ Cointegration brings added information about long-run and short-run correlation between cash and futures prices. Even when the GARCH effect is considered, allowance for the existence of cointegration is argued to be an indispensable component when comparing ex-post performance of various hedging strategies. By using cointegration, investors may obtain added information in forming and progressively re-adjusting hedges. This readjustment may help in maintaining or improving the hedging effectiveness since new information impacts on cash and future prices.

For comparison purpose, this paper also estimates and investigates the hedging effectiveness of the constant minimum variance hedge ratio.⁵ The minimum variance hedge ratio is estimated as the slope coefficient of the following regression:

² Cointegration implies that in a long-run relationship between two or more non-stationary variables, it is required that these variables should not move too far apart from each other. Such non-stationary variables might drift apart in the short run, but in the long run they are constrained. Brenner and Kroner (1995) present a model and conditions under which cash and futures prices may be cointegrated. Yang et al. (2001) present a model and conditions under which cash and future prices of storable commodities may be cointegrated.

³ See Kroner and Sultan (1993), Brenner and Kroner (1995) and Yang et al. (2001) for citation of papers investigating cointegration between cash and futures prices. Baillie and Myers (1991), Covey and Bessler (1992), Fortenbery and Zapata (1993, 1997) provide a study of cointegration between commodities spot and future prices.

⁴ Ghosh (1995), Ghosh and Clayton (1996) and Kroner and Sultan (1993) have shown that hedge ratios and hedging performance may change considerably if cointegration between the cash and futures prices is omitted from the statistical models and estimations.

$$r_t^c = \alpha + \beta r_t^f + \varepsilon_t \quad (1)$$

Where r_t^c and r_t^f are defined as log difference of cash (r_t^c) price, and log difference of futures (r_t^f) prices, β is the constant hedge ratio and ε_t is an error term.

The risk-reducing effectiveness of the time-varying hedge ratios is investigated by checking performance of the ratios in the within sample period (1980-2004) and two out-of-sample periods (2002-2004 and 2003-2004). The hedging effectiveness is estimated and compared by checking the variance of the portfolios created using these hedge ratios. The lower the variance of the portfolio, the higher is the hedging effectiveness of the hedge ratio.

2. Optimal Hedge Ratios

The following section describes the optimal hedge ratio, relying heavily on Cecchetti et al. (1988) and Baillie and Myers (1991). The returns on the portfolio of an investor trying to hedge some proportion of the cash position in a futures market can be represented by:

$$r_t = r_t^c - \beta_{t-1} r_t^f \quad (2)$$

Where r_t is the return on holding the portfolio of cash and futures positions between $t-1$ and t ; r_t^c is the return on holding the cash position for the same period; r_t^f is the return on holding the futures position for the same period; and β_{t-1} is the hedge ratio. The variance of the return on the hedged portfolio is given by

$$\text{Var}(r_t/\Omega_{t-1}) = \text{Var}(r_t^c/\Omega_{t-1}) + \beta_{t-1}^2 \text{Var}(r_t^f/\Omega_{t-1}) - 2\beta_{t-1} \text{Cov}(r_t^c, r_t^f/\Omega_{t-1}) \quad (3)$$

Where Ω_{t-1} presents the information available over the last period. As indicated by Cecchetti et al. (1988), the

⁵ This is done based on the suggestion of the referee.

return on a hedged position will normally be exposed to risk caused by unanticipated changes in the relative price between the position being hedged and the futures contract. This ‘basis risk’ ensures that no hedge ratio completely eliminates risk. The hedge ratio that minimises risk may be obtained by setting the derivative of equation 2 with respect to β equal to zero. The hedge ratio β_{t-1} can then be expressed as:

$$\beta_{t-1} = \text{Cov}(r_t^c, r_t^f / \Omega_{t-1}) / \text{Var}(r_t^f / \Omega_{t-1}). \quad (4)$$

The value of β_{t-1} , which minimises the conditional variance of the hedged portfolio return, is the optimal hedge ratio (Baillie and Myers, 1991). Commonly, the value of the hedge ratio is less than unity, so that the hedge ratio that minimises risk in the absence of basis risk turns out to be dominated by β when basis risk is taken into consideration.⁶

Time-varying optimal hedge ratio can also be based on utility maximization. Based on Myers (1991), under this scenario an individual investor wants to determine the optimal allocation of initial wealth between two investment opportunities: purchase of a risky asset, and purchase of a risk-free asset. There is a futures market in the risky asset and the investor can therefore hedge by selling contracts which mature at or after the period. Using the von Neumann-Morgenstern utility function and a time-varying conditional covariance, Myers (1991) is able to show that optimal hedge ratio is equal to the one presented by equation 4. Myers (1991) defines the optimal hedge ratio as the proportion of the long cash position which should be covered by futures selling. In this model, it is assumed that optimal hedge ratio is preference-free but the demand for the asset depends upon investor risk preferences, as well as on the probability distribution of asset price. Thus, hedge ratio represented by equation 4 can be based on risk minimization or utility maximization.

3. Bivariate GARCH, BEKK GARCH, GARCH-X and BEKK GARCH-X Models

3.1 Bivariate GARCH

⁶ According to Cecchetti et al. (1988), the optimal hedge ratio β can be expressed as $\rho\sigma^c/\sigma^f$, where ρ is the correlation between futures price and cash price, σ^c is the cash standard deviation, and σ^f is the futures standard deviation. Thus, if the futures have the same or higher price volatility than the cash, the hedge ratio can be no greater than the correlation between them, which will be less than unity.

As shown by Baillie and Myers (1991) and Bollerslev et al. (1992), weak dependence of successive asset price changes may be modelled by means of the GARCH model. The multivariate GARCH model uses information from more than one market's history. According to Engle and Kroner (1995), multivariate GARCH models are useful in multivariate finance and economic models, which require the modelling of both variance and covariance. Multivariate GARCH models allow the variance and covariance to depend on the information set in a vector ARMA manner (Engle and Kroner, 1995). This, in turn, leads to the unbiased and more precise estimate of the parameters (Wahab, 1995).

The following bivariate GARCH(p,q) model may be used to represent the log difference of the cash (spot) and futures prices:

$$y_t = \mu + \delta(z_{t-1}) + \varepsilon_t \quad (5)$$

$$\varepsilon_t / \Omega_{t-1} \sim N(0, H_t) \quad (6)$$

$$\text{vech}(H_t) = C + \sum_{j=1}^p A_j \text{vech}(\varepsilon_{t-j})^2 + \sum_{j=1}^q B_j \text{vech}(H_{t-j}) \quad (7)$$

where $y_t = (r_t^c, r_t^f)$ is a (2x1) vector containing the log difference of the cash (r_t^c) price and futures (r_t^f) prices; H_t is a (2x2) conditional covariance matrix; C is a (3x1) parameter vector (constant); A_j and B_j are (3x3) parameter matrices; and vech is the column stacking operator that stacks the lower triangular portion of a symmetric matrix.

The error correction term (z_t) from the cointegration represents the short-run deviations from a long-run relationship between the cash price and the futures price.⁷ A significant and positive coefficient (δ) on the error term implies that an increase in short-run deviations raises the log difference of cash and/or future prices. The

⁷ The following cointegration relationship is investigated by means of the Engle and Granger (1987) method:

$$C_t = \eta + \gamma F_t + z_t$$

Where C_t and F_t are log of cash index and futures price index, respectively. The residuals z_t are tested for unit root(s) to check for cointegration between C_t and F_t . The error correction term, which represents the short-run deviations from the long-run cointegrated relationship, has important predictive powers for the conditional mean of the cointegrated series (Engle and Yoo, 1987). Cointegration is found between the log of cash and futures prices for all five commodities. These results are available on request.

opposite is true if the error term coefficient is negative and significant. Thus the GARCH(1,1) model applied here models the first moment of the bivariate distributions of the variables with a bivariate error correction term (see Kroner and Sultan, 1993).⁸ As advocated by Baillie and Myers (1991, p. 116), it is vital to let the conditional covariance be time-dependent, as in the bivariate GARCH model, rather than constant. This ability of the bivariate GARCH model to have time-dependent conditional variance makes it ideal to provide a time-variant hedge ratio.

Given the bivariate GARCH model of the log difference of the cash and the futures prices presented above, the time-varying hedge ratio can be expressed as:

$$\beta_t = \hat{H}_{12,t} / \hat{H}_{22,t} \quad (8)$$

Where $\hat{H}_{12,t}$ is the estimated conditional variance between the log difference of the cash and futures prices, and $\hat{H}_{22,t}$ is the estimated conditional variance of the log difference of the futures prices from the bivariate GARCH model. Given that conditional covariance is time-dependent, the optimal hedge ratio will be time-dependent.

3.2 Bivariate BEKK GARCH

Lately, a more stable GARCH presentation has been put forward. This presentation is termed by Engle and Kroner (1995) the BEKK model; the conditional covariance matrix is parameterized as

$$\text{vech}(H_t) = C'C + \sum_{K=1}^K \sum_{i=1}^q A'_{Ki} \epsilon_{t-i} \epsilon'_{t-i} A_{Ki} + \sum_{K=1}^K \sum_{i=1}^p B'_{Kj} H_{t-j} B_{Kj} \quad (9)$$

Equations 5 and 6 also apply to the BEKK model and are defined as before. In equation 9 A_{ki} , $i=1, \dots, q$, $k=1, \dots, K$, and B_{kj} , $j=1, \dots, p$, $k=1, \dots, K$ are all $N \times N$ matrices. This formulation has the advantage over the general specification of the multivariate GARCH that conditional variance (H_t) is guaranteed to be positive for all t

⁸ Bera and Higgins (1993) and Engle and Kroner (1995) provide detailed analysis of multivariate GARCH models.

(Bollerslev et al., 1994). The BEKK GARCH model is sufficiently general that it includes all positive definite diagonal representation, and nearly all positive definite vector representation. The following presents the BEKK bivariate GARCH(1,1), with K=1.

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \quad (9a)$$

where C is a 2x2 lower triangular matrix with intercept parameters, and A and B are 2x2 square matrices of parameters. The bivariate BEKK GARCH(1,1) parameterization requires estimation of only 11 parameters in the conditional variance-covariance structure, and guarantees H_t positive definite. Importantly, the BEKK model implies that only the magnitude of past returns innovations is important in determining current conditional variances and co-variances. The time-varying hedge ratio based on the BEKK GARCH model is also expressed as equation 8.

3.3 Bivariate GARCH-X

Lee (1994) provides an extension of the standard GARCH model linked to an error-correction model of cointegrated series on the second moment of the bivariate distributions of the variables. This model is known as the GARCH-X model. According to Lee (1994), if short-run deviations affect the conditional mean, they may also affect conditional variance, and a significant positive effect may imply that the further the series deviate from each other in the short run, the harder they are to predict. Lee (1994, pp. 375-376) indicates that the conditional heteroscedasticity may be modelled with a function of lagged error correction terms if disequilibrium measured by the error correction term is responsible for uncertainty measured by the conditional variance. Given that short-run deviations (error correction term) from the long-run relationship between the cash and futures prices may affect the conditional variance and conditional covariance, then they will also influence the time-varying optimal hedge ratio, as defined in equation 8.

The following bivariate GARCH(p,q)-X model may be used to represent the log difference of the cash prices

and the futures prices:

$$\text{vech}(H_t) = C + \sum_{j=1}^p A_j \text{vech}(\varepsilon_{t-j})^2 + \sum_{j=1}^q B_j \text{vech}(H_{t-j}) + \sum_{j=1}^k D_j \text{vech}(z_{t-1})^2 \quad (10)$$

Once again, equations 5 and 6 (defined as before) also apply to the GARCH-X model. The squared error term (z_{t-1}) in the conditional variance and covariance equation (equation 10) measures the influences of the short-run deviations on conditional variance and covariance.

The inclusion of the error correction term in a GARCH-X specification can be justified in the following manner. Consider a cash security traded in the cash market and a futures contract traded on the basis of the cash security. Since the futures contract is priced off the cash security, the error correction term is given by

$$z_t = C_t - \alpha F_t.$$

Where C_t and F_t represent the log of cash and futures prices, respectively. The error term z_t imposes the long-run cointegration relationship between the cash and futures prices and measures how the dependent variable adjusts to the previous period's deviation from a long-run equilibrium relationship. The parameter α links the log of the cash and the futures prices such that the error correction term is stationary in levels. At any given time the error term (z_t) is expected to be different from its long-run equilibrium level. The expectation of the error term gives the long run equilibrium relationship between the two prices and short-term periods of disequilibrium occur as the observed value of error term varies around its expected value. Therefore, cointegration information relating to the two price series may indeed be significant in modelling the conditional variances and covariances of log of difference of the cash and the futures prices.

As advocated by Lee (1994, p. 337), the square of the error-correction term (z) lagged once should be applied in the GARCH(1,1)-X model. The parameters D_{11} and D_{33} indicate the effects of the short-run deviations

between the cash and the futures prices from a long-run cointegrated relationship on the conditional variance of the residuals of the log difference of the cash and futures prices, respectively. The parameter D_{22} shows the effect of the short-run deviations on the conditional covariance between the two variables. A significant parameter indicates that these terms have significant predictive power in modelling the conditional variance-covariance matrix. Therefore, last period's equilibrium error has significant impact on the adjustment process of the subsequent difference in the price. If D_{11} , D_{33} and D_{22} are significant then optimal hedge, as defined in equation 8, will be affected. In other words, if D_{33} and D_{22} are significant, then H_{12} (conditional covariance) and H_{22} (conditional variance of futures returns) are going to differ from the standard GARCH model H_{12} and H_{22} . In such a case, the GARCH-X time-varying hedge ratio will be different from the standard GARCH time-varying hedge ratio. If the parameters are positive and significant this simply implies as the two prices move apart in the short run the conditional variance and covariance will rise thus increasing the time-varying hedge ratio. Opposite is true when the parameters are negative and significant. The empirical question to investigate is whether a significant influence of the short-run deviations also influences the effectiveness of the time-varying hedge ratio. Such an information may be important to investors looking for the most effective hedge ratio.

3.4 Bivariate BEKK GARCH-X

A similar extension can be made to the standard BEKK GARCH linked to an error-correction model of cointegrated series on the second moment of the bivariate distributions of the variables. Such a model is known as the BEKK GARCH-X. The formulation of the BEKK GARCH(1,1)-X model is given by

$$H_t = C'C + A' \varepsilon_{t-1} \varepsilon'_{t-1} A + B' H_{t-1} B + D' D z_{t-1}^2 \quad (11)$$

Equations 5 and 6 apply to this model also and the variables are as defined in the BEKK GARCH section. Once again, the z_t is the error term from the cointegration tests between the cash and futures prices, and the D is the (1x2) matrix of coefficients. The analysis of the size and sign on the error term coefficients are the same as

described in the bivariate GARCH-X section. The time-varying hedge ratio from the BEKK GARCH-X should differ from the standard BEKK hedge ratio.

If the four time-varying hedge ratios are different, then the interesting empirical question arises: which one is more effective? Also how does the constant minimum variance hedge ratio do against the time-varying hedge ratios? All the above methods of estimating the hedge ratios are applied, and their effectiveness is compared in this paper.

4. Data and Basic Statistics

Daily log difference of the cash (spot) and the futures prices of corn, coffee, wheat, sugar and soybean are used in the empirical tests. All the data are daily and range from August 1980 to July 2004. The effectiveness of the hedge ratio is investigated by comparing the within sample period (August 1980-July 2004) and out-of-sample period performance of the different hedge ratios for two periods, August 2002- July 2004 (two years) and August 2003-July 2004 (one year). The two different lengths of out-of-sample periods are applied to investigate the effect of changing the length on the hedging effectiveness of the hedge ratios.

All futures price indices are continuous series.⁹ The coffee and sugar #11 future prices are from the Coffee, Sugar and Cocoa Exchange (CSCE), the corn, soybean and wheat futures prices are from the Chicago Board of Trade (CBOT). The cash prices are represented by corn spot prices, soybean spot price, wheat #2 spot price, sugar # 11 spot price, and the coffee spot prices. All data are obtained from *Global Financial Data*.

Table 1 (parts A, B and C) shows some of the basic statistics of the four series: log difference of the cash prices and the futures prices, square of the first two series and the cross product of the first two series. The basic statistics are provided for the within sample period (1980-2004) and the two out-of-sample periods, 1980-2002 and 1980-2003. Table 1 part A presents the total period statistics and almost all series are significantly skewed and, as expected, all series are found to have significant and positive kurtosis, implying higher peaks and fatter tails. Thus, the Jarque-Bera statistic shows all series to be non-normal. The statistics from the sub-periods table

⁹ The continuous series is a perpetual series of futures prices. It starts at the nearest contract month, which forms the first values for the continuous series, either until the contract reaches its expiry date or until the first business day of the actual contract month. At this point, the next trading contract month is taken.

1 parts B and C also show similar results. All series are found to be non-normal during the two sub-periods. The mean and variance of all four series seem to stay similar across the three periods. This may imply a lack of structural breaks in the different series.

5. Empirical Results

5.1 Bivariate GARCH, BEKK, GARCH-X and BEKK-X Results

Tables 2, 3, 4 and 5 shows the results from the standard bivariate GARCH(1,1), BEKK(1,1), GARCH-X(1,1) and BEKK-X(1,1) models for with-in sample period, respectively.¹⁰ The results from these tests are quite standard. In most tests, the ARCH coefficients are all positive (A_{11} and A_{33} in the GARCH and GARCH-X tests) and significant, thus implying volatility clustering both in the log difference of cash price and futures price. The ARCH coefficients are also less than unity in all significant cases. The ARCH coefficients (A_{11} and A_{22}) from the BEKK model are close to unity and higher than the other models. The smallest ARCH effects (A_{11} and A_{22}) are found in the BEKK-X tests. In all four models for all commodities the GARCH coefficient is significant and positive implying GARCH effect. A large coefficient of the GARCH term indicates that shocks to conditional variance take a long time to die out and volatility persist. The sign and significance of the covariance parameters indicate positive and significant interaction between the two prices in most cases. All t-statistics are robust to heteroscedasticity. The short-run deviations from a long-run relationship between the cash price and future prices have significant effect on both the mean of cash returns (δ_1) and log difference of futures prices (δ_2) in most of the cases. For the majority of the commodities, the effect on the mean of the cash returns is negative and significant. In the case of log difference of futures prices, the effect is mostly positive and significant except for in the case of the standard GARCH. Thus, an increase in short-run deviations lowers the cash returns but increases the log difference of future prices.

The important part of the GARCH-X and BEKK-X results is the influence of the short-run deviations

¹⁰ In these models, different combinations of p and q may be applied but, as indicated by Bollerslev et al. (1992, p. 10), p=q=1 is sufficient for most financial and economic series. Bollerslev (1988) provides a method of selecting the length of p and q in a GARCH model. Tests in this paper were also conducted with different combinations of p and q, with p=q=2 being the maximum lag length. Results based on log-likelihood function and likelihood ratio tests indicate that the best combination is p=q=1. These results are available on request.

between the cash price and the futures price on the conditional variance and covariance. For GARCH-X, the parameters measuring the effects of the short-run deviations on the conditional variance of cash returns (D_{11}) and log difference of the futures prices (D_{33}) are found to be positive and significant in all tests. A significant effect indicates these terms have potential predictive power in modelling the conditional variance-covariance matrix of the log difference of the cash and futures prices. Therefore, last period's equilibrium error has significant impact on the adjustment process of the subsequent variables. A positive effect of the short-run deviations on the conditional variance implies that as the deviation between the cash and future prices gets larger, the volatility of log difference of the cash and futures prices increases, and prediction becomes more difficult.

Also, in the case of BEKK-X, the significant parameters are found to be positive. The short-run deviation coefficients (D_{11} and D_{33}) are relatively small, as expected. The parameter D_{22} measures the affect of the short-run deviations on the conditional covariance between the two variables. For GARCH-X, only in the case of sugar and corn, D_{22} is found to be significant and positive. The parameter D_{22} is not significant for any commodity, using the BEKK GARCH-X.

To assess the general descriptive validity of the model, a battery of standard specification tests is employed. Specification adequacy of the first two conditional moments is verified through the serial correlation test of white noise. These tests employ the Ljung-Box Q statistics on the standardised (normalised) residuals ($\varepsilon_t/H_t^{1/2}$), standardised squared residuals (ε_t/H_t^2), and the cross-standardised residuals. The latter are the cross product between the standardised residuals of cash and futures. All series are found to be free of serial correlation (at the 5% level). Absence of serial correlation in the standardised squared residuals implies the absence of need to encompass a higher order ARCH process (Giannopoulos, 1995).

Further, the likelihood ratio test (LR) is applied to assess the statistical significance of the incremental explanatory power associated with the general model. In other words, in the LR tests a relatively more complex model is compared to a simpler model to see if it fits a particular dataset significantly better. The LR test is only valid if it used to compare hierarchically nested models. That is, the more complex model must differ from the

simple model only by the addition of one or more parameters. Adding additional parameters will always result in higher likelihood score. However, there comes a point when adding additional parameters is no longer justified in terms of significant improvement in fit of a model to a particular dataset. Thus the LR test is conducted between the standard GARCH and GARCH-X and also between standard BEKK and BEKK-X. The null hypothesis in the LR test is that both models perform the same while the alternate null is that the complex model outperforms the standard model. The LR statistics are provided in tables 4 and 5. In all cases, LR test significantly rejects the null at the 1% level (by means of the χ^2 statistics), indicating that the complex model GARCH-X fits better than the standard GARCH (table 4), and the BEKK-X fits better than the standard BEKK (table 5). The extra parameters in GARCH-X and BEKK-X make their performance superior. This is true during all periods.

5.2 The OLS results

Table 6 presents the OLS estimation of the constant hedge ratio from the minimum variance model of equation 1. The OLS estimate of equation 1 is presented for all three periods. For all commodities except coffee in all three period, the coefficient (the minimum variance hedge ratio) on the futures returns is positive and significantly different from zero. The R^2 is relatively low but the Durbin Watson indicates lack of serial correlation.¹¹ The estimated hedge ratios are quite small in size but this was expected given the application of the daily date.

5.3 Within Sample Period Hedge Ratios Comparison

Comparison between the effectiveness of different hedge ratios is made by constructing portfolios implied by the computed ratios, and the change in the variance of these portfolios indicates the hedging effectiveness of the hedge ratios. The portfolios are constructed as $(r_t^c - \beta_t^* r_t^f)$, where r_t^c is the log difference of the cash (spot) prices, r_t^f is the log difference of the futures prices, and β_t^* is the estimated optimal hedge ratio.¹² The variance of these constructed portfolios is estimated and compared. For example, for comparison between the GARCH and GARCH-X-based portfolios, the change in variance is calculated as $(\text{Var}_{\text{GARCH}} - \text{Var}_{\text{GARCHX}})/\text{Var}_{\text{GARCH}}$.

¹¹ The low R^2 may indicate low level of hedging effectiveness.

Comparison is also provided between the four time-varying hedge ratio-oriented portfolios and an unhedged portfolio. Variance of an unhedged portfolio is presented by the variance of the returns in the cash market.

Table 7 presents the variance of the portfolios and the comparison results for the within sample period (January 1980-July 2004). The table shows the variance of the portfolios estimated using the different types of hedge ratios and the percentage change in the variance of the portfolios constructed. The top part of the table shows the actual variance of the time-varying hedge ratio-oriented portfolios, the constant minimum variance hedge ratio-oriented portfolios and the unhedged portfolio. The second part shows the percentage change in the variance between GARCH-X and the other methods-oriented portfolios. The third part presents the percentage change in the variance between BEKK-X and other methods-oriented portfolios (excluding the GARCH-X). The fourth part presents the percentage change in the variance between BEKK and other methods (excluding the GARCH-X and BEKK-X)-oriented portfolios. The fifth part shows the difference between the GARCH-oriented portfolios, the constant hedge ratio oriented portfolios and the unhedged portfolios. The sixth and last part shows the difference between the constant hedge ratio portfolios and the unhedged portfolio.

Portfolios created using the hedge ratios from the GARCH-X model outperform all other portfolios for all commodities. The differences in the percentage change are quite small, usually less than 5%, except in the case of soybean. For soybean, the GARCH-X time-varying hedge-ratio portfolios outperform the unhedged portfolio by 13.90%, the constant hedge ratio portfolio by 13.51%, the BEKK-X portfolio by 9.86%, the standard BEKK portfolio by 10.70% and the standard GARCH portfolio by 10.28%.

The results for BEKK-X-oriented portfolios are mixed. It does worse than the standard bivariate GARCH for all commodities except for soybean. Once again, the differences are smaller than 5%. The BEKK-X does better than the standard BEKK, the constant hedge ratio and the unhedged portfolio for most commodities. The standard BEKK performs better than the standard GARCH only for corn but does better than the constant hedge ratio and the unhedged for most commodities. The standard GARCH outperforms both the constant hedge ratio and unhedged portfolios for all commodities. The constant hedge ratio portfolios is outperformed by the

¹² In the case of the constant ratio the time subscript does not exist.

unhedged portfolios.

Overall, the GARCH-X portfolios provide the strongest and the standard BEKK the weakest results among the GARCH models. But the standard BEKK does better than the constant ratio portfolios and the unhedged portfolios. Usually, the percentage differences in the portfolio variances are smaller than 5%.

5.3 Out-of-sample Periods Hedge Ratios Comparison

Baillie and Myers (1991) and other papers further claim that the more reliable measure of hedging effectiveness is the hedging performance of different methods for out-of-sample periods. This paper compares the hedging effectiveness of the different methods during two different out-of-sample time periods: from August 2002 to July 2004 (two years), and from August 2003 to July 2004 (one year). Two different lengths of out-of-sample periods are applied to check whether changing the length has any significant effect on the hedging effectiveness of the hedge ratios. In order to investigate the out-of-sample hedging effectiveness of the hedging methods, all GARCH models and the OLS regressions are estimated for the periods January 1980 to July 2002, and January 1980 to July 2003, and then the estimated parameters are applied to compute the hedge ratios and the portfolios for the two out-of-sample periods.¹³ Once again, the variance of these portfolios is compared, and the change in the variance indicates the hedging effectiveness of the hedge ratios.

Table 8 shows the variance of the out-of-sample portfolios and the percentage change in variance of the portfolios from August 2002 to July 2004. The set-up of Table 8 is the same as for Table 7. For most commodities again, the GARCH-X based portfolio outperforms the other model portfolios. The constant ratio portfolio does better than for wheat, sugar and soybean. The BEKK-X outperforms the standard BEKK and the unhedged portfolios, but not the standard GARCH portfolios and constant ratio portfolios in most cases. The standard BEKK is outperformed by the standard GARCH, the constant ratio and the unhedged portfolios for most commodities. The standard GARCH does better than the unhedged portfolio for all commodities except for corn and is outperformed by the constant ratio portfolio for all commodities except for coffee. The constant ratio

¹³ The GARCH estimations for the period 1980-2002 and 1980-2003 are not provided, in order to save space but are available on request. These parameters are similar to the ones estimated for the whole sample period. Once again, cointegration is also found during these periods.

portfolios does worse than the unhedge portfolios. In the cases of sugar and soybean, the differences are sometimes large.

In summary, during the 2-year out-of-sample period, the GARCH-X portfolios provide the strongest and the standard BEKK the weakest results among the GARCH models. Again, the percentage differences in the portfolio variances are usually smaller than 5%. The differences are small except for soybean using the constant hedge ratio portfolios.

Figure 1 presents the estimated and the forecasted corn hedge ratios based on the four GARCH models over the 2-year (2002-2004) out-of-sample period. The estimated hedge ratios are estimated using the four GARCH models and the data for the period 2002-2004. The forecasted hedge ratios during 2002-2004 are forecasted by the GARCH models using data and parameters from GARCH models from the period 1980 -2002. The two hedge ratios based on all GARCH models move together. The GARCH-X estimated and forecasted hedge ratios also tend to move together, but less tightly. Similar graphs of other commodities are not provided to save space, but are available on request. Table 9 shows the results from the shorter out-of-sample (August 2003-July 2004) period. Among the GARCH models, portfolios based on the GARCH-X model again perform best, and the standard BEKK does worst. For wheat, the standard GARCH model-based portfolios do better than other models expect the constant ratio portfolios. The differences are small except for sugar using the constant hedge ratio portfolios.

Changing the length of the out-of-sample period does affect the performance of the hedge ratios. Both the GARCH-X and the BEKK-X show improvement somewhat. The standard GARCH provide similar performance. The BEKK-X performs better than the standard BEKK. The performance of the constant hedge ratio portfolios falls during the one year forecasting period.

Figure 2 presents the estimated and the forecasted corn hedge ratios based on the four GARCH models over the shorter out-of-sample period, August 2003 to July 2004. The estimated and forecasted hedge ratios based on the standard GARCH, GARCH-X and BEKK-X move together. The estimated and forecasted hedge ratios based

on the BEKK model also tend to move together, but not as closely as others. Once again, graphs of other commodities are not provided, to save space, but are available on request.

Of course, with any GARCH method, the hedge portfolio has to be rebalanced frequently. In this paper, the time-varying GARCH hedge ratio changed daily. The trade-off between the risk reduction and the transaction cost will determine the practicality of the GARCH hedging method.¹⁴ According to Myers (1991), since the different GARCH models are more complex to estimate, and since the continual futures adjustments required entail extra commission charges, the extra cost of working with any GARCH model may only be warranted if the investor is extremely risk averse.

6. Conclusion

It is a well-documented claim in the futures market literature that the optimal hedge ratio should be time-varying and not constant. Lately, different versions of the GARCH models have been applied to estimate time-varying hedge ratios for different futures markets. This paper investigates the hedging effectiveness of GARCH estimated time-varying hedge ratios and a constant minimum variance hedge ratio in five agricultural commodities futures: corn, wheat, coffee, sugar and soybean. The time-varying hedge ratios are estimated by means of four different types of GARCH models: the standard bivariate GARCH, bivariate BEKK, bivariate GARCH-X, and bivariate BEKK-X. The constant minimum variance hedge ratio is applied for comparison and is estimated by means of the OLS method. The GARCH-X and the BEKK-X are unique among the GARCH models in taking into consideration the effects of the short-run deviations from a long-run relationship between the cash and the futures price indices on the hedge ratio. The long-run relationship between the price indices is estimated by the Engle-Granger cointegration method. By using cointegration, investors may obtain added information in forming and progressively re-adjusting hedges. The hedging effectiveness is estimated and compared by checking the variance of the portfolios created using these hedge ratios. The lower the variance of the portfolio, the higher is the hedging effectiveness of the hedge ratio.

The empirical tests are conducted by applying daily data. The effectiveness of the hedge ratio is investigated

¹⁴ Park and Switzer (1995) suggest an alternate strategy method that involves less frequent rebalancing, such as rebalancing only when

by comparing the within sample period (August 1980-July 2004) and out-of-sample period performance of the different hedge ratios for two periods, August 2002- July 2004 (two years) and August 2003-July 2004 (one year). The two different lengths of out-of-sample periods are applied to investigate the effect of changing the length on the hedging effectiveness of the hedge ratios.

What do the results show? During the within sample period and the two out-of-sample periods, the GARCH-X-oriented hedge ratio overall performs better than the other GARCH methods, the constant hedge ratio and the unhedged portfolio. The GARCH-X model may be utilised in practical situations to provide greater knowledge of how the individual components in variance-covariance matrix behave over time. Knowledge about the cointegration relationship between the cash and future prices may help investors in forming and re-adjusting hedges in order to improve hedging effectiveness. Among the GARCH models applied, the standard BEKK-oriented hedge ratios provided the worst performance. Also, changing the length of the out-of-sample period from 2 years to 1 year does improve the hedging effectiveness of the GARCH-X and BEKK-X oriented hedge ratios. The performance of the constant ratio portfolios does falls somewhat during the one year forecasting period.

With any GARCH method, the hedge portfolio has to be rebalanced frequently. In this paper, the time-varying GARCH hedge ratio changed every day. The trade-off between the risk reduction and the transaction cost will determine the practicality of the GARCH hedging method. Results in this paper advocate further research in this field. Further research may be conducted using different frequency of the data, different methods of estimation, time period, type of futures markets, for example.

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Table 1
Part A
Basic Statistics of the Total Period (1980-2004)

Variables	Mean	Variance	Kurtosis	Skewness	Jarque-Bera
Log Difference of Cash Price					
Corn	-0.00003	0.00023	6.4595 ^a	-0.1707 ^a	10458.58 ^a
Wheat	-0.00002	0.00030	19.4547 ^a	-0.3470 ^a	94725.77 ^a
Coffee	-0.00024	0.00062	24.104 ^a	1.0591 ^a	146352.07 ^a
Sugar	-0.00010	0.00067	16.1421 ^a	-0.1610 ^a	65156.76 ^a
Soybeans	0.00000	0.00023	9.6940 ^a	-0.3537 ^a	23614.45 ^a
Log Difference of Futures Price					
Corn	-0.00005	0.00021	35.1361 ^a	-1.9629 ^a	312436.96 ^a
Wheat	-0.00005	0.00026	35.1583 ^a	-2.0543 ^a	313194.2 ^a
Coffee	-0.00017	0.00067	8.4794 ^a	0.0589	17975.68 ^a
Sugar	-0.00019	0.00080	10.1253 ^a	0.2661 ^a	25696.93 ^a
Soybeans	-0.00002	0.00019	7.9557 ^a	-0.6270 ^a	16213.67 ^a
Square of Log Difference of Cash Price					
Corn	0.00023 ^a	0.00000	135.5173 ^a	9.6557 ^a	4683683.3 ^a
Wheat	0.00030 ^a	0.000002	1085.92 ^a	29.0089 ^a	295596594 ^a
Coffee	0.0006 ^a	0.00001	779.899 ^a	22.957 ^a	152562103.7 ^a
Sugar	0.00067 ^a	0.000008	1226.732 ^a	30.2520 ^a	377070362 ^a
Soybeans	0.00022 ^a	0.000001	653.500 ^a	20.6059 ^a	107171564 ^a
Square of Log Difference of Futures Price					
Corn	0.00021 ^a	0.000002	2219.1217 ^a	42.2562 ^a	1232705434.3 ^a
Wheat	0.00026 ^a	0.000002	2681.57 ^a	46.2491 ^a	1799542364 ^a
Coffee	0.0006 ^a	0.000004	301.141 ^a	14.057 ^a	22865312.5 ^a
Sugar	0.0008 ^a	0.000008	545.721 ^a	18.2399 ^a	74773152 ^a
Soybeans	0.00019 ^a	0.00000	1091.792 ^a	25.241 ^a	298589677 ^a
Log Difference of Cash Price x Log Difference of Futures Price					
Corn	0.00001 ^a	0.00000	276.6449 ^a	8.9428 ^a	19209876.08 ^a
Wheat	0.00002 ^a	0.00000	48.2379 ^a	0.4954 ^a	581871.87 ^a
Coffee	0.00001	0.000001	232.473 ^a	-2.1527 ^a	13513278.8 ^a
Sugar	0.00007 ^a	0.000001	239.94 ^a	8.024 ^a	14454723.5 ^a
Soybeans	0.00014	0.000004	134.935 ^a	4.6708 ^a	4572949.12 ^a

Note:

a- implies significantly different from zero at 1% level.

Table 1
Part B
Basic Statistics of the Sub Period (1980-2002)

Variables	Mean	Variance	Kurtosis	Skewness	Jarque-Bera
Log Difference of Cash Price					
Corn	-0.00006	0.00023	5.9960 ^a	-0.2614 ^a	8300.18 ^a
Wheat	-0.00007	0.00028	14.7833 ^a	-0.9949 ^a	50982.19 ^a
Coffee	-0.0003	0.00055	29.220 ^a	1.3722 ^a	197350.23 ^a
Sugar	-0.00014	0.00068	17.0673 ^a	-0.1691 ^a	24308.02 ^a
Soybeans	-0.00009	0.00026	10.2546 ^a	-0.3672	24217.59 ^a
Log Difference of Futures Price					
Corn	-0.00006	0.00022	37.1934 ^a	-2.1278 ^a	321110.88 ^a
Wheat	-0.00008	0.00025	39.6601 ^a	-2.3481 ^a	365450.09 ^a
Coffee	-0.00026	0.00061	8.9930 ^a	0.0409	18531.872 ^a
Sugar	-0.00014	0.00083	10.2818 ^a	0.3064 ^a	24308.02 ^a
Soybeans	-0.00007	0.00019	8.4290 ^a	-0.6645 ^a	16683.50 ^a
Square of Log Difference of Cash Price					
Corn	0.00023 ^a	0.00000	112.3772 ^a	8.8423 ^a	2965189.90 ^a
Wheat	0.00028 ^a	0.000001	1218.144 ^a	30.0186 ^a	340819048 ^a
Coffee	0.00055 ^a	0.00001	903.076 ^a	25.3432 ^a	187450609.6 ^a
Sugar	0.00068 ^a	0.000009	1153.27 ^a	29.499 ^a	305540665 ^a
Soybeans	0.00023 ^a	0.000001	629.93 ^a	20.4317 ^a	91301020 ^a
Square of Log Difference of Futures Price					
Corn	0.00022 ^a	0.000002	2068.482 ^a	40.9557 ^a	981876038.9 ^a
Wheat	0.00025 ^a	0.000003	2513.58 ^a	45.008 ^a	1449492287 ^a
Coffee	0.00061 ^a	0.000004	290.049 ^a	13.9176 ^a	19453511.3 ^a
Sugar	0.00083 ^a	0.000008	515.60 ^a	17.815 ^a	61201662 ^a
Soybeans	0.00019 ^a	0.00000	1046.51 ^a	24.9423 ^a	251501861 ^a
Log Difference of Cash Price x Log Difference of Futures Price					
Corn	0.000014 ^a	0.00000	279.7012 ^a	9.0831 ^a	18000699.9 ^a
Wheat	0.000025 ^a	0.00000	47.1679 ^a	1.5465 ^a	511952.57 ^a
Coffee	0.000009	0.000001	260.4146 ^a	-3.0140 ^a	15546607.9 ^a
Sugar	0.00008 ^a	0.000001	227.553 ^a	7.8875 ^a	11921202.8 ^a
Soybeans	0.000015	0.00000	134.011 ^a	4.8000 ^a	4135939.5 ^a

Note:

a- implies significantly different from zero at 1% level.

Table 1
Part C
Basic Statistics of the Sub Period (1980-2003)

Variables	Mean	Variance	Kurtosis	Skewness	Jarque-Bera
Log Difference of Cash Price					
Corn	-0.000006	0.00024	6.5369 ^a	-0.1819 ^a	10267.68 ^a
Wheat	-0.00004	0.00030	21.1106 ^a	-0.3957 ^a	106903.47 ^a
Coffee	-0.00025	0.00061	25.8198 ^a	1.1209 ^a	160897.38 ^a
Sugar	-0.00009	0.00068	16.353 ^a	-0.1614 ^b	64084.85 ^a
Soybeans	-0.00004	0.00023	9.9509 ^a	-0.3616	23844.97 ^a
Log Difference of Futures Price					
Corn	-0.00002	0.00022	35.5884 ^a	-2.0001 ^a	307220.8 ^a
Wheat	-0.00001	0.00025	37.6449 ^a	-2.2029 ^a	344115.11 ^a
Coffee	-0.00018	0.00061	8.6009 ^a	0.0647 ^b	17724.30 ^a
Sugar	-0.00013	0.00081	10.1263 ^a	0.2800 ^a	24638.19 ^a
Soybeans	-0.00003	0.00019	8.1933 ^a	-0.6410 ^a	16474.47 ^a
Square of Log Difference of Cash Price					
Corn	0.00024 ^a	0.00000	131.9454 ^a	9.5566 ^a	4257832.4 ^a
Wheat	0.00030 ^a	0.000002	1099.03 ^a	29.496 ^a	290165764 ^a
Coffee	0.00061 ^a	0.00001	766.1287 ^a	22.8689 ^a	141100837.9 ^a
Sugar	0.00068 ^a	0.000009	1182.29 ^a	29.739 ^a	335678500 ^a
Soybeans	0.00023 ^a	0.000001	638.47 ^a	20.4527 ^a	98048523 ^a
Square of Log Difference of Futures Price					
Corn	0.00022 ^a	0.000002	2141.854 ^a	41.5809 ^a	1100563290 ^a
Wheat	0.00025 ^a	0.000003	2601.43 ^a	45.682 ^a	1623090757 ^a
Coffee	0.00061 ^a	0.00004	292.2138 ^a	13.8835 ^a	20638886.0 ^a
Sugar	0.00082 ^a	0.000008	527.664 ^a	17.965 ^a	67004662 ^a
Soybeans	0.00019	0.00000	1079.04 ^a	25.2478 ^a	279514860 ^a
Log Difference of Cash Price x Log Difference of Futures Price					
Corn	0.00002 ^a	0.00000	269.2571 ^a	8.8677 ^a	17441975.9 ^a
Wheat	0.00002 ^a	0.00000	52.3543 ^a	0.7666 ^a	657142.049 ^a
Coffee	0.00001	0.000001	231.8781 ^a	-2.2117 ^a	12884236.8 ^a
Sugar	0.00007 ^a	0.000001	232.41 ^a	7.9171 ^a	12998707 ^a
Soybeans	0.000014	0.00000	134.252 ^a	4.6932 ^a	4338525.9 ^a

Note:

a - implies significantly different from zero at 1% level.

b - implies significant differently from zero at 5% level.

Table 2
Bivariate GARCH Results

	Corn	Wheat	Coffee	Sugar	Soybean
$\mu_1 \times 10^{-4}$	1.6603 (1.1435)	-1.2614 (-0.6946)	-3.7990 ^a (-2.9788)	-1.1908 (-0.4391)	0.5830 (0.3989)
δ_1	-0.0237 ^a (-6.8022)	0.0047 ^b (2.4816)	0.0044 ^a (4.8838)	0.0017 (0.5955)	0.0193 ^a (4.7828)
$\mu_2 \times 10^{-4}$	-0.0886 (-0.7428)	-3.9093 ^b (-2.4032)	-4.0071 (-1.8570)	-9.4100 ^a (-3.7414)	-2.2384 (-1.7058)
δ_2	0.0724 ^a (24.3000)	-0.0273 ^a (-19.4807)	0.0290 ^a (16.3857)	-0.0547 ^a (-18.1581)	-0.0805 ^a (-25.4300)
$C_1 \times 10^{-4}$	0.0084 ^a (14.1400)	0.0105 ^a (13.4940)	0.0008 ^a (31.5502)	0.0159 ^a (14.6305)	0.0036 ^a (10.9200)
A_{11}	0.1027 ^a (18.3218)	0.0903 ^a (35.9661)	0.0855 ^a (52.2551)	0.0766 ^a (22.3766)	0.0869 ^a (29.5580)
B_{11}	0.8629 ^a (126.3859)	0.8804 ^a (205.0693)	0.9274 ^a (912.8500)	0.9026 ^a (233.6639)	0.9007 ^a (269.4297)
$C_3 \times 10^{-4}$	0.0072 ^a (27.8602)	0.0302 ^a (20.1823)	0.0071 ^a (11.6200)	0.0310 ^a (11.6385)	0.0027 ^a (8.4071)
A_{33}	0.1160 ^a (30.2647)	0.1430 ^a (37.7182)	0.1152 ^a (23.7805)	0.1237 ^a (25.5348)	0.0757 ^a (18.5653)
B_{33}	0.8599 ^a (242.0081)	0.7517 ^a (92.5414)	0.8828 ^a (200.9354)	0.8450 ^a (115.2865)	0.9117 ^a (190.6074)
$C_2 \times 10^{-4}$	0.0001 ^b (1.9675)	0.0007 ^a (3.1037)	0.0026 (0.7602)	0.0001 (0.4466)	0.0004 ^a (3.6458)
A_{22}	0.0215 ^a (10.1367)	0.0641 ^a (28.6130)	0.0197 (1.8079)	0.0270 ^a (5.8311)	0.0584 ^a (19.3277)
B_{22}	0.9694 ^a (388.3794)	0.9188 ^a (384.6335)	-0.6120 (-1.6379)	0.9491 ^a (140.1932)	0.9311 ^a (338.9704)
L	45857.056	43975.865	41572.944	38461.679	46584.466
LB(9) test for Serial Correlation in the Residuals					
$\varepsilon_t/h_t^{1/2}$ - Cash	10.9987	9.8919	6.6305	7.4603	11.5207
ε_t^2/h_t - Cash	10.6232	9.4571	5.4883	9.3698	10.7339
$\varepsilon_t/h_t^{1/2}$ - Futures	5.8925	4.9794	12.2941	7.4530	10.1883
ε_t^2/h_t - Futures	3.3340	5.1328	9.1208	10.3116	10.3786
CSR	8.0457	10.2298	5.4882	10.9081	11.1067

Notes:

a, b & c imply significance at the 1%, 5% & 10% level, respectively.

t-statistics in the parentheses; L=log likelihood function value.

LB=Ljung-Box statistics for serial correlation of the order 9.

ε_t^2/H_t = Standardized Squared Residuals

$\varepsilon_t/h_t^{1/2}$ = Standardized Residuals

Cross Standardized Residuals (CSR) = standardized residuals (cash) x standardized residuals (futures)

Table 3

Bivariate BEKK Results

	Corn	Wheat	Coffee	Sugar	Soybean
$\mu_1 \times 10^{-4}$	-1.0851 (-0.5497)	0.9579 (0.5102)	1.3768 (0.5995)	-3.6440 ^a (-2.7500)	-0.9936 (-0.2435)
δ_1	-0.0062 ^c (-1.7814)	-0.0194 ^a (-6.6360)	-0.0251 ^a (-5.1663)	0.0032 (0.6194)	-0.0029 (-0.9319)
$\mu_2 \times 10^{-4}$	-5.3107 ^c (-1.7663)	-2.2756 (-1.2053)	-0.7400 (-0.3893)	-3.8846 (-1.5250)	-11.2931 ^a (-3.2553)
δ_2	0.0293 ^a (7.0810)	0.0809 ^a (28.3451)	0.0737 ^a (14.2900)	0.0294 ^a (9.1805)	0.0543 ^a (15.2699)
C_{11}	0.0028 ^a (4.7433)	0.0016 ^a (10.9480)	0.0025 ^a (6.8084)	0.0008 ^a (6.5648)	0.0035 ^a (2.8949)
A_{11}	0.2977 ^a (8.6963)	0.2905 ^a (11.7553)	0.2955 ^a (18.2239)	0.2690 ^a (50.1817)	0.2494 ^a (5.3213)
B_{11}	0.9452 ^a (78.3185)	0.9573 ^a (168.415)	0.9428 ^a (123.4078)	0.9692 ^a (2971.411)	0.9608 ^a (61.4838)
C_{22}	0.0037 ^a (9.8381)	0.0014 ^a (8.3648)	0.0023 ^a (4.0340)	0.0022 ^a (5.4498)	0.0029 ^b (2.0185)
A_{22}	0.3273 ^a (9.1173)	0.2645 ^a (15.9791)	0.2968 ^a (7.5842)	0.2769 ^a (9.6238)	0.2286 ^a (3.2090)
B_{22}	0.9205 ^a (73.9110)	0.9602 ^a (200.738)	0.9457 ^a (60.4120)	0.9599 ^a (119.5364)	0.9695 ^a (48.2320)
C_{12}	0.0007 ^a (3.1666)	0.0004 ^a (2.7521)	0.0003 ^c (1.8671)	-0.00002 (-0.0842)	0.00001 (0.0555)
L	33006.140	35568.376	34784.455	30486.062	27405.045
Test for Serial Correlation in the Residuals					
$\varepsilon_t/h_t^{1/2}$ - Cash	10.0391	11.1268	7.8168	6.3129	10.1382
ε_t^2/h_t - Cash	3.5348	12.4398	7.7449	7.5825	10.4509
$\varepsilon_t/h_t^{1/2}$ - Futures	11.2525	8.7245	6.9109	6.1491	10.1382
ε_t^2/h_t - Futures	5.1296	6.5862	2.1600	11.3501	8.8554
CSR	9.9979	2.7034	1.1966	7.8102	11.0624

See notes at the end of table 2.

Table 4
Bivariate GARCH-X Results

	Corn	Wheat	Coffee	Sugar	Soybean
$\mu_1 \times 10^{-4}$	1.4442 (0.9521)	-0.8793 (-0.4653)	-3.9024 ^a (-3.1597)	-0.9717 (-0.3581)	-2.6633 (-1.0940)
δ_1	-0.0215 ^a (-5.0417)	-0.0021 (-0.8004)	0.0007 (0.4412)	-0.0022 (-0.7020)	-0.0007 (-0.0771)
$\mu_2 \times 10^{-4}$	-0.9223 (-0.7154)	-5.3469 ^a (-3.1793)	-4.4945 ^b (-2.0315)	-8.6420 ^a (-3.4131)	-19.6490 ^a (-8.9040)
δ_2	0.0759 ^a (17.3608)	0.0274 ^a (12.2232)	0.0261 ^a (10.3410)	0.0555 ^a (15.9658)	0.1478 ^a (16.6476)
$C_1 \times 10^{-4}$	0.0077 ^a (13.5068)	0.1198 ^a (11.6733)	0.0005 ^a (19.1904)	0.1601 ^a (14.3120)	0.000002 ^a (3.2874)
A_{11}	0.0973 ^a (18.6137)	0.0764 ^a (20.9205)	0.0835 ^a (49.6361)	0.0787 ^a (22.3186)	0.1143 ^a (15.4804)
B_{11}	0.8629 ^a (128.592)	0.8705 ^a (141.8047)	0.9257 ^a (829.1740)	0.9004 ^a (223.7808)	0.8747 ^a (121.2865)
D_{11}	0.0007 ^a (5.1611)	0.0007 ^a (14.2174)	0.00011 ^a (17.9133)	0.00004 (1.1255)	0.0025 ^a (3.6912)
$C_3 \times 10^{-4}$	0.0079 ^a (22.1063)	0.2281 ^a (18.3689)	0.0050 ^a (10.0099)	0.3179 ^a (11.2525)	0.000002 ^a (3.0103)
A_{33}	0.0983 ^a (18.3065)	0.1292 ^a (30.5064)	0.0865 ^a (23.0640)	0.1286 ^a (25.6653)	0.0927 ^a (12.9211)
B_{33}	0.8288 ^a (163.1952)	0.7726 ^a (97.0693)	0.8992 ^a (231.0822)	0.8293 ^a (103.5283)	0.8839 ^a (107.057)
D_{33}	0.0043 ^a (27.0836)	0.0009 ^a (11.9240)	0.0004 ^a (8.8124)	0.00008 ^a (7.7729)	0.0035 ^a (6.1274)
$C_2 \times 10^{-4}$	0.0001 ^c (1.8848)	0.0005 ^c (1.8512)	-0.0022 (-0.6593)	0.0005 (1.0218)	0.000002 ^a (4.2112)
A_{22}	0.0198 ^a (9.5410)	0.0665 ^a (37.7531)	0.0120 (1.3103)	0.0301 (6.3800)	0.0976 ^a (16.8598)
B_{22}	0.9672 ^a (350.255)	0.9068 ^a (330.8771)	-0.6564 ^b (-2.0556)	0.9424 (136.5309)	0.8829 ^a (139.7150)
D_{22}	-0.00004 (-1.3942)	0.00004 (0.7779)	0.0016 ^a (3.8545)	-0.00003 (-0.9667)	-0.0012 ^a (-2.6936)
L	45935.072	44049.782	41708.140	38478.561	16869.323
Test for Serial Correlation in the Residuals					
$\varepsilon_t/h_t^{1/2}$ - Cash	5.3175	11.4825	9.0419	8.1558	3.4670
ε_t^2/h_t - Cash	3.9901	9.4280	6.1916	9.0140	3.8229
$\varepsilon_t/h_t^{1/2}$ - Futures	3.1948	2.9131	11.3499	6.0210	9.7325
ε_t^2/h_t - Futures	3.4460	5.5348	12.4723	8.3245	9.3898
CSR	8.0883	6.6461	4.8309	7.3464	5.6185
LR	156.356 ^{***}	148.172 ^{***}	269.787 ^{***}	34.062 ^{***}	127.245 ^{***}

LR = likelihood ratio test

The null hypotheses of the LR test is tested by means of the χ^2 statistics

*** implies rejection of the null at the 1% level.

See also notes at the end of table 2.

Table 5
Bivariate BEKK-X Results

	Corn	Wheat	Coffee	Sugar	Soybean
$\mu_1 \times 10^{-4}$	0.0001 (0.6117)	-0.8343 (-0.4994)	-3.7502 ^a (-2.9758)	-0.0001 (-0.3064)	1.5508 (1.1179)
δ_1	-0.0217 ^a (-4.4844)	-0.0032 (-1.5677)	0.0011 (0.6315)	-0.0028 (-1.0558)	-0.0167 ^a (-3.5193)
$\mu_2 \times 10^{-4}$	-0.0001 (-0.8175)	-0.5043 ^b (-2.2470)	-4.2079 (-1.4540)	-0.0010 ^a (-3.6195)	-3.2595 ^a (-2.7941)
δ_2	0.0787 ^a (13.9011)	0.0280 ^a (7.3700)	0.0274 ^a (8.4186)	0.0554 (19.6195)	0.0769 ^a (19.6177)
C_{11}	0.0025 ^a (7.9917)	0.0031 ^a (6.3094)	0.0007 ^a (12.3750)	0.0035 ^a (3.8628)	0.0016 ^a (9.8343)
A_{11}	0.2940 ^a (14.2381)	0.2773 ^a (11.1383)	0.2884 ^a (62.4760)	0.2454 ^a (6.8720)	0.2815 ^a (16.8431)
B_{11}	0.9396 ^a (112.095)	0.9378 ^a (79.4085)	0.9634 ^a (984.568)	0.9611 ^a (80.8834)	0.9477 ^a (139.369)
D_{11}	0.0008 ^c (1.6522)	0.0007 ^b (2.1950)	0.0001 ^b (2.3280)	0.00004 (0.5589)	0.0019 ^b (2.4303)
C_{22}	0.0024 ^a (4.1001)	0.0040 ^a (4.9050)	0.0019 ^a (4.9532)	0.0036 ^b (2.0806)	0.0012 ^a (6.2197)
A_{22}	0.2842 ^a (10.0155)	0.3208 ^a (5.3137)	0.2452 ^a (8.0351)	0.2599 ^a (3.3635)	0.2550 ^a (53.6020)
B_{22}	0.9257 ^a (49.0385)	0.9018 ^a (27.9493)	0.9627 ^a (120.457)	0.9565 ^a (34.9261)	0.9546 ^a (2116.33)
D_{33}	0.0044 ^b (2.3193)	0.0010 ^b (2.1913)	0.0004 ^a (3.8324)	0.0004 (1.6258)	0.0018 ^a (9.6741)
C_{12}	0.0002 (1.5054)	0.0008 ^b (2.3889)	-0.0002 (-0.9730)	0.00006 (0.4428)	0.0005 ^a (3.1048)
D_{22}	0.0003 (0.7596)	-0.00002 (-0.8733)	0.00004 (0.9642)	-0.00002 (-0.6519)	-0.0003 (-1.204)
L	34872.835	33091.414	30594.924	27414.323	35638.02
Test for Serial Correlation in the Residuals					
$\varepsilon_t/h_t^{1/2}$ - Cash	5.0870	11.7333	9.7935	9.9070	11.6614
ε_t^2/h_t - Cash	8.4105	9.2800	6.2855	11.1171	15.4775
$\varepsilon_t/h_t^{1/2}$ - Futures	2.3841	5.1006	3.4048	9.0621	8.6266
ε_t^2/h_t - Futures	6.4514	4.1267	10.5593	7.7545	5.4422
CSR	8.2632	9.4667	5.6843	4.0006	8.6776
LR	176.760***	170.548***	5345.258***	18.660***	139.285***

LR = likelihood ratio test

The null hypotheses of the LR test is tested by means of the χ^2 statistics

*** implies rejection of the null at the 1% level.

See also notes at the end of table 2.

Table 6
OLS Regression Results

Period: 1980-2004				
Futures	α	β	R^2	D.W.
Corn	-0.0003 (-0.154)	0.0633 ^a (4.705)	0.035	2.033
Wheat	-0.00002 (-0.08)	0.0844 ^a (6.035)	0.058	2.045
Coffee	-0.0002 (-0.726)	0.0168 (1.288)	0.011	1.978
Sugar	-0.00007 (-0.23)	0.083 ^a (7.0518)	0.080	2.122
Soybeans	0.00002 (0.102)	0.0705 ^a (5.097)	0.042	2.161
Period: 1980-2002				
Corn	-0.00005 (-0.26)	0.0667 ^a (4.820)	0.040	2.010
Wheat	-0.00006 (-0.28)	0.1004 ^a (7.172)	0.090	1.986
Coffee	-0.0003 (-0.898)	0.0150 (1.173)	0.068	1.982
Sugar	-0.0003 (-0.376)	0.0917 ^a (7.520)	0.100	2.106
Soybeans	-0.00005 (-0.29)	0.0743 ^a (5.150)	0.046	2.156
Period: 1980-2003				
Corn	-0.000005 (-0.02)	0.0682 ^a (4.971)	0.041	2.031
Wheat	-0.00004 (-0.19)	0.0957 ^a (6.751)	0.077	2.043
Coffee	-0.0002 (-0.750)	0.0185 (1.413)	0.017	1.979
Sugar	-0.00008 (-0.24)	0.0862 ^a (7.184)	0.087	2.122
Soybeans	-0.00001 (-0.06)	0.0724 ^a (5.114)	0.044	2.158

Notes:

t-statistics in parentheses.

a, b & c presents significance at the 1%, 5% & 10% level.

D.W. = Durbin-Watson statistics

Table 7
With-in Period Portfolio Variance and Percentage Change in the Variance

Hedge Ratios	Corn	Wheat	Coffee	Sugar	Soybeans
GARCH	0.000229	0.000296	0.000615	0.000647	0.000214
BEKK GARCH	0.000234	0.000300	0.000616	0.000653	0.000215
GARCH-X	0.000229	0.000295	0.000615	0.000646	0.000192
BEKK-X	0.000232	0.000297	0.000636	0.000654	0.000213
OLS	0.000232	0.000302	0.000616	0.000668	0.000222
No Hedge	0.000234	0.000304	0.000616	0.000673	0.000223
Percentage Change in the Portfolio Variance between GARCH-X and other methods					
GARCH	0.000	0.338	0.000	0.155	10.280
BEKK GARCH	2.137	1.667	0.162	1.072	10.700
BEKK-X	1.293	0.673	3.302	1.223	9.859
OLS	1.293	2.318	0.162	3.293	13.514
No Hedge	2.137	2.961	0.162	4.012	13.901
Percentage Change in the Portfolio Variance Between BEKK-X and other Methods (excluding GARCH-X)					
GARCH	-1.131	-0.338	-3.414	-1.082	0.467
BEKK GARCH	0.850	1.000	-3.246	-0.153	0.930
OLS	0.000	1.656	-3.247	2.096	4.054
No Hedge	0.850	2.303	-3.246	2.823	4.484
Percentage Change in the Portfolio Variance between BEKK GARCH and other methods (excluding GARCH-X and BEKK-X)					
GARCH	2.137	-1.351	-0.163	-0.927	-0.467
OLS	-0.862	0.662	0.000	2.246	3.153
No Hedge	0.000	1.316	0.000	2.978	3.587
Percentage Change in the Portfolio Variance between Bi-GARCH and No Hedge					
OLS	1.293	1.987	0.162	3.144	3.604
No Hedge	2.137	2.632	0.163	3.863	4.036
Percentage Change in the Portfolio Variance Between OLS and no Hedge					
No Hedge	-0.862	-0.662	0.000	-0.749	-0.450

Notes:

The change in the variance between GARCH and GARCH-X is estimated as $(\text{Var}_{\text{GARCH}} - \text{Var}_{\text{GARCHX}})/\text{Var}_{\text{GARCH}}$. The change in the variance between GARCH and BEKK is estimated as $(\text{Var}_{\text{GARCH}} - \text{Var}_{\text{BEKK}})/\text{Var}_{\text{GARCH}}$. The change in the variance between GARCH and BEKK-X is estimated as $(\text{Var}_{\text{GARCH}} - \text{Var}_{\text{BEKKX}})/\text{Var}_{\text{GARCH}}$. The change in the variance between GARCH-X and BEKK-X is estimated as $(\text{Var}_{\text{BEKKX}} - \text{Var}_{\text{GARCHX}})/\text{Var}_{\text{BEKKX}}$. The change in the variance between GARCH-X and BEKK is estimated as $(\text{Var}_{\text{BEKK}} - \text{Var}_{\text{GARCHX}})/\text{Var}_{\text{BEKK}}$. The change in the variance between BEKK and BEKK-X is estimated as $(\text{Var}_{\text{BEKK}} - \text{Var}_{\text{BEKKX}})/\text{Var}_{\text{BEKK}}$.

Table 8
Out-of-Sample Period (2 Years) Portfolio Variance and Percentage Change in the Variance

Hedge Ratios	Corn	Wheat	Coffee	Sugar	Soybeans
GARCH	0.000275	0.000653	0.001300	0.000587	0.000208
BEKK GARCH	0.000274	0.000683	0.001360	0.000593	0.000211
GARCH-X	0.000271	0.000650	0.001300	0.000585	0.000208
BEKK-X	0.000269	0.000660	0.001350	0.000590	0.000210
OLS	0.000272	0.000640	0.001303	0.000464	0.000196
No Hedge	0.000271	0.000670	0.001300	0.000591	0.000260
Percentage Change in the Portfolio Variance between GARCH-X and other methods					
GARCH	1.455	0.459	0.000	0.341	0.000
BEKK GARCH	1.095	4.832	4.441	1.349	1.422
BEKK-X	-0.743	1.515	3.704	0.847	0.952
OLS	0.368	-1.563	0.230	-26.080	-6.122
No Hedge	0.000	2.985	0.000	1.015	20.000
Percentage Change in the Portfolio Variance Between BEKK-X and other methods (excluding GARCH-X)					
GARCH	2.182	-1.072	-3.846	-0.511	0.952
BEKK GARCH	1.825	3.368	0.735	0.506	0.474
OLS	1.103	-3.125	-3.607	-27.155	-7.142
No Hedge	0.743	1.493	-3.846	0.169	19.230
Percentage Change in the Portfolio Variance between BEKK GARCH and other methods (excluding GARCH-X and BEKK-X)					
GARCH	0.363	-4.594	-4.615	-1.022	-1.442
OLS	-0.735	-6.719	-4.375	-27.802	-7.653
No Hedge	-1.110	-1.941	-4.615	-0.338	-1.442
Percentage Change in the Portfolio Variance between Bi-GARCH and No Hedge					
OLS	-1.103	-2.031	0.230	-26.509	-6.122
No Hedge	-1.476	2.537	0.000	0.677	20.000
Percentage Change in the Portfolio Variance between OLS and No Hedge					
No Hedge	0.368	-4.688	0.230	-27.371	-32.653

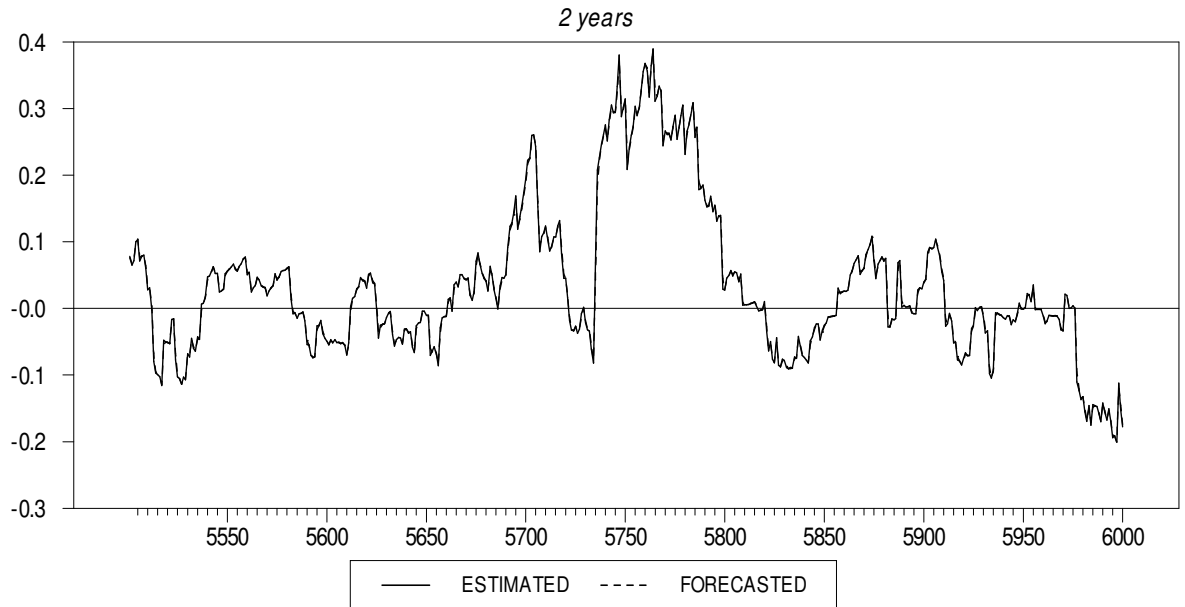
See notes at the end of table 7.

Table 9
Out-of-Sample Period (1 year) Portfolio Variance and Percentage Change in the Variance

Hedge Ratios	Corn	Wheat	Coffee	Sugar	Soybeans
GARCH	0.000169	0.000528	0.000860	0.000460	0.000202
BEKK GARCH	0.000168	0.000548	0.000903	0.000464	0.000205
GARCH-X	0.000167	0.000530	0.000850	0.000458	0.000201
BEKK-X	0.000170	0.000540	0.000890	0.000460	0.000200
OLS	0.000169	0.000520	0.000850	0.000593	0.000206
No Hedge	0.000174	0.000550	0.000900	0.000463	0.000200
Percentage Change in the Portfolio Variance between GARCH-X and other methods					
GARCH	1.183	-0.379	1.163	0.435	0.495
BEKK GARCH	0.595	3.285	5.869	1.293	1.951
BEKK-X	1.765	1.851	4.494	0.435	-0.500
OLS	1.183	-1.923	0.000	22.767	2.427
No Hedge	4.023	3.636	5.555	1.092	-0.500
Percentage Change in the Portfolio Variance Between BEKK-X and other Methods (excluding GARCH-X)					
GARCH	-0.592	-2.272	-3.488	0.000	0.990
BEKK GARCH	-1.190	1.460	1.440	0.862	2.440
OLS	-0.592	-3.846	0.000	22.766	2.913
No Hedge	2.300	1.818	1.111	0.652	0.000
Percentage Change in the Portfolio Variance between BEKK GARCH and other methods (excluding GARCHX)					
GARCH	0.592	-3.788	-5.000	-0.870	-1.463
OLS	0.592	-5.385	-6.235	21.754	0.485
No Hedge	3.448	0.364	-0.333	-0.216	-2.500
Percentage Change in the Portfolio Variance between Bi-GARCH and No Hedge					
OLS	0.000	-1.538	-1.176	22.428	1.942
No Hedge	2.874	4.000	4.444	0.652	-1.000
Percentage Change in the Portfolio Variance between OLS and No Hedge					
No Hedge	-2.959	-5.769	-5.882	21.922	2.943

See notes at the end of table 7.

Corn-Forecasted and Estimated GARCH Hedge Ratio



Corn-Forecasted and Estimated BEKK Hedge Ratio

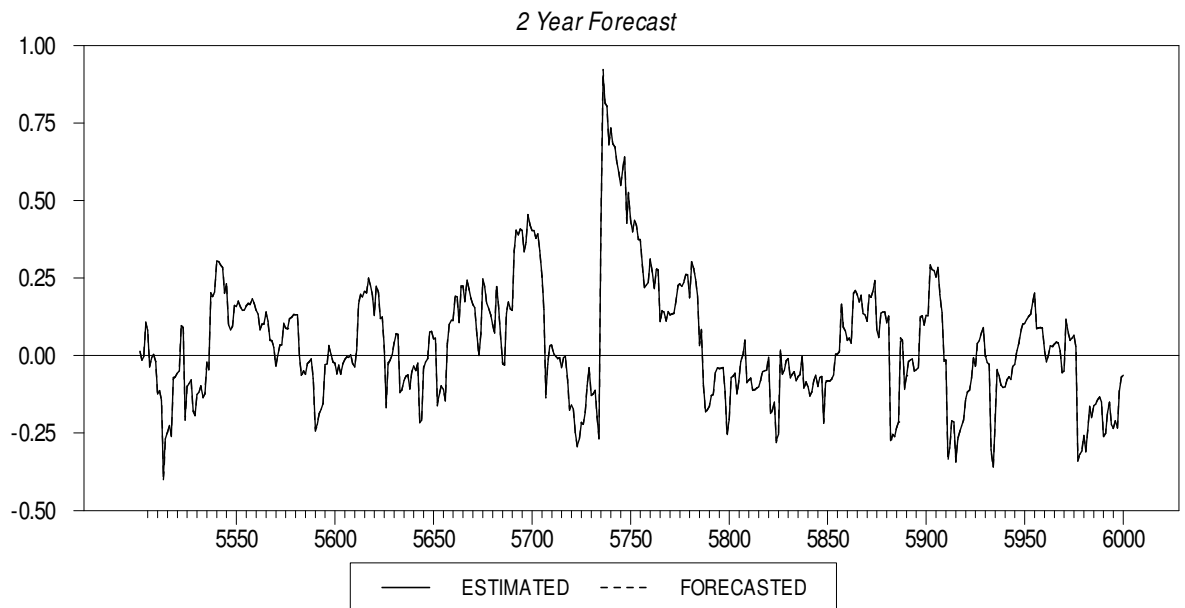
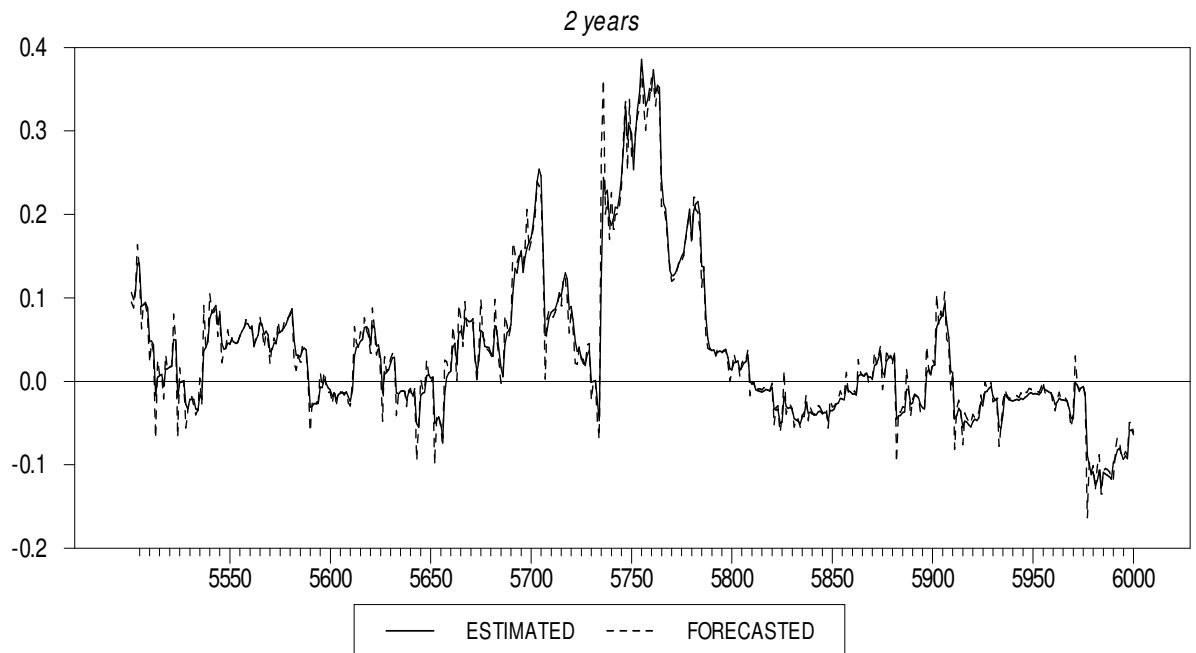


Figure 1

Estimated and Forecasted Hedge Ratios (Aug 2002-July 2004)

The estimated hedge ratios are estimated using the GARCH models during 2002-2004.
The forecasted hedge ratios are forecasted for 2002-2004 using the GARCH models and parameters from 1980-2001.

Corn-Forecasted and Estimated GARCH-X Hedge Ratio



Corn-Forecasted and Estimated BEKK-X Hedge Ratio

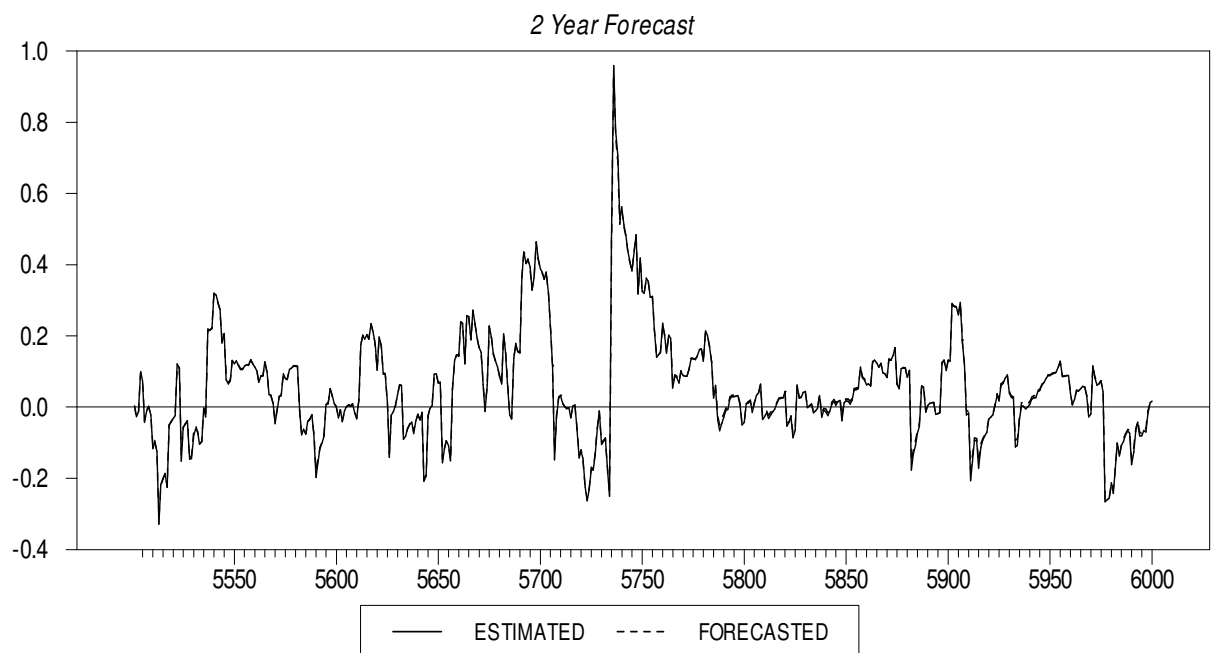
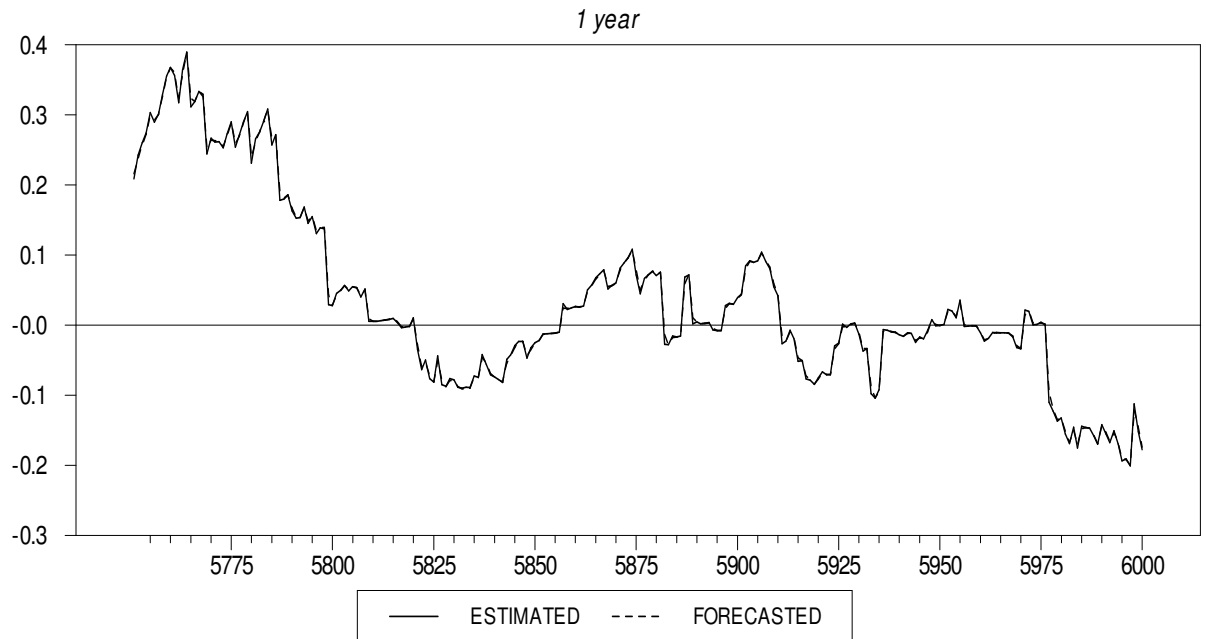


Figure 1

Estimated and Forecasted Hedge Ratios (Aug 2002-July 2004)

The estimated hedge ratios are estimated using the GARCH models during 2002-2004. The forecasted hedge ratios are forecasted for 2002-2004 using the GARCH models and parameters from 1980-2001.

Corn-Forecasted and Estimated GARCH Hedge Ratio



Corn-Forecasted and Estimated BEKK Hedge Ratio

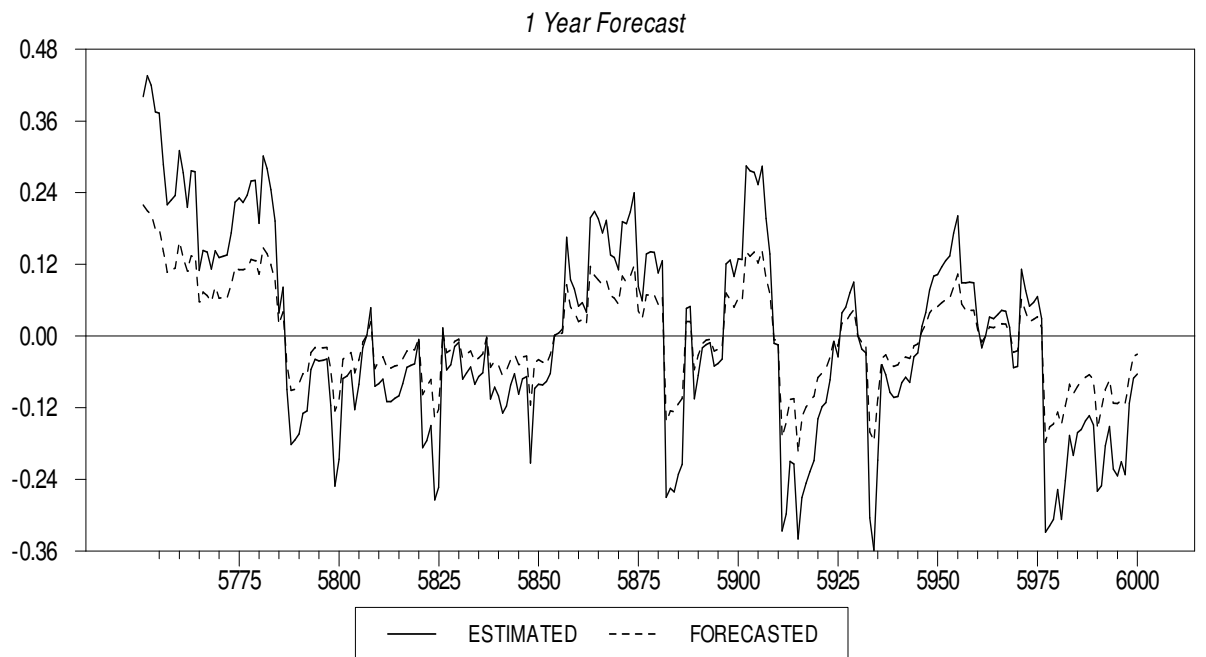


Figure 2

Estimated and Forecasted Hedge Ratios (Aug 2003-July 2004)

The estimated hedge ratios are estimated using the GARCH models during 2003-2004.
The forecasted hedge ratios are forecasted for 2003-2004 using the GARCH models and parameters from 1980-2002.

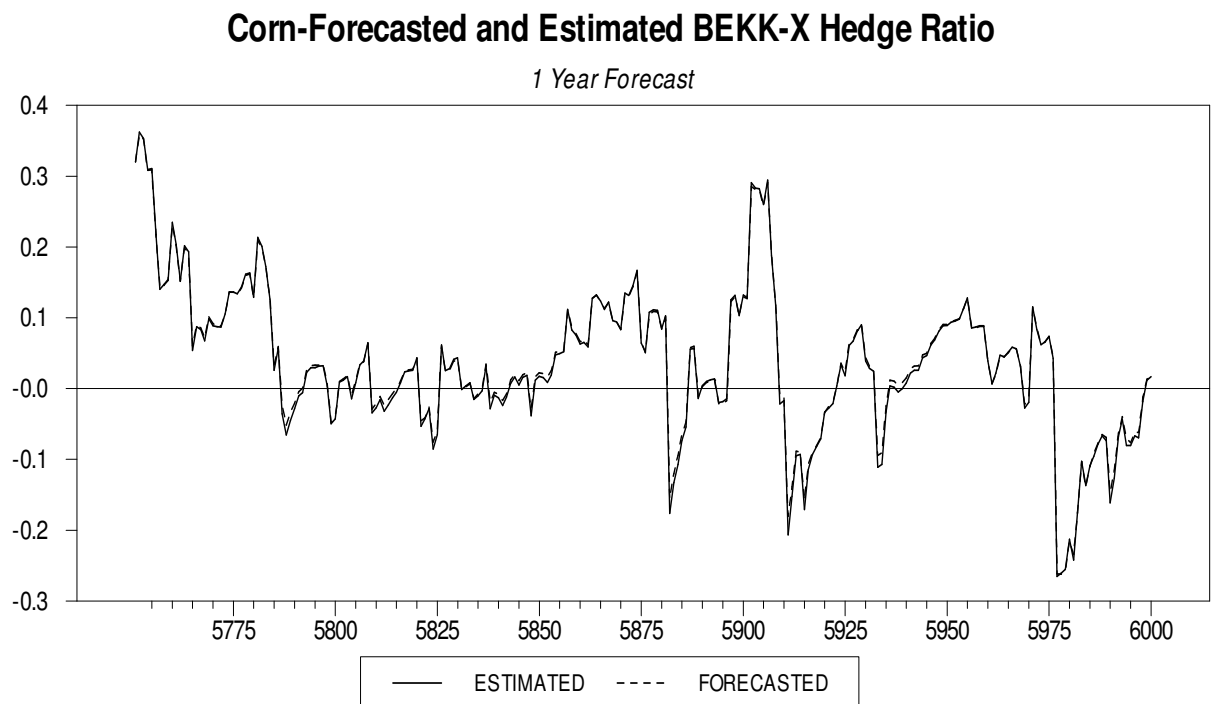
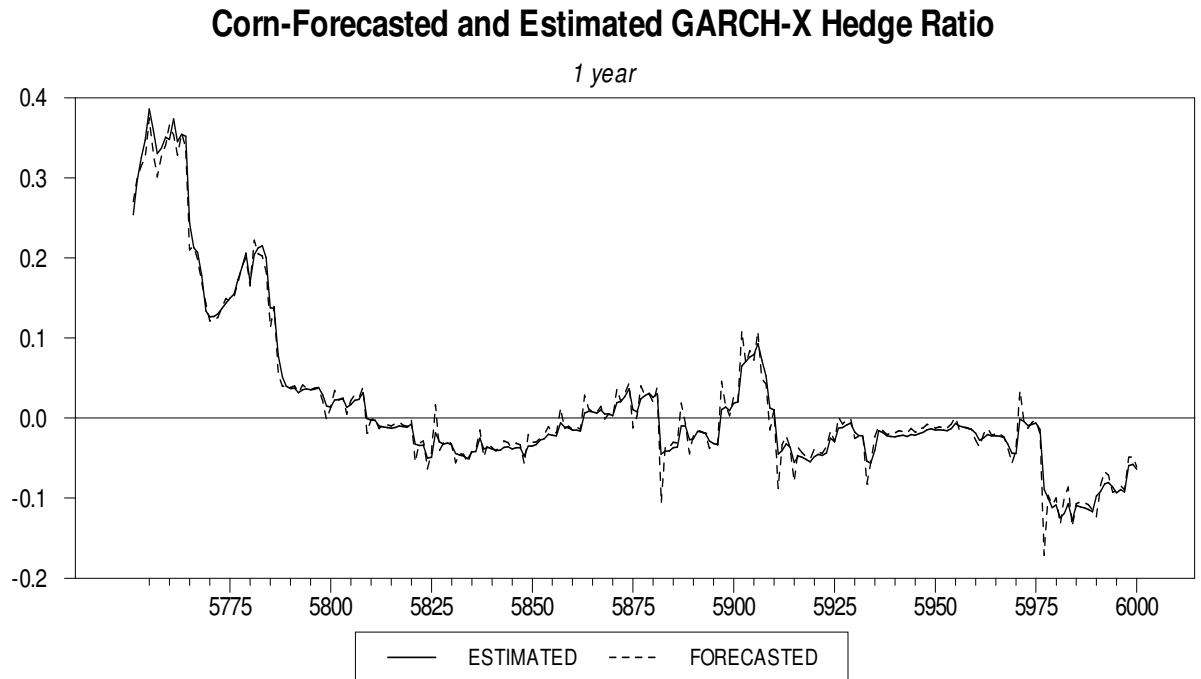


Figure 2
 Estimated and Forecasted Hedge Ratios (Aug 2003-July 2004)
 The estimated hedge ratios are estimated using the GARCH models during 2003-2004.
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