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Photorefractive ring resonator with seeding as an externally driven oscillator

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Abstract

We report on experimental and theoretical studies of a photorefractive ring resonator pumped by a 1.06 μm beam and injected with a weak external, seeding beam. The competition between two dominant gratings that form inside a photorefractive crystal leads to characteristic oscillations in the intensity of the resonating beam. We show that can such a system can be treated as a driven non-linear oscillator.

OCIS terms: 160.5320; 230.4910; 190.5330; 190.7070; 230.5750; 190.3100

Introduction

Ring resonators are commonly used in laser and nonlinear optics. For example, cavities with laser gain media have been extensively studied for their self-adaptive capabilities and pattern formation[1]. However, photorefractive ring resonators[2] offer probably the richest variety of non-linear effects[3,4,5] to study and show unique features of operation. The dynamics of a photorefractive ring resonator system, consisting of a cavity with a photorefractive crystal pumped by a laser beam, has proved, however, to be quite complex[6,7] to analyse and model as it involves considerations of multimode oscillations with their spatial-temporal instabilities and mode competition[8,9]. In particular, studies into spatio-temporal behaviour of ring resonators, based on the photorefractive band-transport model, have predicted that even a single oscillating mode can show self-pulsation and bistability when the cavity is injected with a seeding beam[10]. One of the possible origins of self-pulsing has also been identified and demonstrated experimentally, as competition between gain and loss mechanisms, with different time constants, in a photorefractive material[11]. Externally driven self-pumped phase conjugators also show periodic pulsations and instabilities and numerical modelling confirmed that the competition between self-oscillations and the injected signal[5] gave rise to instabilities.

In this present work we show further evidence of self-pulsations - in a unidirectional ring resonator, based on a two-beam coupling interaction. We also show that a simple, more general than a band transport, model also predicts periodic variations in the output intensity of the resonating beam. These periodic variations can be identified as originating from grating competition – a stationary grating formed by the pump and injected signal and a moving grating between the freely oscillating beam and the pump beam. We show that diffraction of the oscillating beam on both gratings leads to instabilities.

Injected ring resonator

Photorefractive resonators rely on two-beam coupling and/or phase conjugation to provide energy for oscillations inside a cavity. One of the unique features of these resonators is that the oscillation beam can build-up almost regardless of the optical cavity length with frequency determined by the round-trip phase condition[12]. This is due to the appearance of an additional phase shift that originates from photorefractive coupling. This phase shift is, in fact, a function of resonating beam frequency[13] and can be measured[14].

In the typical ring resonator geometry presented in figure 1, a pump beam incident on a photorefractive crystal placed inside the cavity induces scattered light, which can give rise to self-sustained oscillation. Oscillation starts from this scattered light which gets amplified through subsequent two-beam coupling interaction with the pump beam in the photorefractive crystal. An oscillation beam builds up, if the two-beam coupling gain is above threshold (i.e. when gain exceeds losses). This beam can reach high intensity even with a moderate value of two-beam coupling coefficient provided the cavity losses, including the crystal's absorption, are small[15]. In the photorefractive ring configuration, light propagation inside a cavity should be unidirectional as the two-beam coupling gain is also directional and determined by the crystal's symmetry, alignment and the charge transport properties.

When the resonator's gain is below the threshold, the oscillation will decay. This occurs when the two-beam coupling gain (coupling coefficient) is too small or the scattered light is too weak to overcome the cavity losses and crystal's absorption. In this case, the injection of an external weak seeding beam can promote the resonator oscillations by creating additional scattered photons. Such a resonator can be regarded as equivalent to a driven non-linear oscillator and can therefore be approximated by a simple mathematical model.

The model of nonlinear dynamic behaviour of a single-mode seed-injected ring resonator we present here is based on the analysis developed by Anderson and Saxena[16] and also used by Jost and Saleh[17].

For simplicity, let us consider the one-dimensional model of a photorefractive ring resonator with an external seeding beam injected in a single resonator mode. The geometry of such resonator is shown in figure 2. We assume that optical electric field of the pump beam can be presented as:

$$\mathbf{E}_P(\mathbf{r},t)=\mathbf{E}_P(t)\exp[i(\mathbf{k}_P \bullet \mathbf{r}-\omega_P t)]+c.c., \tag{1}$$

where $\mathbf{E}_P(t)$ is the slowly varying pump amplitude, and $\mathbf{k}_P$ and ω_P are its wave vector and angular frequency respectively. The electric field inside the resonator is assumed to consist of two components:

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}_R(\mathbf{r},t)+\mathbf{E}_S(\mathbf{r},t), \tag{2}$$

where $\mathbf{E}_R(\mathbf{r},t)$ and $\mathbf{E}_S(\mathbf{r},t)$ are the passive resonator and seeding beam electric field amplitudes respectively, which can be assumed to have the same form as the pump beam amplitude. $\mathbf{k}$ is the resonator field wave vector, ω its frequency and we assume that $|\mathbf{k}_S|=|\mathbf{k}_P|$.

We have chosen the resonator mode as well as the pump and seeding waves to be a uniform plane wave for simplicity. Also we use the mean field limit in which we neglect the amplitude variation along the cavity length. Moreover, we also assume the weak-field limit, i.e. the total intensity of the resonator field is far less than that of the pump beam $I_S, I_R << I_P$. Finally we take all beams to have the same, extraordinary polarisation. They all propagate at small angles to each other and with respect to the cavity axis. We can also assume that the resonator mode frequency ω is nearly equal to the pump angular frequency ω_p.

The optical field inside a cavity that contains a lossy medium with conductivity σ is given by the following wave equation[18]:

$$\nabla^2 \mathbf{E}_R - \mu_o \sigma \frac{\partial \mathbf{E}_R}{\partial t} - \mu_o \varepsilon \frac{\partial^2 \mathbf{E}_R}{\partial t^2} = -\frac{1}{\varepsilon_o} \nabla(\nabla \cdot \mathbf{P}_{NL}) + \mu_0 \frac{\partial^2 \mathbf{P}_{NL}}{\partial t^2}, \tag{3}$$

where $\mathbf{P}_{NL}(\mathbf{r},t)$ is the non-linear polarisation of the photorefractive medium induced by contributions from all fields inside the crystal. We neglect here the term ($\nabla \cdot \mathbf{P}_{NL} \approx 0$) that is due to the effect of dispersion. Taking into account all the earlier assumptions the nonlinear polarisation can be written as:

$$\mathbf{P}_{NL}(\mathbf{r},t) = 2\varepsilon_0 \{\mathbf{E}_P(\mathbf{r},t) + \mathbf{E}(\mathbf{r},t)\} \Delta n(\mathbf{r},t) \approx 2\varepsilon_0 \mathbf{E}_P(\mathbf{r},t) \Delta n(\mathbf{r},t), \tag{4}$$

where $\Delta n(\mathbf{r},t)$ is the refractive index change in the photorefractive material. This photoinduced change in refractive index Δn is created by the interference pattern between all beam that are present: pump, seeding and the resonator beams :

$$\begin{aligned} I(\mathbf{r},t) &= \frac{1}{2}[\mathbf{E}_P(\mathbf{r},t) + \mathbf{E}_R(\mathbf{r},t) + \mathbf{E}_S(\mathbf{r},t)]^2 \\ &= I_0(t)\left[1 + \left(\frac{\mathbf{E}_P \mathbf{E}_R^*}{I_0} \exp[i(\Delta\mathbf{k} \bullet \mathbf{r} - \Delta\omega\, t)] + c.c\right) + \left(\frac{\mathbf{E}_P \mathbf{E}_S^*}{I_0} \exp[i\Delta\mathbf{k}_P \bullet \mathbf{r}] + c.c\right)\right] \\ &= I_0(t) + I_1(\mathbf{r},t) + I_2(\mathbf{r},t), \end{aligned} \tag{5}$$

where $\Delta \mathbf{k} = \mathbf{k}_P - \mathbf{k}, \Delta \mathbf{k}_P = \mathbf{k}_P - \mathbf{k}_S, \Delta\omega = \omega_P - \omega, I_0(t) = |\mathbf{E}_P|^2 + |\mathbf{E}_R|^2 + |\mathbf{E}_S|^2$, $I_1(\mathbf{r},t)= E_p E_R^* \exp[I(\Delta\mathbf{k}\bullet\mathbf{r}-\Delta\omega t)]+c.c.$ and $I_2(\mathbf{r},t)=E_p E_s^* \exp[i\Delta\mathbf{k}_p.\mathbf{r}]+c.c.$ We have assumed that the interference between the resonator beam and seeding beam as well as terms with higher frequencies such as $2\omega_p$ are negligible.

The modulated terms, namely the second (I_1) and the third (I_2) terms are particularly interesting. I_1 is responsible for creating a moving grating and I_2 for forming a stationary interference pattern between the pump and the seeding beam.

The time development of Δn as a function of intensity modulation can be shown to be equal to:

$$\left[\frac{\partial}{\partial t} + \frac{1}{\tau}\right]\Delta n(\mathbf{r},t) = i\Gamma\{I_1(\mathbf{r},t) + I_2(\mathbf{r},t)\}, \tag{6}$$

where $\Gamma = \gamma n_{st}^3 r_{eff} / 2I_p\tau_c$ and γ is the complex coupling constant, τ is the intensity-dependent response time, which can in principle be complex, $\tau_c = \tau\ I_o/I_p$ is a photorefractive time constant, n_{st} is the background index of refraction, and r_{eff} is the effective electro-optic coefficient of the crystal. We assume the solution of this equation to be a linear superposition of two separate terms:

$$\Delta n(\mathbf{r},t) = Q_1(t)\exp[i(\Delta\mathbf{k}\bullet\mathbf{r} - \Delta\omega t)] + Q_2(t)\exp[i\Delta\mathbf{k}_P \bullet \mathbf{r}] + c.c., \tag{7}$$

where $Q_1(t)$ and $Q_2(t)$ are two slowly varying components of the index grating complex amplitude. Substituting into equation (6) for the refractive index change we obtain the equations for grating amplitudes:

$$\frac{dQ_1}{dt} = -\left[\frac{1}{\tau} - i\Delta\omega\right]Q_1(t) + i\Gamma\left(\mathbf{E}_P\mathbf{E}_R^*\right),$$

$$\frac{dQ_2}{dt} = -\frac{1}{\tau}Q_2(t) + i\Gamma\left(\mathbf{E}_P\mathbf{E}_S^*\right) \qquad (8)$$

These two equations describe the time development of the index grating. The first component of the index grating, with amplitude (Q_1), will oscillate with the frequency difference, $\Delta\omega$, and decay in time when the pump beam is blocked. The other, stationary component (Q_2), decays smoothly in time, when the pump beam is blocked.

Let us consider again the field equation (3). If we assume, for simplicity, that the variations in the field intensity along the direction transverse to the resonator axis are slowly varying as compared to optical wavelengths, we can neglect transverse derivatives. Solving the wave equation (3) for the resonator field gives the following expression:

$$\frac{d\mathbf{E}_R}{dt} = -\frac{1}{2}\cdot\frac{\omega}{Q_R}\mathbf{E}_R - \frac{\mu_0}{2\omega L_R}\operatorname{Im}\left(\int_0^{L_R}\exp\left[-i(\mathbf{kr}-\omega t)\right]\cdot\frac{\partial^2\mathbf{P}_{NL}}{\partial t^2}dz\right), \qquad (9)$$

where L is the resonator length and integration is carried over L. Conductivity $\sigma = \varepsilon_o\omega/Q_R$ is related to the resonator quality factor Q_R.

Substituting expressions for grating components (8) into the resonator field equation (9) we obtain:

$$\frac{d\mathbf{E}_R}{dt} = -\frac{\omega}{2Q_R}\mathbf{E}_R + \alpha\mathbf{E}_P Q_1^* + \beta\mathbf{E}_P Q_2^*\sin(\Delta\omega t), \qquad (10)$$

where $\alpha = \frac{\mu_0 \varepsilon_0 \omega\, l}{L_R}, \beta = \frac{\mu_0 \varepsilon_0 \omega_P^2}{\omega L_R} \frac{2\sin((\mathbf{k}_S - \mathbf{k}) l/2)}{\mathbf{k}_S - \mathbf{k}} \cong \frac{\mu_0 \varepsilon_0 \omega_P^2}{\omega L_R} l$, and l is the length of the crystal.

The normalised form ($E' = E_R / E_P$) for the resonator field can be expressed as:

$$\frac{dE'}{dt} = -\frac{1}{2}\frac{\omega}{Q_R} E' + \alpha Q_1^* + \beta Q_2^* \sin(\Delta\omega t). \quad (11)$$

This is the equation for a driven non-linear oscillator.

The simple analysis of stationary and moving grating in a photorefractive medium we considered here shows that an injection of an external seeding beam into the resonator cavity makes the cavity behave like a driven non-linear oscillator with its output intensity periodically oscillating. The amplitude of these oscillations depends on the stationary grating amplitude and on the beat frequency. In the case when the intensity of the injected beam is zero, then the equation goes to a typical case "free" oscillator, as described in detail in work published earlier [16,17]. The other special case is when $\Delta\omega=0$ and then we observe synchronisation of the resonator and pump oscillations.

Experimental results

In our experiment we used a sample of $Rh{:}BaTiO_3$ pumped by a single-longitudinal mode 1.06 μm miniature diode pumped Nd: YAG laser. The output laser beam was split into pump and seeding beams. Four mirrors, three of them with high reflectivity (99.9%) and one with 90% reflectivity formed a ring resonator. The seeding beam was injected into the cavity through the R=90% mirror.

The pump beam power was kept constant at 100 mW and the seeding beam power was varied in the range from 16 μW to16 mW. Both beams had extraordinary polarisation.

The alignment of the resonator was optimised by examining the intensity of the seeding beam after a single pass inside the resonator and then after multiple passes. Before each new measurement, we erased the remaining grating by uniform illumination of the crystal and its subsequent small rotation.

We determined the maximum level of amplification inside the cavity by optimising the coupling coefficient by changing the incident angle of the pump beam and reorienting the crystal with respect to the resonator's axis. In the crystal sample we used (3200 ppm $Rh:BaTiO_3$), the coupling coefficient was measured to be 7.4 cm^{-1}.

The output beam was coupled out of the resonator via a beam splitter placed inside the cavity. The out-coupled beam was measured on a detector and read via a computer, where the temporal evolution of the output beam could be stored and analysed.

Figure 2 presents the experimental data on the resonator output with the seeding beam present. In both plots the measurement starts with both pump and seeding beams incident. When stable oscillation is established, we block the seeding beam and observe the fast decay of the resonator oscillation.

As can be seen in figure 2a, in addition to the steady-state oscillation we observed periodic behaviour, as expected from our theoretical analysis. The periodic variations in intensity originate from induced grating competition. The injected external beam forms one grating with the pump beam that is stationary (Q_2) and another that is moving (Q_1), as explained in the previous section. The relative strength of the two grating amplitudes vary and can be calculated from equation (8). The resonator beam diffracts from both gratings, and if the oscillating amplitude Q_1 is strong enough, periodic variation in the diffracted resonator beam intensity can be observed. When the pump beam is switched off, the grating will start to decay, but its effect on the resonator's intensity can persist for some time. When the strength of the temporal grating is small compared with the stationary pattern, a stable output (figure 2a) is observed.

As expected, the contribution from stationary and moving gratings have to be comparable in order to induce the small periodic fluctuations. The most regular fluctuations were observed for very weak seeding beams (below 20 μW). Similarly, when the gain and the geometry of the cavity favoured strong free oscillations, i.e. oscillating beams with powers of the order of mWs, the periodic fluctuations features were not clearly distinguishable.

Conclusions

We have carried out a theoretical analysis of a photorefractive ring resonator injected with an additional seeding beam that originates from the same laser as a pump beam. Using a general semi-classical theory of resonators we showed that the expression for the output resonator beam consists of two main contributions: a stationary grating and a grating that oscillates in time with the frequency difference between the pump beam and self-induced cavity oscillations. We have shown that the oscillating grating will cause periodic variation in intensity of the output resonator beam. This theoretical prediction has been confirmed by our experimental results from a ring resonator containing Rh:$BaTiO_3$ crystal pumped by a 1.06 μm beam. The strongest resonator beam with periodic oscillations in intensity was achieved with weak seeding beams.

Acknowledgments

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Figure captions

Figure 1 Typical arrangement for a ring resonator with a photorefractive crystal inside and a seeding beam. M1, M2, M3 and M5 - high reflectivity mirrors, M4 - partially reflective mirror, D- detector, BS – beam splitter.

Figure 2 Temporal evolution of the resonator beam showing the build-up and the decay of oscillation when the pump was turned off: a) stationary output; b) periodic output.

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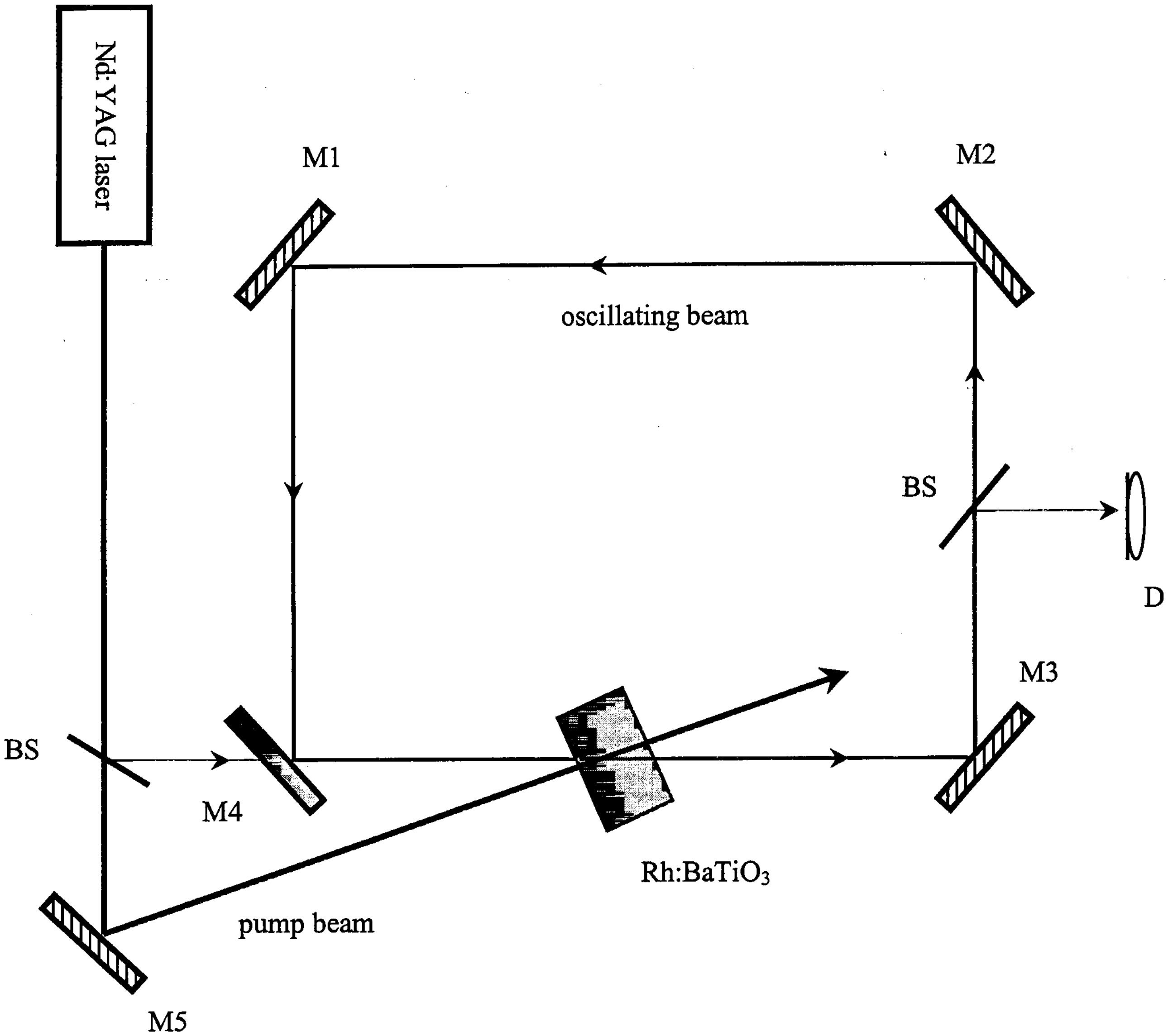

Fig. 1

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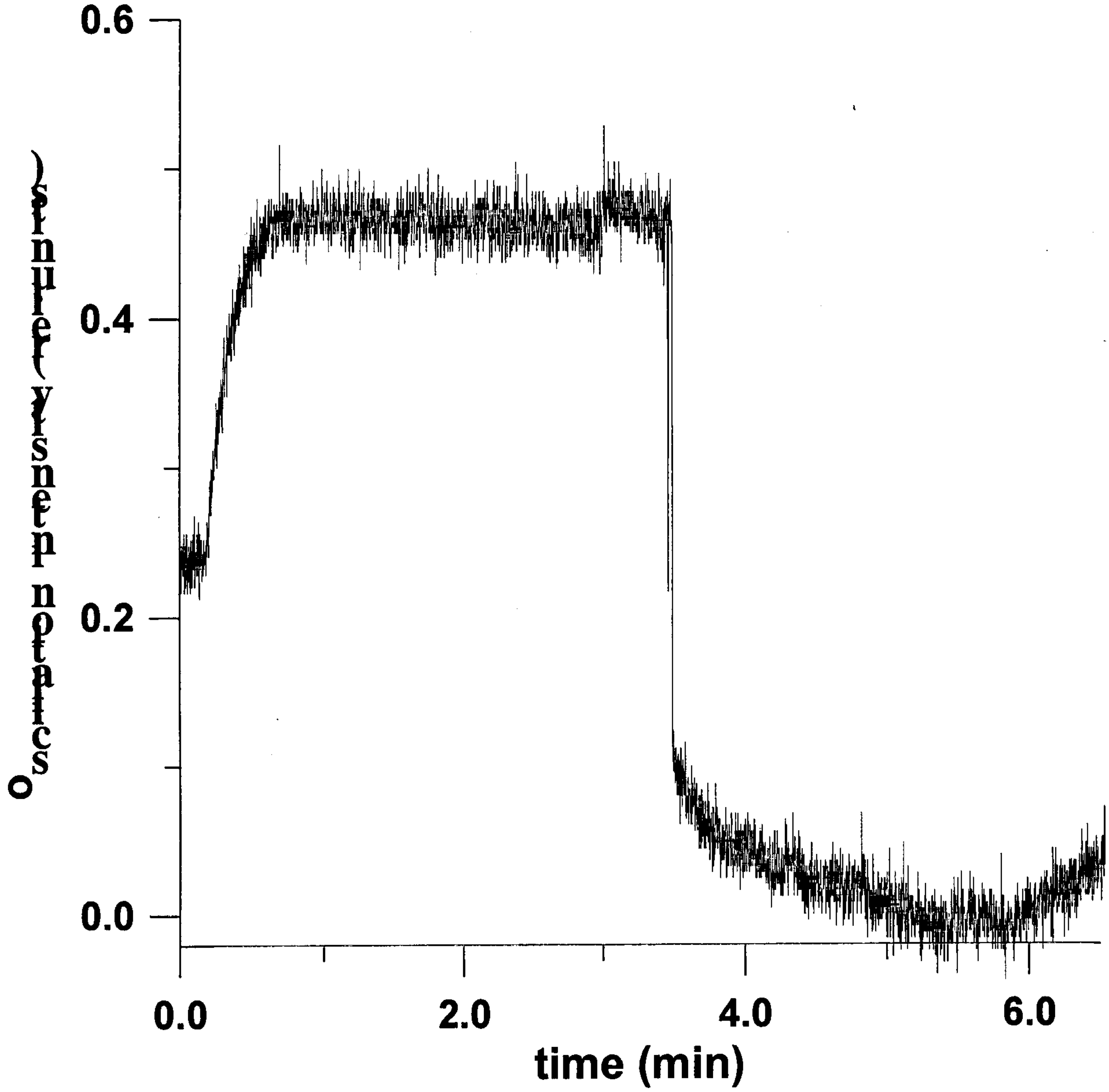

Figure 2a

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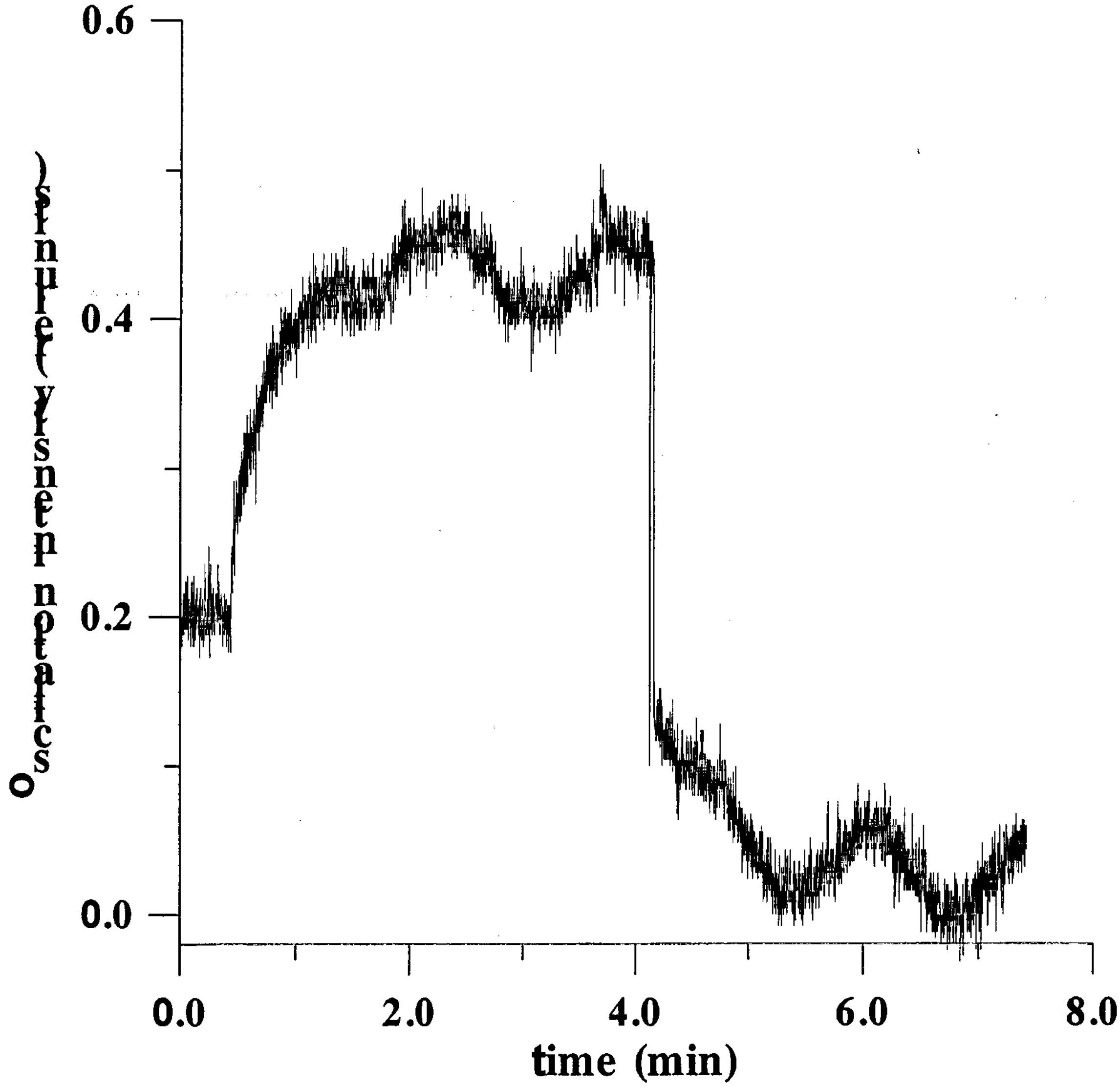

Figure 2b

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