Development of Brown-Roshko structures in the mixing layer behind a splitter plate

Neil D. Sandham and Richard D. Sandberg

Aerodynamics and Flight Mechanics Research Group, School of Engineering Sciences, University of Southampton, Southampton SO17 1BJ, UK
n.sandham@soton.ac.uk r.d.sandberg@soton.ac.uk

A direct numerical simulation has been performed of the near field of a mixing layer formed between a high-speed turbulent stream and a low-speed laminar stream. For improved realism, the simulation includes the splitter plate in the computational domain. The shear layer develops initially from the viscous sublayer of the upstream turbulent boundary layer. A trend towards two-dimensional or weakly oblique structures is observed immediately downstream of the splitter plate. The structures do not show very high coherence levels, suggesting that additional two-dimensional forcing may be present in experiments.

1 Introduction

The experiments of Brown & Roshko [1] can be said to have initiated a new era of research in turbulence, emphasising the role of coherent structures, that continues to the present day. Their experiments illustrated the development of large scale spanwise-coherent rollers. Importantly these structures did not disappear as the Reynolds number increased and thus could not be easily dismissed as a transitional or low Reynolds number phenomenon. Despite the subsequent research, a convincing explanation of the origin of the striking two-dimensionality of Brown-Roshko structures has been elusive. A simple explanation might be that the primary inviscid instability is most unstable as a two-dimensional disturbance and hence the mixing layer, which is convectively unstable, will naturally tend to amplify such structures in preference to oblique waves. However, when one looks closer at the amplification rate curves it is clear that the curves are almost flat for wave angles near zero degrees. Indeed, the temporally-developing simulations of Rogers & Moser [2] only showed clear Brown-Roshko structures when additional two-dimensional forcing was included in the initial condition.

The subject has re-emerged recently since direct numerical simulations of subsonic temporally-developing mixing layers are rather notable for the absence of Brown-Roshko structures (Mathew et al [3]). This is important because in this situation entrainment proceeded more by the old idea of 'nibbling' (small turbulence eddies entraining fluid at the edge of the turbulent region), rather than by 'engulfment' of large lumps of unmixed fluid into the centre of the shear layer, as would be the case if Brown-Roshko structures are active.

Other explanations for the presence of spanwise-coherent structures focus on the presence of a splitter plate in the experiments. This has two interesting consequences. Firstly there will be a small wake behind the trailing edge and such wakes may be absolutely unstable. The presence of a spanwise-coherent resonator due to such an instability may lead to spanwise coherency, although this may only happen for thick splitter plates. Secondly, the trailing edge of the splitter plate is a spanwise line and a receptivity process along that line might provide a bias towards two-dimensional structures, at the expense of oblique modes. Such a receptivity mechanism is only poorly understood at present, and suggests the need for simulations that include the trailing edge itself. One previous simulation of a subsonic mixing layer that includes the trailing edge is that of Laizet & Lambellais [4], who considered the differences between a bevelled and blunt trailing edge, finding a self-excited near wake for the latter configuration.

In the present contribution we modify the trailing-edge aeroacoustics simulation of Sandberg & Sandham [5] to study the initial development of the mixing layer formed after an infinitely thin trailing edge. A stuctural parameter is used to quantify the change in the flow from more streamwise-oriented structures in the turbulent boundary layer upstream of the trailing edge to spanwise-coherent structures immediately downstream of the trailing edge. Results are interpreted in the light of the above discussion.

2 Methodology

The code used for the present study employs fourth order accurate finite differences in space and a fourth order Runge-Kutta time advance with entropy splitting and other conditioning of the compressible Navier-Stokes equations (Sandham et al [6]). When run as a direct numerical simulation, as in the present contribution, the code does not use upwinding, filtering or additional dissipation. Inflow conditions are based on a synthetic turbulence approach, here combined with an inflow fringe zone.

A turbulent boundary layer under a freestream at Mach 0.6 is developed along the upper side of a flat plate, reaching a well-developed state with a Reynolds number of 2300 based on the displacement thickness at the trailing edge. At the end of the plate this turbulent boundary layer encounters a laminar boundary layer with a velocity a tenth that of the upper stream, but with the same temperature and density. All lengths have been normalized with the displacement thickness of the turbulent boundary layer at the trailing

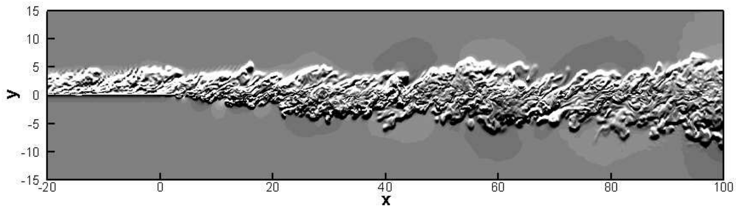

Fig. 1. Contours of vertical density gradient ($\partial\rho/\partial y$), illustrating the flow structure.

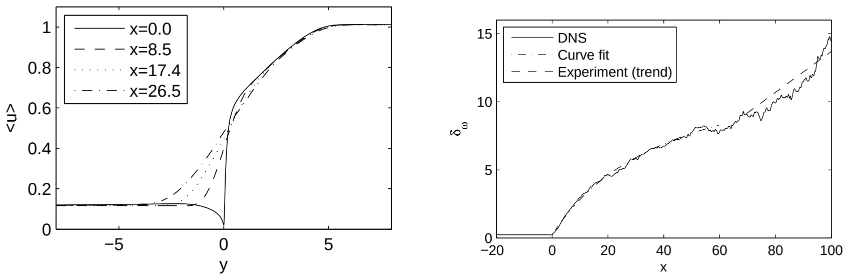

Fig. 2. Velocity profiles in the initial mixing region (left) and thickness of the shear layer (right), including the experimental trend ($d\delta_\omega/dx = 0.15$).

edge. The simulation uses 462 million grid points and has a spanwise domain width equal to 18. The simulation was run on 2048 processors of the UK national high performance computer facility, HECToR.

3 Results

Figure 1 shows a side view of the vertical density gradient for the developed flow. Although the simulation domain extends upstream to $x = -113.3$ and downstream to $x = 200.5$ we focus attention in this paper on the region of flow immediately downstream of the trailing edge. The domain for Figure 1 starts at $x = -20$, where the thick turbulent boundary layer on the upper side of the plate can be seen. The mixing layer originates in the viscous sublayer of the upstream turbulent boundary layer. The vortical flow from the upper side of the plate is gradually entrained into the developing mixing layer.

The change of the flow in the vicinity of the trailing edge is illustrated by means of velocity profiles on figure 2(a) and the streamwise evolution of shear layer thickness on figure 2(b). Velocity profiles are shown at the trailing edge and at locations in the early development of the mixing layer. It can be seen that the flow immediately behind the plate smoothes out rapidly, while it takes some distance downstream for the out region of the turbulent boundary layer to be affected. The vorticity thickness can be seen to rise rapidly from

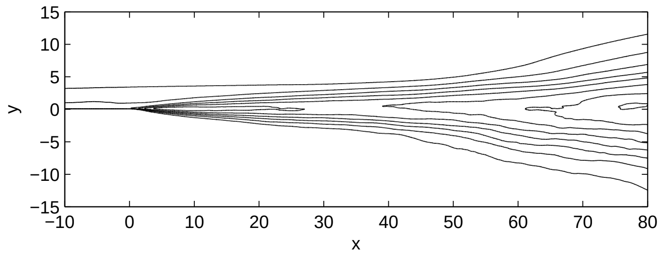

Fig. 3. Contour plot of turbulence kinetic energy.

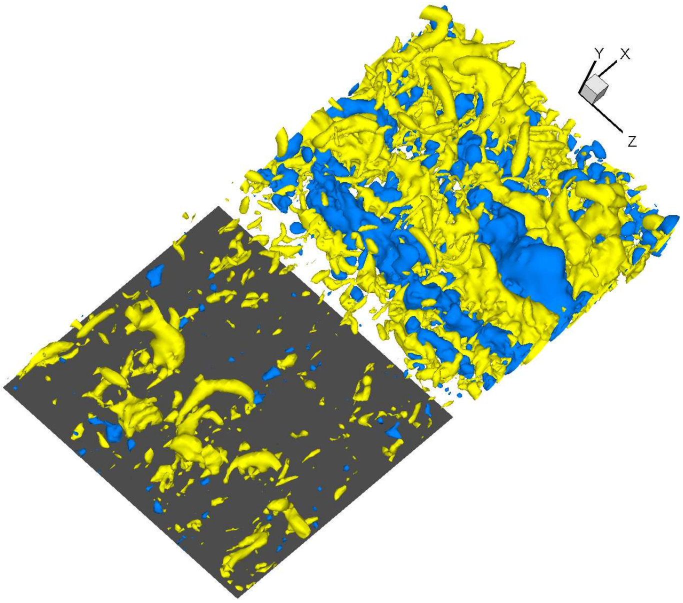

Fig. 4. Pressure fluctuations (dark surfaces are high pressure, light surfaces are low pressure), showing a close-up of the trailing edge and the early formation of spanwise-coherent structures. Flow is from lower left to upper right.

its low value in the upstream turbulent boundary layer. This plot also shows the expected growth rate based on experiments of $d\delta/dx = 0.15$ [1]. A limited statistical sample towards the end of the computational domain prevents a more detailed study of the later evolution towards a self-similar state.

The turbulence development is illustrated by contours of turbulence kinetic energy $k = 0.5\,\langle \rho u_i' u_i' \rangle$ on figure 3. A wedge of high turbulent energy develops just downstream of the trailing edge and spreads downstream. The outer part of the turbulent boundary layer is only entrained by $x = 45$, after which the rate of growth of the mixing layer increases significantly.

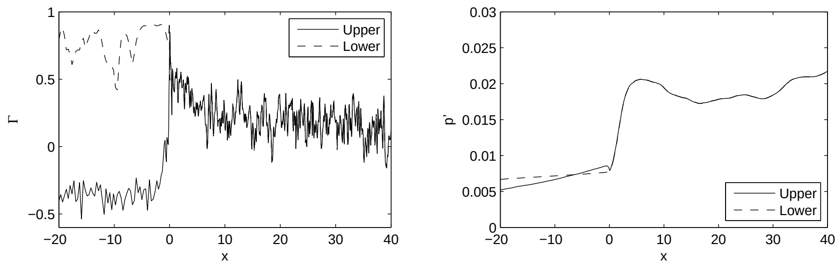

Fig. 5. Structure parameter Γ (left) and root-mean-square pressure fluctuations (right).

Figure 4 shows a close up of the trailing edge region, extending from -$20\delta^*_{TE}$ upstream to $20\delta^*_{TE}$ downstream. Surfaces of constant pressure perturbation, with the light surface locating low pressure and the dark surface high pressure, reveal the presence of coherent structures. Upstream of the trailing edge we see hairpin-type structures typical of the buffer region of a turbulent boundary layer. Within a very short distance of the trailing edge we can see evidence of spanwise organisation. The first high pressure zones downstream of the trailing edge are already preferentially oriented in the spanwise direction, with signs of dislocations across the span. These structures do not yet occupy the whole thickness of the turbulent shear layer and may perhaps be considered precursors of the later Brown-Roshko rollers, already containing the essential structure.

The change in structure near the trailing edge can be quantified by considering a parameter Γ defined by

$$\Gamma = \frac{\langle p'_x \rangle - \langle p'_z \rangle}{\langle p'_x \rangle + \langle p'_z \rangle}, \tag{1}$$

where a subscript denotes differentiation. This measure is equal to 1 for purely spanwise structures and -1 for purely streamwise structures. Figure 5(a) shows the development of Γ in the vicinity of the trailing edge and figure 5(b) shows the corresponding root mean square levels of the pressure. On the upper side of the plate (solid lines) the flow changes from a steamwise orientation of flow structure in the turbulent boundary layer to a marked spanwise orientation $\Gamma \approx 0.5$ after the end of the plate. Thus spanwise coherency in the mixing layer develops very close to the trailing edge. The high turbulence kinetic energy developed in the mixing layer immediately after the trailing edge may have a significant effect on the downstream structure. The spanwise-coherent pressure fluctuations seen on the lower side of the plate upstream of the trailing edge are long wavelength acoustic waves propagating upstream.

4 Conclusion

Direct numerical simulations have been conducted of a spatially developing mixing layer at a high velocity ratio, including the trailing edge in the computational domain. A turbulent boundary layer on the upstream plate undergoes a rapid change in structure as it passes the trailing edge. Associated with this is the development of a region of high turbulence kinetic energy and a change from streamwise- to spanwise-oriented flow structures. The region near the trailing edge appears to provide rapid amplification of a broad band of preferentially spanwise-oriented structures. These may have a strong influence on the subsequent downstream development of the flow. However, the strong spanwise coherency, seen experimentally from early in the shear layer development, is not reproduced. Two possible mechanisms not included in present simulations, but which may be active in experiments, have been identified. The first is a finite thickness splitter plate, which may act as a two-dimensional resonator. The second is the possible presence of two-dimensional acoustic waves in the experiments, which may interact with the splitter plate trailing edge to force a two-dimensional vortical response in the shear layer.

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