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APPLICATION OF THE THEORY OF INTEGRAL EQUATIONS TO THE DESIGN OF A MULTI-CHANNEL REVERBERATION SIMULATOR

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1. Introduction

During the last years the diffusion of multi-channel audio systems has dramatically increased. New products have been introduced in the consumer and high-end market and the research activity in this field has been stimulated by industry. An intense research effort has been dedicated to the study and application of new multi-channel audio technologies [1], [2], [3], [4], [5]. The diffusion of digital technologies which allow the processing of multiple audio signals with small computational cost has given the possibility of implementing a range of scientific theories into real audio systems. In parallel to this technological development, research in mathematics has applied modern theories to the study of electromagnetic and acoustic phenomena. Relevant examples consist of the application of functional analysis to the study of acoustic and electromagnetic scattering [6], [7]. Recent developments have tried to extend this theoretical approach to the physical reconstruction of a sound field with an array of loudspeakers and to near field acoustic holography [8], [9].

In the field of processing of audio signals, a large interest has always been dedicated to the study of digital reverberation. In parallel to the evolution of the well established "reverb engine" based on a delay network, recent scientific developments [10], [11] have allowed the introduction into the market of convolution reverberation systems, which allow the emulation of the reverberation characteristics of real rooms and halls. The underlying theory on which these "convolution reverbs" are based mainly consists in the convolution of the *dry* signal (that means of an audio signal recorded in ideally anechoic conditions) with the impulse response of a real room, which needs to be previously measured or simulated numerically. Most of the available convolution digital reverberation systems are based on single channel processing, even if multi-channel convolution reverberation systems have already been proposed [12].

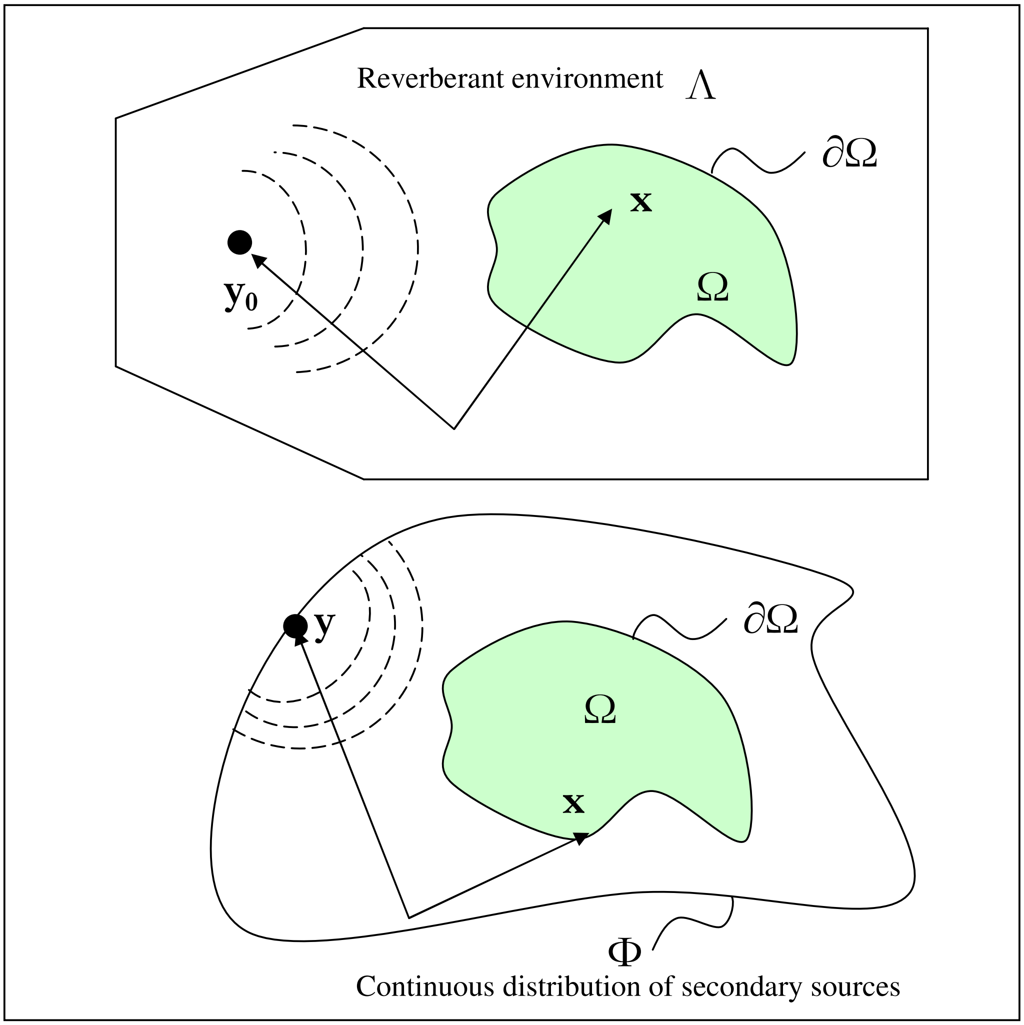

Figure 1: Diagram of the original sound field and reconstructed sound field

This paper introduces the theoretical fundamentals of a new multi-channel convolution reverberation system. The main idea of the proposed approach is to evolve the single channel or binaural rendering of the virtual hall with the physical reconstruction of the reverberant sound field over a bounded region of the space (also called *sweet spot*) using an array of loudspeakers arranged over a three dimensional surface.

The aim of this paper is to provide an introduction to the underlying theory of the system, while the practical implementation of this technique in terms of measurement techniques and design of digital filters is not presented here.

The method presented can be intuitively and briefly summarised as follows: when an acoustic source is radiating sound in a reverberant environment such as a room or a hall, the generated sound field can be represented by the value of the associated acoustic pressure as a function of space and time. It can also be assumed that this function is known on the boundary $\partial\Omega$ of a bounded region of the space Ω, as schematically represented in Figure 1. As discussed in [8], the knowledge of the acoustic pressure $p(\mathbf{x})$ on $\partial\Omega$ fully defines the sound field in the interior region Ω, as long as no sound source or scattering object is contained in Ω. The aim is then to reconstruct the target sound field using a loudspeaker array. The latter is assumed to be constituted by a uniform distribution of monopole sources, later on also called secondary sources, continuously arranged in an anechoic environment on the surface Φ, which can contain a region of the space equal to Ω (see Figure 1). The signals driving the loudspeakers are chosen to be such that the acoustic pressure on the surface $\partial\Omega$ is ideally the same as the target pressure profile $p(\mathbf{x})$. The loudspeaker signals are computed by solving, for each frequency of interest, an integral equation of the first kind. The latter represents an ill-posed inverse problem and therefore an exact solution does not exist. However, it is ideally possible to define an approximate solution to the problem by applying a regularization scheme. It is important to point out that, in the ideal case of a continuous distribution of sources, it is mathematically possible to compute a solution with an arbitrary level of accuracy, at the price of decreasing the stability of the system. In other words, the harder one tries to reduce the reconstruction error, the more the reconstruction will be degraded by the presence of errors in the data.

1. Sound field in a reverberant environment

Let Λ be a region of the space, representing the room or concert hall under consideration. Assuming that a point source is located at $\mathbf{y_0}$, a monochromatic sound field of angular frequency ω generated by this source can be described by the complex function $p(\mathbf{x})$, solution of the inhomogeneous Helmholtz equation

$$\begin{aligned} &\nabla^2 p(\mathbf{x}) + k^2 p(\mathbf{x}) = -\delta(\mathbf{x} - \mathbf{y_0}) \\ &\mathbf{x}, \mathbf{y_0} \in \Lambda \end{aligned} \tag{1}$$

The time dependence $e^{j\omega t}$ has been implicitly assumed and $k = \omega/c$ is the wave number, c being the speed of sound. An explicit solution for equation (1) can be calculated after imposing the impedance condition on the boundary $\partial\Lambda$, and is often called the Green function $G(\mathbf{y_0} \mid \mathbf{x})$. As an example, the walls of the hall could be assumed to be perfectly rigid; this would correspond to imposing Neumann conditions on $\partial\Lambda$

$$\frac{\partial p(\mathbf{x})}{\partial \mathbf{n}} = 0 \qquad \mathbf{x} \in \partial\Lambda \tag{2}$$

The explicit formulation of the Green function largely depends on the geometry of the domain Λ and it is in general a non trivial mathematical task. A general method of calculating the Green function consists of expressing it in terms of the eigenfunctions of the negative Laplacian, but the discussion of this technique is beyond the scope of this paper. For practical purpose and for $\mathbf{y_0}$ given position of the point source, the Green function of an enclosure with complex geometry and complex impedance conditions on the boundary can be computed numerically or measured.

For the case under consideration, it is sufficient to know the value of the Green function for a given source position $\mathbf{y_0}$ and for $\mathbf{x}$ being on $\partial\Omega$. $\partial\Omega$ is the boundary of the bounded and simply connected region Ω, with has boundary of class C^2, which is fully contained in Λ and which does not contain $\mathbf{y_0}$. Under these assumptions, it is obvious that the sound field in Ω satisfies the homogeneous Helmholtz equation. Following [8], it can be shown that the determination of the sound field in Ω from the knowledge of the sound field on the boundary $\partial\Omega$ is a well posed problem with a unique solution, provided that the wave number k does not correspond to one of the eigenvalues k_n of the negative Laplacian

$$\begin{cases} -\nabla^2\psi(\mathbf{x}) = k_n{}^2\psi(\mathbf{x}) & \mathbf{x}\in\Omega \\ \psi(\mathbf{x}) = 0 & \mathbf{x}\in\partial\Omega \end{cases} \tag{3}$$

or, in other words, provided that the angular frequency ω does not correspond to any of the resonance frequency of the cavity Ω with pressure release boundaries.

2. Reconstruction of the reverberant field

Let Φ be the boundary (of class C^2) of a bounded region of the space which can contain Ω (see Figure 1) and assume that a continuous distribution of secondary sources is arranged on Φ. Assume also that the electro-acoustic transfer function of a secondary source located at $\mathbf{y}$ can be described, for a given wave number k, by the free space Green function

$$g\left(\mathbf{y}\,|\,\mathbf{x}\right) = \frac{e^{-jk|\mathbf{x}-\mathbf{y}|}}{4\pi|\mathbf{x}-\mathbf{y}|} \tag{4}$$

In other words, it is assumed that the secondary sources can be assimilated to monopole sources in the free field. Let $a(\mathbf{y})$ be the continuous, complex function representing, for the given wave number, the signal of the secondary sources in the frequency domain. The sound field generated by the secondary sources in Ω can be represented by the integral

$$(Sa)(\mathbf{x}) = \int_\Phi g(\mathbf{y}\,|\,\mathbf{x})a(\mathbf{y})dS(\mathbf{y}) \qquad \mathbf{x}\in\bar{\Omega} \tag{5}$$

Thus, from what has been explained previously, if the secondary sources are driven with a function $a(\mathbf{y})$ such that

$$\int_\Phi g(\mathbf{y}\,|\,\mathbf{x})a(\mathbf{y})dS(\mathbf{y}) = \mathrm{G}(\mathbf{y_0}\,|\,\mathbf{x}) \qquad \mathbf{x}\in\partial\Omega,\ \mathbf{y_0}\notin\Omega \tag{6}$$

where $\mathrm{G}(\mathbf{y_0}\,|\,\mathbf{x})$ is the Green function solution of equation (1), then the sound field generated by the loudspeakers in Ω is the reproduction of the sound field generated by the monopole source at position $\mathbf{y_0}$ in the reverberant environment under consideration. The unknown of the problem is therefore the function $a(\mathbf{y})$, which can be determined by solving the integral equation (6).

3. Solution to the integral equation

Equation (6) is an integral equation of the first kind, and represents therefore an ill-posed problem [6]. This means that, in general, an exact solution to equation (6) does not exist. However, it is possible to compute an approximate solution by performing a singular value decomposition of the operator Φ and then performing the inversion after applying a regularization scheme. The procedure is analogous to that shown in [8], to which the reader is referred for a more in depth discussion.

The adjoint operator S^+ is defined as

$$(S^+ p)(\mathbf{x}) = \int_{\partial\Omega} \overline{g(\mathbf{x}\,|\,\mathbf{y})} p(\mathbf{x}) dS(\mathbf{x}) \qquad \mathbf{y} \in \Phi \tag{7}$$

Then one can solve the eigenvalue problem for the self adjoint, compact operator S^+S

$$(S^+ S a_n)(\mathbf{z}) = \int_{\partial\Omega} \overline{g(\mathbf{x}\,|\,\mathbf{y})} \int_{\Phi} g(\mathbf{x}\,|\,\mathbf{y}) a_n(\mathbf{y}) dS(\mathbf{y}) dS(\mathbf{x}) = \mu_n^{\,2} a_n(\mathbf{y}) \qquad \mathbf{z} \in \Phi \tag{8}$$

The set of eigenfunctions $\{a_n(\mathbf{y})\}$ represents a set of orthogonal functions on Φ. It is also possible to generate a set of orthogonal functions $\{p_n(\mathbf{y})\}$ by letting the operator S act on $\{a_n(\mathbf{y})\}$

$$(S a_n)(\mathbf{x}) = \mu_n p_n(\mathbf{x}) \tag{9}$$

Due to the orthogonality of $\{p_n(\mathbf{y})\}$, which is proved in [8], the target pressure profile on $\partial\Omega$ can be expressed as

$$p(\mathbf{x}) = \sum_{n=1}^{\infty} p_n(\mathbf{x}) \int_{\partial\Omega} \overline{p_n(\mathbf{z})} p(\mathbf{z}) dS(\mathbf{z}) + (Rp)(\mathbf{x}) \tag{10}$$

where $(Rp)(\mathbf{x})$ is the orthogonal projection of $p(\mathbf{x})$ on the null-space of the adjoint operator S^+. If the reconstruction of $p(\mathbf{x})$ is limited to the component of $p(\mathbf{x})$ belonging to the range of S, that is orthogonal to the null-space of S^+, then an approximate solution to equation (6) is given by [6],[8]

$$a(\mathbf{y}) = \sum_{n=1}^{\infty} a_n(\mathbf{y}) \frac{\mu_n}{\mu_n^{\,2} + \beta} \int_{\partial\Omega} \overline{p_n(\mathbf{x})} p(\mathbf{x}) dS(\mathbf{x}) \tag{11}$$

where the regularization parameter β has been introduced in order to obtain a stable solution. This regularisation scheme is known as Tikhonov regularisation [6]. In fact, the ill-conditioning of the inverse problem could also be understood in terms of the roll-off of the singular values μ_n, which in general accumulate at zero.

Provided that the set of functions $\{a_n(\mathbf{y})\}$ is known, equation (11) gives an explicit representation of the signals $a(\mathbf{y})$, which drives the secondary sources in order to recreate the sound field generated by a source in a reverberant environment from the knowledge of the original sound field $p(\mathbf{x})$ on $\partial\Omega$.

4. Conclusions

The basic theory of a multi-channel reverberation has been presented. The proposed method is based on the reconstruction of the sound field generated by a monopole source in a reverberant environment using an array of loudspeakers, ideally constituted by a continuous distribution of monopole sources. The reverberant sound field has been described in terms of a Green function, being the solution of the inhomogeneous Helmholtz equation. The sound field reconstruction problem has been formulated as an integral equation of the first kind. A method for the approximate solution of the inverse problem based on the singular value decomposition of the integral operator and on the Tikhonov regularisation scheme has been illustrated. The proposed solution explicitly shows the relation between the ill-conditioning of the inverse problem and the roll-off of the singular values of the integral operator.

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