nstitute of Sound and Vibration Research Introduction befining the physical sources of sound Non-radiating filter design urces of sound in an axi-symmetric jet Conclusion

Separating propagating and non-propagating dynamics in fluid-flow equations

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Introductio

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How to define the physical sources of sound?

Objectives

- Derive an expression for the physical sources of sound.
- Operation 2 Demonstrate that it is possible to separate the radiating and non-radiating parts of the flow.
- Ompute the physical sources of sound.

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Goldstein's theory Equations

Goldstein's theory



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Goldstein's theory Equations

Goldstein's theory



These sources should be close to the true sources of sound.

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Goldstein's theory Equations

Governing equation for fluctuating quantities

Flow filtering $\mathcal{L}f = \overline{f}$ (1)

Flow decomposition
$$f = \overline{f} + f'$$
(2)

Conservation of mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho \mathbf{v}_j}{\partial \mathbf{x}_j} = \mathbf{0}, \tag{3}$$
$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial \overline{\rho} \overline{\mathbf{v}_j}}{\partial \mathbf{x}_j} = \mathbf{0}. \tag{4}$$

Conservation of mass for fluctuating quantities

$$\frac{\partial \rho'}{\partial t} + \frac{\partial (\rho \mathbf{v}_j)'}{\partial \mathbf{x}_j} = \mathbf{0}.$$

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Governing equation for fluctuating quantities

Conservation of mass for fluctuating quantities

$$\frac{\partial \rho'}{\partial t} + \frac{\partial (\rho v_j)'}{\partial x_j} = 0.$$
(5)

Momentum conservation for fluctuating quantities

$$\frac{\partial(\rho \mathbf{v}_i)'}{\partial t} + \frac{\partial(\rho \mathbf{v}_i \mathbf{v}_j)'}{\partial x_j} + \frac{\partial \mathbf{p}'}{\partial x_i} = \frac{\partial \sigma'_{ij}}{\partial x_j}.$$
 (6)

Taking $\partial(6)/\partial x_i - \partial(5)/\partial t$ gives

$$\frac{\partial^2 \mathbf{p}'}{\partial x_i x_i} - \frac{\partial^2 \rho'}{\partial t^2} + \frac{\partial^2 (\rho \mathbf{v}_i \mathbf{v}_j)'}{\partial x_i \partial x_j} = \frac{\partial^2 \sigma'_{ij}}{\partial x_i \partial x_j}.$$
 (7)

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Governing equation for fluctuating quantities

Favre averaging,
$$\tilde{f} = \overline{\rho f} / \overline{\rho}$$
, (8)

Governing equation

$$\frac{\partial^{2} \boldsymbol{p}'}{\partial x_{i} \partial x_{i}} - \frac{\partial^{2} \rho'}{\partial t^{2}} + \frac{\partial^{2}}{\partial x_{i} \partial x_{j}} (\tilde{\boldsymbol{v}}_{i} \tilde{\boldsymbol{v}}_{j} \rho' + \overline{\rho} \tilde{\boldsymbol{v}}_{j} \boldsymbol{v}_{i}' + \overline{\rho} \tilde{\boldsymbol{v}}_{i} \boldsymbol{v}_{j}') = \frac{\partial^{2} \sigma_{ij'}}{\partial x_{i} \partial x_{j}} + \boldsymbol{s} \quad (9)$$

Source definition

$$\mathbf{s} = -\frac{\partial^2}{\partial x_i \partial x_j} \left(T_{ij} + \rho \mathbf{v}'_i \mathbf{v}'_j + \tilde{\mathbf{v}}_i \rho' \mathbf{v}'_j + \tilde{\mathbf{v}}_j \rho' \mathbf{v}'_i \right)$$
(10)
$$T_{ij} = -\overline{\rho} (\widetilde{\mathbf{v}_i \mathbf{v}_j} - \tilde{\mathbf{v}}_i \tilde{\mathbf{v}}_j).$$
(11)

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Problem description Filter defining properties Local filter Global filter

Problem description Parallel flow



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Problem description Pressure field



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Filter defining properties

Defining properties

Fourier transform

$$f(\mathbf{x}, t) \to F(\mathbf{k}, \omega)$$

 $\overline{f}(\mathbf{x}, t) \to \overline{F}(\mathbf{k}, \omega)$

Non-radiating condition

$$\overline{F}(\mathbf{k},\omega)=0$$
 for $|\mathbf{k}|=rac{|\omega|}{oldsymbol{c}_{\infty}}$

Additional requirement

$$\overline{F}(\mathbf{k},\omega) = F(\mathbf{k},\omega) \text{ for } |\mathbf{k}| \neq \frac{|\omega|}{C_{rec}}$$

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Local filter Filter definition

D'Alembertian filter

$$\bar{f}(\mathbf{x},t) = \left(\frac{1}{c_{\infty}^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)f(\mathbf{x},t),$$

Frequency domain

$$\overline{F}(\mathbf{k},\omega) = \left(|\mathbf{k}|^2 - \frac{\omega^2}{c_{\infty}^2}\right)F(\mathbf{k},\omega)$$

$$\Rightarrow \ \overline{F}({f k},\omega)=0 \quad ext{for} \quad |{f k}|=rac{|\omega|}{c_{\infty}}$$

Results

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Local filter

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Problem description Filter defining properties Local filter Global filter



Results

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Local filter

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Global filter



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Global filter

Gaussian filter

$$W(\mathbf{k},\omega) = \exp\left(-rac{(\mathbf{k}_{\mathbf{x}}-lpha)^2}{2\sigma^2}
ight) + \exp\left(-rac{(\mathbf{k}_{\mathbf{x}}+lpha)^2}{2\sigma^2}
ight)$$



Results

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Global filter

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50 *у*, п 0 -50-5050 100 150 0 *x*, m

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Global filter Results



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Problem description Filter defining properties Local filter Global filter

Global filter Validation

Comparison with analytical result along profile y = 15m



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Flow description Filter design Sound sources

Flow description



Mean flow excited at two frequencies:

$$ω_1 = 2.2,$$
 $ω_2 = 3.4,$ $Δω = 1.2.$

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Flow description Filter design Sound sources

Flow description



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Flow description Filter design Sound sources

Flow description

Hydrodynamic region



Acoustic region



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Flow description

Hydrodynamic region



Acoustic region



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Flow description Filter design Sound sources

Filter design

Tanh filter

$$W(\mathbf{k},\omega) = \frac{1}{2} \left[1 + \tanh\left(\frac{|\mathbf{k}| - k_{co}}{\sigma}\right) \right],$$

$$k_{co} = 1.3, \sigma = 0.2.$$



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Flow description Filter design Sound sources

Filter design

Pressure field p



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Flow description Filter design Sound sources

Filter design





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Flow description Filter design Sound sources

Filter design

Fluctuating pressure p'



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Flow description Filter design Sound sources

Sound sources Using non-radiating filter

Sound source s

Spectrum at (4.0, 0.55)



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Flow description Filter design Sound sources

Sound sources Using time average filter

Sound source s

Spectrum at (5.5, 0.5)



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Flow description Filter design Sound sources

Sound sources Evolution in time

(source)

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Conclusion and future work

Results

- Sound source definition
- Separation possible with convolution filters.
- Clearer physical interpretation of the sources.

Future work

- Mixing-layer and a two-dimensional jet.
- Physical mechanism behind the sound sources.

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