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A Review of Residual Stress Analysis using Thermoelastic Techniques

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Abstract

Thermoelastic Stress Analysis (TSA) is a full-field technique for experimental stress analysis that is based on infra-red thermography. The technique has proved to be extremely effective for studying elastic stress fields and is now well established. It is based on the measurement of the temperature change that occurs as a result of a stress change. As residual stress is essentially a mean stress it is accepted that the linear form of the TSA relationship cannot be used to evaluate residual stresses. However, there are situations where this linear relationship is not valid or departures in material properties due to manufacturing procedures have enabled evaluations of residual stresses. The purpose of this paper is to review the current status of using a TSA based approach for the evaluation of residual stresses and to provide some examples of where promising results have been obtained.

1. Introduction

Residual stresses may be introduced into a component during its entire manufacturing process; it is highly unlikely that a component in-service is free of residual stresses. Common routes for introducing residual stress are: (i) during the material production stage via deformation or thermal treatments, (ii) as a result of non-uniform heating or deformation during component manufacture, or (iii) during assembly as a consequence of welding processes or the interference of multiple parts. It is important to understand how residual stresses are distributed in a component to define its performance characteristics. This allows residual stress to be accounted for at the design stage and considered in the component life cycle. If the residual stress distribution is not known, then structural failure may occur due to the combined effect of the residual and applied stresses [1]. A large amount of tensile residual stress would decrease the applied tensile stress that would normally be required to induce plastic deformation, or indeed failure. Conversely a compressive residual stress would be beneficial, enabling the component to withstand a greater tensile stress; this is used in processes such as shot-peening and cold expansion [2]. At present, there are several techniques available for measuring residual stresses. Destructive methods are not always practical for an in-service industrial environment, while the non-destructive methods are typically expensive and time consuming. X-ray and synchrotron diffraction [3] are commonly employed in a research setting, while ultrasonic techniques [4] have proved useful in a manufacturing and industrial environment. Therefore there is a demand for a cheaper and quicker non-destructive, non-contact, full-field residual stress evaluation technique.

Thermoelastic stress analysis (TSA) [5] has been identified as a possible solution for a robust and portable means of non-destructive residual stress evaluation. TSA is now a well established non-contacting analysis technique that provides full-field stress data over the surface of a cyclically loaded component. TSA is based on the small temperature changes that occur when a material is subject to a change in elastic strain, generally referred to as the 'thermoelastic effect'. When a material is subjected to a cyclic load, the strain induced produces a cyclic variation in temperature. The temperature change (ΔT) can be related to the change in the 'first stress invariant', $\Delta\sigma_{kk}$, or the sum of the principal stresses [5]. An infra-red detector is used to measure the small temperature change, which can then be related to the stress using the following equation:

$$\Delta T = -KT_0 \Delta\sigma_{kk} \tag{1}$$

where T_0 is the absolute temperature and K is the thermoelastic constant, $K = \alpha / (\rho C_p)$, where α, ρ, C_p are the material constants of the coefficient of thermal expansion, mass density and the specific heat at constant pressure, of the material respectively.

The analysis that leads to equation (1) is dependent on three important assumptions [5]:

(i) the material behaviour is linear elastic,
(ii) the temperature changes in the material occur adiabatically,
(iii) the relevant material properties are not temperature dependent.

Belgen [6] first observed that this linear equation was not always correct. As a consequence of the temperature dependence of the thermoelastic constant, Belgen proposed that the temperature change was also dependent on the applied mean stress. Experiments by Machin et al [7] confirmed the existence of a mean stress dependence of the thermoelastic response. The assumptions above were reviewed by Wong et al [8], who later proposed a review of the general theory of the thermoelastic effect that did not rely on the previous assumptions. The idea that the material properties are independent of temperature was rejected. As a consequence, a more complicated nonlinear relationship between the temperature difference and the stress components was derived:

$$\dot{T} = \frac{T_0}{\rho C_p}\left[-\left(\alpha + \left(\frac{\nu}{E^2}\frac{\partial E}{\partial T} - \frac{1}{E}\frac{\partial \nu}{\partial T} \right)\sigma_{kk} \right)\dot{\sigma}_{kk} + \left(\frac{(1+\nu)}{E^2}\frac{\partial E}{\partial T} - \frac{1}{E}\frac{\partial \nu}{\partial T} \right)\sigma_{ij}\dot{\sigma}_{ij} \right] \tag{2}$$

where $\dot{T}$ is the rate of change of temperature, E is Young's modulus of the material, ν is Poisson's ratio, σ_{kk} is the first stress invariant and σ_{ij} is the stress tensor.

Equation (2) is known as the 'revised higher order theory' and accounts for the temperature dependence of the material properties that are contained within the thermoelastic constant. It is important to note that assumptions (i) and (ii) are retained in the derivation of equation (2), as it is the temperature dependence of the elastic properties that provide a route for residual stress measurement. In this paper, three approaches for residual stress measurement based on the thermoelastic response are reviewed. The first two are based on the revised higher order theory given by equation (2) and the third is based on changes in material properties contained within K that allows the extent of plastic deformation to be determined and is based on equation (1). Progress to date is detailed in the paper and the limitations and challenges in each approach are identified.

2. Mean Stress Dependence and the Second Harmonic

For the purpose of simplifying the analysis, the simpler case of uniaxial loading (where $\sigma_{11} = \sigma_{kk}$ and $\sigma_{22} = \sigma_{33} = \sigma_{12} = \sigma_{23} = \sigma_{13} = 0$) is examined, so that equation (2) can be reduced to:

$$\dot{T} = -\frac{T_0}{\rho C_p}\left(\alpha - \frac{1}{E^2}\frac{dE}{dT}\sigma_{11}\right)\dot{\sigma}_{11} \tag{3}$$

Equation (3) shows that the rate of temperature change is a function of the applied stress and its rate of change. In TSA, σ_{11} could be regarded as the mean stress, σ_m, and integrating equation (3) over a period from its initial state to final state provides a relationship between the change in temperature and the change in stress:

$$\Delta T = -\frac{T_0}{\rho C_p}\left(\alpha - \frac{1}{E^2}\frac{dE}{dT}\sigma_m\right)\Delta\sigma_{11} \tag{4}$$

The ‘revised higher order theory’ enables the mean stress dependence of the thermoelastic constant to be accounted for by the temperature dependence of the elastic modulus, and shows that the temperature response is dependent on the mean stress as well as the applied stress as shown in equation (4). Wong et al [9] continued this analysis by considering an experiment whereby a component in a uniaxial system is cyclically loaded about a mean stress, with a sinusoidal stress input such that:

$$\sigma_{11} = \sigma_m + \sigma_a \sin\omega t, \qquad \text{and thus,} \qquad \dot{\sigma}_{11} = \omega\sigma_a \cos\omega t \tag{5}$$

where σ_a is the applied stress amplitude and ω is the frequency of loading.

By substituting equations (5) into equation (3) the following expression was obtained:

$$\dot{T} = -\frac{T_0}{\rho C_p}\left(\alpha - \frac{1}{E^2}\frac{dE}{dT}\left(\sigma_m + \sigma_a \sin\omega t\right)\right)\omega\sigma_a \cos\omega t \tag{6}$$

or,

$$\dot{T} = -\frac{T_0}{\rho C_p}\left[\left(\alpha - \frac{1}{E^2}\frac{dE}{dT}\sigma_m\right)\omega\sigma_a \cos\omega t - \frac{1}{2E^2}\frac{dE}{dT}\omega\sigma_a^2 \sin 2\omega t\right] \tag{7}$$

Then, as before, equation (7) can be integrated over a period of time between $t = 0$ and t to give an expression for the temperature at time, t, as follows:

$$T = -\frac{T_0}{\rho C_p}\left[\left(\alpha - \frac{1}{E^2}\frac{dE}{dT}\sigma_m\right)\sigma_a \sin\omega t + \frac{1}{4E^2}\frac{dE}{dT}\sigma_a^2 \cos 2\omega t - \frac{1}{4E^2}\frac{dE}{dT}\sigma_a^2\right] \tag{8}$$

The first term in equation (8), varying with the frequency of the applied load, is dependent on both the applied mean stress and the stress amplitude. The second term in equation (8) varies at twice the frequency and is proportional to the square of the stress amplitude. The third term is not a function of the loading frequency. In conventional TSA it is the practice to reject all data other than that obtained from the fundamental loading frequency. Wong et al [9] observed that by obtaining data at this frequency and at the second harmonic, it would be possible to derive two simultaneous equations where the two unknowns are the cyclic stress amplitude and the mean stress. Subsequently, it was recognised that thermoelastic stress analysis may potentially yield enough information to provide a technique for deriving the residual stress in a component.

In TSA the practice is to sample the minimum and maximum values of T in each cycle such that ΔT is measured, allowing the following expression to be derived from equation (8):

$$\Delta T = a \sin \omega t - b \cos 2\omega t \tag{9}$$

where $$a = -K_0 T_0 \sigma_a \quad , \quad b = -\frac{K_0 T_0}{4\alpha E^2}\frac{\partial E}{\partial T}(\sigma_a)^2$$

and $$K_0 = \frac{1}{\rho C_p}\left(\alpha - \frac{1}{E^2}\frac{dE}{dT}\sigma_m\right)$$

K_0 is the revised version of the thermoelastic constant that is a function of the mean stress and Young's modulus. Obtaining temperature data at both ω *and* 2ω loading frequencies enables the applied cyclic stress amplitude, σ_a, to be calculated from the 2ω component, and subsequently, the mean stress, σ_m, from the following expressions derived from equation (9):

$$\sigma_a = \sqrt{\frac{-b\rho_0 C_p}{T}\frac{4E^2}{\partial E/\partial T}} \qquad \text{and} \qquad \sigma_m = \left(\frac{a\rho_0 C_p}{T\sigma_a} + \alpha\right)\frac{E^2}{\partial E/\partial T} \tag{12}$$

Validation of the revised thermoelastic theory was provided by Wong et al [9], by comparison of the thermoelastic signal from two uniaxially loaded aluminium specimens. One specimen was undeformed, the other was manufactured curved and straightened to provide a geometrically similar specimen. Strain gauges were used to identify the areas of tensile and compressive stress in the straightened specimen. Results showed that there were significant differences in the thermoelastic data from each specimen; this was attributed to the residual stress and good agreement was made between the data from the strain gauge readings and the equations. Dunn et al [10] confirmed that the mean stress effect is measurable in titanium (Ti-6Al-4V), aluminium (2024) and steel (4340); they observed a declining mean stress dependence of the thermoelastic constant with values of $\partial K \big/ \partial \sigma_m \ K_0^{-1}$ = 0.45, 0.31 and 0.11 GPa^{-1} for each metal respectively. Experimentation on a graphite epoxy composite proved inconclusive in terms of measuring the mean stress effect. The original observations regarding the mean stress effect were conducted on specimens made from a titanium alloy (TIMETAL 21S) [7].

Experiments carried out in [11] did not allow the derivation of the mean stress, however, they did confirm that a significant dependence existed for the two titanium alloys (as above) and also for a nickel alloy (Inconel 718). In further work [12], the thermoelastic signal of TIMETAL was found to vary by 21% over a mean stress range of -300 to +300 MPa. Recent work on Nitinol stents by Eaton-Evans et al [13] has confirmed that for Nickel-Titanium alloy, the thermoelastic constant has a high dependence on the mean stress. It was also shown in [13] that the dependence on mean stress in stainless steel was negligible. While this technique of utilising the mean stress effect does appear to provide a potential route for deriving residual stress, it is not without its limitations. Firstly, inspection of equation (2) highlights the sensitivity of the approach. Equation (2) is dominated by the magnitude of the $\left(1/\alpha E^2 \ \partial E/\partial T \ \sigma_m\right)$ term in comparison with unity. The $\partial E/\partial T$ term must be considerable in magnitude if the mean stress is to have any significant influence on the thermoelastic constant. The influence of the parameter can be estimated by setting $\sigma_m = \sigma_{ult}$, i.e. the maximum residual stress possible. It was found that the effect of this governing parameter was much greater in aluminium than for steel; ignoring this term for steel, gave an error of approximately 1.1% [14]. By comparing this to the natural error in the thermoelastic response due to signal noise (that can be up to 10% depending on surface coating and uniformity), and acknowledging that cyclic stresses used in TSA are not usually in the region of failure, it can be seen that this method could not reliably be used for steel components.

A further limitation of the technique relates to the measurement of the second harmonic component of the thermoelastic signal. Gyekenyesi [12] found that the magnitude of the second harmonic of the temperature variation was approximately 2% of the first harmonic. This was established for titanium, which was one of the most sensitive materials tested. Measurement of this small component is difficult, especially as its relative magnitude is not dissimilar to variations caused by signal noise. Subsequently, confidence in the repeatability of results relating to residual stress is an important consideration. While the mean stress effect has been clearly shown in aluminium, nickel and titanium based alloys, the fact that it cannot be reliably used with steel components severely limits the potential of the mean stress approach to thermoelastic residual stress measurement. This combined with the difficulty in measuring the second order effect, prevents this technique being a practical method of residual stress evaluation in metals thus far. The applicability regarding composite materials is unknown.

3. Mean Stress Effect and the Detector Response

An alternative approach to residual stress analysis which utilises the mean stress effect has been explored by Patterson et al [15]. This technique directly relates the detector response to the principal stresses, and most importantly, does not rely on detection of the small second order component of the thermoelastic response. In the presence of residual stress, the effective mean stress is assumed to be the sum of the applied mean stress, σ_{app}, and the residual stress, σ_{res}, such that the mean stress is $\sigma_m = \sigma_{app} + \sigma_{res}$. For an applied cyclic stress $\sigma = \sigma_m + \sigma_a \sin \omega t$, the thermoelastic signal, S, can be expressed using the analysis in [16] as:

$$\frac{S}{\sigma_a} = b_0 + b_1\sigma_{app} + b_1\sigma_{res} \tag{13}$$

where $b_0 + b_1$ are constants directly calculated from a combination of the material properties and the characteristics of the infra-red detector. It should be noted that $b_0 + b_1$ are only expected to be constant in the elastic region, and thus this approach is limited to evaluating residual stress in this region.

From a plot of $(S/\Delta\sigma)$ as a function of the applied mean stress, σ_{app} (Figure 1), a linear regression yields a graph with gradient b_1, and a y-intercept at $b_0 + b_1 \sigma_{res}$. Thus if b_0 was obtained at the experimental temperature for a material with no residual stress, then σ_{res} could be evaluated from the intercept value.

Initial findings [16] based on the previous studies of Machin et al [7] and Gyekenyesi and Baaklini [11] yielded promising results. Good linearity was observed and the regression lines showed a good fit to the data. However, further work [15] exploring the residual stress around cold expanded holes was inconclusive; good agreement of the TSA data with the known stress distribution was found in some areas, while significant scatter and differences were found in others. These findings bring into question the viability of this approach in its current form. The major disadvantage of this approach is that the constants b_0 and b_1 are functions of the detector response and the component temperature. Thus, if there is to be confidence in the repeatability of tests they must be carried out using the same detector in a temperature controlled environment. A further difficulty is that the thermoelastic response must be recorded over a range of applied mean stresses in order to obtain an accurate linear regression, which may not always be possible. If it is possible to correct for temperature variations as described in [17], more accurate results may be possible. Furthermore, if a radiometrically calibrated infra-red system is available, the values of ΔT can be obtained instead of the signal, S, potentially allowing a higher degree of accuracy. Further work with this approach is required to investigate its viability and accuracy. Since the approach utilises the mean stress effect which is governed by the parameter $\partial E / \partial T$ (see equation (4)), it is unlikely to be practical for steel components as $\partial E / \partial T$ is practically zero for most steels.

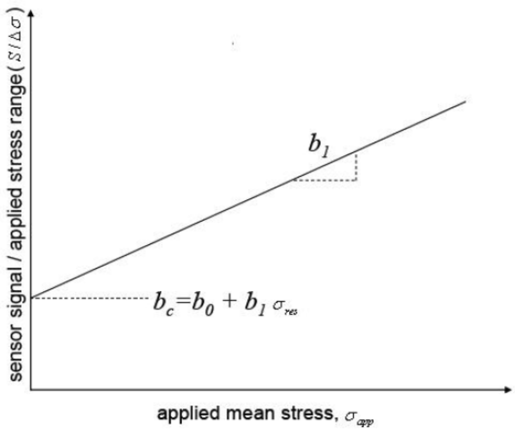
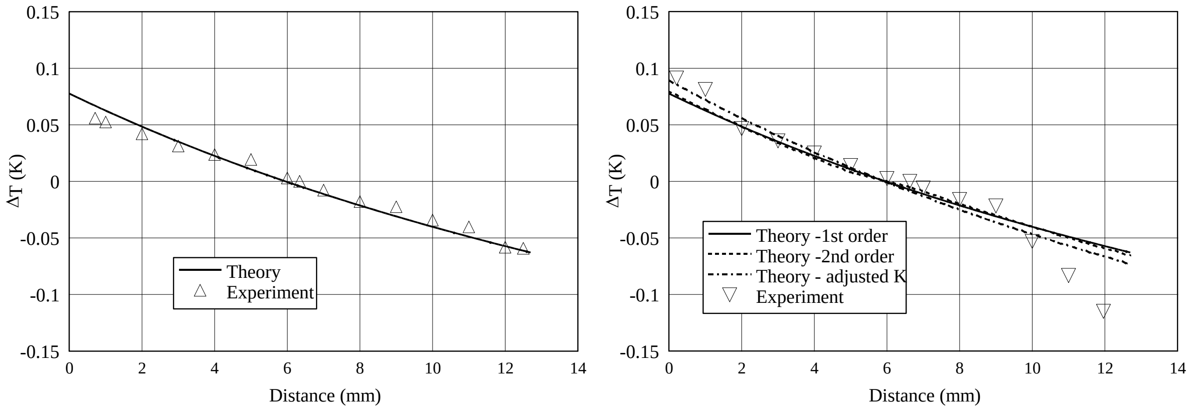


Figure 1: Schematic highlighting the relationships between applied mean stress and the constants b_1 and b_0 using the mean stress method. [15]

4. The Effect of Plastic Deformation on the Thermoelastic Constant

It has been shown [14] that the introduction of plastic deformation modifies the thermoelastic constant in some metals. It has been suggested that this change in thermoelastic constant can be used to estimate the level of plastic strain that a component has induced. Since plastic strain can be directly related to residual stress, there is a clear opportunity to derive a procedure for the assessment of residual stress using TSA, utilising the effect of plastic deformation. Rosenholtz et al [18], and Rosenfield et al [19] have both demonstrated that in steel and aluminium, an application of plastic strain will cause a change in the material property, α, the coefficient of linear thermal expansion. Rosenfield et al [19] also noted that this change in α increases significantly more when subjected to compressive strains, and increases less upon tensile plastic straining.

To indicate if plastic deformation causes a change in thermoelastic constant, a specimen can be loaded in uniaxial tension. If this type of specimen is loaded beyond the material's yield point and then unloaded, it will result in a residual strain; however, there is no residual stress as it can be fully relaxed by the elastic unloading. As a result, σ_m in the modified thermoelastic equation for a uniaxial stress system (equation (4)), becomes zero; therefore any change in the thermoelastic signal would be due to a change in one of the material properties, α, ρ or C_p and not due to a change in the mean stress. Quinn et al [14] conducted tests on steel specimens that had experienced different levels of plastic strain; one specimen was left unstrained, while three specimens were statically strained to give maximum tensile strains of 5%, 6% and 8%, and then unloaded. It was seen that the thermoelastic constant increased from 2.93 x 10^{-6} MPa^{-1} for the unstrained specimen to 3.19 x 10^{-6} MPa^{-1} for the specimen that had experienced 8% plastic strain. The change in the thermoelastic constant is small, but it was repeatable and was seen to increase linearly with the level of plastic strain experienced. Further work in [14] examined a curved beam. One component was machined to shape whilst the other was deformed. The thermoelastic response through the section of both components is shown in Figure 2. It is clear that in the deformed component (Figure 2b) the experimental data shows significant departure from the both the first order and second order theory in the most deformed part of the specimen. For the tensile part of the response this shows good agreement with the modifications observed in K. However, the departures are much greater in the compressive side which are consistent with the observations of [18, 19] where it was shown that exposure to compressive strain changes the coefficient of expansion significantly.



(a) Machined Component (b) Deformed component

Figure 2: Comparison of thermoelastic data to theory for (a) a machined component and (b) a deformed component. [14]

Further work by Quinn et al [20] indicated that the process of strain hardening has a significant effect on the change in thermoelastic constant that can be expected to occur as a result of plastic deformation. It was concluded that the change in thermoelastic constant was dependent on the material dislocation that occurs during strain hardening, and that the change in *K* for a material that does not strain harden would be significantly less than for a material that does. This method shows promise, in that the effect is repeatable and valid for steel components. However, there is a requirement that the residual stress in the component is caused by plastic deformation, and that the material under inspection experiences strain hardening.

5. Conclusions and Future work

Three approaches to residual stress evaluation based on thermoelastic stress analysis have been reviewed. They each show positive results but are not without their limitations, particularly with regards to accuracy and material.

The two approaches utilising the mean stress effect are not appropriate for steel components. Future work will concentrate on quantifying the change in thermoelastic constant resulting from plastic deformation, caused by both mechanical deformation and by heating effects. The variation in thermoelastic response as a consequence of compressive and tensile plastic strain will be explored for steel and aluminium. The possibility of using the revised higher order theory for elastic residual stress will be further explored.

6. Acknowledgement

This work was supported by Airbus UK. The notation adopted in equations (1-8) of section (1) is that of J Eaton-Evans [21].

References

[1] **Withers, P.J.** and **Bhadeshia, H.K.D.H.,** (2001) Overview Residual Stress Part 1 – Measurement Techniques, *Materials Science and Technology*, 17, 355-365.
[2] **Withers, P.J.** and **Bhadeshia, H.K.D.H.,** (2001) Overview Residual Stress Part 2 – Nature and Origins, *Materials Science and Technology*, 17, 366-375.
[3] **Webster, P.J.** and **Withers P.J.,** (2001) Neutron and Synchrotron X-Ray Strain Scanning, *Strain*, 37, 19-33.
[4] **Bray, D.E., Tang, W.** and **Grewal, D.,** (1997) Ultrasonic stress evaluation in a turbine/comprerssor rotor, *J. Testing and Evaluation,* 25, 503-509.
[5] **Dulieu-Barton, J.M.** and **Stanley, P.,** (1998) Development and applications of thermoelastic stress analysis, *J. Strain Analysis*, 33, 93-104.
[6] **Belgen, M.H.** (1968) Infrared radiometric stress instrumentation application range study, *NASA CR-1087.*
[7] **Machin, A.S., Sparrow, J.G.,** and **Stimson, M.G.,** (1987) Mean Stress Dependence of the Thermoelastic Constant, *Strain*, 23, 27-30.
[8] **Wong, A.K., Sparrow, J.G.** and **Dunn, S.,** (1988) On the revised theory of the thermoelastic effect, *J. Physics and Chemistry of Solids*, 49, 395-400.
[9] **Wong, A.K., Dunn, S.A.,** and **Sparrow, J.G.,** (1988) Residual Stress measurement by means of the thermoelastic effect, *Nature*, 332, 613-615.
[10] **Dunn, S.A., Lombardo, D.,** and **Sparrow, J.G.** (1989) The mean stress effect in metallic alloys and composites, *SPIE Stress and Vibration: Recent Developments in Industrial Measurement and Analysis,* 1084, 129-142.
[11] **Gyekenyesi, A.L.,** and **Baaklini, G.Y.,** (1999) Thermoelastic Stress Analysis: The mean stress effect in metallic alloys, *NASA-TM-1999-209376.*
[12] **Gyekenyesi, A.L** (2002) Thermoelastic Stress Analysis: An NDE Tool for Residual Stress Assessment in Metallic Alloys, *J. Engineering for Gas Turbines and Power*, 124, 383-387.
[13] **Eaton-Evans, J., Dulieu-Barton, J.M., Little, E.G.** and **Brown I.A.** (2006) Thermoelastic Studies on Nitinol Stents, *J. Strain Analysis*, 41, 481-495.
[14] **Quinn, S., Dulieu-Barton, J.M.,** and **Langlands, J.M.,** (2004) Progress in thermoelastic residual stress measurement. *Strain*, 40, 127-133.
[15] **Patterson, E. Du, Y.,** and **Backman, D.,** (2008) A new approach to measuring surface residual stress using thermoelasticity, *Proc. SEM XI Int. Congress and Exposition, Orlando, FL.*
[16] **Patterson, E.,** (2007) The potential for quantifying residual stress using thermoelastic stress analysis, *Proc. SEM Conference, Springfield, MA.*
[17] **Dulieu-Barton, J.M., Emery, T.R., Quinn, S.,** and **Cunningham, P.R.** (2006) A temperature correction methodology for quantatative thermoelastic stress analysis and damage assessment. *Materials Science and Technology*, 17, 1627-1637.
[18] **Rosenholtz, J.,** and **Smith, D.,** (1950) The effect of compressive stresses on the linear thermal expansion of magnesium and steel, *J. App. Physics,* 21, 396-399.
[19] **Rosenfield, A. R.** and **Averbach, B. L.,** (1956) Effect of stress on the expansion coefficient, *J. App. Physics,* 27, 154-156.
[20] **Quinn, S., Dulieu-Barton, J.M., Eaton-Evans, J., M., Fruehmann, R.K.,** and **Tatum, P.J,** (2008) Thermoelastic assessment of plastic deformation, *J. Strain Analysis*, 43, 451-468.
[21] **Eaton-Evans, J.** (2006) Thermoelastic stress analysis of intravascular devices, *PhD thesis University of Limerick*.