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Geometrical reasoning in the primary school, the case of parallel lines

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During the primary school years, children are typically expected to develop ways of explaining their mathematical reasoning. This paper reports on ideas developed during an analysis of data from a project which involved young children (aged 5-7 years old) in a whole-class situation using dynamic geometry software (specifically *Sketchpad*). The focus is a classroom episode in which the children try to decide whether two lines that they know continue (but cannot see all of the continuation) will intersect, or not. The analysis illustrates how the children can move from an empirical, visual description of spatial relations to a more theoretical, abstract one. The arguments used by the children during the lesson transcend empirical arguments, providing evidence of how young children can be capable of engaging in aspects of deductive argumentation.

Keywords: mathematics; geometry; primary school; elementary school; ICT; technology; teaching; parallel lines; reasoning; argumentation; proof

Introduction

Across the world, a common aim of primary/elementary school mathematics is to provide a foundation for proof and proving through children being expected to develop ways of explaining their mathematical reasoning. Such learning is fostered as children move into middle school/lower secondary school, with deductive proofs typically being introduced when children are 13-14 years old. What remains a central question for research in this area is how best to develop children's explanations in a way that appropriately supports their growing understanding of the nature of proof and proving in mathematics (Stylianou, Knuth and Blanton, 2009).

This paper reports on ideas developed during a consideration of data from a project which involved young children (aged 5-7 years old) in a whole-class situation using dynamic geometry software (specifically *Sketchpad*). The focus for this paper is a classroom episode in which the children try to decide whether two lines that they know continue (but cannot see all of the continuation) will intersect, or not. The episode relates to two important, and growing, areas of research in primary school education: first, the nature of proof and proving in the elementary grades, and secondly the development of understanding of spatial relations in the early years of school. These areas of research are summarised in the two sections that follow. These summaries are followed by an account of the selected classroom episode, analysed in terms of discursive features that mark the children's transition from an empirical, visual description of spatial relations to a more theoretical, abstract one.

Research on young children and proof

Research has pointed to the abrupt transition that children can encounter as they move from primary school, where proof can be absent, to secondary school mathematics, where it becomes more of a central concern (Balacheff, 1988; Ball *et al*., 2002; Jones and Rodd, 2001; Sowder and Harel, 1998). In order to mitigate the effects of this abrupt transition, several researchers have argued that proof should begin in the early grades, not just in geometry, but also in arithmetic (Bartolini-Bussi, 2009; Stylianides, 2007; Stylianou *et al*, 2009). Further, there is growing evidence that young children can be capable of engaging in deductive reasoning and proving (Carpenter, *et al*., 2003; Galotti *et al*., 1997; Light *et al*., 1989; Maher and Martino, 1996).

What it means to engage in 'proving' requires some explanation, as Jahnke (2007) notes, since a proof must depend on the concept of a theory. For Bartolini-Bussi (2009: 53), in the primary school, theories are "germ theories" that are "based on empirical evidence, with expansive potential to capture more and more principles." In other words, an experimental approach does not necessarily work against the production of general methods and the construction of mathematical proofs. Bartolini-Bussi argues that proving in the early years depends on the teacher being able to lead children from an experimental activity, through discussion, towards general methods and justification, in order to nurture a theoretical attitude.

In a somewhat different approach, Stylianides (2007) argues that proving in the primary grades should satisfy two principles: (a) what he calls the intellectual-honesty principle - the conceptualization that primary school geometry, for example, should be "honest to mathematics as a discipline and honoring of students as mathematical learners" (p. 1); and (b) what he calls the continuum principle - that there should be continuity in this conceptualisation across the different grade levels. Using a case study example, Stylianides draws parallels between a Grade 3 child's argument and Balacheff's (1988) notion of a "thought experiment" which is the highest level of Balacheff's hierarchy of arguments (and which transcends the empirical arguments that are used in lower levels). Here it is worth noting that Balacheff's "thought experiment" describes not only proof, but, perhaps more broadly, argument:

> The thought experiment invokes action by internalizing it and detaching itself from a particular representation. It is still coloured by an anecdotal temporal development, but the operations and foundational relations of the proof are indicated in some other way than by the result of their use. . . (p. 219)

Research on young children and parallel lines

As Bryant (2009: 9), confirms, children's spatial understanding begins early; certainly before the start of formal schooling. By five, according to Bryant, children can take in and remember the orientation of horizontal and vertical lines very well. In contrast, at this age, children have considerable difficulty in remembering either the direction or slope of obliquely-oriented lines. Yet, the research summarised by Bryant indicates that if there are other obliquely oriented lines (in the background) that are parallel to an oblique line, the children's memory of the slope and direction for the oblique line improves dramatically. It seems that children can use the parallel relation between the line that they have to remember and stable features in the background framework to store and recognise information about the oblique line.

Bryant concludes that younger children probably perceive and make use of parallel relations without necessarily being aware of doing so. The implication for teaching is that a key task for the teacher is to transform the children's implicit knowledge of parallel lines into explicit knowledge. A goal of the teaching experiment reported in this paper was to make children's implicit knowledge more explicit by inviting them to reason about the relationships between lines. Further, in keeping with the emphasis on proof and argument in the early years of school, the project followed Bartolini-Bussi in designing classroom tasks that would start experimentally but then provide an opportunity for nurturing a theoretical attitude.

An elementary school classroom episode on parallel lines

The classroom episode analysed in this paper comes from work with Grade 1 and Kindergarten children (aged 5-6) at a pre-K-6 (nursery) University Laboratory school (a school model informed by the original Laboratory School run by John Dewey at the University of Chicago). The school was in an urban area (rated as middle socio-economic status, SES). There were 22 children in the class from diverse ethnic backgrounds and with a wide range of academic attainment (25% being special needs learners). The project entailed working with the children for three days on a selection of geometry topics, each involving the dynamic geometry package *Sketchpad* (Sinclair and Crespo, 2006). Each lesson lasted 30 minutes and was conducted with half the class at a time. The children were seated on a carpet in front of a large screen, with two researchers, and the class teacher, present. The first author conducted the lessons. Each lesson was videotaped and transcribed. The lesson presented in this paper focused on conceptualising intersecting and parallel lines. The children had already had two previous lessons involving the dynamic geometry package *Sketchpad* but had not previously received any formal instruction relating to lines, intersections, or the notion of parallel lines.

The lesson began the children being shown several examples of pairs of lines, where some intersected and others did not. In talking about these examples, the children described the former as "touching," and were offered the more technical word "intersection" which they immediately connected to roads crossings - and, interestingly, cars crashing. The children were then shown two lines that were non-parallel but that did not intersect on the screen (see Figure 1). When asked whether the lines make an intersection, most children responded "no." After a few seconds, one boy said "Oh yes they do." Several students began talking at once, and one said, "Because they go out of the screen." So the instructor adjusted the screen image enough to make the intersection visible.

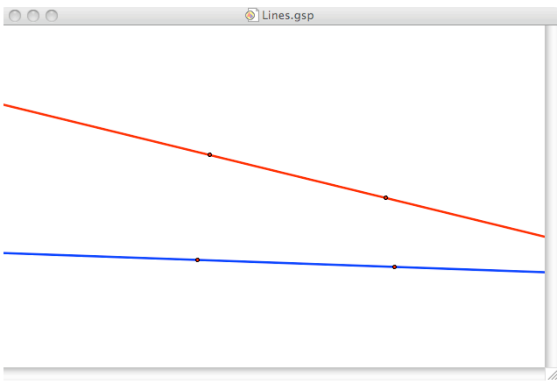

Figure 1: A non-visible intersection

The instructor then dragged the lines even further apart, so that the intersection was again not visible. This time most children said that the lines would intersect. Then a few said that they wouldn't. Jamie (all names are pseudonyms) added, "Because they are too far apart." Other children hedged; "I think they might" said one.

Given the lack of consensus, the instructor probed further.

Instructor:	Can we make some theories about why the lines might intersect?
Robert:	Because it's tilting [*referring to the red – upper – line*].

Zeb disagreed: "The lines, um, can't meet at the edge of the screen because they are too far apart and they can't just like suddenly just have a straight line going down and meet". Then Jamie seemed to change his mind:

Jamie:	Cause they are going like this [*tracing with his index finger two lines coming together*].
Instructor:	But do you think they would ever meet?
Jamie:	Yes, because they are both slanting and the red one is slanting toward the blue one.

Natasha agreed with Jamie, but neither argument seemed to convince Robert:

Natasha:	It's going to always connect somewhere because the red one is slanting so it's going to connect somewhere over here *[pointing toward the outside right of the screen]*.
Instructor:	Even if we can't see it, it's going to connect, intersect somewhere over here?
Robert:	I think it's never going to intersect
Instructor:	Why?
Robert:	Because I just do.
Instructor:	What do you think about the theory that the red is slanting more and more toward the blue?
Robert:	(Standing up) But the blue is also going like this *[using hands and arms to show that both lines are slanting]*.
Instructor:	Oh I see; interesting. So the blue is slanting too.
Robert:	As long as both…., the red's going down, the blue's going down beside it so the line can't just go like that *[bringing his hands together, curving the top one down to touch the bottom one]* and then intersect.

The instructor returned to a configuration where the intersection is visible and showed how both the lines can slant. Then Natasha offered a different reason:

Natasha:	But it's always going to slant because right there *[pointing to the left on the screen]* that's how thick it was so it's always going to slant.

When prompted to repeat her 'theory', Natasha said, "because there *[hand positioned so that her index finger and thumb were a certain distance apart]* isn't the same thickness and it's going to intersect because it always gets smaller." Natasha came to screen and put her index finger on the red line and her thumb on the blue and moved toward the intersection, showing how the separation decreased between her index finger and thumb.

The instructor then dragged the lines to make the intersection non-visible. Jamie explained why the lines will intersect: "Because the red one is slanting enough" *[he gets up to trace to lines off the screen and create their intersection with his fingers]*.

Discussion

At the outset, the children's geometric discourse is about shapes immediately visible to their visual field. So, for example, "line" is a linear segment drawn on the screen. This evolves into an unbounded process that leaves a linear trace, as can be seen in the way the children begin to talk about "they are going like this" and "the red one going down" and the "red one slanting more and more." This change may seem marginal at first, but it marks a significant leap from the geometric discourse of those who are captives of their visual field and speak about static visible objects, to the discourse of possibilities (hypothetical things: "it's going to connect somewhere other here") and abstract objects (the point of intersection) resulting from reified processes.

The role of the instructor is crucial in bringing about the change in discourse, not only in terms of the manipulating of the lines - which go from having visible, to invisible, intersections - but also in terms of modelling the new discourse. The questioning begins with "do the two lines meet?" and then turns into a more hypothetical formulation about "why they might intersect" - the former being about the static, visible lines and the latter going beyond the here-and-now, implying that the 'line' is not just what is contained in one's visual field. This discursive shift is evident in Natasha's statement "It's going to always connect […] so it's going to connect somewhere other there," which involves a hypothetical, dynamic way of talking. The instructor reinforces this way of talking when asking "Even if we can't see it, it's going to connect, intersect somewhere over there?" and when re-voicing the children's dynamic description that "the red is slanting more and more toward the blue."

Toward the end of the classroom episode, the children use the word 'intersection' to describe a place where two lines meet, but this place no longer needs to be visible - indeed, the children show awareness that they do not even need to find where the intersection actually is. Two new gestures are introduced by the children, that of the extending of the lines using one's arms, and that of the 'thickness' (the separation) between the lines – the latter represented as the distance between the thumb and the forefinger. What is more, two new ways of finding out whether two lines intersect are offered by the children: one involves focusing on the slanting of the two lines, and, particularly, whether one of the lines slants 'enough', while the other involves looking at the changing separation of the lines.

Concluding comments

In the classroom episode analysed in this paper, the children were being asked to come up with a method whereby they could predict whether two lines might intersect. Although not explicitly about parallel lines (though the word was eventually introduced to describe lines that the children argued would not intersect), their task involved analysing the relation between lines, and characterising the difference between lines that intersect and lines that do not - a characterisation that forms the basis for the definition of parallelism. Natasha and Jamie both offer arguments that qualify as thought experiments, in Balacheff's sense. The ability of the students to internalize and detach seems to be mediated by the dynamic sketch they used first to experiment with lines and then engage in hypothetical events and relations. Overall, the analysis illustrates how young children can be capable of transcending empirical arguments and engaging in aspects of deductive argumentation.

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References

Balacheff, N. 1988. Aspects of proof in pupils' practice of school mathematics. In D. Pimm (ed.), *Mathematics, Teachers and Children*. London: Hodder and Stoughton. (pp. 216–235).

Ball, D.L., C. Hoyles, H.N., Jahnke, and N. Movshovitz-Hadar. 2002. The teaching of proof. In L.I. Tatsien (ed.), *Proceedings of the International Congress of Mathematicians*. Beijing: Higher Education Press, Vol. III, pp. 907–920.

Bartolini-Bussi, M. 2009. Proof and proving in primary school: an experimental approach. In Fou-Lai Lin et al (eds) *Proceedings of the ICMI Study 19 Conference: Proof and Proving in Mathematics Education*. Taipei, Taiwan: National Taiwan Normal University. Volume 1, pp 53- 58.

Bryant, P. 2009. *Understanding Space and its Representation in Mathematics*. London: Nuffield Foundation.

Carpenter, T.P., M.L. Franke, and L. Levi. 2003. *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Portsmouth, NH: Heinemann.

Galotti, K.M., Komatsu, L.K., and Voelz, S. 1997. Children's differential performance on deductive and inductive syllogisms. *Developmental Psychology*. 33(1): 70-78.

Jahnke, H. N. 2007. Proof and hypotheses. *ZDM - The International Journal on Mathematics Education*. 39: 79–86.

Jones, K. and M. M. Rodd. 2001. Geometry and proof. *Proceedings of the British Society for Research into Learning Mathematics*. 21(1): 95-100.

Light, P., A. Blaye, M. Gilly,. and V. Girotto. 1989. Pragmatic schemas and logical reasoning in 6- to 8-year-old children. *Cognitive Development*. 4: 49-64.

Maher, C.A. and A.M. Martino. 1996. The development of the idea of mathematical proof: A 5-year case study. *Journal for Research in Mathematics Education*. 27: 194–214.

Sinclair, N. and S. Crespo. 2006. Learning mathematics in dynamic computer environment. *Teaching Children Mathematics*. 12: 437-444.

Sowder, L. and G. Harel. 1998. Types of students' justifications. *The Mathematics Teacher*. 91(8): 670–675.

Stylianides, A.J. 2007. The notion of proof in the context of elementary school mathematics. *Educational Studies in Mathematics*. 65: 1–20.

Stylianou, D., E. Knuth, & M. Blanton (Eds.). 2009. *Teaching and Learning Proof across the Grades: a K-16 perspective*. New York, NY: Routledge.

BSRLM geometry working group

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