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Optimizing credit limit policy by Markov Decision Process Models

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Introduction

1.1 Credit card

1.1.1 The credit card culture and market

Credit cards have become one of the most popular consumer credit products. According to the Encyclopedia Britannica, "The use of credit cards originated in the United States during the 1920s, when individual firms, such as oil companies and hotel chains, began issuing them to customers for purchases made at company outlets. The first universal credit card, which could be used at a variety of establishments, was introduced by the Diners Club, Inc., in 1958". There are several reasons accounting for the success and popularity of credit cards. Firstly an increasing number of consumers has a propensity to spend their future income since the late 80s and thus there is a surge on the demand of retail credit. Besides, it is more convenient to using a credit card for shopping than to use cash. The new online shopping era also provides a new platform for credit card development. Credit card issuers, who earn from the merchandisers fee and interest fee, have actively promoted the use of credit cards. They introduce cash reward programs, zero-balance-transfer offers or airline mileage schemes to increase their market share. In short, credit cards have become an important asset of consumers, merchandisers and card issuers.

While the general public may believe credit cards are so widely accepted and therefore will replace other traditional payment means soon or later, credit card issuers indeed face many challenges. First there is intense competition. Having a number of credit card products, it is important for lenders to make the right operation decisions in order to sustain their position in the market. For example, it is critical to decide a proper annual percentage rate (APR). This APR changes the loss and gain of the credit card portfolios. An over-priced APR drives many customers away; conversely, a low APR reduces the profit and increases the expected default. Other than competition driven by other credit card issuers, credit cards have been gradually replaced by other payment cards, such as debit cards or stored value smart cards. Consumers do not need to check or repay their monthly balance bill when using these payment cards. This is thus more convenient especially for consumers having enough capital in their saving accounts.

Although the growth of the credit card market has slowed down in the last few years, the credit card market is still very attractive to credit card issuers and therefore lenders have different strategies to attract new customers or strengthen the relationship with current customers. In this thesis, we look at the most traditional operational policy: increase the credit limit of their current customers. This policy is still widely used by card issuers. A general belief verified by Soman and Cheema (2002) is that increasing the credit limit of a credit card raises the credit card owner's propensity to spend. Consumers assumed lenders have some sophisticated models, which are used to determine appropriate credit limits, but that is not the case in reality.

So how do lenders currently decide on what credit limit to offer a credit card customer?

Often they use classification tree and the return/profit matrices (Trench et al., 2003; Lucas, 2007). Details are described as follows: Lenders require data from past campaigns and use these data to estimate the expected results (expected profit or expected default rate) of adjusting the credit limit. In addition, data related to customer behaviour (such as repayment or purchasing records) and external data (such as credit bureau or marketing data) are used to segment customers into different groups. Finally, organizational data relating to the lender's constraints (e.g. budget or maximum expected default) are also required to set up the decision model.

Using these data, lenders often segment borrowers into different groups with a classification tree and then calculate the corresponding risk/return matrix, i.e. they agree credit limits for each combination of risk band and average balance, which is considered a surrogate for the return to the lender from that customer. This approach is static in that it does not consider whether or how the customers default risk and profitability to the lender will change over time. Nor is there any model to guide what are the optimal credit limits to choose. Lenders have advocated a sequential model rather than the static model (Trench et al., 2003; Lucas, 2007) since the sequential model is able to monitor the change of credit card consumer's lifetime value. Moreover, the credit limit given to each of these risk/reward groups is usually a subjective judgment.

In the last few years lenders began to investigate how to model the problem so as to obtain optimal credit limit policies or optimal interest rate to charge (see Trench et al. (2003)). Their model though does not consider that if the economic situation changes then both the risk and reward of the credit card borrower is likely to change.

1.1.2 Motivations

In this thesis, we propose using Markov Decision Processes (MDP) to improve the credit limit decision. MDP models provide a way of making sequential decisions by considering the evolution of a customer's behaviour over time. It also allows one to calculate the profitability of a credit card customer under the optimal dynamic credit limit policy. One critical condition for using MDP models is having enough data to estimate the chance of transitioning from one state to the others. In the credit card industry consumers records can typically provide over a million observations. This is therefore a viable model to be applied in real credit card pricing models.

We aim to build the MDP model based on the behavioural score. Behavioural scores are used by almost all lenders to assess credit card accounts' default risk. Most lenders have been keeping consumers' behavioural scores for a number of years. Particularly with the advent of the new Basel Accord, lenders are required to keep such data for five years and are encouraged to keep it throughout the whole economic cycle.

Two credit card datasets, one from Hong Kong and one from the United Kingdom, are used throughout this study. The following two sections present the economy and credit card usage rate in these two countries so as to provide some background information for reference.

1.2 Hong Kong

1.2.1 The economy after 1997

Our data covered the period from 2002 to 2007 but we present an overview of the HK economy since 1997. Although these two events were not related, shortly after the sovereignty of Hong Kong returned to China, came the Asian financial turmoil, during which speculators targeted HK with a double market play (the HK currency system and the stock market). The Hang Seng Index, which is the stock market capitalization weighted stock market index representing the 40 largest listed companies in Hong Kong, registered a year-on-year drop of 52.2% in February, 1998. The Hong Kong Government countered this speculative act by intervening in the stock market. However, weakening external exports and local consumption resulted in a sluggish economy. The year-on-year Consumer Price Index (CPI) registered a negative growth in November, 1998 and that was the beginning of the deflation period, which lasted for six years. The Hong Kong economy shone for a while during the e-commerce era in 1999, during which the Hang Seng Index increased by 92% in August, 1999. The myth of e-commerce, however, burst later that year and the Hong Kong economy was marching into recession in 2000. According to the Hong Kong Monetary Authority, there were $73,000$ residential mortgage loans in negative equity which accounted for 16% of the total residential borrowers, and the unemployment rate reached 5.1% in 2001. During 2001 to 2003, there was lack of momentum to change the local economy. What further intensified the worsening economy was the outbreak of Severe Acute Respiratory Syndrome (SARS) in March, 2003. The Gross Domestic Product registered a year-on-year plunge of 7.5% in the second quarter of 2003. In June 2003, the number of residential mortgage loans in negative equity was over $100,000$ and the unemployment rate jumped to 7.9%.

The recovery of the HK economy is mainly due to the booming China market and is

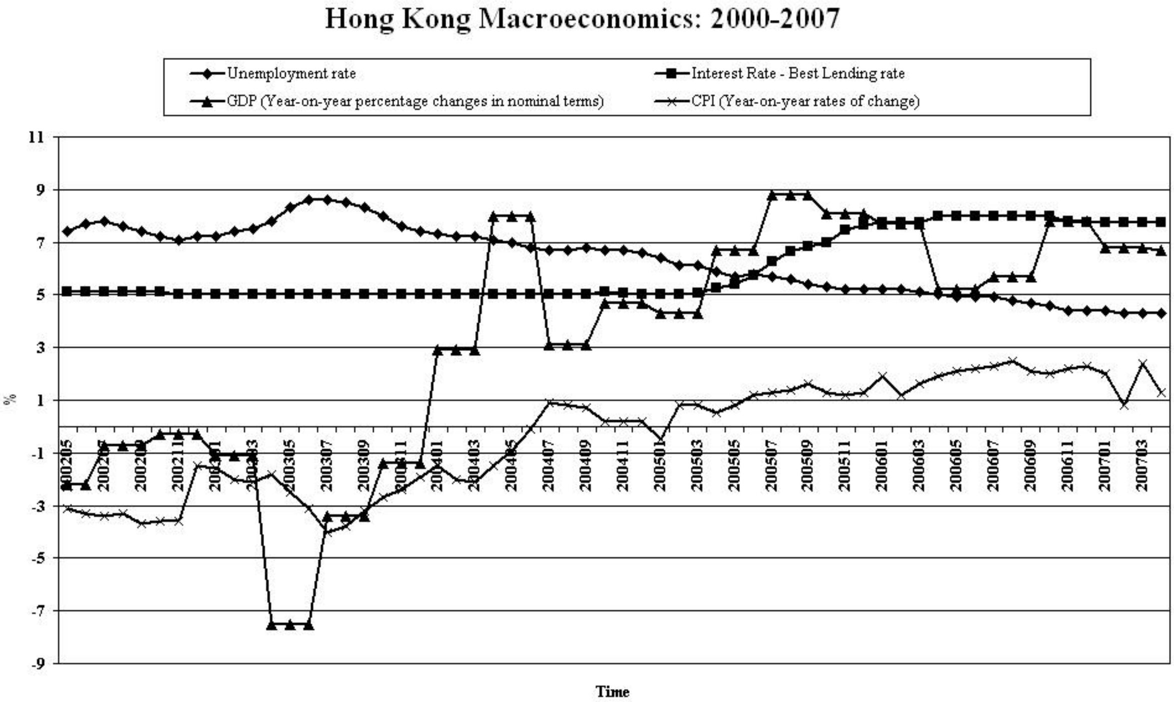


Figure 1.1: Overview of the Hong Kong macroeconomics

benchmarked by the new visa policy where mainland Chinese visitors can enter Hong Kong without traveling visas. In 2004, mainland visitors accounted for 30% of the total visitors and boosted the retail industry. By the end of 2004, the economy of HK finally resumed its momentum resulting in a year-on-year GDP growth rate of 8%. The economy of HK had gradually integrated with the China market and Hong Kong had become the financial center of mainland China, shown by the increasing number of mainland companies listed in the Hong Kong Exchange Board. All key macroeconomics indexes registered a strong growth over 2006. In 2007, there were less than 2,000 residential loans in negative equity and the government surplus was HK$50 billion (a rough average of US$95 per headcount).

1.2.2 The credit card market

Just like many other developed countries, the credit card culture has taken hold in Hong Kong since the 90s, although there is lack of official statistics about the credit card market in the 90s. According to the Hong Kong Monetary Authority, there were 9,217,000 credit cards in circulation in the fourth quarter of 2001 and on average every adult had 1.79 credit card[1]. From the second quarter to the fourth quarter of 2001, the delinquency rate registered a growth from 1.28% to 1.73%. A substantially high credit card charge-off ratio was recorded from 2001 to 2003. In the second quarter of 2002, there was a 6.5% fall in the number of credit cards in circulation. The credit card market gradually stabilised since the fourth quarter of 2004. From 2004 to 2007, the number of credit cards in circulation has increased continuously and the delinquency and charge-off ratio has remained lower than 1%.

Time	No. of credit cards*	Rate of increase	Total receivables at period-end	Average receivables	Delinquent amount	Delinquency ratio %	Charge-off amount	Charge-off ratios %
Q4 2001	9217	-	62050	6732.13	796	1.28	1268	2.14
Q2 2002	9488	2.94	60260	6351.18	1045	1.73	2055	3.41
Q4 2002	8865	-6.57	59247	6683.25	756	1.28	2237	3.78
Q2 2003	8732	-1.5	53985	6182.43	688	1.27	1574	2.9
Q4 2003	8784	0.6	56305	6409.95	519	0.92	1129	2.05
Q2 2004	8933	1.7	53707	6012.2	343	0.64	721	1.34
Q4 2004	9276	3.84	59256	6388.1	259	0.44	534	0.94
Q2 2005	9558	3.04	56992	5962.75	231	0.4	465	0.82
Q4 2005	10095	5.62	68056	6741.56	250	0.37	433	0.68
Q2 2006	10623	5.23	62905	5921.59	251	0.4	510	0.82
Q4 2006	10937	2.96	72211	6602.45	269	0.37	535	0.78
Q2 2007	11320	3.5	69114	6105.48	276	0.4	564	0.83
Q4 2007	11559	2.11	76886	6651.61	269	0.35	504	0.68

Average receivables = Total receivables/No. of credit cards

Total receivables, delinquent amount and charge-off amount are presented in HK$ million.

An account is defined as delinquent if the delinquent dates is more than 90 days.

"'*"' in '000

Accounts is called "'Delinquency"' if it is in arrears for more than 90 days but has not been charged-off by the lender.

Table 1.1: The HK credit cards statistics

One general perception about delinquency and charge-off ratios is they are correlated with the overall economy. It is believed that a deteriorating economy boosts the default

[1]According to the statistics of the 2001 Census provided by the Hong Kong Census and Statistics Department, there were 5,148,653 residences in age 20 or above.

rate. Checking the credit card statistics in Table 1.1, the charge-off ratios during Q4 2001 to Q2 2004 (which roughly covered the recession period) were higher than those of during Q2 2005 to Q4 2007 (which was the good time). Moreover, the credit card data show the credit card take-up rate is moving in the same direction with the economy. This shows the importance of incorporating macroeconomic measurements in credit card pricing models if one would like to understand the behaviour of credit card borrowers.

1.3 United Kingdom

1.3.1 The economy after 1997

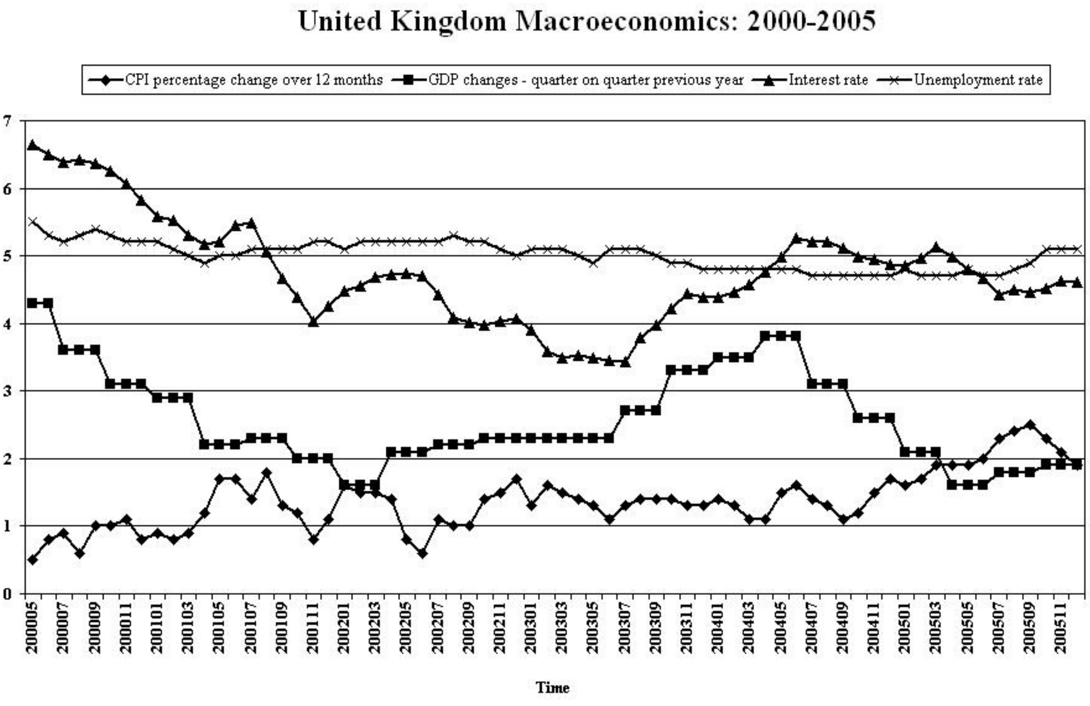


Figure 1.2: Overview of the United Kingdom macroeconomics

The UK sample data consisted of credit card data from 2001 to 2004. Here we present the economy of UK covered by this period from 1997 to 2007 as an overview. Unlike the economy of HK, the UK market was rather stable over the review period. During 1997 to 2004, a mild inflation with values less than 2% was registered. During the period, the year-on-year GDP growth remained less than 4%. After 2005, the year-on-year CPI values have gradually increased due mainly to the rising fuel cost.

1.3.2 The credit card market

Unlike the HK market, we cannot find any source of information about the delinquency ratio or charge-off rate of the UK credit card market. The UK credit card statistics presented in Table 1.2 are provided by The UK Payments Association (APACS). It shows the volumes of total payments and the values of total payments for cash, cheque, debit card and credit card from 2000 to 2006. For example, the first row indicates the number of cash transactions were 28,910 millions and increased by 9% in 2000 compared to the year before.

Year	Cash		Cheque		Debit card		Credit card		Total excluding cash*	
	Number	% change	Number	% change	Number	% change	Number	% change	Number	% change
					Volumes (in millions)					
2000	27910	9	2526	-4.8	2337	13.3	1577	6.9	9887	4.9
2001	27575	-1.2	2401	-4.9	2696	15.4	1695	7.5	10475	6
2002	26459	-4	2247	-6.4	2994	11.1	1825	7.7	10970	4.7
2003	25678	-3	2110	-6.1	3364	12.4	1952	7	11665	6.3
2004	24667	-3.9	1966	-6.8	3690	9.7	2049	5	12496	7.1
2005	23968	-2.8	1845	-6.2	4084	10.7	2007	-2.1	13270	6.2
2006	23069	-3.7	1702	-7.8	4512	10.5	1996	-0.6	13782	3.9
Year	Cash		Cheque		Debit card		Credit card		Total excluding cash*	
	Value	% change	Value	% change	Value	% change	Value	% change	Value	% change
					Values (in £billion)					
2000	261	2.3	1903	-0.4	76	17.5	85	12.3	4011	4.5
2001	268	2.4	1881	-1.2	95	24.9	94	10.2	4266	6.4
2002	267	-0.2	1830	-2.7	108	13.6	103	10.1	4459	4.5
2003	272	2	1772	-3.2	130	21	113	10	4625	3.7
2004	273	0.2	1720	-3	150	15.1	123	8.3	4921	6.4
2005	272	-0.2	1651	-4	171	13.7	124	1	5146	4.6
2006	274	0.8	1620	-1.9	195	14.2	126	1.5	5394	4.8

"‘*"’ including other storage cards.

Table 1.2: The UK credit cards statistics

The UK credit card statistics show that the use of cheque and cash have gradually decreased. On the contrary, the use of plastic card has increased, both in value and volume. It is noticeable that the use of debit card grew sharply. The credit card market is still here to stay but the increment slowed down since 2005. As mentioned, the competition from other plastic cards is one of the challenges faced by the lenders.

1.4 Scope of the study

The objective of this study is to explore the use of Markov Decision Process (MDP) models to support the decision of what credit limit to set for a credit card account so as to maximize the profitability of the account over time. The behavioural score is included in the model's state space. This score is calculated by lenders to assess the default risk of a borrower. One advantage of including this behavioural score in the state space is that the model can be used by almost every lender since this score is universally used in the credit card industry. Besides, the model is able to monitor the default risk and this means a more conservative policy as default risk is involved in the decision.

There are many technical issues about using MDP models in making credit limit decision that have not been addressed in literature. The first is coarse-classifying the state variables. It is common in the consumer credit industry to classify continuous variables into discrete bins. Coarse-classifying the state variables reduces the size of the state space and thus ensures the model's robustness. Another technique in building a MDP model is to look at the order of the Markov Chain. A Markov Chain is called first order Markov if the model's migration depends only on the current status and is independent of the history. Similarly, a n-th order Markov chain requires that the customer migration depends on the current and the previous $n-1$ periods of history. A general perception is that the accuracy of the model improves with the order of the model. However, the weakness of incorporating more history in the state space is that it reduces the model's robustness. Indeed, coarse-classifying and the choice of order are two inseparable issues in defining the MDP model. We will present the details in Chapter 3.

MDP models require numerous data to estimate the transition matrix but this is in general not a problem for the credit card application. The only problem is that the number of movements directly into default from some states is so low (quite possibly zero) that the resultant estimates of zero transition probability of default may affect the structure of the Markov chain. This affects the robustness of the MDP model. This problem of estimating default probabilities in low default portfolios is also recognized in the new Basel Accord. In Chapter 4, we show how one can use conservative estimators for the probability of default for those low default credit card accounts.

The behavioural score band transition matrix has an analogy with the corporate credit rating transition matrix. Since the 90's, there are a number of studies in corporate risk research. One main focus is to look at the impact of economy on credit rating transition. Moreover, central to the new Basel Accord is incorporating the economic environment into credit risk models. In Chapter 5 we present how one can put the macroeconomics measurements into our model and thus can look at the economic environment when one considers adjusting the credit limit of current credit card holders.

In this thesis we use two credit card datasets, one for HK and one for the UK, to generate the empirical results. Since HK experienced a severe economic downturn during 2002 to 2004, it is interesting to see whether and how the economic environment changes the credit migration pattern and credit limit policies. On the other hand, the economic landscape of UK is rather stable over the sampling period. We present some insights about the use of macroeconomic measurements for our model in Chapter 5.

Splitting the population based on their repayment history is presented in Chapter

6. In the credit card industry, borrowers are classified as *transactors* or *revolvers* where a transactor makes full repayment and a revolver carries part of his/her balance to the next month. Revolvers are more profitable than Transactors since lenders earn both interest charges and merchant fee from them. Different borrower types not only change the profit but it is also the fact that Transactors and Revolvers have diverse credit migration patterns. Therefore, it is more sensible to split the dataset by borrower type.

Our model provides insights into the interactions of customer lifetime value, behavioural score and economic environment. This study provides a mechanism to integrate default risk and operational decisions that can be readily applied in the credit capital management.

1.5 Research Questions

In particular, the research questions for this study are:

1. How can lenders use MDP models to adjust the credit limit of current credit card holders?
2. How can lenders use behavioural score as the key parameter in a credit limit decision model?
3. How can lenders incorporate the economy into a credit limit decision model?
4. What is the impact of the economy to the credit card holders' default risk?

We would like to emphasis that there are several reasons of choosing behavioural score as the key parameter. Firstly, behavioural score is the most popular risk indicator used by almost every lenders. Also, it properly captures the repayment behavioural of credit card holders over a period of time. Therefore one can use this single parameter to summarize the behavioural of each card holder as well as build a robust model. Moreover, this model is able to link the default risk (i.e. the behavioural score) and the credit limit decision in a model. Furthermore, building a transition matrix with behavioural score band has an analogy to the corporate credit rating transition. Thus we can compare our result with the corporate risk rating studies. Nevertheless, lenders would not generate the behavioural score for new credit card applicants and therefore the use of this model is limited to the current credit card accounts.

Chapter 2

Literature Review

In this chapter we review the literature on the application of Markov Decision Processes. MDP models have been studied extensively and the aim of this chapter is not to give complete coverage of MDP applications but rather to highlight some applications for illustration. This thesis proposes the use of MDP models in managing credit limits, and thus in the third section we review the relevant literature. We formulate an MDP model where the Markov chain is driven by the credit card accounts' behavioural score migration. There is limited research in the area, whereas many authors have examined bonds' credit rating migration. In the last section we review this literature as a reference.

2.1 The basic components of a Markov Decision Process

A decision maker or controller faces a problem of influencing the behaviour of a probabilistic system as it evolves through time (Puterman, 1994). During these time periods, the decision maker has to make decisions (or actions) to change the behaviour of this probabilistic system. The point of time when he makes a decision refers as the *decision epoch*. The decision maker has to know some information about the (current or past) behaviour of the system. This behaviour of the system is called the *state* in MDP's jargon. For each *state*, there are number of possible actions that can be chosen by the decision maker and the *set* that consists of all these actions is called the *action set*. After the decision maker makes a decision, there is an outcome (or utility) and it is called the *reward*. The state of the system at the next decision epoch is determined by the *transition probability* which is conditional on the current state and the chosen action. Prior to define a MDP model mathematically, we introduce some notations as follows:

$\mathcal{T}$	is the set of possible planning horizon of the problem (indexed by $t = 1, 2, \ldots, T$ where $T = \|\mathcal{T}\|$)
$\mathcal{S}$	is the set of states for the system (indexed by $s = 1, 2, \ldots, S$ where $S = \|\mathcal{S}\|$)
A_s	is the decision set for state s
$p_t(s'\|s, a)$	is the probability of the system changes next stage to s' if the current stage is t, state is s and decision a is chosen. We call this as the transition probabilities
$r_t(s, a)$	is the reward at time t if one applies decision a to state s
$V_t(s)$	is the value function of the system at state s at time t

A general representation of the equation would be as follows:

$$V_t(s) = \max_{a \in A_s} \left\{ r_t(s, a) + \sum_{s' \in \mathcal{S}} p_t(s'|s, a) V_{t-1}(s') \right\}, \forall s \in \mathcal{S} \tag{2.1}$$

The right-hand-side of (2.1) is the expected value over the next t periods if one selects action a at the end of the time period t for a system with state s. The reward function to the decision maker at the end of t is $r_t(s, a)$. The reward on the remaining $t-1$ periods is $V_{t-1}(s')$ if the state at the beginning of the next time period is s'. The chance of moving from the current state to s' is $p_t(s'|s, a)$ and thus the expected reward of moving from s to s' is $p_t(s'|s, a)V_{t-1}(s')$. The total expected profit at the remaining $t-1$ periods is the sum of the expected reward from all the possible state s', i.e. $\sum_{s' \in S} p_t(s'|s,a)V_{t-1}(s')$. The left-hand-side of (2.1) is the value function which measures the overcome outcome from the current time period to the end of the planning period. For more details about the MDP model, please refer to Puterman (1994).

2.1.1 Stationary or Non-stationary

One way to classify the MDP model is whether it is a stationary or non-stationary model. That is to say whether the reward function or transition probability depends on time t, i.e. the reward function and transition probabilities are identical across the whole planning horizon. If it is a stationary model, the transition probability and the reward function are re-defined as:

$p(s'\|s,a)$	is the probability of the system changes next stage to s' if the current state is s and decision a is chosen.
$r_t(s,a)$	is the reward if one applies decision a to state s.

In real application, it is more common to use a stationary model. This is mainly because a non-stationary model requires a substantial amount of data to estimate the parameters (i.e. the transition probability and the reward function) which is inadmissible in most real application. Therefore, just like many authors (White, 1985, 1993), we use a stationary model to formulate the credit limit decision problem.

2.2 General MDP applications

The root of the MDP model can be traced back to the 1940s, when it was developed as a mathematical model for making stochastic sequential decisions during the Second World War (Puterman, 1994). The model was not published until the 50s due to the wartime security requirements. Since then, MDP models have been developed into the most successful sequential decision modelling technique. Until now researchers have continuously applied MDP models in new areas such as internet auction modeling, wireless network planning etc. For a more extensive study, we recommend readers refer to Puterman (1994), White (1985) and White (1993).

A survey conducted by White (1993) has summarized the objectives and results of around one hundred MDP application papers. These applications cover population harvesting, agriculture, water resources, inspection, maintenance and repair, purchasing, inventory and production, finance and investment, queues, sales promotion, search, motor insurance claims, overbooking, epidemics, credit, sports, patient admissions, location, design of experiments and general applications. This shows the diversity of MDP applications.

White (1993) also generalizes some problems of applying MDP in the real world. The first is the Markovian assumption, which assumes the transition probability depends only on the current state. Mathematically, this means

$$p(s_{t+1}|s_0, s_1, \ldots, s_{t-1}, s_t) = p(s_{t+1}|s_t). \tag{2.2}$$

White (1993) suggests this assumption constitutes one of the greatest barriers of applying MDP in reality. A second concern is the infinite horizon of the model. Mathematically, one can prove that there exists an optimal policy for infinite-horizon MDP models. However, it is quite unrealistic to use the optimal infinite-horizon policy in real applications, since the Markovian assumption is likely to hold for a limited period of time only. The optimal solution is, however, just a reference for long term planning.

Two other survey studies conducted by White (1985, 1988) review papers on real MDP applications. "'Real"' is used because the reviewed papers were actually implemented, or at least had an effect on the actual decisions taken.

2.3 Credit limit decision using MDP models

A credit card is a revolving credit product and therefore persistent monitoring of consumption patterns can reduce default rates, increase the profitability, and enhance the customer-company relationship. It is common for lenders to use mathematical models to look at a consumer's consumption pattern and migration behaviour in the credit card industry. In the literature, however, there are limited studies that can be used by credit card lenders directly.

The study by Bierman and Hausman (1970) is the first to use MDP models to make sequential decisions on the credit amount offered. In their paper, they formulate a model such that the creditor makes a decision on whether to offer y amount of credit to a borrower. They assume that y is measured in terms of a multiple of some standard amount. For example, $y = 0.5$ represents offering one-half of the standard amount of credit. Bierman and Hausman assume that the amount of credit offered, y, and the probability of repaying the standard amount, p, have an exponentially declining relationship. Then, the probability of repayment given y amount is granted at time t is $P(repayment|y) = p^y$.

If the event of repayment follows a Beta prior with parameters (r, n) at time t, one can use (r, n) as the state variables at time t in a MDP model and the corresponding probability of repayment is therefore $p^y = \left(\frac{r}{n}\right)^y$ given credit amount y is offered. Suppose $f_{t+1}(p|r, n; y)$ is the revised probability distribution for p when credit amount y has been extended and collected, assuming the prior distribution on p at period t is $f_t(p|r, n)$. Then as shown by Bierman and Hausman (1970), these two density functions possess the following relationship

$$f_{t+1}(p|r, n : y) = f_t(p|r + y, n + y) \tag{2.3}$$

where $f_{t+1}(p|r, n : y) \sim Beta(r + y, n + y)$. Thus, the state variables of the MDP model at $t + 1$ can be updated as $(r + y, n + y)$.

However, if there is no repayment at time t, then (2.3) does not hold anymore. Therefore, to simplify the model formulation, they assume that after any non-repayment period the expected payoff of a borrower is zero and the lender will not offer any credit to this borrower in the future. The credit concept in credit card industry is somewhat different however. Lenders normally do not terminate a credit card account after the first non-repayment. Moreover, Bierman and Hausman (1970) assumes the account either pays back the full amount at the end of the period or pays nothing at all. This is obviously not valid in the credit card industry.

There are various modifications on the Bierman-Hausman model (see review by Rosenberg and Gleit (1994)). Dirickx and Wakeman (1976) show it is still possible to calculate the expected future payoff if there is no repayment at a time period. The difference is that the prior distribution of the probability of repayment does not follow the Beta distribution. The computation, however, is very time consuming and complicated in the sense that it is hard to apply in reality. Srinivasan and Kim (1987) investigates the cash flow timing assumption. The Bierman-Hausman model has a unrealistic assumption in which a lender is able simultaneously to collect its receivables and extend the credit limit. This means a lender receives the payment from a borrower at time t and then this lender can adjust the borrower's credit limit at time t immediately. However, in reality, it takes some time (likely it takes one time period) for the lender to update the credit limit. The modification suggested by Srinivasan and Kim (1987) introduces parameters to bring cash flow timing into consideration. Nevertheless, all of these models assume a stationary environment and none explored the external economic environment.

Trench, Pederson, Lau, Ma, Wang, and Nair (2003) present a MDP model tailored for credit card products. They apply MDP models with the objective of adjusting a consumer's credit card limit or annual percentage rate (APR) to manage the characteristics of consumer lifetime value. They assume a stationary environment. Moreover, their state space does not include the behavioural score, which has become the established way of assessing the default risk of a borrower, and is used by almost every lender. This behavioural score, as we are going to show in the later chapters, is the key state variable. Trench et al. (2003) have not incorporated external environment (i.e. economy) into the model whereas we believe this is essential as promoted by the new Basel Accord.

2.4 The credit transition probability

Not many studies have examined consumer credit risk, whereas numerous studies have examined the behaviour of bond credit migration. Here in this section, we are going to review these studies. The rating of a bond is assessed by rating agencies such as Moody's or Standard & Poor's and is published on a periodical basis, depending on the type of bond. The evolution of a bond rating is important to investors because any possible events, such as downgrade, upgrade, or default, change the bond pricing.

2.4.1 Using the maximum likelihood estimation method to determine the transition matrix

If one is given the transition history of R customers over T periods of time, then the question is how one can estimate the transition probability p_{ij} of transiting from state i to state j via these R customers' transition records. The simplest approach is to assume the transition probabilities from one state to another are the same for all time periods, i.e. the chain is stationary. Let $n_i(r)$ be the number of times that state i appears in the rth customer's transition history during times 0 to $T-1$ and let $n_{ij}(r)$ be the total number of times customer r transits from state i to state j during time 1 to T. Bartlett (1951) and Hoel (1954) showed that the maximum likelihood estimate (MLE) of p_{ij} is

$$\hat{p}_{ij} = \frac{\sum\limits_{r\in R} n_{ij}(r)}{\sum\limits_{r\in R} n_i(r)} \tag{2.4}$$

Indeed, this is the historical average of all transitions. We refer to the transition matrix obtained by this method as an unconditional transition matrix.

2.4.2 Looking at the heterogeneity of credit migration

In the early study it was common to assume the bond rating migration follows a discrete stationary Markov process. That is to say the future rating of a bond depends only on the current rating, and bonds having the same current rating have the same probability of default, upgrade or downgrade. The estimation though is done by the MLE method. Since the early 90s, however, authors have started to document the fact that there are evidently sources of heterogeneity in rating migration. Many researchers find credit migration depends on the age of a bond. Asquith, Mullins, and Wolff (1989) investigate the aging effect in the high-yield bond market (bonds that are rated below investment grade at the time of issue). Their analysis looks at the high-yield bonds' default percentages with respect to bond age. They conclude that the probability of default of a bond for the first several years after issue is lower. Using the average default probability substantially under-estimated the default probabilities of older bonds. Altman and Kao (1991) find this rating drift not only appeared in the high-yield bond category but in all rated bonds.

A second source of heterogeneity is rating momentum. Altman and Kao (1992) show credit rating exhibits significant path dependency such that a bond has a higher mobility if it recently had a rating change. Cantor and Fons (1993) investigate the same context and draw a slightly different conclusion. They test two hypotheses (1) Prob(Upgrade within one year|the bond had been upgraded recently) $\leq$ Prob(Downgrade within one year|the bond had been upgraded recently); (2) Prob(Downgrade within one year|the bond had been downgraded recently $\leq$ Prob(Upgrade within one year|the bond had been downgraded recently). For most of the bond grades, the test rejects the first hypothesis and thus shows that statistically a downgraded bond is more prone to having a subsequent downgrade within one year than an upgrade. The drift in downgrade credit rating is more prominent in default bonds, as shown by Fons (2002). They look at ratings of firms that do eventually default. The results show many of these firms were in the speculative grade (Ba2) five years before default, and the rating had decreased every year. In a more recent study conducted by Hamilton and Cantor (2004), the drift of a credit rating is still preserved.

Many authors have proposed that change in the economy changes the stability of credit transitions. Wilson (1997a,b) and Nickell, Perraudin, and Varotto (2000) are one of the first empirically to investigate in this context. They use the business cycle as an explanatory dummy variable (i.e. recession or expansion) in the ordered probit model to investigate the impact of the economy. Their results show upgrade, downgrade and default probabilities are associated with the macroeconomics. Bangia, Diebold, Kronimus, Schagen, and Schuermann (2002) also find evidence to support the same rating migration pattern. The latest development in modeling is the use of duration analysis, since duration models can capture censored observations (i.e. those not yet default, upgrade or downgrade cases). Kavvathas (2001) and Duffie, Saita, and Wang (2007) use duration analysis with time-dependent macroeconomic covariates to forecast the upgrade, downgrade and default probabilities. In summary these papers support the proposition of "credit rating drift with macroeconomics".

In these studies, the economy is introduced by a dummy explanatory variable to indicate whether the economy was in recession. Not many authors use macroeconomic measurements as explanatory variables to investigate credit migration. The study conducted by Figlewski, Frydman, and Liang (2006) is one of the first to address the issue. They apply a duration analysis model to look at the US corporate bond migration, of which rating upgrade, rating downgrade and default are the observed events and macroeconomics measurements, including interest rates, inflation, GDP growth etc, are the set of time-varying covariates. This framework provides a mechanism to obtain precise coefficient estimates for pricing.

One may not be surprised that industry effect and country of domicile are also the source of heterogeneity in credit rating migration. The methodologies used to examine these effects are similar to those used for examining the impact of business cycle: the ordered probit model (Nickell et al., 2000), and duration analysis (Chava and Jarrow, 2004; Kavvathas, 2001).

2.4.3 More than one Markov chain

Another way of incorporating the population heterogeneity is via the mover-stayer model. Originally introduced by Blumen, Kogan, and McCarthy (1955), Frydman, Kallberg, and Kao (1985) look at the suitability of using the discrete-time mover-stayer model in modelling the payment behaviour of revolving credit accounts. The idea is the credit accounts can be classified into two categories, stayer or mover, with respect to their payment behaviour during the discrete-time planning horizon. The credit migration of these two types of accounts are different and thus the population's credit migration is a mixture of two independent Markov chains. The transition matrix with respect to the stayer's Markov chain is assumed to be an identity matrix. Suppose the transition matrix of those of mover's is M, then the transition matrix of the stayer-mover model is

$$P = SI + (I - S)M \tag{2.5}$$

where $S = diag(s_1, s_2, \dots, s_N)$ represents the proportion of stayers in state i. The question here is how to estimate S and M from the samples? Or in other words, how one can distinguish movers from stayers? Suppose we have the monthly credit and repayment record of R customers across a planning horizon T. If the rth customer had stayed in the credit status i across the whole sampling period, this customer can either be a stayer in state i or a mover who has stayed in state i for time period T . In another paper Frydman (1984) show that there is a recursive method to compute the maximum likelihood estimators for M and S. Frydman et al. (1985) compare the stationary and non-stationary Markov chain models with the mover-stayer model empirically. The results show that mover-stayer model that provides a much better description of the data than do Markov chain models.

A continuous-time analog of the discrete-time mover-stayer model is studied by Frydman and Kadam (2004), in which they model bond rating migration as a continuous-time stochastic process. Again, the bond rating migration embeds two Markov chains. The first evolves according to some infinitesimal generator Q, and that is the Markov chain with respect to the movers. The other's transition probability matrix is an identity matrix I. The transition probability matrix, $P(t)$, of this continuous mover-stayer model on state space S is then defined as

$$P(t) = SI + (I - S)exp(tQ), t \geq 0 \tag{2.6}$$

The algorithm to estimate the parameters in the above equation is presented in Frydman (2005). The results indicate the proposed mixture model statistically dominated the conventional Markov chain model, and it is possible for two individual observations with the same credit rating to have a substantially different future rating distribution.

This literature suggests credit migration is not a pure Markov process. Some kinds of heterogeneity, which could be the age of the bond, the business cycle or industry, are preserved in the population. However, as far as we can ascertain, there are no studies which investigate consumer credit migration, and so no one has examined heterogeneity in consumer credit rating migration.

2.4.4 Bayesian approach

Estimating the probability of default of a loan in the next year has become a necessity for financial institutions that are intending to operate under the new banking regulatory framework called the Basel Accord. Another recent research development in estimating the transition probability, especially for the estimation of the probability of default (PD), is the usage of Bayesian approaches. The maximum likelihood method builds on the assumption that the event (which is the default if one is examining the PD) follows a distribution. However, for data with sparse entries, this assumption is hardly valid. Statisticians suggest the use of the maximum likelihood method to estimate the sparse PD entries results in large estimation errors since the maximum likelihood estimator is suitable for a frequentist estimation framework (i.e. for a dataset that consists of numerous observations). On the other hand, the Bayesian approach only assumes a prior distribution for the parameter and this distribution can be changed by learning from the data. Another advantage for using Bayesian method to estimate the default probability is that it requires a specification of a prior PD, which provides a mechanism to incorporate expert knowledge. Kadam and Lenk (2008) model the changes in credit rating with a discrete space, continuous time, stationary Markov process of which the time spent in a state is not fixed but it is exponentially distributed. They use the Bayesian approach to estimate the credit rating transition matrix. Just like many other authors, Kadam and Lenk (2008) find heterogeneity migration in different industries and countries. Although Bayesian inference is supported by mathematical theory, it can take a lot of computation time to find the estimates. Stefanescu, Tunaru, and Turnbull (2007) develop a statistical model without imposing the Markovian assumption on the transition matrix as they believe "there is considerable evidence support (see the references in the next section) that the Markovian assumption is unrealistic for rating transitions". Their model proposes the default probability depends conditionally on a systematic factor (such as a macroeconomic measurement) and the state of the borrower. The conditional probability follows a certain distribution which depends on the distribution of the idiosyncratic term (a rating class dependent correlation factor). Stefanescu et al. (2007) use the Bayesian framework to estimate the posterior default probability. By using out-of-sample testing, the proposed Bayesian framework generated closer default rates than the traditional latent model.

2.4.5 Low default portfolios

The new Basel accord has highlighted the need to study *low default portfolios* (LDPs). The Accord requires lenders to estimate PDs that are long run, forward looking expected default rates for each grade in each borrower rating model, with an appropriate margin of conservatism (Benjamin, Cathcart, and Ryan, 2006) even if the default rate is low. In credit card portfolios, accounts having good repayment records and so high behavioural scores are possible LDPs. There is no rigid definition on these portfolios. It depends on the lender, the regulator, the type of product or the country of domicile of the lender. However, a common property is one hardly finds any default cases in LDPs. One may not be surprised to have zero or a very low number of defaults in a LDP. Calculating the PDs of LDPs using historical averages inevitably underestimates the potential default risks. Thus, it is necessarily to further explore the estimation of default probabilities of LDPs. Pluto and Tasche (2006) advocate estimating the PDs of the LDPs by using the concept of confidence intervals and making use of the relationship of each grade level.

To begin the discussion, we first introduce some notations. Suppose

J	$=$	number of classes of low default portfolios (indexed by $j = 1, 2, \ldots, J$)
$n(j)$	$=$	total number of observations in the jth class of low default portfolios
n	$=$	total number of low default portfolios (i.e. $\sum_j n(j)$)
$p_D(j)$	$=$	the default probability of the jth class low default portfolios
D	$=$	number of default cases (which is predefined by the lender subjectively)

The credit rating increases in j that is to say the jth class LDP has a better credit quality than the (j-1)th class LDP. Moreover, the (j-1)th class LDP is more likely to default than the jth class, i.e.

$$p_D(j) \leq p_D(j-1), \forall j = 2, \ldots, J \tag{2.7}$$

We specify the PD estimators lies at the confidence region at γ level. According to Pluto and Tasche (2006), there are three methods to estimate this $p_D(J)$.

1. No default and default events are independent

We begin by estimating the default probability, $p_D(J)$, of the Jth class of LDP (i.e. the one with the highest credit quality). Pluto and Tasche (2006) propose the "most prudent estimation" of the value of $p_D(J)$ such that the default probabilities of all classes are equal, i.e.

$$p_D(1) = p_D(2) = \ldots = p_D(J) \tag{2.8}$$

Then the probability of observing not a single default in the Jth class of LDP is $(1 - p_D(J))^n$. Since we assume the estimator of $p_D(J)$ lies in the confidence region at level γ, then mathematically

$$1 - \gamma \leq (1 - p_D(J))^n \tag{2.9}$$

The set of $p_D(J)$ satisfied the inequality (2.9) gives a chance for no default observation no less than $1 - \gamma$. Since our objective is to find the most prudent estimator, we pick the value within this interval such that it gives the *lowest* probability of having no default observation, i.e.

$$1 - \gamma = (1 - p_D(J))^n \tag{2.10}$$

We can thus solve (2.10) to obtain $p_D(J)$. Then we proceed to estimate the default probability of the second highest class, i.e. $p_D(J-1)$. Again, Pluto and Tasche assume that the default probability, $p_D(J-1)$, preserves the following property:

$$p_D(1) = p_D(2) = \ldots = p_D(J-1) \tag{2.11}$$

Then, using the same concept, the estimator of $p_D(J-1)$ is obtained by solving the following equation:

$$1 - \gamma = (1 - p_D(J-1))^{(n-n(J))} \tag{2.12}$$

In general, the following equation is used to estimate the probability of default for the 1st, 2nd, ... (J-1)th class of the LDP portfolios,

$$1 - \gamma = (1 - p_D(j))^{(n - \sum\limits_{k=j+1}^{J} n(k))}, \forall j = 1, 2, \ldots, J-1 \tag{2.13}$$

2. Few defaults and default events are independent

The second method is to assume that there are few defaults and these defaults are independent. There are only two possible outcomes for this default event: default or not default. One therefore can assume that each event follows a Bernoulli distribution. If one assumes (2.8) holds, then the probability of having no more than D defaults is

$$\sum_{k=0}^{D} \begin{pmatrix} n \\ k \end{pmatrix} p_D(J)^k (1 - p_D(J))^{n-k}$$

Similar to (2.10), the estimator of $p_D(j)$ can be found by the following:

$$1 - \gamma = \sum_{k=0}^{D} \begin{pmatrix} n \\ k \end{pmatrix} p_D(J)^k (1 - p_D(J))^{n-k} \tag{2.14}$$

For the rest of the LDP, one can use the following general form to estimate the default probability

$$1-\gamma=\sum_{k=0}^{D}\binom{n-\left(\sum\limits_{m=j+1}^{J}n(m)\right)}{k}p_D(j)^k(1-p_D(j))^{n-\left(\sum\limits_{m=j+1}^{J}n(m)\right)-k},\forall j=1,2,\ldots,J-1 \tag{2.15}$$

3. Few defaults and correlated default events

The third model proposed by Pluto and Tasche is to address the possible time-dependent correlation factor within the spirit of the best-known Merton's Value-at-Risk (VaR) model. To look at the most prudent estimation of the default probabilities of LDP for any given confidence level γ, one has to look at

$$1-\gamma \leq P[\text{No more than } D \text{ defaults observed}] \tag{2.16}$$

The assumption of this model is that there are several default events and there is a systematic correlation factor S_t driving these events in period t. Therefore, the right-hand-side of (2.16) is equivalent to look at

$$\sum_{l=0}^{D} E[P[\text{Exactly } l \text{ borrowers default}|S_t,\ldots,S_T]] \tag{2.17}$$

VaR model looks at the asset value of each portfolio to estimate whether the asset is defaulted. Assume the asset value of the jth portfolio is $V_{j,t}$ and it follows a standard normal distribution. Then the VaR model says this asset is defaulted if $V_{j,t} \leq c$, where c is a real number. Since $V_{j,t}$ is normally distributed, then so as c (Freund, 1992). One can determine the threshold c by

$$c=\Phi^{-1}(p) \tag{2.18}$$

where p is the PD of the value of the jth portfolio falls below c. Note that this p is the parameter we are interested in.

If besides the systematic factor S_t, there is an idiosyncratic component, $\xi_{j,t}$, for a low default portfolio j at time t, then the asset value of the jth portfolio is:

$$V_{j,t} = \sqrt{\zeta}S_t + \sqrt{1-\zeta}\xi_{j,t} \tag{2.19}$$

where S_t and $\xi_{j,t}$ follow the standard normal distribution and ζ is the correlation between different portfolios (Merton, 1974). The systematic correlation is time dependent and it is identical for all portfolios at time t. One can assume that this S_t is the macroeconomic variable at time t. On the other hand, the idiosyncratic factor is different for each portfolios j (for example: the risk factor for the jth portfolio).

By using (2.18), one can expand the default probability of an individual LDP as

$$\begin{aligned}
& P[\text{the } j\text{th portfolio defaults}|S_t, \ldots, S_T] \\
&= P[\min_{t=1,\ldots,T} V_{j,t} \leq \Phi^{-1}(p)|S_t, \ldots, S_T] \\
&= 1 - P[V_{j,1} \geq \Phi^{-1}(p), \ldots, V_{j,T} \geq \Phi^{-1}(p)|S_t, \ldots, S_T] \\
&= 1 - P[\xi_{j,1} \geq G(p,\zeta,S_1), \ldots, \xi_{j,T} \geq G(p,\zeta,S_T)|S_t, \ldots, S_T] \\
&= 1 - \prod_{t=1}^{T}(1 - G(p,\zeta,S_t))
\end{aligned} \tag{2.20}$$

where the function G is defined by

$$G(p,\zeta,S_t) \equiv \Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\zeta}S_t}{\sqrt{1-\zeta}}\right). \tag{2.21}$$

If $\zeta = 0$, then $G(p,0,S_t) = \Phi\left(\frac{\Phi^{-1}(p)-\sqrt{0}S_t}{\sqrt{1-0}}\right) = \Phi\left(\Phi^{-1}(p)\right) = p$. If $\zeta = 1$, then $G(p,1,S_t) = \Phi\left(\frac{\Phi^{-1}(p)-\sqrt{1}S_t}{\sqrt{1-1}}\right) = \Phi(\infty) = 1$. So the function $G(p,\zeta,S_t)$ lies in the interval $[p,1]$. That is to say the function G gives the probability such that the jth portfolio is not defaulted.

If we define $\pi(S_1,\ldots,S_T) \equiv 1 - \prod_{t=1}^{T}(1 - G(p,\zeta,S_t))$, then (2.17) equals to

$$\begin{aligned}
& \sum_{l=0}^{D} E[\text{Exactly } l \text{ borrowers default}|S_t, \ldots, S_T]] \\
&= \sum_{l=0}^{D} \begin{pmatrix} n \\ l \end{pmatrix} E[\pi(S_1,\ldots,S_T)^l(1-\pi(S_1,\ldots,S_T))^{n-l}]
\end{aligned} \tag{2.22}$$

To find an estimator for the default probability, one can thus solve the equation

$$1 - \gamma = \sum_{l=0}^{D} \begin{pmatrix} n \\ l \end{pmatrix} E[\pi(S_1, \ldots, S_T)^l (1 - \pi(S_1, \ldots, S_T))^{n-l}] \tag{2.23}$$

The approach is to simulate all the possible values with respect to $(S_1, \ldots, S_T)$ and then estimate the value of the above equation.

Pluto and Tasche (2006)'s paper has been referenced as a prominent study in the Basel context. There are some extensions of the work over the last few years. The UK regulator (Benjamin et al., 2006) has extended the discussion by looking at the challenges of applying Pluto and Tasche (2006)'s work, including the cutover from LDP to non-LDPs, and the choice of confidence level. However, the paper still has not examined the impact of the economy, whereas the new Basel accords have stated clearly that data history should cover an economic downturn.

2.5 Conclusion summary

In summary, we find there is research opportunity in the following areas:

1. There is limited research in investigating the migration pattern of consumer credit risk. It remains an open question whether the migration pattern preserves the Markovian assumption or consists of some sources of heterogeneity.

2. There is lack of literature investigating the impact of macroeconomic factors on consumer credit risk and consumption behaviour.

3. Although there are few sequential decision models for credit card products have been developed, there is still room for improvement with respect to the Markovity of the model, the way they deal with low default portfolios, and the incorporation of external factors.

Chapter 3

The Basic MDP model

In this chapter we construct a basic MDP model for making sequential credit limit decisions. The focus of this chapter is to look at the basic techniques, including coarse-classifying and order selection, in applying the model to real-life credit card data. We tested the model performance and these techniques with a real-life credit card data set.

3.1 The model

Consider a discrete state, discrete time discounted Markov Decision Process with decision epochs $\mathcal{T}$ (indexed by $t = 1, 2, \ldots, T$) based on a state space S. We use behavioural score as our key state variable in this study. Every month lenders calculate this behavioural score for every borrower. This score shows how likely the borrower is to default in the near future, usually a period of twelve months. Lenders use many variables, including application-form characteristics, credit bureau data, and repayment and usage behaviour of the borrower, to generate this score. These data are obtained from a sample of histories of customers as follows. Lenders choose a particular point of time as the observation point.

A period preceding this point, usually is around twelve to eighteen months, is chosen as the performance period, and the corresponding characteristics (the repayment history and credit bureau data) are collected. A period of time after the observation point, usually twelve months, is chosen as the outcome period. The score is calculated based on a borrower's status at the end of the outcome period. Lenders usually apply logistic regression to predict the status of the borrower at the end of the outcome period (default / not default) using characteristics collected during the performance period. The weighted sum of these characteristics is taken as the behavioural score *score*, and the probability that the borrowers will default (PD) in the outcome period is related to the score by

$$log\left(\frac{1-PD}{PD}\right) = score$$

under the logistic regression model. The higher the score, the lower the default probability.

The second state variable in this study is credit limit as it is obvious that borrowers with different credit limits have heterogeneous behaviour. We split the behavioural score and credit limit into discrete bands (the reason will be explained in the following sections). Therefore, each state in the state space consists of two parts-which behavioural score band the borrower is in, and what is the borrower's current credit limit band.

The state space thus consists of the current credit limit band represent by $\mathcal{L}$ (indexed by $l = 0, 1, \ldots, L$) and the current behavioural score band $\mathcal{I}$ (indexed by $i = 0, 1, \ldots, I$). In our model the actions are limited to keeping the credit limit as is this period or raising it to a higher limit band. This policy of not decreasing credit limits is used by many lenders but the methodology we will describe will not change if this restriction is dropped. Thus with this limitation the action set is defined as $A_l = \{l' : l \leq l'\}$.

Two further elements need to be defined to complete the Markov decision process model. Let $p(i'|l, i)$ be the probability that if l is the current customer's credit limit band and the customer is in behavioural score band i, then the next period the customer will be in behavioural score band i'. Secondly let $r(l, i)$ be the profit obtained in the current period from a customer with credit limit l and in behavioural score band i.

The objective is to maximise the discounted profit obtained from the customer over the next t periods where the discount factor λ describes the time value of money. This leads to the following optimality equation for $V_t(l,i)$, the maximum expected profit over the next t periods that can be obtained from an account which is currently in behaviour score band i, and with a credit limit of l:

$$V_t(l,i) = \max_{l' \in A_l}\{r(l,i) + \lambda \sum_{i'} p(i'|l,i)V_{t-1}(l',i')]\} \tag{3.1}$$

The right-hand-side of (3.1) corresponds to the profit over the next t periods if we change the credit limit to l' from l at the end of the current period for an account with behavioural score state i. We assume it takes one time period for the borrower to become aware of a change in the credit limit as this is usually included in the monthly balance statement sent to the customer. Removing this delay makes no difference to the methodology though of course the optimality equation will be slightly different. The profit to the lender from the credit card at the end of the current period is $r(l,i)$. The $p(i'|l,i)$ is the probability that the behavioural score changes next month to band i'. In that case, the profit on the remaining $t-1$ periods is $V_{t-1}(l',i')$. The discount factor λ is introduced because the subsequent profits in the remaining $t-1$ periods actually occur one period after those used in calculating $V_{t-1}(l',i')$, since that assumes the $t-1$ periods start now. The optimality principle says that the optimal decision l', is the one that maximizes this sum of the future profit, where credit limits can only remain the same or be increased. Note that $V_0(l,i), \forall l,i$ is the boundary condition, that is the customer's profit value at the end of the planning horizon. In the later section, we solve equation (3.1) by looking at the optimal solution for an infinite horizon MDP, i.e.

$$V(l,i) = \max_{l' \in A_l}\{r(l,i) + \lambda \sum_{i'} p(i'|l,i)V(l',i')]\} \tag{3.2}$$

Therefore, we do not need to specify the boundary condition in this study.

3.1.1 Properties

There are some properties which one might expect from (3.1), but these properties only hold with the following assumptions:

A.1. $\sum_{i'=k}^{I} p(i'|l, i+1) \geq \sum_{i'=k}^{I} p(i'|l, i), \ \forall l, i, k = 1, \ldots, I$

A.2. $\sum_{i'=k}^{I} p(i'|l+1, i) \geq \sum_{i'=k}^{I} p(i'|l, i), \ \forall l, i, k = 1, \ldots, I$

A.3. $r(l, i+1) - r(l, i) \geq 0, \ \forall l, i$

A.4. $r(l+1, i) - r(l, i) \geq 0, \ \forall l, i$

A.1 says the higher the current behavioural score, the greater the chance of moving to high behavioural scores. A.2 is a stochastic ordering property that says the higher the credit limit the more likely the borrower is moving to high (good) behavioural scores. A.3 assumes the profit increases as the behavioural score increases. With the same behavioural score, A.4 assumes that the reward in a state with credit limit $l+1$ is higher than that from a state with credit limit l.

One may not be surprised with assumption A.4. This is because a borrower with a high credit limit are more likely to have a high income and essentially they spend more than those with low credit limit. As the lender's reward is roughly proportional to a borrower's monthly spending, the reward increases with the credit limit.

Nevertheless, it might not be reasonable for assumption A.3 to hold in reality. A borrower with high behavioural score rarely has carrying balance. This is because their financial status are good and thus it is very likely that they can repay their monthly balance on time. Having no carrying balance means this borrower does not generate profit for the lender via interest fee. Thus the lender earns merchandiser fees from this borrower only. On the other hand, a borrower with low behavioural score is more likely to keep an amount of carrying balance and has a high tendency to use his/her credit card. Therefore they generate both interest fees and merchandiser fees for the lenders and essentially the profit should be higher.

As a theoretical overview, we first assume the above four assumptions hold to prove the following Lemmas:

Lemma 3.1. If A.1 and A.3 hold, then $V_t(l, i)$ is nondecreasing in i, $\forall l, t$.

Lemma 3.2. If A.2 and A.4 hold, then $V_t(l, i)$ is nondecreasing in l, $\forall i, t$.

Proof. The proof of all lemmas are by induction on t. Assume the equations hold trivially for $t = 0$. For Lemma 3.1, $V_1(l, i) = r(l, i) \geq r(l, i-1) = V_1(l, i-1), \forall i$. Thus the lemma holds for $t = 1$. Assume Lemma 3.1 holds for t, by using

$$\max\{a_1, a_2\} - \max\{b_1, b_2\} \geq \min\{a_1 - b_1, a_2 - b_2\} \tag{3.3}$$

$$\begin{aligned}
V_t(l, i+1) - V_t(l, i) &= \max_{l' \in A_l}\{r(l, i+1) + \lambda \sum_{i'} p(i'|l, i+1)V_{t-1}(l', i')\} - \\
&\quad \max_{l' \in A_l}\{r(l, i) + \lambda \sum_{i'} p(i'|l, i)V_{t-1}(l', i')\} \\
&\geq \min_{l' \in A_l}\{r(l, i+1) - r(l, i) + \lambda \sum_{i'} p(i'|l, i+1)V_{t-1}(l', i') - \\
&\quad \lambda \sum_{i'} p(i'|l, i)V_{t-1}(l', i')\} \\
&\geq \min_{l' \in A_l} \lambda\{\sum_{i'} p(i'|l, i+1)V_{t-1}(l', i') - \sum_{i'} p(i'|l, i)V_{t-1}(l', i')\} \\
&= \min_{l' \in A_l} \lambda\{\sum_{i'} [p(i'|l, i+1) - p(i'|l, i)]V_{t-1}(l', i')\}
\end{aligned}$$

$$
\begin{aligned}
&= \min_{l' \in A_l} \lambda\{p(I|l,i+1) - p(I|l,i)](V_{t-1}(l',I) - V_{t-1}(l',I-1)) \\
&\quad + \sum_{k=I-1}^{I} (p(k|l,i+1) - p(k|l,i))(V_{t-1}(l',I-1) - V_{t-1}(l',I-2)) \\
&\quad + \sum_{k=I-2}^{I} (p(k|l,i+1) - p(k|l,i))(V_{t-1}(l',I-2) - V_{t-1}(l',I-3)) \\
&\quad + \dots \\
&\quad + \sum_{k=2}^{I}(p(k|l,i+1) - p(k|l,i))(V_{t-1}(l',2) - V_{t-1}(l',1)) \\
&\quad + \sum_{k=1}^{I}(p(k|l,i+1) - p(k|l,i))V_{t-1}(l',1)\} \\
&\geq 0
\end{aligned}
$$

given (3.3), A.1, A.3, $\sum_{k=1}^{I}(p(k|1,i+1) - p(k|l,i)) = 0$ and the induction hypothesis.

Then we can prove Lemma 3.2. For $t = 1$, $V_1(l,i) = r(l,i) \geq r(l-1,i) = V_1(l-1,i)$, thus the lemma holds for $t = 1$. Assume Lemma 3.2 holds for t,

we have,

$$
\begin{aligned}
V_t(l+1,i) - V_t(l,i) &= \max_{l' \in A_l}\{r(l+1,i) + \lambda \sum_{i'} p(i'|l+1,i)V_{t-1}(l',i')\} - \\
&\quad \max_{l' \in A_l}\{r(l,i) + \lambda \sum_{i'} p(i'|l,i)V_{t-1}(l',i')\} \\
&\geq \min_{l' \in A_l}\{r(l+1,i) - r(l,i) + \lambda \sum_{i'} p(i'|l+1,i)V_{t-1}(l',i') - \\
&\quad \lambda \sum_{i'} p(i'|l,i)V_{t-1}(l',i')\} \\
&\geq \min_{l' \in A_l} \lambda\{\sum_{i'} p(i'|l+1,i)V_{t-1}(l',i') - \sum_{i'} p(i'|l,i)V_{t-1}(l',i')\} \\
&= \min_{l' \in A_l} \lambda\{\sum_{i'}[p(i'|l+1,i) - p(i'|l,i)]V_{t-1}(l',i')\}
\end{aligned}
$$

$$
\begin{aligned}
&= \min_{l' \in A_l} \lambda\{p(I|l+1,i) - p(I|l,i)](V_{t-1}(l',I) - V_{t-1}(l',I-1)) \\
&\quad + \sum_{k=I-1}^{I} (p(k|l+1,i) - p(k|l,i))(V_{t-1}(l',I-1) - V_{t-1}(l',I-2)) \\
&\quad + \sum_{k=I-2}^{I} (p(k|l+1,i) - p(k|l,i))(V_{t-1}(l',I-2) - V_{t-1}(l',I-3)) \\
&\quad + \ldots \\
&\quad + \sum_{k=2}^{I}(p(k|l+1,i) - p(k|l,i))(V_{t-1}(l',2) - V_{t-1}(l',1)) \\
&\quad + \sum_{k=1}^{I}(p(k|l+1,i) - p(k|l,i))V_{t-1}(l',1)\} \\
&\geq 0
\end{aligned}
$$

given (3.3), A.2, A.4, the induction hypothesis and Lemma 3.1.

These two lemmas say accounts with the highest credit limit and behavioural score are the most profitability accounts. Nevertheless, we will show that there is no guarantee that assumptions A.3 and A.4 hold in practice.

3.2 The UK credit card data

The first credit card dataset used in this study was provided by a UK major financial institute. The dataset consists of 11 attributes, including our two state variables: *credit limit* and *behavioral score*, and covers the period 2001 to 2004 inclusive.

3.2.1 Preprocessing, special accounts and sampling

Accounts opened after January 2001 or with missing values on the account opening date were excluded since we wanted to analyze the behaviour of the cohort of credit card owners who were active in January 2001. We were also interested in the delinquent accounts as one aim is to find the likelihood of credit card account jumping from a *good* status to a delinquency status, which includes *180 days in arrears, charge-off and bankruptcy.* We also recognized cards could move to the *Inactive* or *Closed* states. Accounts classified as *Inactive* (which is defined as credit card which have not been used in the previous twelve months before the sample point) but having entries in the account balance were deleted. We excluded other special accounts (such as Fraud, Stolen, Blocked etc) from our study. The MDP model consists of three special account types: *Bad* (for 180 days in arrears, charge-off or bankruptcy), *Inactive* and *Closed.* There were 50,797 cases in total for analysis.

3.2.2 Deriving the account profit

One critical component in (3.1) is the profit function $r(l, i)$. Every lender has his own tailor-made formula to calculate the profit. Nevertheless, when filing the historical records, some lenders drop this field to save storage space. This is the case for our data provider. This lender provided the historical data but the profit value was not included in the list of characteristics. We thus developed the following method to estimate the account's monthly profit.

$$
\begin{aligned}
\text{Let} \quad f &= \text{the merchandiser rate (in \%)} \\
r &= \text{the monthly percentage rate (MPR) (in \%)} \\
B_t &= \text{credit account balance at the beginning of } t \\
N_t &= \text{new purchase during } t \\
P_t &= \text{repayment by the end of } t
\end{aligned}
$$

Balance at the beginning of t is the summation of balance at the beginning of $t-1$ plus the new purchase during $t-1$ minus the repayment by the end of $t-1$,

$$B_t = B_{t-1} + N_{t-1} - P_{t-1}.$$

In general,

$$\begin{aligned}
\text{profit in period } t-1 &= N_{t-1}f + (B_{t-1} - P_{t-1})r \\
&= N_{t-1}f + (B_{t-1} - B_{t-1} + B_t - N_{t-1})r \\
&= N_{t-1}(f-r) + B_t r
\end{aligned}$$

If we assume $r = f$, we have

$$\text{profit in period } t-1 = B_t r \tag{3.4}$$

In reality, one can find credit cards with annual percentage rate (APR) from 6% to 48% while it is more common to find a credit card with APR 18% to 30%. So r is roughly 1.5% to 2.5%. The merchandise fee f, also varies by merchandiser. For example, the merchandisers fee of travel agencies is around 5%. However, the charges for other retailers are roughly only 2% to 3%. Although f is usually higher than r in reality, as they are in a close magnitude, we think the assumption $r = f$ is acceptable.

So with this assumption the profit is a fixed fraction of the balance at the end of the period. The field monthly balance is luckily available in the credit card dataset and thus throughout the study we used the above estimation as the profit value.

3.3 Coarse-classifying

If we simply include all the state variables, behavioural scores and credit limits, into (3.1), the size of the state space will be substantial. Therefore we divided these two variables into a number of separate groups or bands in order to ensure our model's robustness.

This procedure is called coarse-classifying. A suitable classification is able to maximize difference from one group to next and minimize the difference within a group. As we are modelling the credit card accounts' transition as a Markov chain, we can use the Chi-square test to examine whether the split is good enough. With a good split one can get a good approximation to the Markovian assumption. To check whether the Markov chain satisfies this assumption, for every state, we are interested in whether the hypothesis that the probability of moving from (l_t, i_t) to i_{t+1} is independent of the state at $t-1$, i.e. (l_{t-1}, i_{t-1}). Define $n_t(l_{t-1}, i_{t-1}; l_t, i_t; i_{t+1})$ to be the number of times that a credit account was in state (l_{t-1}, i_{t-1}) at time $t-1$ followed by moving to (l_t, i_t) at time t and i_{t+1} at time $t+1$. Similarly define $n_t(l_t, i_t; i_{t+1})$ to be the number of times that a customer was in state (l_t, i_t) at time t and then moved to behaviour score i_{t+1} at time $t+1$. If we assume the chain is stationary, the estimator for $p(i_{t+1}|l_{t-1}, i_{t-1}, l_t, i_t)$ is:

$$\hat{p}(i_{t+1}|l_{t-1}, i_{t-1}, l_t, i_t) = \frac{\sum_{t=0}^{T-2} n_t(l_{t-1}, i_{t-1}; l_t, i_t; i_{t+1})}{\sum_{t=0}^{T-2} n_t(l_{t-1}, i_{t-1}; l_t, i_t)} \tag{3.5}$$

The Markovity of the chain corresponds to the hypothesis that $p(i_{t+1}|1, 1, l_t, i_t) = p(i_{t+1}|2, 1, l_t, i_t) = \ldots = p(i_{t+1}|L, 1, l_t, i_t) = p(i_{t+1}|1, 2, l_t, i_t) = \ldots = p(i_{t+1}|L, I, l_t, i_t)$, for l_t, i_t, i_{t+1}. To check on the Markovity of state (l_t, i_t), we use the chi-square test (Anderson and Goodman, 1957). Let

$$\chi^2_{(l_t,i_t)} = \sum_{(l_{t-1},i_{t-1})} \sum_{i_{t+1}} \frac{n^*(l_{t-1}, i_{t-1}; l_t, i_t)[\hat{p}(i_{t+1}|l_{t-1}, i_{t-1}, l_t, i_t) - \hat{p}(i_{t+1}|l_t, i_t)]^2}{\hat{p}(i_{t+1}|l_t, i_t)} \tag{3.6}$$

where

$$\hat{p}(i_{t+1}|l_t, i_t) = \frac{\sum_{t=1}^{T-1} n_t(l_t, i_t; i_{t+1})}{\sum_{t=1}^{T-1} n_t(l_t, i_t)} \tag{3.7}$$

and

$$n^*(l_{t-1}, i_{t-1}; l_t, i_t) = \sum_{t=1}^{T-1} n_t(l_{t-1}, i_{t-1}; l_t, i_t) \tag{3.8}$$

Anderson and Goodman (1957) showed that if the chain is Markov (3.6) has a chi-square distribution with $(I-1)(L-1)^2$ degree of freedom, where L is the number of credit limit band and I is the number of behavioural score band.

A traditional approach is to start with a fine classification i.e. with more bands then one really wants and then check if one can combine adjacent bands. Alternatively, one can split the best split into two classes and then splitting one of these into two more until it is not worth splitting further.

In this study, we coarse-classified the behavioural score and credit limit simultaneously. That is we arbitrarily classified behavioural score into five categories. Then we coarse-classified the credit limit. After we found some improvement on the chi-square value (i.e. a split that generates a small chi-square value), we then stopped coarse-classify the credit limit. We then used the latest credit limit split and then classified the credit limit accordingly. Then, we start to coarse-classified the behavioural score. After obtaining a good behavioural score split, we then stopped coarse-classifying the behavioural score and then looked at the split of the credit limit again. This process repeated many times. Here we only reported a summary of the coarse-classify process for illustration.

Initially, our attempt was to classify the credit limit into five bins: £0/missing, £1 to £2000, £2001 to £4000, £4001 to £6000 and £6001+. The chi-square value of this segmentation was extremely large. We believe this is due to the grouping across consumers who have a significant difference in the financial and consumption behaviour. We thus decided to further break down the credit limit to ten groups: £0/missing, £1 to £500, £501 to 1000, £1001 to £1500, £1501 to £2500, £2501 to £3500, £3501 to £4500, £4501 to £5500, £5501 to £7500, £7501+. Upon generating the optimal policy by using this credit limit, we found an unusual pattern on the last two credit limit groups. Studying the transition probability and the profit value, shows that the behaviour of the last two groups are very close to each other, with the optimal values almost the same. Thus we decided to merge the last two credit limit groups to end up with the credit limit states defined in Table 3.1.

Index	Credit limit (in £)	Account Description
0	Closed	Closed
1	1-500	Limit 1
2	501-1000	Limit 2
3	1001-1500	Limit 3
4	1501-2500	Limit 4
5	2501-3500	Limit 5
6	3501-4500	Limit 6
7	4501-5500	Limit 7
8	5501 or above	Limit 8

Table 3.1: List of credit limit status

The monthly generated behavioural score from the lender's internal system ranged from 200 to 780. Accounts with score lower than 365 were labeled as *in risk*. Apart from this, there is no standard rule on classifying behavioural score into discrete bins. We thus first counted the frequency of behavioural scores across four years, divided scores into ten categories, and then allocated every behavioural score record to one behavioural score category (so there are $50,797 * 48$ counts in the frequency table in total). As in finding bins for the credit limit, we monitored the improvement on the chi-square test. The performance with ten behavioural scores was poor even if we aggregated some of the behaviour categories. So we began with just 2 states - one that the account has a behavioural score and the other that it has no behavioural entry. Then we split the behavioural score into categories, in order to improve the model fit of the Markov chains.

We found this was best when the behavioural score was split into four bands. There were three non-behavioural score states - *closed, inactive* and *bad*. The behavioural score status are presented in Table 3.2.

Index	Behavioural score	Account Description
0	-	Closed
1	-	Inactive
2	-	Bad (bankruptcy or charge-off)
3	200-570	Risk Account
4	571-721	Score 1
5	722-742	Score 2
6	743-758	Score 3
7	759+	Score 4

Table 3.2: List of behavioural score status

We found using the second approach is more suitable for the behavioural score binning.

3.4 Choice of Order

A MDP is mth-order if the transition probabilities depend on which state the system is currently in and was in for the previous $m-1$ periods. For a first order Markov chain the transition probability depends only on the current state where for a mth-order Markov chain the transition at time t depends on the states $(i_t, i_{t-1}, \ldots, i_{t+1-m})$ that it occupied for the last m time periods. So, the number of states increases exponentially in m as there are $|S|^m$ states in a mth-order MDP. To test whether a chain satisfies the mth-order Markovity assumption, one can use the chi-square test which is also used to check the homogeneity of a contingency table (Anderson and Goodman, 1957) (as showed in the last section). Test results showed that the transition probabilities in our dataset failed the chi square test (chi-square value = 2357504, degree of freedom = 448, critical value[1] = 498)

[1] at significant level $\alpha = 0.05$

which indicate the Markov chain is not first order. In reality, almost all applications fail to satisfy the first-order Markovity assumption. Since there is a large amount of data it is highly likely that the hypothesis that the system is a first order chain will be rejected. This is because with so much data, one usually can improve the fit beyond what are the narrow significance limits. What is more important is whether there is a significant improvement in the fit, when one uses second or third order Markov chains. So, we tested whether the process is second-order Markov i.e. we redefined the state so that it carried the history of $t-2$ and $t-1$. Although there was an improvement on the chi-square value (chi-square value = 490571, degree of freedom = 28672, critical value[2] = 29067), the hypothesis that the chain was a second-order MDP was also not justified. Using an even higher order Markov chain increases the size of the state space exponentially and so will affect the robustness of the model. So it is a trade off between improvements of fit and increase in size of model. Like many other authors we found the improvement when going to second order or higher order chains is not sufficient to warrant the loss in robustness and simplicity. Therefore, we chose to use first-order to simplify the state space as well as reducing the computational time, and the inaccuracy in doing this is not much greater than using a second order chain.

3.5 Transition matrix, profit function and results

Table 3.3 shows the transition matrix of the credit card data set. The first and second columns are the index for credit limit l, and behavioural score i at time t respectively, and moving to behavioural score i' corresponding to columns three to ten in the table. Note that *Bad* and *Closed* are absorbing states and thus we do not show their transition probability. The last column is the number of transitions corresponding to different initial states (l, i). Table 3.4 shows the average account balance. Note that the account balance

[2]at significant level $\alpha = 0.05$

Credit limit at t (l)	Score at t (i)	Score at (t+1) (i’)								Row Count
		Closed	Inactive	Bad	Risk	Score 1	Score 2	Score 3	Score4	
Limit 1	**Inactive**	2.04	97.4	0.01	-	0.37	0.18	0.03	0.01	108864
	Risk	5.72	-	23.58	27.86	42.86	-	-	-	140
	Score 1	1.4	0.42	0.24	0.17	88.99	8.07	0.72	0.03	57788
	Score 2	1.1	1.43	0.01	-	16.68	72.29	7.95	0.57	28246
	Score 3	0.87	1.59	-	-	4.74	28.06	58.74	6.03	9896
	Score 4	1.02	0.16	-	-	1.18	10.03	38.65	48.99	1876
Limit 2	**Inactive**	2.72	96.57	-	-	0.56	0.15	0.01	0.01	220073
	Risk	2.48	-	24.76	39.61	33.17	-	-	-	202
	Score 1	1.09	0.92	0.28	0.23	85.22	9.91	2.31	0.06	53409
	Score 2	0.81	3.09	0.01	-	11.44	73.46	10.23	0.99	45745
	Score 3	0.69	2.65	-	-	3.13	21.13	62.2	10.23	26438
	Score 4	0.61	0.37	-	-	0.94	4.18	30.6	63.33	10123
Limit 3	**Inactive**	2.34	96.27	-	-	1.1	0.29	0.02	0.01	164251
	Risk	4.49	-	27.76	37.15	30.62	-	-	-	245
	Score 1	1.11	0.62	0.23	0.24	80.58	13.19	3.97	0.09	63390
	Score 2	0.81	3.31	0.01	0.01	10.51	69.85	14.06	1.48	74790
	Score 3	0.68	2.1	-	-	2.96	20.49	61.68	12.12	59337
	Score 4	0.67	0.23	-	-	0.94	3.9	27.95	66.34	29234
Limit 4	**Inactive**	2.2	96.35	-	-	1.03	0.42	0.02	0.01	184735
	Risk	3.46	-	18.44	53.69	24.43	-	-	-	434
	Score 1	1.12	0.46	0.18	0.21	80.18	13.89	3.87	0.13	97345
	Score 2	0.88	2.68	0.01	-	10.63	67.75	16.25	1.83	114607
	Score 3	0.84	1.52	-	-	2.71	18	61.76	15.19	110722
	Score 4	0.66	0.2	-	-	0.8	2.58	23.28	72.51	79909
Limit 5	**Inactive**	1.71	96.33	-	-	1.18	0.75	0.03	0.02	82648
	Risk	2.99	0.38	28.36	44.78	23.51	-	-	-	268
	Score 1	1.29	0.18	0.18	0.16	79.86	14.44	3.73	0.2	92950
	Score 2	1.16	1.7	0.01	-	12.08	63.84	18.36	2.88	95216
	Score 3	1.06	1.34	0.01	-	2.9	15.91	62.45	16.36	107810
	Score 4	0.91	0.2	-	-	0.79	2.05	16.34	79.74	112369
Limit 6	**Inactive**	1.88	96.18	-	-	1.4	0.53	0.04	0.01	69422
	Risk	4.55	-	29.1	35.91	30.46	-	-	-	220
	Score 1	1.12	0.21	0.16	0.16	80.05	14.41	3.73	0.2	92302
	Score 2	1.04	1.67	0.01	-	12.22	64.91	17.79	2.4	95278
	Score 3	1.05	0.97	0.01	-	3.15	16.45	62.91	15.5	100804
	Score 4	1	0.13	0.01	-	0.8	1.99	14.62	81.49	108687
Limit 7	**Inactive**	2.07	96.65	-	-	0.79	0.47	0.04	0.01	97828
	Risk	1.48	-	27.46	34.32	36.77	-	-	-	204
	Score 1	1.07	0.34	0.15	0.15	79.46	15.16	3.41	0.3	91860
	Score 2	1.08	1.42	0.01	-	12.1	64.43	17.65	3.34	98416
	Score 3	1.17	0.9	0.01	-	3.03	15.28	61.6	18.05	105622
	Score 4	1.17	0.13	0.01	-	0.67	1.67	10.09	86.29	179493
Limit 8	**Inactive**	1.47	96.57	0.01	-	0.43	1.38	0.13	0.05	38094
	Risk	4.47	-	33.04	37.5	25	-	-	-	112
	Score 1	1.28	0.03	0.19	0.08	76.18	18.6	3.17	0.51	102040
	Score 2	1.37	0.22	0.01	-	11.15	64.08	18.7	4.5	138835
	Score 3	1.32	0.74	0.01	-	2.51	12.23	61.17	22.06	172165
	Score 4	1.28	0.08	0.01	-	0.5	1.28	7.07	89.82	489326

”‘-”’ represents there is no sample observation.

”‘0”’ represents the transition probability is less than 0.0005.

The transition probabilities of all absorbing states (Closed and Bad) are not shown in the table.

Table 3.3: Transition probability (in percentage)

of Closed and Inactive accounts equals zero.

Credit limit	**Score at t (i)**					
at t (l)	**Bad**	**Risk**	**Score 1**	**Score 2**	**Score 3**	**Score 4**
Limit 1	563	433	316	89	65	52
Limit 2	761	703	490	119	73	54
Limit 3	983	876	570	145	89	73
Limit 4	1658	1451	822	246	139	124
Limit 5	2234	2134	1229	522	235	194
Limit 6	3047	2891	1497	692	351	282
Limit 7	3605	3048	1745	830	467	361
Limit 8	5722	5480	3181	2187	1106	731

Table 3.4: Account average balance

We validated whether the transition probabilities of this credit card data set satisfied the assumptions given in Section 3.1.1. We do not expect the assumptions hold in the special accounts and thus we excluded the special accounts (i.e. Closed, Bad and Inactive) for this validation. We recalculated the transition matrix and the results are presented in column three to seven in Table 3.5. The cumulative row sums are presented in columns eight to twelve in Table 3.5 to check the stochastic ordering properties of Assumptions A.2 and A.1. For example, there are 8.99% accounts with Score 1 and Limit 1 moving to a state with behavioural Score 2 or above in the next month.

Not all transition probabilities satisfy the assumption on A.2. For example, there were 51.73% borrowers with *Limit 7* and behavioural score band *Risk* moving to a state with behavioural score 1 or above whereas there were 40% borrowers where *Limit 8* and behavioural score band *Risk* have the same movement. That implies Lemma 3.2 does not hold in reality and so the optimal profit does not necessarily increase with credit limit.

All transition probabilities satisfy the assumption on A.1. To check whether Lemma 3.1 holds, however, we still need to validate assumption A.3. We calculate the profit with $r = 2$ (i.e. we assume interest rate of 2% per month, which is around the norm

Credit limit at t - (l)	Score at t (i)	Row Percentage Score at $t+1$					Cumulative row sum Score at $t+1$				
		Risk	**Score 1**	**Score 2**	**Score 3**	**Score 4**	**Risk**	**Score 1**	**Score 2**	**Score 3**	**Score 4**
Limit 1	**Risk**	39.4	60.61	0	0	0	100	60.61	0	0	0
	Score 1	0.17	90.85	8.24	0.73	0.03	100	99.84	8.99	0.76	0.03
	Score 2	0	17.11	74.16	8.16	0.59	100	100	82.9	8.74	0.59
	Score 3	0	4.86	28.76	60.21	6.18	100	100	95.15	66.39	6.18
	Score 4	0	1.19	10.15	39.11	49.57	100	100	98.82	88.68	49.57
Limit 2	**Risk**	54.43	45.58	0	0	0	100	45.58	0	0	0
	Score 1	0.24	87.21	10.14	2.37	0.06	100	99.77	12.56	2.43	0.06
	Score 2	0	11.9	76.44	10.65	1.03	100	100	88.11	11.67	1.03
	Score 3	0	3.24	21.86	64.34	10.58	100	100	96.77	74.92	10.58
	Score 4	0	0.95	4.22	30.9	63.95	100	100	99.06	94.84	63.95
Limit 3	**Risk**	54.82	45.19	0	0	0	100	45.19	0	0	0
	Score 1	0.25	82.18	13.45	4.05	0.09	100	99.76	17.58	4.13	0.09
	Score 2	0.01	10.96	72.85	14.66	1.55	100	100	89.04	16.2	1.55
	Score 3	0	3.05	21.07	63.44	12.47	100	100	96.96	75.9	12.47
	Score 4	0	0.95	3.93	28.2	66.94	100	100	99.06	95.13	66.94
Limit 4	**Risk**	68.74	31.27	0	0	0	100	31.27	0	0	0
	Score 1	0.22	81.6	14.13	3.94	0.13	100	99.79	18.19	4.07	0.13
	Score 2	0	11.02	70.24	16.85	1.9	100	100	88.99	18.75	1.9
	Score 3	0	2.78	18.43	63.25	15.56	100	100	97.23	78.8	15.56
	Score 4	0	0.8	2.61	23.48	73.13	100	100	99.21	96.61	73.13
Limit 5	**Risk**	65.58	34.43	0	0	0	100	34.43	0	0	0
	Score 1	0.17	81.18	14.68	3.8	0.2	100	99.84	18.67	3.99	0.2
	Score 2	0	12.43	65.71	18.9	2.97	100	100	87.58	21.87	2.97
	Score 3	0	2.97	16.3	63.98	16.77	100	100	97.04	80.74	16.77
	Score 4	0	0.8	2.07	16.53	80.62	100	100	99.21	97.14	80.62
Limit 6	**Risk**	54.11	45.9	0	0	0	100	45.9	0	0	0
	Score 1	0.16	81.26	14.62	3.78	0.2	100	99.85	18.6	3.98	0.2
	Score 2	0	12.56	66.71	18.28	2.47	100	100	87.45	20.74	2.47
	Score 3	0	3.21	16.79	64.2	15.82	100	100	96.8	80.01	15.82
	Score 4	0	0.81	2.01	14.79	82.41	100	100	99.2	97.2	82.41
Limit 7	**Risk**	48.28	51.73	0	0	0	100	51.73	0	0	0
	Score 1	0.16	80.7	15.4	3.47	0.3	100	99.85	19.16	3.76	0.3
	Score 2	0	12.41	66.08	18.1	3.43	100	100	87.6	21.52	3.43
	Score 3	0	3.1	15.6	62.9	18.43	100	100	96.91	81.32	18.43
	Score 4	0	0.67	1.7	10.23	87.42	100	100	99.34	97.64	87.42
Limit 8	**Risk**	60	40	0	0	0	100	40	0	0	0
	Score 1	0.08	77.33	18.88	3.22	0.52	100	99.93	22.6	3.73	0.52
	Score 2	0	11.33	65.11	19.01	4.57	100	100	88.68	23.57	4.57
	Score 3	0	2.56	12.48	62.46	22.52	100	100	97.45	84.97	22.52
	Score 4	0	0.51	1.3	7.17	91.05	100	100	99.5	98.21	91.05

Table 3.5: Stochastic ordering for the credit card database's transition matrix

for standard credit cards). For Bad accounts, we assumed the loss equals to the account balance (i.e. the lender loss £563 for a Limit1 default account). The values are shown in Table 3.6.

Credit limit	**Score at t (i)**					
at t (l)	**Bad**	**Risk**	**Score 1**	**Score 2**	**Score 3**	**Score 4**
Limit 1	-563	8.66	6.32	1.78	1.3	1.04
Limit 2	-761	14.06	9.8	2.38	1.46	1.08
Limit 3	-983	17.52	11.4	2.9	1.78	1.46
Limit 4	-1658	29.02	16.44	4.92	2.78	2.48
Limit 5	-2234	42.68	24.58	10.44	4.7	3.88
Limit 6	-3047	57.82	29.94	13.84	7.02	5.64
Limit 7	-3605	60.96	34.9	16.6	9.34	7.22
Limit 8	-5722	109.6	63.62	43.74	22.12	14.62

Notes: The monthly profit of Inactive and Closed accounts equals to zero.

Table 3.6: Account monthly profit

Under the same credit limit l, the profit decreases in behavioural score i because a *Good* consumer is more likely to be a *transactor*. Credit card borrowers are classified into two groups: *Transactors* or *Revolvers*. A Transactor makes full repayment and a Revolver carries part of its outstanding balance to the next month. Thus credit lenders only gained interchange fees from the *transactors*. *Revolvers*, on the other hand, are found to be under financial pressure and so have a low behavioural score i. They accumulate a lump sum of debt on their credit accounts and generate higher profit to the lender who is receiving both interest and interchange fees. This means A.3 does not hold and so neither does 3.1 in the Lemma.

Credit limit	**Optimal Policy (Optimal Value)**					
at t	**Inactive**	**Risk**	**Score 1**	**Score 2**	**Score 3**	**Score 4**
$\lambda = 0.995$ (number of iteration:934)						
Limit 1	1(1039)	1(841)	8(2033)	8(1974)	1(1907)	1(1870)
Limit 2	6(885)	2(706)	8(2028)	8(1943)	8(1851)	8(1824)
Limit 3	6(889)	3(752)	8(2033)	8(1937)	8(1852)	8(1823)
Limit 4	6(891)	4(328)	8(2041)	8(1939)	8(1849)	8(1821)
Limit 5	6(893)	5(-141)	8(2053)	8(1947)	8(1843)	8(1816)
Limit 6	6(894)	6(-468)	8(2059)	8(1957)	8(1854)	8(1817)
Limit 7	7(788)	7(-560)	8(2069)	8(1960)	8(1851)	8(1815)
Limit 8	8(719)	8(-1943)	8(2098)	8(2000)	8(1864)	8(1822)
$\lambda = 0.99$ (number of iteration:663)						
Limit 1	1(705)	1(594)	8(1556)	8(1497)	1(1428)	1(1392)
Limit 2	6(611)	2(473)	8(1552)	8(1470)	8(1382)	8(1349)
Limit 3	6(614)	3(497)	8(1555)	8(1465)	8(1382)	8(1347)
Limit 4	6(616)	4(106)	8(1564)	8(1466)	8(1378)	8(1345)
Limit 5	6(617)	5(-325)	8(1575)	8(1474)	8(1374)	8(1340)
Limit 6	6(618)	6(-660)	8(1581)	8(1482)	8(1383)	8(1341)
Limit 7	7(544)	7(-768)	8(1590)	8(1485)	8(1381)	8(1339)
Limit 8	8(501)	8(-2106)	8(1619)	8(1523)	8(1393)	8(1346)

Table 3.7: Optimal policy and valued generated for the credit card database

3.6 Optimal policy

We implemented the value iteration algorithm[3] (Puterman, 1994) in MATLAB to generate the set of optimal policies $O(i, l)$. With ten credit limit states and eight behavioural score states, the size of the state space is eighty in each time period. We use different discount values to check the performance of the model. It took less than 1000 iterations to achieve the optimal solution and the computation time was less than 10 seconds. Table 3.7 presents the result with discount value $\lambda = 0.995$ (a rough estimate with yearly inflation rate 6%) and $\lambda = 0.99$ (a rough estimate with yearly inflation rate 12%). Also, the optimal values of the corresponding optimal policy are presented in the brackets. For example, the optimal policy for a state with Limit 1 and Score 4 is Limit 1 and the corresponding optimal profit is 1870.

For *inactive* accounts, the optimal policy for $s = (l, 1), \forall l = 2, 3, 4, 5$ is to increase the credit limit to $l = 6$. This follows since the optimal value function $V(6, 1)$ is the highest among the *inactive* accounts. It gives some encouragement to start using the account, but since there is no history of repayment does not go to the highest credit level. When the account is in the Risk state, the optimal policy is to keep the credit limit unchanged. The policy here is to keep the credit limit unchanged, because we cannot drop the limit, which is what would be ideal. For the other states - with less risky behavioural score values, the optimal policy is to move to the highest credit limit. The exception is those in the Score3 and Score4 but with the lowest credit limit Limit1. In this case the limit should remain unchanged. This is because the profit is very small with this credit limit

[3] As shown by Puterman (1994), there is an optimal policy for any discounted stationary MDP model. One method to find this optimal policy is by the value iteration algorithm that says the value function (3.1) will converge to the optimal value after a number of iteration. In each iteration, we compare the value functions in two consecutive iterations (i.e. V_t and V_{t-1}). If the differences of these two functions is less than a certain threshold (which we use a value 0.01) then the iteration can be stopped. Then the policy that gives the value function V_t is the optimal policy.

and the chance of moving to a lower behavioural score is substantial. The suspicion is that accounts in this situation must have been unsatisfactory in the past for the credit limit and the behavioural score to be so uncorrelated.

The reason that optimal policies of Score 1 to Score 4 accounts are all Limit 8 can be explained by the probability of default. In the model, the default probabilities of Score 1 to Score 4 accounts are low, therefore the expected loss given default (that is the reward of moving to Bad), $p(D|l,i)V(l,D)$ (where D indicates the account is *Bad*) is very small and thus the model will not be aware the possible loss given default. Whereas the default probabilities of *Risk* accounts are high, the model thus suggests keep these accounts' credit limit unchanged. These results show the model is rather sensitive to the default probabilities.

Whereas if the lender applies these policies (i.e. increase the credit limit of Score 1 to 4 accounts to Limit 8) in a long run, it is possible that many of those starting with low credit limit accounts (such as those Limit 1 to Limit 5) will be default. This is because many of them are not capable to return their balance. In this case, the loss given default of the lender will increase unexpectedly. This shows that this model cannot be applied directly but adjustment should be done. We will further explore the adjustment in the following chapters.

3.7 Conclusions

This chapter demonstrated how the Markov Decision Process model can be used to build a model for adjusting credit card limits. The summary of the results and conclusions are as follows:

- First-order MDP models can balance between achieving robustness and satisfying the Markovian assumption.

- A top down coarse-classifying approach has a more satisfactory performance over the traditional bin-merging approach.

- The model considered in this chapter is a fairly simple one though illustrates that applying MDPs to optimize credit card consumer lifetime value is viable.

Chapter 4

Adjusting the probability of defaults of low default portfolios

Here in this chapter, we are going to adjust the probability of default of the MDP model in order to improve the model performance. In particular, we are trying to find the most prudent estimator for the default probability. We are going to use Pluto and Tasche (2006)'s approach to adjust the default probability. A detail review of their paper can be found in Chapter 2.

4.1 Estimating the probability of Default

Suppose there are $n(l,i)$ accounts in state (l,i) and $D(l,i)$ of them move to default at the next time period, assuming the Markov chain is stationary means the maximum likelihood estimator $\hat{p}_D(l,i)$ for the probability $p_D(l,i)$ is $\frac{D(l,i)}{n(l,i)}$.

In reality, default is a rare event, particularly for high quality portfolios. There may be no examples in the data of transition from certain states (l,i) to the default state

D. Thus it is possible $\hat{p}_D(l, i)$ will be very small or even equal to zero. Putting such estimates into the MDP model leads to apparent "structural zeros" [1] which change the connectedness of the dynamics in the state space. If the probability of default from a given state is zero this can lead to unusual optimal policies because the system wants to move to those apparently "safe" states.

		Transition Probabilities			Profit	Optimal policy (value)
Credit limit	**Score**	**Excellent**	**Good**	**Default**		**at $t = 1$**
	Excellent	0.925	0.075	0	50	50000+ (204)
< 10000	**Good**	0.75	0.25	0	80	50000+ (163)
	Default	0	0	1	-	-
	Excellent	0.98	0.0198	0.0002	80	50000+ (121)
$[10000, 50000)$	**Good**	0.164	0.8333	0.0027	120	< 10000 (175)
	Default	0	0	1	-5000	-
	Excellent	0.9333	0.0653	0.0013	100	< 10000 (122)
50000+	**Good**	0.25	0.74	0.01	150	< 10000 (192)
	Default	0	0	1	-40000	-

Table 4.1: A synthetic example

For example, suppose there are only three behavioural states: Excellent, Good and Bad where Bad is the default state, three credit limit states, < 10000, $[10000, 50000)$ and 50000+, and the corresponding transition probabilities and profit values are described in column three to five in Table 4.1.

We used these transition probabilities and profit values to calculate the optimal policy at $t = 1$ which is listed at the last column of Table 4.1. One can find that the optimal policy for an account having credit limit 50000+ and Excellent behavioural score is to reduce the credit limit to < 10000. This is because the default probability in a state with credit limit < 10000 is zero. The MDP model regards < 10000 as a "'safe policy"' which will not default at all. This state therefore is being chosen by the MDP model, as illustrated in Figure 4.1 (where the arrow indicates the MDP model chooses to decrease

[1] An entry with a structural zero has an expected value of zero, which means that not only did no observations in the dataset at hand fall into that cell, but in fact that no observation could fall into that entry (Berger and Zhang, 2005).

credit card holder's credit limit from 50000+ to < 10000).

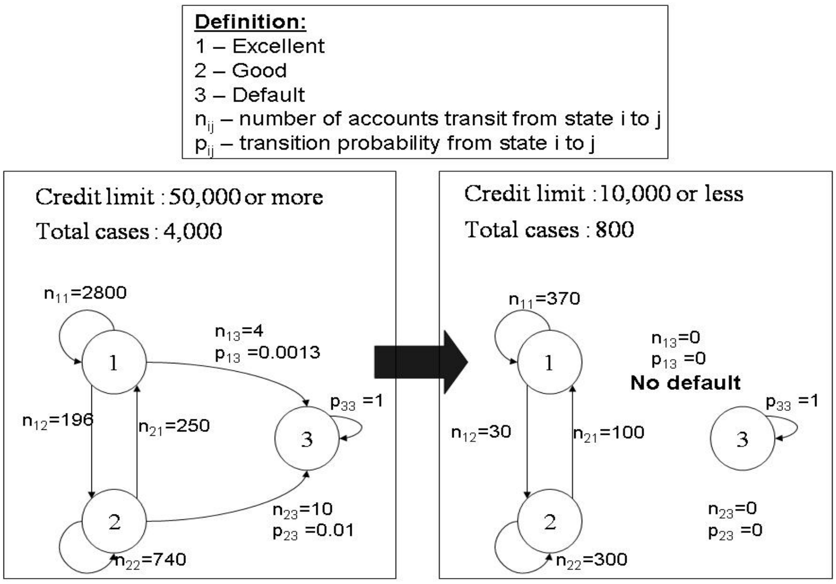


Figure 4.1: An unexpected optimal policy

4.2 Estimating the PDs of low default portfolios

One way to overcome this problem is to take a conservative estimate of the default probabilities, rather than the maximum likelihood estimate. This possibility has been extensively discussed in the context of the Basel Accord where again bank regulators and lenders have been considering the robustness of estimates of default probabilities in low default portfolios.

We follow the approach introduced by Pluto and Tasche (2006) and extended by Benjamin et al. (2006). Firstly we assume the transitions to default are monotonically decreasing as the behavioural score increases, and so if the score bands are labelled with I being the highest quality and there are r low default portfolios, so

$$p_D(l, I) \leq p_D(l, I-1) \leq \ldots \leq p_D(l, I-(r-2)) \leq p_D(l, I-(r-1)) \tag{4.1}$$

So a conservative assumption would be that

$$p_D(l,I) = p_D(l,I-1) = \ldots = p_D(l,I-(r-2)) = p_D(l,I-(r-1)) \tag{4.2}$$

where $I-(r-1)$ is the most risky of the low default portfolios. The second conservative assumption in this approach is not to use the MLE estimate of the default probability, but rather take the lower confidence limit of the default probability. There are D accounts (a subjective choice) defaulting in the next period from all the low default portfolios (the number of low default portfolios is $n \equiv \sum\limits_{m=I-(r-1)}^{I} n(l,m)$ accounts). It is assumed this follows a Binomial distribution $B(n, p_D(l,I))$. One chooses $p_D(l,I)$ to be the highest probability of default, so that the corresponding lower γ-confidence limit is exactly D, i.e. getting a lower number of default this D has a $1-\gamma$ probability of occurring, i.e.

$$1-\gamma = \sum_{k=0}^{D} \binom{n}{k} p_D(l,I)^k (1-p_D(l,I))^{n-k} \tag{4.3}$$

This is how one can obtain the default probability of the (l,I) accounts. One chooses the estimate $\hat{p}_D(l,I)$ in this way for these states (l,I) where the number of actual defaults $D_t(l,I)$ is at or below same agreed value - which might be zero. One would like to use MLE to obtain the estimates of the other transition probabilities $\hat{p}_j(l,I), j \neq D$ from state (l,I). However this would result in the sum of the transition probabilities being greater than 1 and so instead one defines $\gamma(l,I)\hat{p}_j(l,I), \forall j \neq D$ where

$$\gamma(l,I) = \frac{1-\hat{p}_D(l,I)}{\sum_{j\neq D} \hat{p}_j(l,I)} \tag{4.4}$$

For the rest of the LDP, one can use the following general form to estimate the default probability

$$1-\gamma = \sum_{k=0}^{D} \binom{n - \left(\sum\limits_{m=i+1}^{I} n(l,m)\right)}{k} p_D(l,i)^k (1-p_D(l,i))^{n-\left(\sum\limits_{m=i+1}^{I} n(l,m)\right)-k}, \tag{4.5}$$

for all $i = I-(r-1), \ldots, I-1$

For states (l,i) where $i = 1,2,\ldots,I-r$, MLE are used to estimate all transition probabilities.

4.3 Empirical results

We tested the proposed low default portfolios adjustment with the UK data. We have further segmented the population by card type since we want to reduce the heterogeneity of the testing sample. This is based on the assumption that the consumption and migration patterns of borrowers having the other credit card type (i.e. platinum card) are different from those with Master card. Also the reason for restricting the sample to only Master card customers was that they were a homogeneous segment but one where a wide variety of credit limits were applied. The other part of the sample - Platinum card customers, were essentially only given credit limit 5 or higher and so provided no information on what happens with lower credit limits. Table 4.2 shows the transition matrix, Table 4.3 shows the adjusted PDs for the low default accounts and Table 4.4 shows the average account balance. It is proposed to test the model sensitivity by varying r in (3.4) which is the key factor of the MDP reward. Hence the first phase of the analysis has tested the model with $r = 6$, 12, 24 and 30.

There is no standard in selecting the low default accounts. Benjamin et al. (2006) uses 20 default cases as the cutoff value in a corporate rating context. If the credit portfolio has lower than 20 default cases from the sampling period, it is labelled as *low default portfolios*. In all the credit limit bands the number of default cases in Score 2, Score 3 and Score 4 are less than 10, thus we classified them as *low default accounts*. We use a confidence interval 95% in this study since we would like to use a more conservative approach to estimate the probability of default.

Credit limit at t (l)	Score at t (i)	Score at (t+1) (i')								Row Count
		Closed	**Inactive**	**Bad**	**Risk**	**Score 1**	**Score 2**	**Score 3**	**Score 4**	
Limit 1	**Inactive**	1.44	97.94	-	-	0.32	0.27	0.03	0.00	63100
	Risk	6.42	-	24.77	29.36	39.45	-	-	-	109
	Score 1	1.40	0.19	0.24	0.14	89.52	7.87	0.62	0.03	51653
	Score 2	1.09	0.83	0.00	-	18.40	70.54	8.43	0.70	21772
	Score 3	0.84	1.46	-	-	4.85	27.38	58.23	7.24	7831
	Score 4	0.91	0.11	-	-	1.14	8.97	37.42	51.45	1761
Limit 2	**Inactive**	1.28	97.27	-	-	0.29	1.07	0.08	0.01	23733
	Risk	2.83	-	20.75	60.38	16.04	-	-	-	106
	Score 1	1.21	0.08	0.29	0.13	87.33	9.26	1.61	0.09	33988
	Score 2	1.06	0.87	0.01	-	16.08	64.18	15.44	2.36	17483
	Score 3	0.76	1.90	-	-	2.78	14.40	65.03	15.13	17296
	Score 4	0.59	0.14	-	-	0.90	3.02	31.10	64.25	9272
Limit 3	**Inactive**	1.36	96.71	-	-	0.54	1.35	0.04	0.01	30090
	Risk	5.00	-	22.00	48.00	25.00	-	-	-	100
	Score 1	1.39	0.12	0.22	0.14	83.27	12.74	1.98	0.14	35643
	Score 2	1.13	1.15	0.01	0.00	13.75	62.62	18.00	3.33	30150
	Score 3	0.83	1.77	-	-	2.51	13.28	63.55	18.05	38713
	Score 4	0.69	0.11	-	-	0.91	2.95	27.27	68.07	28151
Limit 4	**Inactive**	1.52	96.64	-	-	0.41	1.40	0.03	0.00	50167
	Risk	4.39	-	15.35	63.16	17.11	-	-	-	228
	Score 1	1.34	0.10	0.17	0.14	81.73	14.36	1.97	0.19	61837
	Score 2	1.25	1.07	0.00	-	12.21	63.26	19.08	3.14	60541
	Score 3	1.01	1.50	-	-	2.38	12.81	62.35	19.95	81534
	Score 4	0.67	0.12	-	-	0.76	2.06	22.69	73.70	77658
Limit 5	**Inactive**	1.57	96.63	-	-	0.31	1.45	0.03	0.01	33519
	Risk	4.58	-	30.53	46.56	18.32	-	-	-	131
	Score 1	1.55	0.03	0.17	0.14	81.25	14.84	1.84	0.19	53314
	Score 2	1.59	0.54	0.00	-	12.68	61.58	20.31	3.30	49378
	Score 3	1.25	1.54	0.00	-	2.38	11.97	62.70	20.16	70709
	Score 4	0.91	0.15	-	-	0.79	1.75	19.08	77.32	79164
Limit 6	**Inactive**	1.52	96.79	-	-	0.32	1.33	0.04	0.00	21802
	Risk	6.80	-	28.16	45.63	19.42	-	-	-	103
	Score 1	1.40	0.04	0.15	0.14	80.76	15.38	1.92	0.22	41814
	Score 2	1.43	0.37	-	-	12.77	63.35	19.34	2.74	40299
	Score 3	1.24	1.30	-	-	2.52	11.07	63.86	20.01	57049
	Score 4	0.92	0.11	-	-	0.82	1.54	17.34	79.27	68704
Limit 7	**Inactive**	1.88	96.42	-	-	0.26	1.42	0.02	0.01	17647
	Risk	2.60	-	20.78	50.65	25.97	-	-	-	77
	Score 1	1.25	0.05	0.12	0.12	79.41	16.56	2.19	0.28	33897
	Score 2	1.44	0.27	0.01	-	12.22	63.82	19.16	3.09	36900
	Score 3	1.44	1.27	0.00	-	2.52	10.82	62.74	21.21	53228
	Score 4	1.25	0.13	0.00	-	0.72	1.45	15.51	80.93	74951
Limit 8	**Inactive**	2.01	96.06	0.01	-	0.28	1.60	0.03	0.01	18828
	Risk	4.55	-	42.42	37.88	15.15	-	-	-	66
	Score 1	1.17	0.02	0.13	0.07	78.50	17.75	2.09	0.27	63345
	Score 2	1.26	0.14	0.00	-	11.49	67.37	17.37	2.37	79939
	Score 3	1.26	1.06	0.00	-	2.37	11.28	63.28	20.74	98351
	Score 4	1.23	0.10	0.00	-	0.66	1.30	12.30	84.41	166070

"'-"' represents there is no sample observation.

"'0"' represents the transition probability is less than 0.0005.

The transition probabilities of all absorbing states (Closed and Bad) are not shown in the table.

Table 4.2: Transition probability of Master Card accounts (in percentage)

Credit limit	Score	MLE	Adjusted
Limit 1	**Score 2**	0.00459%	0.0218%
	Score 3	0%	0.0160%
	Score 4	0%	0.0151%
Limit 2	**Score 2**	0.0114%	0.0360%
	Score 3	0%	0.0181%
	Score 4	0%	0.0143%
Limit 3	**Score 2**	0.0133%	0.0304%
	Score 3	0%	0.0133%
	Score 4	0%	0.00944%
Limit 4	**Score 2**	0.00165%	0.00784%
	Score 3	0%	0.00334%
	Score 4	0%	0.00216%
Limit 5	**Score 2**	0.00405%	0.0157%
	Score 3	0.00141%	0.00646%
	Score 4	0%	0.00389%
Limit 6	**Score 2**	0%	0.00743%
	Score 3	0%	0.00308%
	Score 4	0%	0.00180%
Limit 7	**Score 2**	0.00542%	0.0248%
	Score 3	0.00188%	0.0102%
	Score 4	0.00133%	0.00555%
Limit 8	**Score 2**	0.00375%	0.0132%
	Score 3	0.00102%	0.00590%
	Score 4	0.000602%	0.00305%

The second column is the default probabilities calculated with maximum likelihood estimates.

The third column is the default probabilities calculated with adjusted PD method.

Table 4.3: Default Probabilities for Low Default accounts

Credit limit	**Score at t (i)**					
at t (l)	**Bad**	**Risk**	**Score 1**	**Score 2**	**Score 3**	**Score 4**
Limit 1	375	364	327	109	73	50
Limit 2	740	732	572	252	88	64
Limit 3	1066	1077	737	259	106	79
Limit 4	1495	1561	939	369	159	133
Limit 5	2128	2176	1362	668	264	187
Limit 6	3056	2738	1666	918	390	271
Limit 7	3931	3844	1960	1171	503	344
Limit 8	5727	5721	3017	2121	982	728

Table 4.4: Account average balance (Master card only)

4.3.1 Optimal policy generated by value iteration

We used (3.1) to find the optimal credit limit for master card customers. To do so, we implemented the model with the value iteration algorithm Puterman (1994) in MATLAB to generate the set of optimal policies. It took less than 10 seconds to complete the iteration process. We present the result with discount value $\lambda = 0.995$ (a rough estimate with yearly inflation rate 6) in Table 4.5 for illustration. The third to the seventh column present the optimal policies of using different profit values. The entry in column three and row four is 273(7) which indicates, if the profit of an account in a state with Score 1 and Limit 1 is 0.5% (=6%/12) of its monthly balance, the optimal credit limit policy is limit 7, the corresponding optimal value is 273. Since the optimal policies of using different profit values are very similar, we only used the profit function of those calculated by APR equals 24% to test the model performance of adjusting the default probabilities of the low default portfolios, and results are presented in the last column of Table 4.5.

For *inactive* accounts, the optimal policy for $s = (l, 1), \forall l = 1, 2$ is to increase the credit limit to $l = 3$. This follows since the optimal value function $V(3, 1)$ is the highest among the *inactive* accounts. It gives some encouragement to start using the accounts but since there is no history of repayment does not suggest going to the highest credit level. For the *inactive* accounts with credit limit $l = 3, 4, 5, 6$, the optimal policy is to keep the credit limit unchanged. This is because optimal value for $s = (l, 1)$ is decreasing in l with $\forall l = 3, 4, 5, 6$. Since the optimal value for $s = (8, 1)$ is greater than $s = (7, 1)$, the optimal policy for both is to have credit limit $l = 8$. State $i = 2$ is when the accounts are already *bad*, and so the policy is to close them down, and the value function reflects the loss because the account has defaulted. When the account is in the risky state, i.e. $l = 3$, the optimal policy is to keep the credit limit unchanged. The policy here is to keep the credit limit unchanged, because we cannot drop the limit, which is what would be ideal.

Credit Limit l	Score i	APR 6	12	18	24	30	Adjusted PD APR = 24
Limit 1	Inactive	143 (3)	337 (3)	532 (3)	726 (3)	920 (3)	718 (3)
	Risk	23 (1)	253 (1)	483 (1)	713 (1)	943 (1)	706 (1)
	Score 1	273 (7)	683 (8)	1093 (8)	1503 (8)	1913 (8)	1489 (8)
	Score 2	292 (8)	676 (8)	1060 (8)	1445 (8)	1829 (8)	1429 (8)
	Score 3	279 (8)	638 (8)	1002 (1)	1367 (1)	1733 (1)	1352 (1)
	Score 4	274 (8)	628 (1)	987 (1)	1346 (1)	1705 (1)	1330 (1)
Limit 2	Inactive	145 (3)	341 (3)	537 (3)	733 (3)	928 (3)	725 (3)
	Risk	-265 (2)	-92 (2)	82 (2)	256 (2)	429 (2)	250 (2)
	Score 1	273 (7)	683 (8)	1094 (8)	1506 (8)	1917 (8)	1491 (8)
	Score 2	290 (8)	671 (8)	1053 (8)	1434 (8)	1815 (8)	1417 (8)
	Score 3	275 (8)	627 (8)	979 (8)	1331 (8)	1683 (8)	1315 (8)
	Score 4	273 (8)	618 (8)	963 (8)	1308 (8)	1653 (8)	1292 (8)
Limit 3	Inactive	145 (3)	342 (3)	539 (3)	735 (3)	932 (3)	728 (3)
	Risk	-306 (3)	-100 (3)	106 (3)	312 (3)	517 (3)	305 (3)
	Score 1	276 (7)	686 (8)	1096 (8)	1506 (8)	1916 (8)	1491 (8)
	Score 2	289 (8)	667 (8)	1045 (8)	1423 (8)	1801 (8)	1407 (8)
	Score 3	275 (8)	626 (8)	977 (8)	1328 (8)	1679 (8)	1313 (8)
	Score 4	272 (8)	617 (8)	961 (8)	1305 (8)	1650 (8)	1290 (8)
Limit 4	Inactive	137 (4)	321 (4)	504 (4)	687 (4)	871 (4)	680 (4)
	Risk	-466 (4)	-257 (4)	-48 (4)	163 (4)	372 (4)	156 (4)
	Score 1	280 (8)	691 (8)	1102 (8)	1513 (8)	1924 (8)	1499 (8)
	Score 2	290 (8)	667 (8)	1045 (8)	1423 (8)	1801 (8)	1407 (8)
	Score 3	275 (8)	626 (8)	976 (8)	1327 (8)	1678 (8)	1312 (8)
	Score 4	272 (8)	616 (8)	959 (8)	1303 (8)	1647 (8)	1289 (8)
Limit 5	Inactive	135 (5)	313 (5)	492 (5)	671 (5)	849 (5)	663 (5)
	Risk	-1089 (5)	-929 (5)	-769 (5)	-608 (5)	-448 (5)	-613 (5)
	Score 1	282 (8)	694 (8)	1107 (8)	1519 (8)	1931 (8)	1505 (8)
	Score 2	291 (8)	669 (8)	1048 (8)	1427 (8)	1806 (8)	1411 (8)
	Score 3	274 (8)	624 (8)	974 (8)	1324 (8)	1675 (8)	1310 (8)
	Score 4	271 (8)	614 (8)	956 (8)	1299 (8)	1641 (8)	1284 (8)
Limit 6	Inactive	132 (6)	308 (6)	484 (6)	659 (6)	835 (6)	652 (6)
	Risk	-1443 (6)	-1271 (6)	-1100 (6)	-928 (6)	-756 (6)	-933 (6)
	Score 1	285 (8)	699 (8)	1113 (8)	1528 (8)	1942 (8)	1514 (8)
	Score 2	293 (8)	675 (8)	1056 (8)	1438 (8)	1820 (8)	1422 (8)
	Score 3	275 (8)	626 (8)	977 (8)	1328 (8)	1679 (8)	1313 (8)
	Score 4	272 (8)	614 (8)	957 (8)	1300 (8)	1642 (8)	1285 (8)
Limit 7	Inactive	123 (8)	293 (8)	463 (8)	633 (8)	803 (8)	626 (8)
	Risk	-1450 (7)	-1195 (7)	-939 (7)	-684 (7)	-428 (7)	-691 (7)
	Score 1	289 (8)	705 (8)	1121 (8)	1537 (8)	1953 (8)	1522 (8)
	Score 2	294 (8)	677 (8)	1060 (8)	1442 (8)	1825 (8)	1426 (8)
	Score 3	275 (8)	626 (8)	976 (8)	1327 (8)	1678 (8)	1312 (8)
	Score 4	271 (8)	612 (8)	954 (8)	1296 (8)	1637 (8)	1281 (8)
Limit 8	Inactive	123 (8)	293 (8)	463 (8)	633 (8)	804 (8)	627 (8)
	Risk	-3763 (8)	-3615 (8)	-3467 (8)	-3319 (8)	-3171 (8)	-3323 (8)
	Score 1	297 (8)	718 (8)	1139 (8)	1561 (8)	1982 (8)	1546 (8)
	Score 2	300 (8)	690 (8)	1079 (8)	1468 (8)	1857 (8)	1452 (8)
	Score 3	278 (8)	633 (8)	987 (8)	1341 (8)	1695 (8)	1326 (8)
	Score 4	273 (8)	616 (8)	959 (8)	1302 (8)	1645 (8)	1288 (8)

Table 4.5: Optimal policy

For the other states - with less risky behavioral score values, the optimal policy is to move to the highest credit limit, in general. One exception is those in the high behavioral score bands $i = 6, 7$ but with the lowest credit limit $l = 1$. In this case, the limit should remain unchanged. This is because the chance of moving to a lower behavioral score is substantial compared to consumers with higher credit limit but the same behavioral score band. The suspicion is that accounts in this situation must have been unsatisfactory in the past for the credit limit and the behavioral score to be so uncorrelated. However if the APR is 6%, the optimal policy is to increase the credit limit to the highest level since the reward of $l = 1$ is extremely small. The second exception is those in the behavioral band $i = 4$ with credit limit $l = 1, 2, 3$ when APR is 6%. The optimal credit limit is to increase up to $l = 7$. This can be explained by these accounts have a high potential to go default. If the profitability is low, it is better to employ a more conservative policy. As expected, if one adjusted the PDs of Low Default accounts, the optimal value is lower.

4.4 Conclusion

This chapter investigates the use of adjusting PDs on LDPs method in building a MDP model to adjust credit card limit. It turned out that in this case the optimal policy for using a transition matrix with adjusting PDs was the same as those without. However, a model built with non-zero PDs is believed to be a more robust model which avoids constructing a structural zero MDP model.

Chapter 5

Incorporating economic conditions

In this chapter we look at the interaction between credit migration and macroeconomics. We introduce a MDP model with state variables describing the economy. This proposed model is tested with a Hong Kong credit card dataset. Empirical results of the model performance under different economic conditions are presented. This model is useful in real-world applications as it is able to incorporate external factors into a credit card pricing model.

5.1 Models

5.1.1 The MDP model

Consider a discrete discounted MDP model with decision epochs $\mathcal{T}$ (indexed by $t = 1, 2, \ldots, T$) based on a state space S. The state space consists of the current credit limit represented by $\mathcal{L}$ (indexed by $l = 0, 1, 2 \ldots, L$), the current behavioural score $\mathcal{I}$ (indexed by $i = 0, 1, 2, \ldots, I$). The current macroeconomic variables are represented by a $1 \times M$ row vector $\mathbf{M}$, where M is the total number of macroeconomic variables used in the model. As usual, we limit the actions to either keeping the credit limit fixed or increasing to a new credit limit, since most credit card organizations do not want to drop a borrower's credit limit, and thus the action set is defined as $A_l = \{l' : l \leq l'\}$. Define $r(l, i, \mathbf{M})$ to be the profit obtained in the current period by a credit account with a credit limit l, a behavioural score i and macroeconomic variables $\mathbf{M}$.

The objective is to construct a solution to the discounted profit optimality equation where there is a discount factor λ describing the time value of money. This leads to the following optimality equation for $V_t(l, i, \mathbf{M})$, the maximum expected profit over the next t periods that can be obtained from an account which currently has behaviour score i, credit limit l and macroeconomic variables $\mathbf{M}$:

$$\begin{aligned} V_t(l, i, \mathbf{M}) \quad = \quad & \max_{l' \in A_l} \{ r(l, i, \mathbf{M}) + \lambda \sum_{i'} p(i'|i, \mathbf{M}) \\ & \int q_{t-1}(\mathbf{U}|\mathbf{M}) V_{t-1}(l', i', \mathbf{U}) \mathrm{d}\mathbf{U} \} \end{aligned} \tag{5.1}$$

The right-hand-side of (5.1) corresponds to the profit over the next t periods if we change the credit limit to l' at the end of the current period for an account with a behavioural score i and macroeconomic variables $\mathbf{M}$. $p(i'|i, \mathbf{M})$ gives the chance that this behavioural score changes to i', and $q_{t-1}(\mathbf{U}|\mathbf{M})$ is the probability that the current macroeconomic variables change to $\mathbf{U}$. Each entry in the $1 \times M$ row vector $\mathbf{U}$ represents a macroeconomic variable that can take any real value in $(-\infty, \infty)$. The multiple integral includes all possible values of the macroeconomic variables in the next period. The profit to the lender from the credit card borrower in the current period is $r(l, i, \mathbf{M})$ and the profit in the remaining $t-1$ period is $V_{t-1}(l', i', \mathbf{U})$ if the behavioural score changes to i' and the macroeconomic variables change to $\mathbf{U}$. The definition of the discount factor λ and the explanation of the optimality principle are the same as those presented in Chapter 3. $V_0(l, i, \mathbf{M})$, for all $l, i, \mathbf{M}$, are the boundary conditions of (5.1), i.e. the expected return of a customer at the end of the planning horizon. In this study, we assume the boundary conditions equal to zero to simplify the discussion. Whereas it is possible to set up different boundary conditions for different accounts (such as introducing penalty for accounts with low behavioural score etc), we leave it for future research to understand the sensitivity of the model to these boundary conditions.

Given a borrower currently in behavioural score state i, what change in the behavioural score occurs in the next period and what is the impact of the macroeconomic variables on this movement? We can estimate this impact by assuming i', the state the account moves to, is the outcome variable and the set of macroeconomic variables are the explanatory variables in a regression model. As the outcome variable i' is divided into different bands, we analyse the relationship between the outcome and the explanatory variables using logistic regression models (Hosmer and Lemeshow, 2000). Furthermore, the behavioural score band i' can be classified into two groups: ordered and unordered. In this case, we need to use both multinomial and cumulative logistic regression models to analyze the transition probabilities.

We assume in the model that $p(i'|i, \mathbf{M})$ in (5.1) does not depend on the current credit limit l. One might expect borrowers with different credit limits will have different behavioural score movements. If we define the transition probability conditional on the current credit limit (as in (3.1)), the transition probability can capture the state migration better. However, if we define the transition probability conditional on credit limit, behavioural score and macroeconomic variables, the transition probability will cover three dimensions. In this case, the samples break down into very small sets, which complicates our analysis. If we additionally include credit limit in our state space, it is feasible to have borrowers having the same behavioural score but different credit limits going different ways under the same economic conditions. This would mean considering second order effects, where we initially wish to understand the impact of the economy on behavioural score changes. In any event, one of the objectives for this study is to explore whether the lender can use the behavioural score as a key parameter to determine credit limit. The current definition already provides a precise and clear framework for considering the interaction between economy and behavioural score. Therefore we can use the current definition to concentrate on our discussion of economic effects. We therefore leave the possibility of using a transition probability conditional on current credit limit, current behavioural score and current macroeconomic variables as an area for possible future research.

5.1.2 The multinomial logistic regression model

In general, we can classify credit card accounts into three types. If a borrower has terminated the credit card account without incurring any debt, this account is called a *Closed* (represented by C) account. The second type is an *Inactive* (represented by T) account where the credit card account has not been active for a period of time. These Closed or Inactive accounts generate a small monthly loss since every account incurs an operational cost. An account that is neither Closed nor Inactive is termed *Active* (represented by A). Active accounts bring profit, either gains or losses, to the lender, and thus accounts in default or in arrears are also termed Active. We can estimate the relationship between these account types and the impact of the economy through

$$\log\left(\frac{p(A|i,\mathbf{M})}{p(C|i,\mathbf{M})}\right) = \beta^0_{(A,C|i,\mathbf{M})} + \mathbf{M}\beta_{(A,C|i,\mathbf{M})} \tag{5.2}$$

$$\log\left(\frac{p(T|i,\mathbf{M})}{p(C|i,\mathbf{M})}\right) = \beta^0_{(T,C|i,\mathbf{M})} + \mathbf{M}\beta_{(T,C|i,\mathbf{M})} \tag{5.3}$$

$$\log\left(\frac{p(A|i,\mathbf{M})}{p(T|i,\mathbf{M})}\right) = \beta^0_{(A,T|i,\mathbf{M})} + \mathbf{M}\beta_{(A,T|i,\mathbf{M})} \tag{5.4}$$

Note that entries in the $\mathbf{M} \times 1$ column vector $\beta_{(A,C|i,\mathbf{M})}$ are the regression coefficients corresponding to the macroeconomic variables $\mathbf{M}$, similarly for $\beta_{(T,C|i,\mathbf{M})}$ and $\beta_{(A,T|i,\mathbf{M})}$. The assumption in (5.2) is that the log of the ratio of the number of accounts in state i which become Inactive to the number of accounts which become Closed is a linear function of the macroeconomic variables. One can derive the outcome probabilities (listed below) by (5.2)-(5.4).

$$
\begin{aligned}
p(C|i,\mathbf{M}) &= \frac{1}{1+\exp\left(\beta^0_{(A,C|i,\mathbf{M})}+\mathbf{M}\beta_{(A,C|i,\mathbf{M})}\right)+\exp\left(\beta^0_{(T,C|i,\mathbf{M})}+\mathbf{M}\beta_{(T,C|i,\mathbf{M})}\right)} \\
p(A|i,\mathbf{M}) &= \frac{\exp(\beta^0_{(A,C|i,\mathbf{M})}+\mathbf{M}\beta_{(A,C|i,\mathbf{M})})}{1+\exp\left(\beta^0_{(A,C|i,\mathbf{M})}+\mathbf{M}\beta_{(A,C|i,\mathbf{M})}\right)+\exp\left(\beta^0_{(T,C|i,\mathbf{M})}+\mathbf{M}\beta_{(T,C|i,\mathbf{M})}\right)} \\
p(T|i,\mathbf{M}) &= \frac{\exp(\beta^0_{(T,C|i,\mathbf{M})}+\mathbf{M}\beta_{(T,C|i,\mathbf{M})})}{1+\exp\left(\beta^0_{(A,C|i,\mathbf{M})}+\mathbf{M}\beta_{(A,C|i,\mathbf{M})}\right)+\exp\left(\beta^0_{(T,C|i,\mathbf{M})}+\mathbf{M}\beta_{(T,C|i,\mathbf{M})}\right)}
\end{aligned}
$$

It is simple to verify that the sum of the above equations is equal to 1.

5.1.3 The cumulative logistic regression model

The Active accounts include those with behavioural score and those which have just defaulted. There is an obvious ordering of creditworthiness in such accounts, starting with those in the highest behavioural score band, and going down the score bands to end with the default accounts. Hence we can use cumulative logistic regression to exploit this ordering.

Define $P(j|i,\mathbf{M}) \equiv \sum_{k\leq j} p(k|i,\mathbf{M})$ as the cumulative probability that a borrower currently with a behavioural score i moves to a behavioural state j or lower if the macroeconomic variables take values $\mathbf{M}$. The relationship between the transition probabilities and the macroeconomic variables is estimated using the cumulative logistic regression models, i.e.

$$
\log\left(\frac{P(j|i,\mathbf{M})}{1-P(j|i,\mathbf{M})}\right) = \gamma^0_{(j|i,\mathbf{M})}+\mathbf{M}\mathbf{\Gamma}_{(i,\mathbf{M})}, \qquad \forall j=2,\ldots,I-1 \tag{5.5}
$$

In (5.5), the left-hand-side of the equation is the log of the odds of the expected number of borrowers moving to a state with behavioural score band j or lower compared with the the expected number of borrowers moving to a state with behavioural score band higher than j. For different score bands j, the equation fits the same vector of coefficients $\mathbf{\Gamma}_{(i,\mathbf{M})}$

but different intercepts $\gamma^0_{(j|i,\mathbf{M})}$. This means the impact of macroeconomic variables is the same for a state with initial behavioural score band i.

Note that for any Markov chain model, the row sum of the transition matrix must add up to 1. Using $p_k(A|i,\mathbf{M})$ and $P(j|i,\mathbf{M})$, we calculate the entries by

$$p(i'|i,\mathbf{M}) = [P(i'|i,\mathbf{M}) - P(i'-1|i,\mathbf{M})]\, p(A|i,\mathbf{M}), \forall i, i', \mathbf{M} \quad (5.6)$$

Since the intercept $\gamma^0_{(j|i,\mathbf{M})}$ of the cumulative logistic regression model in (5.5) increases in j, i.e. $\gamma^0_{(j+1|i,\mathbf{M})} \geq \gamma^0_{(j|i,\mathbf{M})}, \forall j$ (Hosmer and Lemeshow, 2000), the cumulative probability also increases in j, i.e. $P(j+1|i,\mathbf{M}) \geq P(j|i,\mathbf{M}), \forall j$. Therefore the transition probability in (5.6) must be non-negative since $P(i'|i,\mathbf{M}) - P(i'-1|i,\mathbf{M}) \geq 0$.

5.2 The Hong Kong Data

5.2.1 Sampling and data preparation

The MDP model is applied to credit card data from a major Hong Kong financial institution. The data consist of the credit card histories and characteristics of over 1,400,000 credit accounts for each of 60 months (May, 2002 to April, 2007). Fields used in this study are account balance, account repayment, credit limit, account written-off record, behavioural score and credit limit changed-date.

There are three criteria for sample selection. First, because our interest is to look at the impact of economy, the sample period should encompass both expansion and recession. Second, a minimal sample size threshold has to be met at every month to ensure statistical reliability of the estimates. Third, the sample should cover both new and current customer's migration behaviour to ensure the sample is representative.

To achieve these criteria, we allow the sample to vary over time, incorporating new credit card holders and including those defaulted or closed accounts. We extract random samples every month and estimate that month's behavioural score migration by looking at the behavioural score of these accounts in the current month and the next month. This procedure ensures that the sample size is always large enough to estimate the impact of the monthly economy, and includes all special (defaulted, inactive or closed) accounts.

We would like to ensure the sample period consists of different economic conditions. As generally agreed by economists, recession is two consecutive quarters of negative GDP growth Knoop (2004). Looking at the quarterly GDP growth of HK (refer to the graph in Chapter 1), *recession* in this study refers to the period from May 2002 to March 2004, and *expansion* refers to the period from April 2004 to April 2007.

There is a field *credit limit changed-date* which shows the most recent date of the account's credit limit being adjusted. The consumption patterns of accounts having had their credit limit adjusted recently are somewhat different from the others. These accounts contribute a high profit and thus skew the values of the profit function $r(l, i, \mathbf{M})$. So the question here is whether we should exclude these accounts from our sample? However, if we do so, there may not have enough special accounts in the samples since lenders like to adjust the credit limit of special accounts (such as Inactive accounts). Special accounts are interesting portfolios to look at and we would like to include them in our study. We thus use a *two-phase sampling method* to ensure there is a certain number of special accounts in the sample but also possibly minimize the impact of the current credit limit policy. In the first-phase, we extract a random sample of 50,000 accounts over the sampling period in each monthly dataset. We have checked that these samples include all new, existing, inactive, in arrears, default and closed accounts from each month during the sampling period. Then we look at the non-special accounts (not inactive, default, in arrears, closed accounts) in the first-phase samples and replace those which are credit limit adjusted accounts[1]. The replacements are accounts having the same characteristics (i.e. same credit limit and behavioural score in the same month) but whose credit limit has not been changed in the last three months. A remark here is the field *credit limit changed-date* can be used to understand the lender's current policy. We can therefore compare the model and the current policy and this would be included as a future research of how to develop measurements to validate the performance of the MDP model. We delete some of the ambiguous, missing and special accounts that we are not interested in covering in this study, which account for less than 0.2% of the raw samples. We are left with 2,994,584 transitions for analysis.

5.2.2 Special accounts and coarse-classifying accounts

Each month an account is given a behavioural score or possibly put in a special state. We define a Closed account to include all accounts where the credit card service terminated with zero account balance. A credit card account which has never been activated or has not been used in the last twelve months is called Inactive. A credit card account which was newly opened in the last two months before the sample point (and so does not have enough data to merit a behavioural score) is called New. Since there is only small number (less than 1%) of New accounts in the sample, we grouped them with the Inactive accounts to simplify the discussion. A 3+ Cycle account is one in which the account has been in arrears for 3 or more months but the lender has not yet written the account off.

There are four possible reasons to write-off an account, bankruptcy, charge-off, revoked and 3+cycles delinquent. A written-off account is followed-up by the debt collection department. Such written-off accounts may repay all, part or none of the outstanding debt. Even when the account makes full repayment, the time spent in the collection process is variable and could be several years. It is important to estimate the average

[1] Accounts' credit limit have been increased in the past 3 months.

future repayment amount of different default accounts because it affects the values of the profit function in (5.1). There is very little research (Matuszyk et al., 2007) on estimating the loss given default of revolving credit products. So we use a simple approach to compare the debt repayment ratios. Define $R \equiv \frac{B_{t+1}-B_{t+24}}{B_{t+1}}$ where B_{t+1} is the current balance of an account at default in month t. The repayment ratio R is defined as the proportion of the debt repaid to the lender after two years. For reasons of confidentiality, we cannot show the exact repayment ratio, but the results showed one of the forms of default had a high repayment ratio which was significantly different from the others. We call this default account state Bad2, and group the rest of the three forms of written-off together into one default state, called Bad1. So in our cost function the loss generated by accounts in Bad1 is higher than those of Bad2. In total we have five special behavioural score states.

We then divided the behavioural score and credit limit into different groups. We used a classification tree to split the behavioural scores into ten groups. The target field (or dependent variable) in the classification tree is the monthly profit of every borrower. Thus this split is independent of the macroeconomic measurements, and we only take an average on all economic conditions when we are splitting the behavioural score into different bands. For reasons of confidentiality, we do not disclose the precise behavioural score and credit limit bands. We use Score1 to Score10 to represent the behavioural score where Score1 are those with lowest behavioural score and Score10 are those with highest. Similarly, we split the credit limits into 10 groups and use Limit1 to Limit10 to represent the credit limit with Limit1 as the lowest credit limit band.

5.2.3 Macroeconomic variables

There are a broad range of macroeconomic measurements whereas we only select those closely related to the consumer market. The five macroeconomic variables used in this study are listed in Table 5.1.

Factors	Description and data sources
Consumer price index (CPI)	We use the month-on-month rate of change of the seasonal adjusted consumer price index (CPI). CPI is an indicator consolidating the prices of commodities, petroleum, food and transportation. It is considered a good index to show inflation in the economy.
Gross domestic product (GDP)	We use the quarter-to-quarter rate of change of the seasonal adjusted gross domestic product (GDP). GDP measures the total market value of goods and services produced in a country. We include it as the index for production of the overall economy.
Best lending rate (Int)	We use the month-on-month difference in the Best lending rate. Best lending rate is designed by the Hong Kong Monetary Authority and it is used as a basis reference for residential mortgage interest rates. It reflects the financial stress of homeowners. We use the month-on-month difference rather than the actual interest rate. What is important is the relative position of someone compared to the previous time. The month-on-month differences reflect recent market changes, which are reflected in borrowers' confidence. As we use the lag of macroeconomic variables in the model (details are explained in the next section), this lag gives some indication of the interest rate position compared with when the credit card was first issued/used.
Stock Return (Sto)	We calculate the Stock return at month t by $Stock = \ln(\frac{H_{t-1}}{H_t})$ where H_t is the monthly closed value of the Hang Seng Index.
Unemployment rate (Une)	We use the month-on-month difference to measure the labour market performance in Hong Kong. This measurement reflects recent market changes, which are reflected in borrowers' confidence.

Table 5.1: Description and sources of the macroeconomic factors

We first test the importance of macroeconomic variables individually to understand their impact. Consequently, we incorporate two or more macroeconomic variables into the logistic regression model so as to look at the impact on the economy (referred to as multivariate logistic regression models in the following discussion).

It is important to test whether the macroeconomic variables are highly correlated. If we incorporate two highly correlated macroeconomic measurements in the regression model, multicollinearity will be present. We explain the impact of multicollinearity as follows. Hypothetically, we put two measurements X and Y into a multivariate regression model to predict Z. The coefficient estimates of X and Y are β_x and β_y respectively. The usual interpretation of these coefficient estimates is "'if we hold the variable Y unchanged, Z increases/decreases β_x units for a unit increase of X"'. However, if X and Y are highly correlated, the assumption of "'if we hold the variable Y unchanged and a unit increase of X"' does not stay true. Although it will not reduce the predictive power of the model as a whole, it is not cost-effective to put many correlated measurements into a multivariate regression model. Therefore, it is better to avoid the presence of such correlation.

Factors	$H_0 : \rho = 0$ (no correlation)					Mean	S.D.
	CPI	GDP	Int	Sto	Une		
CPI	1	.0545 (.6793)	.1283 (.3286)	.0107 (.9357)	−.4658 (.0002)	.0267	.3178
GDP		1	.0626 (.6346)	.0747 (.5706)	−.3621 (.0045)	1.5350	1.5805
Int			1	.0312 (.8132)	−.0575 (.6627)	.0437	.1260
Sto				1	−.0358 (.7862)	.0100	.0432
Une					1	−.0467	.1620

Table 5.2: Correlation analysis of the macroeconomic factors

The result of the correlation analysis is presented in Table 5.2. The null hypothesis of this correlation analysis is "'there is no correlation between the measurements"'. We will not reject this hypothesis if the p-value is greater than 0.01. We observe that Une is substantially correlated with CPI and GDP. It is thus more sensible to separate Une from CPI and GDP. We cannot reject the null hypothesis between CPI and GDP, and thus these two macroeconomic measurements can be used as explanatory variables in a multivariate logistic regression model. *Stock* can be used as a measurement in any multivariate logistic regression model since it is not highly correlated with any others. As shown in Table 5.2, Int is not highly correlated with any other measurements. However, this interest rate is controlled by the financial authority, and we thus decided to exclude it from the sub-models. Having the four measurements, we divide them into two sub-models as listed below.

Model A: CPI, GDP, Sto

Model B: Une, Sto

5.2.4 The format and lag of macroeconomic variables

The impact of economic factors are not instantaneous and consumers likely take some time to absorb or digest the consequence of a change. We thus allow the lag of macroeconomic variables x to enter our transition matrix. There is no definite length or mathematical formulation for lag variables. As a reasonable assumption, we use exponentially declining weights on these lag variables (Figlewski et al., 2006). The lag macroeconomic variables in month t are defined as

$$m_t = \frac{\sum_{j=1}^{k+1} w^j x_{t-j+1}}{\sum_{j=1}^{k+1} w^j} \tag{5.7}$$

This is the weighted sum of the macroeconomic variables from month $t-k$ to t with weight $w \in (0,1]$, and the weighted value w^j decreasing in j exponentially. The lag value is the mean of the previous k months' macroeconomic variables when $w = 1$, and a weight

w close to zero implies (5.7) weights recent macroeconomic variables heavily. If $k = 0$, the macroeconomic variable is independent of its historical values. We test several decay factors: $w = 0.2, 0.5, 0.8, 1$. A lag window of 12 months or less is used in our study since consumers are far more sensitive to the economy than corporations. We test *CPI*, *GDP*, *Int* and *Une* with $k = 1, 3, 6, 12$. The impact of stock market is believed to be instantaneous, therefore we choose $k = 1, 2, 3$ for *Sto*.

5.3 Empirical results

We use the credit card data from May 2002 to April 2006 as training samples and those from May 2006 to April 2007 as an out of time hold-out testing sample.

5.3.1 Unconditional transition matrices and profits

Score i							Score i' at $t+1$									Row
at t	Closed	Inactive/New	Bad1	Bad2	3+Cycle	Score1	Score2	Score3	Score4	Score5	Score6	Score7	Score8	Score9	Score10	Count
Inactive/New	1.01	**81.98**	0	0	-	-	0.04	0.53	5.04	3.21	1.69	0.82	1.44	1.2	3.03	173635
3+Cycle	31.08	-	29.05	4.5	2.93	0.45	12.16	19.37	0.45	-	-	-	-	-	-	444
Score1	2.35	-	30.97	5.55	3.16	14.5	40.55	2.81	0.11	-	-	-	-	-	-	2848
Score2	1.24	0	1.81	0.85	0.61	3.82	**65.77**	25.4	0.5	0	-	-	-	-	-	48634
Score3	0.61	0.14	0.24	0.13	0.01	0.18	4.45	**78.14**	15.3	0.54	0.19	0.04	0.02	0.01	0	312913
Score4	0.58	0.21	0.04	0.02	-	0	0.19	8.69	**75.38**	10.27	1.5	1.36	1.48	0.21	0.08	486989
Score5	0.56	0.28	0.02	0	-	-	0.03	2.22	12.37	**59.82**	7.01	5.85	7.03	3.67	1.12	269203
Score6	0.66	0.62	0.01	0	-	-	0.02	1.24	5.84	13.42	**52.74**	7.29	12.25	2.74	3.16	131118
Score7	0.53	0.37	0.01	0	-	-	0.01	1.05	4.85	10.26	7.08	**50.47**	10.61	10.69	4.07	137946
Score8	0.32	0.18	0.01	0	-	0	0	0.59	2.8	4.49	6.59	6.06	**62.48**	9	7.48	261466
Score9	0.28	0.11	0.01	0	-	-	0	0.35	1.72	2.24	1.44	6.15	8.51	**69.43**	9.76	252975
Score10	0.19	0.01	0.01	0	-	-	0	0.16	1.25	0.66	0.53	1.01	6.3	7.2	**82.68**	317098

"'-"' represents there is no sample observation.

"'0"' represents the transition probability is less than 0.0005.

A bold value indicates the transition frequency is greater than 50% .

The transition probabilities of all absorbing states (Closed, Bad1 and Bad2) are not shown in the table.

Table 5.3: Unconditional transition matrix (in percentage)

Table 5.3 shows the unconditional transition matrix for all the training samples. Each entry represents the sample frequency of accounts with initial behavioural score band i (indexed by the first column in the table) moving to a state with behavioural score i'

(indexed by the second row in the table) divided by the total number of accounts with initial behaivoural score band i (given in the last column of the table).

This transition matrix is mainly dominated by the diagonal entries. This excludes 3+Cycle accounts of which 31.08% move to Closed and 29.05% move to Bad1. Accounts with a behavioural score state Score1 are more likely to move to a state with Score2 (40.55%) or Bad1 (30.97%). The volatility of score transition is clearly higher for accounts with Score 5 to Score 8, since the percentages of these accounts remaining in the same behavioural score band are less consistent.

	Limit1	Limit2	Limit3	Limit4	Limit5	Limit6	Limit7	Limit8	Limit9	Limit10
Closed	10.28	17.56	54.38	98.41	5.74	59.78	140.01	55.35	104.78	298.31
Inactive/New	-8.17	-6.48	-6.53	-5.85	-2.02	-0.27	-0.49	-4.19	-6.1	-5.76
Bad1	-7642.68	-11048.99	-15837.72	-24031.79	-21109.75	-31625.49	-38639.66	-47651.89	-61405.35	-97164.11
Bad2	-3858.67	-5839.01	-8148.58	-11487.51	-13576.19	-16099.02	-20042.11	-24408.86	-34081.19	-52407.39
3+Cycle	-635.15	-670.12	-1097.4	-1125.18	-951.49	-954.55	-1149.93	-2139.17	-711.03	-257.61
Score1	-699.48	-1117.09	-1535.14	-2025.38	-2003.03	-2480.33	-3147.94	-3714.42	-5209.01	-9184.49
Score2	204.8	255.44	369.75	483.82	559.34	701.14	926.46	1245.03	1384.91	2414.72
Score3	151.44	186.99	281.22	392.45	214.38	460.81	592.7	697.27	899.82	1618.16
Score4	29.84	40.09	78.73	128.99	71.18	162.82	223.58	272.92	392.85	979.24
Score5	7.74	5.14	16.21	27.85	24.52	38.93	56.3	69.68	112.78	262.2
Score6	7.09	0.03	5.2	11.98	11.57	18.63	26.55	33.44	57.02	145.3
Score7	2.45	-3.39	3.16	5.69	14.82	17.49	26.22	38.48	65.03	152.27
Score8	-6.81	-7.64	-4.13	-0.3	7.49	10.55	17.98	23.87	44.73	103.44
Score9	-7.45	-8.24	-5.23	-2.3	4.28	6.17	11.56	21.86	35.09	78.36
Score10	-8.71	-5.83	-4.23	-2.4	7.24	8.37	11.42	22.02	32.85	83.54

The first column indexes the Score status and the first row indexes the Credit Limit statues

All values are in HK dollar (£1 $\approx$ HK$15)

For all absorbing states (Closed, Bad1, Bad2), we use the profit value in the month of the account being written-off or closed as the profit.

Table 5.4: Average monthly profit

Looking at the profit values shown in Table 5.4 there is no consistent trend across Inactive accounts. Of course one may say Inactive accounts are not important since they generate a loss less than HK$9 per month. Indeed, this loss is the cost of funds for account operation. Just like the UK credit card dataset, account profit increases with credit limit and decreases with behavioural score.

Score i	Macroeconomics Measurement	Format (k,w)	Log(A/C) β	Log(T/C) β	Log(A/T) β	-2Log(likelihood)	Implications
Inactive/New	CPI	(9,1)	1.43*	0.7124*	0.7177*	176876	A>T>C
3+Cycle	CPI	(12,1)	2.0016**	/	/	541	A>C
Others	CPI	(12,0.8)	0.4637*	1.1336*	-0.67*	197760	T>A>C
Inactive/New	GDP	(3,1)	-0.1574*	-0.1079*	-0.0495*	177044	C>T>A
3+Cycle	GDP	(N,N)	N	/	/	/	/
Others	GDP	(12,1)	0.0787*	0.3152*	-0.2365*	197753	T>A>C
Inactive/New	Int	(6,1)	2.1035*	0.1492	1.9543*	176057	A>T,A>C
3+Cycle	Int	(N,N)	N	/	/	/	/
Others	Int	(12,1)	1.1083*	1.9803*	-0.8721*	197746	T>A>C
Inactive/New	Sto	(1,1)	0.7488	-0.6218	1.3707*	177187	A>T
3+Cycle	Sto	(N,N)	N	/	/	/	/
Others	Sto	(3,0.8)	2.275*	4.5227*	-2.2476*	197837	T>A>C
Inactive/New	Une	(9,1)	-1.7367*	-1.2592*	-0.4775*	177149	C>T>A
3+Cycle	Une	(N,N)	N	/	/	/	/
Others	Une	(9,1)	-0.8548*	-2.6242*	1.7695*	197613	C>A>T

”/” represents there is no observation in the data.

”‘N”’ represents the stepwise multinomial logistic regression cannot find any significant explanatory macroeconomic variable.

”‘*”’ indicates the parameter is significant at 99% level.

”‘**”’ indicates the parameter is significant at 95% level.

The first column is the index of the initial score state i where ”‘Others”’ refers to accounts with ordinary behavioural score (Score1 to Score10).

The best fit macroeconomic variables (discussed in Section 5.2.4) are presented in column three.

The estimated parameters are presented in columns four to six.

-2log(likelihood) ratios which are used to measure the model fit statistics are presented in column seven.

Log(A/C) represents $\log\left(\frac{p(A|i,\mathbf{M})}{p(C|i,\mathbf{M})}\right)$ in (5.2)

Log(T/C) represents $\log\left(\frac{p(T|i,\mathbf{M})}{p(C|i,\mathbf{M})}\right)$ in (5.3)

Log(A/T) represents $\log\left(\frac{p(A|i,\mathbf{M})}{p(T|i,\mathbf{M})}\right)$ in (5.4)

Table 5.5: Summary of the multinomial logistic model estimates

	Model A (CPI, GDP, Stock)									
	Log(A/C)			Log(T/C)			Log(A/T)			-2Log(likelihood)
	CPI	GDP	Stock	CPI	GDP	Stock	CPI	GDP	Stock	
Inactive	1.2654*	-.1456*	/	.5378*	-.0938*	/	.7276*	-.0518*	/	176642
Others	.5226*	**-.00946**	2.4463*	.6908*	.2069*	5.1219*	-.1683	-.2164*	-2.6756*	197632

	Model B (Unemployment rate, Stock)						
	Log(A/C)		Log(T/C)		Log(A/T)		-2Log(likelihood)
	Stock	Unemployment	Stock	Unemployment	Stock	Unemployment	
Others	-2.5503*	1.8117*	4.4443*	-2.6175*	-2.5503*	1.8117*	197564

"'*"' indicates the parameter is significant at 99% level.

The models are developed only if there is significant variable found in Table 5.5.

Table 5.6: Summary of multinomial logistic model estimates of Model A and Model B

5.3.2 Estimates for the multinomial logistic regression

We estimate the way the economy impacts on the transition probabilities in two stages. Firstly we estimate the transition probability of moving to Inactive, Active and Closed accounts (refer to Section 5.1.2). Then having the transition probability of moving to the Active state, we calculate the transition probability of moving to a particular state (ref to Section 5.1.3). To begin the discussion, we first look at the frequency distribution of Closed, Inactive and Active accounts. The second and third columns in Table 5.3 show the percentage of accounts with score band i moving to Closed and Inactive states respectively. One may observe that for Score1 to Score10 accounts the frequency distribution of moving to the Closed and Inactive states are rather similar. We therefore merge these accounts to perform the analysis so as to ensure the multinomial regression has enough observations to generate significant results. Thus we have three initial behavioural score states: Inactive/New, 3+Cycle and Others, where "'Others"' represents the merged score state.

Table 5.5 summarizes the multinomial logistic regression results. The first column is the initial behavioural score state. Second column is the macroeconomic variable being used in the logistic regression and the third column is the best fit format of the corresponding macroeconomic variable. The first row of the result in Table 5.5 shows $(9, 1)$ is the best fit format which means that using the mean of the previous 9 month's CPI variables provides the best measurement to look at the impact of CPI on Inactive/New accounts. Columns four to six are the regression coefficients of the log odds in the logistic regression. Log(A/C) equals 1.43 in the first row of the table, which mathematically means that $\frac{\text{P(moving to an Active account|An Inactive/New account)}}{\text{P(moving to a Close account|An Inactive/New account)}} = e^{1.43CPI+Intercept}$. In words, if there is a unit increase in the CPI, there are $4.17(\approx e^{1.43})$ more Inactive/New accounts which become Active accounts for every Inactive/New account that becomes a Closed account. Column seven shows the -2log(likelihood) ratios which are used to measure the model fit statistics. This measurement is proportional to the sample size and a smaller magnitude indicates a better fit.

The last column "Implication" describes the rate of increase of accounts moving to Closed(C), Inactive(T) and Active(A) if there is a unit increase of the macroeconomic variable. There is an implication only if the coefficient estimate β is significant at the 95% level. The Implication column summarizes the impact of the economic variable on the movements for that type of account. The first row of the Implication is "'$A > T > C$"' which means that for Inactive/New accounts, the increase in the CPI will increase the numbers that next month become Active. Also the increase in the CPI has a lower impact on those who stay Inactive/New and in fact they drop but the largest drop will be in those who will close their account.

We first examine the variable format. In most cases, the "'mean"' of macroeconomic variables is the best fit format which implies consumers react fairly and consistently across sub-periods. One can compare the lag window used by Figlewski et al. (2006) who choose $w = 0.88$ and $k = 18$ while looking at the corporate bond rating.

We first look at the impact of CPI and Interest Rate on account movement. Note that the economy experiences inflation when the interest rate or CPI goes up. The implications of the CPI and Interest Rate models for Inactive/New borrowers are ”‘$A > T > C$”’ and ”‘$A > T, A > C$”’ respectively which indicate more of these borrowers require credit during inflation, whereas for Others borrowers, the implications for both the CPI and Interest Rate models are ”‘$T > A > C$”’, which indicates borrowers reduce their borrowing during inflation. CPI is highly associated with 3+Cycle accounts and the impact lasts for a year. This indicates when inflation comes, not many 3+Cycle accounts are able to repay their debt and close their credit card accounts.

GDP going up indicates the economy is doing well. The results show the demand for credit reduces when the HK market is doing well, since the implications for Inactive/New and Others borrowers are ”‘$C > T > A$”’ and ”‘$T > A > C$”’ respectively.

The implication for Others borrowers in the Stock model is ”‘$T > A > C$”’ which indicates more Others borrowers reduce the use of their credit card when HK's stock market index increases. If the unemployment rate goes up, the implications for Inactive/New and Others accounts are ”‘$C > T > A$”’ and ”‘$C > A > T$”’, that is, more borrowers close their credit card accounts.

Table 5.6 shows the results of Model A and B (as defined in Section 5.2.3). Note that we use the best combination of the macroeconomic variables where individual effects are found in Table 5.6 in Models A and B. When comparing the parameter estimates between Table 5.5 and 5.6, only one parameter changes sign (highlighted in bold). This sign-changed parameter is associated with GDP and the corresponding magnitude is small and thus there is no significant impact on the overall model.

5.3.3 Estimates for the cumulative logistic regression

We begin the discussion in this section by examining the result of using a single explanatory variable. The regression results shown in Table 5.7 use Score10 as the reference category. So a negative β means more borrowers move to a state with a higher behavioural score whenever the explanatory macroeconomic variable increases one unit.

Initial i	CPI			GDP			Interest rate			Stock			Unemployment		
	(k,w)	β	-2Log(L)	(k,w)	β	-2Log(L)	(k,w)	β	-2Log(L)	(k,w)	β	-2Log(L)	(k,w)	β	-2Log(L)
3+Cycle	(6,1)	-1.7764**	848	(3,1)	0.1119**	855	(6,1)	-5.2234*	843	N	N	N	(9,.8)	1.7812**	854
Inactive	(12,1)	-6.4673*	101530	(12,1)	-0.9764*	106849	(6,.8)	-9.9927*	99844	(2,.2)	7.8655*	110248	(12,1)	6.016*	106520
Score1	(12,1)	-0.9668*	7747	(12,1)	-0.1098**	7758	(12,1)	-1.8927*	7746	(2,.2)	-3.0851**	7749	(12,1)	1.5825*	7739
Score2	(6,1)	-1.0983*	87751	(12,1)	-0.1692*	88073	(6,.8)	-3.4636*	87881	(1,1)	-0.6749*	88225	(12,1)	1.7241*	87916
Score3	(6,1)	-1.2022*	430217	(12,1)	-0.1863*	432117	(12,1)	-2.5571*	431345	(3,.8)	-3.2326*	432565	(12,1)	2.0705*	430555
Score4	(12,1)	-1.0462*	845059	(12,1)	-0.1251*	846900	(12,1)	-1.844*	844964	(3,.8)	-5.8616*	845166	(12,1)	2.2039*	842108
Score5	(12,1)	-0.5408*	730487	(12,1)	-0.0745*	730792	(12,1)	-0.9525*	730495	(3,.8)	-3.0535*	730480	(12,.8)	1.2364*	729680
Score6	(9,1)	-0.8268*	385572	(12,1)	-0.1345*	385961	(12,1)	-1.7219*	385425	(3,.8)	-3.6684*	385876	(12,.8)	1.6059*	385116
Score7	(12,1)	-1.5348*	428421	(12,1)	-0.1968*	429894	(12,1)	-1.8635*	429469	(3,.8)	-4.1701*	430120	(12,1)	2.252*	427946
Score8	(6,1)	-1.7928*	680673	(12,1)	-0.4607*	683939	(12,1)	-2.9207*	684962	(3,.8)	-3.0955*	689597	(12,1)	3.5824*	680296
Score9	(12,1)	-1.9308*	549049	(12,1)	-0.1276*	553407	(12,1)	-2.6738*	550782	(1,1)	-4.1888*	551750	(12,1)	2.6247*	549195
Score10	(9,1)	-2.9673*	428366	(12,1)	-0.6563*	433237	(12,1)	-2.5793*	434126	(3,.8)	-15.8883*	433483	(12,1)	3.8103*	427040

"'*"' indicates the parameter is significant at 99% level.

"'**"' indicates the parameter is significant at 95% level.

"'N"' represents the stepwise cumulative logistic regression cannot find any significant explanatory macroeconomic variable.

Table 5.7: Summary of cumulative logistic model estimates

The number of borrowers moving to a high behavioural score increases with inflation. This is different from the general perception which holds that inflation is an indicator of more challenging times ahead and thus the risk score of consumers will be lower, and the results in corporate research (Figlewski et al., 2006) which indicates inflation is associated with increased risk of a rating downgrade. Our results indicate that behavioural score and CPI move in the same direction. This is because the behavioural scores of borrowers go up during expansion, and so does CPI. One can observe in Figure 1.1 that the year-on-year CPI percentage change was positive from 2005 to 2007. This result shows one should use not only inflation or deflation to predict the direction of the behavioural score movement but also add other economic indicators to help make such estimates.

The coefficient estimates of the GDP variable are fairly similar and the best combination of the macroeconomic variable is the mean of the previous twelve months. So if GDP goes up, borrowers' behavioural score improves gradually and it takes one year for credit card holders to gain the real benefit. Similar results hold in the corporate risk research which says there is a lower number of credit rating downgrade (Figlewski et al., 2006) or higher default risk (Helwege and Kleiman, 1997) when GDP grows.

Consumers' scores tend to increase when interest rate goes up as all coefficient estimates with respect to the Interest Rate model are positive. The coefficient estimate of 3+Cycle is high and the lag is only six months. This shows interest rate has a more severe impact on those in arrears than for standard behavioural score accounts. Moreover, interest rate has a significant impact on Inactive accounts. These people tend to activate their credit card account when the interest rate goes up.

The estimates of the Stock variable show stock market performance is a key indicator. It is however rather hard to find a consistent trend in the regression coefficients. It is evident that there is a huge dependence between Score 10 and Stock market, with regression parameters $\beta = -15.8883$. So if stock market goes up, people's behavioural score improves. (A similar result found by Figlewski et al. (2006) which show reduced credit rating downgrade if the stock market is doing well.) Conversely, when it goes down, their score goes down and it is those in scoreband 10 (the highest) who are most hit.

Unlike the findings in Figlewski et al. (2006) which show labour market have volatile impact on the credit rating, our finding show that the effect of the labour market is clear in our parameter estimates. The parameters are positive and thus indicate the behavioural scores are moving inversely with unemployment rate. The coefficient estimates for Inactive borrowers are significantly higher than those of the other accounts. This indicates that borrowers tend to use their credit cards when the labour market is not doing well.

Table 5.8 shows the cumulative regression results of the multivariate model. Putting

Initial i	Model A				Model B		
	CPI β	GDP β	Stock β	-2Log(L)	Stock β	Unemployment β	-2Log(L)
3+Cycle	-1.6538**	0.0811	N	846	N	N	N
Inactive	-6.2968*	-0.1634*	9.9704*	100046	12.1121*	6.7087*	104552
Score1	-0.9267**	-0.0314	-3.3048*	7731	-2.4069**	1.3978*	7731
Score2	-1.1747*	-0.0227	-1.8722*	87659	-0.2523	1.7104*	87914
Score3	-1.4405*	**0.0025**	-5.874*	428739	-1.8103*	1.9667*	430410
Score4	-0.9516*	-0.0183**	-5.6601*	842854	-4.0862*	1.9673*	841003
Score5	-0.448*	-0.0351*	-3.148*	729955	-3.1522*	1.2489*	729141
Score6	-0.9523*	**0.0114**	-4.3578*	385007	-3.4841*	1.5761*	384748
Score7	-1.366*	-0.0359*	-3.105*	428121	-1.463*	2.1433*	427886
Score8	-1.7587*	-0.1467*	-6.4164*	677831	**0.0597**	3.5864*	680296
Score9	-1.9944*	**0.0805***	-3.3807*	547479	-3.0181*	2.36*	548145
Score10	-3.7399*	**0.3292***	-2.1302*	427866	-0.1625	3.7916*	427040

"‘*"’ indicates the parameter is significant at 99% level.

"‘**"’ indicates the parameter is significant at 95% level.

"‘N"’ represents the stepwise cumulative logistic regression cannot find any significant explanatory macroeconomic variable.

Table 5.8: Summary of cumulative logistic model estimates - Model A and Model B

more than one variable into the cumulative logistic regression changes the signs of some regression coefficients (highlighted in bold). This is because macroeconomic variables are correlated with each other (as presented in Table 5.2). These collinear variables contain the same information about the behavioural score migration. When one puts all these variables together in a multivariate regression, the coefficient parameters are adjusted in order to give a precise estimation of the behavioural score.

For example, say we examine the linear relationship between the dependent variable Z and independent variables X and Y. The equations describing the relationships between the dependent variable and each independent variable are:

$$z = 10 + 0.5x, \quad z = 10 + 0.01y \tag{5.8}$$

Hypothetically, if x and y are independent from each other, then the equation line describes the relationship between z and x and y is:

$$z = 10 + 0.5x + 0.01y \tag{5.9}$$

One can interpret this equation as "‘if the value of x is unchanged and the value of y

increases one unit, the value of z increases 0.5."' However, if x and y are correlated, any change in y essentially changes the value of x. In that case, if we still use the above equation to estimate z, then the value of z is too high or too low. Therefore if one puts x and y together as independent variables of a regression analysis, the coefficient estimates of x and y are different to those presented in (5.9) and are as follows:

$$z = 10 + (0.5 + \delta_1)x + (0.01 + \delta_2)y, \text{ where } \delta_1, \delta_2 \in \Re. \tag{5.10}$$

If one would like to observe the actual impact of the independent variable, s/he should use (5.8). Whereas if the objective is to predict the value of z from x and y, equation (5.10) is used.

Note that as the magnitude of the coefficient estimates of these macroeconomic variables is small, it is possible that any adjustment in these coefficient estimates could change their sign. Model A and Model B can be used later as a prediction of the behavioural score migration whereas one should not use the models' coefficient estimates to investigate the impact of each individual macroeconomic measurement.

5.3.4 Comparing transition matrices

Suppose, at time t, the probability distribution of the behavioural score state is $x(t)$ (a row vector with dimension $1 \times I$) and the behavioural score transition matrix is P_t (a $I \times I$ matrix). Then the probability distribution of the behavioural score state at $t+1$ is given by

$$x(t+1) = x(t)P_t$$

Since we assume the behavioural score migration is a Markov chain, we can use the following equation:

$$x(T) = x(1) \prod_{t=1}^{T} P_t \tag{5.11}$$

to estimate the probability distribution of the behavioural score state at time T. What we are concerned with is the accuracy of using our model's transition matrix and this essentially leads us to the comparison of the model's transition matrix with the empirical transition matrix. Moreover, we propose that estimating the migration of the behavioural score through a transition matrix conditional on macroeconomic measurements has a better performance than through an unconditional transition matrix. This proposition will be verified in this section.

Score	$x(1)$	x(12)								
		Real	Unconditional	CPI	GDP	Int	Sto	Une	ModelA	ModelB
Expansion										
Closed	0	5.91	5.23	5.15	5.81	5.58	5.72	5.6	4.89	5.54
Inactive/New	8.09	1.59	1.6	1.48	1.66	1.74	1.68	1.67	1.37	1.74
Bad1	0	0.82	0.82	0.35	0.76	0.73	0.75	0.55	0.3	0.54
Bad2	0	0.41	0.29	0.12	0.27	0.26	0.27	0.2	0.1	0.19
3+Cycle	0.03	0.02	0.01	0	0.01	0.01	0.01	0.01	0	0.01
Score1	0.05	0.06	0.08	0.02	0.07	0.04	0.07	0.05	0.02	0.04
Score2	0.27	0.32	1.39	0.44	1.3	0.84	1.22	0.88	0.33	0.82
Score3	4.17	4.18	9.95	4.71	9.66	7.23	8.86	7.52	4.04	7.15
Score4	16.12	15.58	16.97	10.72	16.82	15.82	15.73	14.9	10.09	14.49
Score5	8.74	7.66	10.31	7.05	10.25	9.7	9.66	9.42	6.49	9.3
Score6	4.48	4.67	5.39	3.68	5.4	4.68	5.08	4.93	3.35	4.93
Score7	6.18	7.42	5.97	4.47	6.05	5.29	5.59	5.76	4.08	5.73
Score8	12.11	12.62	12.17	11.37	12.71	11.87	11.66	12.88	10.52	12.94
Score9	8.69	8.08	12.21	15.44	12.58	14.03	12.26	14.2	14.7	14.59
Score10	31.08	30.66	17.6	35	16.64	22.18	21.42	21.43	39.72	22
Chi-square Value			8263	5142	9177	4403	4846	4817	7084	4578
Recession										
Closed	0	6.14	6.29	6.85	6.62	6.49	6.15	6.71	6.63	6.49
Inactive/New	7.03	1.56	1.3	1.53	1.24	1.41	1.44	1.38	1.38	1.33
Bad1	0	2.03	2.19	3.27	2.73	2.44	2.01	2.66	2.8	2.43
Bad2	0	0.72	0.77	1.16	0.98	0.88	0.73	0.96	1	0.88
3+Cycle	0.08	0.02	0.02	0.03	0.03	0.03	0.02	0.03	0.03	0.03
Score1	0.14	0.11	0.13	0.19	0.18	0.17	0.13	0.18	0.21	0.17
Score2	4.13	3.08	2.23	3.18	3.01	2.63	2.26	2.94	3.37	2.85
Score3	22.55	18.88	13.51	17.87	16.99	15.5	14.23	17.32	18.47	17.32
Score4	19.1	18.51	18.64	22.86	22.53	21.72	20.08	22.85	22.51	22.77
Score5	11.52	12.11	9.35	11.09	11.35	10.97	10.51	10.87	11.14	10.85
Score6	6.43	6.76	4.12	5.06	5.27	5.17	5.23	5.04	5.35	5.06
Score7	5.87	6.12	4.13	5.07	5.26	5.32	5.51	5.15	5.24	5.15
Score8	16.26	15.38	7.32	9.16	9.44	9.86	10.81	9.36	8.98	9.48
Score9	6.84	8.46	5.94	7.29	7.77	8.59	10.15	7.56	7.36	7.81
Score10	0.05	0.04	3.75	5.38	6.61	8.82	10.72	7	5.53	7.38
Chi-square Value			9679	6030	6045	6898	7674	6444	5883	6441

Table 5.9: Behavioural score state distribution

To compare these transition matrices, we look at the probability distribution of the behavioural score at the beginning of the testing period. 49,577 random samples are extracted from that month (May 2006) and the behavioural score state distribution is listed in the second column of Table 5.9. We then calculate the "'Real"' transition matrix for the following 12 months (May 2006 to April 2007) and use (5.11) to calculate the probability distribution in month 12 (i.e. April 2007). The outcome is listed in column 3 of Table 5.9. We calculate $x(12)$ with the unconditional transition matrix and the model's transition matrix with results listed in columns 4 to 11. For comparison, we extracted 49,319 random samples from January 2003 to repeat the exercise for the recession period. The results are listed in the bottom part of Table 5.9. Note that the recession samples are extracted from the in-sample period.

The chi-square test checks if real outcomes have the same distribution as those estimated using the model. In a chi-square test, one compares the probability distribution of the expected (E) and the observed (O) outcomes via the equation $\chi^2 = \sum_i \frac{(O-E)^2}{E}$. The expected outcome here is the probability distribution estimated with the model and the observed outcome is the real probability distribution in the testing sample. Suppose the probability distribution in month 12 estimated with the empirical transition matrix $x_i(12)$, and the probability distribution in month 12 estimated with the unconditional matrix or the model's transition matrix is $y_i(12)$. We perform the test with a null hypothesis $x_i(12) = y_i(12), \forall m$. We look at the chi-square value of

$$\chi^2 = \sum_i n \frac{(x_i(12) - y_i(12))^2}{y_i(12)} \tag{5.12}$$

where n is the total number of testing samples, and check whether this value falls inside or outside the significant value for a χ^2 test with $14 \times 15 = 210$ degrees of freedom. The results are listed in the row labelled "'Chi-square value"' in Table 5.9. Since the cutoff value for the chi-square distribution with a significant level of 0.0001 is 294.9, we reject the null hypothesis for all the models. Note that the chi-square values with respect to the models (except those of GDP in Expansion) are lower than those for the unconditional transition matrix. Moreover, during expansion, the value for Int model and Model B are fairly low compared with the unconditional.

5.3.5 Estimates for the linear regression analysis

We use linear regression analysis to determine the relationship between profit and macroeconomic variables. The mathematical model for this exercise is

$$r(l,i,\mathbf{M}) = \alpha_{(l,i)} + \mathbf{M}\beta_{(l,i)} \tag{5.13}$$

where $\beta_{(l,i)}$ is a single regression coefficient if we consider only one macroeconomic variable (i.e. $\mathbf{M}$ is a 1×1 row vector) in the model, and if we look at more than one macroeconomic variable $\beta_{(l,i)}$ is a column vector.

		Parameter α					Parameter β				
		CPI	GDP	Int	Sto	Une	CPI	GDP	Int	Sto	Une
Limit1	3+Cycle	-589.476**	-652.867**	-678.858**	N	-621.158**	540.0183**	12.1933	1935.966**	N	-570.256
	Inactive	-7.314**	-8.6143**	-8.7991**	-8.102**	-8.1227**	9.7712**	0.3303	12.6303**	-14.6442**	N
	Score1	-721.331**	-603.633**	-697.424**	-715.883**	-696.929**	-155.335	-83.1265**	250.3812	2145.11**	-5.2385**
	Score2	210.9161**	171.3952**	204.3485**	203.1618**	218.9636**	49.0768	27.6404**	66.4266	208.0974	-62.8712
	Score3	155.957**	139.9364**	151.6328**	152.6542**	151.2403**	46.4859**	9.123**	25.9001	-209.489**	-411.143**
	Score4	34.0376**	33.1016**	29.8555**	32.6485**	29.742**	45.9363**	-2.3676**	-1.4438	-358.891**	8.1219
	Score5	9.1264**	5.3085**	6.6943**	8.489**	7.547**	28.28**	1.6021	37.6679**	-96.8838**	34.3265**
	Score6	9.4106**	-1.1335	5.5024**	8.4817**	6.9143**	42.5901**	6.0814**	49.826**	-180.874**	-5.4055
	Score7	4.0183**	-15.7759**	-0.3522	2.1165**	0.9024	54.7493**	11.8198**	65.5427**	82.061**	-11.0281
	Score8	-7.2939**	-16.2333**	-9.3383**	-6.757**	-7.8107**	26.5937**	6.5408**	38.9994**	-5.6362	-66.0161**
	Score9	-6.602**	-25.0574**	-9.7546**	-7.5314**	-9.0821**	42.1238**	11.1475**	54.824**	38.5941**	-37.106**
	Score10	-10.4682**	-18.4682**	-10.9743**	-9.2372**	-10.1097**	20.6459**	5.8946**	21.8557**	102.7364**	-57.5532**
Limit10	3+Cycle	-240.145**	-259.441**	-253.987**	N	-255.299**	105.5777**	1.2171	925.3441	N	-13.9376
	Inactive	-3.6775**	-10.6024**	-7.2353**	-5.8255**	-5.4558**	22.5889**	3.6549**	30.6156**	10.7277	-121.131
	Score1	-9714.84**	-8457.12**	-8950.95**	-9393.02**	-9172.78**	-6020.89	-587.256	-8629.38	29437.71	-28.3977**
	Score2	2164.712**	2746.148**	2518.092**	2431.85**	2366.31**	-2106.1**	-260.719	-11469**	-1248.94	6161.068
	Score3	1655.627**	1395.439**	1591.255**	1631.801**	1606.639**	632.7056**	158.4026**	1228.397**	-1237.93	2284.35
	Score4	1044.634**	541.4482**	822.3373**	1006.78**	904.4936**	2304.4**	287.5503**	3174.84**	-2991.74**	-1251.11**
	Score5	300.6129**	228.2479**	233.0438**	266.3223**	251.3212**	591.8813**	22.6957**	1105.934**	-411.736**	-2318.13**
	Score6	169.7808**	82.3828**	129.103**	144.4935**	139.0193**	339.0175**	46.7821**	741.9142**	88.1014	-316.074**
	Score7	144.8015**	176.4515**	153.566**	152.5672**	156.3525**	-106.086**	-15.9125	-65.1358	-33.239	-318.152**
	Score8	112.6287**	64.5296**	101.1547**	103.0252**	106.99**	122.6226**	29.9533**	172.8561**	62.9689	213.8856**
	Score9	82.3667**	72.3827**	75.9774**	77.8565**	76.1055**	68.3721**	3.718	97.8609**	46.4252	-221.822**
	Score10	79.1144**	67.3448**	79.9896**	82.0303**	78.7824**	51.6226**	9.5638**	36.5684**	275.961**	-60.4351**

"'**"' indicates the parameter is significant at 95% level.

"'N"' represents the cumulative logistic regression cannot find any significant explanatory macroeconomic variable and therefore we did not build up the corresponding regression model.

The regression model is to test the relationship between the profit value and the macroeconomics variables (mathematically,$r(l,i,\mathbf{M}) = \alpha + \beta\mathbf{M}$)

Table 5.10: Summary of regression estimates

Here we only present the results with respect to Limit1 and Limit 10 in Table 5.10 and 5.11 respectively for illustration. One point of note is that the R-square of these regression models are small (less than 0.05). This is because we use the whole samples ($N = 2,994,584$) to generate the regression estimates. Thus deviations from the regression estimates can be explained by the unaccounted random heterogeneity of the population.

		Model A				Model B		
		Intercept	CPI	GDP	Sto	Intercept	Sto	Une
Limit1	3+Cycle	-608.665**	541.825**	13.3096	N	N	N	N
	Inactive	-4.5132**	14.7537**	-1.6772**	-20.402**	-8.0167**	-20.7451**	-7.1456**
	Score1	-604.204**	90.9678	-85.0342**	2035.695**	-721.827**	2235.316**	N
	Score2	168.8085**	7.5201	28.8467**	263.2898**	218.2551**	65.7681	129.5972
	Score3	150.3217**	31.9605**	4.0654	-155.113**	152.9342**	-219.71**	-405.564**
	Score4	61.8805**	90.846**	-14.8484**	-417.7**	32.5289**	-347.256**	-9.1437
	Score5	15.6126**	39.7014**	-3.1942**	-139.568**	8.2783**	-97.8224**	10.1872
	Score6	15.7792**	51.7884**	-3.32	-179.321**	8.2836**	-187.12**	-6.2457
	Score7	-4.49	45.5685**	5.4855**	-52.4032	1.059	-85.7943**	-15.2094
	Score8	-10.5848**	21.5499**	2.2791**	11.276	-7.4663**	-47.9994**	-74.2411**
	Score9	-17.8757**	27.1885**	6.9507**	-2.8365	-9.0958**	-12.9029	-39.9121**
	Score10	-14.0573**	11.7416**	2.5647	19.9089	-10.0135**	51.4547**	-59.0239**
Limit10	3+Cycle	-242.582**	139.4636**	5.3337	N	N	N	N
	Inactive	-5.1008	20.1671**	0.8903	3.6767	-5.3841**	-10.78	-19.8931**
	Score1	-11750.3**	-9279.42	1215.44	34300.96	-9390.48**	31043.92	-29.3218**
	Score2	2055.639**	-2528.27**	88.67	-3918.34	2379.206**	-880.349	7319.155
	Score3	1536.965**	485.1551**	77.0519**	144.0313	1625.127**	-1719.76**	2245.768
	Score4	1185.994**	2496.369**	-62.9488**	-4353.19**	942.9864**	-4993.49**	-1301.79**
	Score5	432.7177**	768.8106**	-75.7037**	-736.595**	255.1174**	-367.19	-2549.94**
	Score6	205.6745**	412.684**	-25.0685**	344.8193	138.3998**	68.635	-312.652**
	Score7	153.294**	-94.5249**	-4.9552	-16.6526	155.2951**	133.2475	-317.88**
	Score8	96.5516**	108.2475**	10.4781	209.8475	108.6581**	-211.486	220.7216**
	Score9	87.9927**	75.9425**	-3.5223	44.2822	75.7165**	39.4081	-238.652**
	Score10	84.6163**	48.2214**	-3.5326	141.1312	79.0458**	123.4093	-59.4235**

”‘**”’ indicates the parameter is significant at 95% level.

”‘N”’ represents the cumulative logistic regression did not find any significant explanatory macroeconomic variable and therefore we did not build up the corresponding regression model.

Table 5.11: Summary of regression estimates - Model A and Model B

We hardly find any definite trend across different behavioural score groups. For example, if Unemployment rate goes up, most people decrease their spending (such as accounts with credit limit 1 and Score 10) but people with Score 8 and Limit 10 increase their spending. But we can still draw several conclusions, as follows:

- If CPI, GDP or Interest rate goes up, the profitability of credit card accounts is higher. Moreover, this profit decreases with behavioural score.
- If Unemployment goes up, the profitability of credit card accounts decreases and the rate of decrease is higher in the low behavioural score group.
- There is no definite pattern for the impact of Stock market.

When the model consists of more than one macroeconomic measurement, the most significant results are those with respect to CPI and Unemployment. This is consistent with the logistic regression results. It is thus more sensible to use a model with either CPI or Unemployment rate as the lead variable.

5.3.6 Optimal policy

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Credit limit 1															
3+Cycle	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	1
Score1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Score2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	1
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	1
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	1
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	1
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	1
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	1
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	1
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	1
Credit limit 6															
3+Cycle	6	6	6	6	6	6	6	6	6	6	6	6	6	6	9
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score1	6	6	6	6	6	6	6	6	6	6	6	6	6	6	9
Score2	6	6	6	6	6	6	6	6	6	6	6	6	6	6	9
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9

Table 5.12: Summary of optimal policy for accounts (for the first month of the period)

We use the transition matrices and profit functions conditional on the macroeconomic climate during the out-of-sample period to generate the optimal policy[2] for the MDP

[2] We use backward iteration to generate the optimal policies during Expansion (May, 2006 to April, 2007) and Recession (Jan, 2003 to Dec, 2003). Thus there are different optimal policies every month. We present the optimal policy of the first month of that period. That is if the lender predicts that the economy will experience expansion (or recession) in the coming year, what is the optimal policy in the

model in (5.1). We present the results of accounts having credit limit band 1 and band 6 in Table 5.12 (Refer to Appendix D for the rest of the results).

The optimal policy for accounts in arrears (3+Cycle) and having the lowest behavioural score (Score1) is to keep the current credit limit. This is what one would expect as these accounts have a high default risk. The optimal policy for accounts having Score3 or above is to increase their credit limit to the highest band. This can be explained by the default probabilities. Since the probability of default is low, the model selects the highest credit limit band in order to maximize the profit. A similar observation holds for Inactive accounts.

To understand the effect of the economy on the optimal policy, we test the model performance during the recession period. Note that this recession period (January 2003 to December 2003) is an in-sample period. The results for this in-sample recession period are the same as those for the out-of-sample expansion period.

The transition matrices of recession and expansion are significantly different from each other. It is surprising to find that the optimal policy stays the same in recession as in expansion. We can explain this result with the profit function $r(l, i, \mathbf{M})$. The former increases in l for all i, i.e. profit is proportional to the credit limit. The MDP model always increases borrowers' credit limit to maximize the profit. If the expected loss given default is large, the MDP model might not choose a high credit limit because it may lead to a greater loss when in default. In our model, the loss when in default (i.e. $r(l, D, \mathbf{M})$) is very small so the decision does not change at all. Therefore, even though there is difference in the transition probabilities for recession and expansion, the model still generates the same result.

To understand the impact of the loss values, we change $r(l, D, \mathbf{M})$ to generate the

first month of this period?

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
							Credit limit 1								
3+Cycle	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Inactive	10	10	10	10	10	10	10	7	7	7	7	7	7	7	2
Score1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
Score2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
Score3	10	8	10	8	10	10	10	1	1	1	1	1	1	1	2
Score4	10	10	10	10	10	10	10	7	7	7	7	7	7	7	2
Score5	10	10	10	10	10	10	10	7	7	7	7	7	7	7	2
Score6	10	10	10	10	10	10	10	7	7	7	7	7	7	7	2
Score7	10	10	10	10	10	10	10	7	7	7	7	7	7	7	2
Score8	10	10	10	10	10	10	10	7	7	7	7	7	7	7	2
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	2
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	2
							Credit limit 6								
3+Cycle	6	6	6	6	6	6	6	6	6	6	6	6	6	6	10
Inactive	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score1	6	6	6	6	6	6	6	6	6	6	6	6	6	6	10
Score2	6	6	6	6	6	6	6	6	6	6	6	6	6	6	10
Score3	10	8	10	8	10	10	10	6	6	6	6	6	6	6	10
Score4	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score5	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score6	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score7	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score8	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

Table 5.13: Summary of optimal policy for accounts (loss equals to the credit limit)

optimal policy. We set the loss of an account equal to its credit limit. For example, say the credit limit of Limit 1 is $HK\$10000$, if an account with Limit 1 defaults, the loss $r(l, D, \mathbf{M})$ is equal to $-HK\$10000$. Note that the losses during expansion or recession are the same. The result is presented in Table 5.13. The results for recession and expansion are very different in that the credit policy during Recession is far more conservative. For example, the model suggests keeping the credit limit of an account with a Score 3 and Limit 1 unchanged during recession whereas the model suggests increasing the credit limit of these portfolios to Limit 10 during expansion. These results show the model can generate different policies in different economic conditions provided that one uses a high enough default value. Thus one way a lender can be conservative with the optimal credit limit policy is to assume default values which are higher than the historical average.

Indeed, lenders have different credit limit policies in different economic conditions in the credit card industry. After the global financial crisis in September 2008, many credit card issuers reduced risk by decreasing credit limits on active cards (Business Finance Week, 2009).

5.4 Incorporating macroeconomic variables into the UK data

We apply the model to the UK data. The objective is to contrast and compare the use of macroeconomic measurements in credit card model with two datasets to make sure that some properties of the models might hold in general while others depend on the particular economy under investigation. We will not discuss the estimates in depth and instead only present the critical estimates. The full results are presented in the Appendix B.

We present the estimates of the multinomial and cumulative logistic regression coefficient estimates of the UK dataset in Tables 5.14, B.3, 5.15 and B.4.

We first compare the results in Table 5.14 with those in Table 5.5. For the CPI model, the implications for the HK and UK data are "‘$A > T > C$"’ and "‘$A > C > T$"’ for inactive accounts respectively. These indicate that when the economy experiences inflation, more Inactive accounts activate their credit cards. On the other hands, when there is inflation, the Others accounts in HK and UK react differently. In HK, more Others borrowers reduce the use of their credit cards since the implication is "‘$T > A > C$"’. The UK borrowers react more extremely to inflation as many of them close their accounts (since the implication is "‘$C > A > T$"’) when there is inflation. The increase of interest rate is also an indicator for inflation and therefore the results in the Interest Rate model

Score i	Macroeconomics Measurement	Format (k,w)	Log(A/C) β	Log(T/C) β	Log(A/T) β	-2Log(likelihood)	Implications
Inactive	CPI	(9,1)	1.9152*	-9.3047*	11.2199*	222005	A>C>T
Risk	CPI	(N,N)	N	/	/		-
Others	CPI	(12,.8)	-2.333*	-3.6692*	1.3362*	255310	C>A>T
Inactive	GDP	(6,1)	-3.436*	-0.3594*	-3.0765*	219995	T>C>A
Risk	GDP	(12,.8)	0.585**	/	/	1592	A>C
Others	GDP	(9,1)	-0.2084*	0.3474*	-0.5558*	255449	T>C>A
Inactive	Int	(1,1)	6.5946*	6.1976*	0.397**	228573	A>T>C
Risk	Int	(9,1)	-9.5591**	/	/	1589	C>A
Others	Int	(12,1)	-10.5283*	-7.0736*	-3.4547*	255240	C>T>A
Inactive	Sto	(2,.8)	-18.5047*	-26.1677*	7.663*	223946	C>A>T
Risk	Sto	(2,0.2)	-5.8758**	/	/	1586	C>A
Others	Sto	(3,.8)	3.8009*	3.5201*	0.2809	255660	A>T>C
Inactive	Une	(1,1)	-0.7448*	3.3103*	-4.0551*	219815	T>C>A
Risk	Une	(9,1)	-9.5591**	/	/	1589	C>A
Others	Une	(12,1)	-10.5283*	-7.0736*	-3.4547*	255240	C>T>A

"/" represents there is no observation in the data.

"'N"' represents the stepwise multinomial logistic regression cannot find any significant explanatory macroeconomic variable.

"'*"' indicates the parameter is significant at 99% level.

"'**"' indicates the parameter is significant at 95% level.

The first column is the index of the initial score state i where "'Others"' refers to accounts with ordinary behavioural score (Score1 to Score4).

The best fit macroeconomic variables (discussed in Section 5.2.4) are presented in column three.

The estimated parameters are presented in column four to six.

-2log(likelihood) ratios which are used to measure the model fit statistics are presented in column seven.

Log(A/C) represents $\log\left(\frac{p(A|i,\mathbf{M})}{p(C|i,\mathbf{M})}\right)$ in (5.2)

Log(T/C) represents $\log\left(\frac{p(T|i,\mathbf{M})}{p(C|i,\mathbf{M})}\right)$ in (5.3)

Log(A/T) represents $\log\left(\frac{p(A|i,\mathbf{M})}{p(T|i,\mathbf{M})}\right)$ in (5.4)

Table 5.14: Summary of the multinomial logistic model estimates for the UK dataset

are very similar to those of the CPI model. When the interest rate goes up, more Inactive borrowers in HK or UK require additional credit since more of them are moving to an active status. The Others borrowers in UK tend to close their account whenever the interest rate increases but those in HK tend to remain inactive.

The way HK and UK Inactive borrowers react to GDP is very similar. When GDP goes up (i.e. the economy is doing well), more HK Inactive borrowers close their accounts (since the implication is ”‘$C > T > A$”’) and more UK Inactive borrowers remain inactive (since the implication is ”‘$T > C > A$”’). In other words, the Inactive borrowers do not need credit when the economy is doing well. For the Others borrowers, the implications of the HK and UK data are ”‘$T > A > C$”’ and ”‘$T > C > A$”’ indicating more of them move to an Inactive status. So in summary, these results show borrowers do not want credit during good times.

The borrowers' reaction to Stock market is different in the HK and UK markets. When there is bull market, more Inactive borrowers in HK activate their credit card immediately (since the lag=1 and the implication is ”‘$A > T$”’). Conversely, in UK, more Inactive borrowers close their accounts when the stock market is doing well. For the Others borrowers in HK, they tend to reduce borrowing with their credit card when the Hang Seng index increases, whereas those in UK keep their current status unchanged (i.e. remain Active).

The reaction to the labour market is quite similar in these two credit card datasets. The HK borrowers, either the Inactive or Others, close their credit cards when the unemployment rate increases. In the UK market, Inactive borrowers remain as inactive but Other borrowers tend to close their credit card account when the unemployment rate goes up. We performed multinomial analysis for Model A and Model B and the results are presented in the Appendix B for reference.

Initial i	CPI			GDP			Interest rate			Stock			Unemployment		
	(k,w)	β	-2Log(L)	(k,w)	β	-2Log(L)	(k,w)	β	-2Log(L)	(k,w)	β	-2Log(L)	(k,w)	β	-2Log(L)
Inactive	(6,1)	-1.0618**	133333	(3,1)	-0.2249**	13332	(3,1)	-1.3373**	13340	(2,0.2)	-1.6606**	13337	(3,1)	-1.3373**	13340
Risk	(9,1)	1.5535**	7382	(9,1)	0.7646*	7354	(12,1)	6.8403**	7376	(3,0.8)	3.286**	7380	(12,1)	6.8403**	7376
Score1	(1,1)	0.0881*	385808	(12,1)	-0.3608*	385442	(6,0.8)	0.8759*	385830	(1,1)	-0.8003*	385797	(6,0.8)	0.8759*	385830
Score2	(1,1)	-0.1732*	559078	(9,1)	-0.2894*	558900	(12,1)	-2.5087*	559150	(1,1)	-1.2616*	559112	(12,1)	-2.5087*	559150
Score3	(1,1)	-0.2399*	604810	(9,1)	-0.185*	605137	(1,1)	0.6274*	605191	(2,0.5)	-1.4517*	605161	(1,1)	0.6274*	605191
Score4	(1,1)	-0.3256*	364450	(12,1)	0.7598*	363388	(9,1)	-5.9774*	364486	(3,0.8)	3.9488*	364727	(9,1)	-5.9774*	364486

”‘*”’ indicates the parameter is significant at 99% level.

”‘**”’ indicates the parameter is significant at 95% level.

Table 5.15: Summary of cumulative logistic model estimates for the UK dataset

We look at the results in Table 5.7 and 5.15 to compare the behavioural score migration of credit card borrowers in HK and UK. Since most of the coefficient estimates with respect to the CPI model in both markets are negative, these datasets show borrowers' behavioural score increases when there is inflation. All coefficient estimates with respect to the GDP model are negative. These indicate that the behavioural score of borrowers increases when the economy is doing well (which is shown by the increase in GDP).

The borrowers in HK and UK react differently with respect to increasing Interest Rate. Most of the coefficient estimates of the Interest Rate model of the UK dataset are positive. These indicate that borrowers' behaivoural score reduces when there is inflation. Conversely, the behavioural score of the HK borrowers increases when the Interest Rate goes up since all coefficient estimates of the Interest Rate model of the HK dataset are negative.

Most of the coefficient estimates with respect to the Stock market model in both markets are negative. This indicates the behavioural score of borrowers improves when there is bull market. One surprising result of the Unemployment model in UK is that there are negative coefficient estimates. These indicate borrowers' move to a state with high behavioural score when the unemployment rate increases. Note, however, that the labour market in UK was rather stable over the sampling period and thus this result might not reflect the actual impact of unemployment rate on the UK market.

5.5 Macroeconomic measurements

5.5.1 Consumer Price Index

CPI is one of the mostly quoted macroeconomic variables and reflects whether the economy is experiencing inflation or deflation. In corporate research, CPI is not the key macroeconomic measurements in understanding the credit rating migration. Study conducted by credit rating agencies (such as Moody's Investor) has seldom incorporated CPI as a macroeconomic indicators. Figlewski et al. (2006) found that the influence of inflation on credit rating upgrade is inconsistent in sign and not significant but it is significant in reflecting the credit rating downgrade. However, in our study, most of the regression estimates for the HK and UK data of the CPI model are significant at 1% level. Note that the CPI model and Model A generate rather satisfactory results. We therefore propose using CPI as one of the key components for credit card pricing models.

For CPI, we expect positive coefficients in the cumulative logistic regression models, i.e. prices and behavioural scores should be moving in opposite directions. The assumption is that inflation indicates tough times ahead and therefore the capital level of credit card borrowers reduces. However, the results have the opposite sign. That is, when prices go up, more credit card borrowers move to a state with high behavioural score. This observation can be found in both HK and UK datasets, i.e. this observation holds in periods of both expansion and recession.

Looking at the overall economy during the sampling period, both regions experienced inflation during expansion, and the HK market was in deflation during recession. It is certainly true that credit card borrowers' behavioural scores will be downgraded during downturns. It happened that during downturns, prices were going down in the HK region and so were the credit card accounts' behavioural score. The implication is that there is no definite sign for the CPI measurement. Prices can go up during a recession (e.g. In 1970s, US was in inflation during recession of which CPI rose more than 10%). Using the percentage difference (year-to-year or month-to-month) of CPI can reflect the financial stress on households, but not the direction of the economy. One possibility is to use a model with both GDP and CPI as explanatory variables. GDP is the best measurement to reflect the direction of the economy. Indeed, Model A, which includes both CPI and GDP as explanatory variables, has a very satisfactory performance.

One final remark about the use of CPI is the lag in CPI. It appears that the CPI variables should have a short lag. This can be explained as follows. Since Governments are using a set of household commodity prices to measure the CPI, it reflects the instantaneous financial stress on a consumer, and thus the reaction time is rather short.

5.5.2 Gross Domestic Product

GDP is designed to measure the overall output of an economy. This overall output includes the production of listed corporates, private companies, consumers and households. It is not limited to measuring output in the consumer spectrum. GDP is widely used in the corporate risk research (Fons, 1991; Achary et al., 2004; Pesaran et al., 2006)

The coefficient estimates in the GDP model are found to have the right sign (Figlewski et al., 2006): negative implies that behavioural score goes up with the GDP. The only exception is the coefficient estimates ($\beta = 0.7598$) with respect to accounts in a state with Score4 in the UK dataset. Indeed the behaviour of accounts with the highest behavioural score are generally different from the rest of the accounts. This can be explained through the same argument presented in Chapter 6: Revolvers in the state with the highest behavioural score have a volatile migration pattern.

The lag of GDP is usually twelve months since it takes some time for consumers to absorb the changes in the economy. Since all coefficient estimates of the GDP models are negative which is an expected sign, GDP can be used as a good indicator of the economy. We therefore recommend keeping it in any credit card pricing model.

5.5.3 Interest Rate

"'Interest Rate"' as used in this thesis are the Best Lending Rate and the base rate benchmarked by the Hong Kong Monetary Authority and the Bank of England respectively. The question is whether the Interest rate variable is a good measurement to be used in the credit card pricing model. Interest rate is a monetary policy controlled by the central bank and is not driven by the market, and this is therefore a variable that reflects the economy as a whole. However it can influence consumer loan interest rates directly, or indirectly via interbank lending rates and cost of funding. The interaction between behavioural score migration and the Interest rate variable is worth exploring for two reasons. First, if a mortgage borrower chooses a tracker mortgage (i.e. the interest rate of the mortgage varies throughout the repayment periods), the Interest Rate indicates the mortgage borrower's financial stress. Secondly, Interest Rate can be used as an indicator to reflect the overall movement of the economy. We recommend using Interest Rate as a single explanatory variable but do not associate it with any other macroeconomic measurements, since we wish to avoid having the explanatory variables highly correlated, leading to multicollinearity in the logistic regression model.

In our models, the coefficient estimates of the Interest Rate model are highly significant. For the HK dataset results, the magnitudes and the signs (all are negative) of the coefficient estimates in the Interest Rate model are very stable. These results indicate the behavioural score goes up when the interest rate is raised. In corporate risk research, however, the default probability increase when the interest rate goes up Figlewski et al.

(2006). If we examine the impact of interest rate on the UK dataset, there are positive and negative signs for the regression coefficients in the UK dataset. Aside from the Risk accounts, the magnitude of the negative regression coefficients estimates are very small. For the Risk accounts, it is to be expected that the results are relatively volatile. Thus we conclude that in general more accounts move to a higher behavioural score group when the Interest Rate goes up.

5.5.4 Stock Market

We expect the performance of the Stock Market to have two major impacts on credit card borrowers. The first is essentially the real wealth term. Any increase or decrease of the Stock Index affects the capital and property level of consumers who have invested on the stock market. An increase in real wealth leads to an upgrade in behavioural score. On the other hand, a bear market reduces these consumers' savings and so increases their default probabilities, and thus it is to be expected their behavioural scores will be downgraded. Another possible impact of the Stock Market is on the consumer's confidence index. This is particularly true for the HK market where the stock market is the key industry.

The importance of the Stock Market is reflected in both datasets. First, across all behavioural score levels, the coefficient estimates of the Stock model are highly significant. Despite this, it is hard to find a definite trend across different behavioural score groups. A possible explanation is that investment in the Stock market is not proportional to behavioural score but rather depends on the individual. However, in general, accounts in a state with the highest or the lowest behavioural score are highly sensitive to the Stock Market.

5.5.5 Unemployment Rate

The last macroeconomic variable examined in this study is the Unemployment rate which reflects the condition of the labour market. Unemployment rate is not commonly used in corporate research to reflect the condition of the economy. Only Figlewski et al. (2006) use the unemployment to understand the migration of credit rating and they found that have volatile impact on the credit rating transition. We believe stock market is a critical indicator in consumer research because this is directly related to consumers. The expected sign for this measurement is positive, indicating that the score migration is moving inversely with unemployment rate.

The HK dataset has the right sign whereas there is a conflicting finding on the UK dataset. When the unemployment rate goes up, more people in the highest behavioural score group remained in the same behavioural score group. This can be explained in that the unemployment rate over the UK sampling period is very stable (around 5%). Having this stable unemployment rate makes it hard to generate a very significant result to reflect the actual impact of the unemployment rate.

5.6 Conclusions

The model built in this chapter has shown that including macroeconomics measurements in credit limit decisions adds another dimension - flexibility to reflect the performance of the economy. In particular, using the transition probabilities generated with the macroeconomic variables can describe the behavioural score movement better than using the unconditional transition probabilities. Also we compared the use of macroeconomic variables in credit card models with the UK and HK datasets. The table below summarizes the use of the macroeconomic variables.

Macro. variable	Lag	Weight	Notes
CPI	6 months	1	Critical measurements but cannot reflect the direction of the economy
GDP	12 months	1	Not directly related to the credit card market but it is a good indicator to show the direction of the economy
Interest Rate	12 months	1	Use individually
Stock market	3 months	0.8	A critical indicator to show the financial stress and future confidence of credit card holders
Unemployment rate	12 months	1	Significant results for recession period

Table 5.16: Summary of using macroeconomic variables in the model

Chapter 6

Transactors vs Revolvers

This chapter looks at the heterogeneity of behavioural score migration with respect to the repayment pattern. We classify the credit card accounts into transactors and revolvers and adjust the model proposed in Chapter 5 for analysis. Note that since we have limited data in the UK dataset, in this chapter, we only analyze the Hong Kong credit card dataset and empirical results for the performance of the model during expansion and recession are presented.

6.1 Definition

In common with many other researchers (Frydman et al., 1985; Thomas et al., 2002), we segment the credit card population into smaller groups in order to enhance the model's ability to forecast the credit card accounts' future behaviour. In the credit card industry, borrowers are classified as *Transactors* or *Revolvers*, where a Transactor makes full repayment and a Revolver carries part of their outstanding balance to the next month. Revolvers are preferred by lenders since these consumers pay both financial charges and merchandise fees. Different borrower types not only have different profitability, but it is also the case that Transactors and Revolvers have diverse behavioural score migration patterns, as we will show in later sections.

The key to classifying borrowers into *Transactors*(indexed by $n = 0$) and *Revolvers* (indexed by $n = 1$) is to look at whether (1) borrowers had any balance carried forward or (2) were in arrears. This classification was done by looking at a number of variables: current balance, repayment amount, date of last repayment, membership fee and behavioural score. As it might not be representative to look at repayment history in a single month only, we instead examine the half year repayment history of an account.

Let t be the time that the sample has been selected. The six months preceding this point is our observation period (i.e. $t-6$ to $t-1$). If the borrower was in arrears at any point during the observation period, we classified this borrower as a *Revolver*. Otherwise, we looked at the borrower's repayment pattern. If this borrower was not able to repay the full balance in any month during the observation period, we also classified this borrower as a *Revolver*. So what remains in the dataset are those able to repay all of their balance during the whole observation period, and these are the *Transactors*. Note that if a borrower has not activated his account six month preceding the sample point (i.e. his account balance at $t-6$ to $t-1$ =0), he is also a *Transactor*.

Below is an algorithm that we used to classify borrowers into *Transactors* and *Revolvers*:

Step 0. Set $t' = t$; define $n_t \equiv$ borrower type at month t (where $n_t = 0$ if the borrower is a transactor at month t, and $n_t = 1$ otherwise); goto Step 2.

Step 1. Set $t' = t' - 1$. If $t = t' - 7$, goto Step 7; Otherwise, goto Step 2.

Step 2. If the account is in arrears at t', $n = 1$, goto Step 8; Otherwise, goto Step 3.

Step 3. If the current balance at t' is equal to or less than zero, $n_{t'} = 0$, goto Step 1; Otherwise, goto Step 4.

Step 4. If the current balance at t' is equal to the membership fee, $n_{t'} = 0$, goto Step 1; Otherwise, goto Step 5.

Step 5. If the repayment at t' is greater than or equal to the current balance at t', $n_{t'} = 0$, goto Step 1; Otherwise, goto Step 6.

Step 6. $n_{t'} = 1$, goto Step 1.

Step 7. If $\sum_{t'=t-6}^{t} n_{t'} \leq 0$, then $n = 0$; Otherwise, $n = 1$. Goto Step8.

Step 8. Stop.

Step 1 iterates through the half year before the sample point. If a credit card account was in arrears, Step 2 classifies this account as a Revolver. A credit card account having zero balance is essentially a Transactor, as stated in Step 3. Lenders in Hong Kong commonly agree to waive the credit card account's membership fee if the credit card holder makes such a request. Normally the fee will be reimbursed into the cardholder's account in the next statement. In Step 4, we thus assume an account having current balance equal to the membership fee is a Transactor. If a credit card holder repaid the full balance, Step 5 classifies this cardholder as a Transactor. Step 7 calculates the number of months that this account was a Revolver. If a credit card account was a Revolver for one or more month, the algorithm classifies this account as a Revolver. Otherwise, this account is assumed to be a Transactor.

6.2 Borrower type distribution

Figure 6.1 shows the borrower type distribution during the sampling period. Revolvers accounted for 35% to 45% of the total borrowers. During a recession, borrowers are very

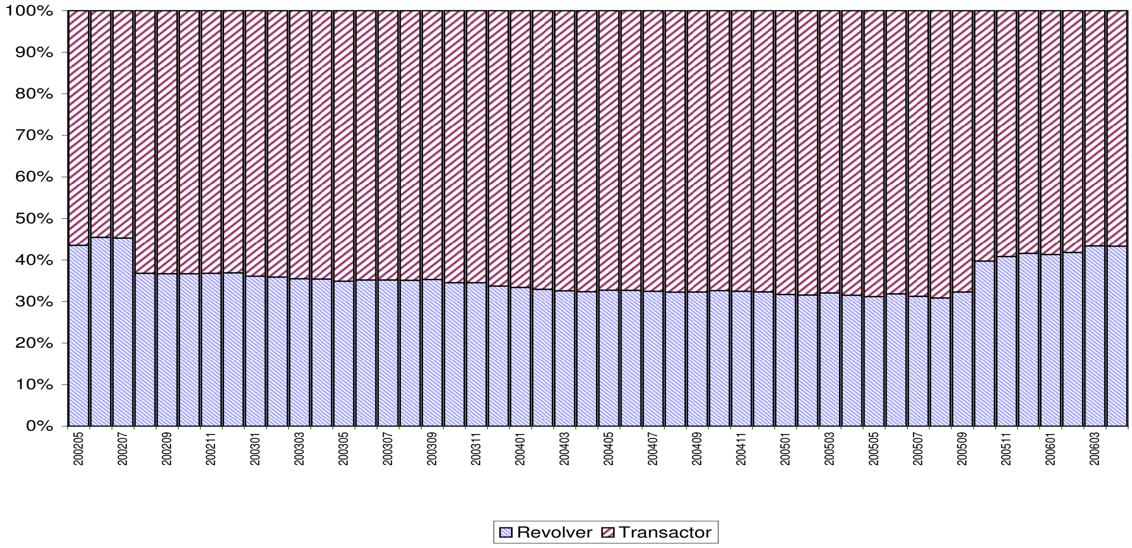


Figure 6.1: Account type distribution (In-sample period)

careful over borrowing on a credit card. Unless there is no other credit means, borrowers seldom carry any balance on their credit card. Therefore the percentage of Revolvers was low during recession. Conversely, during good times, borrowers are more willing to spend with their credit card even if they are not capable of repaying the balance immediately. Because these borrowers are optimistic about the economy they do not mind to spending their future income (i.e. borrowing with a credit card).

Type n at t	Type n' at $t+1$		Row Count
	Transactors	Revolvers	
Transactors	95.17	4.83	1871169
Revolvers	15.18	84.82	524100

Table 6.1: Unconditional transition probabilities of borrower type (with respect to the training sample period)

The unconditional probability of moving between borrower type is listed in Table 6.1. 84% revolvers remain revolvers one period later. For Transactors this month on month estimation was 95.2%. One can assume the event of changing account type follows a

Geometric distribution. We first look at the monthly event of changing from a Transactor to a Revolver. The probability of a Transactor changing to a Revolver is $p = 0.0483$. The probability that a borrower remains a Transactor in the 1st to the (k-1)th month and changes from a Transactor to a Revolver in the k-th month is:

$$P(X = k) = (1 - p)^{k-1}p$$

The expected value for the above probability distribution function is $E[X] = \frac{1}{p}$. Therefore, the average number of months before a Transactor becomes a Revolver is $20.7 (= 0.0483^{-1})$ months. Similarly, Revolvers take $6.6 (= 0.1518^{-1})$ months on average to become Transactors.

6.3 The model

There are two possible approaches to model the borrower type: including the borrower type in the state space or not. However, in view of the transition probability in Table 6.1, it is not very likely for a borrower to migrate from one borrower type to the others. Therefore, we decided to fit two different MDP models: one for Transactors and one for Revolvers. The approach of including the borrower type, on the other hand, is a possible extension for future research. Once an account has been predefined as a Transactor or Revolver, the assumption is that the borrower status of this account will not change in the planning horizon. The model is similar to (5.1), except in that we have an additional index n ($n = 1, 2$) for the borrower type. This leads to the following optimality equation for $V_t^n(l, i, \mathbf{M})$, the maximum expected profit over the next t periods that can be obtained from an account which currently has a behaviour score i, a credit limit l, a borrower-type n and macroeconomic variables $\mathbf{M}$:

$$\begin{aligned} V_t^n(l, i, \mathbf{M}) &= \max_{l' \in A_l} \{r^n(l, i, \mathbf{M}) + \lambda \sum_{i'} p^n(i'|i, \mathbf{M}) \\ &\qquad \int q_{t-1}(\mathbf{U}|\mathbf{M}) V_{t-1}^n(l', i', \mathbf{U}) \mathrm{d}\mathbf{U}\} \end{aligned} \tag{6.1}$$

The $p^n(i'|i,\mathbf{M})$ gives the probability that this behavioural score changes to i', and $q_{t-1}(\mathbf{U}|\mathbf{M})$ is the probability that the current macroeconomic variable changes to $\mathbf{U}$. The profit to the lender from the credit card borrower in the current period is $r^n(l,i,\mathbf{M})$ and the profit in the remaining $t-1$ periods is $V^n_{t-1}(l',i',\mathbf{U})$ if the behavioural score changes to i' and the macroeconomic variables change to $\mathbf{U}$. The definition of the discount factor λ and the explanation of the optimality principle are the same as presented in (5.1). Similar to the model presented in Chapter 5, $V^n_0(l,i,\mathbf{M})$, for all $l,i,\mathbf{M}$, are the boundary conditions of (6.1), i.e. the expected return of a customer at the end of the planning horizon. In this study, we assume the boundary conditions equal to zero to simplify the discussion. Whereas it is possible to set up different boundary conditions for different accounts (such as introducing penalty for accounts with low behavioural score etc), we leave it for future research to understand the sensitivity of the model to these boundary conditions.

Given a borrower of type n, currently in behavioural score state i, what change in the behavioural score occurs in the next period and what is the impact of the macroeconomic variables on this movement? We use the same approach to estimate the transition probabilities and profit function as presented in Chapter 5.

6.4 Empirical results

We used the Hong Kong credit card data for empirical study. The definition of Expansion and Recession, and the testing and training periods are the same as those defined in Chapter 5.

6.4.1 Unconditional transition matrices and profits

Table 6.2 shows the unconditional transition matrices for Transactors (the top part of the table) and Revolvers (the bottom part of the table). After classifying the data into Transactors and Revolvers, there are two inadmissible transitions: Transactors to 3+ Cycle, Revolver to Inactive. 3+cycle accounts (i.e. in arrears for 3 months or longer) have carrying balance in their account and therefore our algorithm will classify these accounts as Revolvers. Since Revolvers must have some carrying balance in their accounts and thus their accounts are always activated, they can never move to an Inactive state. However, we can have *New Revolvers*. A credit card account which was newly opened in the last two months before the sample point is called *New*. We grouped these *New* accounts with the Inactive accounts to simplify the discussion. *New Transactors* are those able to pay back their balance during their short customer lifetime, whereas *New Revolvers* are those having carry balance in the first two months of their lifetime with the lender. The frequency distribution of Transactors' behavioural score spreads quite evenly as shown in the last column of the table. Conversely, there are a large number of Revolvers having a behavioural scoreband 3 to 5.

These transition matrices are mainly dominated by the diagonal entries. One exception is New Revolvers. These accounts borrowed at the beginning of their lifetime, therefore it is not surprising to see more than half of them moved to a state with a low score band (Score4 or lower). An account with Score1 is more likely to move to a state with Score2 or a state with Bad1, especially for Revolvers.

The volatility of score transition is clearly higher for Revolvers than for Transactors such that the percentages remaining in the same behavioural score band for Transactors are higher than those for Revolvers, especially for a state with behavioural Score6 to Score9. On the other hand, the diagonal entry of Score1 or Score3 Revolvers is higher than those for Transactors. This implies Transactors with low behavioural scoreband have

Score i at t	Score i' at $t+1$															Row
	Closed	Inactive/New	Bad1	Bad2	3+Cycle	Score1	Score2	Score3	Score4	Score5	Score6	Score7	Score8	Score9	Score10	Count
								Transactors								
Inactive/New	1.02	**82.99**	0	0	-	-	0.01	0.15	4.63	3.17	1.64	0.76	1.39	1.22	3.02	171115
Score1	1.25	-	25	2.5	-	26.88	38.13	5	1.25	-	-	-	-	-	-	160
Score2	2.12	0.03	1.04	0.34	-	3.09	**69**	23.08	1.28	0.03	-	-	-	-	-	2977
Score3	1.43	0.82	0.06	0.03	-	0.06	1.29	**74.47**	18.88	1.75	0.94	0.16	0.09	0.03	0	55205
Score4	0.82	0.49	0.01	0	-	0	0.08	3.63	**76.19**	11.35	1.96	2.12	2.83	0.38	0.14	208900
Score5	0.77	0.46	0.02	0	-	-	0.02	1.7	9.04	**59.33**	8.06	7.54	7.47	4.14	1.45	165033
Score6	0.77	0.78	0.01	-	-	-	0.01	0.71	3.4	13.77	**53.78**	7.67	13.75	2.01	3.33	104841
Score7	0.59	0.44	0.01	0	-	-	0	0.55	2.78	9.83	7.6	**51.45**	11	12.16	3.58	116257
Score8	0.36	0.21	0.01	0	-	0	0	0.25	1.74	3.27	7.12	6.59	**64**	9.93	6.52	220698
Score9	0.29	0.12	0.01	0	-	-	0	0.13	0.85	1.09	1	6.38	8.59	**71.72**	9.82	233977
Score10	0.21	0.01	0.01	0	-	-	-	0.06	0.45	0.18	0.42	0.76	5.57	7.65	**84.68**	264797
								Revolvers								
New	-	13.17	0.08	0.16	-	-	2.66	26.83	33.1	5.6	5.12	4.84	4.37	0.28	3.81	2520
3+Cycle	31.08	-	29.05	4.5	2.93	0.45	12.16	19.37	0.45	-	-	-	-	-	-	444
Score1	2.42	-	31.32	5.73	3.35	13.76	40.7	2.68	0.04	-	-	-	-	-	-	2688
Score2	1.18	-	1.86	0.88	0.65	3.87	**65.56**	25.55	0.45	0	-	-	-	-	-	45657
Score3	0.43	-	0.28	0.15	0.01	0.21	5.13	**78.92**	14.53	0.29	0.03	0.01	0	0	0	257708
Score4	0.4	-	0.06	0.03	-	0	0.28	12.48	**74.77**	9.45	1.16	0.79	0.47	0.09	0.03	278089
Score5	0.23	-	0.02	0	-	-	0.05	3.05	17.65	**60.6**	5.35	3.18	6.34	2.94	0.6	104170
Score6	0.23	-	0.01	0	-	-	0.03	3.34	15.6	12.01	48.61	5.77	6.26	5.66	2.48	26277
Score7	0.18	-	0.01	-	-	-	0.03	3.76	15.92	12.61	4.3	45.23	8.51	2.78	6.68	21689
Score8	0.11	-	-	-	-	-	0.01	2.45	8.5	11.12	3.69	3.2	**54.27**	3.97	12.68	40768
Score9	0.12	-	0.01	-	-	-	0.03	3.08	12.44	16.39	6.92	3.22	7.54	41.25	9.02	18998
Score10	0.1	-	0.01	0	-	-	0	0.62	5.28	3.1	1.06	2.31	10.02	4.93	**72.57**	52301

"‘-"’ represents there is no sample observation.

"‘0"’ represents the transition probability is less than 0.0005.

A bold value indicates the transition frequency is greater than 50% .

The transition probabilities of all absorbing states (Closed, Bad1 and Bad2) are not shown in the table.

There is no 3+Cycle Transactors.

Table 6.2: Unconditional transition matrix (in percentage) for Transactors and Revolvers

a high probability of moving to a higher behavioural score state.

There is a larger behavioural score state movement for Revolvers than for Transactors. For example, 11.12% Revolvers with Score8 move to a state with Score5 in the next month, however, it is much less likely that Transactors will have such movement (only 3.27%). Moreover, it is noticeable that more Transactors than Revolvers move to a state with higher behavioural score in the next month. The number of default accounts (moving to Bad1 or Bad2) is higher for Revolvers than Transactors, as may be expected.

Table 6.3 shows the profit value of Transactors and Revolvers. There is no consistent trend for Inactive accounts if we look at the magnitudes of the profit values. Of course one may say Inactive Transactors are not important accounts since they generate a loss less than HK$11 per month. The loss is the cost of funding for account operation. However, New Revolvers generate a profit of on average HK$182 per month. Note that the profit

	Limit1	Limit2	Limit3	Limit4	Limit5	Limit6	Limit7	Limit8	Limit9	Limit10
	Transactors									
Closed	71.54	94.81	198.91	308.33	105.74	215.04	438.92	272.94	483.64	1057.77
Inactive/New	-10.75	-10.1	-9.97	-8.86	-5.76	-5.98	-5.18	-6.09	-8.08	-5.94
Bad1	-4472.83	-6000.56	-7038.7	-15959.43	-11803.34	-22019.16	-16774.42	-13915.23	-20945.72	-41227.9
Bad2	-2402.74	-6809.58	-5926.3	-11199.04	-15131.59	-6824.92	-16926.6	-23376.59	-31816.03	-25834.61
Score1	-515.09	-611.54	-1316.97	-978.37	-1820.19	-1530	-1520.76	-1111.14	-1751.72	-5975.94
Score2	207.11	285.63	463.95	562.84	436.29	791.46	967.87	888.83	650.08	1216.29
Score3	24.41	26.39	57.79	90.57	21.71	92.15	118.32	132.96	158.6	358.1
Score4	-6.3	-2.58	1.22	5.93	13.77	18.11	24.9	39.68	59.46	138.17
Score5	-2.62	-4.36	0.86	6.32	10.45	11.07	21.29	30.99	51.22	108.12
Score6	1	-5.86	-2.6	3.64	1.03	7.59	9.44	15.55	31.14	63.58
Score7	-3.74	-9.04	-4.63	-1.48	6.52	7.46	17	28.22	48.48	115.91
Score8	-11.87	-12.49	-9.79	-6.08	-2.68	0.27	6.02	13.14	30.09	84.23
Score9	-12.21	-12.9	-10.21	-7.98	-4.36	-2.57	2.52	10.8	23.16	63.88
Score10	-13.3	-11.57	-9.01	-6.96	-1.65	0.82	3.81	12.55	22.59	71.34
	Revolvers									
Closed	-13.14	-15.54	-11.29	-8.65	-14.93	-9.88	-7	-8.43	-17.02	62.69
New	161.35	171.21	194.26	156.24	222.91	254.17	233.11	145.33	271.48	21.86
Bad1	-7828.04	-11363.53	-16486.3	-24312.91	-22590.3	-32337.07	-40516.15	-51693.26	-65367.88	-103788.13
Bad2	-3895.07	-5830.98	-8274.66	-11502.82	-13484.69	-16408.15	-20200.52	-24537.89	-34343.81	-54801.32
3+Cycle	-635.15	-670.12	-1097.4	-1125.18	-951.49	-954.55	-1149.93	-2139.17	-711.03	-257.61
Score1	-709.45	-1137.16	-1546.51	-2068.89	-2022.05	-2569.78	-3214.09	-3920.44	-5575.69	-9426.65
Score2	204.64	253.53	362.36	478.14	567.48	695.55	924.13	1264.85	1416.39	2477.14
Score3	181.12	229.7	328.46	445.73	312.51	539.04	678.61	807.06	1026.91	1764.97
Score4	80.39	87.23	143.38	212.88	136.13	258.48	337.3	425.47	579.58	1296.55
Score5	35.62	27.36	43.27	60.9	48.47	76.37	98.9	127.35	193.44	450.07
Score6	36.76	27.96	35.14	42.32	71.9	65.3	88.8	103.91	141.88	388.53
Score7	37.29	36.1	45.6	42.16	70.82	70.24	69.51	92.65	141.58	313.29
Score8	21.46	20.53	22.68	26.3	78.15	61.2	74.88	86.54	128.61	214.26
Score9	45.53	45.25	48.88	61.51	131.85	115.58	116.02	163.87	181.04	284.54
Score10	15.69	23.99	17.51	17.69	55.27	42.86	47.67	71.47	87.83	158.78

The first column indexes the Score status and the first row indexes the Credit Limit statues

All values are in HK dollar (£1 $\approx$ HK$15)

For all absorbing states (Closed, Bad1, Bad2), we use the profit value (derived by the lender) in the month of the account being written-off or closed.

Table 6.3: Average monthly profit for Transactors and Revolvers

with respect to New Revolvers with credit limit band 10 is low, only HK$21. In the samples, there are 24 New Revolvers with credit limit band 10 of which 13 of these borrowers were in arrears. That is, these 13 borrowers did not repay anything in their second or third month-on-book. In the profit value defined by the lender, the provision is higher for those in arrears than those not. Moreover, the provision decreases with month-on-book (i.e. the longer credit history, the lower the provision). The provision for these New in arrear Revolvers therefore is very high and skews the average profit.

Profit increases with credit limit if the account has a behavioural scoreband 2 or above and decreases with credit limit if the account has a behavioural state Score1 or 3+Cycle. These observations hold for both Transactors and Revolvers. For Transactors with a behavioural state Score 2 or above, the profit is low. Roughly speaking, these profit values decrease with behavioural score. For Revolvers with Score 2 or above, the profit value is much higher than those of Transactors. Since the number of Revolvers with Score 6 or above is low, the corresponding profit values fluctuate.

Given the same credit limit and behavioural score status, Revolvers in general generate a much higher profit than Transactors. For example, Revolvers with a credit limit 10 and behavioural score 10 add HK$158.78 monthly profit. This amount is double the profit of a Transactor in the same state (HK$71.34). This is because the accumulated revolving balance contributes interest to the profit.

The profit values with respect to different borrower types are different and thus justify our argument for segmenting the samples by repayment pattern.

6.4.2 Estimates for the multinomial logistic regression model

Table 6.4 summarizes the multinomial logistic regressions results for Transactors and Revolvers.

We first examined the behaviour of Inactive Transactors. The Implications for the CPI and Interest Rate models are ”‘$A > T > C$”’ and ”‘$A > T, A > C$”’ respectively, which both indicate more Inactive Transactors activate their credit cards when these two measurements increase. Since the increase of CPI and Interest Rate indicates the economy is in an inflationary period, these results show during inflation the demand for credit cards increases. More Inactive Transactors activate their credit cards when the Stock market is doing well and the reaction time with respect to changes of the Stock Index is quite instantaneous with the lag equal to 1. Conversely, when GDP goes up, the Implication ”‘$C > T > A$”’ indicates more Inactive Transactors close their credit card accounts. Since GDP is an index to show the economy’s performance, this result shows the demand for

Score i	Macroeconomics Measurement	Format (k,w)	Log(A/C) β	Log(T/C) β	Log(A/T) β	-2Log(likelihood)	Implications
			Transactors				
Inactive/New	CPI	(9,1)	1.4277*	0.7147*	0.713*	168893	A>T>C
Others	CPI	(12,0.8)	0.4732*	1.1082*	-0.635*	150157	T>A>C
Inactive/New	GDP	(3,1)	-0.1609*	-0.1077*	-0.0532*	169017	C>T>A
Others	GDP	(12,1)	0.1116*	0.3073*	-0.1958*	150143	T>A>C
Inactive/New	Int	(6,1)	2.0642*	0.1454	1.9188*	168166	A>T, A>C
Others	Int	(12,1)	1.0369*	2.0645*	-1.0276*	150157	T>A>C
Inactive/New	Sto	(1,1)	0.8107	-0.607	1.4177*	169174	A>T
Others	Sto	(3,0.8)	3.0142*	4.5483*	-1.5341**	150204	T>A>C
Inactive/New	Une	(9,1)	-1.8346*	-1.2699*	-0.5647*	169109	C>T>A
Others	Une	(9,1)	-1.1578*	-2.6118*	1.454*	150011	C>A>T
			Revolvers				
New	CPI	(6,1)	/	/	1.2436*	1944	A>T
3+Cycle	CPI	(12,1)	2.0016**	/	/	541	A>C
Others	CPI	(12,1)	0.556*	/	/	43173	A>C
New	GDP	(4,1)	/	/	0.2085**	1956	A>T
3+Cycle	GDP	(N,N)	N	/	/	/	/
Others	GDP	(N,N)	N	/	/	/	/
New	Int	(9,1)	/	/	1.7144**	1950	A>T
3+Cycle	Int	(N,N)	N	/	/	/	/
Others	Int	(12,1)	1.2527*	/	/	43148	A>C
New	Sto	(2,0.2)	/	/	8.2053*	1932	A>T
3+Cycle	Sto	(N,N)	N	/	/	/	/
Others	Sto	(N,N)	N	/	/	/	/
New	Une	(1,1)	/	/	-1.0578**	1950	T>A
3+Cycle	Une	(N,N)	N	/	/	/	/
Others	Une	(12,1)	-0.5505**	/	/	43187	C>A

"/" represents there is no observation in the data.

"'N"' represents the stepwise multinomial logistic regression cannot find any significant explanatory macroeconomic variable.

"'*"' indicates the parameter is significant at 99% level.

"'**"' indicates the parameter is significant at 95% level.

The first column is the index of the initial score state i where "'Others"' refers to accounts with ordinary behavioural score (Score1 to Score10).

The best fit macroeconomic variables (discussed in Section 5.2.4) are presented in column three.

The estimated parameters are presented in column four to six.

-2log(likelihood) ratios which are used to measure the model fit statistics are presented in column seven.

Log(A/C) represents $\log\left(\frac{p(A|i,\mathbf{M})}{p(C|i,\mathbf{M})}\right)$ in (5.2)

Log(T/C) represents $\log\left(\frac{p(T|i,\mathbf{M})}{p(C|i,\mathbf{M})}\right)$ in (5.3)

Log(A/T) represents $\log\left(\frac{p(A|i,\mathbf{M})}{p(T|i,\mathbf{M})}\right)$ in (5.4)

Table 6.4: Summary of the multinomial logistic model estimates for Transactors and Revolvers

credit reduces when the economy is expanding. More Inactive Transactors close their account when the Unemployment rate increases. This is because borrowers are cautious in borrowing during bad times.

For the Others Transactors, the implication for the CPI and Interest Rate models is ”‘$T > A > C$”’ which indicates that these borrowers reduce the use of their credit cards when these measurements increase. This result shows borrowers who have used credit cards are very caution in borrowing when there is inflation. The implication for the Stock market model is also ”‘$T > A > C$”’ which indicates these borrowers reduce their spending when there is a bull market. The implication for the GDP model is the same, which indicates when economy is doing well, borrowers reduce their spending with credit cards. The implication for the Unemployment model is identical to those of Inactive Transactors, that is these borrowers tend to close their credit card accounts when there is stress in the labour market.

The coefficient estimates of the models with respect to the New Revolvers reflects the lender's decision only. These New Revolvers had no repayment record and therefore the lender did not have enough information to generate a behavioural score for these accounts. After two to three months, the lender was able to generate a score for them and those the probability of moving from New to Active was an operational decision.

Only CPI has significant impact on the distribution of 3+Cycle borrowers moving to the Active, Inactive and Closed state. The implication is ”‘$A > C$”’, that is, more 3+Cycle borrowers remain active if there is inflation. Or in other words, when there is inflation, many of them are not able to repay their debt in full.

For Others Revolvers, the implication for the CPI and Interest Rate models is ”‘$A > C$”’ which indicates these borrowers keep their carrying balance when there is inflation. Conversely, if the unemployment rates goes up, i.e. the economy is in bad times, the implication is "'$C > A$"'. This result shows Others Revolvers want to payoff their debt and so as reduce their spending.

Sixty-three percent of the data are Transactors, thus the parameter estimates of All samples (presented in Chapter 5) are the same as those of Transactors. These results show it is necessary to split the dataset in order to understand the finer details of the behavioural score migration of different borrower types.

Model A (CPI, GDP, Stock)										
	Log(A/C)			Log(T/C)			Log(A/T)			-2Log(likelihood)
	CPI	GDP	Stock	CPI	GDP	Stock	CPI	GDP	Stock	
Transactors										
Inactive	1.2601*	-.1494*	/	0.5401*	-0.0937*	/	.7206*	-.0556*	/	168643
Others	.4104*	.0471**	3.2039*	0.6776*	0.2079*	5.2561*	-0.2672**	-0.1607*	-2.0521**	150049
Revolvers										
New	/	/	/	/	/	/	0.6591**	**-0.0293**	6.2860*	1928

Model B (Unemployment rate, Stock)							
	Log(A/C)		Log(T/C)		Log(A/T)		-2Log(likelihood)
	Stock	Unemployment	Stock	Unemployment	Stock	Unemployment	
Transactors							
Others	2.6314*	-1.1041*	4.5818*	-2.6044*	-1.9504*	1.5002*	149958
Revolvers							
New	/	/	/	/	6.327*	-0.684	1930

"'*"' indicates the parameter is significant at 99% level.

"'**"' indicates the parameter is significant at 95% level.

The models are developed only if there is significant variable found in Table 6.4.

Table 6.5: Summary of multinomial logistic model estimates of Model A and Model B for Transactors and Revolvers

When comparing the parameter estimates between Table 6.4 and 6.5, there is one sign-changed parameter (highlighted in bold). This is because there are correlations among macroeconomic measurements. This does not change our conclusions about each individual macroeconomic variable, since one should only use Model A and Model B to predict the behavioural score in the next month, but not to look at the impact of each macroeconomic variable (as explained in Chapter 5).

6.4.3 Estimates for the cumulative logistic regression

Initial i	CPI			GDP			Interest rate			Stock			Unemployment		
	format	β	-2Log(L)	format	β	-2Log(L)	format	β	-2Log(L)	format	β	-2Log(L)	format	β	-2Log(L)
Transactors															
Inactive /New	(12,1)	-6.7626*	90454	(12,1)	-1.0338*	95740	(6,.8)	-10.6821*	88405	(2,.2)	8.7253*	99060	(12,1)	6.1264*	95680
Score1	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
Score2	(6,1)	-1.1155*	4766	(12,1)	-0.1719**	4786	(12,1)	-1.9464**	4790	(3,.8)	3.0606**	4791	(9,1)	1.6271*	4779
Score3	(1,1)	0.2402*	78654	(9,1)	-0.0464*	78720	(6,.8)	-1.0807*	78699	(3,.8)	-2.4181*	78690	(1,1)	-0.214*	78721
Score4	(6,1)	-0.6927*	358061	(2,.8)	0.0847*	358405	(3,1)	0.4255*	358744	(3,.8)	-2.7931*	358558	(12,1)	1.0019*	358405
Score5	(12,1)	-0.4758*	452934	(12,1)	-0.0408*	453113	(12,1)	-0.8188*	452977	(1,1)	-1.2288*	453009	(3,1)	0.6434*	452716
Score6	(6,1)	-0.5952*	294268	(9,1)	-0.0684*	294580	(12,1)	-1.7539*	294154	(3,.8)	-2.3313*	294553	(12,.8)	1.1221*	294252
Score7	(12,1)	-1.3732*	348100	(12,1)	-0.1568*	349143	(1,1)	-1.3968*	348825	(3,.5)	-2.8758*	349280	(12,1)	1.9061*	348043
Score8	(6,1)	-1.838*	545258	(12,1)	-0.4574*	548000	(12,.8)	-2.9134*	549964	(3,.8)	-3.0677*	552732	(12,1)	3.6516*	545299
Score9	(12,1)	-2.0559*	460572	(12,1)	-0.1364*	464680	(12,1)	-2.846*	462499	(1,1)	-4.8054*	462693	(12,1)	2.9001*	460294
Score10	(9,1)	-3.7254*	305240	(12,1)	-0.8775*	309471	(12,1)	-2.8825*	312621	(3,.8)	-18.5641*	311055	(12,1)	4.8722*	302997
Revolvers															
3+Cycle	(6,1)	-1.7764**	848	(3,1)	0.1119**	855	(6,1)	-5.2234*	843	N	N	N	(9,.8)	1.7812**	854
New	(12,1)	-6.0597*	6651	(12,1)	-0.8582*	7018	(6,.8)	-8.1252*	6714	(1,1)	3.2333**	7274	(12,1)	5.6492*	6932
Score1	(12,1)	-1.121*	7269	(12,1)	-0.1207*	7284	(12,1)	-1.8704*	7274	(2,.5)	-3.4411**	7276	(12,1)	1.6939*	7263
Score2	(6,1)	-1.0972*	82876	(12,1)	-0.1688*	83178	(6,.8)	-3.608*	82973	(1,1)	-0.7666*	83316	(12,1)	1.723*	83028
Score3	(6,1)	-1.4185*	345203	(12,1)	-0.2419*	347142	(12,1)	-2.8117*	346541	(3,.8)	-2.9047*	348086	(12,1)	2.3614*	345653
Score4	(12,1)	-2.0709*	462674	(12,1)	-0.2984*	466338	(12,1)	-2.9479*	463883	(3,.8)	-6.621*	466882	(12,1)	3.2658*	460547
Score5	(12,1)	-1.2195*	268739	(12,1)	-0.155*	269524	(12,1)	-2.085*	268642	(3,.8)	-5.8526*	269147	(12,1)	2.3445*	268014
Score6	(9,1)	-2.2351*	82911	(12,1)	-0.5398*	83277	(12,1)	-2.8916*	83258	(3,.8)	-7.7378*	83872	(12,1)	3.6088*	82633
Score7	(9,1)	-2.7243*	70587	(12,1)	-0.3816*	71638	(12,1)	-2.9425*	71305	(3,.8)	-5.2799*	71999	(12,1)	2.9623*	70996
Score8	(12,1)	-5.9702*	113322	(12,1)	-1.0071*	119054	(12,.8)	-6.4278*	116474	(3,.8)	-2.6835*	122116	(12,1)	5.7196*	117738
Score9	(12,1)	-4.8335*	62555	(6,1)	0.2139*	65905	(6,1)	-8.1506*	61011	(1,1)	0.6409*	66004	(12,1)	1.9513*	65692
Score10	(12,1)	-1.5928*	108006	(6,1)	2.2258*	102641	(6,1)	-3.8963*	106356	(3,.8)	-6.1142*	108710	(1,1)	-1.6228*	108388

”‘*”’ indicates the parameter is significant at 99% level.

”‘**”’ indicates the parameter is significant at 95% level.

”‘N”’ represents the stepwise cumulative logistic regression cannot find any significant explanatory macroeconomic variable.

Table 6.6: Summary of cumulative logistic model estimates for Transactors and Revolvers

Table 6.6 presents the results of the cumulative logistic regression for Transactors and Revolvers. The top of this table presents the results for Transactors and the bottom of this table presents those of Revolvers.

We first examine the impact of CPI. Except for those of Transactors with Score 3, the coefficient estimates of borrowers with other scorebands are negative, which indicates more borrowers move to a state with a high behavioural score if CPI goes up. This means the default probabilities of borrowers decrease during inflation. The coefficient estimate of Inactive Transactor is -6.7626, which indicates the behavioural score of these accounts have a high increment when there is inflation. This is because after these Inactive Transactors resume using their credit cards, it is likely the lender allocates them high scores as these borrowers have no debt history. The best fit format of the CPI variable for Transactors is in general shorter than those for Revolvers indicating inflation has a greater short-term impact on Transactors than Revolvers.

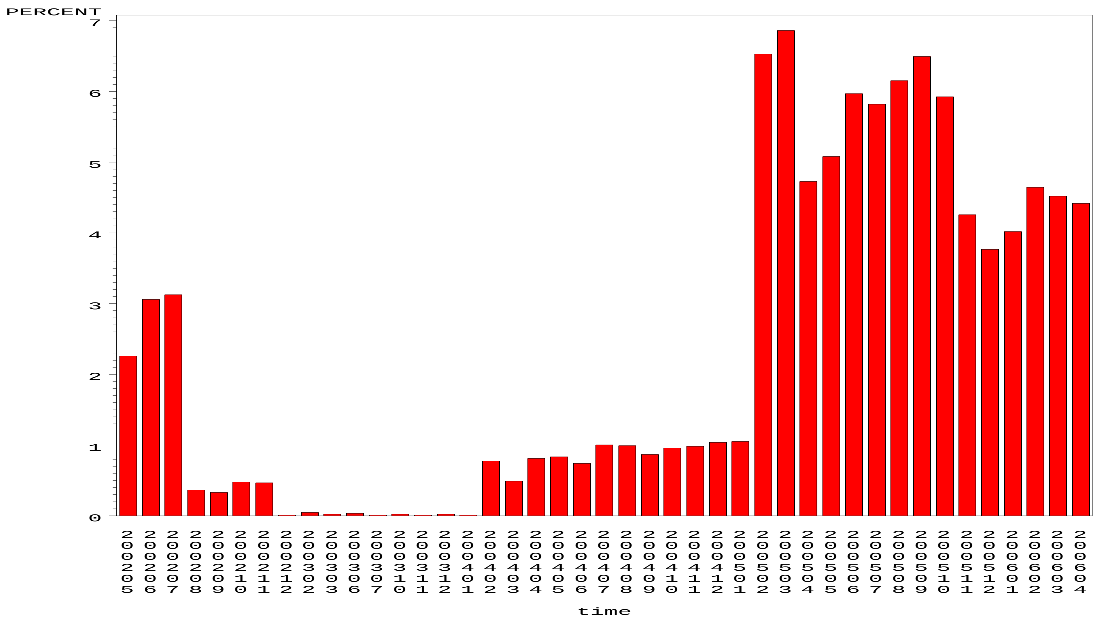


Figure 6.2: The "'time"' distribution of Revolvers in a state with Score 10 at t move to a state with Score 9 or lower

The coefficient estimates with respect to the GDP model are positive which indicates borrowers move to a state with higher behavioural score when the economy is doing well. This is with the exception of Revolvers in a state with Score 10. When GDP increases, these accounts move to a state with lower behavioural score when GDP goes up. Looking at the data, there were 14344 Revolvers in a state with Score 10 which moved away from their current behavioural score state. 61.08% of these accounts has just become Revolvers at time t. That is, these accounts repaid their balance in month $t-5, t-4, t-3, t-2, t-1$ (when t is the sampling time) and had some carrying balance in month t. Our algorithm classifies these accounts as Revolvers. As shown in Figure 6.2, the majority of these accounts are found during Expansion. There are two reasons to explain why the logistic regression model generates a positive regression parameter ($\beta = 2.2258$) for Revolvers with

Score 10. The first is that Revolvers with Score 10 have a very good repayment history over the last half year and therefore the scoring system gives them a high behavioural score. However, such borrowers' financial status is not very strong. Once there is any over-spending, these Revolvers are not capable of paying off their balance. This happens when the economy is doing well, when consumers tend to increase their spending. The second reason is the lender's policy which might change becuase of GDP increases. This is something that is out of the scope of this research as we do not have a full access to the behavioural score definition and the operational decisions of the lender.

The fact that the coefficient estimates of the Interest Rate model are negative indicates the behavioural score of borrowers increases if interest rate increases. One observation is that Interest rates put high pressure on Revolvers. The regression parameters of the Interest rate model with respect to Revolvers are higher than those of the Transactors. Borrowers take different time periods to react to changes in the economy. For example, Transactors in a state with Score 4 (where lag=3) and Score 7 (where lag=1) react to the Interest Rate quite instantaneously. However, the lag of the Interest Rate model for Transactors in a state with Score 9 and 10 is 12 months. In general, the impact of Interest Rates on Revolvers lasts longer than it does with Transactors.

The regression estimates of the Stock market model show the behavioural score migration is volatile with respect to the different bands. It is hard to find a consistent trend among different behavioural score states. New Revolvers are much more likely to move to a state with low behavioural score if the Stock variable is going up. This finding may suggest that borrowers having a poor financial record were betting on the stock market with money borrowed from credit cards.

The effect of the labour market is clear in our parameter estimates. The parameters of the Une model are mostly positive and statistically significant, i.e. the behavioural scores are moving in the opposite direction to unemployment rate. An exception, however, is

Initial i	Model A				Model B		
	CPI β	GDP β	Stock β	-2Log(L)	Stock β	Unemployment β	-2Log(L)
Transactors							
Inactive/New	-6.6344*	-0.1735*	11.2484*	88774	13.1573*	6.8782*	93563
Score1	N	N	N	N	N	N	N
Score2	-1.0792*	-0.0111	0.2521	4766	**1.4089**	1.4945**	4778
Score3	0.3608*	-0.1226*	-2.31*	78509	-2.3993*	-0.2078*	78672
Score4	-0.9296*	0.1018*	-5.2144*	356878	**-2.3834***	0.9174*	358234
Score5	-0.4808*	**0.0021**	-1.2261*	452792	-1.3505*	0.6641*	452545
Score6	-0.721*	-0.0047	-3.6397*	293962	-2.4183*	1.1352*	294108
Score7	-1.3047*	-0.0102	-2.3016*	347932	-1.2025*	1.8201*	348000
Score8	-1.8272*	-0.1676*	-6.88*	542404	-0.2025	3.6378*	545297
Score9	-2.078*	**0.0669***	-4.0183*	458706	-3.5998*	2.5935*	458966
Score10	-3.9639*	**0.163***	-2.6328*	305100	**0.3265**	4.9097*	302997
Revolvers							
3+Cycle	-1.6538**	0.0811	N	846	N	N	N
New	-5.7948*	-0.1473*	4.9853*	6613	7.316*	6.205*	6863
Score1	-1.1047**	-0.0175	-3.6097**	7255	-2.4547**	1.4874*	7257
Score2	-1.1779*	-0.0228	-1.9478*	82782	-0.3309	1.7018*	83025
Score3	-1.5812*	-0.0217**	-5.61*	344068	-0.9189*	2.3006*	345623
Score4	-1.8219*	-0.0495*	-4.8807*	461748	-2.0901*	3.0991*	460395
Score5	-1.2448*	**0.0137**	-5.7539*	268081	-4.0585*	2.1349*	267703
Score6	-1.9326*	-0.0813*	-5.9487*	82698	-0.9585*	3.5245*	82629
Score7	-3.0652*	**0.1967***	-0.7281	70487	**4.3441***	3.4206*	70931
Score8	-7.4596*	**0.4503***	**8.0063***	112562	**11.8904***	6.8287*	117044
Score9	-5.0535*	0.2151*	4.3363*	62258	2.5384*	2.222*	65630
Score10	-3.1794*	2.2038*	**16.7296***	101499	-12.9723*	-2.4137*	107675

”‘*”’ indicates the parameter is significant at 99% level.

”‘N”’ represents the stepwise cumulative logistic regression cannot find any significant explanatory macroeconomic variable.

Table 6.7: Summary of cumulative logistic model estimates for Transactors and Revolvers- Model A and Model B

Transactors with Score3. Its coefficient equals -0.214(<.0001). A second exception is Revolvers with Score10. These indicate borrowers in these states tend to reduce consumption instantaneously in preparation for the tough times ahead. In summary, the impact of macroeconomic variables on behavioural score migration is more volatile for Revolvers than for Transactors. That the magnitudes of the regression estimates are in general higher for Revolvers indicates Revolvers are more sensitive to the economy. It is also noticeable that the impact of macroeconomic variables is less marked on those with lower score or in 3+Cycle.

The coefficient estimates of the macroeconomic models with respect to Transactors are very similar to the results presented in 5.7. This is because more than 60% of the samples are Transactors, showing that splitting the samples into Transactors and Revolvers can provide more insight into the behavioural score migration.

Table 6.7 shows the cumulative regression results with more than one explanatory variable. Putting more than one variable into the cumulative logistic regression changed the signs of several coefficients. This is due to the correlation among macroeconomic variables, although these estimates do not change the conclusions with respect to the impact of each macroeconomic variable (explained in Chapter 5).

6.4.4 Comparing transition matrices

We use equation (5.12) to compare the behavioural score states forecast by the transition matrix conditional on what actually happened in the economy. Note that our credit card dataset consists of a lot of samples (i.e. $n >>$ in equation (5.12)), thus obtaining a χ^2 value that does not reject the null hypothesis $x_i(12) = y_i(12)$ is very unlikely. The focus in this section, however, is to use the χ^2 value as a tool for comparison rather than as an indication of whether or not we need to reject our proposed hypothesis.

The chi-square value (= 14601) for the probability distribution estimated with the unconditional transition matrix is very large, which implies the matrix unsuccessfully reflects the real behavioural score transition path, especially for Revolvers. For most of the portfolios, the fitness ratio of using the unconditional transition matrix to estimate the probability distribution is less informative. However, it is remarkable to note that the unconditional matrix estimates the default probability quite accurately.

Among all the conditional matrices, those built with CPI (i.e. CPI model or Model A) perform generally well. This is particularly the case for Revolvers during Expansion. The chi-square value of the Pearson chi-square test for CPI model and Model A are 1115 and 1278 respectively, which is ten times lower than for the other conditional transition matri-

Score	$x(1)$	x(12)								
		Real	Unconditional	CPI	GDP	Int	Sto	Une	ModelA	ModelB
Transactors - Expansion										
Closed	0	7.59	5.74	5.8	6.56	6.35	6.43	6.26	5.68	6.18
Inactive/New	13.77	3.05	2.71	2.62	2.98	3.12	3	2.95	2.5	3.08
Bad1	0	0.19	0.15	0.09	0.15	0.15	0.14	0.12	0.08	0.12
Bad2	0	0.06	0.03	0.02	0.03	0.03	0.03	0.02	0.01	0.02
3+Cycle	0	0	0	0	0	0	0	0	0	0
Score1	0	0.01	0.01	0	0.01	0.01	0.01	0.01	0	0.01
Score2	0.05	0.06	0.14	0.07	0.15	0.13	0.14	0.12	0.06	0.11
Score3	0.49	0.75	2.61	1.33	2.66	2.71	2.47	2.17	1.13	2.08
Score4	8.16	6.89	9.77	4.4	9.76	9.73	9.15	7.87	3.77	7.7
Score5	6.31	4.37	8.75	4.16	8.68	7.88	7.95	7.18	3.76	7.01
Score6	3.82	3.87	6.26	3.08	6.18	5.49	5.64	5.08	2.79	5.02
Score7	8.27	10.69	7.53	4.13	7.54	6.54	6.65	6.54	3.76	6.4
Score8	12.42	13.85	15.25	10.84	15.84	13.79	13.77	14.74	10.13	14.59
Score9	10.49	10.6	17.21	17.86	17.76	16.83	16.31	19.05	17.28	19.37
Score10	36.23	38.01	23.84	45.61	21.69	27.25	28.3	27.87	49.05	28.29
Chi-square value			5278	5224	6359	3950	3440	3714	6742	3683
Transactors - Recession										
Closed	0	6.56	7.51	7.7	7.57	7.26	6.87	7.7	7.58	7.37
Inactive/New	11.03	2.58	2.57	2.68	2.19	2.46	2.51	2.45	2.44	2.35
Bad1	0	0.24	0.26	0.34	0.31	0.28	0.26	0.31	0.33	0.29
Bad2	0	0.08	0.06	0.08	0.07	0.07	0.06	0.07	0.08	0.07
3+Cycle	0	0	0	0	0	0	0	0	0	0
Score1	0.02	0.01	0.01	0.02	0.02	0.02	0.01	0.02	0.02	0.02
Score2	0.33	0.32	0.23	0.34	0.31	0.28	0.24	0.3	0.35	0.29
Score3	6.7	5.41	3.6	4.97	4.38	4.2	3.89	4.52	5.09	4.65
Score4	17.19	14.92	12.16	16.16	14.25	13.7	12.48	15.1	15.54	15.07
Score5	11.95	12.41	9.94	12.34	12.73	11.32	10.09	11.88	12.87	11.84
Score6	9	9.79	6.61	7.84	8.19	7.43	7	7.61	8.62	7.63
Score7	8.56	9.12	7.62	8.59	8.88	8.29	8.08	8.54	9.07	8.56
Score8	24.65	24.26	14.76	15.85	16.04	15.25	15.85	15.74	15.54	15.72
Score9	10.5	14.16	15.59	14.14	14.55	14.91	16.63	14.25	13.82	14.39
Score10	0.07	0.06	19.06	8.94	10.51	14.53	16.03	11.52	8.65	11.75
Chi-square value			9177	4428	4830	6652	7400	5345	4329	5384
Revolvers - Expansion										
Closed	0	3.6	3.87	3.92	3.76	4.35	3.77	4.43	3.88	3.6
New	0.32	0	0	0.09	0.06	0.06	0.08	0.06	0.09	0.07
Bad1	0	1.72	2.17	0.72	1.9	2	1.94	1.38	0.61	1.33
Bad2	0	0.9	0.83	0.28	0.73	0.77	0.75	0.54	0.24	0.52
3+Cycle	0.08	0.05	0.04	0.01	0.03	0.02	0.03	0.02	0.01	0.02
Score1	0.11	0.13	0.24	0.05	0.21	0.13	0.21	0.14	0.04	0.13
Score2	0.58	0.68	4.14	1.09	3.71	2.51	3.6	2.6	0.88	2.48
Score3	9.21	8.98	26.18	11.37	25.35	19.5	23.89	20.82	10.35	20.35
Score4	27	27.21	31.93	24.71	32.43	34.5	31.65	32.82	24.12	32.75
Score5	12.06	12.17	12.83	14.57	13.27	16.72	13.56	14.79	14.32	15.1
Score6	5.39	5.6	3.16	4.48	3.4	4.42	3.45	4	4.58	4.12
Score7	3.32	3.27	2.31	3.91	2.44	3.33	2.56	3.03	3.85	3.13
Score8	11.69	10.51	4.92	10.58	5.21	6.24	5.55	6.62	10.46	6.85
Score9	6.22	4.81	2.03	5.87	2.1	2.36	2.3	2.63	6.38	2.69
Score10	24.03	20.37	5.35	18.36	5.4	3.09	6.66	6.11	20.2	6.86
Chi-square value			14601	1115	13880	23556	10368	10100	1278	8465
Revolvers - Recession										
Closed	0	5.21	5.72	5.49	6.01	5.24	5.51	5.22	5.39	5.91
New	0.08	0	0	0.01	0.01	0.01	0.02	0.01	0.02	0.02
Bad1	0	5.33	5.49	8.15	6.66	5.95	4.83	6.41	7.14	5.95
Bad2	0	1.89	2.03	2.94	2.46	2.18	1.8	2.35	2.62	2.22
3+Cycle	0.22	0.05	0.05	0.07	0.07	0.06	0.05	0.07	0.08	0.06
Score1	0.36	0.31	0.33	0.45	0.44	0.4	0.32	0.41	0.51	0.39
Score2	10.72	8.27	5.45	7.33	7.13	6.28	5.44	6.72	8.17	6.53
Score3	50.04	42.97	29.65	35.18	34.35	32.51	30.87	34.35	36.93	34.42
Score4	22.42	22.09	30.12	28.39	28.1	31.31	30.55	29.98	27.14	30.06
Score5	10.79	9.78	10.33	7.42	7.66	9.52	9.87	8.06	6.84	7.88
Score6	1.98	1.48	2.34	1.48	1.56	2.02	2.17	1.67	1.3	1.61
Score7	1.2	0.98	1.64	0.95	1.01	1.36	1.52	1.12	0.88	1.12
Score8	1.7	1.07	3.07	1.25	1.47	1.91	3.08	1.77	1.2	1.95
Score9	0.47	0.53	1.27	0.45	0.65	0.62	1.24	0.68	0.41	0.71
Score10	0.02	0.01	2.52	0.44	2.42	0.64	2.73	1.17	1.37	1.17
Chi-square value			2614	1060	1319	1460	2428	1221	974	1263

Table 6.8: Behavioural score state distribution - Transactors and Revolvers

ces. One weakness of these two models, however, is the estimation of default probability. They over-estimate the number of default cases during Recession and under-estimate the number of default cases during Expansion. Nevertheless, the overall performance of the CPI model and Model A are very satisfactory.

6.4.5 Estimates for the regression analysis

We use regression analysis to determine the relationship between profit and macroeconomic variables, results are presented in the Appendix C. Here we only present the results in Table 6.9 and 6.10, with respect to Limit1 and Limit 10, for illustration.

One observation is that the impact of macroeconomic measurements on the profit function is more consistent for Transactors than for Revolvers. For example, if there is a unit change of GDP, Transactors tend to slightly increase (all with positive regressors (β) and referring to significant parameters only) their credit card usage, whereas Revolvers with different behavioural score have different consumption patterns (borrowing either more or less). One may say that it is hard to convince a 'Good' customer to borrow even if the economy is doing well.

Notice that the regression parameters of Revolvers with Score10 are different from those in the lower behavioural group. This can be explained by the fact that these accounts do not have a very strong financial foundation. For example, say the time we extracted these samples is t. They may be able to pay off their full balance in $t-6, t-5, \ldots, t-2$, and therefore the scoring system generates a high score to these "'good"' borrowers. When these borrowers are not able to repay their balance in $t-1$, these borrowers are classified as Revolvers according to our definition (which is presented at the beginning of this Chapter). However, as these borrowers make full payment during most of the observation period (which is used to extract the performance data to calculate the behavioural score),

		Parameter α					Parameter β				
		CPI	GDP	Int	Sto	Une	CPI	GDP	Int	Sto	Une
		Transactors									
Limit1	Inactive	-9.358**	-14.8223**	-11.5503**	-10.827**	-10.5777**	15.914**	2.9951**	16.1011**	17.3277**	-21.5767**
	Score2	214.6951**	208.129**	205.2988**	209.3069**	206.5986**	75.089	-0.793	523.0692	-157.812	N
	Score3	24.8458**	24.9604**	22.1858**	24.956**	24.4365**	4.8301	-0.4366	184.3264**	-52.0418	-40.6215
	Score4	-4.8397**	-6.0348**	-7.5699**	-6.3205**	-6.1334**	17.8509**	-0.1778	43.8172**	2.4805	1.6774
	Score5	-1.9213**	-6.1493**	-2.7885**	-2.4709**	-3.1029**	11.4855**	2.3231**	8.7718	-19.6276	-28.841**
	Score6	2.9041**	-7.5404**	-0.4239	2.171	0.6502	45.5707**	6.3531**	53.3613**	-135.123**	-13.6567**
	Score7	-1.7064	-23.0082**	-23.0082**	-4.0648**	-5.1249**	53.8285**	12.6105**	12.6105**	81.0685**	-20.4707
	Score8	-11.9178**	-20.6051**	-14.8709**	-11.9918**	-12.7439**	24.1727**	6.211**	40.1437**	13.4285	-69.9504**
	Score9	-11.1518**	-25.8332**	-13.9263**	-12.2655**	-13.4582**	37.9413**	8.6816**	49.481**	28.7733**	-40.4746**
	Score10	-14.6815**	-21.2082**	-14.8309**	-13.8328**	-14.6263**	16.9856**	4.7531**	16.2516**	106.3518**	-48.9259**
Limit10	Inactive	-4.2438**	-10.1484**	-6.9549**	-6.0087**	-5.6751**	18.3828**	3.1775**	21.1403**	11.6257	-24.9206**
	Score2	1282.258	301.6541	1310.976	1534.14	1176.502	598.1707	657.951	-28897.1	-24816.7	-24.6766**
	Score3	353.145**	302.3895**	304.9737**	350.7289**	357.8236**	-74.8055	41.8075	2088.507**	499.1023	-4394.49
	Score4	161.8748**	118.7261**	122.5792**	146.4554**	140.8982**	291.9732**	14.6326**	506.351**	-698.417**	-9.8448
	Score5	125.998**	82.3991**	105.4503**	107.1216**	104.1268**	200.7662**	17.4211**	216.5456**	105.712	-557.864**
	Score6	72.6411**	53.1556**	61.0598**	65.3069**	61.7984**	117.3595**	7.4978	278.0578**	-183.169	-96.5053**
	Score7	112.3033**	114.4006**	114.4006**	117.3629**	118.6302**	-48.1817	0.9946	0.9946	-152.211	-116.473**
	Score8	95.4003**	45.5795**	81.7571**	83.6035**	90.7196**	111.0142**	30.8684**	157.9425**	92.5254	129.2794
	Score9	66.3793**	59.7475**	62.6707**	62.5675**	62.4455**	39.1742**	2.5632	58.5047**	115.5095**	-248.182**
	Score10	66.8314**	55.9455**	66.671**	69.5453**	66.5147**	50.5256**	8.9331**	49.8775**	297.0855**	-38.4349**
		Revolvers									
Limit1	3+Cycle	-589.476**	-652.867**	-678.858**	N	-621.158**	540.0183**	12.1933	1935.966**	N	-570.256
	Inactive	132.1707**	284.7269**	196.0249**	157.0641**	145.8868**	-410.845**	-100.204**	-459.434**	-1038.64**	N
	Score1	-726.414**	-625.223**	-705.839**	-723.087**	-705.997**	-119.812	-73.1968**	357.6259	2335.848**	578.0048**
	Score2	210.5769**	168.7747**	204.5245**	202.977**	220.3502**	47.0052	29.8137**	18.5524	230.1352**	-80.5748
	Score3	186.1071**	165.388**	181.5737**	181.3783**	182.2409**	51.2494**	12.5236**	51.0053**	-55.781	-426.789**
	Score4	79.6252**	100.2156**	81.7486**	83.1569**	80.5202**	-11.5397	-14.0637**	-77.2417**	-578.707**	-39.1395**
	Score5	35.9541**	36.0818**	35.0729**	37.2323**	36.6675**	20.1712	-0.3082	10.5024	-216.928**	112.4593**
	Score6	36.7358**	45.2547**	37.0719**	37.2412**	36.942**	-5.0708	-5.8936	-5.339	-176.594	35.7146**
	Score7	38.7511**	43.0125**	39.4874**	37.1168**	37.7011**	-34.1999	-3.5275	-30.6547	70.6016	21.5649
	Score8	28.4333**	48.2992**	39.309**	21.422**	25.6156**	-96.5318**	-16.416**	-122.897**	6.2876	9.2566
	Score9	53.1967**	2.4119	80.0707**	45.1257**	43.4504**	-115.319**	27.0243**	-214.098**	93.5587	72.2372**
	Score10	15.5305**	-98.1335**	26.5883**	15.4645**	12.5569**	1.6361	75.6476**	-55.1338**	40.1743	-34.5416
Limit10	3+Cycle	-240.145**	-259.441**	-253.987**	N	-255.299**	105.5777**	1.2171	925.3441	N	-215.179**
	Inactive	73.6662	-74.1425	-55.7597	28.0795	29.0486	613.2376	70.6857	1029.174**	-358.868	-121.131
	Score1	-9933.37**	-8476.72**	-9229.76**	-9686.91**	-9416.96**	-6023.89	-776.353	-6874.85	34086.14	-598.258
	Score2	2208.515**	2833.378**	2569.097**	2474.082**	2425.416**	-2254.66**	-281.603	-10402.3**	225.6363	7252.259
	Score3	1796.247**	1567.981**	1745.413**	1772.047**	1753.169**	559.0514**	138.9789**	846.7927**	-668.273	2337.611
	Score4	1292.784**	1022.114**	1146.718**	1314.451**	1224.759**	1819.893**	170.5733**	2311.776**	-2182.06**	-1170.59**
	Score5	473.3263**	433.0826**	396.6362**	457.5807**	442.5904**	656.3879**	11.1738	1228.398**	-719.708	-1552.14**
	Score6	394.5633**	267.3071**	352.9707**	376.381**	380.0842**	320.2606**	83.797**	594.369**	1498.907**	-250.546**
	Score7	294.1524**	447.209**	318.5244**	304.8226**	316.841**	-491.539**	-88.3283**	-188.986	1830.378**	-595.896**
	Score8	229.8247**	355.0325**	281.1884**	214.5434**	232.498**	-446.4**	-89.6367**	-565.364**	-48.3109	337.8152**
	Score9	288.0819**	259.5032**	347.0945**	285.0907**	274.9195**	-210.36**	15.6453	-590.396**	-146.499	428.0257**
	Score10	153.4814**	-129.938**	197.7582**	157.1834**	155.2927**	88.6204	190.1618**	-224.685**	810.3713**	-249.969**

"‘**"’ indicates the parameter is significant at 95% level.

"‘N"’ represents the cumulative logistic regression cannot find any significant explanatory macroeconomic variable and therefore we did not build up the corresponding regression model.

Table 6.9: Summary of regression estimates for Transactors and Revolvers

		Model A				Model B		
		Intercept	CPI	GDP	Sto	Intercept	Sto	Une
					Transactors			
Limit1	Inactive	-11.4582**	12.2693**	1.2734**	11.1029**	-10.573**	-0.8916	-21.6581**
	Score2	238.3766**	112.6692	-16.129	64.7147	208.4051**	-117.793	N
	Score3	27.9148**	6.6524	-1.911	-46.0476	24.9998**	-52.8317	-27.4848
	Score4	-4.515**	19.4735**	-0.4032	39.8692**	-5.9586**	-16.9758	2.0733
	Score5	-4.1082**	8.1843	1.402	-18.1955	-2.9621**	-16.1005	-29.7001**
	Score6	0.9893	39.4744**	1.7204	-74.9322	1.8286	-136.317**	-13.2394**
	Score7	-13.2194**	38.3787**	7.1712**	-6.9909	-5.028**	-46.282	-20.9284
	Score8	-15.934**	18.8902**	2.6268**	36.2742	-12.5477**	-24.5137	-74.4376**
	Score9	-18.204**	28.7944**	4.3406**	-7.3982	-13.4724**	-15.6905	-41.7794**
	Score10	-16.8842**	6.8419	1.6422	59.0312**	-14.4869**	66.758**	-50.6974**
Limit10	Inactive	-5.9539**	15.4533**	1.0608	6.2169	-5.6286**	-7.0448	-28.8285**
	Score2	-141.07	-3089.53	1014.917	-30780.8	1498.545	-22923.6	-25.2851**
	Score3	233.7135**	-172.097	79.4694	481.2159	350.4425**	499.435	-1253.47
	Score4	146.3401**	282.2437**	12.0842**	-110.884	152.3616**	-953.459**	-10.1771
	Score5	133.0962**	212.1593**	-4.7122	92.797	102.7878**	129.3594	-589.847**
	Score6	81.8955**	130.2314**	-6.065	18.0567	63.3873**	-165.545	-99.4686**
	Score7	100.658**	-65.5663	7.6289	-131.817	119.4381**	-93.7268	-114.741**
	Score8	70.3278**	87.1549**	16.8302**	236.953	92.3652**	-184.583	125.2321
	Score9	65.2711**	40.4923**	-0.0989	119.3744**	61.2412**	112.7179**	-263.467**
	Score10	70.4766**	37.9339	-2.169	201.4575**	66.968**	203.7054**	-36.4956**
					Revolvers			
Limit1	3+Cycle	-608.665**	541.825**	13.3096	N	N	N	N
	Inactive	167.8658**	-422.192**	-13.556	-1026.07**	145.4118**	-399.854	534.1021**
	Score1	-618.367**	95.5701	-78.853	2258.543**	-730.538**	2481.055**	N
	Score2	164.095**	0.1869	32.0385**	280.561**	219.5362**	77.8121	153.8681
	Score3	175.4652**	35.3775**	7.2632**	-5.3627	183.1084**	-120.418	-419.947**
	Score4	118.9694**	71.8018**	-21.8699**	-624.427**	82.5486**	-433.6**	-49.5764**
	Score5	47.5622**	44.2644**	-6.1175	-264.945**	37.6115**	-181.116**	74.2642**
	Score6	52.8225**	31.8727	-10.7361	-154.47	37.2402**	-161.6	21.9561
	Score7	26.1025**	-76.1248**	8.5149	251.5856	38.0187**	148.1691	4.78
	Score8	21.1587	-135.932**	4.6599	383.2693**	25.4402**	257.2657**	24.9192
	Score9	4.6298	-138.492**	30.6452**	281.0833**	43.7447**	74.662	98.4003**
	Score10	-97.6155**	-16.0179	75.9123**	117.7843	13.1861**	-149.392	-24.2986
Limit10	3+Cycle	-242.582**	139.4636**	5.3337	N	N	N	N
	Inactive	-7.5464	308.5877	30.6448	174.8047	32.1094	-181.775	-234.509**
	Score1	-11856.8**	-9271.27	1099.105	39992.58	-9700.95**	37496.12	-590.779
	Score2	2095.061**	-2566.44**	85.8759	-2384.6	2415.72**	662.0061	8976.736
	Score3	1691.844**	448.9804**	65.4526	516.8665	1765.585**	-1212.85	2370.186
	Score4	1549.22**	2200.589**	-136.945**	-4496.65**	1247.104**	-4110.39**	-1212.58**
	Score5	688.9073**	946.6731**	-127.631**	-1083.39**	451.1442**	-940.449**	-1798.14**
	Score6	414.4702**	371.947**	-22.0442	1598.199**	374.2596**	810.889	-293.009**
	Score7	280.6083**	-528.01**	1.3939	2163.552**	305.5247**	3086.183**	-543.039**
	Score8	227.0274**	-500.162**	-1.2231	1129.02**	230.2443**	1152.272**	619.8885**
	Score9	256.4719**	-217.257**	19.8249	0.0047	274.7839**	-366.065	533.0457**
	Score10	-133.609**	6.5101	191.2582**	822.6668	155.893**	-444.124	-289.206**

"‘**"’ indicates the parameter is significant at 95% level.

"‘N"’ represents the cumulative logistic regression cannot find any significant explanatory macroeconomic variable and therefore we did not build up the corresponding regression model.

Table 6.10: Summary of regression estimates for Transactors and Revolvers-Model A and Model B

the scoring system still classifies these borrowers as "'Good"' customers and thus gives a high score to these Revolvers. Indeed, for all the analyses presented in this section, Revolvers in a state with high behavioural score react uniquely.

6.4.6 Optimal policy

We use the real macroeconomic variables during the testing sample period to generate the profit value and the transition matrices of that period. Here we report the optimal policy[1] of accounts having credit limit band 1 and band 6 in Table 6.11. The rest of the results can be found in Appendix D

In summary, the optimal policy of Transactors is very similar to those presented in Chapter 5. There are some differences on the Score2 accounts' policies whereas the model suggests increase most accounts' credit limit to the highest level (i.e. Limit 10). Moreover, the optimal policy for expansion and recession are identical, except for Revolvers in the state with Score 2 and limit 1. The optimal policy in the CPI model for these borrowers during expansion is to increase the credit limit to Limit 10, however, the optimal policy during recession is Limit 8. Similarly, the optimal policy in the CPI model for Revolvers with Score 2 and Limit 6 is to increase their credit limit to Limit 8 during recession whereas the optimal policy for these borrowers during expansion is Limit 10.

The optimal policy of accounts in arrears (3+Cycle) and having the lowest behavioural score (Score1) is to keep the current credit limit. This is what one would expect as these accounts have a high default risk. For Inactive accounts, the proposed optimal policy is

[1]We use backward iteration to generate the optimal policies during Expansion (May, 2006 to April, 2007) and Recession (Jan, 2003 to Dec, 2003). Thus there are different optimal policies every month. We present the optimal policy of the first month of that period. That is if the lender predicts that the economy will experience expansion (or recession) in the coming year, what is the optimal policy in the first month of this period?

	Transactor							Unconditional	Revolver							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB		CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Expansion - Credit Limit 1																
3+Cycle					-				1	1	1	1	1	1	1	1
Inactive/New	10	10	10	10	10	10	10	8	9	9	9	9	9	9	9	1
Score1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	2
Score2	7	7	7	7	7	7	7	8	10	8	8	8	8	8	8	2
Score3	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score4	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score5	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score6	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score7	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score8	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score9	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score10	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Recession - Credit Limit 1																
3+Cycle					-				1	1	1	1	1	1	1	1
Inactive/New	10	10	10	10	10	10	10	8	9	9	9	9	9	9	9	1
Score1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	2
Score2	7	7	7	7	7	7	7	8	8	8	8	8	8	8	8	2
Score3	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score4	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score5	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score6	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score7	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score8	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score9	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score10	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Expansion - Credit Limit 6																
3+Cycle					-				8	8	8	8	8	8	8	10
New	10	10	10	10	10	10	10	8	9	9	9	9	9	9	9	6
Score1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score2	7	7	7	7	7	7	7	8	8	8	8	8	8	8	8	10
Score3	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Recession - Credit Limit 6																
3+Cycle					-				8	8	8	8	8	8	8	10
New	10	10	10	10	10	10	10	8	9	9	9	9	9	9	9	6
Score1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score2	7	7	7	7	7	7	7	8	10	8	8	8	8	8	8	10
Score3	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10

Table 6.11: Summary of optimal policy for Transactors and Revolvers

to increase the credit limit in the hope these customers will start to use their credit cards. Transactors are safe portfolios and thus the model suggests increasing their credit limit to the highest level. The New Revolvers have a volatile behavioural score migration. The model suggests increasing their credit limit to Limit9 rather than the highest credit limit level.

One observation concerns the optimal policy for accounts with a behavioural score state 2. The model suggests increasing Transactors' credit limit in such a state with Limit 1 and behavioural scoreband 2 to Limit 7. However, the optimal policy for Revolvers in a state with Limit 1 and behavioural scoreband 2 is Limit 8. Thus the credit limited offered to Revolvers is higher than those of Transactors.

The optimal policy of accounts having Score3 or above is to increase their credit limit to the highest band for both Transactors and Revolvers. The same observation was found in the previous chapter.

	Transactor							Unconditional	Revolver							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB		CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Expansion - Credit Limit 1																
3+Cycle					-				1	1	1	1	1	1	1	1
Inactive	10	10	10	10	10	10	10	8	9	9	9	9	9	9	9	1
Score1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	2
Score2	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	2
Score3	4	4	4	4	4	4	4	8	10	8	10	8	8	10	8	2
Score4	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score5	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score6	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score7	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score8	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score9	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Score10	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	2
Recession - Credit Limit 1																
3+Cycle					-				1	1	1	1	1	1	1	1
Inactive	7	7	7	7	7	7	7	8	3	3	3	3	3	3	3	1
Score1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	2
Score2	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	2
Score3	3	3	3	3	3	3	3	8	1	1	1	1	1	1	1	2
Score4	10	10	10	10	10	10	10	8	4	4	4	4	4	4	4	2
Score5	10	10	10	10	10	10	10	8	7	7	7	7	7	4	4	2
Score6	10	10	10	10	10	10	10	8	7	7	7	7	7	7	7	2
Score7	10	10	10	10	10	10	10	8	6	6	6	6	6	6	7	2
Score8	10	10	10	10	10	10	10	8	7	7	7	7	7	7	7	2
Score9	10	10	10	10	10	10	10	8	6	5	6	7	6	7	6	2
Score10	10	10	10	10	10	10	10	8	5	5	7	5	5	5	5	2
Expansion - Credit Limit 6																
3+Cycle					-				6	6	6	6	6	6	6	10
Inactive	10	10	10	10	10	10	10	8	9	9	9	9	9	9	9	6
Score1	6	6	6	6	6	6	6	8	6	6	6	6	6	6	6	10
Score2	6	6	6	6	6	6	6	8	6	6	6	6	6	6	6	10
Score3	7	7	7	7	7	7	7	8	10	8	10	8	8	10	8	10
Score4	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	8	10	10	10	10	10	10	10	10
Recession - Credit Limit 6																
3+Cycle					-				6	6	6	6	6	6	6	10
Inactive	7	7	7	7	7	7	7	8	6	6	6	6	6	6	6	6
Score1	6	6	6	6	6	6	6	8	6	6	6	6	6	6	6	10
Score2	6	6	6	6	6	6	6	8	6	6	6	6	6	6	6	10
Score3	6	6	6	6	6	6	6	8	6	6	6	6	6	6	6	10
Score4	10	10	10	10	10	10	10	8	6	6	6	6	6	6	6	10
Score5	10	10	10	10	10	10	10	8	7	7	7	7	7	7	7	10
Score6	10	10	10	10	10	10	10	8	7	7	7	7	7	7	7	10
Score7	10	10	10	10	10	10	10	8	6	6	6	6	6	6	7	10
Score8	10	10	10	10	10	10	10	8	7	7	7	7	7	7	7	10
Score9	10	10	10	10	10	10	10	8	6	7	6	7	6	7	6	10
Score10	10	10	10	10	10	10	10	8	7	6	7	7	7	6	7	10

Table 6.12: Summary of optimal policy for Transactors and Revolvers (loss equals to the credit limit)

To understand the impact of the loss value on the optimal policy, we changed the profit value for the default states (i.e. Bad1 and Bad2). The new default values equal the credit limit of the credit card, i.e. the loss of a default account with credit limit HK$1000 is -HK$1000. The results are presented in Table 6.12.

The optimal policies with respect to Transactors are almost the same during recession and expansion. The exception is Transactors in state Score 3. The MDP model offers a more conservative policy during recession. For example, the optimal policy for Transactors with credit limit 1 and behavioural score 3 is limit 4 during expansion whereas the optimal policy for these borrowers during recession is limit 3 instead.

On the other hand, the optimal policies for Revolvers during expansion and recession exhibit huge differences. The optimal policy for Revolvers during Expansion are almost identical to those of Transactors, whereas, during the bad times, the model offers a very low credit limit for Revolvers. For example, in the CPI model, the optimal policy for Revolvers in a state with credit limit 1 and behavioural score 10 during recession is to increase the credit limit to limit 5.

6.5 Conclusions

The model built in this chapter has shown that segmenting the population into Transactors and Revolvers yields more insight about the behavioural score migration pattern and the profitability pattern.

The results presented in Table 6.12, i.e. changing the loss to the credit limit, show two issues about the use of MDP model in adjusting the credit limit of credit card borrowers:

1. Splitting borrowers into Transactors and Revolvers allows the model to generate different policies for two very different borrower types.
2. Although both probability of default and loss function changes the value of expected loss (since expected loss = probability of default $\times$ loss function), the later is the key value in estimating the expected loss. Thus it has a major effect on the credit limit policy which is derived from the model.

Chapter 7

Conclusion

The aim of this thesis is to develop and to further research the application of the Markov Decision Process model to a credit card pricing model and to demonstrate both the method and the process of incorporating macroeconomic measurements into the model. This concluding chapter summarizes the findings and the contributions of this research and discusses possible future areas of research for the application.

7.1 Summary

7.1.1 Building a model for making sequential credit limit decisions

This thesis identifies the importance of replacing the conventional static decision model with a MDP model in pricing the credit limit of credit cards. The MDP model is able to derive a sequence of policies to maximize profit, leading to a competitive alternative to the current model. The second advantage of using the MDP model is that it can incorporate other mathematical models. In Chapter 5, we demonstrated how one can use logistic regression models to estimate the interaction between the economy and credit card accounts.

7.1.2 Using behavioural score as the key parameter

Behavioural score has been used by lenders for at least the past twenty years in monitoring the default risk of current credit customers. This score is tailor-made by every lender, and characterizes credit behaviour while estimating the possible default risk of every individual customer. This study demonstrated using behavioural score to simplify the state space of an MDP model. Using behavioural score as a key parameter not only simplifies the model development, but also dynamically links the default risk with a profit model.

7.1.3 Lack of samples: low default portfolios

Researchers in the credit card industry usually have no problem in overall sampling since the numbers of credit card holders, and therefore the quantities of available samples, are enormous. This, though, is not always true of the default cases. It is possible for lenders to have no default observations during the sampling period, particularly for credit card accounts with good credit history. In Chapter 4, we used the method proposed by Pluto and Tasche (2006) to adjust the default probabilities of some credit card accounts. The results show that the method does not change the optimal policy. Nevertheless, it is still worth incorporating such adjustments in default probabilities, as it guarantees the connectedness of the states and prevents formulating a structural zero MDP model.

7.1.4 Segmentation by repayment behaviour

In common with many other authors, we segment the population by looking at the repayment behaviour of the credit card users. *Transactors* and *Revolvers* have significantly different behavioural score migration patterns as shown in Chapter 6. The segmentation enhances the performance of MDP models in adjusting the credit card limit of current credit card customers, since it provides a better estimation on the path of behavioural score migration.

7.1.5 Credit card accounts and the economy

Results in this thesis show there are significant interactions between the economy and the riskiness of credit card accounts. Our model is able to take the economy into consideration when one is making a credit limit decision. We also explored the adequacy of using different macroeconomic variables, the reaction time, the distribution of account types under different macroeconomic conditions, and the possibility of using more than one macroeconomic variable.

7.2 Contributions to knowledge

This thesis has developed and has explored the context of using a MDP model for building a credit card pricing model. All the stages of the model development: parameter selection, coarse-classifying, choice of order, dealing with low default portfolios, movements of inactive, closed and active accounts, the interaction with logistic regression model, the impact of the economy on credit card holders's behaviour in two different countries, account segmentation, assessment of model performance; can be accomplished using the methods proposed in this thesis. This is the first research to provide such an in-depth study in the context. This study has demonstrated the impact of economy on the credit card accounts, and hence provides empirical evidence to encourage lenders to use macroeconomic measurements for pricing models.

7.3 Research limitation

Behaviour score is the key parameter in this thesis, although we do not have knowledge regarding how the lender generates this score. The transition matrix in our dataset has preserved a lot of the lender's operation policy. If the information were available, we could adjust the definition of the score bands accordingly.

7.4 Suggestions for future research

7.4.1 Estimation of the default value

Further improvement on the model performance can be done by looking at the default amount of different Bad accounts. This default amount is an important component both in our model and in reality. In our MDP model, the expected loss value (=default probability $\times$ the loss given default [1]) drives the optimal policy. We have looked at the possibility of adjusting the expected loss when in default. The results presented in Table 5.13 and

[1] This loss given default (called LD) is the loss value, i.e. the profit at default such as for example $r(l, D)$ and $r(l, D, \mathbf{M})$ as defined in Chapters 3 and 5 respectively. This value is thus different from the loss given default (LGD) defined in the capital formula of the new Basel Accord where LGD is a fraction of credit exposure that will not be recovered in the event of default. That is if the total borrowing of a credit card holder is B, then the two variables can be related as follow: $LGD = \frac{LD}{B}$.

6.12 in Chapters 5 and 6 respectively show the optimal policies are very sensitive to the expected loss given default. Questions that have not been studied are: how sensitive is the MDP model to this expected loss given default? Should one adjust the probability of default or the loss given default? Up to now there is no literature that considers the expected loss of credit card products. This is despite the fact that such work would be of practical use for lenders since LGD is a critical component in the new Basel accord's capital formula.

7.4.2 Simulating the economy

We used the out-of-sample period macroeconomic variables to test the performance of the MDP model in Chapter 5. This solution is an optimal policy built on a finite-horizon MDP model conditional on the actual macroeconomic measurements observed. One can use simulation to estimate possible future macroeconomic measurements and put these into the MDP model.

This is how corporate credit risk models are often being validated in practice. Some credit rating agencies (Office of Compliance Inspections and Examinations, United States Securities and exchange commission, July 2008) use a Monte Carlo method to simulate a time series of macroeconomic variables. Using these macroeconomic variables, the credit rating agencies create a loss distribution to predict the loss given default or the probability of default. One can use the same approach to simulate $q_{t-1}(\mathbf{U}|\mathbf{M})$ in equation (5.1) and then use the simulated macroeconomic variables to predict the transition probabilities $p(i'|i, \mathbf{M})$ and the reward function $r(l, i, \mathbf{M})$. In this way the MDP model in (5.1) will generate a set of MDP policies that can be readily used by lenders in the future.

7.4.3 Measurement of the model's performance

In Section 5.3.4 of Chapter 5 we used a chi-square test to compare the predicted and actual transition probabilities in order to check whether using macroeconomic variables to estimate the transition probabilities lead to a reasonable model. The observation window we used was twelve months, but is this an optimal period? Should we instead look at a shorter period, say six months, since consumers are sensitive to the economy? Besides these open questions that have not been explored, it is possible that there are other assessments can be carried out by a researcher to understand the MDP model's performance. One possible approach is to compare the difference between the following:

1. actual profit value of actual policy
2. model profit value of actual policy
3. model profit value of optimal policy
4. actual profit value of optimal policy

Say we extract N random samples in the out-of-time sampling period, then we can calculate or estimate the actual profit value. Then (1.) and (2.) can be used to compare the actual profit and the model profit so as to examine the performance of the regression model. Comparing (2.) and (4.) or (1.) and (4.) can assess the difference in using the actual policy and optimal policy. Also, one can determine which of these gives the most applicable policy.

7.4.4 Using Bayesian inference to estimate the default probability

The transition probabilities in this study use maximum likelihood estimates (MLE), i.e. taking an average of the history data. Researchers (Kadam and Lenk, 2008; Stefanescu et al., 2007) suggest that using the maximum likelihood method to estimate the sparse default probability entries results in large estimation errors, since MLE is suitable for a frequentist estimation framework, i.e. for a dataset consisting of numerous observations. By contrast, the Bayesian approach gives a greater mathematical basis for estimating the default probability. The idea is presented as follows. Assume the probability of default given a credit card holder in a state s (which can depend on behavioural score, credit limit, or other characteristics of the credit card holder) is $Pr(D|s)$. Then, according to Bayes' rule, this probability can be written as:

$$Pr(D|s) = \frac{Pr(s|D)Pr(D)}{Pr(s)} \tag{7.1}$$

where $Pr(D)$ is the unconditional prior probability, $Pr(s|D)$ is the posterior probability that the credit card holder is in a state s given s/he defaults, and $Pr(S)$ is called the marginal probability of S. If we have a dataset with the credit card holders' histories, we can estimate the posterior probability. Say there are N_D default cases and $N_D(s)$ of them are in default with a state s, then the posterior probability $Pr(s|D)$ is equal to $\frac{N_D(s)}{N_D}$. The marginal probability is calculated as the sum of the product of all probabilities of any state s_i and corresponding conditional probabilities $Pr(s_i)$, i.e. $Pr(s) = \sum_i Pr(s|s_i)Pr(s_i)$. One can then set the prior probability $Pr(D)$ to a certain value and calculate $Pr(D|s)$ in the first iteration. In the second iteration, one uses the computed posterior equation $Pr(s|D)$ as the prior posterior and repeats the calculation process. There is mathematical evidence to show one can find a posterior probability close to the real conditional probability after several iterations. However, researchers warn that the value of this posterior probability is sensitive to the prior probability.

Therefore, possible areas to explore in using Bayesian inference to estimate the default probability are: (1) The sensitivity of the default probabilities to the prior probabilities. (2) The difference in using the maximum likelihood method and the Bayesian method to estimate the default probabilities. (3) How to incorporate macroeconomic variables in estimating the default probabilities with Bayesian Inference.

Appendix A

UK Data

This Appendix provides some information about the UK data samples, including, the field specification, frequency distribution, coarse-classifying and chi-square goodnes-of-fit test results. All of these are excluded from the main thesis contents and are provided here as additional information for readers.

The file specifiication provided by the UK data provider.

I. Fields definition

Name	Description	Contents	Format	
prodid	MasterKey for linkage		n	8
advlim	Advised Credit Limit	Pounds	n	7
act_bal	Current balance	Pounds.Pence (eg \pounds1 = 1.00)	n	11.2
coff_code	Charge-off Reason Code		c	2
days_del	Days delinquent	Numeric length 3 (eg 1 = 001)	n	3
ext_stat	External status	ext_stat	c	1
accopen	Open date	CCYYMMDD	c	8
sortcode	The bank sortcode of the cardholder's checking account-holding branch unless an alternative sort code is used for direct debit payments		c	6
cusbehsc	Visa Behavioural score	cusbehsc	c	3
prodtype	Product type		c	3
attrsc	Attrition score		c	3

II. Further details

External Status (ext_stat)

Z	Charge Off
B	Blocked / Bankrupt
L	Lost
U	Stolen
U	5 + Cycles Delq
U	Never Active
U	Inactive
U	Transferred Account
A	Auth Prohibited
C	Closed
E	Revoked
F	Frozen
I	Interest Prohibited
*	Designated Scores (the system does not calculate a behaviour score but uses the designated score during processing)
**	Fixed Scores (the system does not calculate a behaviour score but assigns a fixed score to be used in processing)
***	Optional Scores (the behaviour score is added to the exception score to calculate an optional score)

Coff_Code (following a 'Z' charge off)

00	Awaiting insolvency details
01-05	In house debt collectors (pre 1995)
07	Weekly fixed payers (or cash payers)
74	Fixed payers using Baines \& Ernst
75	Fixed payers using Gregory Pennington
76	All other fixed payers using external bureaux
88	Stolen charge off
89	Bankrupt charge off
90	Deceased
93	Being referred for or waiting abandon
94	Account outplaced to debt collector
95	Account outplaced to debt collector

Prodtype

ART	ART Cards
GCC	Gold Credit Card
GMC	Gold Mastercard
HN	Harvey Nichols
NT	National Trust
MC	Master Card
Pl	Platinum
Pr	Premier
V	Visa

Behavioural Score (cusbehsc)

NB Behaviour score forecasts the probability of accounts to become, within the next six months, 3 or more cycles delinquent (excluding fraudulent losses), or bankruptcy, where the customer's outstanding debt is \pounds10 or greater.

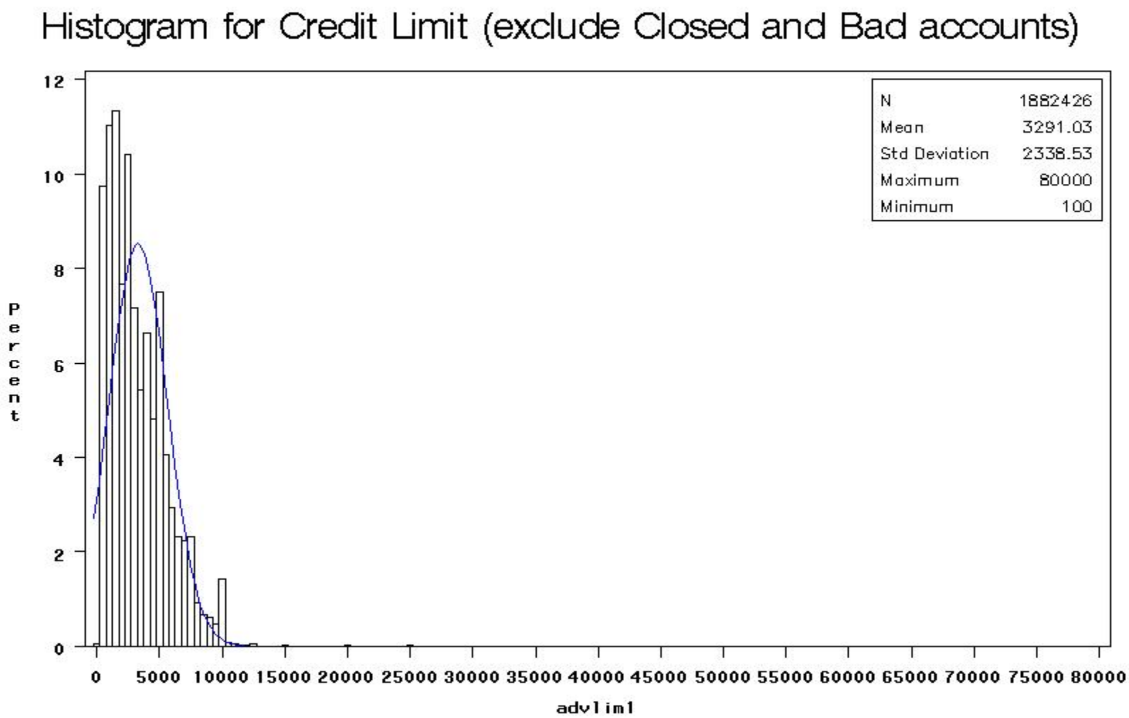
Score	Odds (/1)
780	3840
765	1920
750	960
735	480
720	240
705	120
690	60
675	30
660	15
645	7.5
630	3.75
615	1.87
600	0.93
585	0.46
570	0.23

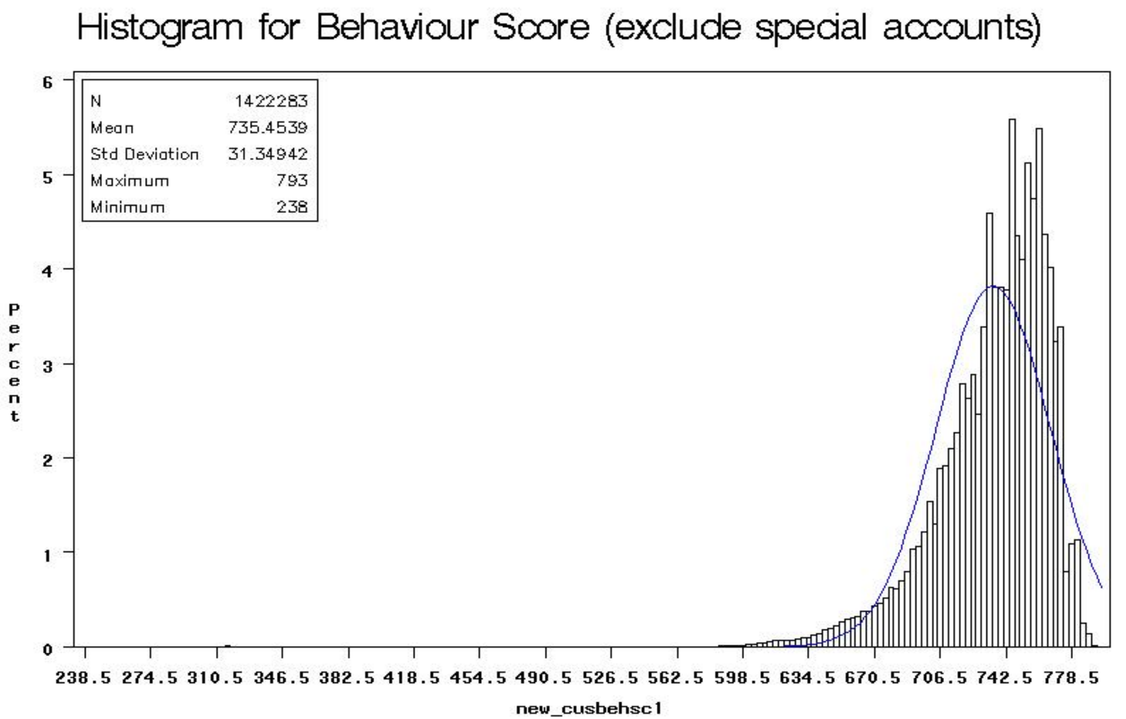
Exclusions to Behaviour Scores

0	New account which has not yet cycled, so score not yet generated
1	Deceased
2	Not used
3	Not used
4	Bankrupt
5	Written Off
6	Not used
7	Involuntarily closed more than 6 months ago and balance = zero
8	Voluntarily closed more than 6 months ago and balance = zero
9	Attrition Score exclusion - Involuntarily closed more than 6 months ago and balance = zero
10	Attrition Score exclusion - Voluntarily closed more than 6 months ago and balance = zero
11	Never Active
12	Inactive 12+ months
13	Recently Reactivated in the last 2 months
14	Recently Acquired (less than 3 cycles on the books)

NB Behaviour score forecasts the probability of accounts to become, within the next six months, 3 or more cycles delinquent (excluding fraudulent losses), or bankruptcy, where the customer's outstanding debt is \pounds10 or greater.

	Score	Odds
Payment Projection	365	16
	350	8
	335	4
(These scores are applied if the account is under some stress, i.e. having these scores means the account is of worse condition than the rest). These scores predict the likelihood of payment.	320	2
	305	1
	290	0.5
	275	0.25
	260	0.125
	245	0.06
	230	0.03
	215	0.015
	200	0.0075


Histogram for Credit Limit (exclude Closed and Bad accounts)
N 1882426
Mean 3291.03
Std Deviation 2338.53
Maximum 80000
Minimum 100
Percent
advlim1


Histogram for Behaviour Score (exclude special accounts)
N 1422283
Mean 735.4539
Std Deviation 31.34942
Maximum 793
Minimum 238
Percent
new_cusbehsc1

Appendix B

UK Data - incorporating macroeconomic variables

B.1 Unconditional transition matrices

Score i	Score i' at $t+1$								Row
at t	Closed	Inactive	Bad	Risk	Score1	Score2	Score3	Score4	Count
Inactive	2.22	**96.03**	-	-	1	0.68	0.06	0.01	458817
Risk	5.09	-	19.79	**53.68**	21.44	-	-	-	3951
Score 1	0.76	0.37	0.08	0.49	**84.25**	11.73	2.2	0.13	379242
Score 2	0.69	1.68	0	-	12.15	**67.81**	15.69	1.98	323448
Score 3	0.78	1.46	-	-	2.77	15.27	**64.13**	15.59	322344
Score 4	0.71	0.2	0	-	0.84	1.85	17.27	**79.14**	307728

"‘-"’ represents there is no sample observation.

"‘0"’ represents the transition probability is less than 0.0005.

A bold value indicates the transition frequency is greater than 50% .

The transition probabilities of all absorbing states (Closed and Bad) are not shown in the table.

Table B.1: Unconditional transition matrix (in percentage) for the UK data

The unconditional behavioural score transition matrix is diagonally dominated. Note that accounts in the Risk behavioural score state preserve the highest default probabilities (= 19.79%). 96.03% Inactive accounts remain Inactive after one time period which is much higher than those (= 81.96%) of the HK dataset. The mobility of Score1 and Score4 accounts are low since around 80% of them remain in the same behavioural score state in the subsequency month.

Note that we use the account balance presented in Table 3.4 and $r = 0.02$ to estimate the unconditional account profit with result present in Table B.2.

Credit limit	Score at t (i)							
at t (l)	Close	Inactive	Bad	Risk	Score 1	Score 2	Score 3	Score 4
Limit 1	0	0	-563	8.66	6.32	1.78	1.3	1.04
Limit 2	0	0	-761	14.06	9.8	2.38	1.46	1.08
Limit 3	0	0	-983	17.52	11.4	2.9	1.78	1.46
Limit 4	0	0	-1658	29.02	16.44	4.92	2.78	2.48
Limit 5	0	0	-2234	42.68	24.58	10.44	4.7	3.88
Limit 6	0	0	-3047	57.82	29.94	13.84	7.02	5.64
Limit 7	0	0	-3605	60.96	34.9	16.6	9.34	7.22
Limit 8	0	0	-5722	109.6	63.62	43.74	22.12	14.62

The profit value of Closed and Inactive are assumed to be 0.

Table B.2: Profit value used in the UK macroeconomic model

B.2 Logistic regression estiamtes

B.3 optimal policy

	Model A (CPI, GDP, Stock)									
	Log(A/C)			Log(I/C)			Log(A/I)			-2Log(likelihood)
	CPI	GDP	Stock	CPI	GDP	Stock	CPI	GDP	Stock	
Inactive	1.527*	-2.9093*	-15.0743*	-5.8577*	-.1453*	-20.8934*	7.3846*	-2.7641*	5.8191*	209550
Risk	N	.4150	-6.0157**	N	N	N	N	N	N	1584
Others	-2.5268*	-.2913	4.7335*	-4.1679*	.3324*	3.7540*	1.6411	-..6237*	.9795**	254776

	Model B (Unemployment rate, Stock)						
	Log(A/C)		Log(I/C)		Log(A/I)		-2Log(likelihood)
	Stock	Unemployment	Stock	Unemployment	Stock	Unemployment	
Inactive	-26.7510*	-1.4272*	-26.4975*	2.6415*	-.2535	-4.0687*	215664
Risk	-6.5421**	-.6713	N	N	N	N	1587
Others	2.2037*	-9.8242*	2.3607*	-6.3339*	-.1570	-3.4903*	255204

"'*"' indicates the parameter is significant at 99% level.

The models are developed only if there is significant variable found in Table 5.14.

Table B.3: Summary of multinomial logistic model estimates of Model A and Model B for the UK dataset

Initial i	Model A				Model B		
	CPI β	GDP β	Stock β	-2Log(L)	Stock β	Unemployment β	-2Log(L)
Inactive	-1.0534**	-0.197**	-2.0944**	13310	-1.6245**	-1.2918**	13332
Risk	1.4063**	0.7397*	1.8988	7343	5.231*	9.7446*	7356
Score1	0.0738*	-0.3407*	-0.4804*	385384	-0.7727*	0.8082*	385771
Score2	-0.1822*	-0.2762*	-1.1101*	558446	-1.3893*	-2.8125*	558877
Score3	-0.236*	-0.1604*	-1.1193*	604509	-1.3517*	0.5744*	605040
Score4	-0.3473*	0.7012*	3.0011*	362294	3.7205*	-5.714*	364023

"'*"' indicates the parameter is significant at 99% level.

"'**"' indicates the parameter is significant at 95% level.

Table B.4: Summary of cumulative logistic model estimates - Model A and Model B for the UK dataset

	Limit 1								Limit 6							
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	Unconditional	CPI	GDP	Int	Sto	Une	ModelA	ModelB	Unconditional
Inactive	8	8	8	8	8	8	8	1	8	8	8	8	8	8	8	6
Risk	1	1	1	1	1	1	1	1	6	6	6	6	6	6	6	6
Score 1	8	8	8	8	8	8	8	1	8	8	8	8	8	8	8	6
Score 2	8	8	8	8	8	8	8	1	8	8	8	8	8	8	8	6
Score 3	8	8	8	8	8	8	8	1	8	8	8	8	8	8	8	6
Score 4	8	8	8	8	8	8	8	1	8	8	8	8	8	8	8	6

Table B.5: Summary of optimal policy for accounts - UK data

Appendix C

HK Data - Regression analysis estimates

		Parameter α					Parameter β				
		CPI	GDP	Int	Sto	Une	CPI	GDP	Int	Sto	Une
	3+Cycle	-589.476**	-652.867**	-678.858**	N	-621.158**	540.0183**	12.1933	1935.966**	N	-570.256
	Inactive	-7.314**	-8.6143**	-8.7991**	-8.102**	-8.1227**	9.7712**	0.3303	12.6303**	-14.6442**	**
	Score1	-721.331**	-603.633**	-697.424**	-715.883**	-696.929**	-155.335	-83.1265**	250.3812	2145.11**	-5.2385**
	Score2	210.9161**	171.3952**	204.3485**	203.1618**	218.9636**	49.0768	27.6404**	66.4266	208.0974	-62.8712
	Score3	155.957**	139.9364**	151.6328**	152.6542**	151.2403**	46.4859**	9.123**	25.9001	-209.489**	-411.143**
	Score4	34.0376**	33.1016**	29.8555**	32.6485**	29.742**	45.9363**	-2.3676**	-1.4438	-358.891**	8.1219
Limit1	Score5	9.1264**	5.3085**	6.6943**	8.489**	7.547**	28.28**	1.6021	37.6679**	-96.8838**	34.3265**
	Score6	9.4106**	-1.1335	5.5024**	8.4817**	6.9143**	42.5901**	6.0814**	49.826**	-180.874**	-5.4055
	Score7	4.0183**	-15.7759**	-0.3522	2.1165**	0.9024	54.7493**	11.8198**	65.5427**	82.061**	-11.0281
	Score8	-7.2939**	-16.2333**	-9.3383**	-6.757**	-7.8107**	26.5937**	6.5408**	38.9994**	-5.6362	-66.0161**
	Score9	-6.602**	-25.0574**	-9.7546**	-7.5314**	-9.0821**	42.1238**	11.1475**	54.824**	38.5941**	-37.106**
	Score10	-10.4682**	-18.4682**	-10.9743**	-9.2372**	-10.1097**	20.6459**	5.8946**	21.8557**	102.7364**	-57.5532**
	3+Cycle	-626.062**	-599.913**	-679.67**	N	-642.025**	351.0271	-44.6027	455.0281	N	-22.4374**
	Inactive	-5.0507**	-7.9122**	-7.3581**	-6.4536**	-6.2965**	13.2526**	1.1079	21.6564**	-10.8732	-1319.97**
	Score1	-1148.5**	-1043.2**	-1118.63**	-1136.19**	-1109.98**	-192.747	-67.6749	-111.288	1672.044**	-7.3596
	Score2	264.7964**	215.9097**	253.6813**	248.777**	283.6798**	72.6912	32.9342**	405.2293**	863.8926**	-144.695
	Score3	192.0291**	171.4079**	187.1244**	188.087**	186.959**	44.0972**	12.7606**	11.5623	-202.13**	-766.036**
	Score4	40.9139**	52.7938**	40.5213**	45.3432**	38.6813**	7.3377	-9.3941**	-127.425**	-565.121**	0.8099
Limit2	Score5	7.953**	0.0887	4.8382**	5.832**	4.993**	30.7415**	3.5491**	28.0398**	-79.135**	118.409**
	Score6	4.356**	-4.8428**	-0.1898	1.2384	0.0138	36.5942**	3.8789**	46.8611**	-108.295**	-6.6612
	Score7	-1.0722	-8.316**	-3.5105**	-3.4044**	-3.4831**	29.2822**	3.2203**	46.2254**	6.8096	-6.272
	Score8	-6.4254**	-15.4633**	-8.7241**	-7.843**	-7.5174**	18.4352**	5.9746**	39.9121**	17.3056	-15.3321**
	Score9	-6.6017**	-17.7164**	-9.2646**	-8.2178**	-9.2628**	35.4062**	5.8404**	47.3968**	39.797**	-38.1734**
	Score10	-7.0973**	-16.6989**	-8.0128**	-6.0441**	-6.934**	26.4566**	7.0022**	30.7315**	127.9152**	-41.3865**
	3+Cycle	-989.195**	-1136.83**	-1104.38**	N	-1030.51**	706.1175	23.519	769.1358	N	-31.9354**
	Inactive	-6.475**	-4.6693**	-6.709**	-6.434**	-6.6281**	0.6317	-1.3634	3.361	-31.9727**	-2132.62
	Score1	-1716.39**	-1236.55**	-1540.17**	-1557.97**	-1569.79**	-1295.25**	-263.751**	-803.118	1910.321	12.4114**
	Score2	369.447**	338.8211**	371.7739**	365.1708**	391.0807**	-2.4762	24.9793	-304.985	506.3753**	891.8996
	Score3	289.6987**	262.6102**	281.7647**	282.5025**	282.4171**	83.7289**	14.6654**	83.1577**	-186.016**	-720.234**
	Score4	83.214**	88.0112**	79.6965**	84.4742**	78.5674**	47.4088**	-6.6752**	-88.6037**	-642.181**	-53.3118**
Limit3	Score5	17.9186**	19.5953**	15.7523**	17.0184**	16.8862**	21.8749**	-2.348**	25.1277**	-89.2109**	88.5878**
	Score6	7.7848**	2.2786	4.8348**	6.5429**	5.258**	28.4721**	2.2045**	24.9532**	-129.436**	26.1784**
	Score7	5.0626**	1.7619	2.8122**	3.3124**	3.1409**	26.0454**	0.936	30.3204**	-37.4448	5.4467
	Score8	-3.425**	-12.6923**	-5.4167**	-4.1391**	-4.5637**	20.6049**	6.1542**	33.6746**	0.7478	-2.8378
	Score9	-3.5313**	-12.9644**	-6.427**	-5.3119**	-6.1341**	34.7457**	4.9408**	45.0668**	26.066**	-39.5137**
	Score10	-5.3107**	-11.5459**	-5.9052**	-4.5254**	-5.1681**	18.4768**	4.532**	21.2502**	96.7612**	-40.9441**
	3+Cycle	-945.672**	-1076.64**	-1157.37**	N	-1077.63**	1269.029	-49.8221	2065.578	N	-20.6226**
	Inactive	-5.5725**	-3.9283**	-5.8877**	-5.7857**	-5.843**	4.068	-1.3703	0.5784	-18.3762	-1300.57
	Score1	-2148.32**	-1843.04**	-2025.22**	-2108.08**	-2031.75**	-938.065	-152.253	144.1903	5955.255**	5.308
	Score2	490.1844**	453.7079**	484.9757**	467.5699**	506.4127**	53.9895	23.9981	-166.806	1615.81**	271.7233
	Score3	404.0841**	349.4323**	393.458**	391.7773**	396.0295**	124.3296**	33.2751**	338.1869**	107.0566	-890.28**
	Score4	140.8944**	132.6267**	128.9788**	137.6178**	129.0818**	137.7224**	-2.5855	0.7597	-986.887**	-197.654**
Limit4	Score5	30.5246**	31.8296**	27.3693**	29.4827**	28.7371**	34.7817**	-2.7926**	23.8346**	-168.679**	57.9466**
	Score6	14.0936**	10.4936**	11.7072**	13.4771**	12.2233**	24.2457**	1.1181	18.2516	-157.936**	36.0953**
	Score7	8.0351**	2.5898	5.2967**	5.9782**	5.6681**	28.9675**	2.1484**	33.9845**	-47.4308	20.93**
	Score8	0.1461	-4.688**	-1.1495**	-0.0428	-0.4861	15.8232**	3.0939**	22.6206**	-26.9824	-7.3142
	Score9	-0.6688	-6.8434**	-2.9297**	-2.4196**	-2.6965**	25.8443**	3.0224**	27.019**	21.8131**	-14.2116**
	Score10	-3.3185**	-8.7855**	-3.8036**	-2.5788**	-3.2207**	18.2606**	3.9659**	19.4721**	74.7146**	-25.5901**
	3+Cycle	-945.515**	-802.499**	-882.934**	N	-932.662**	62.2539	-131.802	-1587.17	N	-19.4895**
	Inactive	-2.2263**	1.3654	-1.6996	-1.767**	-1.7012**	-3.6881	-2.3094**	-4.7774	-35.5171	-737.782
	Score1	-2251.46**	-1205.55**	-2035.52**	-2044.49**	-2085.98**	-1522.59	-738.981**	-1595.95	10965.77**	18.6776**
	Score2	510.5147**	652.2264**	566.4721**	554.9855**	551.5548**	-401.283**	-75.1041	-956.848	943.6692	1573.496
	Score3	231.3896**	190.6841**	217.0372**	222.2184**	216.0518**	143.8175**	19.215**	349.8029**	-827.028**	217.1357
	Score4	83.423**	53.4722**	70.3467**	74.8049**	71.7204**	118.3834**	13.1977**	116.6653**	-496.075**	-64.3555
Limit5	Score5	30.582**	17.2835**	24.0543**	25.9325**	24.201**	58.7165**	5.3123**	88.1385**	-173.915**	-53.6535**
	Score6	19.0442**	2.8328	11.6755**	12.207**	11.5934**	74.3722**	6.9955**	107.8187**	-132.952**	-25.9792**
	Score7	17.8957**	11.4343**	14.4151**	14.792**	15.0444**	31.317**	2.4388	81.0709**	4.48	-35.76**
	Score8	10.5721**	-2.8325**	7.1729**	8.1033**	8.953**	33.223**	8.4088**	57.0638**	-90.2641**	-21.8892**
	Score9	5.4394**	-0.7406	3.2053**	4.3778**	3.0344**	22.164**	3.0823**	57.3311**	-19.3784	-49.1071**
	Score10	5.281**	-3.9227	5.3086**	6.7478**	5.4111**	26.344**	6.7921**	21.6329**	120.6405**	-36.3958**

		Parameter α					Parameter β				
		CPI	GDP	Int	Sto	Une	CPI	GDP	Int	Sto	Une
Limit6	3+Cycle	-508.098	-688.78	-1104.25	N	-1164.35**	6633.009**	-186.757	4107.207	N	-31.7124**
	Inactive	-1.0862	0.7375	0.1632	-0.1324	-0.1058	-11.8323	-0.7199	-6.7376	-23.992	-11904.3**
	Score1	-2515.16**	-2286.71**	-2495.5**	-2572.16**	-2467.54**	-290.63	-158.563	2367.511	8187.108**	40.8168
	Score2	676.7212**	620.4059**	708.274**	694.733**	729.2421**	-199.988	64.5777**	-1990.31**	669.7515	-808.247
	Score3	478.817**	430.446**	461.5582**	468.6492**	462.3007**	175.1175**	23.9029**	172.5263**	-992.565**	-1046.49**
	Score4	172.0612**	178.9124**	163.7332**	173.4889**	163.0257**	109.4686**	-11.4791**	-64.143**	-1386.12**	-70.828
	Score5	43.9762**	38.5399**	37.5593**	42.2786**	39.261**	59.0452**	0.2736	85.4159**	-342.138**	162.5558**
	Score6	21.9435**	18.6484**	17.7159**	19.3756**	19.0001**	42.7683**	-0.0146	71.5229**	-113.814**	15.1427
	Score7	18.0439**	17.6003**	17.1187**	17.8016**	17.5037**	6.4667	-0.0773	30.5337**	-44.9522	29.8625**
	Score8	11.8025**	3.3006	9.7959**	11.0039**	10.7174**	22.4509**	5.4128**	31.3645**	-52.0622	6.1826
	Score9	7.1632**	2.0303	5.1896**	6.2743**	5.0549**	19.3671**	2.5553**	43.847**	-16.3329	-33.4768**
	Score10	6.9717**	1.4104	7.2869**	8.0616**	7.0955**	19.1321**	4.2111**	12.2621**	72.9115**	-31.2708**
Limit7	3+Cycle	-830.459	-1163.27	-1365.21**	N	-1030.38**	2396.046	6.6174	6196.54	N	-21.9343**
	Inactive	-0.359	1.4401	-0.5381	-0.8284	-0.4003	2.1175	-1.3398	0.7539	54.1875	-6391.13
	Score1	-3392.29**	-2923.1**	-3145.27**	-3265.73**	-3141.54**	-2047.59	-173.829	-1508.37	11376.15**	7.4244
	Score2	905.9658**	966.4549**	936.964**	900.2062**	945.2012**	-172.954	-30.8935	-2309.12**	2280.26**	-448.071
	Score3	604.7205**	554.8105**	593.3521**	595.9859**	594.613**	120.8463**	29.2311**	340.6315**	-391.72**	-833.646**
	Score4	240.9439**	226.9219**	220.5536**	233.9156**	224.014**	227.9975**	-2.329	157.4529**	-1297.85**	-112.464**
	Score5	62.1948**	59.8344**	54.7077**	59.106**	56.6781**	69.5945**	-2.5125	96.9507**	-300.731**	62.1585**
	Score6	30.64**	25.911**	25.1379**	27.343**	26.7104**	53.8907**	0.4862	99.559**	-108.88**	17.0336
	Score7	27.1338**	26.8905**	26.1951**	27.0694**	26.1934**	10.0336	-0.4766	2.0475	-118.113	12.9972
	Score8	19.0762**	13.0239**	17.4441**	18.6232**	18.1042**	20.2629**	3.6632**	24.163**	-85.7033**	10.8532
	Score9	13.5392**	8.1045**	10.4276**	11.4632**	10.581**	31.5097**	2.2169**	53.4034**	11.6922	-25.8729**
	Score10	10.2608**	4.2116**	10.4661**	11.1127**	9.9417**	16.7178**	4.3402**	11.3447**	74.5552**	-35.7157**
Limit8	3+Cycle	-2144.76	-2131.94	-2514.97	N	-2111.47**	-148.928	-3.2807	8205.221	N	-25.8514**
	Inactive	-3.2183**	-6.0418**	-4.8185**	-4.4627**	-4.3178**	13.9362**	1.2899	10.8248	36.4478**	-18398**
	Score1	-5027.6**	-2261.52**	-3734.34**	-3862.88**	-3891.53**	-9368.74**	-1210.41**	-7660.38	15415.82	-12.9756**
	Score2	1202.352**	1306.167**	1251.657**	1225.626**	1248.733**	-351.537	-47.4049	-1117.98	1887.308	7180.274**
	Score3	720.7597**	629.6955**	696.1256**	713.765**	700.0589**	249.4964**	51.9031**	472.9266**	-1632.36**	-161.838
	Score4	305.2036**	210.7888**	260.7896**	287.253**	270.7977**	442.1478**	43.9412**	565.6882**	-1608.52**	-245.093**
	Score5	78.8684**	67.0888**	66.7311**	72.6089**	69.1487**	107.8272**	1.8028	181.5699**	-294.1**	-199.515**
	Score6	37.251**	40.0893**	31.6459**	33.9932**	34.305**	52.4318**	-4.8237	138.4763**	-75.3867	-20.2275
	Score7	38.8395**	45.0727**	38.0491**	37.4937**	38.4726**	4.6925	-4.3848	29.5663	128.8815**	39.7979**
	Score8	25.7848**	14.5799**	22.9859**	23.5221**	24.5265**	25.3955**	7.1053**	51.1299**	36.1787	-0.4385
	Score9	23.8254**	18.5318**	20.2788**	21.8634**	20.6102**	36.6471**	2.0696	69.5003**	-0.1815	-51.2053**
	Score10	20.4893**	14.6569**	20.5259**	21.7695**	20.6253**	19.4827**	4.3733**	16.5691**	51.2985	-35.3494**
Limit9	3+Cycle	-997.349	-687.895	-731.143	N	-381.319	-1321.46	-13.3359	2009.37	N	-21.7985**
	Inactive	-5.1101**	-7.3397**	-6.7067**	-6.1963**	-6.0521**	10.3995	0.9251	12.8196	16.3163	-5376.57
	Score1	-6001.03**	-4495.27**	-5220.29**	-5230.12**	-5333.63**	-5322.68	-619.261	-6718.16	2568.676	-4.3588
	Score2	1270.641**	1598.576**	1420.356**	1356.85**	1386.496**	-881.894**	-171.135	-6403.25**	2717.896	3114.584
	Score3	922.7394**	834.3853**	899.0795**	918.9792**	901.8153**	253.5054**	49.5194**	265.949	-1972.14**	-53.896
	Score4	442.424**	295.0313**	371.0868**	407.8786**	388.042**	725.3986**	68.2671**	905.1133**	-1818.51**	-229.305**
	Score5	125.4756**	115.5729**	108.2568**	117.3322**	112.3539**	154.7852**	-1.9433	273.7494**	-461.073**	-397.546**
	Score6	63.7718**	57.9703**	53.7867**	56.4694**	57.3653**	81.9112**	-0.7135	255.5579**	72.9987	-15.993
	Score7	60.9132**	75.6143**	66.0033**	65.1748**	65.6528**	-50.3114**	-7.1551	-81.7367**	-19.813	22.5776
	Score8	47.4081**	29.3496**	43.6836**	43.1832**	45.9933**	33.4689**	11.8408**	77.1735**	175.4604**	68.6362**
	Score9	38.0209**	29.9548**	33.5227**	35.1129**	33.8**	46.2171**	3.2534**	80.7326**	-2.4016	-81.8838**
	Score10	31.8487**	29.1436**	32.3057**	32.8256**	31.946**	12.8051	2.1932	6.1067	4.7462	-43.5881**
Limit10	3+Cycle	-240.145**	-259.441**	-253.987**	N	-255.299**	105.5777**	1.2171	925.3441	N	-13.9376
	Inactive	-3.6775**	-10.6024**	-7.2353**	-5.8255**	-5.4558**	22.5889**	3.6549**	30.6156**	10.7277	-121.131
	Score1	-9714.84**	-8457.12**	-8950.95**	-9393.02**	-9172.78**	-6020.89	-587.256	-8629.38	29437.71	-28.3977**
	Score2	2164.712**	2746.148**	2518.092**	2431.85**	2366.31**	-2106.1**	-260.719	-11469**	-1248.94	6161.068
	Score3	1655.627**	1395.439**	1591.255**	1631.801**	1606.639**	632.7056**	158.4026**	1228.397**	-1237.93	2284.35
	Score4	1044.634**	541.4482**	822.3373**	1006.78**	904.4936**	2304.4**	287.5503**	3174.84**	-2991.74**	-1251.11**
	Score5	300.6129**	228.2479**	233.0438**	266.3223**	251.3212**	591.8813**	22.6957**	1105.934**	-411.736**	-2318.13**
	Score6	169.7808**	82.3828**	129.103**	144.4935**	139.0193**	339.0175**	46.7821**	741.9142**	88.1014	-316.074**
	Score7	144.8015**	176.4515**	153.566**	152.5672**	156.3525**	-106.086**	-15.9125	-65.1358	-33.239	-318.152**
	Score8	112.6287**	64.5296**	101.1547**	103.0252**	106.99**	122.6226**	29.9533**	172.8561**	62.9689	213.8856**
	Score9	82.3667**	72.3827**	75.9774**	77.8565**	76.1055**	68.3721**	3.718	97.8609**	46.4252	-221.822**
	Score10	79.1144**	67.3448**	79.9896**	82.0303**	78.7824**	51.6226**	9.5638**	36.5684**	275.961**	-60.4351**

"'**"' indicates the parameter is significant at 95% level.

"'N"' represents the cumulative logistic regression did not find any significant explanatory macroeconomic variable and therefore we did not build up the corresponding regression model.

Table C.1: Summary of regression estimates - All accounts

		Model A				Model B		
		Intercept	CPI	GDP	Sto	Intercept	Sto	Une
Limit1	3+Cycle	-608.665**	541.825**	13.3096	N	N	N	N
	Inactive	-4.5132**	14.7537**	-1.6772**	-20.402**	-8.0167**	-20.7451**	-7.1456**
	Score1	-604.204**	90.9678	-85.0342**	2035.695**	-721.827**	2235.316**	**
	Score2	168.8085**	7.5201	28.8467**	263.2898**	218.2551**	65.7681	129.5972
	Score3	150.3217**	31.9605**	4.0654	-155.113**	152.9342**	-219.71**	-405.564**
	Score4	61.8805**	90.846**	-14.8484**	-417.7**	32.5289**	-347.256**	-9.1437
	Score5	15.6126**	39.7014**	-3.1942**	-139.568**	8.2783**	-97.8224**	10.1872
	Score6	15.7792**	51.7884**	-3.32	-179.321**	8.2836**	-187.12**	-6.2457
	Score7	-4.49	45.5685**	5.4855**	-52.4032	1.059	-85.7943**	-15.2094
	Score8	-10.5848**	21.5499**	2.2791**	11.276	-7.4663**	-47.9994**	-74.2411**
	Score9	-17.8757**	27.1885**	6.9507**	-2.8365	-9.0958**	-12.9029	-39.9121**
	Score10	-14.0573**	11.7416**	2.5647	19.9089	-10.0135**	51.4547**	-59.0239**
Limit2	3+Cycle	-585.009**	208.5566	-37.4409	N	N	N	N
	Inactive	-3.021	17.2884**	-1.1942	-19.595	-6.1889**	-20.1344	-16.6725**
	Score1	-1069.43**	3.007	-59.7989	1586.559	-1136.74**	1677.501	-9.4564**
	Score2	220.0369**	76.311	31.3206**	988.5898**	276.8933**	636.1877**	10.0691
	Score3	177.955**	17.6377	9.7483**	-156.497**	188.7584**	-219.738**	-715.107**
	Score4	76.0658**	55.7579**	-17.8776**	-594.684**	43.5325**	-482.073**	-17.3886
	Score5	9.8436**	33.0026**	-0.6454	-87.1908**	5.6809**	-80.2522**	87.3712**
	Score6	9.5179**	42.4824**	-2.9813**	-64.5747**	1.2223	-108.71**	-7.4207
	Score7	0.3425	35.4409**	-0.5245	-71.4291**	-3.4406**	-36.4373	-6.7118
	Score8	-11.4763**	15.2922**	3.1508**	59.9963**	-7.5036**	-1.1396	-18.9229**
	Score9	-10.1276**	30.7434**	2.0412**	6.1946	-9.2408**	6.3306	-38.2155**
	Score10	-8.806**	26.03**	1.1479	-31.191	-6.9629**	-13.1093	-40.6134**
Limit3	3+Cycle	-1052.03**	941.7419	59.0208	N	N	N	N
	Inactive	-2.444	8.265	-2.4217**	-34.3048**	-6.5448**	-22.3432	-33.4118**
	Score1	-1486.23**	-815.308	-160.813	1594	-1611.79**	2662.409**	10.1679**
	Score2	317.4881**	-37.1288	34.6823**	532.3446**	387.8702**	285.8476	1154.09**
	Score3	284.6593**	74.2603**	3.5075	-53.6023	284.5959**	-254.691**	-699.069**
	Score4	127.4118**	111.3452**	-22.9033**	-704.149**	83.9284**	-591.733**	-72.0217**
	Score5	32.0797**	42.7734**	-7.962**	-118.541**	17.659**	-87.0516**	51.916**
	Score6	15.4157**	37.7212**	-4.3594**	-98.1163**	6.5987**	-129.214**	25.7219**
	Score7	11.7368**	39.6653**	-3.5368**	-97.1034**	3.3158**	-52.1142**	5.0956
	Score8	-8.3981**	15.8564**	3.1669**	38.4705**	-4.3233**	-23.8591	-7.6083
	Score9	-4.5098**	33.2682**	0.5707	4.1381	-6.1327**	-1.6456	-40.4914**
	Score10	-6.0823**	15.2226**	0.5623	17.6181	-5.0916**	35.4277	-41.1163**
Limit4	3+Cycle	-920.178**	1219.531	-33.3509	N	N	N	N
	Inactive	-0.4125	12.946**	-3.1901**	-22.7675	-5.7924**	-14.9712	-16.5542**
	Score1	-2183.64**	-779.548	-19.5777	5727.639**	-2125.74**	6185.316**	3.9555
	Score2	451.3021**	123.1834	23.1753	1784.51**	489.9385**	1435.758**	617.4167
	Score3	373.2426**	93.4942**	20.1861**	294.7278**	396.7029**	-88.4005	-810.209**
	Score4	211.7135**	241.5758**	-37.0922**	-1106.06**	137.6432**	-990.439**	-204.056**
	Score5	51.0527**	63.8856**	-11.4332**	-205.203**	30.3608**	-168.035**	-3.5286
	Score6	23.3347**	35.8111**	-5.2244**	-135.382**	13.661**	-154.133**	35.9105**
	Score7	12.783**	38.009**	-2.4219	-86.1834**	6.0534**	-67.0661**	19.0207**
	Score8	-0.1242	15.453**	0.1848	-0.2289	-0.1338	-39.8768**	-12.7843
	Score9	-0.0183	26.4977**	-0.4466	11.55	-2.7289**	8.1716	-16.2336**
	Score10	-2.8188	20.5519**	-0.3609	-13.9298	-3.2117**	3.7946	-24.8543**
Limit5	3+Cycle	-769.365**	272.9027	-137.934	N	N	N	N
	Inactive	2.7956	3.7032	-2.9519**	-39.3451	-1.5335	-27.2237	-19.0573**
	Score1	-1291.91**	196.9436	-663.221**	9717.775**	-2205.01**	12679.89**	17.0422**
	Score2	545.9735**	-316.745	-22.6108	605.2108	543.7436**	1044.02	2922.102
	Score3	235.6661**	117.1674**	-1.4885	-589.947**	225.9745**	-900.913**	300.6919
	Score4	97.697**	142.023**	-5.7315**	-566.81**	76.5677**	-605.114**	-117.383**
	Score5	38.3018**	70.563**	-3.565**	-201.886**	25.6428**	-182.426**	-96.5012**
	Score6	35.2062**	104.9034**	-9.9133**	-148.68**	12.3622**	-159.843**	-29.3204**
	Score7	22.4324**	38.7438**	-2.6821	-14.6294	15.2445**	-29.8359	-42.3498**
	Score8	4.8487**	23.4219**	4.2702**	-63.0498**	10.6163**	-175.104**	-24.3924**
	Score9	3.901	21.6352**	1.024	-31.7859**	3.101**	-44.6827**	-65.0516**
	Score10	2.8079	20.7485**	1.7391	7.602	5.4716**	25.9001	-40.9192**

		Model A				Model B		
		Intercept	CPI	GDP	Sto	Intercept	Sto	Une
Limit6	3+Cycle	-345.834	6524.186	-119.17	N	N	N	N
	Inactive	-2.9116	-14.6866	1.2381	-17.996	-0.121	2.8196	-28.831**
	Score1	-2503.66**	-132.184	-67.4852	8015.084**	-2567.55**	8093.916**	41.0226
	Score2	478.8903**	-434.262**	131.3076**	529.0762	724.5984**	411.3549	-224.918
	Score3	483.3035**	146.2577**	-1.3067	-733.443**	472.7053**	-1115.31**	-1020.13**
	Score4	261.9121**	255.725**	-46.669**	-1565.21**	172.9195**	-1299.98**	-146.375**
	Score5	68.5404**	92.4267**	-12.7139**	-382.89**	42.5865**	-341.519**	72.5234**
	Score6	44.7768**	83.9837**	-14.2859**	-110.013**	19.6304**	-101.487**	14.2059
	Score7	20.8475**	10.9922	-1.4312	-50.6945	17.783**	-41.1708	27.0152**
	Score8	9.0425**	17.8501**	2.0254	-23.7889	11.5483**	-91.1369**	3.24
	Score9	6.2287**	18.8114**	0.6491	-23.1287	5.1618**	-32.3009**	-39.5187**
	Score10	7.2528**	20.5476**	-0.2094	-9.0606	7.1077**	5.359	-33.9829**
Limit7	3+Cycle	-1059.57	2982.995	152.5289	N	N	N	N
	Inactive	2.7199	6.5472	-2.1806	50.8028	-0.7395	62.422**	-21.334**
	Score1	-3650.07**	-2096.99	108.102	10822.18**	-3268.64**	11440.48**	12.2251
	Score2	906.769**	-20.5257	-6.6506	2246.812**	917.9968**	2112.431**	157.2895
	Score3	581.5112**	80.131**	16.0288	-192.64	599.2633**	-494.363**	-705.406**
	Score4	347.0937**	393.5285**	-56.5265**	-1573.26**	233.9609**	-1324.1**	-141.89**
	Score5	94.8789**	115.9212**	-18.0761**	-356.334**	59.4762**	-300.478**	-23.633
	Score6	57.6491**	102.8407**	-17.0824**	-95.924	27.4399**	-104.844**	16.7451
	Score7	33.1031**	19.4993	-2.9712	-127.552	27.0417**	-115.121	10.505
	Score8	18.8891**	17.8827**	0.3806	-60.6855**	19.0991**	-126.343**	2.6228
	Score9	16.0155**	34.8415**	-1.4783	4.4491	10.5893**	-1.3173	-35.0281**
	Score10	7.7825**	11.2387	1.6976	9.246	9.9178**	-10.4336	-35.8016**
Limit8	3+Cycle	-2112.39	-192.886	-15.4273	N	N	N	N
	Inactive	-2.6391	14.757**	-0.5307	32.2535	-4.5271**	30.7924	-27.0271**
	Score1	-4951.97**	-8842.9**	-111.798	13733.71	-4098.57**	18420.54**	-11.0018
	Score2	1185.416**	-268.518	7.9273	1633.306	1226.951**	1874.96	8381.789**
	Score3	696.3652**	148.4006**	20.9316	-1224.69**	719.1002**	-1796.29**	-52.3273
	Score4	369.7983**	536.5971**	-28.8909**	-1890.65**	286.4162**	-1872.52**	-323.358**
	Score5	113.4349**	154.8078**	-18.7729**	-357.354**	72.0911**	-293.149**	-299.978**
	Score6	80.8013**	129.8853**	-27.2736**	-46.8335	34.7314**	-62.4035	-19.2657
	Score7	48.1802**	17.5237	-6.1183**	111.1322	37.5349**	137.369**	38.4378**
	Score8	21.7388**	23.7121**	2.4145	79.1348**	24.6369**	-10.5262	8.2672
	Score9	27.6218**	41.4743**	-2.1348	-12.5671	20.6898**	-14.1191	-51.9083**
	Score10	19.0635**	20.1031**	0.9157	-34.1268	20.5838**	-17.4744	-36.3593**
Limit9	3+Cycle	-974.648	-1841.69	-78.05	N	N	N	N
	Inactive	-4.1104	11.963	-0.6901	12.5558	-6.1468**	14.1577	-23.7645**
	Score1	-6282.51**	-5773.58	180.5866	759.4945	-5370.46**	3576.29	-3.2694
	Score2	1355.524**	-638.983	-58.7816	1935.15	1352.158**	2759.962	3300.319
	Score3	904.7019**	144.369**	18.113	-1622.53**	924.092**	-2191.38**	144.9341
	Score4	546.2507**	887.237**	-50.9863**	-2384.61**	405.7426**	-2341.19**	-342.529**
	Score5	186.0956**	238.8553**	-33.3559**	-581.144**	116.9284**	-460.602**	-534.151**
	Score6	109.3008**	165.1629**	-29.6632**	115.3004	56.7801**	81.5924	-15.0704
	Score7	64.174**	-45.482**	-1.9494	2.4894	65.2703**	56.1837	24.5028
	Score8	36.4316**	28.3304**	6.6229**	222.3889**	44.9979**	100.1392**	72.3244**
	Score9	41.5669**	50.7752**	-1.9919	-12.9432	33.9204**	-17.8146	-74.6216**
	Score10	32.7045**	20.4853	-0.6741	-64.2824	31.8206**	-52.8263	-44.7226**
Limit10	3+Cycle	-242.582**	139.4636**	5.3337	N	N	N	N
	Inactive	-5.1008	20.1671**	0.8903	3.6767	-5.3841**	-10.78	-19.8931**
	Score1	-11750.3**	-9279.42	1215.44	34300.96	-9390.48**	31043.92	-29.3218**
	Score2	2055.639**	-2528.27**	88.67	-3918.34	2379.206**	-880.349	7319.155
	Score3	1536.965**	485.1551**	77.0519**	144.0313	1625.127**	-1719.76**	2245.768
	Score4	1185.994**	2496.369**	-62.9488**	-4353.19**	942.9864**	-4993.49**	-1301.79**
	Score5	432.7177**	768.8106**	-75.7037**	-736.595**	255.1174**	-367.19	-2549.94**
	Score6	205.6745**	412.684**	-25.0685**	344.8193	138.3998**	68.635	-312.652**
	Score7	153.294**	-94.5249**	-4.9552	-16.6526	155.2951**	133.2475	-317.88**
	Score8	96.5516**	108.2475**	10.4781	209.8475	108.6581**	-211.486	220.7216**
	Score9	87.9927**	75.9425**	-3.5223	44.2822	75.7165**	39.4081	-238.652**
	Score10	84.6163**	48.2214**	-3.5326	141.1312	79.0458**	123.4093	-59.4235**

”‘**”’ indicates the parameter is significant at 95% level.

”‘N”’ represents the cumulative logistic regression did not find any significant explanatory macroeconomic variable and therefore we did not build up the corresponding regression model.

Table C.2: Summary of regression estimates - Model A and Model B - All accounts

		Parameter α					Parameter β				
		CPI	GDP	Int	Sto	Une	CPI	GDP	Int	Sto	Une
Limit1	Inactive	-9.358**	-14.8223**	-11.5503**	-10.827**	-10.5777**	15.914**	2.9951**	16.1011**	17.3277**	-21.5767**
	Score2	214.6951**	208.129**	205.2988**	209.3069**	206.5986**	75.089	-0.793	523.0692	-157.812	**
	Score3	24.8458**	24.9604**	22.1858**	24.956**	24.4365**	4.8301	-0.4366	184.3264**	-52.0418	-40.6215
	Score4	-4.8397**	-6.0348**	-7.5699**	-6.3205**	-6.1334**	17.8509**	-0.1778	43.8172**	2.4805	1.6774
	Score5	-1.9213**	-6.1493**	-2.7885**	-2.4709**	-3.1029**	11.4855**	2.3231**	8.7718	-19.6276	-28.841**
	Score6	2.9041**	-7.5404**	-0.4239	2.171	0.6502	45.5707**	6.3531**	53.3613**	-135.123**	-13.6567**
	Score7	-1.7064	-23.0082**	-23.0082**	-4.0648**	-5.1249**	53.8285**	12.6105**	12.6105**	81.0685**	-20.4707
	Score8	-11.9178**	-20.6051**	-14.8709**	-11.9918**	-12.7439**	24.1727**	6.211**	40.1437**	13.4285	-69.9504**
	Score9	-11.1518**	-25.8332**	-13.9263**	-12.2655**	-13.4582**	37.9413**	8.6816**	49.481**	28.7733**	-40.4746**
	Score10	-14.6815**	-21.2082**	-14.8309**	-13.8328**	-14.6263**	16.9856**	4.7531**	16.2516**	106.3518**	-48.9259**
Limit2	Inactive	-8.1219**	-14.091**	-11.102**	-10.1444**	-9.5484**	18.2597**	3.0907**	25.0308**	14.6506**	-20.9184**
	Score2	289.8143**	242.0597**	286.134**	286.8734**	283.1466**	40.4023	32.9164	-1152.34**	-103.223	-21.8822**
	Score3	26.3903**	26.2133**	24.8612**	27.2768**	26.4012**	-0.0373	0.1426	210.4964**	-85.9032	-248.67
	Score4	-1.5818**	-3.9434**	-3.311**	-2.3408**	-2.4362**	10.1072**	0.8766**	26.4915**	-19.9878	0.4241
	Score5	-1.881**	-12.1945**	-4.4648**	-4.2842**	-4.8897**	23.9422**	5.5345**	25.0127**	-8.0681	-14.1826**
	Score6	-2.6588**	-10.7984**	-5.9572**	-5.236**	-5.8322**	28.2007**	3.8319**	66.1978**	-49.4039**	-14.8323**
	Score7	-6.1359**	-15.844**	-15.844**	-9.1164**	-9.2333**	34.4386**	4.4393**	4.4393**	43.8606**	-30.6598**
	Score8	-10.7851**	-20.1698**	-14.1881**	-12.8352**	-11.9347**	18.7773**	6.0761**	55.7667**	26.8414**	-29.2916**
	Score9	-11.3048**	-20.4963**	-13.5231**	-12.8791**	-13.6347**	29.3463**	4.6907**	42.2189**	28.9285**	-45.741**
	Score10	-12.4343**	-20.0643**	-13.0621**	-11.6999**	-12.4242**	21.6162**	5.4864**	25.7519**	106.4391**	-33.0294**
Limit3	Inactive	-9.0723**	-12.966**	-10.5602**	-10.0083**	-9.87**	10.8734**	2.1938**	10.8919**	11.7751	-26.261**
	Score2	470.4698**	419.1515**	465.3528**	460.8675**	462.3923**	62.3058	34.2486	352.1531	267.1929	-13.0756**
	Score3	58.7913**	48.9819**	52.4457**	57.4394**	57.4241**	11.7879	6.7893**	505.6216**	37.82	-203.35
	Score4	2.5209**	0.1477	0.0846	1.4486**	1.3687**	15.2687**	0.7085	35.9123**	-20.0766	-16.6286
	Score5	0.9526	0.7481	0.8897	1.0302	0.6933	1.0632	0.075	-3.5087	-20.5889	-23.7989**
	Score6	0.144	-7.9471**	-3.046**	-1.8781**	-2.8182**	30.5367**	3.9987**	46.4761**	-60.916**	-4.6088
	Score7	-2.6821**	-8.9212**	-8.9212**	-4.6177**	-4.721**	24.235**	2.8808**	2.8808**	-3.0583	-27.8481**
	Score8	-8.5285**	-19.0399**	-11.5325**	-9.8986**	-9.8875**	20.5909**	6.869**	43.1866**	10.0323	-14.9587**
	Score9	-8.2397**	-17.6257**	-11.2203**	-10.2992**	-11.0278**	34.4971**	4.7475**	50.8222**	28.9538**	-49.7056**
	Score10	-9.9903**	-15.6743**	-10.3986**	-9.2717**	-10.0052**	17.4219**	4.0723**	20.1802**	79.6426**	-40.9681**
Limit4	Inactive	-8.0829**	-10.6389**	-9.2882**	-8.8991**	-8.8815**	11.3409**	1.2664	6.6292	10.704	-20.1711**
	Score2	580.5261**	645.1414**	564.7131**	523.2566**	559.7276**	173.1152	-62.6429	1030.974	3692.64**	-13.8164**
	Score3	93.624**	73.8099**	80.6233**	90.6042**	89.3534**	36.4812**	12.8093**	758.3996**	-3.6507	-466.681
	Score4	6.9065**	5.1354**	4.4184**	6.6518**	5.9942**	11.6123**	0.5229	46.8384**	-63.876**	-52.6561**
	Score5	5.8605**	7.8595**	6.7827**	6.5855**	6.4336**	-5.1347	-1.0703	-44.0861**	-27.7984	-10.3743
	Score6	6.0991**	-1.5732	3.4025**	5.0798**	3.5422**	28.0586**	3.9198**	26.4609	-129.451**	3.2668
	Score7	1.3069	-7.362**	-7.362**	-1.6182**	-1.5441**	31.748**	4.1104**	4.1104**	20.54	-13.2032
	Score8	-5.0511**	-13.6118**	-8.3337**	-6.0817**	-6.2846**	19.36**	5.4427**	55.8938**	0.133	-27.6145**
	Score9	-5.9705**	-14.2714**	-8.7549**	-8.1656**	-8.538**	29.0709**	4.1819**	41.1157**	31.4603**	-36.8086**
	Score10	-7.9436**	-13.3569**	-8.24**	-7.1764**	-7.9723**	18.2476**	3.8649**	19.275**	67.5204**	-36.7466**
Limit5	Inactive	-5.2493**	-8.6954**	-6.0904**	-5.8275**	-5.9766**	9.2991**	1.9962**	4.8605	8.8389	-19.7078**
	Score2	396.9551**	610.4533**	435.1468**	394.5717**	452.6859**	-504.586	-126.075	249.7886	5967.692**	-12.2216**
	Score3	21.5028**	27.7366**	19.2635**	26.8637**	22.4234**	-2.761	-4.4829	373.118**	-400.046**	1050.087
	Score4	16.4576**	11.0399**	13.9747**	15.1895**	14.2779**	30.6081**	1.9212	-6.4409	-162.482**	18.92
	Score5	13.7008**	4.3969**	10.4795**	10.9441**	10.5195**	28.4928**	4.4748**	27.486**	-68.885**	-32.3004**
	Score6	7.1815**	-3.5486**	1.6003	1.6112	1.0474	64.067**	3.4555**	87.6415**	-113.277**	2.3286
	Score7	11.031**	0.8566	0.8566	6.5044**	6.7726**	44.7999**	4.0552**	4.0552**	2.3206	-36.244**
	Score8	1.78**	-14.1332**	-3.8257**	-2.4136**	-0.3591	38.8759**	9.6943**	89.8309**	-39.5352**	-34.6465**
	Score9	-3.0315**	-8.1032**	-5.1756**	-4.3674**	-5.2923**	23.363**	2.2988**	55.9055**	1.051	-57.9172**
	Score10	-2.8398**	-7.7303**	-2.664**	-1.946**	-2.8613**	16.5783**	3.6692**	12.5814**	73.2528**	-27.9559**

		Parameter α					Parameter β				
		CPI	GDP	Int	Sto	Une	CPI	GDP	Int	Sto	Une
Limit6	Inactive	-5.0654**	-11.682**	-6.2592**	-6.1384**	-6.0716**	13.1195**	4.0798**	4.4023	28.1097**	-20.3303**
	Score2	798.4906**	904.1937**	793.2447**	750.6962**	789.9947**	60.6417	-86.0941	1288.266	3623.488	-24.0779**
	Score3	93.7078**	90.0878**	82.5341**	97.3925**	92.4565**	18.213	1.5308	910.6243**	-440.304**	-620.814
	Score4	19.1074**	20.0117**	18.0304**	19.8349**	18.1603**	13.2221**	-1.2987	2.215	-162.317**	9.9324
	Score5	11.8653**	8.1591**	11.0805**	11.5056**	11.062**	7.8275	2.0788	-1.4234	-46.2752**	-13.1363
	Score6	11.6925**	6.2141**	7.239**	8.1204**	7.4894**	53.5799**	0.9999	94.4371**	-74.8848	-0.2736
	Score7	9.1921**	2.0328	2.0328	7.0979**	7.3823**	19.157**	3.7641**	3.7641**	47.2735	-9.0411
	Score8	2.3002**	-8.9166**	-1.0602	0.3744	1.0971	24.2425**	7.1366**	51.6985**	-11.5321	-20.6808
	Score9	-1.5331**	-5.2445**	-3.2002**	-2.5752**	-3.548**	18.5708**	1.6463**	34.597**	0.042	-48.9865**
	Score10	-0.3935	-5.6431**	-0.0077	0.6141	-0.4604	16.6647**	3.8512**	10.3495**	45.6913	-27.5091**
Limit7	Inactive	-4.796**	-6.9238**	-4.9222**	-5.387**	-5.2873**	6.3437	1.2144	-3.7398	33.4397**	-20.2818**
	Score2	988.0649**	977.8119**	973.1486**	953.5261**	967.1668**	198.0311	-7.6709	994.9007	982.1918	-9.459
	Score3	121.3373**	94.861**	100.5414**	121.724**	117.0625**	39.5185**	17.3957**	1454.118**	-305.378	-185.381
	Score4	25.3278**	28.0477**	24.1255**	26.0744**	24.9073**	5.8997	-2.1201	21.2825	-111.182	-41.3156
	Score5	21.4944**	23.3819**	21.4422**	21.5648**	21.4497**	2.071	-1.4917	-24.2657	-31.4798	-4.2068
	Score6	13.4615**	10.2226**	8.8977**	9.6444**	9.1581**	51.9554**	-0.5749	111.1097**	-26.8124	4.7353
	Score7	20.6579**	11.0503	11.0503	17.0736**	17.1099**	37.4967**	4.2493	4.2493	-9.4062	-27.2458**
	Score8	7.986**	0.0776	5.3398**	6.3633**	6.5817**	23.8267**	4.5579**	27.3927**	-44.2718	-53.4693**
	Score9	4.4434**	-2.0257	1.6093**	2.3942**	1.6147**	27.708**	2.919**	55.9493**	15.1256	-33.8947**
	Score10	2.9486**	-1.2324	3.2653**	3.625**	2.4966**	12.1281**	2.9762**	7.027	39.8247	-33.8828**
Limit8	Inactive	-5.3817**	-8.1607**	-6.3627**	-6.2838**	-6.2046**	10.1028**	1.446	4.8073	26.8351	-20.6314**
	Score2	893.7369**	828.6126**	889.2721**	923.5811**	869.4649**	64.1701	43.3656	-1338.26	-3190.11	-11.9124**
	Score3	134.5798**	115.2487**	123.5843**	138.6577**	132.7955**	20.4274	13.5209	713.4185**	-410.522	-1194.95
	Score4	43.6824**	28.4299**	39.036**	42.0441**	40.2491**	47.8634**	8.5384**	20.7838	-191.265	-6.6114
	Score5	32.9024**	26.847**	31.1544**	31.6084**	30.4153**	18.6631	2.9094	-23.6028	-63.4921	-90.0144**
	Score6	18.8977**	15.0227**	14.7128**	16.2977**	15.36**	51.502**	0.3569	153.229**	-102.989	-14.4146
	Score7	29.5575**	29.4294**	29.4294**	27.0209**	27.7827**	16.8693	-0.8035	-0.8035	137.9336**	-8.0557
	Score8	16.227**	2.839	11.6028**	13.0464**	14.5645**	31.1588**	8.1287**	72.5373**	9.153	-28.2128
	Score9	12.362**	8.8393**	9.669**	10.7123**	9.9718**	26.9219**	1.2153	58.6839**	10.1441	-65.4158**
	Score10	11.1868**	6.1896**	11.4401**	12.2934**	11.2129**	16.3669**	3.6936**	12.8908**	46.7322	-23.1106**
Limit9	Inactive	-6.2866**	-11.4879**	-8.8061**	-8.1405**	-7.8555**	18.8702**	2.5439**	15.4097**	10.3955	-18.4695**
	Score2	622.1653**	1582.336**	588.4421**	598.1492**	636.2804**	-278.653	-753.834	-8646.7	6543.304	-20.4634**
	Score3	164.2332**	119.0215**	142.2669**	162.0555**	155.3411**	77.7303**	29.9648**	1430.096**	-261.851	3808.997
	Score4	64.2759**	49.8404**	55.7312**	63.5252**	60.403**	55.8156**	7.2022**	122.8581**	-358.721**	-110.528
	Score5	56.0484**	48.7774**	50.8857**	51.4353**	50.471**	48.2225**	1.7158	50.5724	-21.9758	-97.8063**
	Score6	35.3341**	35.188**	30.7968**	30.9046**	31.4464**	50.8367**	-2.8783	139.0191**	31.82	-20.0601
	Score7	48.4184**	44.3225**	44.3225**	47.6512**	48.2815**	-0.7448	2.8092	2.8092	99.5117	22.0018
	Score8	34.153**	11.8476**	28.2779**	28.8855**	32.3958**	39.0366**	14.4948**	107.9113**	129.7395**	-17.513
	Score9	25.6891**	18.8816**	21.9919**	23.0382**	22.121**	36.8507**	2.7006**	75.4326**	13.4391	-95.152**
	Score10	21.1077**	15.2989**	22.0347**	22.4274**	20.7414**	17.6021**	4.2099**	6.4683	28.6924	-34.5357**
Limit10	Inactive	-4.2438**	-10.1484**	-6.9549**	-6.0087**	-5.6751**	18.3828**	3.1775**	21.1403**	11.6257	-24.9206**
	Score2	1282.258	301.6541	1310.976	1534.14	1176.502	598.1707	657.951	-28897.1	-24816.7	-24.6766**
	Score3	353.145**	302.3895**	304.9737**	350.7289**	357.8236**	-74.8055	41.8075	2088.507**	499.1023	-4394.49
	Score4	161.8748**	118.7261**	122.5792**	146.4554**	140.8982**	291.9732**	14.6326**	506.351**	-698.417**	-9.8448
	Score5	125.998**	82.3991**	105.4503**	107.1216**	104.1268**	200.7662**	17.4211**	216.5456**	105.712	-557.864**
	Score6	72.6411**	53.1556**	61.0598**	65.3069**	61.7984**	117.3595**	7.4978	278.0578**	-183.169	-96.5053**
	Score7	112.3033**	114.4006**	114.4006**	117.3629**	118.6302**	-48.1817	0.9946	0.9946	-152.211	-116.473**
	Score8	95.4003**	45.5795**	81.7571**	83.6035**	90.7196**	111.0142**	30.8684**	157.9425**	92.5254	129.2794
	Score9	66.3793**	59.7475**	62.6707**	62.5675**	62.4455**	39.1742**	2.5632	58.5047**	115.5095**	-248.182**
	Score10	66.8314**	55.9455**	66.671**	69.5453**	66.5147**	50.5256**	8.9331**	49.8775**	297.0855**	-38.4349**

"'**"' indicates the parameter is significant at 95% level.

"'N"' represents the cumulative logistic regression did not find any significant explanatory macroeconomic variable and therefore we did not build up the corresponding regression model.

Table C.3: Summary of regression estimates - Transactors

		Model A				Model B		
		Intercept	CPI	GDP	Sto	Intercept	Sto	Une
*	Inactive	-11.4582**	12.2693**	1.2734**	11.1029**	-10.573**	-0.8916	-21.6581**
	Score2	238.3766**	112.6692	-16.129	64.7147	208.4051**	-117.793	N
	Score3	27.9148**	6.6524	-1.911	-46.0476	24.9998**	-52.8317	-27.4848
	Score4	-4.515**	19.4735**	-0.4032	39.8692**	-5.9586**	-16.9758	2.0733
	Score5	-4.1082**	8.1843	1.402	-18.1955	-2.9621**	-16.1005	-29.7001**
Limit1	Score6	0.9893	39.4744**	1.7204	-74.9322	1.8286	-136.317**	-13.2394**
	Score7	-13.2194**	38.3787**	7.1712**	-6.9909	-5.028**	-46.282	-20.9284
	Score8	-15.934**	18.8902**	2.6268**	36.2742	-12.5477**	-24.5137	-74.4376**
	Score9	-18.204**	28.7944**	4.3406**	-7.3982	-13.4724**	-15.6905	-41.7794**
	Score10	-16.8842**	6.8419	1.6422	59.0312**	-14.4869**	66.758**	-50.6974**
*	Inactive	-9.6596**	15.4874**	0.9488	4.5733	-9.5076**	-7.4903	-13.4506**
	Score2	229.5678**	-32.4656	39.0759	81.2359	278.7623**	328.6386	-22.6654**
	Score3	27.2902**	-0.8532	-0.0614	-87.5154	27.289**	-85.9872	-291.766
	Score4	-3.0351**	10.2524**	0.9546**	-1.3387	-2.1042**	-26.7702	0.66
	Score5	-8.8837**	12.9034**	4.1488**	-1.5622	-4.8265**	-6.7938	-15.2025**
Limit2	Score6	-4.5396**	26.4932**	1.183	12.8542	-5.2636**	-45.1036**	-14.7799**
	Score7	-7.4306**	33.2642**	0.7936	-12.0964	-9.2257**	-7.567	-30.2099**
	Score8	-16.3087**	16.3193**	3.4278**	76.0547**	-12.039**	7.9093	-30.0573**
	Score9	-14.0838**	25.8516**	1.6001**	1.7736	-13.6262**	2.4146	-45.4544**
	Score10	-12.6885**	22.6151**	0.1504	-15.0766	-12.4422**	-7.4659	-32.7389**
*	Inactive	-10.7899**	7.8501**	1.0616	6.3293	-9.8681**	-0.5015	-27.0971**
	Score2	411.8736**	16.5201	36.4301	532.861	456.2234**	510.3485	-13.1257**
	Score3	49.4775**	4.9526	6.2044	73.1301	57.0309**	41.6279	-239.389
	Score4	1.4271	15.7392**	0.6651	10.9915	1.746**	-33.1078	-16.827
	Score5	1.6234	2.0237	-0.2797	-21.2691	0.8725	-19.7551	-25.1816**
Limit3	Score6	-1.302	28.9518**	0.924	5.7559	-2.1548**	-55.2181**	-4.2457
	Score7	-2.3854	25.7776**	-0.0246	-32.5666	-4.6183**	-28.4017	-27.0626**
	Score8	-15.5996**	14.2887**	4.5041**	54.7111**	-9.7382**	-13.2995	-17.5913**
	Score9	-9.2428**	32.7705**	0.5628	8.3766	-11.0303**	2.5339	-50.1759**
	Score10	-10.6471**	15.4214**	0.4555	7.4277	-9.9652**	16.8391	-40.7071**
*	Inactive	-7.2087**	12.66**	-0.5701	4.6752	-8.8777**	-1.1136	-18.2486**
	Score2	737.4926**	645.3233**	-120.482	4624.642**	507.4964**	4621.78**	-13.9165**
	Score3	81.2191**	27.63**	8.4092	73.3454	89.3014**	5.7165	-870.067
	Score4	6.2193**	9.5883**	0.6851	-46.6896	6.8041**	-70.3389**	-52.6792**
	Score5	7.9118**	-2.321	-1.0552	-29.7434	6.7282**	-28.7597	-13.2103
Limit4	Score6	4.2659	21.4701**	1.616	-80.3688**	4.9743**	-128.276**	3.8966
	Score7	0.3548	30.25**	0.5543	3.9813	-1.4816**	-9.4204	-12.3936
	Score8	-9.7469**	14.9253**	2.9485**	37.0242**	-6.0557**	-22.844	-28.3432**
	Score9	-7.6635**	26.2463**	0.9096	22.5241**	-8.5994**	13.7181	-37.8108**
	Score10	-8.1003**	18.5199**	0.0973	-6.0123	-7.9553**	6.4242	-35.5675**
*	Inactive	-7.1295**	6.5489	1.1326	8.8427	-5.9961**	3.1158	-18.9816**
	Score2	472.305	-89.3181	-59.2603	5562.311	407.0812**	5436.591	-12.0381**
	Score3	35.6166**	0.913	-6.2328	-423.79**	27.456**	-396.28**	563.4893
	Score4	14.2468**	25.3934**	1.9928	-122.666**	16.2135**	-198.914**	17.0048
	Score5	12.1681**	26.0626**	1.3095	-71.8373**	11.0063**	-68.8031**	-44.4135**
Limit5	Score6	12.0357**	68.8429**	-3.1235**	-50.2859	1.7178	-129.709**	2.0867
	Score7	15.3386**	51.6189**	-2.5389	-11.9581	7.0394**	-38.4751	-40.0156**
	Score8	-4.6171**	30.6466**	4.5938**	3.8611	1.003	-127.827**	-37.8249**
	Score9	-2.9718	23.9459**	0.0213	-12.0273	-5.2621**	-17.1686	-70.3269**
	Score10	-2.2512	16.7127**	-0.3853	9.9748	-2.8224**	15.0063	-29.6748**

		Model A				Model B		
		Intercept	CPI	GDP	Sto	Intercept	Sto	Une
*	Inactive	-11.269**	2.6304	3.7948**	30.1363**	-6.1411**	13.0431	-18.6635**
	Score2	1002.299**	648.3062	-143.491	4638.61	734.4388**	4834.706	-23.1249**
	Score3	103.6091**	18.5247	-3.5756	-425.293**	97.7139**	-440.566**	-1112.71
	Score4	21.6806**	7.5589	-1.0106	-142.851**	20.0451**	-174.277**	10.1687
	Score5	10.1893**	4.813	1.2782	-44.8223**	11.5139**	-46.3047**	-20.8458
Limit6	Score6	20.3479**	65.0636**	-5.6353**	-4.1638	8.0249**	-77.2	0.2347
	Score7	4.9675	13.0412	2.296	46.744	7.1411**	32.3401	-10.2134
	Score8	-3.9135**	16.9798**	4.204**	21.2787	1.7293**	-61.3186**	-18.5596
	Score9	-1.0774	19.3093**	-0.226	-7.2416	-3.5007**	-12.8636	-53.2866**
	Score10	-1.7167	16.5275**	0.8579	-23.8279	-0.5005	-15.8561	-28.5711**
*	Inactive	-6.2059**	3.8204	0.7343	32.4462	-5.4406**	28.606	-22.0389**
	Score2	1036.574**	355.988	-42.6738	1571.089	948.9186**	1228.234	-7.2713
	Score3	105.4007**	22.4744	12.678	-220.827	120.4654**	-305.182	-269.568
	Score4	28.8792**	2.2252	-1.8697	-98.2443	26.1536**	-116.39	-41.2833
	Score5	26.307**	9.1832	-2.6972	-35.709	21.7421**	-32.0047	-9.203
Limit7	Score6	24.6361**	68.6549**	-7.4614**	42.6171	9.3888**	-29.9656	5.0352
	Score7	21.3802**	38.4778	-0.3737	-12.4996	17.5607**	-53.4876	-27.476**
	Score8	6.8614**	21.9639**	0.7991	-8.9695	7.3903**	-88.5413**	-56.9328**
	Score9	3.9585**	27.0434**	0.2194	11.6228	1.5882**	3.939	-40.628**
	Score10	1.1901	9.3634	1.1639	-3.6835	2.4408**	-22.5314	-33.6405**
*	Inactive	-6.214**	8.7956	0.387	25.2039	-6.3489**	21.4313	-23.1601**
	Score2	859.9291	-192.854	37.1118	-3431.84	895.6995**	-2129.56	-10.5347
	Score3	122.3396**	1.1542	11.904	-351.856	138.4856**	-410.833	-1007.66
	Score4	33.4883**	39.8575**	8.2529**	-109.443	43.1382**	-230.312**	-6.9752
	Score5	32.4987**	17.1476	0.6015	-62.6586	31.0428**	-61.6575	-97.2502**
Limit8	Score6	29.7688**	64.0968**	-6.69**	-35.5046	16.0985**	-104.32	-13.6931
	Score7	32.9796**	22.1864	-2.7254	127.4327**	26.8156**	123.6376**	-9.0948
	Score8	11.6055**	28.6259**	2.9159	66.5748**	15.1115**	-46.9523	-21.3188
	Score9	15.3585**	30.6195**	-1.7313	1.1328	9.9586**	2.1701	-68.6524**
	Score10	9.2006**	13.4436	1.3091	-4.5199	11.2297**	6.6827	-22.9635**
*	Inactive	-6.3104**	18.7997**	-0.0083	4.8743	-7.8336**	-3.2922	-17.7248**
	Score2	2039.398**	1934.912	-1006.95**	6274.578	593.5789**	5463.228	-20.7176**
	Score3	138.1412**	52.8303	19.2122	-82.7055	158.8928**	-270.416	3627.327
	Score4	56.6748**	40.9801**	7.0962**	-279.206**	65.2521**	-414.622**	-110.947
	Score5	64.7544**	61.1711**	-4.98	-32.2981	50.6602**	-18.4693	-114.045**
Limit9	Score6	50.6175**	73.538**	-10.0578**	101.2758	31.1876**	36.2594	-19.7691
	Score7	39.6195**	-13.1296	4.6166	109.7103	47.583**	91.0122	22.5024
	Score8	18.6675**	27.8591**	10.0041**	186.6794**	31.8888**	45.7102	-12.1779
	Score9	27.497**	39.1601**	-1.0779	6.4311	22.0986**	3.0783	-91.7331**
	Score10	17.8883**	15.2534	2.0842	-33.4373	20.6535**	-34.9298	-34.3554**
*	Inactive	-5.9539**	15.4533**	1.0608	6.2169	-5.6286**	-7.0448	-28.8285**
	Score2	-141.07	-3089.53	1014.917	-30780.8	1498.545	-22923.6	-25.2851**
	Score3	233.7135**	-172.097	79.4694	481.2159	350.4425**	499.435	-1253.47
	Score4	146.3401**	282.2437**	12.0842**	-110.884	152.3616**	-953.459**	-10.1771
	Score5	133.0962**	212.1593**	-4.7122	92.797	102.7878**	129.3594	-589.847**
Limit10	Score6	81.8955**	130.2314**	-6.065	18.0567	63.3873**	-165.545	-99.4686**
	Score7	100.658**	-65.5663	7.6289	-131.817	119.4381**	-93.7268	-114.741**
	Score8	70.3278**	87.1549**	16.8302**	236.953	92.3652**	-184.583	125.2321
	Score9	65.2711**	40.4923**	-0.0989	119.3744**	61.2412**	112.7179**	-263.467**
	Score10	70.4766**	37.9339	-2.169	201.4575**	66.968**	203.7054**	-36.4956**

”‘**”’ indicates the parameter is significant at 95% level.

”‘N”’ represents the cumulative logistic regression did not find any significant explanatory macroeconomic variable and therefore we did not build up the corresponding regression model.

Table C.4: Summary of regression estimates - Model A and Model B - Transactors

		Parameter α					Parameter β				
		CPI	GDP	Int	Sto	Une	CPI	GDP	Int	Sto	Une
	3+Cycle	-589.476**	-652.867**	-678.858**	N	-621.158**	540.0183**	12.1933	1935.966**	N	-570.256
	Inactive	132.1707**	284.7269**	196.0249**	157.0641**	145.8868**	-410.845**	-100.204**	-459.434**	-1038.64**	**
	Score1	-726.414**	-625.223**	-705.839**	-723.087**	-705.997**	-119.812	-73.1968**	357.6259	2335.848**	578.0048**
	Score2	210.5769**	168.7747**	204.5245**	202.977**	220.3502**	47.0052	29.8137**	18.5524	230.1352**	-80.5748
	Score3	186.1071**	165.388**	181.5737**	181.3783**	182.2409**	51.2494**	12.5236**	51.0053**	-55.781	-426.789**
Limit1	Score4	79.6252**	100.2156**	81.7486**	83.1569**	80.5202**	-11.5397	-14.0637**	-77.2417**	-578.707**	-39.1395**
	Score5	35.9541**	36.0818**	35.0729**	37.2323**	36.6675**	20.1712	-0.3082	10.5024	-216.928**	112.4593**
	Score6	36.7358**	45.2547**	37.0719**	37.2412**	36.942**	-5.0708	-5.8936	-5.339	-176.594	35.7146**
	Score7	38.7511**	43.0125**	39.4874**	37.1168**	37.7011**	-34.1999	-3.5275	-30.6547	70.6016	21.5649
	Score8	28.4333**	48.2992**	39.309**	21.422**	25.6156**	-96.5318**	-16.416**	-122.897**	6.2876	9.2566
	Score9	53.1967**	2.4119	80.0707**	45.1257**	43.4504**	-115.319**	27.0243**	-214.098**	93.5587	72.2372**
	Score10	15.5305**	-98.1335**	26.5883**	15.4645**	12.5569**	1.6361	75.6476**	-55.1338**	40.1743	-34.5416
	3+Cycle	-626.062**	-599.913**	-679.67**	N	-642.025**	351.0271	-44.6027	455.0281	N	-97.1895**
	Inactive	143.3644**	278.8875**	198.5966**	171.2019**	157.5448**	-322.463**	-84.131**	-417.076**	-671.19	-1319.97**
	Score1	-1165.64**	-1053.82**	-1139.04**	-1154.54**	-1131.35**	-176.911	-75.772	-133.668	1672.525	569.7727**
	Score2	263.1256**	214.7522**	251.4231**	246.9607**	284.1307**	73.6651	32.5161**	519.643**	883.9659**	-117.922
	Score3	236.4344**	206.5417**	230.6621**	229.5655**	232.9979**	58.3425**	19.0849**	70.8619**	33.4127	-786.001**
Limit2	Score4	83.3711**	119.6494**	87.5449**	92.815**	84.4358**	-37.8212**	-23.7338**	-200.094**	-887.039**	-85.6539**
	Score5	28.1622**	31.6026**	27.6523**	28.9032**	27.6479**	12.5546	-2.9512	-11.4431	-206.502**	203.0517**
	Score6	25.1702**	64.0061**	28.6533**	28.5732**	29.4349**	-66.5041**	-24.1028**	-34.2657	-150.604	37.916**
	Score7	35.1698**	41.3571**	37.4351**	35.8431**	36.1907**	-41.1995**	-3.4872	-58.2612**	-203.686**	127.6465**
	Score8	27.8177**	65.0963**	40.4629**	18.848**	27.5288**	-156.733**	-28.1978**	-158.41**	230.0776**	47.8762**
	Score9	50.8879**	-1.1484	74.9865**	45.0136**	43.9284**	-114.41**	29.0376**	-212.146**	57.7143	141.0646**
	Score10	24.9834**	-88.2682**	36.975**	23.7248**	22.0248**	-11.9678	73.9445**	-68.8702**	64.8588	-23.7659
	3+Cycle	-989.195**	-1136.83**	-1104.38**	N	-1030.51**	706.1175	23.519	769.1358	N	-79.5334**
	Inactive	152.0291**	385.0957**	235.0581**	194.0832**	172.4272**	-591.304**	-149.365**	-609.972**	-1746.87**	-2132.62
	Score1	-1728.38**	-1274.52**	-1553.13**	-1565.29**	-1581.01**	-1272.6**	-241.526**	-803.251	1837.044	1003.571**
	Score2	360.9427**	333.9845**	364.7962**	357.4635**	384.9157**	-11.5798	23.0242	-373.32**	552.5418**	821.7551
	Score3	337.8628**	304.0449**	329.1054**	328.981**	331.3989**	92.2305**	19.3145**	88.534**	-81.1016	-721.576**
Limit3	Score4	142.2967**	175.8483**	145.5521**	149.6669**	143.6979**	-13.4745	-22.8672**	-171.799**	-883.88**	-115.249**
	Score5	44.3334**	52.0154**	43.5489**	45.5086**	43.7816**	18.8717**	-6.1434**	-8.2368	-241.706**	188.3631**
	Score6	33.2041**	75.3055**	37.3402**	35.8812**	38.0534**	-87.7459**	-26.5161**	-65.6476**	-158.238	46.9222**
	Score7	44.9916**	66.6265**	46.7044**	45.7435**	46.5086**	-34.8524**	-13.7581**	-39.3326	-134.567	151.9286**
	Score8	30.7364**	62.6682**	42.4832**	21.4444**	28.9532**	-145.024**	-25.0098**	-149.923**	180.4596**	63.8178**
	Score9	55.1728**	20.6549**	83.491**	49.0906**	51.3765**	-154.44**	17.8408**	-244.105**	-50.9809	118.1039**
	Score10	17.3497**	-82.2507**	24.025**	17.0127**	16.7889**	2.4999	65.7149**	-36.1597**	210.34**	55.0849**
	3+Cycle	-945.672**	-1076.64**	-1157.37**	N	-1077.63**	1269.029	-49.8221	2065.578	N	-68.3446**
	Inactive	134.4958**	293.1784**	198.5781**	157.0243**	150.7184**	-431.05**	-103.698**	-493.223**	-874.149	-1300.57
	Score1	-2230.47**	-1850.13**	-2068.79**	-2164.99**	-2085.45**	-1240.88**	-183.853	122.4531	7974.574**	701.1201**
	Score2	483.1007**	440.6321**	479.216**	462.1288**	501.4871**	41.6681	29.9936	-158.961	1579.645**	685.3529
	Score3	456.9509**	398.2886**	446.6615**	443.8943**	451.1081**	120.8149**	36.7053**	301.5618**	314.3507**	-885.957**
Limit4	Score4	218.2107**	243.5684**	214.3208**	221.6059**	213.9816**	76.8602**	-21.2658**	-78.9762**	-1242.85**	-266.026**
	Score5	63.4668**	66.4545**	60.6767**	64.3198**	61.3303**	44.901**	-3.9451	6.353	-339.441**	162.3963**
	Score6	40.6956**	87.1991**	44.251**	42.1616**	44.623**	-96.6955**	-29.1289**	-52.4156**	44.5634	42.6131**
	Score7	40.8362**	62.3461**	42.3728**	42.384**	42.7358**	-41.9802**	-13.3732**	-11.8836	-175.732	136.3879**
	Score8	34.1145**	80.9962**	47.1143**	26.2859**	33.9535**	-160.015**	-34.6502**	-165.107**	2.3579	69.9161**
	Score9	62.1984**	43.0225**	91.6405**	61.4656**	62.9238**	-149.096**	11.9091**	-254.697**	66.133	159.0859**
	Score10	17.0265**	-71.7853**	21.0372**	17.8274**	18.551**	19.593**	59.2561**	-21.063**	243.3034**	74.899**
	3+Cycle	-945.515**	-802.499**	-882.934**	N	-932.662**	62.2539	-131.802	-1587.17	N	-86.8016**
	Inactive	183.1788**	461.8471**	280.2913**	223.2145**	223.3194**	-700.818**	-177.735**	-702.365**	-999.518	-737.782
	Score1	-2193.28**	-1252.13**	-2038.77**	-2055.69**	-2097.34**	-992.822	-718.475**	-627.634	11552.46**	1141.889**
	Score2	519.5173**	653.6486**	576.1555**	563.7293**	560.7166**	-385.076**	-70.2121	-1304.35**	815.676	1243.851
	Score3	341.4394**	244.7057**	316.8096**	316.5162**	325.0376**	237.0486**	56.6535**	477.34**	-517.463**	179.9627
Limit5	Score4	149.6001**	128.5132**	134.478**	139.855**	136.1966**	153.2525**	5.4253	178.16**	-660.122**	-357.039**
	Score5	55.0133**	40.7192**	46.5975**	51.2724**	48.4507**	77.1402**	5.6263**	116.4199**	-318.074**	-20.1855
	Score6	72.7076**	74.1391**	71.069**	72.0926**	72.4376**	17.3414	-1.6671	26.9628	-71.4334	-24.5588
	Score7	66.4047**	70.9708**	70.4781**	70.9422**	71.6923**	-57.2328**	-0.1141	50.942	443.0271**	-34.4906
	Score8	89.9701**	226.243**	133.0355**	83.2877**	98.7727**	-462.907**	-95.5751**	-497.599**	-738.631**	-28.4357
	Score9	138.1397**	94.0728**	192.7188**	132.454**	132.0767**	-363.142**	23.3403**	-556.463**	-197.945	492.5526**
	Score10	53.4816**	-128.794**	95.3129**	53.7785**	51.0369**	21.5349	122.3565**	-212.903**	339.9258**	4.6902

		Parameter α					Parameter β				
		CPI	GDP	Int	Sto	Une	CPI	GDP	Int	Sto	Une
Limit6	3+Cycle	-508.098	-688.78	-1104.25	N	-1164.35**	6633.009**	-186.757	4107.207	N	-181.37**
	Inactive	207.3904	535.7143	355.0599**	276.3184	292.3012**	-1207.02	-199.97	-976.378	-2151.66	-11904.3**
	Score1	-2517.83**	-2544.59**	-2588.44**	-2653.19**	-2542.35**	442.7493	-20.6895	2786.287	9051.084**	2738.085**
	Score2	668.6921**	604.7199**	703.323**	689.1259**	723.6226**	-219.248	72.8644**	-2285.17**	661.6692	-1716.2
	Score3	560.6524**	491.8027**	539.7969**	543.9694**	544.7303**	208.8659**	37.4567**	158.4942**	-699.441**	-1010.2**
	Score4	261.5233**	312.5654**	260.4673**	268.1331**	259.8994**	43.0642**	-37.5938**	-123.834**	-1676.24**	-229.743**
	Score5	80.638**	82.9332**	74.2809**	82.8068**	77.1835**	66.4993**	-4.5954	72.3217**	-656.388**	296.477**
	Score6	63.7998**	109.2658**	70.2012**	65.8335**	66.4577**	-98.8086**	-30.4576**	-95.9485**	-125.336	67.0196**
	Score7	64.897**	96.175**	71.4259**	70.3212**	69.8857**	-101.394**	-18.0868**	-80.4476**	-103.021	145.2503**
	Score8	76.7843**	166.0883**	104.3113**	62.6419**	81.3003**	-354.808**	-65.8407**	-351.655**	-203.387	66.3159**
	Score9	117.9139**	75.6676**	159.8568**	115.6049**	114.2267**	-233.935**	24.5022**	-410.989**	-8.8528	378.5472**
	Score10	42.095**	-94.4886**	66.3592**	41.9652**	40.9738**	10.8063	90.5989**	-129.396**	269.0831**	-34.0276
Limit7	3+Cycle	-830.459	-1163.27	-1365.21**	N	-1030.38**	2396.046	6.6174	6196.54	N	-133.332**
	Inactive	220.1053**	427.9145**	271.688**	226.4517**	241.8533**	-310.582	-134.478**	-352.804	563.027	-6391.13
	Score1	-3453.24**	-2928.26**	-3211.23**	-3381.25**	-3208.4**	-2015.37	-221.142	-1444.06	16612.56**	696.7501**
	Score2	900.4301**	966.1416**	934.9342**	898.3457**	944.0526**	-198.471	-32.4527	-2554.29**	2307.596**	-387.374
	Score3	691.8947**	629.6459**	679.1546**	679.4322**	683.547**	132.1378**	37.9557**	258.5684**	-104.018	-862.006**
	Score4	347.4891**	382.7152**	335.1165**	346.5519**	339.8841**	167.0017**	-30.7803**	94.6255**	-1433.13**	-248.706**
	Score5	104.5996**	105.3119**	96.2287**	105.237**	99.5089**	86.1631**	-4.5484	93.1708**	-647.095**	208.3689**
	Score6	87.0896**	132.7649**	92.617**	90.1064**	89.3736**	-75.2282**	-31.0026**	-78.9929**	-246.845	54.4705**
	Score7	63.2127**	108.763**	71.1618**	69.9407**	68.8862**	-129.362**	-26.9323**	-106.264	-318.264	136.5544**
	Score8	90.7943**	180.7055**	113.3235**	76.1858**	93.2992**	-366.678**	-66.5851**	-323.91**	-221.327	191.8066**
	Score9	118.8952**	125.4346**	166.8602**	115.2208**	116.1713**	-233.169**	-5.9626	-479.343**	160.2878	357.559**
	Score10	46.1196**	-100.445**	65.724**	46.896**	46.01**	26.7321**	97.4694**	-104.903**	442.2681**	4.5407
Limit8	3+Cycle	-2144.76	-2131.94	-2514.97	N	-2111.47**	-148.928	-3.2807	8205.221	N	-159.045**
	Inactive	174.0745**	124.4976	130.7941**	140.3386**	149.334**	335.3393	15.4821	186.3951	367.6366	-18398**
	Score1	-5127.42**	-2611.67**	-3939.17**	-4096.83**	-4081.22**	-8674.06**	-1096.89**	-7325.15	17496.17	-291.721
	Score2	1221.021**	1327.336**	1270.985**	1243.177**	1270.172**	-353.794	-48.6526	-1082.83	2089.465	6295.343
	Score3	831.3344**	730.0231**	805.9993**	819.3484**	811.7843**	260.6189**	59.0617**	394.9048**	-1311.31**	-224.258
	Score4	441.5572**	441.654**	411.8866**	435.2897**	426.7831**	331.0442**	-10.7445	460.7792**	-1473.03**	-366.367**
	Score5	135.0847**	133.8592**	120.6261**	129.9514**	127.3387**	129.8528**	-4.4628	222.3231**	-263.821**	60.3813
	Score6	102.7731**	145.8867**	106.3849**	103.6869**	103.6312**	-23.9317	-30.5342**	-57.9062	31.1209	-0.4084
	Score7	87.6932**	122.4062**	93.2632**	92.1609**	92.6017**	-96.2078**	-19.9921**	-47.999	465.6923**	92.3045**
	Score8	95.637**	173.5025**	126.9482**	81.9356**	98.1386**	-340.62**	-56.3397**	-357.397**	698.2946**	39.5971
	Score9	164.5732**	151.4455**	215.3614**	162.97**	158.1443**	-176.962**	7.5662	-498.817**	250.8851	292.9201**
	Score10	68.7325**	-129.474**	89.1784**	71.074**	71.2833**	58.0314**	132.956**	-105.908**	536.7743**	-186.929**
Limit9	3+Cycle	-997.349	-687.895	-731.143	N	-381.319	-1321.46	-13.3359	2009.37	N	-215.412**
	Inactive	179.9119	485.8094**	308.2664**	254.3207**	207.6774**	-689.412	-179.189	-582.995	1425.938	-5376.57
	Score1	-6218.56**	-4915.09**	-5589.38**	-5589.55**	-5687.66**	-4309.35	-576.814	-6077.92	1981.737	1107.2
	Score2	1301.003**	1601.852**	1454.13**	1390.38**	1421.578**	-881.902**	-148.49	-6811.43**	2447.198	2631.335
	Score3	1046.631**	966.1418**	1026.628**	1042.182**	1029.964**	222.2956**	45.8896**	86.6749	-1673.78**	-177.554
	Score4	603.169**	573.3785**	556.3224**	589.7596**	577.9524**	559.5894**	4.0796	691.9489**	-1552.15**	-311.516**
	Score5	202.1713**	212.2478**	185.8749**	202.0496**	194.9607**	149.4958**	-12.9216	257.1659**	-858.91**	-67.364
	Score6	145.672**	151.6507**	133.0885**	140.8912**	142.0693**	78.4912**	-7.0594	190.7325**	120.3753	90.0317**
	Score7	125.9778**	213.753**	145.4574**	141.1333**	141.1483**	-283.87**	-48.929**	-310.901**	204.3843	-72.0838
	Score8	137.8173**	246.7808**	180.9571**	122.0906**	139.6815**	-424.868**	-77.1658**	-478.117**	1048.53**	261.1408**
	Score9	181.2363**	186.8364**	241.6393**	180.2449**	177.0277**	-202.611**	-3.5916	-572.244**	225.198	314.7024**
	Score10	85.6312**	-145.169**	105.5818**	87.6935**	88.2817**	52.7686**	152.7884**	-107.837**	485.5899**	-165.349**
Limit10	3+Cycle	-240.145**	-259.441**	-253.987**	N	-255.299**	105.5777**	1.2171	925.3441	N	-215.179**
	Inactive	73.6662	-74.1425	-55.7597	28.0795	29.0486	613.2376	70.6857	1029.174**	-358.868	-121.131
	Score1	-9933.37**	-8476.72**	-9229.76**	-9686.91**	-9416.96**	-6023.89	-776.353	-6874.85	34086.14	-598.258
	Score2	2208.515**	2833.378**	2569.097**	2474.082**	2425.416**	-2254.66**	-281.603	-10402.3**	225.6363	7252.259
	Score3	1796.247**	1567.981**	1745.413**	1772.047**	1753.169**	559.0514**	138.9789**	846.7927**	-668.273	2337.611
	Score4	1292.784**	1022.114**	1146.718**	1314.451**	1224.759**	1819.893**	170.5733**	2311.776**	-2182.06**	-1170.59**
	Score5	473.3263**	433.0826**	396.6362**	457.5807**	442.5904**	656.3879**	11.1738	1228.398**	-719.708	-1552.14**
	Score6	394.5633**	267.3071**	352.9707**	376.381**	380.0842**	320.2606**	83.797**	594.369**	1498.907**	-250.546**
	Score7	294.1524**	447.209**	318.5244**	304.8226**	316.841**	-491.539**	-88.3283**	-188.986	1830.378**	-595.896**
	Score8	229.8247**	355.0325**	281.1884**	214.5434**	232.498**	-446.4**	-89.6367**	-565.364**	-48.3109	337.8152**
	Score9	288.0819**	259.5032**	347.0945**	285.0907**	274.9195**	-210.36**	15.6453	-590.396**	-146.499	428.0257**
	Score10	153.4814**	-129.938**	197.7582**	157.1834**	155.2927**	88.6204	190.1618**	-224.685**	810.3713**	-249.969**

"'**"' indicates the parameter is significant at 95% level.

"'N"' represents the cumulative logistic regression did not find any significant explanatory macroeconomic variable and therefore we did not build up the corresponding regression model.

Table C.5: Summary of regression estimates - Revolvers

		Model A				Model B		
		Intercept	CPI	GDP	Sto	Intercept	Sto	Une
	3+Cycle	-608.665**	541.825**	13.3096	N	N	N	N
	Inactive	167.8658**	-422.192**	-13.556	-1026.07**	145.4118**	-399.854	534.1021**
	Score1	-618.367**	95.5701	-78.853	2258.543**	-730.538**	2481.055**	**
	Score2	164.095**	0.1869	32.0385**	280.561**	219.5362**	77.8121	153.8681
	Score3	175.4652**	35.3775**	7.2632**	-5.3627	183.1084**	-120.418	-419.947**
	Score4	118.9694**	71.8018**	-21.8699**	-624.427**	82.5486**	-433.6**	-49.5764**
Limit1	Score5	47.5622**	44.2644**	-6.1175	-264.945**	37.6115**	-181.116**	74.2642**
	Score6	52.8225**	31.8727	-10.7361	-154.47	37.2402**	-161.6	21.9561
	Score7	26.1025**	-76.1248**	8.5149	251.5856	38.0187**	148.1691	4.78
	Score8	21.1587	-135.932**	4.6599	383.2693**	25.4402**	257.2657**	24.9192
	Score9	4.6298	-138.492**	30.6452**	281.0833**	43.7447**	74.662	98.4003**
	Score10	-97.6155**	-16.0179	75.9123**	117.7843	13.1861**	-149.392	-24.2986
	3+Cycle	-585.009**	208.5566	-37.4409	N	N	N	N
	Inactive	189.2889**	-207.703**	-18.256	-841.548	158.0485**	-273.315	-103.952**
	Score1	-1060.6**	59.6019	-75.791	1577.996	-1157.45**	1709.482	548.6295**
	Score2	220.1287**	79.5819	30.4345**	1005.514**	277.2872**	641.19**	51.2366
	Score3	215.9044**	33.9892**	14.2666**	97.4808	233.5655**	-74.1046	-732.616**
	Score4	142.0171**	66.8011**	-30.9698**	-900.343**	89.3444**	-660.296**	-92.3926**
Limit2	Score5	42.583**	36.1439**	-7.823**	-223.005**	28.8394**	-171.807**	148.5383**
	Score6	68.3405**	14.5559	-26.4364**	-57.3343	28.9076**	177.3458**	25.7403**
	Score7	15.4296	-75.7836**	12.5052**	-80.2311	36.1356**	-37.5137	144.6918**
	Score8	7.5003	-202.21**	12.0991**	452.6161**	25.547**	560.1229**	43.6548**
	Score9	3.0382	-131.338**	29.8312**	250.0075**	44.0947**	40.6436	183.6706**
	Score10	-87.6937**	-50.5872**	75.2592**	405.3528**	22.3513**	-120.602	-17.7913
	3+Cycle	-1052.03**	941.7419	59.0208	N	N	N	N
	Inactive	165.9347**	-186.717	14.7108	-1605.18**	173.8763**	-588.027	-86.08**
	Score1	-1532.15**	-864.562	-137.775	1686.05	-1621.72**	2785.214	934.2601**
	Score2	310.8982**	-41.6908	33.5048**	569.4752**	381.2785**	321.4003	1113.146**
	Score3	326.5779**	78.759**	7.571	53.2953	333.2871**	-224.083**	-696.407**
	Score4	209.1642**	100.6655**	-35.9076**	-945.157**	148.5349**	-692.161**	-133.18**
Limit3	Score5	70.8626**	58.751**	-15.3817**	-255.294**	45.5443**	-204.794**	136.453**
	Score6	68.8822**	-20.6663	-22.4853**	-29.7424	37.3277**	253.8777**	34.4336**
	Score7	73.1546**	22.8066	-17.7454**	-34.3276	46.5699**	130.9396	175.7676**
	Score8	16.475**	-194.656**	8.2284**	563.0012**	27.8029**	553.003**	78.0898**
	Score9	25.086**	-163.284**	18.9033**	129.8123**	51.3849**	5.6192	167.8888**
	Score10	-82.3675**	-54.6953**	67.2446**	578.201**	16.7471**	25.3674	55.7885**
	3+Cycle	-920.178**	1219.531	-33.3509	N	N	N	N
	Inactive	91.852**	-60.3289	42.1013	-723.943	150.9966**	-162.014	-66.583**
	Score1	-2294.13**	-1123.34	-12.2761	7764.791**	-2203.11**	8635.538**	684.181**
	Score2	428.5635**	85.5809	33.7619	1729.388**	485.2536**	1394.35**	1247.61
	Score3	418.8322**	85.8355**	24.8266**	477.2646**	450.8351**	36.9563	-806.398**
	Score4	318.8417**	249.4466**	-54.5668**	-1415.21**	221.3421**	-1129.59**	-263.182**
Limit4	Score5	95.5524**	91.1487**	-18.449**	-343.01**	64.3319**	-314.978**	78.5357**
	Score6	78.9495**	-34.572	-24.7287**	255.4603**	43.6417**	558.9888**	25.5303
	Score7	63.4335**	7.1216	-13.8836**	-72.1157	42.7023**	87.1951	193.5289**
	Score8	38.1884**	-179.798**	-3.9704	526.2762**	33.1843**	528.0651**	79.4172**
	Score9	37.1442**	-173.356**	16.0788**	289.9013**	63.231**	189.3944**	208.9965**
	Score10	-74.0727**	-23.8917**	61.4816**	461.6603**	18.5651**	-68.8148	98.2221**
	3+Cycle	-769.365**	272.9027	-137.934	N	N	N	N
	Inactive	262.6267**	-546.564**	-35.2317	-1192.37	223.3615**	-156.734	-92.2444**
	Score1	-1039.71	1045.434	-777.735**	10776.75**	-2234.41**	13722.69**	1124.494**
	Score2	550.6622**	-313.086	-19.8767	481.3653	553.9236**	898.5671	2847.921
	Score3	299.1041**	174.3174**	29.957**	-151.279	334.4695**	-894.162**	250.6417
	Score4	199.5949**	248.7282**	-26.0238**	-899.09**	140.8766**	-790.218**	-428.32**
Limit5	Score5	71.8385**	101.6588**	-8.4337**	-354.92**	51.6435**	-365.025**	-84.3134**
	Score6	93.8858**	58.6397	-14.2119	-68.6091	73.4291**	-230.701	-47.1616**
	Score7	22.0937	-163.519**	27.3063**	524.6352**	70.0199**	534.808**	-58.5782
	Score8	63.8511**	-508.136**	17.3331**	59.9017	98.0551**	225.4379	30.8688
	Score9	101.958**	-368.273**	22.211**	105.6775	131.431**	-213.279	512.8538**
	Score10	-127.401**	-32.2552	121.8999**	449.6275**	51.286**	-78.4752	-21.8486

		Model A				Model B		
		Intercept	CPI	GDP	Sto	Intercept	Sto	Une
Limit6	3+Cycle	-345.834	6524.186	-119.17	N	N	N	N
	Inactive	281.1815	-447.246	-4.797	-2477.5	300.7125**	-890.979	-185.472**
	Score1	-2612.06**	411.0281	5.7844	9056.063**	-2629.5**	8454.624**	2683.382**
	Score2	449.026**	-486.338**	146.0205**	507.7865	718.991**	400.8091	-1138.6
	Score3	547.6933**	173.6623**	9.5823	-392.699**	553.5335**	-988.743**	-983.535**
	Score4	395.1878**	292.0996**	-73.0854**	-1897.89**	267.249**	-1369.6**	-303.786**
	Score5	126.251**	128.5769**	-24.374**	-696.426**	82.941**	-627.037**	184.2939**
	Score6	97.2443**	-39.3629	-22.7909**	83.7152	65.45**	285.579	34.8309**
	Score7	58.6893**	-116.512**	3.7285	83.7882	69.7187**	124.7742	171.6415**
	Score8	57.8361**	-399.848**	11.2345	427.8048**	79.8845**	517.6081**	79.4703**
	Score9	81.0031**	-236.469**	22.4322**	147.6694	114.1796**	-36.3704	420.9147**
	Score10	-94.7551**	-46.5511**	91.6715**	581.0585**	41.175**	-82.9317	-37.6512
Limit7	3+Cycle	-1059.57	2982.995	152.5289	N	N	N	N
	Inactive	290.804**	-252.648	-39.0711	637.402	228.2105**	1283.484	-138.568**
	Score1	-3669.69**	-1869.2	55.7608	16071.87**	-3394.18**	16979.09**	819.8292**
	Score2	899.0643**	-48.0382	-4.5081	2252.139**	917.2043**	2133.256**	629.9873
	Score3	654.2682**	87.2445**	24.9646**	114.9726	686.5436**	-325.701	-731.745**
	Score4	505.193**	436.8226**	-87.6908**	-1840.85**	346.8539**	-1264.91**	-270.143**
	Score5	155.9881**	155.6847**	-28.3943**	-687.085**	105.3363**	-628.415**	112.0829**
	Score6	136.963**	13.7704	-33.5237**	-58.5163	89.0744**	60.8203	25.0912
	Score7	67.847**	-117.378	-2.6905	-96.0805	68.3755**	302.2479	142.0301**
	Score8	64.3109**	-436.401**	16.2448	623.919**	92.4551**	604.5043**	223.3428**
	Score9	121.2361**	-244.844**	-2.3455	304.4313**	115.9034**	177.3192	410.5843**
	Score10	-101.906**	-54.7328**	99.5656**	828.4548**	46.0176**	-7.1304	22.2996
Limit8	3+Cycle	-2112.39	-192.886	-15.4273	N	N	N	N
	Inactive	102.9617**	252.71	32.249	693.1349	145.619**	261.0052	-159.511**
	Score1	-5128.89**	-8225.29**	-78.8903	15676.39	-4336.69**	21487.68**	-279.206
	Score2	1206.075**	-255.791	6.1552	1849.095	1245.819**	2064.93	7816.043**
	Score3	792.8551**	162.2731**	28.8046**	-880.502**	827.7735**	-1592.63**	-100.682
	Score4	604.8882**	605.2473**	-90.224**	-2120.31**	434.6698**	-1538.63**	-448.835**
	Score5	195.1599**	216.4419**	-35.2534**	-350.264**	129.8289**	-276.277**	-48.7089
	Score6	196.7721**	128.3917**	-64.5441**	272.0128	102.1154**	200.8641	-14.8143
	Score7	102.6552**	-85.1462	-10.1239	652.3412**	91.705**	779.48**	108.278**
	Score8	52.6418**	-459.364**	23.9228**	1401.827**	93.1178**	1581.571**	118.3002**
	Score9	152.1814**	-196.167**	6.7144	403.7163**	158.1037**	88.4952	429.395**
	Score10	-132.997**	0.9267	134.956**	613.3233**	71.4108**	-188.107	-177.913**
Limit9	3+Cycle	-974.648	-1841.69	-78.05	N	N	N	N
	Inactive	79.0348	-723.959	89.6216	1420.622	177.3356	2001.705	-229.429**
	Score1	-6214.67**	-4277.77	-5.3334	991.7727	-5719.12**	3240.537	1215.797
	Score2	1337.244**	-722.717	-25.9695	1601.754	1390.65**	2444.817	2838.352
	Score3	1024.322**	124.0834	19.8272	-1387.88**	1049.334**	-1997.61**	-8.3596
	Score4	814.595**	913.9354**	-117.983**	-2594.38**	586.7395**	-1836.91**	-428.859**
	Score5	300.8787**	288.1512**	-55.5135**	-977.471**	202.4467**	-817.594**	-201.669**
	Score6	246.8666**	238.6387**	-69.7213**	374.949	141.8171**	30.0755	47.9862
	Score7	112.8593**	-320.168**	6.7665	528.5107	138.5816**	1103.714**	-69.9198
	Score8	109.4955**	-548.353**	11.6802	2107.534**	133.0201**	2136.621**	371.7614**
	Score9	184.6312**	-218.902**	-2.9291	382.8187**	176.942**	81.0702	503.2141**
	Score10	-148.263**	-11.5441	155.0121**	666.4523**	88.3943**	-260.525	-157.102**
Limit10	3+Cycle	-242.582**	139.4636**	5.3337	N	N	N	N
	Inactive	-7.5464	308.5877	30.6448	174.8047	32.1094	-181.775	-234.509**
	Score1	-11856.8**	-9271.27	1099.105	39992.58	-9700.95**	37496.12	-590.779
	Score2	2095.061**	-2566.44**	85.8759	-2384.6	2415.72**	662.0061	8976.736
	Score3	1691.844**	448.9804**	65.4526	516.8665	1765.585**	-1212.85	2370.186
	Score4	1549.22**	2200.589**	-136.945**	-4496.65**	1247.104**	-4110.39**	-1212.58**
	Score5	688.9073**	946.6731**	-127.631**	-1083.39**	451.1442**	-940.449**	-1798.14**
	Score6	414.4702**	371.947**	-22.0442	1598.199**	374.2596**	810.889	-293.009**
	Score7	280.6083**	-528.01**	1.3939	2163.552**	305.5247**	3086.183**	-543.039**
	Score8	227.0274**	-500.162**	-1.2231	1129.02**	230.2443**	1152.272**	619.8885**
	Score9	256.4719**	-217.257**	19.8249	0.0047	274.7839**	-366.065	533.0457**
	Score10	-133.609**	6.5101	191.2582**	822.6668	155.893**	-444.124	-289.206**

"'**"' indicates the parameter is significant at 95% level.

"'N"' represents the cumulative logistic regression did not find any significant explanatory macroeconomic variable and therefore we did not build up the corresponding regression model.

Table C.6: Summary of regression estimates - Model A and Model B - Revolvers

Appendix D

HK Data - Optimal Policy

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Credit limit 2															
3+Cycle	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	10
Score2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 3															
3+Cycle	3	3	3	3	3	3	3	3	3	3	3	3	3	3	10
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	10
Score2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 4															
3+Cycle	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Score2	4	4	4	4	4	4	4	5	5	5	5	5	5	5	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 5															
3+Cycle	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Score2	6	5	5	5	5	6	5	5	5	5	5	5	5	5	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Credit limit 7															
3+Cycle	7	7	7	7	7	7	7	7	7	7	7	7	7	7	10
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	7	7	7	7	7	7	7	7	7	7	7	7	7	7	10
Score2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 8															
3+Cycle	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 9															
3+Cycle	9	9	9	9	9	9	9	9	9	9	9	9	9	9	10
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	9	9	9	9	9	9	9	9	9	9	9	9	9	9	10
Score2	9	9	9	9	9	9	9	9	9	9	9	9	9	9	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 10															
3+Cycle	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score2	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

Table D.1: Summary of optimal policy for All accounts (Results of Credit Limit 1 and Limit 6 are presented in Table 5.12)

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Credit limit 2															
3+Cycle	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Inactive	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	10
Score2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	10
Score3	10	8	10	8	10	10	10	2	2	2	2	2	2	2	10
Score4	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score5	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score6	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score7	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score8	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 3															
3+Cycle	3	3	3	3	3	3	3	3	3	3	3	3	3	3	10
Inactive	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	10
Score2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	10
Score3	10	8	10	8	10	10	10	3	3	3	3	3	3	3	10
Score4	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score5	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score6	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score7	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score8	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 4															
3+Cycle	4	4	4	4	4	4	4	4	4	4	4	4	4	4	10
Inactive	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score1	4	4	4	4	4	4	4	4	4	4	4	4	4	4	10
Score2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	10
Score3	10	8	10	8	10	10	10	4	4	4	4	4	4	4	10
Score4	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score5	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score6	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score7	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score8	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 5															
3+Cycle	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Inactive	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Score2	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Score3	10	8	10	8	10	10	10	6	6	6	6	6	6	6	10
Score4	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score5	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score6	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score7	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score8	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Credit limit 7															
3+Cycle	7	7	7	7	7	7	7	7	7	7	7	7	7	7	10
Inactive	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score1	7	7	7	7	7	7	7	7	7	7	7	7	7	7	10
Score2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	10
Score3	10	8	10	8	10	10	10	7	7	7	7	7	7	7	10
Score4	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score5	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score6	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score7	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score8	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 8															
3+Cycle	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Inactive	10	10	10	10	10	10	10	8	8	8	8	8	8	8	10
Score1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score3	10	8	10	8	10	10	10	8	8	8	8	8	8	8	10
Score4	10	10	10	10	10	10	10	8	8	8	8	8	8	8	10
Score5	10	10	10	10	10	10	10	8	8	8	8	8	8	8	10
Score6	10	10	10	10	10	10	10	8	8	8	8	9	8	8	10
Score7	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score8	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 9															
3+Cycle	9	9	9	9	9	9	9	9	9	9	9	9	9	9	10
Inactive	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score1	9	9	9	9	9	9	9	9	9	9	9	9	9	9	10
Score2	9	9	9	9	9	9	9	9	9	9	9	9	9	9	10
Score3	10	9	10	9	10	10	10	9	9	9	9	9	9	9	10
Score4	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score5	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score6	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score7	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score8	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 10															
3+Cycle	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score2	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

Table D.2: Summary of optimal policy for All accounts (loss equals to the credit limit) and the results of Credit Limit 1 and Limit 6 are presented in Table 5.13)

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
							Credit limit 2								
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	8
Score2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
							Credit limit 3								
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	8
Score2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
							Credit limit 4								
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	8
Score2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
							Credit limit 5								
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	8
Score2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Credit limit 7															
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
Score2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Credit limit 8															
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
Score2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Credit limit 9															
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score1	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
Score2	10	9	10	9	9	10	9	9	9	9	9	9	9	9	9
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Credit limit 10															
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score2	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

Table D.3: Summary of optimal policy for Transactors (Results of Credit Limit 1 and Limit 6 are presented in Table 6.11)

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Credit limit 2															
Inactive	10	10	10	10	10	10	10	7	7	7	7	7	7	7	8
Score1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	8
Score2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	8
Score3	4	4	4	4	4	4	4	3	3	3	3	3	3	3	8
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Credit limit 3															
Inactive	10	10	10	10	10	10	10	7	7	7	7	7	7	7	8
Score1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	8
Score2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	8
Score3	4	4	4	4	4	4	4	3	3	3	3	3	3	3	8
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Credit limit 4															
Inactive	10	10	10	10	10	10	10	7	7	7	7	7	7	7	8
Score1	4	4	4	4	4	4	4	4	4	4	4	4	4	4	8
Score2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	8
Score3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	8
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Credit limit 5															
Inactive	10	10	10	10	10	10	10	7	7	7	7	7	7	7	8
Score1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	8
Score2	6	6	6	6	6	6	6	6	6	6	6	6	6	6	8
Score3	7	7	7	7	7	7	7	6	6	6	6	6	6	6	8
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
							Credit limit 7								
Inactive	10	10	10	10	10	10	10	7	7	7	7	7	7	7	8
Score1	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8
Score2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8
Score3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
							Credit limit 8								
Inactive	10	10	10	10	10	10	10	8	8	8	8	8	8	8	8
Score1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
Score2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
Score3	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8
							Credit limit 9								
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score1	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
Score2	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
Score3	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9
							Credit limit 10								
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score2	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

Table D.4: Summary of optimal policy for Transactors accounts (loss equals to the credit limit) and the results of Credit Limit 1 and Limit 6 are presented in Table 6.12)

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Credit limit 2															
3+Cycle	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Inactive	9	9	9	9	9	9	9	9	9	9	9	9	9	9	2
Score1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	10
Score2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 3															
3+Cycle	3	3	3	3	3	3	3	3	3	3	3	3	3	3	10
Inactive	9	9	9	9	9	9	9	9	9	9	9	9	9	9	3
Score1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	10
Score2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 4															
3+Cycle	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Inactive	9	9	9	9	9	9	9	9	9	9	9	9	9	9	4
Score1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Score2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 5															
3+Cycle	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Inactive	9	9	9	9	9	9	9	9	9	9	9	9	9	9	5
Score1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Score2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Credit limit 7															
3+Cycle	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Inactive	9	9	9	9	9	9	9	9	9	9	9	9	9	9	7
Score1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 8															
3+Cycle	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Inactive	9	9	9	9	9	9	9	9	9	9	9	9	9	9	8
Score1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 9															
3+Cycle	9	9	9	9	9	9	9	9	9	9	9	9	9	9	10
Inactive	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
Score1	9	9	9	9	9	9	9	9	9	9	9	9	9	9	10
Score2	10	9	10	9	10	10	10	10	10	10	10	10	10	10	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Credit limit 10															
3+Cycle	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score2	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

Table D.5: Summary of optimal policy for Revolvers (Results of Credit Limit 1 and Limit 6 are presented in Table 6.11)

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Credit limit 2															
3+Cycle	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Inactive	9	9	9	9	9	9	9	3	3	3	3	3	3	3	2
Score1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	10
Score2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	10
Score3	10	8	10	8	8	10	8	2	2	2	2	2	2	2	10
Score4	10	10	10	10	10	10	10	4	4	4	4	4	4	4	10
Score5	10	10	10	10	10	10	10	7	7	7	7	7	4	4	10
Score6	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score7	10	10	10	10	10	10	10	6	6	6	6	6	6	7	10
Score8	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score9	10	10	10	10	10	10	10	6	5	6	7	6	7	6	10
Score10	10	10	10	10	10	10	10	5	5	7	5	5	5	5	10
Credit limit 3															
3+Cycle	3	3	3	3	3	3	3	3	3	3	3	3	3	3	10
Inactive	9	9	9	9	9	9	9	3	3	3	3	3	3	3	3
Score1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	10
Score2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	10
Score3	10	8	10	8	8	10	8	3	3	3	3	3	3	3	10
Score4	10	10	10	10	10	10	10	4	4	4	4	4	4	4	10
Score5	10	10	10	10	10	10	10	7	7	7	7	7	4	4	10
Score6	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score7	10	10	10	10	10	10	10	6	6	6	6	6	6	7	10
Score8	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score9	10	10	10	10	10	10	10	6	5	6	7	6	7	6	10
Score10	10	10	10	10	10	10	10	5	5	7	5	5	5	5	10
Credit limit 4															
3+Cycle	4	4	4	4	4	4	4	4	4	4	4	4	4	4	10
Inactive	9	9	9	9	9	9	9	5	5	5	5	5	5	5	4
Score1	4	4	4	4	4	4	4	4	4	4	4	4	4	4	10
Score2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	10
Score3	10	8	10	8	8	10	8	4	4	4	4	4	4	4	10
Score4	10	10	10	10	10	10	10	4	4	4	4	4	4	4	10
Score5	10	10	10	10	10	10	10	7	7	7	7	7	4	4	10
Score6	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score7	10	10	10	10	10	10	10	6	6	6	6	6	6	7	10
Score8	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score9	10	10	10	10	10	10	10	6	5	6	7	6	7	6	10
Score10	10	10	10	10	10	10	10	5	5	7	5	5	5	5	10
Credit limit 5															
3+Cycle	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Inactive	9	9	9	9	9	9	9	5	5	5	5	5	5	5	5
Score1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Score2	5	5	5	5	5	5	5	5	5	5	5	5	5	5	10
Score3	10	8	10	8	8	10	8	6	6	6	6	6	6	6	10
Score4	10	10	10	10	10	10	10	6	6	6	6	6	6	6	10
Score5	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score6	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score7	10	10	10	10	10	10	10	6	6	6	6	6	6	7	10
Score8	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score9	10	10	10	10	10	10	10	6	5	6	7	6	7	6	10
Score10	10	10	10	10	10	10	10	5	5	7	5	5	5	5	10

	Expansion							Recession							Unconditional
	CPI	GDP	Int	Sto	Une	ModelA	ModelB	CPI	GDP	Int	Sto	Une	ModelA	ModelB	
Credit limit 7															
3+Cycle	7	7	7	7	7	7	7	7	7	7	7	7	7	7	10
Inactive	9	9	9	9	9	9	9	7	7	7	7	7	7	7	7
Score1	7	7	7	7	7	7	7	7	7	7	7	7	7	7	10
Score2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	10
Score3	10	8	10	8	8	10	8	7	7	7	7	7	7	7	10
Score4	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score5	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score6	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score7	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score8	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score9	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Score10	10	10	10	10	10	10	10	7	7	7	7	7	7	7	10
Credit limit 8															
3+Cycle	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Inactive	9	9	9	9	9	9	9	8	8	8	8	8	8	8	8
Score1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	10
Score3	10	8	10	8	8	10	8	8	8	8	8	8	8	8	10
Score4	10	10	10	10	10	10	10	8	8	8	8	8	8	8	10
Score5	10	10	10	10	10	10	10	8	8	8	8	8	8	8	10
Score6	10	10	10	10	10	10	10	8	8	8	8	8	8	8	10
Score7	10	10	10	10	10	10	10	8	8	8	8	8	8	8	10
Score8	10	10	10	10	10	10	10	8	8	8	8	8	8	8	10
Score9	10	10	10	10	10	10	10	8	8	8	8	8	8	8	10
Score10	10	10	10	10	10	10	10	8	8	8	8	8	8	8	10
Credit limit 9															
3+Cycle	9	9	9	9	9	9	9	9	9	9	9	9	9	9	10
Inactive	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
Score1	9	9	9	9	9	9	9	9	9	9	9	9	9	9	10
Score2	9	9	9	9	9	9	9	9	9	9	9	9	9	9	10
Score3	10	9	10	9	10	10	10	9	9	9	9	9	9	9	10
Score4	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score5	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score6	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score7	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score8	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score9	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Score10	10	10	10	10	10	10	10	9	9	9	9	9	9	9	10
Credit limit 10															
3+Cycle	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Inactive	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score2	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score4	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score5	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score6	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Score10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

Table D.6: Summary of optimal policy for Revolvers accounts (loss equals to the credit limit) and the results of Credit Limit 1 and Limit 6 are presented in Table 6.12)

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