A NEW ANALYSIS OF DEBRIS MITIGATION AND REMOVAL USING NETWORKS

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ABSTRACT

Modelling studies have shown that the implementation of mitigation guidelines, which aim to reduce the amount of new debris generated on-orbit, is an important requirement of future space activities but may be insufficient to stabilise the near-Earth debris environment. The role of a variety of mitigation practices in stabilising the environment has been investigated over the last decade, as has the potential of active debris removal (ADR) methods in recent work. We present a theoretical approach to the analysis of the debris environment that is based on the study of networks, composed of *vertices* and *edges*, which describe the dynamic relationships between Earth satellites in the debris system. Future projections of the 10 cm and larger satellite population in a non-mitigation scenario, conducted with the DAMAGE model, are used to reconstruct a network in which vertices represent satellites and edges encapsulate conjunctions between collision pairs. The network structure is then quantified using statistical measures, providing a numerical baseline for this future projection scenario. Finally, the impact of mitigation strategies and active debris removal, which can be mapped onto the network by altering or removing edges and vertices, can be assessed in terms of the changes from this baseline. The paper introduces the network methodology, highlights the ways in which this approach can be used to formalise criteria for debris mitigation and removal. It then summarises changes to the adopted network that correspond to an increasing stability and changes that represent a decreasing stability of the future debris environment.

Keywords: Space debris, active debris removal, complexity science, networks, network theory

1. INTRODUCTION

Recent modelling studies have shown that the implementation of mitigation guidelines, which aim to reduce the amount of new debris generated on-orbit, is an important requirement of future space activities but may be insufficient to stabilise the near-Earth debris environment. In fact, even if there were no further launches into space it is suspected that the space debris population would continue to increase (Figure 1) due to persistent collision activity involving Earth satellites already on-orbit [1,2].

The role of a variety of mitigation practices in stabilising the environment has been investigated over the last decade, as has the potential of active debris removal (ADR) methods in recent work by Liou and Johnson [3]. Their findings indicate that the removal of space debris from the environment can improve the reliability of future space systems. However, given the technological, financial and political constraints on the implementation of these schemes it is important to identify effective mitigation and removal strategies, based on reliable, robust criteria and appropriate performance metrics. In previous works, the selection of these criteria and metrics has been accomplished on a relatively *ad-hoc* basis, using high-definition models of the space debris environment. Thus, whilst they have been empirically validated, potentially optimal solutions may still remain unidentified.

[insert Figure 1 about here]

This preliminary work looks at how data generated by simulation models in two case studies can be presented and analysed using network theory. Then, by using statistics that quantify the topology of these networks, the aim is to explore their role in characterising the space debris system (the ‘topology’ and dynamics of the space debris environment) in this new context. By adopting and developing this approach, we hope to identify and formalise reliable criteria for debris mitigation and removal.

2. COMPLEX NETWORKS

Complexity science has grown from systems theory as a contrasting approach to reductionism. Thus, instead of trying to understand the fundamental simplicity of the laws of nature, complexity science aims to encapsulate the complexity in nature [4]. Complex systems typically contain a number of simple entities that often display *emergent*, *collective properties* that are unpredictable at lower levels of granularity [5]. It is the simple, local but numerous interactions of the entities that allow the complex behaviour to emerge independently of the selection of finely-tuned parameters of the system [6]. Put more simply, a system is complex if, "the whole is more than the sum of its parts." [7] The application domains for complexity science are diverse, ranging from biology and social science to economics and information technology, but rigorous and universal definitions remain unidentified.

Graph theory has emerged as an effective way of understanding complex systems. Since the 1990s, there has been increasing interest in complex networks and graph theory (or "network theory") due to the growing availability of network data from the internet and biological, social and transport networks. In graph theory, networks are described using *vertices*, which represent components of the system, and *edges*, which describe the connections between the components. The number of vertices, n, gives the *order*, and the number of edges – less than $n(n-1)/2$ – gives the *size* of the network. In the context of the space debris 'system' we use vertices to represent orbiting objects and edges to denote interactions (conjunction events) between them. Different network types exist, including *random*, *scale-free*, *temporal*, *weighted* and *directional*, and some are discussed in more detail below.

Networks can be described mathematically* using an adjacency matrix,

$$\mathbf{A} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}, \tag{1}$$

where a connection (edge) from vertex i to vertex j is recorded by a '1' in the ith row and the jth column. In *weighted* networks a real number is attached to each edge, giving a real adjacency matrix. For example, in a social network edges can carry a weight representing how well two people know one another [8]. The potential to incorporate collision probabilities and other metrics of conjunctions via weights within this type of network is discussed later.

Edges can also be directed, pointing in only one direction, in networks representing email messages or telephone calls, for example. These *directed* networks, in general, have an asymmetric adjacency matrix, which describes the direction of information flow, whilst *undirected* networks have a symmetric adjacency matrix. Whilst

* See Nomenclature at end of paper

directed networks are certainly of interest in the context of the space debris system, they are considered to be beyond the scope of this preliminary work. As such, we refer to a network from this point as being undirected.

Networks are typically quantified using statistical measures of the network as a whole and by using related measures that capture the lower-level topology and dynamics. A fundamental descriptor of a network is the *degree centrality*, or simply *degree*. The degree, k_i , of a vertex *i* is the number of edges connected to *i* and can be easily determined from the adjacency matrix,

$$k_i = \sum_j^n a_{ij} \ . \tag{2}$$

The *network* degree is simply the average, or expected value, of k_i ,

$$\overline{K} = \langle k_i \rangle = \frac{1}{n} \sum_i^n k_i \ . \tag{3}$$

Again, in the context of our space debris network, objects with high degree correspond to objects that approach many other objects whilst on-orbit whereas those with low degree show little interaction with the rest of the population. *Scale-free networks*, comprising many low-degree vertices and relatively few high degree vertices, or *hubs* (Figure 2), are characterised by degree distributions that follow a power law (i.e. show no characteristic scale) and demonstrate dynamical behaviour typical of systems in a critical state [9, 10]. At the critical state there is no correspondence between the details of a perturbation and the system's response. For example, launching another payload onto orbit may cause nothing to happen, or it may cause a "collision cascade" that affects the whole environment, if it is in this critical state. It is therefore of interest to establish whether the space debris system follows similar behaviour. As such this statistic may provide key information about the expected dynamics of the space debris system.

[insert Figure 2 about here]

The network *clustering coefficient*, or *transitivity*, provides a measure of the density of connections between neighbouring vertices. There are numerous ways to calculate a clustering coefficient and we adopt a simple approach that utilises the elements of the adjacency matrix [11]. The clustering coefficient of a vertex *i*, given by

$$c_i = \frac{2}{k_i(k_i - 1)} \sum_{j \neq k}^{n} \sum_{k}^{n} a_{ij}\, a_{ik}\, a_{jk} \,, \tag{4}$$

quantifies how close the vertex and its neighbours are to being part of a *clique* [12]. For example, in social networks, a clique represents a group of people who all know each other (Figure 3). Networks that preserve the power law degree distribution have a clustering coefficient independent of degree.

[insert Figure 3 about here]

As before, the expected value of c_i provides a measure of the overall clustering behaviour of the whole network,

$$\overline{C} = \langle c_i \rangle = \frac{1}{n} \sum_{i}^{n} c_i \,. \tag{5}$$

Assortativity, which is also known as *degree correlation*, describes the correlation between the degree of a vertex and the degree of its neighbours. Networks in which highly connected vertices are linked to other vertices with a high degree are termed *assortative*, whilst networks in which highly connected vertices are only linked to vertices with a low degree are termed *disassortative*. A way to measure assortativity is to determine the correlation coefficient between the degrees of pairs of connected vertices,

$$r = \frac{(\mathbf{X} - \mu\mathbf{1})(\mathbf{Y} - \mu\mathbf{1})^T}{\|\mathbf{X} - \mu\mathbf{1}\| . \|\mathbf{Y} - \mu\mathbf{1}\|} \,, \tag{6}$$

where **X** is a $1 \times n(n\text{-}1)$ vector,

$$\mathbf{X} = (\mathbf{x}_i)_{i=1\ldots n} , \tag{7}$$

with the 1×(*n*-1) vector,

$$\mathbf{x}_i = (k_i)_{i=1\ldots n,\, i\neq j} . \tag{8}$$

The 1×*n*(*n*-1) vector **Y** is defined in a similar fashion,

$$\mathbf{Y} = (\mathbf{y}_i)_{i=1\ldots n} , \tag{9}$$

and,

$$\mathbf{y}_i = (k_j)_{j=1\ldots n,\, i\neq j} . \tag{10}$$

Finally, the value μ is the mean of all values in the vector **X**,

$$\mu = \frac{1}{\sum_i \sum_j a_{ij}} \sum_s^{\sum_i \sum_j a_{ij}} x_s . \tag{11}$$

The assortativity coefficient, *r*, lies between -1 (disassortative) and 1 (assortative). Social and collaboration networks typically show assortative mixing (i.e. positive values of assortativity) whereas biological and technological networks demonstrate disassortative mixing (i.e. negative values of assortativity). Using this classification scheme it is possible that the space debris system, being derived from technological processes, would show disassortative connections, i.e. a few high degree vertices (hubs) connected to many low degree vertices, similar to the World Wide Web. However the relationship described by the edges in our space debris network (“conjunction”) closely resembles the relationships used to define social networks (“knows”), so it is also possible for the network to show some assortative mixing.

The concept of network *resilience* is related to degree distributions and correlations [12]. A network is *resilient* if the removal of vertices causes little or no loss in the functionality (i.e. the connectivity) of the network as a whole. Due to their degree distribution, scale-free networks are characterised by their resilience to random removal of nodes, and their vulnerability to any targeted attacks. In contrast, random networks are equally sensitive to both forms of disruption. Thus, the properties of assortativity are useful in the understanding of network resilience. For instance, the removal of a portion of a network’s vertices may correspond to retrieving, disposing or “graveyarding” individual satellites or debris, in an attempt to stabilise the environment. If the space debris system demonstrates assortative mixing, collisions involving high degree objects are likely to “cascade” to other high degree objects. Alternatively, if the system is disassortative then mitigation or removal strategies that specifically target the high degree vertices may quickly disrupt the collisional network.

[insert Figure 4 about here]

A statistic that complements those based on degree, is the *geodesic distance*, or *shortest-path* distance. The distance from a source vertex to any other vertex in the network is simply the number of edges traversed in order to move from the source to the target. Thus, the geodesic distance is the minimum number of edges between the source and target vertices. The geodesic distance for a network can be determined using a *breadth-first search* (Figure 4). Breadth-first search is a general technique for traversing a network and can be used to compute a variety of network measures [13]. Beginning at a source vertex, the breadth-first search visits all the neighbouring (i.e. connected) vertices in turn, and then visits each of their neighbours, and so on until a target vertex is reached. The method makes use of a queue to store a list of vertices to be visited. If the queue is empty, every vertex in the network has been visited and the search is terminated with the target not found.

The mean shortest-path distance between a vertex and all other vertices reachable from it provides a measure of the *closeness* of one vertex to another. It is essentially a measure of how long it will take information to spread from a given vertex to others in the network. A particular type of network, called a *small-world network*, is characterised by small average distances, as most vertices can be reached from every other by a small number of steps, but most vertices are not neighbours of one another (Figure 5). Small-world networks are similar to random networks in that there is a degree of randomness in the connections between vertices, but they are typically more robust to the random removal of vertices. Ideally, the space debris system would not share these small-world characteristics, as this would suggest that the outcome of a single collision could affect every point in the environment even if there were relatively few connections between objects. Rather, objects in the space debris system would ideally have closeness values that are high, as this would be indicative of a more stable environment.

[insert Figure 5 about here]

The closeness for a vertex *i* is the mean shortest-path distance from *i* to all other vertices in the network, given by

$$D_i = \frac{1}{n-1}\sum_{j}^{n} d_{ij} \ , \tag{12}$$

where d_{ij} is the shortest-path distance between vertices *i* and *j*, and the closeness statistic for the network is,

$$\overline{D} = \langle D_i \rangle = \frac{1}{n}\sum_{i}^{n} D_i \ . \tag{13}$$

The *efficiency* of a vertex *i* is,

$$e_i = \frac{1}{n-1}\sum_{j}^{n} \frac{1}{d_{ij}} \ , \tag{14}$$

and the equivalent statistic for the network as a whole is,

$$\overline{E} = \langle e_i \rangle = \frac{1}{n}\sum_{i}^{n} e_i \ . \tag{15}$$

If a vertex is situated on many paths between other vertices then it is said to have high *betweenness*, and has a role in connecting different parts (possibly different communities) of the network. There are a number of ways to measure betweenness [8]. The measure we adopt here is the *shortest-path betweenness* (also called *betweenness centrality*), which quantifies the extent to which a vertex lies on the paths between others. The shortest-path betweenness of a vertex *i* is defined to be,

$$B_i = \sum_s^n \sum_t^n \frac{\sigma_{st}(i)}{\sigma_{st}} \text{ for } s \neq i \neq t \,. \tag{16}$$

The betweenness centrality is calculated in two phases. The first phase computes distances and shortest-path counts using a breadth-first search (as in Figure 4). The second phase visits all the vertices to accumulate values for B_i. The accumulation process is completed using the following steps:

1. A variable b_i^s, taking the initial value 1, is assigned to each vertex *i*.
2. Going through the vertices *i* in order of their distance from the source vertex *s*, starting from the furthest. The value of b_i^s is added to the corresponding variable on the predecessor (or parent) vertex of *i*, i.e. the vertex connected to *i* and closer to vertex *s*. If *i* has more than one predecessor, b_i^s is divided equally between them.
3. Go through all vertices in this fashion and record the value b_i^s for each vertex *i*. Repeat the entire calculation for every source vertex *s*.
4. The betweenness for each vertex *i* is then obtained as

$$B_i = \sum_s^n b_i^s \,. \tag{17}$$

The *relative* betweenness centrality provides a normalised measure of the betweenness,

$$\hat{B}_i = \frac{2}{(n-1)(n-2)} B_i \,. \qquad (18)$$

Finally, the betweenness centrality of the network is given by the mean of the vertex betweenness,

$$\overline{B} = \langle B_i \rangle = \frac{1}{n} \sum_{i}^{n} B_i \,, \qquad (19)$$

and the relative betweenness centrality of the network is calculated in a similar fashion,

$$\overline{\hat{B}} = \langle \hat{B}_i \rangle = \frac{1}{n} \sum_{i}^{n} \hat{B}_i \,. \qquad (20)$$

Betweenness is a useful measure of the vulnerability of a network vertex or edge and, hence, provides important information about resilience. For example, the removal of a vertex with high betweenness is likely to disrupt the dynamics of flow across the network significantly. In the context of space debris, targeting objects with high betweenness for removal may be as efficient at stabilising the environment as removing objects with a high degree (i.e. a hub).

2.1 Examples

To illustrate the results of applying the formulae above to a network, the statistics calculated for a simple network (Figure 6) are listed in Table 1. This network has five vertices and five edges (i.e. order and size of five) connected according to the adjacency matrix,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}, \qquad (21)$$

such that there are two paths, B-C-D and B-E-D, which have the same geodesic distance.

[insert Figure 6 about here]

[insert Table 1 about here]

The statistics in Table 1 indicate a network with low connectivity ($\overline{K} = 2, \overline{C} = 0$) and disassortative mixing (r = -0.67). The tendency is thus for vertices with low degree to be connected to vertices with higher degree. The statistics for the example network may be compared with corresponding statistics computed for the real-world networks highlighted in Table 2 [11]. These real-world networks exhibit a broad range of values that seem to depend on the type of network. For example, biological networks and technological networks (to a lesser amount) tend to demonstrate disassortative mixing whereas social networks show assortative mixing. Further, the biological and technological networks generally have lower levels of clustering than the social networks [11].

The example network has some similarity with the technological and biological networks by virtue of its disassortative nature.

[insert Table 2 about here]

The statistics calculated for individual vertices in the example network (Table 3) indicate that vertex B plays an important role in connecting different parts of the network. This vertex has high betweenness ($B = 11$) and the mean shortest-path distance (i.e. closeness) to other vertices in the network is low ($D = 1.4$) compared with the other vertices. Clearly, all paths from vertex A to other vertices have to pass through B. The importance of vertex A, on the other hand, is much less than that of vertex B. In this particular network, there is a strong correlation between the degree of the vertex and its betweenness, although this is not the case with all networks.

[insert Table 3 about here]

3. METHOD AND DATA

The data used in the following case studies were generated using the University of Southampton's debris model, Debris Analysis and Monitoring Architecture for the Geosynchronous Environment (DAMAGE). DAMAGE is a three-dimensional model that was initially aimed at simulating debris within the geosynchronous orbital regime but has since been upgraded to allow investigations of the full low Earth orbit (LEO) to geosynchronous earth orbit (GEO) debris environment. As with other evolutionary models, DAMAGE is able to simulate the historical and future debris populations ≥ 10 cm using a Monte Carlo (MC) approach, whereby multiple projection runs are performed to establish reliable statistics on the outcome. Projections covering the historical period from 1957 to 2001 employ launch and fragmentation information from ESA's Database and Information System Characterising Objects in Space (DISCOS) and historical monthly averaged solar flux F10.7 values combined with the CIRA-72 atmospheric model for atmospheric drag calculation. Future projections use statistics derived from DISCOS data covering the period 1996 – 2001 to simulate future launch and explosion activity, with a long-term F10.7 projection based on a repeating sine function. The historical and future fragmentation events are simulated using the NASA Standard Breakup Model [14] and non-fragmentation sources of debris, except mission-related objects included in DISCOS, are not considered. All objects are propagated forwards using a semi-analytical orbital propagator that includes Earth's J_2, J_3, $J_{2,2}$, luni-solar gravitational perturbations, solar radiation pressure (with cylindrical Earth shadow) and atmospheric drag. Collision probabilities are estimated using a fast, pair-wise algorithm based on the 'Cube' approach adopted in NASA's LEO-to-GEO Environment Debris model (LEGEND) [15].

A 'business as usual (BAU)' (2000 – 2040) scenario (described in Table 4) was used by DAMAGE to provide data for this investigation. The BAU scenario is commonly used as a baseline for comparison with debris mitigation case studies and was chosen for this reason. The effective number of objects ≥ 10 cm in LEO for 60 MC runs of the BAU scenario is shown in Figure 7. The curve shows a quadratic increase in the effective number of objects, driven by a rising number of collisions that is characteristic of this scenario.

[insert Table 4 about here]

For each MC run, DAMAGE recorded information about all the conjunction events occurring in the projection period, between objects ≥ 10 cm. This information included the identification, mass, size and orbit of each object, as well as the collision probability and energy. A dedicated module within DAMAGE was then used to calculate the network statistics described above, although computer implementation and memory requirements currently constrain the network order to <1,000 vertices for betweenness and shortest-path distance statistics. A freely available software tool, entitled 'Cytoscape', was used to display the resulting networks [16].

[insert Figure 7 about here]

Two case studies are presented to highlight particular results and issues discovered in our preliminary investigation of the use of network theory in understanding the space debris system. In the first case study, networks were generated from BAU data using the assumption that satellites (payloads, rocket bodies, mission-related debris, explosion fragments and collision fragments) are represented by vertices in the network. If two satellites occupy the same 'cube' in any simulation time-step then an edge, representing a conjunction, is inserted into the network to connect the vertices corresponding to those satellites. In the second case study, DAMAGE BAU data were used to construct two multi-relational networks featuring three types of vertex and three corresponding types of edge (Figure 8). The vertices described *intact objects* (payloads and rocket bodies labelled using their international designation, e.g "1978 100D"), *fragments* generated by the break-up model in DAMAGE (with labels derived from the parent object's international designation, e.g. "1978 100D-F15"), or *launch groups* (with labels based on the international designation launch year and number, e.g. "1978 100"). DAMAGE added the additional launch group and heritage data automatically for every object involved in a conjunction event, and labelled the relationships accordingly. Consequently, each conjunction event introduced up to six vertices into the network. The edges represented the corresponding relationships, "*conjunction*" (as in the first case study), "*is a fragment of*", or "*is a member of*".

The networks constructed in both case studies were *undirected* and *unweighted*. That is, the edges in the networks were bi-directional and data from the conjunction events (collision probabilities and energies, etc.) were not used to apply weightings to them. Finally, in order to avoid exacerbating the complexity of the networks it was decided to use the same future launch traffic in each MC run.

[insert Figure 8 about here]

4. RESULTS AND DISCUSSION

4.1 Case Study 1

As DAMAGE uses a Monte-Carlo simulation approach to build a reliable measure of the collision 'potential' of each object in the environment, networks describing the interactions between objects were necessarily constructed by combining the data generated in each MC run. Figure 9 shows how the complexity of the networks grew when increasing numbers of MC runs were used. A 'tipping' point in the complexity of the resulting networks appears to have been reached after combining data from only three MC runs. Table 5 lists the corresponding statistics for networks selected from Figure 9.

The results in Table 5 demonstrate that the selected networks have degree $\overline{K} \approx 2$. In other words, an 'average' vertex in these networks (representing an 'average' object in the debris environment) is typically connected to two other vertices such that together they form a chain. In the networks generated from one (Figure 10) or two MC runs (Figure 11), these chains are short. In these cases, one might conclude that there would be no major collision feedback processes if a 'business as usual' approach to space activities was taken and this argument is supported by the lack of clustering ($\overline{C} = 0$). However, combining one additional MC run resulted in a connected network with over 5,000 vertices. In addition, there is disassortative mixing ($r \approx -0.1$, i.e. a tendency for low degree vertices to connect to high degree vertices) at a level similar to other technological systems, such as the Internet. Consequently, it is likely that these networks would be relatively robust to the random removal of vertices (i.e. random removal of objects from orbit). An efficient "attack" on these networks would target particular vertices, as demonstrated in Table 6 for the network generated from two MC runs (Figure 11).

[insert Figure 9 about here]

[insert Table 5 about here]

[insert Figure 10 about here]

[insert Figure 11 about here]

[insert Table 6 about here]

From the vertices selected, it is clear that the number of conjunction events involving an object (i.e. the degree of its vertex) provides a mechanism for identifying objects that are important to the network. Targeting just a few objects with a high degree (e.g. "I", "Z" and "AM" in Figure 11) would efficiently destroy the network. These particular vertices also demonstrate an importance due to their high betweenness and efficiency, and their low closeness statistics (compared to the network averages in Table 5), with object "I" having a betweenness value 33 times higher than object "A", for example. Clearly, there are many possible paths through the network that pass through the vertices representing these objects. However, it is also apparent from Table 6 that object "X" plays an important role in this network too. Even though it is only connected to two other objects ($k = 2$) it effectively provides a route for the progression of several conjunction events. The shortest-path betweenness for "I" is only 1.3 times higher than the same value for object "X" and, in addition, the vertex representing object "X" is 'closer' ($D = 4.2$) and is more 'efficiently' connected ($E = 0.24$) to other vertices in the network compared with the vertex corresponding to object "I" ($D = 4.4$, $E = 0.23$), and the 'typical' or 'average' vertex (Table 5). The vertices "X" and "Y" essentially connect two communities of objects in the debris environment centred, in the network, on object "I" and object "Z". Hence, removing either "X" or "Y" would result in the connection between these two communities being broken and the network would be severely compromised.

The use of the shortest-path betweenness (betweenness centrality) to identify vulnerable points in the network above, follows the strategy of removing *edges* according to the Newman-Girvan algorithm for community detection [17], which has also been shown to be an efficient way of damaging a network [18]. In this approach, the steps are:

1. Calculate the betweenness of all edges in the network.
2. Remove the edge with the highest betweenness.
3. Recalculate the betweenness for the remaining edges.

4. Continue at (2) until no edges are left.

In the case of the network in Figure 11, the vertices “X” and “Y” correspond to fragments from the historical break-ups of 1978 100D (SL-14 rocket body) and 1981 31A (Cosmos 1261). As such, the removal of these objects from the environment would represent a significant challenge! Similarly, the high-degree vertex “Z” represents a fragment from a Thor Ablestar rocket explosion, although “I” and “AM” correspond to intact objects (1973 109B and 1978 34B, both SL-8 rocket bodies). In this case, targeting 1973 109B and 1978 34B for removal would damage much of the connectivity of the network in Figure 11 as these objects also correspond to high-degree vertices (hubs) and have high betweenness values. In fact, removing these two vertices reduces the maximum number of connected vertices from 52 to 15 and the network betweenness centrality from $\overline{B} = 324.54$ to $\overline{B} = 27.3$, indicating a significant drop in the connectivity.

The first case study has demonstrated the potential for network theory to aid the identification of criteria for debris removal. However, it is apparent that not all debris objects are possible targets for removal and this information needs to be incorporated into a network before reliable criteria can be established. To address this issue, a study of multi-relational networks was undertaken.

[insert Figure 12 about here]

4.2 Case Study 2

The multi-relational networks generated from one BAU MC run are shown in Figure 12. The largest network contains > 5,000 vertices and has indications of disassortative mixing, whereby most vertices are low degree and there are hubs having a much higher number of connections. Unfortunately, statistics could not be generated for this network because of computational constraints associated with the prototype DAMAGE code. However, a second network, generated from 25 years of data in one MC run, was successfully analysed (Table 7) and is shown in Figure 13.

[insert Figure 13 about here]

The multi-relational network in Figure 13 exhibits some similarity with the networks generated in the first case study, being characterised by a low degree ($\overline{K} = 2$), a lack of clustering ($\overline{C} = 0$) and negative assortativity ($r = -0.12$). In contrast with the previous networks, however, the multi-relational network has longer path lengths ($\overline{D} = 18.0$), as a result of the additional vertices representing launch groups and parent objects. The effect of these additional vertices is to highlight the relationships between fragments and their parent objects. In particular, the multi-relational network shows clearly the role of explosion events in the growth of the space debris population. For example, if we take the vertex labelled "D" in Figure 13 (corresponding to a rocket body, designation 1987 068B) we can see that this object is not involved in a conjunction event in the simulation, but as the parent object of many fragments that *are* involved in conjunction events it plays a central role in the multi-relational network. Hence, the historical passivation of this object (if it had been possible) would have removed a significant portion of the vertices and their edges from the network, thus reducing the number of conjunction events in the debris environment. Similarly, it is possible to see that three objects on the same launch and connected to vertex "C" (launch number 1993 030) were involved in conjunction events in the simulation, either directly or by generating fragments which interact with other objects.

[insert Table 7 about here]

The statistics and a visual inspection of Figure 13 suggest that the multi-relational network is similar to other technological networks and would be robust to the random removal of vertices but vulnerable to the targeted removal of vertices. The analysis of individual vertex statistics (in Table 8) indicated that targeting high degree objects, such as 1987 068B (vertex "D"), was an appropriate method for disrupting the network. However, just as it was for the networks in the first case study, the strategy of targeting vertices with high betweenness centrality (e.g. vertices "B" and "C") also provides an efficient way to destroy the network. The advantage with the multi-relational network is that we have relational information that can assist with the identification of a suitable mitigation approach.

[insert Table 8 about here]

5. CONCLUSIONS

A new method for investigating the space debris environment, in particular debris mitigation and removal strategies, has been introduced. The method is based on network theory and its associated mathematics. Using data from future 'business as usual' projections by the University of Southampton's debris model, DAMAGE, several networks were constructed and then quantified in a preliminary study.

At the outset of this work, it had been hoped that a classification of the space debris system, using the simulation data and network theory, would be possible. Following the investigations outlined above, it became apparent that this system demonstrates a range of characteristics that, in the first instance, defy a convenient classification and require further investigation. However, a number of key features have been discovered:

1. The space debris environment likely demonstrates disassortative mixing. That is, most objects interact with only one or two other objects whilst on orbit, but a relatively few highly connected objects will exist. These "hubs" are potential candidates for mitigation or removal, but they also represent a possible threat to the stability of the environment if they remain.
2. As a result of the disassortative mixing, the space debris environment is likely to be robust to the random removal of objects from orbit. A targeted approach to debris removal is expected to be much more efficient at breaking down the connections between objects and stabilising the environment.
3. The fragmentation of orbiting objects, through explosive or collisional break-up, is akin to 'seeding' the environment with highly connected hubs. Again, prevention of these fragmentation events by passivation or collision avoidance would disrupt the connectivity of the system.
4. Targeting objects for removal based on the number of connections they have with other objects is just one of several methods that could be utilised. In particular, the betweenness centrality statistic, which effectively measures the importance of an object to a *sequence* of conjunction events, may provide a useful alternative.

The concept of network resilience has been introduced, whereby a network is resilient if the removal of vertices causes little or no loss in the functionality (i.e. the connectivity) of the network as a whole. In the context of space debris mitigation, resilience is *undesirable*. That is, the aim of mitigation is to remove objects or to destroy the connections between them such that the environment becomes stable. Using network theory, we have shown the potential to quantify the resilience of the space debris environment using statistics and, hence, the possibility to reliably assess the impact of debris mitigation.

The use of weighted networks, which employ collision probability, mass and energy data to define connection weights, is an objective of future work. In the networks presented above, edges corresponding to conjunction events were all assumed to be of equal importance. Consequently, a conjunction event involving two small fragments approaching with a relative speed of 50 m/s was treated as being equivalent to an event involving two intact rocket bodies approaching one another at 10 km/s. Clearly these two events would result in very different outcomes for the debris environment if collisions actually occurred. The discriminating power of a weighted network combined with statistics that incorporate the weights is thus necessary to achieve the goal of formalising criteria for debris mitigation and removal. In addition, there is an issue related to the temporal nature of debris interaction that will also be investigated using the concept of temporal networks.

The results presented above demonstrated that the complexity of the networks increased dramatically when the data from more than two MC runs were combined and even before different traffic models were included. In order to achieve the goal of formalising mitigation criteria, the computational expense associated with the calculation of the network statistics will need to be reduced. This may be achieved by selecting and coding efficient algorithms.

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NOMENCLATURE

$\mathbf{A}$ = Adjacency matrix

a_{ij} = Adjacency matrix element (describing the connection between vertices *i* and *j*)

$\overline{B}$ = Network betweenness centrality

$\hat{B}_i$ = Relative betweenness centrality of vertex *i*

B_i = Betweenness centrality of vertex *i*

$\overline{C}$ = Network clustering coefficient

c_i = Clustering coefficient for vertex *i*

$\overline{D}$ = Network Closeness

D_i = Mean closeness of vertex *i*

d_{ij} = Shortest-path distance between vertices *i* and *j*

$\overline{E}$ = Network efficiency

e_i = Efficiency of vertex *i*

$\overline{K}$ = Network degree or degree centrality

k_i = Degree of vertex *i*

n = Number of vertices in network

r = Network assortativity

σ_{st} = Number of shortest paths between vertices *s* and *t*

$\sigma_{st}(i)$ = Number of shortest paths between vertices *s* and *t* passing through vertex *i*

TABLE CAPTIONS

Table 1. Example network statistics.

Table 2. Statistics for published networks [11]. The precise formulae used for some statistics (*) may vary from those used here.

Table 3. Vertex statistics from the example network.

Table 4. Description of the ‘business as usual’ (2000 – 2040) scenario.

Table 5. Network statistics for networks generated by combining multiple DAMAGE MC runs. Blank entries indicate unavailable data.

Table 6. Vertex statistics for vertices selected from a network generated by combining two DAMAGE MC runs.

Table 7. Network statistics for the multi-relational network generated from one short DAMAGE MC run of the BAU scenario.

Table 8. Selected vertex statistics from the multi-relational network generated from a short DAMAGE BAU MC run. Objects are identified using their international designation or a derivative indicating the heritage of the object.

Table 1

Statistic	**Symbol**	**Value**
Order	n	5
Degree	$\overline{K}$	2.0
Clustering coefficient	$\overline{C}$	0.0
Assortativity	r	-0.67
Closeness	$\overline{D}$	1.72
Efficiency	$\overline{E}$	0.59
Betweenness	$\overline{B}$	6.4
Relative betweenness	$\hat{\overline{B}}$	1.07

Table 2

Network	**Type**	$\overline{K}$	$\overline{C}\,*$	$r\,*$	$\overline{D}\,*$
Company directors	Social	14.4	0.88	0.28	4.6
Physics co-authorship	Social	9.27	0.56	0.36	6.19
Internet	Technological	5.98	0.39	-0.19	3.31
Train routes	Technological	66.79	0.69	-0.03	2.16
Marine food web	Biological	4.43	0.23	-0.26	2.05
Neural network	Biological	7.68	0.28	-0.23	3.97

Table 3

Vertex	*A*	*B*	*C*	*D*	*E*
Degree	1	3	2	2	2
Clustering coefficient	0.0	0.0	0.0	0.0	0.0
Closeness	2.2	1.4	1.6	1.8	1.6
Efficiency	0.45	0.71	0.63	0.56	0.63
Betweenness	4.0	11.0	6.0	5.0	6.0
Relative betweenness	0.67	1.83	1.0	0.83	1.0

Table 4

Parameter	**Value**
Projection period	1 January 2000 – 1 January 2040
Traffic model (2000 – 2040)	Based on launch statistics for 1996-2001
Future explosions (2000 – 2040)	Based on explosion statistics for 1996 - 2001
Time-step	5 days
Minimum object size	10 cm
Collision prediction: cube size	10 km

Table 5

Statistic	**Symbol**	**1 MC**	**2 MC**	**3 MC**
Maximum order	n	12	52	5537
Degree	$\overline{K}$	1.83	1.96	2.08
Clustering coefficient	$\overline{C}$	0.0	0.0	0.0
Assortativity	r	-0.07	-0.13	-0.11
Closeness	$\overline{D}$	3	6.36	
Efficiency	$\overline{E}$	0.35	0.16	
Betweenness	$\overline{B}$	33	324.54	
Relative betweenness	$\overline{\hat{B}}$	0.6	0.26	

Table 6

Vertex	*A*	*I*	*X*	*Z*	*AM*
Degree	1	7	2	4	5
Clustering coefficient	0.0	0.0	0.0	0.0	0.0
Closeness	9.5	4.4	4.2	4.1	4.6
Efficiency	0.1	0.23	0.24	0.24	0.22
Betweenness	51	1689	1339	1741	1133
Relative betweenness	0.04	1.32	1.05	1.37	0.89

Table 7

Statistic	**Symbol**	**Value**
Order	n	222
Degree	$\overline{K}$	2.0
Clustering coefficient	$\overline{C}$	0.0
Assortativity	r	-0.122
Closeness	$\overline{D}$	18.048
Efficiency	$\overline{E}$	0.058
Betweenness	$\overline{B}$	3974.428
Relative betweenness	$\overline{\hat{B}}$	0.163

Table 8

Vertex	*A*	*B*	*C*	*D*	*E*
Object Designation	1988 002G	1988 102J	1993 030	1987 068B	1992 093B-F135
Object type	Intact object	Intact object	Launch group	Intact object	Fragment
Degree	2	2	3	15	2
Clustering coefficient	0.0	0.0	0.0	0.0	0.0
Closeness	15.19	13.50	20.53	12.13	23.85
Efficiency	0.07	0.07	0.05	0.08	0.04
Betweenness	660	12912	14034	34947	1096
Relative betweenness	0.03	0.53	0.58	1.44	0.05

FIGURE CAPTIONS

Figure 1. Effective number of objects $\geq$ 10 cm for the projection period 1957 – 2200 assuming no new launches after 1 January 2000.[2]

Figure 2. Random (left) versus scale-free (right) networks. Connections between vertices in the random network follow a Binomial distribution, whereas the distribution is described by a power law for scale-free networks [9, 10].

Figure 3. An illustration of a social clique. Nodes A to F represent people and the connections between them represent the relationship "knows". In this illustration, each person knows every other person in the network.

Figure 4. Geodesic distance can be measured using a breadth-first search.

Figure 5. An illustration of networks differing by the amount of randomness in their connectivity.

Figure 6. Example network showing the degree of each vertex.

Figure 7. Effective number of objects $\geq$10 cm in LEO (average from 60 MC runs with DAMAGE) for a 'Business as usual' scenario.

Figure 8. Vertex and edge types used in the multi-relational networks of the second case study. Vertex labels are based on international designations for space objects.

Figure 9. Networks formed from different (a) one MC run, (b) two MC runs, (c) three MC runs, and (d) four MC runs of the BAU scenario displayed using Cytoscape's organic 'y-layout'. The highlighted networks are used in the first case study and two are shown in detail in Figures 10 and 11.

Figure 10. Network formed from one MC run of the BAU scenario.

Figure 11. Network formed from two MC runs of the BAU scenario. Labels are used to identify vertices in Table 6.

Figure 12. Multi-relational network formed from one BAU MC run. Edges represent conjunctions, fragment parentage or launch group associations.

Figure 13. Multi-relational network formed from a short (2001 – 2026) DAMAGE BAU MC run. Vertex and edge types are described in Figure 8. Statistics for the vertices labelled A to E are shown in Table 8.