PREDICTING FAN NOISE PROPAGATION IN AEROENGINE BYPASS DUCTS

by

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A thesis submitted in partial fulfillment for the degree of Doctor of Engineering

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To the people in my life who have always encouraged me to reach for the stars:

My dad, Jerry, my mum, Tricia, and my fiancé, Ian.
This thesis explores the prediction of rearwards fan noise propagation within the bypass duct and its radiation into the far field. Two recently developed models: B-induct and GXMunt, are exploited in application to real engine bypass ducts and their performance is evaluated. These methods are an improvement on current industry standards, allowing realistic duct geometry and flow conditions to be modelled with reasonable computation and time demands. The main focus is on the model b-induct.

B-induct predictions for bypass attenuation are integrated into an industry standard whole engine model, and predictions of far-field noise are obtained for a modern high bypass-ratio engine. These predictions compare more favourably with measured data from full-scale static engine tests than similar predictions made using a standard uniform rectangular duct model for the bypass attenuations, indicating that b-induct is an improvement over the current model.

Initial studies on the effect of duct geometry on noise propagation suggest a noise benefit for a duct with higher curvature when compared to a typical Baseline design. This suggestion is confirmed using measured data from zero-flow rig tests. Predictions for three-dimensional duct geometries are also performed to show the effect of scattering due to bifurcations within the duct.

B-induct allows for the specific bypass geometry and liner positions to be taken into account when performing impedance optimisations. A new optimisation procedure is proposed in which b-induct predictions are used within an existing whole aircraft noise prediction model. This procedure is used to select liner impedances for a modern engine bypass duct.

B-induct is demonstrated to be a promising new tool within the engine design process, for both analysis of the impact of rear fan noise on whole engine noise, and assessment of potential low noise bypass configurations.
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I, CLAIRE ROISIN MCALEER, declare that the thesis entitled,

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Nomenclature

$(R, \theta, \psi)$ Spherical coordinate system

$(r, \theta, z)$ Cylindrical coordinate system

$(x, y, z)$ Cartesian coordinate system

$\alpha_j$ incident amplitude of radial mode order $j$

$\beta = \sqrt{1 - M^2}$

$\Delta PWL$ sound power level attenuation

$\Delta SPL$ sound pressure level attenuation

$\delta^c, \Delta^c$ calibration factor

$\epsilon_1, E$ error

$\gamma$ ratio of specific heats, $C_P/C_V$

$\lambda_{mn}$ eigenvalue

$\omega$ angular frequency

$\omega^{co}_{mn}$ modal cut-off frequency

$\Phi$ total velocity potential

$\phi$ velocity potential, no subscript: unsteady/ acoustic component, subscript: 0, mean flow component, $e$, within an element, $i$, at element node $i$

$\psi_s$ cone of silence angle

$\psi_{mn}$ modal propagation angle relative to duct axis

$\rho$ density, no subscript: unsteady/ acoustic component, subscripts: 0, mean flow component, $T$, total

$\textbf{C}$ acoustic damping term

$\textbf{F}$ forcing term
NOMENCLATURE

**K**  acoustic stiffness term

**M**  acoustic mass term

**n, n**  unit vector perpendicular to a surface

**v**  velocity, *no subscript:* unsteady/ acoustic component, *subscripts:* 0, mean flow component, T, total

**x; ξ**  local coordinate system

**ε**  variable parameter for duct curvature and duct contraction studies

**ξ_{mn}**  modal cut-off ratio

**a**  radius of cylindrical duct or outer wall radius of annular duct

**A, B**  amplitude coefficients

**a_{mn}^{±}**  modal wave amplitude, *superscripts:* + travelling in the positive z-direction, - travelling in the negative z-direction

**A_{ij}**  modal amplitude transmission coefficients

**b**  inner wall radius of annular duct

**c**  sound speed, *no subscript:* unsteady/ acoustic component, *subscripts:* 0, mean flow component, T, total

**C_{P}**  specific heat capacity at constant pressure

**C_{V}**  specific heat capacity at constant volume

**d**  liner cell depth

**D_{mn}**  modal directivity function

**f**  frequency

**h**  duct hub-to-tip ratio, b/a

**I**  acoustic intensity

**i**  $\sqrt{-1}$

**k**  acoustic wavenumber

**k_{co}^{mn}**  modal cut-off wavenumber

**k_{z}, k_{zm}^{±}**  axial wavenumber, *superscripts:* + wave travelling in the positive z-direction, - wave travelling in the negative z-direction

**k_{rmn}**  radial wavenumber
\textit{L/D} \quad \text{industry model parameter to account for amount of liner present in duct: liner length, } L, \text{ divided by duct height, } D

\textit{LI} \quad \text{liner insertion loss}

\textit{M} \quad \text{Mach number}

\textit{m} \quad \text{azimuthal mode order, or liner facing sheet mass inertance}

\textit{minj} \quad \text{notation to signify a mode with azimuthal order } i \text{ and radial order } j

\textit{N} \quad \text{shape function, or total number of cut-on modes}

\textit{n} \quad \text{radial mode order, or number of nodal points or shape functions within an element}

\textit{N_{mn}} \quad \text{modal normalisation factor}

\textit{p} \quad \text{pressure, } \text{no subscript: unsteady/ acoustic component, } \text{subscripts: } 0, \text{ mean flow component, } T, \text{ total}

\textit{POA} \quad \text{liner facing sheet porosity (percentage open area)}

\textit{PW L_{tr}} \quad \text{transmitted sound power level at duct exhaust}

\textit{R} \quad \text{acoustic resistance}

\textit{S} \quad \text{denotes surface, or factor to account for wavefront stretching}

\textit{T_{ij}} \quad \text{power transmission coefficients}

\textit{U}, \textit{v} \quad \text{uniform axial flow velocity magnitude}

\textit{V} \quad \text{denotes volume}

\textit{v_n} \quad \text{normal component of the acoustic particle velocity at the duct wall}

\textit{W} \quad \text{sound power}

\textit{W}, \textit{w} \quad \text{weighting function or test function}

\textit{X} \quad \text{acoustic reactance}

\textit{Z} \quad \text{specific acoustic impedance}
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In undertaking an EngD, I hoped to gain insight and contribute to industrially focussed research. Under the guidance of Jeremy Astley (ISVR) and Andrew Kempton (Rolls-Royce), who understood my desire to be involved in collaborative, goal-oriented work, I have achieved these aims, and gained valuable experience both academically and personally. I am very grateful to them both. Special thanks are also due to Rie Sugimoto, responsible for the development of the b-induct codes and the integration of b-induct and GXMunt, for always being available for discussion, and whose help, advice and attention to detail have been absolutely invaluable.

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Claire Roisin McAleer, March 2009
Chapter 1

Introduction

1.1 Problem Overview

The introduction of the jet engine into commercial service in the 1950s has greatly facilitated both international business and travel, and has consequently improved the quality of life throughout the world. However, these benefits come with the cost of increased levels of noise pollution, which can be both irritating and harmful to health. Over the years, increasing aircraft numbers and frequency of flights has caused large public reaction to the noise around major airports.

In order to tackle the problem of aircraft noise, in 2001, the International Civil Aviation Organisation (ICAO) adopted a “balanced approach” where the noise problem is addressed by the study of four principal elements: noise reduction at source (quieter aircraft), land-use planning and management, noise abatement operational procedures and operating restrictions. For the aircraft industry, the first of these solutions: reducing aircraft noise at source, has the most impact, as it is the requirement that industry meets (or surpasses) the necessary standards for aircraft noise.

There has been great effort to reduce aircraft noise at source over the years, and significant improvements have been made so far, especially due to the introduction of the high bypass ratio engine in the 1960s. Modern aircraft are consequently about 20dB quieter than those of the 1960s (based on lateral noise level measurements), corresponding to a 99% reduction in sound power [1].

However, both local noise restrictions at major airports and individual aircraft noise certification levels are becoming increasingly stringent. In order to set targets for aircraft noise reduction, the Council of ICAO sets noise certification standards that aircraft built today are required to meet, with the most recent “Chapter 4” standard applicable since January 1st 2006 [2]. These noise certification levels are determined based on ground level measurements taken at three different reference conditions in the aircrafts.
flight path. These reference conditions are termed: Approach, Lateral (or Sideline) and Flyover (or Cutback), and are defined as shown in figure 1.1.

Recently the Advisory Council for Aeronautics Research in Europe (ACARE) has defined the long term objective for a halving in perceived aircraft noise from the year 2000 to the year 2020. In order to meet these targets, progress in the field of noise reduction needs to continue.

There are various mechanisms for aircraft noise production. The two main sources of aircraft noise are the airframe itself (undercarriage/ high-lift devices etc), and the aircraft engines.

Turbofan engines generate noise from several components (figure 1.2). The operation of a turbofan engine involves a rotating fan causing air to flow into the engine, where it is then compressed in various stages before being mixed with fuel and burned. Hot air from this process expands and is expelled from the back of the engine as an exhaust jet, driving a turbine as it travels, which in turn drives the fan through a common shaft. Of these mechanisms, the principal sources of noise are the fan, the core (compressor and combustor), the turbine and the jet. Each of these sources need to be individually addressed to make the engine significantly quieter.

Different sources of noise are dominant at different aircraft operating conditions. Figure 1.3 shows the noise source distribution at each operating condition for a typical large high bypass ratio engine. At approach, as the engines are operating on a low power setting the airframe noise tends to dominate the noise spectrum. However, at Cutback
Figure 1.2: Sources of noise generation within a turbofan engine.

and Sideline engine power settings are higher and thus engine noise dominates over airframe noise. This thesis focuses on the reduction of noise generated by the engine.

For a high bypass ratio turbofan engine a large proportion of the air ingested by the fan is funneled around the core of the engine through a bypass duct (see figure 1.2), allowing the required thrust to be produced at slower jet speeds. As a consequence, jet noise is greatly reduced and a significant portion of the thrust is delivered from the fan stage. Although no single noise source is dominant over all engine operating conditions, as turbofan bypass ratios become higher, jet noise is decreasing and fan noise is thus becoming the dominant source, both at cutback and sideline. It is for this reason that reduction of fan noise is the topic of this investigation.

Fan noise is comprised of discrete tone peaks, containing several correlated interaction modes, superimposed on a broadband base level generated by turbulence and containing multiple uncorrelated modes. This noise is radiated both forwards through the engine intake, and rearwards through the bypass duct. The work within this thesis concerns the prediction of rearwards fan noise propagation within the bypass duct, and its radiation into the far-field.
1.1.1 Reducing rearwards propagating fan noise

For rearwards radiating fan noise, duct geometry and mean flow conditions both have a significant effect on the far field directivity, as does the distribution of any acoustic treatment within the duct. Attenuation of fan noise within the bypass duct can greatly reduce the amount of noise radiated into the far field, and at present the use of acoustic liners within the duct is one of the most important techniques for achieving this.

To test the benefits of a potential low noise design experimentally is time consuming and costly, and it is therefore necessary to develop other methods of assessment. Computational modelling of noise propagation using numerical prediction techniques is much less expensive and more efficient, and the use of computational models can provide a means of investigating the physics of noise related problems at scales and locations that are impractical to examine by experiment.

For a bypass prediction method to give accurate predictions it must include the effects of duct geometry, flow and acoustic treatment, as well as accurately modelling the acoustic propagation. This work involves the use of propagation and radiation codes for rear fan noise that have been recently developed at the ISVR. The model b-induct is a shell code for making predictions of in-duct propagation for realistic bypass geometries, liner configurations and flow fields, using the commercially available Finite Element code ACTRAN/TM [3]. The code GXMunt provides an analytic solution for the radiation of noise from an idealised exhaust geometry through the bypass shear layer and into the far-field. These two models are presented in more detail in chapter 3.
Chapter 1 Introduction

In order to have confidence in a prediction method it must be thoroughly assessed against experimental data, ensuring that it provides accurate predictions before it can be used as a tool for examining novel noise reduction concepts. This thesis provides assessment of the codes b-induct and GXMunt, and their suitability for use in the engine design process. The codes are also used to examine the effects of duct geometry and liner impedance on propagation within the bypass duct.

1.2 Motivation, aims, and claim to originality

The Engineering Doctorate research programme deals with problems of industrial interest. In industry design choices need to be made within a limited time frame, and due to such constraints it is common to use simplified or idealised prediction models. The challenge is therefore to improve on prediction accuracy without sacrificing time. Accordingly, the focus for this work is to apply the most up to date numerical and theoretical approaches for predicting bypass noise, within the engine design process and under realistic industrial timescales. This research has been performed in collaboration with Rolls-Royce, with the interest of moving towards the integration of a more realistic bypass propagation and radiation model into their whole aircraft noise prediction system: the “Noise Key System”.

B-induct and GXMunt are two recently developed models for predicting propagation and radiation of rearwards fan noise. They are an improvement on current industry standard models because they capture the realistic geometry and flow conditions of the problem. Another advantage of the b-induct shell code is its automatic pre and post-processing capability, allowing time-efficient predictions to be made. The aim of this work is to exploit these prediction methods within real engine bypass ducts, and to obtain a better understanding of the mechanisms behind noise propagation, so that ultimately these models can be used to improve the design process, leading to future low noise engines.

For a model to be used in the design process, it must fulfil some important criteria: Whilst ultimately, it must be usable within a whole engine model in order to predict whole engine noise, it also must be capable of being used as a stand-alone prediction tool, to make fast and reliable assessments of potential low noise bypass configurations. The work within this thesis addresses both of these criteria, with the main focus being on the model b-induct.
1.2.1 Original contributions

1.2.1.1 Assessment of performance of b-induct within a whole engine model:

- Application of b-induct results within an industry standard whole engine model.
- Analysis of whole engine predictions made using b-induct, when compared against measured data from full-scale static engine tests.
- Demonstration of the improved performance of b-induct versus a current industry standard, as a bypass model to be used in whole engine predictions.
- Identification of areas requiring further attention for the prediction of rearwards propagating fan noise within a whole engine model.

1.2.1.2 Assessment of performance of b-induct, as a stand-alone application, against measured data from rig-scale tests on realistic bypass geometries:

- Evaluation of industrially feasible bypass geometries, including a novel low noise design within the ISVR no-flow facility.
- Use of b-induct in combination with GXMunt to provide far-field predictions for the above geometries, and evaluation of these predictions with respect to test data.
- Demonstration of the acoustic benefit of a Highly Curved duct over a Baseline configuration.
- Use of in-duct radial microphone array measurements to re-construct relative amplitudes of source modes.

1.2.1.3 Use of b-induct as a prediction tool to aid in the design of low noise bypass ducts:

- Investigation of the effect of bypass geometry, incorporating both axisymmetric and three-dimensional (3D) features, on noise propagation.
- Demonstration of improved liner attenuation as a consequence of increasing duct curvature.
- B-induct predictions are made for real engines, taking into account the actual bypass duct geometry and specific liner positions, which typical industry models don’t allow for.
• Integration of b-induct attenuations into a liner impedance optimisation procedure, which incorporates a whole engine model and uses aircraft Effective Perceived Noise Level (EPNL) as a cost function. This improved procedure allows an entire set of bypass liners to be optimised within an industrially acceptable timescale, whilst also accounting for the effect of duct geometry.

• Use of this new process to optimise the impedances of each of the liners within the bypass duct for three different real engine bypass configurations, including the selection of liner impedances for a current new engine.

The extensive use of the b-induct model in the applications described above has also provided a useful “Beta-testing” environment for the code, which is continually being improved.

1.3 Planning and progress

The EngD scheme involves research of particular interest to, and in close collaboration with an industrial sponsor. The aim is to tackle problems of industrial relevance, gaining experience of management and research in a commercial context. Approximately half of the time is spent working within the company.

This EngD project was undertaken in association with Rolls-Royce. The company’s interest in this work is to reduce the rearwards fan noise contribution to overall aircraft EPNL and hence meet aircraft certification requirements. This is the common theme of the studies presented. Due to the nature and timescale of industrial research, the work is broad in context, focusing on several projects at once.

In particular, much of this work has been undertaken as part of the UK TSB\textsuperscript{1}-led programme ANDANTE (Aircraft Noise Disturbance Alleviation by Novel TEc hnology). Within this programme, the author project managed Work Package 3.2: “Design and test of a low noise bypass duct”, and was responsible for such things as: planning the test schedule, selecting duct configurations to examine, undertaking risk analysis, performing mitigating actions, producing deliverables and reporting at Quarterly Review Meetings. During the time spent at Rolls-Royce, the author also reported on the status of the curved duct technology as part of a Critical Capability Acquisition Review.

\textsuperscript{1}The Technology Strategy Board is an executive body established by the Government to drive innovation. It promotes and invests in research, development and the exploitation of science, technology and new ideas for the benefit of business - increasing sustainable economic growth in the UK and improving quality of life. For more information visit www.innovateuk.org
Throughout the EngD, regular meetings have been held in order to review progress and plan ongoing and future work. An overview of the work plan undertaken over the 4 years is shown in figure 1.4.2.

1.4 Outline of contents

This thesis is structured as follows: A review of the literature is presented in chapter 2, providing an overview of methods used for obtaining solutions to aeroacoustic problems.

In chapter 3 the dominant physics for modelling fan noise propagation through and radiation from a bypass duct is reviewed. The Finite Element Method (FEM) is presented as a numerical solution to this problem for non-uniform ducts, and the model b-induct, for predicting propagation through a realistic bypass geometry, is described. The model GXMunt, an analytic solution for predicting noise radiation from an annular jet pipe is also introduced.

An assessment of the use of b-induct predictions within a whole engine model is given in chapter 4 by comparing whole engine predictions made incorporating b-induct attenuations with measured data from full-scale engine tests.

In chapter 5, initial predictions examining the effect of axisymmetric duct geometry on noise propagation within the bypass duct are presented, and a set of zero-flow rig-scale tests performed at the ISVR for two geometries of interest: a typical bypass geometry and a novel highly curved geometry, are described in chapter 6. Measured data from the tests on axisymmetric builds is compared with predictions in chapter 7 to continue the examination of the effect of axisymmetric duct geometry on noise propagation, and also to make an assessment of the acoustic benefit of the highly curved configuration. In chapter 8 the effect of 3D features within the bypass duct (such as the pylon and lower bifurcation) on the propagation and radiation of rear fan noise is investigated, by assessing predicted and measured data for non-axisymmetric builds.

In chapter 9 the use of b-induct as a tool within the engine design process is further evaluated by the incorporation of b-induct predictions into a liner impedance optimisation methodology. This procedure is then used to optimise the liner impedances for several real engines, in order to maximise the sound absorption within the bypass duct. Overall conclusions and suggestions for future work are presented in chapter 10.

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2Some aspects of work undertaken, such as the intake code development and rig predictions, were reported in the author’s mini-thesis [4] but are not reported here.
Figure 1.4: Research workplan. Tasks in dark grey undertaken at ISVR. Tasks in light grey undertaken at Rolls-Royce.
Chapter 2

Literature Review

2.1 Introduction

Aeroacoustic problems are those involving the generation and propagation of sound in moving flows. For the aeroacoustics of aircraft engines in particular, there are many methods which are used for obtaining solutions to such problems. In the 1960s and 70s, theoretical models for idealised geometries and flow fields were developed, many of which are still in use today. These include duct eigenvalue solutions, mode matching techniques, simple radiation models and ray theory. A review of these theoretical models is given in section 2.2.

More recently, the development of computational methods involving the use of numerical techniques has enabled solutions to be obtained for more realistic geometries and flows. These numerical methods can be grouped broadly into source models (e.g. Direct Numerical Simulation (DNS), Large Eddy Simulation (LES) and Reynolds Averaged Navier-Stokes (RANS) calculations), although these will not form a significant part of this discussion, and propagation models (commonly based on solution of the Linearised Euler Equations (LEEs) and their various subsets), which will be discussed in more detail. A review of these numerical models is given in section 2.3.

To investigate the aeroacoustics of aircraft engines, each of these methods have their advantages and disadvantages. As techniques have evolved that allow for the solution of a more accurate description of the problem, the amount of effort needed to obtain a solution has also increased. Therefore, in choosing a technique to use it is important to consider the particular problem to be solved in terms of the physical effects and numerical accuracy required, the time limitations, and the computational resources available.
2.2 Theoretical approaches

For theoretical duct propagation models to predict noise attenuation in turbofan ducts, the typical starting point is the simplification of the inlet or bypass geometry to axisymmetric cylindrical or annular ducts of infinite length and axial uniformity, for which the mean flow field is uniform, and the in-duct pressure field can be decomposed in terms of duct modes. An overview of some of these duct acoustics methods is given by Eversman in reference [5]. The consideration of the sound field within an aeroengine duct in terms of acoustic modes was pioneered in the 1960s by Tyler and Sofrin [6], and many of the theoretical models established since then for predictions of duct noise propagation are based on this foundation.

For a particular duct section, the solution of the convected Helmholtz equation with the appropriate boundary conditions can be obtained using an eigenvalue solver and used to determine the modes present in the duct. The investigation of finite length liners can then take place by matching modal expansions at the interface between different axial segments, as first demonstrated for the aeroengine duct problem by Lansing and Zorumski [7], and used more recently by McAlpine et al. [8] and Astley et al. [9, 10].

The downside of the mode-matching approach is that simple analytic solutions are only available for certain geometries and flow conditions, such as circular, annular and rectangular ducts with uniform mean flow and liners which are uniform in the circumferential or transverse directions. Also, whilst rigid duct modes can be easily defined, the determination of lined duct modes is more difficult, particularly when flow is present, meaning that the development of robust methods which are able to resolve all of the modes all of the time is not a simple matter. Despite this, mode-matching methods are computationally inexpensive and thus lend themselves well to the optimisation of liners within aeroengine duct systems [11, 12, 13].

Recently, Rienstra [14] has derived an exact solution for lined circular and annular ducts whose diameter, mean flow and impedance are slowly varying in the axial direction, the calculation complexities of which are no greater than for classical uniform duct models. This work was initially extended to hard walled ducts of elliptic cross-section with axially varying eccentricity, by Peake and Cooper [15]. Rienstra then extended the solution further to any hollow or annular duct with arbitrary cross-section and impedance which are slowly varying in the axial direction [16]. Whilst Rienstra’s work allows ducts of axially non-uniform geometry to be considered, the calculation of the eigensolutions for arbitrary cross-sections is still not straightforward.

The use of a modal representation within the duct also lends itself to integration with analytic radiation models at the duct exit. For these radiation models, a simplification of the exhaust geometry must also be used. Homicz and Lordi [17] and also Rice [18, 19, 20] used mode-theory approaches where the analytic solution to the pressure field
within a duct is determined in terms of a summation of duct modes, and the radiative
directivity pattern is examined using both mode cut-off ratio and mode propagation
angle as parameters. In particular, Rice proposed that for uniform Mach number both
inside and outside of the duct, the resultant axial propagation angle inside the duct
coincides with the angle of the principal far-field radiation lobe.

More recent analytic radiation models are those of Munt [21, 22], who considered the
problem of a circular jet pipe with a bypass shear layer, and of Rienstra [23], who
derived the general solution for a semi-infinite duct with an infinite centrebody, using
the Wiener-Hopf technique. This solution thus incorporated the effect of an exhaust
centrebody, but for the case of uniform flow across the jet and bypass stream. By
extending the work of Munt and Rienstra, Gabard [24] has recently developed an exact
solution for a semi-infinite annular duct with an infinite centrebody and also a bypass
shear layer, allowing for the prediction of far-field bypass noise radiation. As a General
eXtension of the Munt solution, Gabard’s model is known as “GXMunt”, and is utilised
within the work carried out for this thesis. A more complete description of the GXMunt
model will therefore be given further on.

The final theoretical method to mention is the ray theory approach, as used by Kempton
[25, 26, 27] and Dougherty [28], where both the in-duct and external solutions can be
examined using ray tracing techniques. Ray theory has the advantage of being able
to model the propagation of sound along variable area ducts, attenuation by acoustic
liners, refraction by intake and exhaust flows and scattering by turbulence, making it
a useful tool within part of a complete prediction scheme. However, the ray acoustics
approach does not model the effects of diffraction and interference, so to provide accurate
predictions, it is necessary to find a way to incorporate these effects into the model
[29]. Also, whilst the ray acoustics solutions are independent of frequency, the physical
approximation is only valid at high frequencies, where the wavelength concerned is
much shorter than the length scale of the problem geometry. Whilst this restricts its
application at lower frequencies, ray theory is particularly useful in the high frequency
regime where the use of mode theory techniques is limited due to the large number of
modes excited and an excessive number of grid points would be required for numerical
analysis.

2.3 Numerical approaches

Aeroacoustics problems are by nature very different from standard aerodynamics and
fluid mechanics problems. For instance, the energy level of the mean flow field is usually
much larger than the energy level of the acoustic field, and it is necessary to model small
amplitude fluctuations on large scale flows. Also, aeroacoustics problems tend to involve
multiple scales, where the acoustic wavelength is greater than the length scale of the fluid dynamic source but smaller than the length scale of the model geometries involved.

Conventional Computational Fluid Dynamics (CFD) schemes designed for application to fluid problems are generally not adequate for modelling acoustic propagation. Standard CFD schemes tend to be low order numerical methods and are therefore very dissipative, such that any acoustic disturbances are damped, and have poor dispersion characteristics, resulting in a breakdown in coherence. To model the acoustic disturbances accurately it is critical to use high order numerical methods specially designed to reduce these dispersion and dissipation errors. Consequently, an independent development of high order methods to solve problems of aeroacoustics has occurred. These approaches are known as Computational Aeroacoustics (CAA).

2.3.1 Non-linear schemes

The most complex of these computational approaches involves the solution of the full Navier-Stokes (N-S) equations for a compressible fluid, without approximation. This approach is called Direct Numerical Simulation (DNS) and allows for the calculation of the entire range of spatial and temporal scales of turbulence. DNS is therefore a useful prediction tool for fundamental research in turbulence. However, because all scales are captured high order resolution is critical, and both the number of mesh points and the number of time steps required increase with Reynolds number, making the computational cost of DNS extremely high. For this reason DNS is only practically applied to low Reynolds number flows for jet noise simulation [30] or to small problem domains, as used by Tam and Kurbatskii [31] to explore the mechanisms behind acoustic energy dissipation at the mouth of a liner.

In aeroacoustics, turbulence is the principal source of broadband noise, and the modelling of turbulent flows is therefore necessary to understand the mechanisms of broadband noise generation and propagation. However, because DNS is so computationally expensive it is presently not considered feasible for the large problem domains and high Reynolds number flows associated with aero-engines, resulting in the need to develop turbulence models for practical applications. For the higher Reynolds number flows associated with aero-engine jet noise Large Eddy Simulation (LES) is common, where the large scale structures are still captured using DNS, but the effect of the smaller scale turbulence is modelled using a sub-grid scale model [32, 33]. In this way, LES is less computationally demanding than DNS, although as the computational requirements are still large it is usually only used for source calculation in the near field.

For a method that is computationally cheaper still, the solution of the Reynolds-Averaged Navier-Stokes (RANS) equations combined with turbulence modelling is possible. To obtain the RANS equations the flow variables in the N-S equations are separated into
a mean part and a fluctuating part. A time-average is then taken and the Reynolds stress term in the resulting equations is approximated by a turbulence model. In this way a solution is found for the steady flow field and the overall turbulence properties. Because the solutions for turbulence obtained using RANS methods are time-averaged rather than instantaneous, RANS methods tend to be lower order and provide a lower level of detail than LES. However, RANS solutions for turbulence intensity can still be used as an input to an acoustic source model for predictions of jet noise, as shown by Self [34] and Omais et al. [35].

In order to simplify the physics of the problem somewhat, if viscosity and thermal conduction effects are neglected, the N-S equations convert to the Full Euler equations. High order schemes are still necessary to avoid dispersion and dissipation errors, and the French Aerospace Centre ONERA have developed a CAA code named sAbrinA.v0 which solves the Full Euler equations using a high order finite difference scheme. This code is used in reference [36] for predictions of aft fan noise emitted by a full three-dimensional (3D) exhaust (with pylon and internal bifurcations included in the model).

Ozyoruk and Long also use a high order finite difference scheme for solution of the Full Euler equations in reference [37]. These calculations are performed for an intake geometry, where the modelling of non-linear effects is important, especially at high power settings of the engine, due to high inlet Mach numbers and large acoustic perturbations. Due to the large computational requirements, calculations are carried out on parallel processors. The same method is also used in reference [38] with an extension to full engine configurations.

2.3.2 Linear schemes

2.3.2.1 Rotational flows

For flows where non-linear effects are less significant, such as those associated with an engine bypass duct, it is common to search for solutions to the Linearised Euler Equations (LEEs), where solutions for the flow terms are assumed to take the form of an unsteady perturbation upon a mean flow term, and 2nd order or higher terms within the equations are neglected. Schemes which solve the LEEs can thus no longer model non-linear effects, but can still deal with arbitrarily sheared or rotational flows, making them suitable for bypass and exhaust flow problems.

There are a wide range of different approaches used to obtain solutions to the LEEs. To model the problem geometry, both structured and unstructured grids can be used. Structured grid methods involve the use of templates which relate the solution at adjacent points, and although it is easier to implement high order methods using structured grids, the large templates necessary lead to long computation times. Typical examples of
structured grid methods include finite difference, Dispersion Relation Preserving (DRP) [39] and compact schemes.

Unstructured grid methods define the order of the scheme within each individual element, making the implementation of high order methods slightly more complex. The use of unstructured grid methods such as the Finite Element Method (FEM) is better for modelling complex geometries, and although typical FEMs are restricted to irrotational flows there are some kinds of FEM that do permit rotational mean flows such as Discontinuous Galerkin Method (DGM) and Streamline Upwind Petrov-Galerkin (SUPG) schemes.

Also, because the Euler equations are now in linear form they can be decomposed into Fourier components and hence solved in either the time domain or the frequency domain. Both of these solution techniques still suffer from dispersion and dissipation errors as a consequence of resolving the waves numerically, and therefore high order schemes are still necessary to reduce these and ensure accurate interpolation. A review of the most common schemes and a discussion of the advantages and disadvantages of time domain and frequency domain methods is given below.

2.3.2.2 Time domain vs frequency domain

Time domain solutions offer a complete set of space-time data for an acoustic phenomenon and thus can be used to investigate noise generation and propagation mechanisms in much the same way as experimentation, but without many of the associated time and cost constraints. The Discontinuous Galerkin Method (DGM) is a time domain approach that uses Finite Elements (FEs) and allows a discontinuity of the numerical solution at element interfaces. DGM schemes are high order and simulate wave propagation with small levels of dissipation and dispersion, as examined by Ainsworth [40] and Hu and Atkins [41]. Also, due to the use of FEs, unstructured meshes can be used for the spatial discretisation, allowing complex geometries and problems of multiple scales to be easily modelled [42, 43]. Chevaugeon et al. [44] have exploited these advantages in the development of a commercial Runge-Kutta DGM code, ACTRAN/DGM, focusing the use on large-scale industrial aero-acoustics applications.

Finite Difference schemes are also common time domain approaches. In particular the Dispersion Relation Preserving (DRP) high order Finite Difference scheme of Tam and Webb [39] is quite popular. The same scheme is also pursued by Zhuang and Chen, [45] and Lockard et al. [46], with some modifications, and by Bailly and Juvé [47], who perform calculations for a sheared mean flow and add non-linear terms to the LEEs to suppress the growth of hydrodynamic instabilities. These instabilities are physical and known as Kelvin-Helmholtz instabilities, generated in the presence of parallel shear flows. In the complete physical problem these particular instabilities are limited and
modified by non-linear and viscous effects, but as these are not modelled within the framework of the LEEs, the instability wave solution can grow exponentially and completely overwhelm the acoustic solution. This effect is non-physical, and thus much effort has also been applied to produce schemes in which these Kelvin-Helmholtz instabilities can be suppressed.

For instance, Zhang et al. [48] use a high-order prefactored compact scheme to obtain a time domain solution to the LEEs, and remove the mean flow gradient terms from the LEEs in order to prevent the occurrence of the Kelvin-Helmholtz instabilities. The same scheme is also used by Richards et al. for axisymmetric geometries in reference [49], and in reference [50] the scheme is extended to fully 3D geometries, but in this case the instabilities are suppressed in a different way by the removal of vortical modes. The advantage of compact schemes such as these is that they use smaller templates for the numerical interpolation than conventional finite difference methods, and are therefore attractive due to the lower computational memory required.

Time domain methods such as those described above have the advantage of low storage requirements and suitability for parallelisation. However, it is difficult to formulate impedance boundary conditions in the time domain [51, 52, 53] and therefore one of the weaknesses of time domain approaches is the ability to adequately represent acoustic liners. Because liners have frequency dependent impedance characteristics it is much easier to represent them using frequency domain methods.

To combine both approaches, Ozyoruk [54] obtains a solution for the LEEs in the frequency domain but introduces a pseudo-time derivative so that a time-marching Finite Difference technique can still be used. Similarly, Lan [55] and Breard [56] describe a frequency domain solution using a pseudo-time derivative and the DRP Finite Difference scheme developed by Tam in reference [39]. This is originally presented for the axisymmetric case in references [55] and [57], and then extended to the 3D case in reference [58].

Schemes such as these exploit the advantages of both time domain and frequency domain approaches because it is possible to use a frequency-dependent impedance condition to represent the acoustic liners, but still use a parallel computing approach for the solution. However the Kelvin-Helmholtz instabilities mentioned previously are still present in the solution and it is again necessary to use techniques that suppress these.

Agarwal [59] has shown that one way to ensure the removal of the Kelvin-Helmholtz instabilities is by solution of the LEEs in the frequency domain using a direct solver. Ozyoruk et al. [60] and Tester et al. [61] also exploit this approach in a frequency domain propagation model named “FLESTURN” in which a direct LEE solver is employed to solve the discretised equations using parallel computation.
Agarwal’s findings are also exploited by Zhao and Morris [62, 63], who develop a frequency domain methodology for the solution of the LEE using the Streamline Upwind Petrov-Galerkin method (SUPG). As with the DGM, the SUPG is a type of FEM that does not require the usual assumptions of either axisymmetric or irrotational mean flow, and can use unstructured grids to model complex geometries. Rao and Morris [64] compare both of these methods in the frequency domain and find the SUPG method to be more computationally efficient than the DGM.

Frequency domain schemes are generally less computationally demanding than the equivalent time domain methods for solving axisymmetric problems, and therefore tend to be attractive for industrial applications. However, frequency domain solutions do scale poorly with problem size, and hence tend to have large storage requirements and longer computation times for problems in three-dimensions. Furthermore, whilst frequency domain schemes can solve for multiple modes simultaneously, they are limited by only being able to examine one frequency at a time. Time domain schemes on the other hand can solve for multifrequency sources in a single simulation. This makes time domain methods more suitable for broadband problems whilst frequency domain schemes are preferable for predicting tone noise.

2.3.2.3 Irrotational flows - Finite Element Methods

For further simplification of the problem, the physics must again be considered. The Linearised Euler Equations support three modes of waves: acoustic, vortical and entropic disturbances, and can therefore model any acoustic-aerodynamic interactions that may occur. By regarding an isentropic, adiabatic gas and assuming irrotational flow, both vortical and entropic disturbances are neglected, and the flow can be expressed in terms of a velocity potential, given by the gradient of the flow term. By also assuming an acoustic perturbation that is time-harmonic, the LEEs are simplified such that there is now only one 2nd order Helmholtz-like equation for this velocity potential, which can be solved in the frequency domain.

Parrett and Eversman [65] use a Finite Element/ Wave Envelope Element formulation developed by Astley [66] to obtain solutions to the velocity potential formulation for a turbofan inlet problem. This model is then extended to the exhaust problem in [67], where the presence of the jet requires certain conditions to be imposed on acoustic radiation across the shear layer.

There are also various codes in industrial use which aim to provide solutions for this velocity potential. The NASA Eversman code [68], as used by Abdelhamid [69] uses FEs for the near field and Mapped Infinite Wave Envelope Elements, as developed in reference [70] for the radiation problem.
The ACTRAN/AE and ACTRAN/TM codes ([71] and [3] respectively) are commercial tools developed by Free-Field Technologies, Belgium, and also obtain solutions to the velocity potential equation in the frequency domain, by use of the Finite Element Method (FEM), with Infinite Elements (IEs) used to map the far-field. Astley et al. use this code in reference [72] to model tone propagation from a turbofan inlet, where the order of the Infinite Elements used is varied until a converged solution is obtained.

Sugimoto [73, 74, 75] also uses ACTRAN/TM as a prediction method within a shell code called b-induct, designed for automating the procedure of obtaining solutions of noise propagation in axisymmetric bypass ducts. A three-dimensional version of this code has also been developed [76, 77] for modelling non-axisymmetric features within the duct. The b-induct model has been used extensively within this thesis and hence further details of these codes will be provided later.

2.3.2.4 Boundary Element Methods

Within industry it is common to develop codes that will simplify the prediction process and make it more efficient to investigate low noise concepts within the engine design process. One common simplification is to use the Boundary Element Method (BEM), which discretises the problem on the boundary of the domain, and requires the use of a Green’s function [78]. One such code is the ACTI3S BEM code, developed by EADS Common Research Centre, France (EADS-CRC-F) and used by Airbus, France [79, 80]. This model provides a frequency domain solution to the 3D homogeneous Helmholtz equation using the BEM and is used for problems involving isolated engines. For the assessment of acoustic effects involved in airframe-engine interaction the larger computational domain results in the need for more efficient computation. To achieve this, the ACTI3S code has been modified to incorporate a Fast Multipole Method (FMM) algorithm, resulting in a new code called ACTIPOLE [81], which provides a great reduction of data storage requirements and computational time.

BEM codes such as these are able to simulate realistic 3D geometries using surface meshes, but are restricted to uniform (or nearly uniform) base flows. Despite this limitation they are still useful tools for parametric investigation of geometric effects, or to make preliminary assessments of a new design before the use of more sophisticated numerical methods.

2.3.2.5 Parabolic Equations

Another industrial code, used by Boeing, is the CDUCT code for ducted fan noise propagation, developed by Dougherty [82] and revised by Lan [83]. The CDUCT code is based on a parabolic approximation to the convected Helmholtz Equation, which
simplifies the problem by neglecting reflected waves, and provides a solution for one-way wave propagation. The same parabolic approximation theory model is also used by NASA as the propagation module within a modular ducted fan noise propagation and radiation code called CDUCT-LaRC [84].

These approaches afford very efficient propagation calculations, allowing solutions for complex, 3D geometries to be provided at relatively low computational costs, such that both the CDUCT and CDUCT-LaRC codes satisfy many of the requirements of an industrial design tool. However, the simplification of the physics by neglecting rearwards propagating waves leads to a reduction in accuracy of the model, especially for problems where reflection and/or scattering of acoustic waves becomes important. As such, Nark [85] suggests that the CDUCT-LaRC model is used to identify preliminary low noise designs and then higher fidelity methods (such as FEM) are then used to further refine the results.

2.4 Summary of prediction techniques

Overall, prediction methods for investigating the aeroacoustics of aircraft engines can range from simple theoretical models, which are computationally inexpensive and idealise the problem geometry or approximate the physics in some way, to more complex numerical methods which can provide a more accurate description of the physics or geometry, but require large computational resources in terms of time and memory. The choice of method will depend on both the accuracy of the solution required, and any practical considerations necessary. For industrial applications, the available computational resources and time constraints are both factors which greatly influence the method chosen for the design or analysis of low noise aeroengines.

2.5 Experimental approaches

In the preceding sections discussion has focussed on the various approaches used to model the aeroacoustics of aircraft engines. Simulation of the generation and propagation of noise is important because experiments are time consuming and costly, and if accurate models with which to investigate potential low noise technologies can be produced, the need for testing decreases. However, experimentation is still essential: Firstly because it provides data with which to evaluate the accuracy of predictions and hence validate new models; Secondly, because measured data allows further analysis of specific effects and can be used to confirm the benefits of low noise technologies suggested by initial simulations.
The validation of a new model usually involves several stages, whereby the model is tested against experimental data for increasingly complex geometries and flow fields, and increasingly realistic noise sources. In this way, at each stage confidence in the prediction accuracy of the model is improved. The sections below review recently conducted experimental work, starting with tests of simple geometries using broadband, multimodal noise sources, and then moving onto more complex geometries, and noise sources that are more representative of those within a real engine.

2.5.1 Broadband sources

As part of the European research programme SILENCE(R), tests have been performed at the ISVR no-flow facility, using a broadband source and examining both lined radial and circumferential splitters. Measurements of attenuation provided by incorporating lined radial splitters within a uniform duct have been compared with predictions using a 3D version of the b-induct code [76], and showed a level of attenuation similar to that expected due to the additional liner area. Configurations involving a lined circumferential splitter were also tested, and in this case measurements of attenuation were greater than would be expected simply in terms of the overall liner area increase [74, 86].

The effect of liner on the exhaust afterbody has also been tested at the ISVR no-flow facility [87], again using a broadband source, as part of the European TURNEX programme. These tests indicate that up to 3dB sound power level insertion loss can be achieved with an afterbody liner, in addition to that achieved with conventional bypass liners, and ACTRAN FEM predictions compare well with the measured data. The ISVR no-flow facility has also been used to obtain measured data for lined and unlined configurations of a uniform annular duct, for both a straight inner wall and an inner wall incorporating a turbine hump [74]. This data showed good agreement with predictions of sound pressure directivity obtained using b-induct + GXMunt.

All of the tests discussed above have been carried out using a multimodal source, which is representative of the broadband base level generated by turbulence at the fan. Enghardt et al. [88] evaluate different analysis techniques to decompose such a broad-band ducted sound field into its’ modal constituents. Tests were performed in the DLR no-flow rig for a cylindrical duct, the in-duct sound field being acquired by means of a wall-flush-mounted microphone array consisting of 4 circumferential arrays spaced axially along the duct. Two different techniques involving the relative spectral cross-correlations of the sensors are examined and these techniques are benchmarked with regard to their ability to distinguish coherent and incoherent modes.
2.5.2 Single mode sources

For experiments using modal sources, representative of the content of specific rotor-alone or rotor-stator interaction tones, the NASA Curved Duct Test Rig (which can be used with flow) incorporates an adaptive control system to generate a tone in the duct with a target mode shape. This facility uses a rectangular duct approximation to an annular duct geometry, and has been used to test both straight and curved duct sections [89, 90]. Results are compared with numerical FEM predictions, and also used to examine the effect of duct curvature on liner attenuation.

Mode generation and detection techniques have also been developed by EADS-IW. A mode synthesiser consisting of a circular phased array of actuators is used to generate particular azimuthal modes, and an in-duct circumferential array of flush-mounted transducers is used to determine the azimuthal mode spectra in the duct. In the far-field, a traversable circular microphone array is also used for modal analysis. These mode generation and detection techniques were tested initially in the NLR small anechoic wind tunnel facility KAT [91], and then applied in the examination of 3D engine exhaust geometries at the Qinetiq jet noise test facility NTF, as part of the European TURNEX programme [92, 93]. In addition to examining the effects of various geometric configurations on noise propagation, the Qinetiq test data was used to validate numerical methods applied within TURNEX (such as the high-order finite-volume method used by Kok [94]), where in-duct measurements serve as the input for the prediction models and the far-field data is used for validation.

2.5.3 Realistic sources

Whilst speaker arrays are a simple, inexpensive approach to simulating source noise, it is important to examine sources that are as realistic as is practically achievable. Whilst the optimum case would be a full-scale fan installed within an engine, a simplified situation is to use a rig-scale fan mounted in a test assembly.

The NASA Active Noise Control Fan (ANCF) test rig [95] consists of a uniform duct, with a source comprising a rotating fan and stator vanes, where in-duct rakes can be used to provide complex mode amplitudes at planes close to the inlet and exhaust. SPL directivities in the rear arc from experiments performed using this rig, have previously been compared to b-induct + GXMunt predictions for validation purposes, and shown to give good agreement [75]. Likewise, the Rolls-Royce AneCom test rig, which similarly incorporates a rotating fan source, has been used to investigate the effect of various intake configurations, including fancase liner splices and a zero spliced liner, on forward propagating fan noise [96]. Experimental data from these tests has also been used to assess the FEM intake code ANPRORAD [97].
The ultimate aim of computational modelling is to provide accurate noise predictions for real engines. Accordingly, measured data from full scale engine tests, in addition to assessing actual noise levels generated by the engine, is invaluable in providing insight into the accuracy of the models. For example, Lan uses data from NASA tests on a Pratt & Whitney JT15D turbofan engine, acquired both statically in a wind tunnel and in flight, to validate a frequency domain LEE solver [57]. Similarly, Han compares measurements of in-duct, near-field and far-field SPLs from static engine tests with numerical RANS CFD simulations, and reports good agreement [98].

2.6 Summary of experimental approaches

To be completely certain of the noise profile of a real full scale engine, there is no substitute for measurement. However, experiments involving real engines are both costly and time consuming, and therefore in the design process it is important to utilise methods which can predict the overall noise as accurately as possible. To assess the accuracy of these prediction models it is necessary to perform validation in stages, where increasingly complex geometries and increasingly realistic sources are used. A variety of test facilities suitable for addressing such complexities in the experimental design are available, and have been used for this purpose.
Chapter 3

Methods

3.1 Problem definition

To model noise propagation within an aircraft bypass duct, it is first necessary to understand the content and structure of the noise source at the fan plane - the “source model” - and then to obtain a representation of how the sound from this source propagates within the duct - the “propagation model”. Once the sound field reaches the duct exit a “radiation model” is also necessary to represent how the noise propagates into the far-field. Whilst a detailed examination of the source model is beyond the scope of this thesis, within this chapter, the current best methods for predicting propagation are reviewed in section 3.3 and those for predicting radiation are reviewed in section 3.5.

3.2 Governing Physics

Flow acoustics is governed by the equations of fluid dynamics, which are derived from three fundamental physical principles: Conservation of mass, which leads to the “Continuity Equation”, conservation of momentum, which leads to the “Momentum Equations” or “Navier-Stokes Equations”, and conservation of energy which leads to the “Energy Equation”. All of these equations, together with the equation of state define the “Full Navier-Stokes Equations” for a compressible fluid.

The Navier-Stokes equations include the effects of viscosity, where the transport phenomena of friction and thermal conduction are included. These transport phenomena are dissipative - they always increase the entropy of the flow. Mass diffusion (due to concentration gradients of different chemicals) is not modelled. If the terms in the Navier-Stokes equations involving viscosity and thermal conductivity are neglected (ie. if we simply drop all of the terms involving friction and thermal conduction) the Navier-Stokes Equations can be simplified to the Euler Equations. Thus the Euler Equations
are the governing equations for an unsteady, three-dimensional, compressible, inviscid flow.

By making the assumption that all flow quantities can be written as the sum of steady mean flow components and unsteady perturbations, the values for total density, \( \rho_T \), total pressure, \( p_T \), total sound speed, \( c_T \), and total flow velocity, \( \mathbf{v}_T \), can be written as:

\[
\rho_T(x, t) = \rho_0(x) + \rho(x, t) \tag{3.1}
\]

\[
p_T(x, t) = p_0(x) + p(x, t) \tag{3.2}
\]

\[
c_T(x, t) = c_0(x) + c(x, t) \tag{3.3}
\]

\[
\mathbf{v}_T(x, t) = \mathbf{v}_0(x) + \mathbf{v}(x, t) \tag{3.4}
\]

where \( \rho_0, p_0, c_0 \) and \( \mathbf{v}_0 \) are the density, pressure, sound speed and flow velocity of the mean flow and \( \rho, p, c \) and \( \mathbf{v} \) are their unsteady counterparts.

Using these expressions within the Euler Equations, and neglecting any second order (or higher) terms results in a set of equations where only the linear/first order terms remain. These are known as the Linearised Euler Equations (LEEs):

**Linearised Continuity Equation:**

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho_0 \mathbf{v} + \mathbf{v}_0 \rho) = 0. \tag{3.5}
\]

**Linearised Momentum Equation:**

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}_0 \cdot \nabla \mathbf{v} + \frac{1}{\rho_0} \nabla p + \mathbf{v} \cdot \nabla \mathbf{v}_0 - \frac{\rho}{\rho_0^2} \nabla p_0 = 0. \tag{3.6}
\]

At this stage the linearised equations support acoustic, vortical and entropic disturbances. However, if an adiabatic process is considered (which in the absence of dissipation is also isentropic), the entropic disturbances are excluded. For this case, the energy equation and the equation of state reduce to a single isentropic relationship between pressure and density such that for the mean flow:

\[
c_0^2 = \gamma \frac{p_0}{\rho_0} \tag{3.7}
\]

and for the unsteady perturbations:

\[
p = c_0^2 \rho, \tag{3.8}
\]

where \( c_0 \) is the local speed of sound in the mean flow, and \( \gamma \) is the ratio of specific heats, \( C_P/C_V \) (heat capacity at constant pressure/heat capacity at constant volume), and is equal to 1.4 for dry air at 20°C.
For a uniform flow field where both the mean flow $v_0$ and the unsteady perturbation flow $v$ are irrotational, the vortical disturbances are now also excluded, and the flow can be expressed in terms of a velocity potential, $\phi$ such that:

$$v_T = \nabla \Phi,$$  

(3.9)

$$v_0 = \nabla \phi_0,$$  

(3.10)

$$v = \nabla \phi,$$  

(3.11)

where $\Phi$ denotes the total velocity potential, $\phi_0$ denotes the velocity potential of the mean flow and $\phi$ denotes the velocity potential of the acoustic perturbation.

If we substitute these expressions for $v$ into the linearised continuity and momentum equations, we obtain the acoustic field equations in terms of velocity potential:

**Acoustic Continuity Equation (Irrotational):**

$$\frac{\partial p}{\partial t} + \nabla \cdot (\rho_0 \nabla \phi + \rho \nabla \phi_0) = 0.$$  

(3.12)

**Acoustic Momentum Equation (Irrotational):**

$$p = -\rho_0 \left( \frac{\partial \phi}{\partial t} + \nabla \phi_0 \cdot \nabla \phi \right).$$  

(3.13)

In order to determine a particular solution to the acoustic field equations in terms of pressure or density, specific boundary conditions must also be defined in the vicinity of solid objects such as a duct wall.

For a rigid, impervious surface a “hard-wall” boundary condition is defined such that both the mean flow and acoustic velocities are tangential to the wall, i.e.:

$$v_0 \cdot n = 0,$$  

(3.14)

and

$$v \cdot n = \nabla \phi \cdot n = 0,$$  

(3.15)

where $n$ is a unit vector perpendicular to the surface of the wall.

For a lined wall, which is impervious to the mean flow, but forms a porous, locally reacting impedance surface for the acoustic field, the acoustic particle velocity is restricted to the direction normal to the wall (see figure 3.1).

The impedance boundary condition defines the relationship between the acoustic pressure and the normal component of the acoustic particle velocity at the duct wall, $v_n$,.
such that:

\[ \frac{p}{v_n} = Z, \]  

(3.16)

where \( Z \) is the specific acoustic impedance of the wall.

When flow is present this relationship holds at the inner (no-slip) surface of a boundary layer. However, in order to apply such a boundary condition to the acoustic problem, equation (3.16) must be projected through the boundary layer and applied in the acoustic domain where an inviscid “slip” condition exists at the wall. When the boundary layer is infinitesimally small, and for time harmonic acoustic perturbations of the form \( p(x, t) = p(x)e^{i\omega t} \), (where \( \omega \) denotes angular frequency) Myers [99] has shown that this is equivalent to the following boundary condition on the acoustic pressure:

\[ \mathbf{v} \cdot \mathbf{n} = \nabla \phi \cdot \mathbf{n} = \left( \frac{p}{Z} \right) + \left( \frac{1}{i\omega} \right) \mathbf{v}_0 \cdot \nabla \left( \frac{p}{Z} \right) - \left( \frac{p}{i\omega Z} \right) \mathbf{n} \cdot (\mathbf{n} \cdot \nabla) \mathbf{v}_0. \]  

(3.17)

It should be noted that the above boundary condition assumes a vanishingly thin non-viscous boundary layer, which is physically unrealistic. Moreover, the application of this boundary condition has been shown to lead to instabilities [100]. In spite of these limitations, the Myers boundary condition has been widely used in the absence of better models. Recently however, Aurégan [101] and Bramley [102] have presented a model which includes the effect of viscosity in the boundary layer, and is therefore more physically realistic. Such models present an opportunity for further understanding of the underlying physics and eventual improvement of prediction accuracy.

The implementation of appropriate boundary conditions, specific to a problem, allows the particular solutions for the acoustic field to be determined. This will be shown in the case of a uniform duct with hard walls in section 3.3.2 and for a uniform duct with lined walls in section 3.3.3.
3.3 Propagation Models

3.3.1 Propagation within a uniform axial flow

For propagation of acoustic disturbances within an axially uniform sub-sonic flow of velocity magnitude, \( U_0 \), or Mach number \( M = U_0/c_0 \) along the duct axis (z-direction), the mean flow velocity can be written:

\[
v_0 = \nabla \phi_0 = (0, 0, U_0),
\]

and the convected derivative can now be written:

\[
\left( \frac{\partial}{\partial t} + v_0 \cdot \nabla \right) = \left( \frac{\partial}{\partial t} + U_0 \frac{\partial}{\partial z} \right).
\] (3.19)

Equations (3.5), (3.6) and (3.8) can then be combined to yield the Convected Wave Equation:

\[
\nabla^2 p - \frac{1}{c_0^2} \left( \frac{\partial}{\partial t} + U_0 \frac{\partial}{\partial z} \right)^2 p = 0,
\] (3.20)

or:

\[
\nabla^2 p - \frac{1}{c_0^2} \left( \frac{\partial^2}{\partial t^2} + 2U_0 \frac{\partial^2}{\partial t \partial z} + U_0^2 \frac{\partial^2}{\partial z^2} \right) p = 0.
\] (3.21)

For an acoustic disturbance which is time-harmonic with angular frequency \( \omega \), the acoustic pressure can be written:

\[
p(x, t) = p(x) e^{i\omega t},
\] (3.22)

and the Convected Wave Equation reduces to the Convected Helmholtz Equation:

\[
\nabla^2 p - M^2 \frac{\partial^2 p}{\partial z^2} - 2ikM \frac{\partial p}{\partial z} + k^2 p = 0,
\] (3.23)

where \( k = \omega/c_0 \) is the wavenumber of the acoustic disturbance.

By substituting the expression for \( p \) in (3.13) into equation (3.23) it can be shown that the same equation holds for the acoustic velocity potential, \( \phi \). In Cartesian coordinates \((x, y, z)\) this would be expressed as:

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + (1 - M^2) \frac{\partial^2 \phi}{\partial z^2} - 2ikM \frac{\partial \phi}{\partial z} + k^2 \phi = 0,
\] (3.24)

or in cylindrical coordinates \((r, \theta, z)\):

\[
\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + (1 - M^2) \frac{\partial^2 \phi}{\partial z^2} - 2ikM \frac{\partial \phi}{\partial z} + k^2 \phi = 0.
\] (3.25)
In all equations from this point, the symbol \( p \) will be used to represent the acoustic pressure in terms of the spatial coordinates only (ie. \( p = p(\mathbf{x}) \)).

### 3.3.2 Propagation in a uniform duct with hard walls

For a rigid cylindrical duct of circular cross-section (with radius \( a \)), as shown in figure 3.2, separable solutions to the Convected Helmholtz Equation (3.25) can be sought. Sinusoidal variation of angular frequency \( m \) is assumed in the circumferential direction, and sinusoidal variation with wavenumber \( k_z \) is assumed along the duct axis. In this way a separable solution for \( \phi \) can be given by:

\[
\phi(r, \theta, z) = \phi(r)e^{im\theta}e^{-ik_zz},
\]

and this expression for \( \phi \) can be substituted into equation (3.25) to obtain a solution of the form:

\[
\phi_{mn}(r, \theta, z) = A_{mn}^\phi N_{mn}|m|J_{|m|}(k_{rmn}r)e^{im\theta}e^{-ik_{zmn}z},
\]

where \( J_{|m|} \) is the Bessel function of order \( m \), \( A_{mn}^\phi \) is the amplitude of the wave, \( N_{mn} \) is a normalisation factor, \( k_{rmn} \) is the wavenumber in the radial direction, and \( k_{zmn} \) is the axial wavenumber. These are related by:

\[
k_{zmn}^\pm = \frac{-kM \pm \sqrt{k^2 - \beta^2 k_{rmn}^2}}{\beta^2},
\]

where \( \beta \) is given by:

\[
\beta = \sqrt{1 - M^2}.
\]
Equivalently, using equation (3.13), the solution in terms of acoustic pressure is given by:

\[ p_{mn}(r, \theta, z) = A^p_{mn}N_{mn}J_{|m|}(k_{rmn}r)e^{im\theta}e^{-ik_{zmn}z}, \quad (3.30) \]

where \( A^\phi_{mn} \) and \( A^p_{mn} \) are related through:

\[ A^\phi_{mn} = \frac{A^p_{mn}}{-i\rho_0c_0(k - Mk_{zmn})}. \quad (3.31) \]

The values of \( k_{rmn} \) in equation (3.27) are obtained by applying the hard wall boundary condition at \( r = a \):

\[ \frac{\partial \phi}{\partial r} \bigg|_{r=a} = 0, \quad (3.32) \]

such that:

\[ J'_{|m|}(k_{rmn}a) = 0, \quad (3.33) \]

and for a particular value of \( m \), the \( n^{th} \) root (or eigenvalue), \( \lambda_{mn} \), of:

\[ J'_{|m|}(\lambda_{mn}) = 0 \quad (3.34) \]

gives the related value of \( k_{rmn} \) by:

\[ k_{rmn} = \frac{\lambda_{mn}}{a}. \quad (3.35) \]

There are an infinite number of such solutions, for each integer value of \( m \). These solutions define duct modes which either propagate or decay. At a fixed axial position, \( z \), angular traveling waves (or spinning modes) of the form:

\[ p \propto e^{ \pm im\theta} \quad (3.36) \]

are observed, while at a fixed circumferential position, \( \theta \), axial traveling waves of the form:

\[ p \propto e^{-ik_{zmn}z} \quad (3.37) \]

are observed. A given duct mode is therefore a combination of a spinning mode and an axial traveling wave.

The values \( m \) and \( n \) are referred to as the circumferential and radial orders of the mode and the parameters \( k_{zmn} \) and \( k_{rmn} \) are known as the axial and radial wavenumbers.

Thus, the general solution for the acoustic velocity potential field within the duct can be expressed in terms of a summation of all duct modes:

\[ \phi(r, \theta, z) = \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} J_{|m|}(k_{rmn}r)e^{im\theta}(a^+_{mn}e^{-ik_{zmn}z} + a^-_{mn}e^{-ik_{zmn}z}), \quad (3.38) \]
Figure 3.3: Annular duct with outer radius $a$ and inner radius $b$

where for a particular mode, $(m, n)$, $k_{zmn}^+$ corresponds to a wave traveling along the positive $z$-direction with amplitude $a_{mn}^+$, and $k_{zmn}^-$ corresponds to a wave traveling along the negative $z$-direction with amplitude $a_{mn}^-$. 

A similar solution can be found for a cylindrical duct of annular cross-section, with inner radius $b$ and outer radius $a$ (as shown in figure 3.3):

$$
\phi(r, \theta, z) = \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} (AJ_{|m|}(krmnr) + BY_{|m|}(krmnr)) e^{im\theta}(a_{mn}^+ e^{-ik_{zmn}^+z} + a_{mn}^- e^{-ik_{zmn}^-z}).
$$

(3.39)

However, in this case, both Bessel functions, $J_{|m|}$, and Neumann functions, $Y_{|m|}$, of order $m$ are present in the solution. In this case, the radial wavenumbers, $k_{rmn}$, are found by applying the hard wall boundary condition (3.32) at both the inner and outer wall, resulting in the eigenvalue equation:

$$
J'_{|m|}(\lambda_{mn}h)Y_{|m|}(\lambda_{mn}h) - J'_{|m|}(\lambda_{mn}h)Y'_{|m|}(\lambda_{mn}) = 0,
$$

(3.40)

the roots of which $(\lambda_{mn})$ lead to the values of $k_{rmn}$ as before (equation (3.35)). In equation (3.40), $h$ is referred to the “hub-to-tip ratio”, defined as the ratio $b/a$.

In both cases (ie. for both a cylindrical and an annular duct) once the radial wavenumbers are known, the axial wavenumbers, $k_{zmn}^+$ for modes propagating in the positive axial direction, and $k_{zmn}^-$ for modes propagating in the negative axial direction, can be found from equation (3.28), which is known as the dispersion relation:

$$
k_{zmn}^+ = \frac{-kM + \sqrt{k^2 - \beta^2 k_{rmn}^2}}{\beta^2},
$$

(3.41)

$$
k_{zmn}^- = \frac{-kM - \sqrt{k^2 - \beta^2 k_{rmn}^2}}{\beta^2}.
$$

(3.42)
From these relationships we can see that if the value of $\sqrt{k^2 - \beta^2 k_{mn}^2}$ is positive, then the longitudinal wavenumber will be entirely real, in which case the mode will propagate along the duct. However, if the value of $\sqrt{k^2 - \beta^2 k_{mn}^2}$ is negative the longitudinal wavenumber will have an imaginary part, resulting in an evanescent wave that decays as it propagates along the duct. These decaying waves are said to be “cut-off”, and the cut-off frequency, $\omega_{co}^{mn}$, defined as the minimal frequency at which a given mode, $(m,n)$, will propagate, can be expressed in terms of the equivalent cut-off wavenumber:

$$k_{co}^{mn} = \frac{\omega_{co}^{mn}}{c_0} = k_{mn} \sqrt{1 - M^2}.$$  

(3.43)

Thus, although the solution for the acoustic velocity potential or the acoustic pressure field within the duct is defined as the summation of an infinite set of duct modes with varying radial and circumferential orders, such as that shown in equation (3.30), whether each individual mode is actually present within the duct will depend on the source at the duct inlet, and also the boundary conditions both along the duct walls and at the duct exit.

The “cut-off ratio”, $\xi_{mn}$, of a mode (also sometimes referred to as the “cut-on ratio”) is defined as the ratio of the free-space wavenumber of the mode to $k_{co}^{mn}$ [103]:

$$\xi_{mn} = \frac{k}{k_{co}^{mn}}.$$  

(3.44)

Thus, for a hard-walled duct, if:

- $\xi_{mn} < 1$, the mode is cut-off and decays exponentially,
- $\xi_{mn} = 1$, the mode is just cut-on,
- $\xi_{mn} > 1$, the mode is cut-on and propagates without decay.

Another physical picture of the propagation within the duct (which is exact in a 2D duct but approximate in a cylindrical duct with a circular cross-section) [5, 19] is to view the acoustic field due to each mode as the interference of plane waves propagating at an angle $\psi_{mn}$ to the duct axis, and therefore reflecting from the duct walls, as shown in figure 3.4. This is a useful representation of the modal propagation, as the angle $\psi_{mn}$ has been shown to govern the location of the principal lobe of the far-field radiation for both rectangular ducts with uniform flow [104, 105] and for circular ducts with zero flow [17]. For the zero-flow case, $\psi_{mn}$ is directly related to the cut-off ratio of the mode by [103, 17]:

$$\psi_{mn} = \sin^{-1}\left(\frac{1}{\xi_{mn}}\right).$$  

(3.45)

Thus, a mode which is well cut-on ($\xi_{mn} >> 1$) will have a small propagation angle, and travel close to the duct axis. As the cut-on ratio decreases the mode will propagate at a steeper and steeper angle until, when $\xi_{mn} = 1$, the modal propagation angle equals $90^\circ$, and the mode will not propagate along the duct axis.
3.3.3 Propagation in a uniform lined duct

For a cylindrical duct of circular cross-section with the same geometry as before, (see figure 3.2), but with lined rather than rigid walls, the same separable solutions to the Con- vested Helmholtz Equation as shown in equation (3.27) apply, and equation (3.28) still holds for the relationship between the axial wavenumber $k_{zm}$ and the radial wavenumber $k_{rm}$. However, for the lined case, the boundary condition which must be applied to find the appropriate values of $k_{rm}$ is now different. In the case of uniform axial flow of magnitude $U_0$, the Myers boundary condition (3.17) simplifies to:

$$\frac{\partial \phi}{\partial r} = -\frac{\rho_0}{Z} \left( i\omega \phi + U_0 \frac{\partial \phi}{\partial z} \right) - \left( \frac{1}{i\omega} \right) U_0 \frac{\partial}{\partial z} \left( \frac{\rho_0}{Z} \left( i\omega \phi + U_0 \frac{\partial \phi}{\partial z} \right) \right),$$  \hfill (3.46)

and the application of this boundary condition at the duct wall, $r = a$, leads to a second relationship between $k_{zm}$ and $k_{rm}$:

$$k_{rm} \frac{J'(k_{rm}a)}{J(k_{rm}a)} = -\frac{\rho_0 i\omega}{Z} \left( 1 - \frac{U_0 k_{zm}}{\omega} \right)^2 .$$  \hfill (3.47)

Thus, for a lined duct, the solutions for $k_{rm}$ are coupled to the solutions for $k_{zm}$, and therefore equation (3.47) must be solved simultaneously with the dispersion relation (given by equation (3.28)) in order to obtain values for the radial and axial wavenumbers. Also, because the value for impedance, $Z$, is complex, values of $k_{rm}$ and $k_{zm}$ will also be complex. For a lined duct mode, there is therefore no meaning to the term “cut-off”, because all modes will decay as they propagate along the duct at a rate depending on the magnitude of the imaginary part of the axial wavenumber, $\text{Im}\{k_{zm}\}$.

3.3.4 Propagation within non-uniform ducts

For simple uniform duct geometries with uniform axial flow, as seen in sections 3.3.2 and 3.3.3, it is possible to derive analytic solutions for the pressure field within the duct in terms of rigid-wall or lined-wall duct modes with particular mode shapes and corresponding wavenumbers. However, the complexity of the propagation problem is increased when considering non-uniform duct geometries and mean flows, such as those
found in an aero-engine bypass. In such cases, although the same fundamental physical laws are obeyed, it is less straightforward to derive a solution analytically for the pressure field within the duct. For these more complex problems, the governing equations must be solved numerically rather than analytically.

Common numerical solutions apply the technique of discretisation, where the differential equation which governs the variation of pressure within the duct is replaced by a set of algebraic equations, solutions to which can be found by matrix manipulation. (Examples of such methods are the Finite Difference Method, the Method of Weighted Residuals, and the Finite Element Method.) Numerical methods such as these can be computationally expensive in terms of CPU-time and memory requirements. However, these methods are often the only way to obtain solutions for problems involving non-uniform geometries and mean flows and/or variations in impedance. The work within this thesis focuses on the use of the Finite Element Method (FEM) to obtain a solution to a Weak Variational Form of the combined acoustic field equations.

3.3.4.1 The Weak Formulation

For a non-uniform duct geometry which is axisymmetric, the acoustic field can be represented as periodic in the circumferential direction. Acoustic perturbations in pressure, density and velocity potential that are harmonic in time with frequency $\omega$ can then be written in a cylindrical coordinate system as: $p(r, z)e^{i(\omega t + m\theta)}$, $\rho(r, z)e^{i(\omega t + m\theta)}$ and $\phi(r, z)e^{i(\omega t + m\theta)}$ respectively.

Using these expressions for $p$, $\rho$ and $\phi$, the acoustic continuity equation for irrotational flow (3.12) can be written as:

$$i\omega \rho + \nabla \cdot (\rho_0 \nabla \phi + \rho \mathbf{v}_0) = 0.$$  \hspace{1cm} (3.48)

Consider a general axisymmetric domain with volume, $V$, bounded by the surface, $S$, as shown in figure 3.5. For situations where an exact solution may not be easily obtained, rather than requiring that the solution satisfies the differential equation at every point in the domain (which is too “strong” a constraint), instead an approximate solution for $\phi$ is sought by restating the acoustic continuity equation in a weak, or variational formulation. If $\phi$ is considered to be an approximate solution then equation (3.48) becomes:

$$i\omega \rho + \nabla \cdot (\rho_0 \nabla \phi + \rho \mathbf{v}_0) = \epsilon_1 \neq 0,$$  \hspace{1cm} (3.49)

where $\epsilon_1$ is the error.
A solution for $\phi$ is then sought by requiring the average of the weighted residual of the continuity equation over the whole domain to be zero \([70]\):

$$\int_V W \left( i\omega \rho + \nabla \cdot (\rho_0 \nabla \phi + \rho v_0) \right) dV = 0, \forall W; \quad (3.50)$$

where $W$ is the weighting function, or test function.

This can be rearranged by using the divergence theorem to give:

$$\int_V (\nabla W \cdot (\rho_0 \nabla \phi + \rho v_0) - Wi\omega \rho)dV = \int_S W (\rho_0 \nabla \phi + \rho v_0) \cdot n dS, \forall W, \quad (3.51)$$

where $n$ is a unit vector normal to the surface, pointing into the domain (figure 3.5). Both equations (3.50) and (3.51) are weak formulations of the acoustic continuity equation and are required to hold true for all possible weighting functions.

By combining the acoustic momentum equation for irrotational flow (3.13), with the equation of state (3.8) it is possible to obtain an expression for the acoustic density amplitude in terms of the velocity potential:

$$\rho = -\frac{\rho_0}{c_0^2} (i\omega \phi + v_0 \cdot \nabla \phi). \quad (3.52)$$

This expression is then substituted into equation (3.51) and rearranged to give a weak formulation for the Conveceted Wave Equation:

$$\int_V \frac{\rho_0}{c_0^2} \left( c_0^2 \nabla W \cdot \nabla \phi - (v_0 \cdot \nabla W) (v_0 \cdot \nabla \phi) \right) dV$$

$$+ \int_V \frac{\rho_0}{c_0^2} \left( i\omega |W (v_0 \cdot \nabla \phi) - (v_0 \cdot \nabla W) \phi| - \omega^2 W \phi \right) dV$$

$$= \int_S \frac{\rho_0}{c_0^2} \left( c_0^2 W \nabla \phi - v_0 W (v_0 \cdot \nabla \phi) - i\omega v_0 W \phi \right) \cdot n dS, \forall W. \quad (3.53)$$
The volume integral terms in equation (3.53) provide information as to how $\phi$ behaves within the domain, and the surface integral term provides information as to how $\phi$ behaves on the domain boundary. For a hard-wall boundary, both $v_0 \cdot n$ and $\nabla \phi \cdot n$ are zero, and the entire right hand side of equation (3.53) vanishes. For a lined wall, the Myers boundary condition as shown in equation (3.17) applies, and this expression for $\nabla \phi \cdot n$ can therefore be substituted into the surface integral.

The weak formulation of the equation is required to hold true for all test functions, $W$, in which case it must also be true within any part of the domain. If the weak formulation could be solved exactly then we would obtain an exact, rather than an approximate value for $\phi$. The reason that this solution method yields an approximate solution for $\phi$ is in the way in which the solution to the weak formulation is sought by discretising the domain of the problem, such as in the Finite Element Model.

### 3.3.4.2 The Finite Element Model

To form a Finite Element model, the entire domain of the problem is divided up into a number of sub-volumes called Finite Elements. These elements can be various shapes ranging from linear and quadrangular to hexahedral [3]. Each element is defined by nodes at the element vertices or along the element sides, where each node is associated with a value of the velocity potential, $\phi$ (which is unknown). Figure 3.6 shows an example of the discretisation of a volume $V$ into Finite Elements in this way. Within equation (3.53), the integral over the whole domain volume, $V$, now becomes a summation of the volume integrals for each element, and the integral over the whole domain surface, $S$, becomes a summation of the surface integrals for each element (but only for the element surfaces on the external boundary of the domain).

By discretising the domain into elements, an expression for the velocity potential within each element, $\phi_e$, can be obtained:

$$
\phi_e = \sum_{i=1}^{n} N_i^e (\xi) \phi_i,
$$

(3.54)
where ξ is the local coordinate system, n is the number of nodal points or shape functions within the element, \( \phi_i \) are the values of \( \phi \) on the element nodes, and \( N_i^e(\xi) \) is the value of the element shape function corresponding to node \( i \) at spatial location \( \xi \).

If the contributions from each element shape function, \( N_i^e \), are assembled into a global shape function \( N_i \), then a global expression for \( \phi \) within the whole domain can be written:

\[
\phi = \sum_{i=1}^{N} N_i \phi_i, \quad (3.55)
\]

where the summation is now over the total number of nodes, \( N \), within the domain. The weighting function, \( W \), can also be written in the same form [106]:

\[
W = \sum_{j=1}^{N} N_j w_j, \quad (3.56)
\]

where \( w_j \) are the values of \( W \) at each nodal point and \( N_j \) is the same global shape function as used for the velocity potential (the Galerkin method).

These expressions for \( \phi \) and \( W \) can then be substituted into the weak formulation of the Convected Wave Equation (3.53), such that the integrations in the weak formulation are carried out as the sum of integrations over individual elements. In this way, by assembling the contributions from each of the elements in the problem domain, it is possible to obtain a global matrix equation of the form:

\[
(K + i\omega C - \omega^2 M)\Phi = F, \quad (3.57)
\]

The vector \( \Phi \) is the only unknown and contains the values for the velocity potential at all nodes in the problem domain, such that the solution of equation (3.57) provides a solution for the velocity potential at all nodes in the domain. \( K \), \( C \), \( M \) and \( F \) are the acoustic stiffness, damping and mass matrices and forcing vector respectively, and are determined purely by the shape functions and boundary conditions for the problem, and contain contributions from all elements in the domain:

\[
K = \sum_e K^e, \quad (3.58)
\]

\[
C = \sum_e C^e, \quad (3.59)
\]

\[
M = \sum_e M^e, \quad (3.60)
\]

\[
F = \sum_e F^e, \quad (3.61)
\]

where \( \sum_e \) denotes assembly of the individual element matrices: \( K^e \), \( C^e \), \( M^e \) and \( F^e \) in the usual way [107].
Matrices $K$ and $M$ receive contributions from the volume integral terms in equation (3.53), whilst $C$ and $F$ receive contributions from the surface integral terms. In particular, $C$ contains information regarding damping due to the impedance boundary conditions imposed, and $F$ contains information about the acoustic source (in terms of modal boundary conditions at each end of the duct).

### 3.3.4.3 Resolution requirements

When discretising the domain of the problem into a FE mesh, it is necessary to ensure that there are enough nodal points within the domain to model accurately the pressure variation. This is known as a resolution requirement. Nyquist’s criterion within signal processing would suggest that to resolve a sine wave of a certain frequency, $f_0$, it is necessary to sample at a minimum sample frequency of $2f_0$. Applying this to the spatial resolution within a FE mesh would result in 2 nodes per wavelength. However, in order to properly represent the spatial variation of a sine wave it is generally accepted that a minimum of 6 nodes per wavelength is necessary within the FE mesh (see figure 3.7). In this way, for problems involving higher frequencies, finer mesh resolutions are required. This results in larger matrix equations, and can therefore be computationally expensive, in terms of CPU memory requirements and run times. This is obviously a weakness of the FEM. However, it’s major strength is that its application to problems with complex geometries or impedance boundary conditions allows for the numerical solution of such problems that would not be possible analytically.

![Figure 3.7: Representation of a sine wave using 6 nodes per wavelength](image)
3.4 Modelling propagation within engine bypass ducts

3.4.1 ACTRAN/TM

ACTRAN/TM [3] is a commercial Finite Element code for analysing the tonal noise components from rotating machinery, and contains features such as acoustic finite and infinite elements and convected wave propagation. Within this application the Myers impedance boundary condition can be accommodated, and the acoustic source can be defined in terms of a non-modal excitation (such as a velocity boundary condition), or a modal boundary condition, where the acoustic field is described in terms of a set of rigid wall duct modes as derived in section 3.3.2. ACTRAN analysis can be performed for two-dimensional (2D) problems, or axisymmetric geometries, where only a 2D mesh in the $r - z$ plane is required and a pressure solution of the form $p(r, z)e^{i(\omega t + m\theta)}$ is assumed, and also for fully three-dimensional (3D) geometries, where a 3D mesh is necessary to model any non-axisymmetric effects. Information regarding the problem specifications is given by the user in an input file, and ACTRAN produces an output file containing details of the predicted acoustic solution (in terms of nodal pressures or modal intensities).

3.4.2 B-induct

In order to obtain a solution for the acoustic pressure field within a realistic bypass duct geometry the generation of input files, ACTRAN execution, and extraction of results can be a potentially time consuming process for the user. Therefore within the ISVR a shell code named b-induct (written in Fortran) has been developed, (Sugimoto, 2004 [108]), in order to automate ACTRAN analysis, and speed up both the pre- and post-processing phases.

B-induct is a prediction tool for sound propagation within axisymmetric bypass duct geometries, such as that shown in figure 3.8. B-induct automatically generates an ACTRAN input file from a set of data supplied by the user, executes ACTRAN analysis, and then extracts the results from the ACTRAN output files. Details of the bypass duct geometry and liner configuration must be specified by the user in an input text file for b-induct. Because the duct geometry is assumed to be axisymmetric any circumferential liner variations are neglected, as are the pylon and lower bifurcation that would be present in a realistic bypass duct. The computational domain for the analysis extends from an “inlet” plane, defined as the plane to the rear of the outlet guide vane (OGV), to an “exhaust” plane, defined at the duct outlet (see figure 3.8). A second input text file must also be created containing values of density, sound speed and Mach number at both the inlet and exhaust planes, the frequencies at which the problem is to be solved, and the resolution required for the FE mesh.
B-induct reads these input files and uses a spline interpolation function to interpolate the geometry. A FE mesh is created (figure 3.9) using quadrilateral elements, and the mesh is sized in order to provide adequate resolution at the frequency of interest (usually with 10 nodes per wavelength). A compressible Euler solver within b-induct computes the mean flow within the duct such that the mean flow velocity is specified at each nodal point in the FE mesh. Because ACTRAN uses the analytic expression for hardwall duct modes with uniform flow, the mean flow field is required to be axial and uniform at the duct ends where a modal boundary condition is applied. Within the b-induct flow solver parallel flow is therefore assumed at the inlet, and plug flow is assumed at the exhaust.

An ACTRAN input file is generated which contains details of the mesh geometry and flow. Impedance boundary conditions can be applied on the duct walls in order to model any acoustic liners present within the duct. Modal boundary conditions are also applied at each duct end. At the inlet, a source consisting of all cut-on modes is assumed, and each mode is assigned unit intensity. Reflected modes are also permitted to propagate back along the duct. At the exhaust plane, an anechoic boundary condition is imposed such that the sound field is assumed to consist of only downstream propagating modes (see figure 3.8). This is physically equivalent to placing an infinitely long hard walled
annular duct at this section, and any reflections which occur at the bypass exhaust are neglected.

When b-induct is coupled to the GXMunt radiation model (described in section 3.6), the effect of reflections at the duct termination is included in the far-field predictions. For a multimodal, equal energy per mode source the total transmitted sound power predicted using in-duct analysis only has been compared with the total radiated sound power predicted using b-induct + GXMunt, and seen to be very similar [74]. This indicates that, for the in-duct predictions, the effect of omitting the reflected waves at the exhaust is minimal.

B-induct executes ACTRAN analysis, and predicts the propagation, reflection and attenuation of each incident mode as it propagates down the bypass duct, for each frequency specified. From the ACTRAN output files, information about the modal content at the inlet and exhaust planes is extracted, and a sequence of power transmission coefficients $T_{ij}$ are generated, which define the transmitted power in mode $j$ at the exhaust plane due to an incident mode, $i$, of unit power at the inlet plane. Alternatively, modal amplitude transmission coefficients $A_{ij}$ are generated, which define the transmitted amplitude of mode $j$ at the exhaust plane due to an incident mode, $i$, of unit amplitude at the inlet plane. Transmission files containing this information are produced for each frequency. From these transmission files, parameters such as the total incident modal intensity at the inlet plane and the total transmitted modal intensity at the exhaust plane can be extracted.

Because b-induct performs ACTRAN analysis assuming an axisymmetric bypass duct, it is unable to predict the effect of any non-axisymmetric structures (such as pylons or splitters) or liner configurations on the noise propagation. In order to assess the effects of these non-axisymmetric configurations it is necessary to generate a full 3D mesh (see figure 3.10) and execute 3D ACTRAN analysis. To facilitate this process, a 3D version of the b-induct code has also been developed [76, 77, 109]. This version allows for the incorporation of 3D duct features such as hardwall patches and splitters into the finite element model, and solutions for the 3D problem are then found using ACTRAN/TM in the same way. However, due to the larger number of degrees of freedom of 3D problems, the CPU and memory requirements are greatly increased.
3.5 Radiation Model

In order to accurately model radiation from the bypass duct into the far field, the effects of the external mean flow field on the acoustic propagation must be taken into account. Ideally, the external geometry of the engine must also be modelled, and any reflections that take place at the bypass exhaust must be included. This would require a propagation model in the duct which is fully coupled with a radiation model to the far field, which satisfies the Sommerfeld radiation condition of only outward propagating waves at infinity.

In the absence of mean flow, it is possible to formulate such a model for the entire domain (both the in-duct and external regions) using ACTRAN, where the in-duct and near-field regions are modelled using Finite Elements, and the non-reflecting boundary condition for the radiation problem is modelled using a set of Infinite Elements on the external boundary of the model. Infinite Elements (IEs) are unbounded FEs onto which the infinite domain is mapped \cite{70}. The interpolation of pressure within an IE follows a multipole expansion of the prescribed order to simulate radiation out to infinity (and hence satisfy the Sommerfeld condition). For the no-flow case, FE/IE models such as this can provide accurate solutions for the acoustic field.

However, when flow is introduced the complexity of the problem increases due to the presence of the bypass mixing layer which must now also be modelled. Since ACTRAN/TM only provides a FEM solution based on the assumption of irrotational flow it therefore cannot represent propagation within the bypass shear layer, which is inherently rotational.

Sugimoto et al. \cite{73} dealt with this by calculating the far-field directivity for the no-flow problem using ACTRAN/TM and then using a Lorentz transform to introduce the effect of uniform mean flow, and a Snell’s Law adjustment to model the effect of refraction by
Figure 3.11: Definition of the cone of silence: At the boundary between higher and lower Mach numbers, rays are refracted away from the axis. The refraction of ray 2 (originally travelling parallel to the exhaust axis) defines a “cone of silence” at an angle $\psi_s$ to the axis.

Figure 3.12: Representation of the bypass shear layer as an infinitesimally thin vortex sheet between two shear flows of Mach numbers $M_1$ and $M_2$.

the shear layer. As Snell’s Law is based on ray theory this method provides reasonable results for high frequencies. However, it does not account for diffraction and therefore this model does not resolve the sound field in the cone of silence along the exhaust axis (see figure 3.11).

Another way to model the bypass shear layer using ACTRAN/TM is to use viscoelastic membrane elements to define an infinitesimally thin vortex sheet between two irrotational shear flows (see figure 3.12). Whilst this approach accounts for the effect of refraction, the representation of the bypass shear layer as a sheet does not allow the effects of mixing to be taken into account, and coupling of the vortical and acoustic disturbances is not modelled.

For a full propagation and radiation solution that allows the bypass mixing layer to be accurately represented, ACTRAN/DGM could be used. ACTRAN/DGM is a time domain Finite Element scheme based on the Discontinuous Galerkin Method (DGM). It provides a full solution to the LEEs, and therefore can account for the effect of vorticity within the flow. The disadvantage of LEE time domain solutions such as this lies in their
susceptibility to hydrodynamic instabilities, and the fact that they are computationally more intensive.

Whilst modelling the entire domain (both in-duct and external regions) allows for the solution of both the propagation and radiation problems simultaneously, the large size of the domain (and hence large number of nodes involved) leads to solutions which are computationally expensive, especially for high frequencies or non-axisymmetric geometries.

An alternative to modelling the propagation and radiation problems simultaneously is to use separate propagation and radiation models such that the in-duct and external calculations are performed separately. In this approach the sound field at the exit plane of the bypass duct is “weakly coupled” to a radiation model. This is done by using the computed modal amplitudes at the bypass exhaust as an input to the radiation model for the calculation of far-field directivity.

This approach raises again the problem of determining an accurate duct termination condition. To overcome this, it is common to assume that there are no reflections at the duct exit, or that the termination is anechoic. For unlined ducts at relatively high frequencies, and for modes that are not near cut-off, reflections from open ends are small and such an assumption is a valid approximation. The same can be said for lined ducts, where the incident amplitudes are considerably reduced before reaching the open end. In both cases the application of a non-reflecting boundary condition at the duct exit is therefore a reasonable approximation.

3.6 Modelling radiation from an engine bypass exhaust

Having separated the propagation and radiation models it is now possible to consider the radiation model alone. For the purpose of this work an analytic solution for the sound radiation from a semi-infinite annular jet pipe will be used. This model has been developed by Gabard [24], and is an extension of of the work of Munt [21, 22] and Rienstra [23]. The model is known as “GXMunt” (Generalised eXtension of the Munt solution).

The configuration used for this radiation model is a cylindrical duct with an annular cross-section (see figure 3.13). The duct is semi-infinite in the axial direction (for \( z < 0 \)), has a zero-thickness outer wall at radius \( R_0 \), and an infinite centre body of radius \( R_1 \) along the axis. All surfaces are rigid and impervious. The ambient flow in the outer region, \( r > R_0 \), is uniform, axial and steady, with velocity \( v_0 \), density \( \rho_0 \), sound speed, \( c_0 \), and Mach number \( M_0 = v_0/c_0 \). The duct carries a jet which is also uniform, axial and steady, with velocity \( v_j \), density \( \rho_j \), sound speed, \( c_j \), and Mach number \( M_j = v_j/c_0 \). Downstream of the duct exit, \( (z > 0) \), the jet is separated from the ambient flow by
Figure 3.13: Geometry used for the radiation model: a semi-infinite annular jet pipe.

For each normal mode of the duct \((m, n)\), an analytic solution for the far-field acoustic pressure is derived by applying the Wiener-Hopf technique \([24]\). When a number of such modes are present, this gives a total far-field pressure amplitude \(p\) (in terms of spherical coordinates \(R, \theta\) and \(\psi\)):

\[
p(R, \theta, \psi) = \frac{1}{R} \sum_{m,n} B_{mn}^+ D_{mn}(\psi) e^{i\omega RS(\psi)/c_0 + im\theta},
\]

(3.62)

where \(B_{mn}^+\) is the amplitude of mode \((m, n)\) at the duct exhaust, and \(D_{mn}(\psi)\) is the directivity function of the mode \((m, n)\) for a unit pressure mode amplitude given by the Wiener-Hopf solution. The factor \(S(\psi)\) accounts for the wavefront stretching introduced by the ambient mean flow:

\[
S(\psi) = \sqrt{1 - M_0^2 \sin^2 \psi - M_0 \cos \psi} / \sqrt{1 - M_0^2}.
\]

(3.63)

The relationship between the cylindrical coordinates \((r, \theta, z)\) used for the in-duct problem and the spherical coordinates \((R, \theta, \psi)\) used for the external problem is defined by \(z = R \cos \psi\) and \(r = R \sin \psi\). An illustration of this relationship is given in figure 3.14.

In order to automate the prediction procedure for radiation from a realistic bypass duct, Gabard’s MATLAB program known as “GXMunt” has been used. This automatically calculates the directivity function \(D_{mn}(\psi)\) for each mode specified by the user. These directivity functions can then be coupled with modal amplitudes at the bypass exhaust, \(B_{mn}^+\), predicted by b-induct, so that the overall far-field pressure distribution for a given bypass configuration is obtained (equation (3.62)).
The semi-infinite annular duct geometry used within GXMunt (shown in figure 3.13) is obviously a simplification of the realistic bypass duct geometry, and reflection effects from the external geometry of the nacelle are therefore not accounted for. Also, because the bypass shear layer is idealised by a vortex sheet, the effects of mixing are not accounted for. In contrast, a full FE/IE model would provide a more accurate representation of the geometry, but would not be appropriate for shear flow, and a full LEE model would provide an accurate representation of the mixing layer, but would be much more computationally intensive. Overall, one of the major benefits of using GXMunt as a radiation model is that it allows the propagation and radiation calculations to be performed separately, and reduces the overall time needed for computation of a solution. Therefore, in all of the work contained within this thesis, b-induct is used as the propagation model, and GXMunt is used as the radiation model (coupled to the b-induct results at the exhaust).
Chapter 4

Assessing b-induct for use within the engine design process

4.1 Introduction

In the previous chapter, the prediction method b-induct was introduced as an automated shell code for predicting noise propagation within axisymmetric bypass ducts. In order to use b-induct as a prediction tool within the industrial engine design process, it is first necessary to ensure its capability to make accurate predictions for bypass noise propagation.

Previous validations of b-induct have shown good agreement, in the no-flow case, with measured sound power level (PWL) attenuations obtained at the ISVR for both a 1:6 rig-scale uniform annular duct geometry, and a duct with a turbine hump [74]. Also, Sugimoto [75] has shown b-induct to give good correspondence with experimental data from the NASA Active Noise Control Fan (ANCF) test rig for a uniform annular duct with flow. In each of these validation cases, the conditions are idealised: the source content is known and the geometries used are simplifications of a real bypass duct geometry. In reality, aeroengine bypass ducts are much more complex - both in terms of geometry and also liner positioning. Therefore, for b-induct to be demonstrated as a useful industrial design tool, it needs to be shown to be capable of making accurate predictions for realistic bypass configurations. Also, as noise certification requirements are defined for the entire aircraft, it is necessary to integrate these predictions for the real bypass configuration into a model for whole engine noise. As yet this has not been demonstrated.

In this chapter, the performance of b-induct as a design tool for whole engine configurations is examined. The model is used to predict bypass attenuations for a real bypass
configuration, and the results are integrated into a whole engine model, and compared with measured data from a full-scale static engine test.

4.2 The Engine Design Process

In assessing the performance of b-induct against whole engine data, it is important to consider the new issues introduced. Firstly, for a real engine the modal source content at the entrance to the bypass duct is unknown. B-induct assumes equal energy in each cut-on mode, which is a common assumption for rearwards propagating fan noise, although in reality the source excited by the fan will contain certain modes that are excited more than others. Secondly for whole engine predictions, it is necessary to include noise due to other engine sources, the exact content of which is also unknown, and hence some assumptions must also be made about these other sources. These unknowns provide increased opportunity for errors in the prediction, and therefore must be taken into consideration in interpreting the results.

Because of this increased opportunity for errors, it is often not possible to produce accurate predictions for whole engine noise that align with real engine data, based solely on simulation. So how can these discrepancies be dealt with? In the design stages of a new engine, it is common practice within industry to use prediction models to predict the change in noise level from a similar engine configuration for which the noise level is already known, to the new configuration of interest. In fact, it is often the case that the ability to accurately predict incremental changes in performance due to modifications to a base configuration is more critical from a practical standpoint than accurate predictions for the base configuration itself. Two generalised examples of this approach are described below.

An overview of the first approach is shown in figure 4.1. The noise level for a current engine (engine 1) with no liners is known, and a prediction of the noise level for this current engine can then be compared such that the difference between the predicted and measured levels provides a calibration factor, \( \delta_c \). If this calibration factor is assumed to be the same for a new engine prediction, then it can be added onto the predicted noise level for the new engine (engine 2) to obtain a calibrated prediction for engine 2.

Figure 4.2 shows an overview of a slightly different approach, where engine 1 and engine 2 have different acoustic liners. In this case engine 1 incorporates bypass liners, and the actual noise level for this configuration of engine 1 with liners is known. By adding the predicted attenuations for the engine 1 bypass liners (\( \Delta_1 \)) onto this measured level, a prediction for the noise level of engine 1 with no liners is obtained. Taking this value as a starting point for the new engine calculations, if this level is then adjusted by a source delta, \( \delta_{1-2} \) that accounts for the predicted difference in noise source between engine 1 and engine 2, a prediction for the noise level of engine 2 without liners is obtained. Then
the predicted attenuation of the engine 2 bypass liners ($\Delta_2$) can be deducted from this level to obtain a value for the noise level of engine 2 in the lined configuration.

The use of both of the methods described here is common within the design process for a new engine, and although measured data is used to calibrate the predictions the process still relies on accurate prediction methods. In addition to this, due to the requirement for performing multiple iterations during the design process, it is also highly beneficial if these predictions can be delivered rapidly. For rearwards propagating fan noise, a bypass prediction model that gives reliable predictions in short timescales (less than a day) is therefore a requirement.

A common standard of model used in industry for predicting bypass duct liner attenuations is an analytical solution for the propagation within a rectangular duct approximation to an annular duct, which is only valid at large hub-to-tip ratios. This model was originally used to generate a set of attenuation tables for specific values of liner impedance, frequency, Mach number, duct average height, and liner length normalised to duct height ($L/D$). These tables could subsequently be referred to as required. This
Chapter 4 Assessing b-induct for use within the engine design process

approach shall be referred to as the “industry” model. Since the attenuation tables were only generated once, the majority of the computational cost associated with obtaining predictions using this model is “non-recurring”, the additional cost being only that associated with accessing and interpolating the tabulated data in order to obtain values of attenuation for liners of interest. In this way, attenuations can be obtained for several liners, at multiple frequencies and engine operating conditions in only a few minutes.

As a numerical model, b-induct is inherently not as fast as the industry model, however it is still capable of making predictions for bypass attenuation in timescales which are acceptable for industrial design iterations. For example, for a full-scale engine with a given liner configuration operating at the cutback condition, prediction of attenuations at 1/3 octave centre frequencies from 50Hz to 4kHz takes approximately 14 hours. Although b-induct is not as fast as the industry model it is much more physically realistic, as described in detail below. Furthermore, the industry model is subject to errors introduced by interpolation of the tabulated data. Given these two factors, one might expect that whole engine predictions made using b-induct as a bypass model would be an improvement. The difference in performance of these two methods is also examined in this chapter. But firstly the differences between the two bypass models must be fully described.

4.3 Prediction of Bypass Attenuations

4.3.1 In-duct Predictions: Industry model vs. B-induct

A schematic comparison between the industry model and b-induct is shown in figure 4.3. The most obvious difference between them is the geometry each model uses to represent the bypass duct. The industry model is a rectangular duct approximation to an annular duct, and therefore does not account for the way in which the propagation might be affected by the curvature of the duct. Conversely, although b-induct is an axisymmetric representation and doesn’t account for any 3D features, an exact cross-section of the bypass duct can be modelled, and therefore b-induct takes account for the way in which the geometry of the duct will affect the propagation.

Figure 4.3: Comparison between b-induct and industry bypass models
The way in which liner attenuations are added in each model is also a significant difference. B-induct can model the exact positions of any liners within the bypass duct (figure 4.3(a)), whereas the industry model represents each liner in turn assuming it is distributed evenly between both the inner and outer duct walls (figure 4.3(b)). Both models assume an equal distribution of energy between all cut-on modes at the duct inlet. However, because the industry model does not account for the relative positions of the liners within the duct, the attenuation due to each liner is calculated separately, assuming this same equal energy per mode energy distribution at the beginning of each liner. This is physically unrealistic because the modal energy distribution incident on any liner will obviously be determined by the amount that each mode has been attenuated by the previous liner. In this way, b-induct is again much more physically realistic, because by modelling all liners at once the distribution of energy amongst modes at the start of each liner will be much more accurate.

In order to model the amount of liner present, the industry model uses the parameter: \( L/D \) (liner length, \( L \), divided by duct height, \( D \)) and this parameter can either be given as a value for each liner \( i \), \((L/D)_i\), or as a value for all liners together, \((L/D)_{\text{Total}}\). To calculate the total attenuation of a given liner configuration, firstly, for each liner, \( i \), the PWL attenuation, \( \Delta_i \) dB, is obtained assuming that the amount of liner is \((L/D)_{\text{Total}}\), and then the contributions due to each liner are added depending on the proportionate area of that liner \((L/D)_i/(L/D)_{\text{Total}}\). In this way, for a set of \( n \) liners, the total PWL attenuation due to all liners within the bypass duct, \( \Delta_{\text{Total}} \) dB, is calculated as shown in equation (4.1).

\[
\Delta_{\text{Total}} = \frac{(L/D)_1}{(L/D)_{\text{Total}}} \Delta_1 + \frac{(L/D)_2}{(L/D)_{\text{Total}}} \Delta_2 + \ldots + \frac{(L/D)_n}{(L/D)_{\text{Total}}} \Delta_n.
\]  

(4.1)

In contrast, because b-induct models the exact duct geometry and liner configuration, it is more physically realistic and the \( L/D \) parameter is not necessary. To calculate the total PWL attenuation using b-induct, a prediction of the transmitted PWL at the exhaust is obtained for the hardwall duct, \( \text{PWL}_{\text{tr}}(\text{HW}) \), and also for the duct with all liners present, \( \text{PWL}_{\text{tr}}(L) \), and the difference between them is taken:

\[
\Delta_{\text{Total}} = \text{PWL}_{\text{tr}}(\text{HW}) - \text{PWL}_{\text{tr}}(L).
\]  

(4.2)

### 4.3.2 Radiation Predictions

For the whole engine prediction process it is necessary to provide bypass attenuations in terms of SPL directivities rather than PWL values. One way of doing this, which is widely used in industry, is to multiply the PWL attenuation, \( \Delta_{\text{Total}} \), by an empirical directivity factor which varies with angle in the far-field. A typical empirical directivity pattern obtained in this way is shown in figure 4.4 (where \( 0^\circ \) is defined as the intake
Chapter 4 Assessing b-induct for use within the engine design process

For the purpose of the whole engine predictions performed within this chapter, this particular empirical directivity pattern has been applied to both the industry model and the b-induct PWL attenuations. Predictions of SPL directivity have also been obtained using the b-induct results coupled with the GXMunt radiation model described in the previous chapter, but these have not been included in the whole engine prediction model. The whole engine predictions using both the industry model and b-induct with empirical directivities are compared with measured data in section 4.4 and a comparison of the empirical and GXMunt directivity patterns is made in section 4.5.

4.4 Comparison of predicted and measured data

The test set-up for a set of full-scale static engine tests is shown in figure 4.5. Sound Pressure Levels (SPLs) are recorded at each microphone in an array 150ft from the engine-axis. Microphones in a starboard array are spaced every 10° from 10° to 160°, relative to the intake axis, whilst microphones in a port array are spaced every 5° from 10° to 160°. For a typical modern high-bypass ratio engine, measured data of this form was available in 1/3 octave frequency bands from 50Hz to 10kHz for two different bypass configurations.

The first of these is a fully lined bypass configuration, the b-induct model for which is shown in figure 4.6. Any liner interruptions within the real engine have been accounted for by modifying the start and end positions of the liners in the b-induct model to ensure that the exact area of each liner is represented. For this configuration, liners 1 and 2 are engine liners whilst liners 3-5 are nacelle liners. We shall refer to this as the “lined”
configuration. The b-induct model for the second bypass configuration is shown in figure 4.7. In this case, all of the nacelle liners have been removed and only the engine liners remain. We shall refer to this configuration as the hardwall nacelle configuration, or “hardwall” for short.

Figure 4.5: Overview of engine test set-up

Figure 4.6: B-induct model for “Lined” bypass configuration

Figure 4.7: B-induct model for “Hardwall” bypass configuration
4.4.1 Comparison of predicted bypass deltas with measured whole engine data

B-induct was used to make predictions of the transmitted PWL at the exhaust for each of these two bypass configurations and the total PWL attenuation due to the insertion of the nacelle liners was obtained using equation (4.2). Values for this liner insertion loss were obtained at 1/3 octave centre frequencies from 50Hz to 10kHz, for the engine running at two different operating conditions (approach, and cutback). From these PWL attenuations, SPL attenuations were then calculated using the empirical directivity pattern described in section 4.3.

To compare these predictions of bypass liner insertion loss to the measured engine data it is necessary to observe at an angle in the far field where rearwards propagating fan noise would provide a significant contribution to the overall engine noise. This region is usually within the range 110°-130°. Figure 4.8 shows the b-induct predicted bypass liner insertion loss spectrum at the 110° microphone for the cutback condition, along with the equivalent industry model prediction, and also with the measured data for the whole engine.

By comparing the predictions for the attenuation of rearwards propagating fan noise within the bypass duct with the measured data we see that there is generally good agreement in the low frequency range, but at the peak frequencies for attenuation both b-induct and the industry model predict larger deltas than are actually observed. The reason for this is that both prediction methods are only modelling fan noise propagating down the bypass duct, whereas the measured SPLs at the microphone array will have contributions from all sources of noise within the engine, including turbine, combustor and jet noise. All of these noise sources will have a different directivity pattern, and only the fan noise travelling downstream through the bypass duct will be attenuated by the liners. Any noise radiated at 110° that has not travelled through the bypass duct will not be attenuated by the liner. Therefore, even if the presence of the liner causes a reduction in the amount of rearward radiated fan noise as predicted by either b-induct or the industry model, unless these other sources of noise are significantly lower than the fan noise, they will “contaminate” the measured data and the overall measured deltas will not be the same as those predicted.

To obtain good agreement between the predicted and measured attenuations it is therefore necessary to include these other sources in the prediction model. Aerospace companies commonly have standard processes for making whole engine predictions by integrating results from different prediction models for different parts of the engine. One such standard industry process was used for the case studied here, to obtain whole engine predictions, and to incorporate b-induct predicted bypass liner attenuations for the

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1 where the value for liner insertion loss at 10kHz was extrapolated from the predicted value at 8kHz
prediction of rearwards radiating fan noise. In this way, whole engine predictions incorporating b-induct attenuations could be compared to the equivalent predictions made using the industry model attenuations, and also to the measured data, whilst taking into account the impact of other noise sources.

### 4.4.2 Prediction of whole engine noise

For a prediction of whole engine noise, knowledge of the levels (or at least relative levels) of the different noise sources within the engine is required. Predictions of the noise due to each engine source can be made using the existing prediction models, and these can then be calibrated with measured data from a static test on the engine in the fully hardwall configuration (i.e. with no liners present). Using an empirical process based on past engine data, the measured data is split into the components that are believed to have come from each different engine source (jet, intake, bypass and core sources) such that the prediction for each source can be compared to the measured level.

Any inconsistencies between predicted and measured source levels are rectified by a series of calibration correction factors, $\Delta C_s$, calculated for each 1/3 octave. These corrections are calculated for each source, at each microphone angle and engine power setting, by
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1. \( \Delta C \) calculated from hardwall data:

For each source
(fan, jet etc)

\[
\begin{array}{c}
\text{Source prediction (Hardwall)} \\
\text{Measured engine data (Hardwall)} \\
\text{Source correction factor } \Delta C \\
\end{array}
\]

difference

2. \( \Delta C \) incorporated into future predictions.

Figure 4.9: Ideal source correction process.

taking the difference between the predicted and measured levels for that source. Once the correction factors have been obtained, they are then incorporated into any future predictions for the engine as a source level correction to account for the inaccuracy in the model.

By using measured data from static tests on fully hardwall engine configurations it can be reasoned that any discrepancy between the prediction and the measured data is due to an inaccuracy in the prediction of the source. A schematic for this ideal process is shown in figure 4.9. However, due to the costs involved for full scale static engine tests, it is usual to test only final engine configurations where the liners are installed. This means that in practise measured data from hardwall configurations tends not to be available, and the measured lined data is used instead to calculate the appropriate correction factors for use in the predictions. For the engine examined here, although hardwall data is available, in order to make results as applicable as possible data from the production lined engine is used.

When using lined data, the measured data is split into components that are believed to have come from each different engine source in the same way as before, and these levels are compared to source level predictions where predicted liner attenuations have now also been applied. The correction factor for each source is also calculated in the same way as before, and is again taken to be an inaccuracy in the prediction of the source, and added on as a source level correction in any future predictions. A schematic for this process is given in figure 4.10.

If the liner attenuation prediction was perfectly accurate, this \( \Delta C \) calculated would indeed be entirely due to an inaccuracy in the source prediction. However, in reality any discrepancy between the predicted and measured data could be due to either an inaccurate source prediction or an inaccurate liner attenuation prediction. In an ideal scenario, data from tests on the hardwall engine configuration could be used to determine
where the inaccuracy lies, although there is usually limited or no data available for this purpose. Thus, the strength of using measured data from the lined configuration to calculate source correction factors relies heavily on the accuracy of the liner prediction model.

4.4.3 Comparison of whole engine predictions with whole engine measured data

For the same modern high-bypass ratio engine as in section 4.4.1, the standard whole engine prediction process was used to make whole engine predictions using predicted bypass liner attenuations from both the industry bypass model and from the b-induct model. The source correction factors were calculated using measured data from the lined configuration as within the standard practise. A comparison between the predictions and the measured data for the lined case is shown in figure 4.11 in the form of the frequency spectrum at the 110° microphone, for the cutback condition as examined previously.

As can be seen, both the industry model and b-induct predictions agree well with the measured data. This is as might be expected because the source correction factors are
calculated using the measured data for the lined case, and will therefore produce lined predictions that show good agreement with the lined data.

The equivalent predicted and measured frequency spectra at the 110° microphone for the hardwall case at cutback are shown in figure 4.12, and the liner insertion loss spectra (the difference between the hardwall and lined spectra) for the same case are shown in figure 4.13.

For both figures 4.12 and 4.13, the predicted spectra using both bypass models show good agreement with the measured data at low frequencies, where the liner is not very effective, but at higher frequencies there is less agreement between the predicted and measured data. At 1250Hz there is a significant difference between the measured liner insertion loss and the prediction incorporating the industry model bypass deltas (figure 4.13), with the industry model overpredicting by almost 7dB. The corresponding b-induct method still overpredicts the liner insertion loss at 1250Hz, but the difference is much less at only 3dB.

The reason for this large difference between the whole engine predictions obtained using b-induct and those obtained using the industry model is due to the way in which the correction factors are calculated and applied. To calculate the predicted rearwards fan
noise for the lined case, either the industry model bypass deltas or the b-induct bypass
deltas (as shown in figure 4.8) are applied to the predicted source level, and then the
correction factor $\Delta C$ is calculated. As the industry bypass delta at 1250Hz is much larger
than the b-induct bypass delta, the predicted bypass level using the industry model will
be lower, and the $\Delta C$ value will be consequently higher. Because this larger $\Delta C$ is
then added on as a source correction in the hardwall case, the whole engine prediction
at 1250Hz using the industry model is consequently larger than the prediction using
b-induct. The fact that the b-induct whole engine prediction provides much closer
agreement to the measured data in the region between 1-2kHz implies that the b-induct
prediction for bypass liner insertion loss is more accurate than that of the industry model
in this region. It can also be seen that the b-induct predictions capture more closely the
shape of the measured data than the corresponding industry model predictions.

For the hardwall bypass configuration, figure 4.12, between 315Hz and 500Hz the two
peaks in the measured data due to buzz-saw noise, produced as a consequence of the
super-sonic blade tip speeds, are not seen in the predicted data. The reason for this is
also due to the way in which the correction factors are calculated and applied, but in
this case a different model is used for predicting the attenuation due to the intake liner.
Figure 4.13: Liner insertion loss frequency spectra at 110° microphone, at cutback.

Figure 4.14 shows the whole engine predictions for liner insertion loss compared to the measure data at the 110° microphone for the engine in the approach configuration. As for the cutback case, at low frequencies both the industry model and the b-induct predictions are similar and show reasonable agreement with the measured data. In this case, as there is no buzz-saw noise present at this engine condition, the shapes of the predicted curves also agree with the measured data. The two attenuation peaks observed in the measurement, at approximately 1250Hz and 3150Hz are also captured by both bypass models, however the peak at 1250Hz is overpredicted by both models and the peak at 3150Hz is underpredicted by both models. At 1250Hz b-induct overpredicts the attenuation by just under 2dB compared to 5dB by the industry model, and at 3150Hz b-induct underpredicts the attenuation by 2dB compared to 3dB by the industry model. Overall, the b-induct prediction is again closer to the measured data than the industry model prediction.
In summary, although there are differences observed between the whole engine predictions and the measured data, the b-induct predictions are in closer agreement with the measurements than the industry model predictions are. This seems reasonable given that b-induct is more physically realistic than the industry model, and should therefore provide more accurate predictions for the bypass liner attenuation.

The improvement of the b-induct predictions over the industry model predictions is an encouraging result for the future use of b-induct as a tool within the engine design process. However, the discrepancy between the predictions and the measured data, especially at peak attenuation frequencies, still requires explanation. Due to the complexity of the whole engine prediction model, and the way in which source correction factors are applied, it is difficult to judge whether these differences are due to inaccuracies in the bypass liner prediction model, or in the source model itself. In order to understand the reason for these discrepancies the whole engine prediction process needs to be fully explored, and this is a significant task for future work.
4.5 Radiation model

4.5.1 Comparison of B-induct + GXMunt fieldshapes with measured fieldshapes

The whole engine predictions presented within this chapter have all been obtained using bypass PWL attenuations to which an empirical directivity factor (see section 4.3) has been applied. The empirical directivity factor has been obtained based on previous measured data and is independent of flow Mach number and temperature, the main factors that cause refraction of sound as it propagates from the nozzle across the shear layer.

In order to incorporate the effect of the shear layer, predictions of far field directivity for rearwards radiated fan noise can be obtained by using the b-induct predicted modal amplitudes at the bypass exhaust coupled to the GXMunt radiation model described in section 3.6. For the far field predictions the b-induct model assumes a source content of incoherent modes incident at the duct inlet (each with unit intensity). Within GXMunt any coherence between modes at the duct exhaust that have originated from the same incident mode is accounted for. A schematic of this process is shown in figure 4.15 where two incoherent modes $m1n1$ and $m1n2$ are incident at the duct inlet\(^2\).

By using the previous b-induct results coupled to GXMunt in this way, predicted field-shapes for both the hardwall and lined bypass configurations of the aeroengine were obtained at each 1/3 octave centre frequency. For the cutback condition, the fieldshapes at 1250Hz and 4000Hz are shown in figure 4.16.

For both the hardwall and lined bypass configurations, the model predicts that the rearwards radiated fan noise increases in the rear arc until it reaches a sharp peak at an angle between 120° and 125°, after which it drops off steeply. This rapid reduction in SPL corresponds to the region of the “cone of silence” predicted by Snell’s law where,

\[^2\text{The notation: } m_{ij}\text{ signifies a mode with azimuthal order } i \text{ and radial order } j, \text{ and shall be used throughout this thesis.}\]
for a ray travelling parallel to the exhaust axis, the effect of refraction at the bypass shear layer causes the path of the ray to move away from the axis (see figure 3.11). The reason that there is still a finite amount of sound energy observed at angles above 120° is because the GXMunt radiation model predicts that some energy will be diffracted into the cone of silence.

The measured fieldshapes for both the hardwall and lined builds at 1250Hz and 4000Hz are shown in figure 4.17. As the measured data is for the whole engine, where all noise sources are present, and the predictions are only for rearwards radiated fan noise, assuming a source of all cut-on modes present with unit intensity, the scales of figures 4.16 and 4.17 are not the same, and whilst it is not feasible to compare absolute values it is still possible to make some comments on the shapes of the curves.

On examination of the measured data, the noise in the rear arc peaks between 125° and 130° and then drops off at higher angles, showing reasonable agreement with the GXMunt prediction for the cone of silence. However, although the peak angle is well
predicted, for 1250Hz (figure 4.17(a)) the sound reduction after 120° is not as steep as the GXMunt prediction, and although it is more visible at the higher frequency of 4000Hz (figure 4.17(b)) where the Snell’s law approximation becomes more appropriate, it is still not as pronounced as in the predictions. The reason for this is because GXMunt models the bypass shear layer as an infinitesimally thin vortex sheet, and thus predicts a correspondingly sharp cone of silence. The effects of mixing are not accounted for, and in reality the shear layer will spread out downstream such that the cone of silence becomes less abrupt and more smeared, as seen in the measured data. Another reason for the difference seen in the amount of noise reduction after the peak is that the predictions are only for rearwards radiating fan noise, whereas the measured data contains other noise sources such that a steep drop off in fan noise may not be observed.

4.5.2 Comparison of GXMunt and empirical directivity patterns

In order to observe the effect that using the GXMunt directivity pattern rather than the empirical directivity pattern for the bypass attenuations has on the whole engine predictions, it would be ideal to incorporate the b-induct + GXMunt predictions into the whole engine prediction model. Unfortunately, due to the availability of the whole engine prediction model, at this stage it has not yet been possible to achieve this. However, it is still interesting to compare the bypass attenuation fieldshapes themselves. By taking the difference between the predicted hardwall and lined SPLs in figure 4.16 we obtain the b-induct + GXMunt prediction of the insertion loss due to the nacelle liners. This can then be compared to the equivalent liner insertion loss prediction made using b-induct with the empirical directivity pattern (b-induct + empirical). Figure 4.18 shows a comparison of these fieldshapes at 1250Hz and 4000Hz for the cutback condition.

It can be seen that although the fieldshapes for both the hardwall and the lined bypass configurations predicted by GXMunt (figure 4.16) follow approximately the same shape and are relatively smooth, the difference between them (figure 4.18) is not quite

![Figure 4.18: Predicted bypass liner insertion loss fieldshapes. Cutback condition.](image-url)
as smooth. GXmunt predicts that for both frequencies the liner insertion loss will peak at angles perpendicular to the duct axis, which seems reasonable given that these are the propagation angles for modes nearer cut-off that are likely to be the most attenuated. The least liner insertion loss is predicted to occur around 120° - 125° which also corresponds to the angle of the predicted cone of silence seen in figure 4.16. This also seems reasonable because the modes propagating at this angle will be those that were initially travelling along the duct axis prior to refraction by the shear layer, and hence are those which are well cut-on.

For both the GXMunt and empirical directivity predictions the same b-induct result has been used for the propagation prediction, and therefore the same transmitted power is present at the exhaust in both cases. If all of this power is then radiated into the far-field, the area under both the GXMunt and empirical curves should be the same. However, the area under the empirical curve is less than the area under the GXMunt curve because the empirical directivity pattern does not conserve the sound power radiated into the far-field, whereas the GXMunt prediction does.

It is clear from the difference between the GXMunt and empirical directivity patterns that the incorporation of the GXMunt directivities within the whole engine model is likely to have an impact on the prediction. However, due to the complexity of the whole engine prediction process it is difficult to make assumptions as to what this effect might be. An investigation into whether the incorporation of the GXMunt directivities would provide any better agreement with the measured engine data is only possible if the whole engine predictions are actually generated, and this is an interesting topic for future work.
Chapter 5

Axisymmetric Geometry Effects
Part I: Initial Simulations

5.1 Introduction

Having examined the use of b-induct for predicting bypass attenuations within a whole engine model in chapter 4, it is also of interest to examine the use of b-induct as a tool within the engine design process to assess potential low noise bypass duct configurations.

In order to reduce rearwards propagating fan noise within the bypass duct there are two main methods commonly used. Firstly the use of acoustic liners within the duct in order to absorb sound as it propagates can give significant reductions in SPL. The optimisation of liner impedances is already common within the design stages of a new engine and in chapter 9 a new method for optimisation of liner impedances is presented. Secondly the modification of the duct geometry itself can influence the reflection and scattering of sound as it propagates and hence can potentially be used to advantage in increasing the effectiveness of the liners.

In this chapter the influence of axisymmetric duct geometry on the in-duct propagation and attenuation is investigated. Potential low noise bypass duct geometries of interest are introduced and predictions for initial liner configurations are made in sections 5.2 and 5.4. Parametric studies are also performed (section 5.3) in order to investigate how varying the curvature and contraction of a duct systematically can influence the propagation and attenuation of noise.

Results from these initial investigations are used to select two bypass geometries to manufacture and test in the ISVR no-flow rig. These tests are described in chapter 6 and measured data is compared with predictions to continue the investigation of axisymmetric geometry effects in chapter 7. In chapter 8 the influence of 3D features such as the pylon and lower bifurcation is also examined.
5.2 Predictions for Initial Configurations

To begin with, two initial geometries of interest were considered (figure 5.1): a “Baseline” geometry, based on a typical modern aeroengine bypass duct, and a novel, “Highly Curved” (or “HC”) bypass duct which is significantly shorter than the Baseline, and hence benefits from reduced weight and drag. These designs are two extremes of realistic bypass geometry and b-induct was used to make predictions of the transmitted PWL at the exhaust plane ($PW_{Ltr}$) for each, using different configurations of liner within the duct, in order to assess the difference in attenuation provided. An inlet Mach number of 0.45, which is representative of the cutback condition, was used throughout.

For the Highly Curved geometry, predictions of $PW_{Ltr}$ were made in the hardwall case, and in the “Fully Lined” case where the total surface area of the duct on both the outer and inner wall is lined. The liner was chosen to have a non-dimensional acoustic resistance, $R = 1$, depth, $d = 0.0254m$, and mass inertance, $m = 0.01397m$, similar to that used in a modern bypass duct. Predictions of $PW_{Ltr}$ were also made for the Baseline geometry in the hardwall case, the “Fully Lined” case, and in the case where the start position of the liner on each wall was modified to provide the same area of liner as in the Highly Curved “Fully Lined” case.

The hardwall transmission loss for each geometry was obtained by subtracting the transmitted PWL at the exhaust plane in the hardwall case from the incident PWL at the inlet plane. These plots are shown in figure 5.2. The liner insertion loss for each configuration was obtained by subtracting the transmitted PWL at the exhaust plane in the lined case from the transmitted PWL at the exhaust plane in the hardwall case. Plots of liner insertion loss are shown in figure 5.3 for each geometry in the fully lined case and
also for the Baseline geometry with the same area of liner as the “Fully Lined” Highly Curved duct.

The total lined transmission loss for each configuration was obtained by subtracting the transmitted PWL at the exhaust in the lined case from the incident PWL at the inlet plane. These plots are shown in figure 5.4.

5.2.1 Initial geometries results and discussion

The predicted hardwall transmission loss (figure 5.2) for each geometry is a measure of how much power is reflected back down the duct by the duct geometry. Both the plots for the Baseline duct and the Highly Curved duct oscillate at low frequencies ($f$), before levelling out at $f > 1.6$kHz, whereafter the Highly Curved geometry appears to reflect about 0.25dB more than the Baseline geometry. The low frequency behaviour can be understood in terms of resonance characteristics, with the peaks of the hardwall transmission loss curves occurring at resonant frequencies of the duct. (For instance, for the shorter HC geometry, peaks at 250, 500 and 1000Hz could be attributed to an axial resonance, whereas axial resonances for the longer Baseline duct occur at the lower frequencies of 200, 400 and 800Hz.) At higher frequencies ray theory approximations are more appropriate, such that if the ray paths are traced along the duct there will be some that result in reflection of the sound back down the duct. This behaviour is frequency independent and thus the curves for hardwall transmission loss level out at
these higher frequencies. The higher predicted transmission loss of the Highly Curved geometry may be expected because as it is more highly curved than the Baseline, the line of sight from the inlet plane to the exhaust plane is reduced and results in more reflections as the sound travels down the duct.

Figure 5.3 shows the liner insertion loss deltas for each configuration. This is a measure of the amount of power that is attenuated by the liner on the duct walls. The “Fully Lined Highly Curved” case and the “Baseline - same lined area as Highly Curved” both have the same area of liner, and figure 5.3 shows that for \( f > 1300 \text{Hz} \), this area of liner attenuates more sound within the Highly Curved geometry than it does within the Baseline geometry, with about 3.5dB being the largest difference in attenuation at 1600Hz. The “Fully Lined Baseline” case has a greater area of liner than the “Fully Lined Highly Curved” case, and thus may be expected to provide more attenuation. Indeed, below about 1500Hz the Fully Lined Baseline case does provide more attenuation than the Highly Curved case, with a maximum difference of about 4dB at 1250Hz. However, between 1500Hz and 3150Hz the Highly Curved case provides up to 1dB greater attenuation, despite having a smaller area of liner. This may be due to the larger number of reflections within the highly curved geometry causing the sound to be incident on the liner a greater number of times as it travels down the duct, hence providing a larger attenuation.

The total transmission loss for each geometry in the lined case is shown in figure 5.4. This total lined transmission loss is the amount of power that is lost as the sound
travels down the duct both due to reflections by the geometry and attenuation by the liner. Thus the lined transmission loss could also be obtained by adding the hardwall transmission loss and the liner insertion loss. Figure 5.4 shows again that with the same area of liner, above about 1300Hz, the Highly Curved geometry attenuates more sound than the Baseline geometry, with a maximum difference of about 4dB at 1600Hz. The “Fully Lined Baseline” with a greater area of liner provides greater attenuation than the “Fully Lined Highly Curved” before about 1500Hz but as before, the larger number of reflections caused by the Highly Curved geometry causes a greater lined transmission loss at higher frequencies.

Overall these initial predictions show that for a given area of liner, the Highly Curved geometry provides greater attenuation than the less curved Baseline duct above about 1300Hz, and even provides greater attenuation than the Baseline with a larger area of liner between 1500Hz and 3700Hz. Thus, for a given area of liner, the Highly Curved geometry has acoustic benefits over the Baseline geometry, in addition to the low weight and drag benefits mentioned earlier.
5.3 Parametric studies

Two parametric studies were carried out to assess how the curvature of the duct and the contraction of the duct respectively would affect the sound propagated. Within each of these studies, either the duct curvature or contraction were defined using a parameter, which was then varied systematically in order to obtain several different geometries of duct. Using an inlet Mach number of 0.45, predictions were made of the transmitted PWL at the exhaust plane for each of these intermediate geometries. Comparisons were then made of the hardwall transmission loss, liner insertion loss and lined transmission loss in the same way as before to investigate how these different geometries affect the propagation of sound within the duct.

5.3.1 Duct Curvature

As a starting point for this study, two ducts with different extremes of curvature were defined by a set of curves and coordinates. Firstly the inner and outer walls of the Highly Curved geometry were defined by the curves $r_1(x)$ and $r_2(x)$ and the radii of the inner and outer walls at the inlet were defined as $r_A$ and $r_B$ respectively. This is shown in figure 5.5, where $A(x)$ is the Highly Curved cross-sectional area at point $x$.

![Figure 5.5: Highly Curved geometry defined as a set of curves and coordinates.](image)

A second geometry, the “endpoint” geometry (figure 5.6), with no curvature was also defined, by setting the inner wall at a constant radius, equal to $r_A$, and setting the outer wall $r_{outer}(x)$ (equation (5.1)) such that the cross-sectional area of the endpoint duct at each $x$-coordinate was the same as the cross-sectional area of the Highly Curved duct, $A(x)$, at that point.

$$r_{outer}(x)^2 = r_2(x)^2 - r_1(x)^2 + r_A^2$$  \hspace{1cm} (5.1)
Starting from these two extreme geometries, a set of intermediate geometries (figure 5.7) was obtained in the following way:

The inner wall $r_1'(x)$ was defined by:

$$r_1'(x) = \varepsilon r_1(x) + (1 - \varepsilon)r_A,$$  \hspace{1cm} (5.2)

such that when $\varepsilon = 0$, $r_1'(x) = r_A$, and when $\varepsilon = 1$, $r_1'(x) = r_1(x)$. With the constraint that $A(x)$ is constant, the outer wall becomes defined by:

$$r_2'(x)^2 = r_{outer}(x)^2 + r_1'(x)^2 - r_A^2. \hspace{1cm} (5.3)$$

Thus $\varepsilon = 1$ corresponds to the Highly Curved geometry, $\varepsilon = 0$ corresponds to the Endpoint geometry, and by decreasing $\varepsilon$ in steps of 0.2 a set of geometries with intermediate curvatures is generated (figure 5.8).
Figure 5.8: Intermediate geometries obtained by varying $\varepsilon$ from 1 (Highly Curved) to 0 (Endpoint), to vary duct curvature.

Figure 5.9: Hardwall Transmission Loss $\Delta PWL$s for a set of geometries with varying curvature.

Using b-induct, predictions of the transmitted PWL at the exhaust plane were made for each geometry from $\varepsilon = 1$ to $\varepsilon = 0$, and curves for the hardwall transmission loss, liner insertion loss and lined transmission loss (for a given area of liner) were obtained for each geometry (figures 5.9, 5.10 and 5.11).

At low frequencies below about 1500Hz, the hardwall transmission loss for each geometry is frequency dependent, with the peak amounts of sound power reflected occurring at resonant frequencies of the duct cavity. At higher frequencies however, the curves level out such that each geometry reflects an approximately constant sound power for all
frequencies. From figure 5.9 it can be seen that hardwall transmission loss increases with increasing duct curvature such that for the most highly curved duct the power loss at high frequencies is about 1dB more than for the straight duct.

The predictions for liner insertion loss (figure 5.10) show that, at low frequencies the amount of sound absorbed by the liner is slightly greater for a less curved duct, and at higher frequencies ($f > 1500\text{Hz}$) the amount of sound absorbed by the liner is slightly greater for a duct with higher curvature. This crossover at 1500Hz is not easily explainable, however the most notable point to make is that the maximum difference in liner insertion loss for all the different geometries is about 1dB, and in general the similarity of the curves indicates that duct curvature does not have a very large effect on liner attenuation.

Both the sound reflected due to the geometry and the sound attenuated by the liner add together to give a prediction for the lined transmission loss for each geometry (figure 5.11). It can be seen that the overall sound lost as it propagates down the duct is almost independent of duct curvature for frequencies below 1500Hz, but above this increasing duct curvature can lead to an increase in sound attenuation within the duct, with the most highly curved duct, ($\varepsilon = 1$), attenuating up to 2dB more sound than the straight duct, ($\varepsilon = 0$).
5.3.2 Duct Contraction

To perform a parametric study into how duct contraction affects the propagation and attenuation of sound, the Highly Curved geometry was again used as a starting point. As before, the inner and outer walls were defined as \( r_1(x) \) and \( r_2(x) \) respectively, and the duct cross-sectional area at point \( x \) was designated by \( A_s(x) \) (figure 5.12). The curve defining the duct centre line was calculated using equation (5.4) and this was used as a measure of the duct curvature.

\[
\overline{r}_s(x) = \frac{1}{2} [r_1(x) + r_2(x)] \quad (5.4)
\]

A set of intermediate geometries with varying contraction were obtained by using equations (5.5) and (5.6) to define the inner and outer walls respectively, and varying the parameter \( \varepsilon \) (figure 5.13). By this method, for each intermediate geometry, the duct length, \( L \), and the duct curvature \( \overline{r}_s(x) \) were kept the same as for the Highly Curved geometry, and the duct cross-sectional area was defined by equation (5.7). By increasing \( \varepsilon \) from 0 to 0.1, geometries with increasing contraction were produced, and by decreasing \( \varepsilon \) from 0 to -0.2, geometries with expanding cross-section were produced. This set of geometries is shown in figure 5.14.
Using b-induct, predictions of the transmitted PWL at the exhaust plane were made for each geometry from $\varepsilon = 0.1$ to $\varepsilon = -0.2$, and curves for the hardwall transmission loss, liner insertion loss and lined transmission loss were obtained for each geometry, where the same area of liner was used for each duct (figures 5.15, 5.16 and 5.17).

The hardwall transmission loss curves (figure 5.15) again show resonant behaviour in the lower frequency region, which starts to level out above about 2kHz. The $\varepsilon = -0.2$ geometry with the largest expansion provides the lowest transmission loss, as this is the geometry with the best line of sight from inlet to exhaust and consequently there are
Figure 5.14: Intermediate geometries obtained by varying $\varepsilon$ from -0.2 to 0.1 to vary duct contraction.

Figure 5.15: Hardwall Transmission Loss $\Delta PW Ls$ for a set of geometries with varying contraction.

less reflections of sound. The $\varepsilon = -0.1$ geometry provides the next lowest amount of reflection, and the $\varepsilon = 0$, 0.05 and 0.1 geometries all provide the greatest amount of reflection with only about 0.1dB difference between them. Overall, the greatest difference in the amount of sound power reflected by the different geometries is only about 0.35dB at 2kHz, indicating that the effect of duct contraction on the hardwall transmission loss is very small.
In the lower frequency range, below 1300Hz, the effect of duct contraction on liner insertion loss is fairly minimal, with the $\varepsilon = -0.2$ geometry providing the largest liner attenuation, followed by the $\varepsilon = -0.1$, 0, 0.05 and 0.1 geometries respectively (figure 5.16), and with the maximum difference being about 1.5dB at 1kHz. At frequencies
above about 1700Hz, liner insertion loss increases for ducts with increasing contraction, with the \( \varepsilon = 0.1 \) geometry providing the greatest liner insertion loss, and a maximum difference of about 3dB obtained between the \( \varepsilon = 0.1 \) and \( \varepsilon = -0.2 \) curves at 2kHz. At peak attenuation, the most contracted geometry, \((\varepsilon = 0.1)\), does not provide the largest attenuation. In order to investigate this further, predictions could be made for a greater number of frequency points around 1600Hz, as it may be found that the \( \varepsilon = 0.1 \) duct does actually provide the maximum peak attenuation, but that this peak occurs at a slightly different frequency. Overall it can be seen that duct contraction does have an effect on the attenuation of the liner within the duct, notably more so than the curvature of the duct.

The combined effect of both the reflections due to the duct geometry and the attenuation of the liner is captured by the lined transmission loss curves (figure 5.17). These show that in the low frequency range, duct contraction does not have a very large effect on sound attenuation within the duct, however above 1300Hz ducts with increasing contraction provide increasing attenuation. Overall the maximum difference in sound attenuated by the geometries with different contractions is just over 3dB at 2kHz.

### 5.3.3 Parametric studies conclusion

Overall these parametric studies have shown that by increasing the curvature or the contraction of a duct, the line of sight from source to exhaust is reduced and thus a greater number of reflections take place. Because of this there is a higher rate of incidence of sound on the duct walls, and the liner within the duct provides a greater attenuation. The curvature study has suggested that for a given surface area of liner, ducts with a higher curvature would attenuate more sound than those with a lower curvature for frequencies above 1.5kHz. The contraction study has suggested that a duct with a contraction would attenuate more sound than a duct with an expansion, although further investigation into this is required to produce any quantitative results. Thus, increasing the curvature or contraction of a duct decreases the amount of sound power transmitted down the duct, although the differences provided are still quite small.

In the case of a realistic engine, several other factors must be taken into account when considering the potential curvature and contraction of a duct. A certain mass flow rate through the bypass duct is necessary in order to produce the required thrust, which affects the possible cross-sectional area of the duct. Also if modifications to the curvature or contraction of the duct were to increase the speed of the air flow through the bypass duct, the speed of the cold jet emanating from the rear of the bypass duct would increase, leading to an overall increase in jet noise. These factors act as restrictions on the potential geometries for a realistic bypass duct, along with the necessary positioning of duct hardware, and must also be considered in the design of a low noise duct.
5.4 Predictions for Further Configurations

In order to examine potential low noise designs which also satisfy other industrial requirements it was chosen to compare two realistic bypass duct configurations. In this case the novel Highly Curved geometry from section 5.2 was compared with a different Baseline, which we shall label “B2”. These two geometries are shown in figure 5.18, with the initial Baseline geometry from section 5.2 also on the same plot for comparative purposes. As can be seen, the B2 design is much more highly curved than the similar length initial Baseline, although slightly less so than the shorter HC geometry.

Within each of the HC and B2 geometries, a realistic configuration of liners was considered, with positions and impedances specified as they would be in the engine. Figure 5.19 shows the liner configuration for the B2 geometry, consisting of a Rear Fancase (RFC) liner and an Inner Fixed Structure (IFS) liner on the inner bypass wall, and an RFC liner, an Outer Fixed Structure (OFS) liner and an OFS tapered liner on the outer bypass wall. The initial liner specifications for the B2 design are listed in table 5.1.

As the HC geometry is a shorter duct, there are no RFC liners present, and the configuration of liners in the HC geometry consists of two IFS liners, and one OFS liner (see figure 5.20). The initial liner specifications for the HC duct are listed in table 5.2.

Figure 5.18: Comparison of Highly Curved, Baseline and B2 geometries.
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Figure 5.19: Liner configuration within the B2 geometry.


The b-induct method was used to make predictions of the liner insertion loss for these two initial bypass duct configurations at each of three conditions: Approach, Cutback and Sideline. These predictions are shown in figure 5.21.

<table>
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<th>Wall</th>
<th>liner area (in²)</th>
<th>SDOF/ DDOF</th>
<th>FCD (in)</th>
<th>FPO (%)</th>
<th>SCD (in)</th>
<th>SR (cgs rayl)</th>
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<td>0.9</td>
<td>7.5</td>
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</tbody>
</table>

Figure 5.21: B-induct predictions of Liner Insertion Loss, $\Delta PWL$, for B2 and HC realistic liner configurations.

In the lower frequency range (below 1000Hz) the liner insertion losses for the B2 and HC are very similar. However, above 1000Hz the B2 liner configuration provides a greater insertion loss than the HC liner configuration, with a maximum difference of about 4dB at peak attenuation frequencies (1600Hz - 2000Hz), which levels off to a difference of approximately 2dB at higher frequencies.

In this case, similar to the initial study comparing the HC and the Baseline, the HC geometry has much less liner area than the B2 (13586 in² (8.8 m²) compared to 23503 in² (15.2 m²)), and this reduced area contributes to a lower liner insertion loss, as would be expected. However, in contrast to the initial geometry study, the difference in curvature between the HC and B2 geometries is not enough to counteract this decrease in attenuation. Figure 5.22 is a plot of the predicted hardwall transmission loss, and shows that, at high frequencies, where ray theory approximations are valid, the amount
Figure 5.22: B-induct predictions of Hardwall Transmission Loss, $\Delta PWL$, for B2 and HC geometries.

of power reflected down the duct is similar for both the B2 and HC geometries (with a maximum difference of approximately 0.4dB). Thus, the increased curvature of the HC geometry is not enough to provide a larger amount of reflected power, and also the improvement in liner performance within this geometry is not significant enough to overcome the reduction in attenuation due to the decreased liner area.

5.5 Conclusion

The parametric studies demonstrate that modifying the curvature or contraction of a bypass duct can influence the amount of sound attenuated by the liner within the duct. Whilst it may be possible to suggest an optimal low noise bypass design on the basis of parametric studies such as these, other factors must also be taken into account within the engine design.

In section 5.2 two realistic geometries of bypass duct were considered: a Baseline geometry, based on a typical modern aeroengine bypass duct, and a novel, Highly Curved (or HC) bypass duct which is significantly shorter than the Baseline, and hence benefits from reduced weight and drag. This initial study showed that the HC geometry provides larger attenuations when the same area of liner is used within each duct, and also provides similar levels of attenuation to the Baseline when both are fully lined despite the HC having a reduced amount of liner. This observation implies an inherent noise attenuation benefit of the HC design.
Within the further geometry study (section 5.4), a different Baseline, B2, was compared to the HC design and realistic liner configurations were considered. In this case the predicted attenuations for the B2 configuration were larger than those for the HC configuration. This is because the B2 geometry is already similar in curvature to the HC geometry, so that the dominant effect is that of liner area.

In summary it has been seen that, for a given liner area, there is a geometry effect provided by increasing the curvature of the bypass duct that can lead to improved liner attenuation. However, for shorter ducts the increase in liner attenuation provided by this curvature effect is sometimes not enough to counteract the decrease in attenuation due to the reduced liner area available. The novel Highly Curved design is shorter and so has associated weight and drag benefits, although whether there is also a noise benefit depends on the Baseline geometry and liner configuration used for comparison.

To evaluate these predictions and to provide further data for the study of the effects detailed above, two geometries were selected for manufacture and rig-scale testing. Because of the predicted noise benefit provided by the Highly Curved design compared to the initial Baseline design, these two geometries were chosen. Whilst the initial predictions performed so far for these geometries have considered the same liner area within each, practical considerations in aero-engine design (for example positioning of bleed valves and other duct hardware) limit the area of liner that is actually available within the duct. Accordingly, the rig tests account for this with realistic areas of liner being used for both geometries.
Chapter 6

ANDANTE Tests

6.1 Introduction

In the previous chapter two different realistic bypass geometries were introduced: a Baseline geometry, based on a typical modern aeroengine bypass duct, and a novel, Highly Curved (or HC) bypass duct which is significantly shorter than the Baseline, and hence benefits from reduced weight and drag. Initial predictions of liner insertion loss for these geometries suggested that the more highly curved duct could provide a noise benefit. In order to validate this result, and also to provide a database against which various prediction methods may be benchmarked, the two bypass geometries were manufactured on a rig scale, and a series of no-flow duct acoustic measurements were taken in the ISVR no-flow facility using both broadband and tone sources. This chapter consists of a description of the experimental rig and test procedure, with details of the various duct builds and source content. Results from these tests are presented along with predictions in chapter 7.

6.2 The Test-Rig and Test Procedure

6.2.1 Description of Test-Rig

The chamber set-up for the no-flow tests is shown in figure 6.1. The duct system is mounted in a pair of heavy, acoustically sealed doors between the ISVR small reverberation and large anechoic chambers. The duct inlet extends a short distance into the reverberation chamber (see figure 6.2), where noise is generated by a source (either broadband or tone) which then propagates through the test duct, and into the anechoic chamber.
Figure 6.1: Layout of test rig

Figure 6.2: View of duct from reverberation chamber
The basic duct extends about 4m into the anechoic chamber (see figure 6.3) and consists of a cylindrical duct of 397mm diameter, with a solid centrebody of radius 120mm which starts approximately 3.5m along the section of the duct within the anechoic chamber as a tapered conical section, as shown in figure 6.4. The centrebody is held in place by eight 6mm diameter support bolts at 0°, 90°, 180° and 270°. This basic duct remained unchanged throughout the tests, whilst various combinations of hardwall and lined inner and outer duct sections were attached onto the end of this existing duct (see section 6.3), in order to obtain the realistic bypass geometries that were tested.

A polar microphone array of radius 3.95m, and consisting of 25 microphones, is used to record the far field Sound Pressure Levels (SPLs) at 5° angles from 0° (on-axis) to 120°. This polar array consists of a flexible steel rod to which the microphones are attached, which is then suspended from the anechoic chamber ceiling, and the position and shape of which are adjusted using a system of guy ropes. The floor of the anechoic chamber was almost completely removed except for a walkway immediately beneath the duct.
6.2.2 The Microphone Array

The microphones used in the polar array are Brüel and Kjaer type 4189 “Falcon” half-inch pre-polarised measurement microphones, with consecutive serial numbers from #2285284, for the 0° microphone, to #2285309 for the 120° microphone. These microphones were powered by ISVR-built amplifiers set to +30dB gain. A view of the microphone array can be seen in figure 6.5.
6.2.3 Acoustic Excitation

For each build that was tested, far field SPL measurements were taken using both a broadband source and individual tone sources at various frequencies. To generate the broadband source, two loudspeakers (Electovoice T251, rated at 400W r.m.s. and with additional high frequency horns) in the reverberation chamber, were driven by two independent spectrally-shaped white noise signals via a Crown DC300 power amplifier. This arrangement generates a near-diffuse field in the reverberation chamber, which gives an in-duct sound field that can be closely approximated by an assumption of equal energy in all cut-on modes [110]. For the broadband tests, the level of excitation was monitored and recorded using a microphone on a stand in the reverberation chamber (of the same type as the far-field microphones).

For the tone tests, the section of duct protruding into the reverberation chamber was modified to incorporate a ring of 30 circumferential piezoelectric in-house-modified loudspeakers, spaced at equal angles around the duct inlet (see figure 6.6). For a specific frequency, by applying a different phase to each loudspeaker signal, this arrangement was used as a mode synthesiser to generate an azimuthal mode of the required order. No radial mode modulation was applied, resulting a modal source consisting of a single azimuthal order with all cut-on radial modes present. A total of 36 different modal sources were used for the tone tests. In order to reduce the number of test points required modal sources at 2 or 3 different individual frequencies were combined into one multiple-tone signal. By ensuring that the frequencies used within this tone signal were not multiples, sums or differences of each other, the resulting sound field could be decomposed into the individual sound fields at each frequency. The matrix of tones used for these tests is contained in appendix A.

Measurements taken using a ring of 30 circumferential microphones (Panasonic WM61) in a plane 20cm behind the loudspeaker ring were used to obtain information about the source level and also the purity of each azimuthal mode generated. A radial microphone array of 8 (Panasonic WM61) microphones mounted in place of one of the centrebody support bolts was also used to obtain information about the radial mode purity (see figure 6.7).

6.2.4 Data Acquisition and Processing

Two Sony SIR1000 multi-channel digital tape recorders were used to store the time histories from the microphones. A bandwidth of 20kHz was set, such that each tape recorder was capable of storing data on 32 channels. For all test points, signals from the 25 far-field array microphones plus a voice identification track were recorded. For the broadband tests, the level of excitation was monitored by recording data from the reverberation chamber microphone, and for the tone tests the level of excitation was
Figure 6.6: View of mode synthesiser from reverberation chamber

monitored by recording data from the 30 circumferential microphones and 8 radial microphones.

For each test point, a 1-minute recording was stored. The temperature and humidity of the air in the anechoic chamber were noted at the start of each test, and each of the microphones was calibrated at the start of the tests. The far-field, reverberation chamber and radial microphones were all calibrated to the same level using a pistonphone, whereas the 30 circumferential microphones were calibrated relative to each other using a loudspeaker signal consisting of the 10 tones to be tested.

The microphone data was post-processed on a PC using Matlab software. For the broadband test points, the frequency spectrum for each far-field microphone in third octave bands from 400Hz to 16kHz was obtained. For the tone test points, far-field directivity plots were obtained at each frequency, along with plots to show circumferential mode amplitude and radial mode purity.

For the broadband results, the SPL at the reverberation chamber microphone can be used as a source correction level, and for the tone results, the SPLs from the 30 circumferential microphones can be used.
6.2.5 Leakage and Signal-to-Noise Ratio

Due to the low levels of sound radiated by some of the test builds, especially at wide angles, and when absorbent lining is present, it is important to minimise any leakage in sound from the reverberation chamber to the anechoic chamber via any path other than radiation from the open duct-end. To achieve this, mineral-loaded rubber dead-sheet was wrapped around the entire duct (except the exit section), and was also used to seal any potential leakage areas between the reverberation chamber and anechoic chamber. The level of leakage was tested by sealing a 25mm thick wooden plate against the open duct end and comparing sound levels at the far-field array to those for builds that were expected to yield high attenuation. The level of background noise was also tested during the shakedown, and it was ensured that both the signal-to-noise ratio and the amount of leakage were at acceptable levels.

6.3 Descriptions of Test Builds

The two geometries tested are representative of realistic bypass duct geometries with very different duct curvatures. The Baseline geometry has a curvature typical of modern
high-bypass ratio bypass ducts, and the novel Highly Curved (HC) architecture is, as the name suggests, more highly curved. The HC duct is also significantly shorter than the Baseline, (in effect the Rear Fancase (RFC) section has been removed), which results in weight and drag benefits for the engine, but reduces the area available in the bypass duct where liners can be placed.

Both of these geometries were manufactured on a 1 : 4.2 (rig : full) scale, by scaling the inner wall of the full-scale duct geometry to match the inner wall of the existing rig section. A tapered outer wall transition segment was manufactured to bring the existing outer section to the same scale. The inner wall for each geometry was manufactured as 4 conical sections plus a conical afterbody (figure 6.8) and the outer wall was manufactured as 4 square cross-sections each with a conical hole hollowed through (figure 6.9). All rig parts were manufactured in both a hardwall (HW) and a lined version so that various combinations of HW and lined sections could be tested. The liner used consisted of a 6mm deep honeycomb cell structure with a wire mesh facing sheet, the in-situ, zero-flow non-dimensional acoustic resistance of which was measured as 1.2 using a hand-held acoustic impedance meter (Brüel and Kjaer Portable Impedance Meter System, type 9737).

**Figure 6.8:** Inner wall sections (left → right: Baseline lined, HC lined, Baseline HW, HC HW)
In order for the exhaust planes of both rig geometries to fall at the centre of the polar microphone array it was necessary to translate the Baseline geometry backwards such that both the HC and Baseline exhausts were axially aligned. This results in the RFC section for the Baseline being at the location of the tapered transition segment. Figure 6.10 shows the set-up of the rig geometries.

For each geometry, a HW datum build was tested where all 4 inner and outer sections were HW. (Build #1 is the Baseline HW datum, and build #16 is the HC HW datum). Lined datum builds were also tested, where realistic amounts of liner were used for each geometry. For the Baseline lined datum (build #3), representing the tapered transition section as the RFC, sections 1 – 3 as the Thrust Reverser Unit (TRU) and section 4 as the nozzle, and assuming that the whole RFC and 2/3 of the TRU could be lined, it was chosen to line sections 1 – 3 and keep section 4 HW. (As the tapered transition segment could not be lined, the RFC liner was simulated by lining section 1). Similarly for the HC lined datum (build #18), 2/3 of the TRU (sections 2 and 3) were lined, and the nozzle (section 4) was kept HW. In this way, the areas of liner present in the lined datum Baseline and HC ducts were 0.67m$^2$ (1037in$^2$) and 0.5m$^2$ (780in$^2$) respectively. For all datum builds a HW afterbody was used. As the HC bypass duct is shorter than the Baseline, the afterbody for the HC configuration is longer than for the Baseline.
Lined afterbodies were also tested for both geometries (Baseline build #2 and HC build #17).

To investigate the effect of HW patches within the duct, two axisymmetric HW patches extending around the whole circumference of the duct, and covering the first half of each of sections 2 and 3, were inserted in the HC geometry on both the inner and outer walls (build #25). For this build, section 1 was also lined such that the total area of liner present was the same as for the lined datum build.

As well as the axisymmetric duct configurations, various non-axisymmetric configurations were tested using pylons and HW patches. For the Baseline geometry, measurements were taken both with HW and with lined bifurcations in both the upper and lower duct sections (builds 4-7). Measurements were also taken with a HW patch taped over the liner on the outer wall in section 1, with the patch starting about a third of the way along section 1, and extending along about half the length and around a quarter of the circumference of the section (builds 8-14). For the HC geometry, measurements were taken both with a HW and with a lined single vertical bifurcation (builds 19-25). For all of the non-axisymmetric duct configurations, several builds were tested where the orientation of the pylons or HW patch was varied in order to build up more of a 3D picture of the far field directivity pattern.

A full matrix describing all the builds that were tested is contained in appendix B, along with diagrams that show the particular build configurations. The builds are numbered 1–26, and for the broadband tests, each test point number corresponds to the equivalent
build number tested with a broadband source. For the tone tests the test points are numbered 1 – 364, each of which corresponds to a particular build number and tone number. The full matrix for the tone tests is contained in appendix C.

6.4 Summary

During the no-flow rig tests a large amount of data was collected. Overall 26 builds were tested, each with a broadband source and 36 individual tone sources, such that in total there are 962 points in the complete test matrix. It has not been possible to examine all of this data within this thesis, so only specific results are presented along with predictions in the next two chapters. Although not all of the data has been examined thus far it provides a good database against which various prediction methods may be benchmarked in future.
Chapter 7

Axisymmetric Geometry Effects
Part II: Testing of Predictions

7.1 Introduction

In chapter 5 the influence of axisymmetric duct geometry on the in-duct propagation and attenuation of fan noise was investigated, and parametric studies indicated that modifying the curvature or contraction of a bypass duct can influence the amount of sound attenuated by the liner within the duct. Two particular bypass geometries of interest were considered: a typical “Baseline” geometry, and a novel “Highly Curved” (or “HC”) design, and initial predictions for these, with larger amounts of liner than would be realistic in an aeroengine, suggested a potential acoustic benefit for the Highly Curved design over the Baseline. To confirm this result rig tests were performed, as described in chapter 6, where the two duct geometries were manufactured on a rig scale and realistic amounts of liner were used within each geometry.

In this chapter the study of axisymmetric geometry effects continues. Measured data from the rig tests is compared to both b-induct predictions for the in-duct results, and b-induct + GXMunt predictions for the far-field results. The purpose of comparing the test results to these predictions is twofold: Firstly, the examination of how well the predictions fit the measured data provides an indication of the accuracy of both the b-induct and GXMunt prediction models. Secondly, the measured data allows further analysis of the effect of duct geometry on sound attenuation, and may confirm the initial indication of the acoustic benefits of the novel Highly Curved design. Both of these results will also offer further assessment of the value of b-induct as a tool within the engine design process.
7.2 Measured vs Predicted data using a multimodal source

To investigate the effect of geometric changes to an axisymmetric duct on noise propagation, data was evaluated for the hardwall and lined datum builds (figures 7.1 and 7.2 respectively) for both the Baseline and Highly Curved ducts. For the lined builds the liner used consisted of a 6mm deep honeycomb cell structure with a wire mesh facing sheet, whose zero-flow non-dimensional acoustic resistance and mass inertance were measured as $R = 1.2$ and $m = 0.003$ m respectively using a hand-held acoustic impedance meter. As can be seen from figure 7.2 the Highly Curved lined datum has approximately $2/3$ the liner area of the Baseline lined datum. Results are presented here using a multimodal source at third-octave centre frequencies from 400Hz to 16kHz (rig-scale, where rig : full scale is $1 : 4.2$).

7.2.1 In-duct results

To examine the effect of duct geometry alone on noise propagation, values for the transmitted sound power level at the exhaust plane of the duct ($PW_{Ltr}$) can be compared for each of the geometries in the hardwall case. Figure 7.3 shows a plot of $\frac{PW_{L_{tr}}}{HC} - PW_{L_{tr}}^{\text{Base}}$. 

![Figure 7.1: Hardwall datum builds for Baseline (left) and Highly Curved (right).](image1)

![Figure 7.2: Lined datum builds for Baseline (left) and Highly Curved (right).](image2)
against frequency for both the measured and predicted data, where $\Delta \text{PW} L_{tr}$ is defined as $\text{PW} L_{tr}$ for the Baseline duct minus $\text{PW} L_{tr}$ for the Highly Curved duct:

$$\text{Base} \Delta \text{PW} L_{tr} = \text{PW} L_{tr}(\text{Baseline}) - \text{PW} L_{tr}(\text{Highly Curved}).$$  

(7.1)  

(Details of the calculation of $\text{PW} L_{tr}$ for the measured data are given in appendix E.)

In figure 7.3, where $\text{Base} \Delta \text{PW} L_{tr}$ is positive the acoustic power transmitted by the Highly Curved duct is less than that transmitted by the Baseline. At frequencies above 3150Hz this is the case for both the predicted and the measured Deltas and it is observed that there is an acoustic benefit provided by the Highly Curved duct over the Baseline in the hardwall case, of up to about 0.4dB in the higher frequency range. For frequencies below 2000Hz the effect of the hardwall duct geometry on the transmitted power seems to be less significant. This result may be expected because according to ray-theory there is an acoustic benefit of a more highly curved duct due to the reduction in line-of-sight through the duct, and this is more valid at higher frequencies. The effect of duct geometry is less relevant at lower frequencies, where the longer wavelength sound field is less sensitive to changes in geometry, and the predictions again capture this behaviour well.

The largest discrepancy between the predicted and the measured data occurs at 2500Hz, where the model predicts there to be more power transmitted through the Highly Curved than through the Baseline, contrary to all of the other high frequency points. This is perhaps due to a resonance associated with the duct geometry occurring at 2500Hz.
such that sound is well transmitted through the Highly Curved geometry, and it is encouraging that this downward peak also occurs in the measured data, although at the slightly lower frequency of 2000Hz. For future work it may be worth examining the behaviour in this region in more detail to investigate whether this is a physical effect. Apart from in this 2000Hz frequency region, the predictions compare well to the measured data and although the shapes of the Baseline-predicted and Highly Curved-predicted curves differ, the scale of the graph is small such that over most of the frequency range the predictions are within 0.3dB of the measured values.

When liners are used within the duct, the overall transmission loss of sound as it propagates along the duct is a combination of both the transmission loss due to the hardwall geometry, as seen above, and also the insertion loss due to the attenuation by the liner itself. This liner insertion loss, \( LI \), is defined as the difference in \( PWL_{tr} \) for the hardwall and the lined case for each geometry:

\[
LI = \frac{HW}{Lined} \Delta PWL_{tr} = PWL_{tr}(hardwall) - PWL_{tr}(lined).
\]  

(7.2)

Figure 7.4 shows b-induct predictions of \( LI \), compared to the measured values for each geometry.

It can be seen from figure 7.4 that the predictions capture well the shape of the measured data, although there is a notable difference in levels specifically at the peak attenuation frequencies. The reasons for this difference are explored in section 7.2.2.
Despite the reduced liner areas in the Highly Curved duct, both the predicted and the measured data indicate that the Highly Curved geometry provides similar attenuation to the Baseline geometry, with somewhat larger liner insertion loss at higher frequencies, and slightly less at lower frequencies. This can be seen more clearly in figure 7.5 in which $\Delta LI_{HC} - \Delta LI_{Base}$ is plotted against frequency. In this case, $\Delta LI_{HC} - \Delta LI_{Base}$ is defined as the liner insertion loss provided by the Highly Curved duct ($LI_{HC}$) minus the liner insertion loss provided by the Baseline duct ($LI_{Base}$): 

$$\Delta LI_{HC} - \Delta LI_{Base} = LI_{HC} - LI_{Base}. \quad (7.3)$$

The predictions again capture well the shape of the measured data. The negative values of $\Delta LI_{HC} - \Delta LI_{Base}$ indicate that the Baseline geometry provides a larger insertion loss than the Highly Curved geometry at low frequencies, whereas at high frequencies positive values of $\Delta LI_{HC} - \Delta LI_{Base}$ indicate a noise benefit provided by the Highly Curved duct. This result is in agreement with that seen in figure 7.3: at lower frequencies the effect of duct geometry is less significant and therefore the Baseline geometry with the larger amount of liner provides more attenuation, whereas at higher frequencies the effect of geometry is more significant such that the larger amount of reflections within the HC causes greater attenuation, and the increased curvature compensates for the reduced liner area.
7.2.2 Discussion of discrepancy in attenuation levels

In order to examine the reason that the predicted liner insertion loss values in figure 7.4 are so much larger than the measured values at peak frequency there are several factors to consider: the effect of modal energy distribution, the effect of liner impedance and the effect of bandwidth. In what follows, each of these is discussed in turn.

7.2.2.1 Modal energy distribution

Reference [110] shows the equivalence of an equal energy per mode sound field and isotropic noise within a circular duct, therefore an assumption of equal energy per mode is not unreasonable. However, it is possible that the assumption of an equal energy per mode distribution at the inlet of the duct model is inappropriate in this case, because although the portion of the duct protruding into the reverberation chamber is circular, the inclusion of the centrebody shown in figure 7.6 causes the duct geometry to change from circular to annular. It is possible that this change in geometry could cause the modal energy distribution within the duct to change such that at the b-induct model inlet the assumption of equal energy per mode is inappropriate.

As the discrepancy in figure 7.4 is such that b-induct has over-predicted the amount of attenuation at peak frequency, it is likely that within the prediction model there is more energy in the least cut-on (and most highly attenuated) modes than is actually present in the tests. In order to investigate the effect of the modal energy distribution at the inlet on the predicted attenuation, a b-induct prediction of liner insertion loss was performed using a different modal energy distribution.

At the peak attenuation frequency of 5kHz, 69 modes are calculated to be cut-on at the inlet plane of the model, and within the original b-induct predictions a unit intensity is assigned to each of these modes. Figure 7.7 is a plot of these original incident modal intensities.
To explore the significance of modal energy distribution, a “top-hat” source distribution was selected. For this profile, lower order modes with a cut-on ratio greater than 1.5 were assigned an intensity of 2, and higher order modes with a cut-on ratio less than 1.5 were assigned an intensity of 1. The value 1.5 was chosen to ensure an approximately even split of modes at each of the two intensities. This new incident modal intensity distribution is shown in figure 7.8.

Using this new modal energy profile at 5kHz at the inlet, an ACTRAN prediction of the transmitted power level at the exhaust \( P_{WL_{tr}} \) was made for the Baseline geometry in both the hardwall and lined duct configurations, and the value of liner insertion loss
calculated as before. This value is plotted in figure 7.9 along with the b-induct predicted liner insertion loss spectrum and that of the measured data.

As seen from figure 7.9, using the “top hat” modal energy distribution at the inlet provides a value of liner insertion loss at 5kHz of 21.26dB, which is even higher than the equal energy per mode prediction of 20.55dB and still significantly higher than the measured value of 14.08dB. This is unusual given that for the new modal energy distribution there is less energy in the modes with lower cut-on ratios, which we would usually expect to be the most attenuated. To explore the reasons for this effect, a plot of the transmitted modal intensities at the exhaust for the lined configuration is examined (figure 7.10). From this it can be seen that at 5kHz some of the more cut-on modes (eg. $m5n2$) are actually being attenuated more than some of the less well cut-on modes (eg. $m11n2$). This is an interesting observation which has also been noted in other studies [73]. In this instance, this effect explains why the “top hat” energy distribution actually provides a higher liner insertion loss, although the physical reason for this effect is unclear, and further analysis is required for it to be fully understood.

In order to obtain a predicted liner insertion loss that more closely matches the level of the measured data it is necessary to perform more experiments using different incident modal energy distributions. (For example, in this case perhaps an “inverse top-hat” profile could be used, where more energy is present in the higher order modes.) However, even if a source profile were found that predicts the correct level of attenuation, an explanation as to how this source distribution could have been generated within the
rig tests would still be necessary. Perhaps a more appropriate approach would be to model the entire duct domain, starting from the circular portion of duct within the reverberation chamber, where the use of an equal energy per mode assumption would be more accurate.

From this investigation it is clear that the modal energy distribution used at the inlet has a small effect on the predicted liner attenuation. But an inappropriate modal energy distribution is not the only possible reason for the discrepancy in liner insertion loss observed.

### 7.2.2.2 Liner impedance

Another potential reason for the overprediction of the liner insertion loss could be that the impedance used in the model is wrong. For the b-induct predictions, the values of impedance, $Z$, are input in terms of the non-dimensional liner resistance, $R$ and reactance, $X$, where $Z = R + iX$. The value for $X$ is calculated using $X = km - \cot(kd)$, where $k$ is the wavenumber, $d$ is the liner cell depth, and $m$ is the liner facing sheet mass inertance. For the rig builds the value for $d$ is known to be $d = 0.006m$, and in order to determine the values of $R$ and $m$ to use within the b-induct model, impedance measurements were made using both a standing wave tube and a hand held impedance meter (Brüel and Kjaer Portable Impedance Meter System, type 9737). From these tests the resistance of the liner was measured as $R = 1.2 \pm 0.15$ and the mass inertance was determined to be $m = 0.003m \pm 0.0015m$.

The original liner insertion loss predictions shown in figure 7.4 are made using the values $R = 1.2$ and $m = 0.003m$. In order to ascertain how the error in the impedance...
measurement (±0.15 for $R$ and ±0.0015m for $m$) could affect the predicted liner insertion loss, b-induct predictions of liner insertion loss at the peak attenuation frequency, 5kHz, were made for the Baseline geometry using the combinations of $R$ and $m$ shown in Table 7.1. Liner 1 (L1) is the original liner used for the b-induct predictions (with $R = 1.2$ and $m = 0.003$m). The liner insertion loss predicted for each of these liners is plotted in Figure 7.11 along with the original b-induct prediction for liner 1 and the measured data.

Figure 7.11 shows that with a resistance error of ±0.15 and a mass inertance error of ±0.0015m a range in peak attenuation of approximately 2.5dB can be caused. Whilst it is clear from this that errors in the measured impedance can result in errors in the liner insertion loss prediction, the measured value of attenuation at 5kHz still does not fall inside the predicted error in Figure 7.11, and so the discrepancy cannot be readily explained in this way.

**Table 7.1:** Table of $R$ and $m$ combinations used for liner insertion loss predictions at 5kHz.

<table>
<thead>
<tr>
<th>m</th>
<th>R</th>
<th>1.05</th>
<th>1.2</th>
<th>1.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>L5</td>
<td>0.0015</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L4</td>
<td>0.003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L6</td>
<td>0.0045</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L2</td>
<td>0.003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L1</td>
<td>0.003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L3</td>
<td>0.0045</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L7</td>
<td>0.003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L8</td>
<td>0.0015</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L9</td>
<td>0.003</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 7.11:** Predicted vs measured liner insertion loss for Baseline geometry, using different liner impedances (L1 → L9) at 5kHz.
7.2.2.3 Bandwidth

As well as considering sources of error in the prediction of liner insertion loss, it is important to consider sources of error in the examination of the measured data. In the measurements, the time histories of the signal at the far-field microphone locations are recorded, and these data are post-processed to give 1/3 octave band SPLs by integrating the signal over the 1/3 octave band width. So far these have been compared to the SPLs at 1/3 octave centre frequencies predicted using b-induct, and thus the quantities compared are not the same. Despite this, such comparison of 1/3 octave band levels and narrowband levels at the centre frequencies is quite common because these levels are often reasonably consistent as long as the spectrum curve is smooth. However, the difference between these values can be non-negligible in the vicinity of a steep peak or drop. Therefore, to analyse the frequency spectrum in more detail it is necessary to examine the narrowband measured data. The measured narrowband liner insertion loss spectra for both the Baseline and HC geometries are shown in figure 7.12 along with the 1/3 octave band measurements and predictions as before.

As can be seen from figure 7.12, on the whole the measured narrowband attenuation spectra follow the same form as the measured 1/3 octave band attenuation spectra. However, there is a significant difference between the measured narrowband and 1/3 octave band attenuation spectra at the frequencies where maximum attenuation occurs, with almost a 3.5dB difference between the peaks for the HC geometry, and with the narrowband data providing much better agreement with the b-induct predictions than the 1/3 octave data.
Close examination of the narrowband data also indicates that the peak attenuation occurs at a different frequency for each of the Baseline and HC ducts. This is an interesting observation since, given that the liner impedance is the same in both cases, it indicates that duct geometry is significant in determining the frequency at which maximum attenuation occurs. In this case the possible cause may lie in the reduction in duct height along the axis. However, more detailed analysis is required for this effect to be fully understood. In figure 7.12 the narrowband measured Baseline peak attenuation occurs at 4675Hz whereas the HC peak attenuation occurs at 5650Hz. As the b-indent predictions have been made at each 1/3 octave centre frequency, these peaks were not captured in the original prediction. In an attempt to capture these different peak attenuation frequencies using b-indent, further predictions of the liner insertion loss were made for frequencies 4675Hz, 5300Hz and 5650Hz.

These values have been added to the predicted attenuation spectra in figure 7.13 which shows that the addition of the extra data points does indeed affect the capture of the different peak attenuation frequencies for the Baseline and HC geometries. However, the inclusion of these extra frequency predictions has also resulted in peak attenuation predictions that are even higher than before, so that the HC attenuation is overpredicted by about 4dB and the Baseline by almost 11dB. Although the shape of the attenuation spectra is now captured well, it is still necessary to explain this difference in level. It appears that the liner within the rig tests is not as effective at attenuating sound as the liner used within the prediction model. The reason for this is unclear, although possible causes may be that the amount or impedance of the liner used within the prediction model does not accurately represent that used within the rig tests.
7.2.2.4 Summary

These studies have highlighted the importance of the narrowband data, since the examination of 1/3 octave band data can clearly result in loss of information, especially in the region of peak attenuation. The b-induct predictions capture well the shape of the measured data, however there is a significant discrepancy in the levels of the predicted and measured attenuations. It has been observed that both the distribution of energy amongst incident modes and the impedance used within the model can affect the prediction of liner insertion loss, although the liner insertion loss predictions appear to be more sensitive to the impedance used than to the incident modal energy distribution used. Assuming that the liner present in the rig tests is appropriately effective, and that there is no discrepancy between the amounts of liner in the measurements and predictions, the most likely remaining cause for the discrepancy in levels between the measured and predicted attenuations is the error in the impedance used.

7.2.3 Far-field results

In order to make far-field predictions to compare with the measured data, the b-induct predictions of the amplitudes of the transmitted modes at the duct exhaust were used as inputs to the GXMunt radiation model. The predicted fieldshapes were then calibrated to the level of the measured data at each frequency so that the total radiated power in the prediction would be the same as that of the measurement. This was done for both the hardwall data and the lined data (ie. the hardwall prediction was calibrated to the level of the hardwall measured data and the lined prediction was calibrated to the level of the lined measured data.)

The predicted and measured fieldshapes for the Baseline geometry are shown for the hardwall case in figure 7.15 and for the lined case in figure 7.16. For the HC geometry the predicted and measured fieldshapes for the hardwall case are shown in figure 7.17 and for the lined case in figure 7.18. For the HC geometry, the sharp peak at 27° in the directivity plots for 3150Hz is non-physical, and is due to a known error in the GXMunt MATLAB calculation, which is to be corrected in the future development of this code.

From the plots of the predicted and measured fieldshapes for both the Baseline and HC geometries, it can be seen that for the lower frequencies the predictions do not capture the measured fieldshapes as well as they do at the higher frequencies. The reason for this is that the diffuse field within the reverberation chamber is more likely to produce an equal energy per mode distribution in the duct at higher frequencies than at lower frequencies. For the higher frequencies, where the assumption of equal energy per mode is more valid, the b-induct + GXMunt predictions capture well the shape of the far-field directivity. Any differences are likely due to the fact that the GXMunt radiation model is not an exact representation of the rig geometry. For instance, the outer wall of the
GXMunt model at the duct exhaust is a uniform cylindrical duct, whereas in the rig tests the outer wall at the duct exhaust is flanged (figure 7.14). Also, the GXMunt model has an infinitely long afterbody such that the pressure along the duct axis is undefined, whereas the afterbody of the rig geometry is finite.

![Diagram](image)

(a) GXMunt model

(b) Rig geometry

**Figure 7.14:** Comparison between GXMunt model and rig geometry at the exhaust.
Figure 7.15: Predicted vs. measured far-field directivities for Baseline hardwall configuration.
(i) 3150Hz

(j) 4000Hz

(k) 5000Hz

(l) 6300Hz

(m) 8000Hz

(n) 10000Hz

(o) 12500Hz

(p) 16000Hz

(i) 3150Hz

(j) 4000Hz

(k) 5000Hz

(l) 6300Hz

(m) 8000Hz

(n) 10000Hz

(o) 12500Hz

(p) 16000Hz
Figure 7.16: Predicted vs. measured far-field directivities for Baseline lined configuration.
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(i) 3150Hz
(ii) 4000Hz
(iii) 5000Hz
(iv) 6300Hz
(v) 8000Hz
(vi) 10000Hz
(vii) 12500Hz
(viii) 16000Hz
Figure 7.17: Predicted vs. measured far-field directivities for Highly Curved hardwall configuration.
(i) 3150Hz
(j) 4000Hz
(k) 5000Hz
(l) 6300Hz
(m) 8000Hz
(n) 10000Hz
(o) 12500Hz
(p) 16000Hz
Figure 7.18: Predicted vs. measured far-field directivities for Highly Curved lined configuration.

(a) 500Hz  

(b) 630Hz  

(c) 800Hz  

(d) 1000Hz  

(e) 1250Hz  

(f) 1600Hz  

(g) 2000Hz  

(h) 2500Hz
(i) 3150Hz

(j) 4000Hz

(k) 5000Hz

(l) 6300Hz

(m) 8000Hz

(n) 10000Hz

(o) 12500Hz

(p) 16000Hz
7.3 Measured vs Predicted data using a single azimuthal mode source

7.3.1 Calculation of incident modal amplitudes

Whilst the modal energy distribution for the broadband tests is unknown, data is also available for tests using individual azimuthal mode sources, generated by the mode generator as described in section 6.2.3. For such sources, measurements of pressure are available at each of the 8 microphones in the in-duct radial array (see figure 7.19), and these can be used to reconstruct information about the relative amplitudes of the radial modes present. The way in which this can be done is as follows:

The bypass model domain for these calculations is shown in figure 7.20. The model inlet again starts at the widest part of the centrebody cone, at the point where the duct becomes uniform annular, and 8 field-points for the ACTRAN prediction are located at the positions of the radial microphones. B-induct was used to generate the FE mesh and relevant input files, and an initial ACTRAN prediction was made for each radial mode, $n_j$, incident at the inlet with an amplitude of $1 + 0i$. The predicted pressure $p_{in,j}$ at each radial microphone $i$ due to each incident radial mode $n_j$ was then extracted.

Figure 7.19: In-duct radial microphone array
Assuming that the total pressure at each radial microphone $i$ can be expressed as a sum of the pressures due to each radial mode, an expression for the total calculated pressure at each radial microphone, $P_{ci}^k$, can be written:

$$P_{ci}^k = \alpha_1 p_{in1} + \alpha_2 p_{in2} + \ldots + \alpha_N p_{inN} = \sum_{j=1}^{N} \alpha_j p_{inj}, \quad (7.4)$$

where $N$ is the total number of radial modes cut-on, and $\alpha_j$ gives the amount of each radial mode present.

This expression for the total calculated pressure, $P_{ci}^k$, can then be used along with the value for the total measured pressure, $P_{mi}^i$, at each radial microphone $i$, such that a least squares fitting can be performed (as described in appendix D) in order to calculate the value of $\alpha_j$ for each radial mode and determine the relative amounts of each radial mode present in the rig test.

These values of $\alpha_j$ can then be used as the incident amplitudes for each radial mode in a second ACTRAN prediction, such that total calculated pressures at the radial microphone array for this prediction should be comparable to those measured. Also, for this second ACTRAN prediction the amplitude of each transmitted mode present at the exhaust plane can be calculated and these transmitted modal amplitudes can then be used with the GXMunt radiation model in order to predict the overall far-field directivity pattern to compare with the measured data.

### 7.3.2 Results

The particular azimuthal modes chosen to examine must be as pure as possible, i.e. free from contamination by other azimuthal modes. In order to test for purity, measured data from the in-duct circumferential microphone array can be examined. Within the rig tests, the microphones in the circumferential array were only calibrated relative to
each other, rather than to an external source level, so although it is not possible to know the absolute levels of the individual azimuthal modes it is still possible to observe their levels relative to each other. In doing this, it was clear that not all of the test-points for the individual mode tests were completely free from contamination. Thus, to ensure that the intended azimuthal mode was as pure as possible, the selection criteria of at least 10dB protrusion above all other modes was set. In this way, two particular azimuthal modes were selected for examination: a mode with multiple radial orders cut-on: $m = 10$ at 6300Hz with radial orders $n = 1, 2, 3$, and a mode with only a single radial order cut-on: $m = 12$ at 4000Hz with radial order $n = 1$. Results are presented here for the Baseline geometry in the hardwall configuration, for which plots of the relative azimuthal mode amplitudes recorded at the circumferential array are shown in figure 7.21(a) for the $m = 10$ mode source at 6300Hz and figure 7.21(b) for the $m = 12$ mode source at 4000Hz.

7.3.2.1 6300Hz, $m = 10$

For the $m = 10$ azimuthal mode at 6300Hz there are 3 radial modes present. Using the technique described above the amplitude of each of these modes at the inlet plane was calculated. A plot of the equivalent intensity of each of these modes is shown in figure 7.22, and clearly indicates the dominance of the first radial mode. ACTRAN predictions of the pressure at each radial microphone were then made using this new modal energy distribution. The prediction for pressure at the radial microphone array due to each individual radial mode is shown in figure 7.23, and the prediction for the total pressure due to all radial modes is compared to the measured data in figure 7.24, where the measured pressures are all normalised relative to a pressure of $1 + 0i$ at the outer wall microphone.
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Figure 7.22: Calculated incident radial mode intensities for \( m = 10 \) at 6300Hz.

![Figure 7.22](image)

(a) Real part of pressure at each radial microphone
(b) Imaginary part of pressure at each radial microphone

Figure 7.23: Predicted pressures at the in-duct radial microphone array due to each radial mode, for \( m = 10 \) at 6300Hz.

![Figure 7.23](image)

(a) Real part of pressure at each radial microphone
(b) Imaginary part of pressure at each radial microphone

Figure 7.24: Measured vs fitted total pressures at the in-duct radial microphone array for \( m = 10 \) mode at 6300Hz.

![Figure 7.24](image)

(a) Real part of pressure at each radial microphone
(b) Imaginary part of pressure at each radial microphone
Using the ACTRAN predictions for the amplitude of each radial mode at the exhaust, a prediction of the far-field directivity was made using the GXMunt radiation model, where the far-field modal pressures were summed coherently. This prediction is compared to the measured directivity in figure 7.25.

As can be seen from figure 7.24, the predictions of the pressures at each radial microphone seem to agree well with the measured data. For the far-field directivity (figure 7.25) the predictions capture the general fieldshape, especially the peak radiation angle, quite well. At angles where there is a difference in level between the predicted and measured SPL it is possible that there is some contamination due to other azimuthal modes within the measured data. For instance, the predicted SPL at 0° is extremely low, as would be expected for a pure $m = 10$ mode on-axis, whereas the large SPL measured on-axis, and at low angles, suggests the presence of the plane wave and low order modes within the tests. The data from the circumferential microphone array in figure 7.21(a) shows that the amplitude of the $m = 10$ source mode is roughly 10 times the amplitude of the $m = 0$ mode, which would provide a level of contamination on-axis 20dB lower than the $m = 10$ peak. Another possible source of contamination is the scattering of the $m = 10$ mode caused by the presence of the support bolts within the duct.
7.3.2.2 4000Hz, $m = 12$

For the $m = 12$ azimuthal mode at 4000Hz there is only 1 radial mode cut-on. The incident amplitude of this mode at the inlet plane was found to be $0.26 + 0.31i$. ACTRAN predictions of the pressure at the radial microphone array using this incident amplitude are shown in figure 7.26 along with the measured pressures.

In the same way as before, a prediction of the far-field directivity was made using the GXMunt radiation model and the ACTRAN predicted modal amplitude at the exhaust plane. This prediction is compared to the measured directivity in figure 7.27.

**Figure 7.26**: Measured vs fitted total pressures at the in-duct radial microphone array for $m = 12$ mode at 4000Hz.

**Figure 7.27**: Measured vs predicted total far-field directivity for $m = 12$ mode at 4000Hz. Baseline hardwall case.
The predictions of pressure at the radial microphone array for the \( m = 12 \) mode at 4000Hz (figure 7.26) do not agree as well with the measured pressures as the predictions for the \( m = 10 \) mode at 6300Hz. This is most likely because it is easier to fit \( P_i^c \) to \( P_i^m \) when there are three parameters that can be varied (\( \alpha_1, \alpha_2 \) and \( \alpha_3 \)) than when there is only one parameter that can be varied (\( \alpha_1 \)). Furthermore, the existence of contaminant modes in the measured data would make it impossible to achieve a perfect fit using only a single mode within the predictions.

For the far-field predictions, (figure 7.27), the general shape of the measured data is captured quite well, although the peak radiation lobe for the measured data occurs at a higher angle than the predicted peak. The ray theory prediction of the propagation angle for the \( m = 12, n = 1 \) mode is approximately 53\(^\circ\), which is in closer agreement with the ACTRAN + GMUmt predicted peak than with the measured peak. Therefore it is possible that there are some other high order modes present in the measured data such that the measured peak is shifted to a higher angle.

The lack of agreement between the predicted and measured SPLs at low angles is once again most likely due to the presence of lower order modes such as the plane wave mode in the measured data. Because the \( m = 12 \) mode is not well cut-on, it has a high propagation angle and therefore the effects of contaminant low order modes are significant for low angle measurements. On examination of the relative modal amplitude measurements in figure 7.21(b) it is clear that the \( m = 0 \) mode alone does not account for this level of contamination at low angles. Indeed, even a combination of several low order modes seems unlikely to have caused this much contamination given the relatively high amplitude of the \( m = 12 \) mode at source. This again implies the existence of an additional source of contamination, such as scattering due to the support bolts within the duct.

### 7.4 Summary

For the axisymmetric geometries, both the predicted and the measured data indicate that the Highly Curved geometry provides a similar attenuation to the Baseline geometry despite having only approximately two-thirds of the liner area. This result indicates that weight and drag benefits for the engine can be achieved by using a shorter, more highly curved bypass, with no significant detriment in noise. This is a promising result and, as such, further studies are to be made for this geometry under more realistic conditions: within a modular fan rig, with flow.

The importance of narrowband data has been demonstrated by comparing the attenuation spectra against 1/3 octave band data, which does not sufficiently capture the location and shape of the attenuation peak. Whilst the shapes of the measured and predicted attenuation curves are very similar there are significant differences in level.
Possible causes for the discrepancy are an incorrect assumption of modal energy distribution, or an inappropriate liner impedance used in the calculation. For the studies presented here, liner insertion loss predictions appear to be more sensitive to the latter than the former. Analysis of the distribution of energy amongst transmitted modes has highlighted an interesting observation of the apparent attenuation of lower order modes over higher order modes. The physical reason for this effect is unclear, and further analysis is required for it to be fully understood.

Far-field SPL predictions using b-induct + GXMunt have been compared to measured fieldshapes. For multimodal sources, in general the predictions capture the fieldshapes quite well, although some differences are observed probably because the GXMunt model is not an exact representation of the rig geometry. For single azimuthal mode sources, measured pressures at an in-duct radial microphone array have been used to calculate the amplitudes of the incident radial modes. Far-field predictions of SPL made using these incident radial mode amplitudes again capture the shape of the measured data quite well, although some differences are observed possibly due to the presence of other azimuthal mode orders within the measured data. For future experiments, it would be useful to refine the mode generation technique in order to ensure the production of pure azimuthal modes with minimal contamination. It would also be beneficial to obtain absolute values of SPL for each of the modes generated, by calibrating the circumferential microphones against an external reference source. The suggested scattering due to the support bolts could also be investigated using a 3D model.
Chapter 8

Geometry Effects - 3D

8.1 Introduction

Whilst an axisymmetric computational model which uses only a 2D domain (in the $r-z$ plane) can be used to make predictions for an axisymmetric geometry, in reality aircraft engines lack axial symmetry. For example, within the intake, axial liner splices may be present due to the use of separate panels around the circumference of the inlet, and scattering between azimuthal modes will occur at these circumferential discontinuities. Similarly, the presence of hardware within the bypass duct, such as the Thrust Reverser Unit (TRU), bleed valves, bifurcations and the pylon, means that in practice the bypass duct is truly three-dimensional. These features scatter the incident sound field into a wide range of radial and azimuthal modes. It is therefore necessary to use 3D models to evaluate the effects of such non-axisymmetric features within the engine.

The usefulness of 3D models is evidenced in the assessment of the scattering caused by axial liner splices within the inlet [111], and the acoustic benefits of zero splice technology [112, 113]. 3D models have also been used for bypass duct configurations which lack axial symmetry. Sugimoto [76] has previously used 3D b-induct to examine modal scattering by a rectangular hardwall patch, representing a liner interruption due to the installation of hardware, in a uniform annular lined duct. In the same paper, the attenuation due to lined radial splitters in a uniform annular duct is also reported. Chen [50] has also used 3D numerical simulations to examine a generic engine bypass geometry with four bifurcations within the duct, and observed the way in which a circumferentially rotating mode is diffracted by the bifurcations, setting up a standing wave interference pattern in the downstream region.

Two significant geometric features within the bypass duct are the pylon and lower bifurcation (figure 8.1), and it is important to understand how these features influence noise propagation within the duct. The inclusion of a pylon or bifurcation necessarily results
in the reduction of liner area available on the bypass walls. However, if the bifurcation itself can be lined this loss can be offset, and in some cases the total liner area can even be increased. Despite this, the angle of incidence of the sound waves on the bifurcation and on the bypass duct walls will not be the same, and it has been suggested that lining the bifurcation is less effective than lining the bypass wall [76]. Furthermore, bifurcations can result in the scattering of incident sound into modes which may be attenuated to a greater or lesser extent. Given the complexity of these effects and their potential interactions, it is important that a 3D propagation model adequately captures all the relevant factors.

In addition to the influence of these structures on the in-duct pressure field, since the pylon extends beyond the bypass exhaust, there is also a lack of axial symmetry in the external domain, and it is necessary to also consider the effect of this extension on the radiated directivity pattern. Recent experiments have been performed for realistic 3D exhaust geometries, incorporating the pylon, as part of the European TURNEX project [92, 93]. Azimuthal mode analysis performed in the far-field using a large circular microphone array showed that the installation effect of the pylon was to introduce an asymmetry to the modal content in the far-field. Similarly, Redonnet [36, 114] has carried out extensive numerical simulations for a realistic exhaust geometry with flow, and has also observed a highly asymmetric directivity pattern. In particular, it is seen that as well as the effect of the pylon on the in-duct sound field, once the sound exits the duct the pressure pattern continues to be modified by the existence of the pylon, such that the pattern downstream of the exhaust is very different to that at the bypass exit.
Modelling of 3D problems requires large computational memory and processing power. Additionally, creating a mesh model for numerical analysis of a complex 3D geometry can be time consuming. The advantage of a shell code such as 3D b-induct is the automatic pre- and post-processing capability it provides. This feature reduces the time required to build input files and extract results, thereby increasing the overall efficiency of the prediction process.

Within this chapter, predictions made using the recently developed 3D b-induct code are presented, and compared with measured data from the rig tests described in chapter 6. These results are presented for two non-axisymmetric geometries, using both multimodal and single mode sources.

### 8.2 Selected Geometries

As part of an investigation into the effect of duct geometry on noise propagation, a set of rig tests were performed in the ISVR no-flow chamber. Two bypass duct geometries of interest: a typical “Baseline” geometry, and a novel “Highly Curved” (or “HC”) design, were manufactured on a 1 : 4.2 (rig : full) scale. The axisymmetric lined datum configuration for each of these geometries is shown in figure 8.2 where the liner area within the HC duct is approximately 2/3 the liner area within the Baseline duct. In order to examine the effect of changes to an axisymmetric duct, measured data from these axisymmetric builds was compared to b-induct predictions in chapter 7.

As well as testing axisymmetric duct configurations, various 3D builds were also tested (the full test matrix for these builds is given in appendix B). For this investigation, the builds selected for examination are the Baseline axisymmetric lined datum configuration with a pylon incorporated in the duct at 0° and a lower bifurcation at 180° (figure 8.3), and the Highly Curved axisymmetric lined datum configuration with a single pylon at 0° (figure 8.4). For simplicity in what follows, the term “bifurcation” shall be used when referring to pylons as well as to lower bifurcations. Within each geometry, both hardwall
and lined bifurcations were tested, where the liner used on the bifurcations was the same as the liner used on the duct walls. This liner consists of a 6mm deep honeycomb cell structure with a wire mesh facing sheet, whose zero-flow, non-dimensional acoustic resistance and mass inertance were measured as $R = 1.2$ and $m = 0.003$ respectively.

As described in chapter 6, the basic test rig geometry consists of a uniform annular duct, onto which a tapered rig section was added to achieve the correct hub-to-tip ratio. The specific rig bypass geometries under test were then attached to this base platform. For each geometry the domain for the computational model therefore consists of both the basic test rig section, and the bypass geometry under examination. The FE mesh generated using 3D b-induct for the Baseline geometry is shown in figure 8.5, and the mesh generated for the Highly Curved geometry is shown in figure 8.6. In each case the bypass duct domain has been extended by a straight annular duct section in order to include the entire axial extent of the bifurcation within the FE domain. This ensures a complete hardwall annulus at the exit plane of the model, which is necessary for the application of modal boundary conditions within ACTRAN. The lack of influence of the pylon in the final section of the model means that the modal distribution at the trailing
edge of the pylon will continue to propagate unperturbed in this section, and this will be reflected in the resultant pressure field. However, as the axial extent of the imposed annular duct section is very small, it is assumed that the forcing of an annular modal structure at the exhaust will not significantly influence the prediction.

Figure 8.5: FE mesh generated by 3D b-induct for the Baseline geometry with two bifurcations at 0° and 180°. Model domain consists of basic test rig section + bypass geometry.

Figure 8.6: FE mesh generated by 3D b-induct for the Highly Curved geometry with a single bifurcation at 0°. Model domain consists of basic test rig section + bypass geometry.
8.3 In-duct results

8.3.1 Multimode source

Values for the transmitted sound power level at the duct exhaust, \( (PW L_{tr}) \) were calculated for the axisymmetric lined datum with no bifurcations (“NoBF”), the lined datum with hardwall bifurcations, (“HWBF”), and for the lined datum with lined bifurcations, (“LBF”), for each geometry, using a multimode source. For each geometry, by taking the difference between \( PW L_{tr} \) for the lined datum with no bifurcations (NoBF) and the lined datum with hardwall bifurcations (HWBF), we obtain a value for the sound power level transmission loss due to the insertion of the hardwall bifurcations:

\[
\Delta PW L_{HWBF}^{NoBF} = PW L_{tr, \text{lined datum}(NoBF)} - PW L_{tr, \text{lined datum}(HWBF)}.
\] (8.1)

Similarly, by taking the difference between \( PW L_{tr} \) for the lined datum with no bifurcations (NoBF) and the lined datum with lined bifurcations (LBF), we obtain a value for the sound power level transmission loss due to the insertion of the lined bifurcations:

\[
\Delta PW L_{LBF}^{NoBF} = PW L_{tr, \text{lined datum}(NoBF)} - PW L_{tr, \text{lined datum}(LBF)}.
\] (8.2)

In order to obtain the same Deltas using the experimental data, it is necessary to calculate values of \( PW L_{tr} \) from the measured SPLs, which involves performing a surface integration as described in appendix E. For this calculation, it would be ideal to have far-field SPL measurements from microphones located at all points over a hemispherical surface, by rotating the polar array through 360° of azimuthal angle. However, due to practical constraints this was not possible, and measurements were only taken at a limited number of azimuthal angles.

For the Baseline geometry, measurements were taken with the two bifurcations in two orthogonal planes (0°-180° and 90°-270°), and for the Highly Curved geometry, with the single bifurcation in three different orientations (90°, 180° and 270°). (This simulates rotating the polar microphone array, see figure 8.7). In both cases some variation in SPL directivity with azimuthal angle was seen, indicating a lack of azimuthal symmetry. However, without further measurements available to obtain a more accurate value of \( PW L_{tr} \), approximations of \( PW L_{tr} \) were calculated by using the average of the SPLs obtained from the measurements at each orientation.

These values of \( PW L_{tr} \) were then used to determine the measured Deltas due to the insertion of the bifurcations, as described in equations (8.1) and (8.2). Figure 8.8(a)
Figure 8.7: For Baseline geometry, bifurcations positioned at 0° and 180° corresponds to polar microphone array in orientation (1) (corresponding to azimuthal angle 90°) and bifurcations positioned at 90° and 270° corresponds to polar microphone array in orientation (2) (corresponding to azimuthal angle 0°).

shows a plot of these measured Deltas for the Baseline geometry together with the 3D b-induct predictions, and figure 8.9(a) for the Highly Curved geometry\(^1\).

As a second comparison of predictions with the measured data, by taking the difference between \(PW_{L_{tr}}\) for the lined datum with hardwall bifurcations (HWBF) and the lined datum with lined bifurcations (LBF), we obtain a value for the reduction in sound power level due to the insertion of the liner on the hardwall bifurcations:

\[
\frac{HWBF}{LBF} \Delta PWL = PW_{L_{tr}, \text{lined datum (HWBF)}} - PW_{L_{tr}, \text{lined datum (LBF)}}. \quad (8.3)
\]

This can also be written as:

\[
\frac{HWBF}{LBF} \Delta PWL = \frac{NaBF}{LBF} \Delta PWL - \frac{NaBF}{HWBF} \Delta PWL. \quad (8.4)
\]

Figure 8.8(b) is a plot of this insertion loss achieved by lining the two bifurcations in the Baseline geometry, and figure 8.9(b) is a plot of the insertion loss achieved by lining the single bifurcation in the HC geometry.

As can be seen from figures 8.8(a) and 8.9(a), the 3D b-induct prediction gives reasonably good agreement with the measured data for the insertion loss due to the bifurcation(s), although at two frequencies there are some deviations where the prediction displays a

\(^1\)Due to the computational cost of the 3D ACTRAN calculations, predictions were only performed up to a frequency of 6300Hz.
somewhat oscillatory behaviour. The reason for these deviations is not immediately obvious. To determine whether the discrepancies seen at 3150Hz and 5kHz are specific to unique frequencies or are indicative of deviations over a broader frequency range, further predictions could be made at interim frequencies (using a smaller frequency step) to better resolve the shape of the predicted spectra.

Also, as discussed in the previous chapter, because the 1/3 octave band measured data are obtained by integrating the signal over the 1/3 octave band width, whilst the predictions are made at discrete 1/3 octave centre frequencies, the quantities compared are not the same. As seen in chapter 7 it is possible that the exact location and shape of the peak is not adequately captured using 1/3 octave band measured data. Thus for future assessment, the measured data could be re-processed to give narrowband spectra, which would provide a more accurate comparison between the predicted and measured data.

It is useful to note, that despite the discrepancies observed in the plots of insertion loss
(a) Delta PWL due to insertion of hardwall ($\Delta PWL_{HWBF}$) and lined ($\Delta PWL_{LBF}$) bifurcation in Highly Curved lined datum.

(b) Delta PWL due to application of liner to bifurcation ($\Delta PWL_{HWBF}$) in Highly Curved duct.

**Figure 8.9:** Effect of bifurcation on power transmission loss within Highly Curved duct.

due to the bifurcation(s), figures 8.8(b) and 8.9(b) show that the b-induct predictions of the attenuation due to the liner on the bifurcations are in good agreement with the measurements, where both the shape and level of the measured data are captured well.

Overall, insertion losses due to the bifurcations in the Baseline geometry are seen to be larger than those for the Highly Curved geometry (8.8(a) vs. 8.9(a)), with a maximum insertion loss due to the lined bifurcations of around 1dB for the Baseline duct and around 0.5dB for the Highly Curved duct. This difference might be expected due to the fact that there are two bifurcations in the Baseline case versus one bifurcation in the Highly Curved case. The larger insertion loss for the hardwall bifurcations (shown in blue) in the Baseline is due to the increased scattering effect of the second bifurcation, and the larger insertion loss for the lined bifurcations (shown in pink) is also due to the extra area of liner associated with the second bifurcation. Figures 8.8(b) and 8.9(b) show more clearly this extra attenuation due to the liner on the bifurcations.
8.3.2 Single mode source

In order to observe in more detail the scattering due to bifurcations within the duct, it is useful to examine predictions using a single mode source. In this example a single high order mode is chosen with azimuthal order, \( m = 9 \) and radial order, \( n = 1 \) at a frequency of 3150Hz. This mode is incident with unit intensity at the duct inlet.

Figure 8.10(a) shows the predicted transmitted modal intensities at the exhaust plane for the Baseline lined datum, and figure 8.10(b) for the case where two hardwall bifurcations are present. For the lined datum, the \( m9n1 \) mode is present at the exhaust with an intensity of approximately 0.04W/m\(^2\), whereas the presence of the two hardwall bifurcations causes scattering of this mode into azimuthal orders with \( m + 2j \) (where \( j \) is an integer), such that the resultant intensity of the transmitted \( m9n1 \) mode is reduced by about 50% to just below 0.02W/m\(^2\).

Figure 8.11(a) shows the transmitted modal intensities at the exhaust plane for the Highly Curved lined datum, and figure 8.11(b) for the case where a single hardwall bifurcation is present, for the same mode (\( m9n1 \)) incident with unit intensity at the duct inlet. [Note that the y-axis scale of these plots is double that of figure 8.10.] In this case, for the lined datum the \( m9n1 \) mode is present at the exhaust with an intensity of approximately 0.16W/m\(^2\) and has not been attenuated by the shorter liner in the Highly Curved duct as much as the same mode has been by the longer liner in the Baseline geometry. The presence of the single hardwall bifurcation causes scattering of this mode into azimuthal orders with \( m + 1j \), such that the resultant intensity of the \( m9n1 \) mode is reduced by approximately 25% to just above 0.12W/m\(^2\). It could be reasoned that the percentage of the transmitted modal intensity in the axisymmetric lined case that is scattered into other azimuthal modes is proportional to the number of bifurcations within the duct, as with 2 bifurcations the \( m9n1 \) mode is reduced by 50% whereas with 1 bifurcation it is only reduced by 25%. However, more data would need to be examined to conclude whether this effect is the same at all frequencies or for all modes.
(a) Transmitted modal intensities at the exhaust plane for a single incident mode, \( m9n1 \) at 3150Hz for Highly Curved lined datum.

(b) Transmitted modal intensities at the exhaust plane for a single incident mode, \( m9n1 \) at 3150Hz for Highly Curved lined datum with a single hardwall bifurcation.

**Figure 8.11:** Modal intensities at the exhaust plane for Highly Curved duct. [Note that the y-axis scale of these plots is double that of figure 8.10.]

Whilst figures 8.10(b) and 8.11(b) show that the bifurcations can scatter the incident \( m = 9 \) mode into modes with different azimuthal orders, the presence of negative order modes, rotating in the opposite direction from the incident positive \( m \) number mode, indicates that a circumferential reflection also takes place at the bifurcations. Figure 8.12 shows the in-duct pressure contours for the same incident \( m9n1 \) mode at 3150Hz within the Highly Curved geometry with a hardwall bifurcation, and the reflective effect of the bifurcation is also clearly seen here: The incident \( m = 9 \) mode is visualised, rotating in an anticlockwise direction (when looking down the inlet), and is reflected by the bifurcation, generating a counter-rotating \( m = -9 \) mode, which in turn interferes with the initial incident mode to create a standing wave pattern ahead of the bifurcation. A shadow region of low pressure is visible behind the bifurcation where the incident mode cannot pass.

**Figure 8.12:** Sound pressure contours within the Highly Curved duct with hardwall bifurcation, for a single incident \( m9n1 \) mode at 3150Hz. (Mode rotation anticlockwise when viewed from inlet).
8.4 Far-field results

8.4.1 Single mode source

For the same single incident mode ($m9n1$ at 3150Hz with unit intensity), modal pressure amplitudes at the exhaust, predicted by 3D b-induct, were used in conjunction with modal far-field directivity patterns predicted by GXMunt. In this way, resultant far-field directivity patterns for the pressure magnitude, $|p|$, on the surface of a hemisphere with the same radius as the test microphone array, (3.95m) were produced for each of the different builds. These plots are shown in figure 8.13 for the Baseline geometry and figure 8.14 for the Highly Curved geometry. For the Highly Curved geometry, the sharp peak in the directivity plots at $27^\circ$ polar angle is, as before, non-physical and due to a known error in the GXMunt MATLAB calculation, which is to be corrected in the future development of this code.

In each of figures 8.13 and 8.14, the mode is spinning clockwise when looking from the exhaust to the inlet, and the scattering by the bifurcations in each geometry can be seen to affect the far-field directivity pattern. For the Baseline geometry the mode is reflected by the bifurcations at $0^\circ$ and $180^\circ$, and standing wave patterns are consequently seen in the regions $0^\circ$-$30^\circ$ and $180^\circ$-$210^\circ$. For the Highly Curved geometry a standing wave pattern is evident in the $0^\circ$-$30^\circ$ region due to reflection from the single bifurcation at $0^\circ$. The effect of lining the bifurcations is seen to reduce the amplitude of the pressure directivity in some regions and to increase the amplitude in other regions, although, as seen from the previous in-duct calculations (figures 8.8 and 8.9) the overall effect of lining the bifurcations is to reduce the sound power transmitted.

8.4.1.1 Comparing predictions with measured data

In order to compare the agreement of these b-induct + GXMunt predictions with the measured data for the 3D builds, it is necessary to review the available measured data: For the Baseline build, measurements were taken with the bifurcations within the duct in the $0^\circ$ and $180^\circ$ orientation and also in the $90^\circ$ and $270^\circ$ orientation, with the location of the polar microphone array kept constant. If the bifurcations had instead been kept fixed in the same orientation within the duct, and the polar array moved circumferentially around the duct, this would correspond to measurements taken with the polar array in the $90^\circ$ azimuthal orientation (shown as (1) in figure 8.15) and also in the $0^\circ$ azimuthal orientation (shown as (2) in figure 8.15).

From the b-induct + GXMunt far-field results for the Baseline geometry, predictions of SPL around $360^\circ$ of azimuthal angle have been extracted at polar angles of $30^\circ$, $60^\circ$ and $90^\circ$. This was done for both the axisymmetric lined build (figure 8.13(a)), and the lined build with two hardwall bifurcations (figure 8.13(b)).
(a) Lined datum  
(b) with two hardwall bifurcations  
(c) with two lined bifurcations.

Figure 8.13: Plots of $|p|$ in the far-field at a radius of 3.95m, for Baseline geometry with a single incident mode of unit intensity: $m9n1$ at 3150Hz. View into exhaust.

By taking the difference between these two sets of azimuthal SPLs, we obtain a $\Delta SPL$ that gives the SPL increase due to the insertion of the hardwall pylons within the axisymmetric lined duct. Plots of this predicted $\Delta SPL$ are given in figure 8.16 for the three different polar angles, and the corresponding measured data points are also shown on these plots. Due to the axisymmetry of the builds, the measured data from the polar array oriented at $0^\circ$ azimuthal angle has also been plotted at $180^\circ$, and similarly the measured data from the polar array at $90^\circ$ has also been plotted at $270^\circ$.

From figure 8.16 we can see that the predicted delta is highest at the lowest polar angle of $30^\circ$. This is because at small angles the prediction for the axisymmetric case gives very low pressure for a pure $m = 9$ mode, which radiates at a higher angle. (For a pure $m = 9$ mode, the predicted pressure on axis would be zero and thus the predicted SPL on-axis would tend to minus infinity). Because the predicted SPL at small angles for the axisymmetric case is low, when compared to the predicted SPL for the build with
bifurcations, the $\Delta SPL$ is predicted to be large. The reason that this large $\Delta SPL$ is not observed in the measured data is possibly due to contamination from lower order modes, such that the $m = 9$ mode source is not perfectly pure, resulting in a non-zero on-axis pressure for the axisymmetric case.

At the higher angles of $60^\circ$ and $90^\circ$, there is better agreement between the overall level of the measured and predicted $\Delta SPLs$ for the small number of measured data points available. Future work could involve the examination of lower order modes, (for example $m = 2$ or 3), such that the far-field standing wave pattern observed would only have 2 or 3 peaks and troughs, allowing for easier comparison of predictions with this small number of measured data points. Better still, measurements could be taken at an increased number of azimuthal angles, for instance every $5^\circ$ of azimuthal arc, such that predicted results could be plotted against a larger number of measurement points.
Figure 8.15: Measured data recorded with polar microphone array in two different orientations: (1) corresponding to azimuthal angle 90° and (2) corresponding to azimuthal angle 0°.

Figure 8.16: Azimuthal variation of $\Delta SPL$ due to insertion of hardwall pylons in axisymmetric lined duct. For polar angles of 30°, 60° and 90°.
8.5 Summary

It has been demonstrated that bifurcations have a scattering effect that is visible both in the noise propagation within the duct and also in the far-field radiation. For each geometry, the incorporation of lined bifurcations has been shown to provide an increase in PWL attenuation compared to the axisymmetric lined build. In the cases examined here the addition of two lined bifurcations within the Baseline provides a greater attenuation than the addition of one lined bifurcation within the Highly Curved, with a maximum of around 1dB for the Baseline duct and around 0.5dB for the Highly Curved duct. However, from a holistic point of view the acoustic benefits of including extra lined bifurcations must obviously be weighed up against the associated weight and drag penalties.

For a single incident azimuthal mode source, within the duct reflection by the bifurcations causes the generation of a counter-rotating mode such that a standing wave pattern is set-up in the region ahead of the bifurcation, and a shadow region is observed behind the bifurcation. This standing wave pattern is also propagated into the far-field, where predictions of the azimuthal variation of the increase in SPL due to the insertion of the hardwall pylons within the axisymmetric lined duct agree more closely with the overall level of the measured data at higher polar angles rather than lower polar angles. This is possibly due to the higher degree of contamination of the source with azimuthal modes of lower order, at lower polar angles. In order to make accurate comparisons of the predicted and measured azimuthal fieldshapes it would be useful to obtain measurements at an increased number of azimuthal angles.
Chapter 9

Liner Impedance Optimisation

9.1 Introduction

Having discussed the effect of geometry on noise propagation within the bypass duct in the previous chapters, it is also important to consider the effect of the acoustic liner configuration. In order to maximise the reduction of overall aircraft noise, the choice of liner impedance is an important aspect of the engine design process. The liner configuration selected must maximise the attenuation over a wide range of frequencies.

In order to achieve the maximum benefit over all relevant engine conditions, multiple liners with different impedances are necessary. The positioning of these liners can also be used to improve noise reduction. For example, judicious placement of liners with the appropriate impedances can result in scattering of lower order modes into higher order modes which are more readily attenuated by downstream liners. Scattering between azimuthal modes will occur at circumferential liner discontinuities, whereas scattering between radial modes will occur at axial liner discontinuities. Given the computational cost of predictions for fully 3D models, usually impedance optimisations are only performed for ducts with axially varying liners.

The selection of the most appropriate liner configuration requires a suitable method for predicting the attenuation resulting from any given arrangement. Various analytical and numerical methods are available for this purpose. The most common analytical methods used for optimisation involve mode-matching schemes, where acoustic pressure and axial particle velocity are matched at the liner interfaces for uniform sections. Such analytical techniques are computationally inexpensive and hence a large number of design variables can be considered in an optimisation. For instance, the impedances and lengths of a set of liners can be varied [13, 115], or the order of set of liners with given impedances can be varied [12]. Whilst such analytical techniques are advantageous in terms of computation time, they are limited to highly idealised geometries.
Numerical methods, in contrast, can be computationally expensive and often require parallel computing capability, but they do have the advantage of being able to deal with realistic geometries. In such instances it is often the case that positions of liners are dictated by other engineering design considerations, in which case liner position is no longer a variable that can be manipulated. Hence many numerical approaches consider a set of liners with predetermined positions, and optimise impedance parameters only [116, 117, 118].

Whichever prediction methodology is selected, it is necessary to define a “cost function” which specifies exactly what quantity is to be optimised. For example, a liner configuration that maximises sound power transmission loss within the bypass duct may not be the best choice for minimising total perceived aircraft noise, since frequencies to which the human ear is more sensitive may not be adequately addressed. Having chosen an appropriate cost function it must be evaluated across a range of design variable combinations, to identify the specific configuration which yields the optimal value. In order to do this the prediction process must be integrated with an optimisation methodology, the selection of which requires several important considerations: the method must adequately sample the design space; it must reliably find the global optimum in the presence of local optima; and of particular relevance for the design of new engines within industry, it must be computationally efficient.

There are several software packages in existence which have previously been used for cost function optimisation [115, 118, 116]. Whilst these packages have the advantage of being able to automatically sample a wide range of design space, it is important to note that the automatic selection of an optimum without any “engineering judgement” should be used with caution.

For the present work an optimisation methodology that was currently in use within industry was chosen, and axisymmetric b-induct predictions were incorporated into this pre-existing framework. The use of b-induct allows the effect of the bypass geometry and liner positions to be considered within the optimisation, and the industry optimisation method was selected because it has several benefits. Firstly, the methodology incorporates a whole engine prediction model and allows for the calculation of the Effective Perceived Noise Level (EPNL) for the entire aircraft. Since the ultimate objective is to design low noise aircraft, EPNL is arguably the most appropriate cost function to use. Secondly, the final selection of the optimum configuration involves consideration of three different certification conditions (Approach, Cutback and Sideline) and hence requires a reasonable amount of “engineering judgement”. This ensures a robust design which provides maximum benefit across the range of operating conditions. Finally, the optimisation process is sufficiently fast that the selection of impedances for an entire bypass configuration can be made in an industrially acceptable timescale (typically 1-2 weeks for a full-scale engine). This optimisation process is described in detail in the following sections.
9.2 Optimisation of bypass liner impedances using b-induct

Figure 9.1 shows a given bypass geometry with a set of liners, labelled 1-4, whose positions are pre-determined based on other design constraints. In order to select the appropriate impedance configuration that provides the maximum reduction of aircraft noise, the impedance of each liner is optimised individually and in turn.

In order to optimise the impedance of a single liner, b-induct is used to make predictions of the attenuation due to the whole bypass liner configuration at different values of the impedance of the liner in question, and in this way generate a set of attenuation tables (described in section 9.2.1). These b-induct attenuation tables are then used in an optimisation process (described in section 9.2.2), from which an optimum impedance for the liner is selected. This whole process can then be repeated for each liner in turn until optimum impedances for all of the bypass liners have been chosen.

9.2.1 Generation of b-induct attenuation tables

For a bypass configuration as shown in figure 9.1, the “Baseline” or starting point impedance of each liner \( j \) can be written as \( Z_{j\text{ initial}} \), and the objective is to optimise the impedance of each of these liners in turn. Starting with the impedance optimisation for liner 1, the impedances of each of the other liners are held at their initial values \( Z_{j\text{ initial}} \), and the impedance of liner 1 is set to a value \( Z_1 = R_1 + iX_1 \). A b-induct prediction can then be made at each 1/3 octave centre frequency to obtain the attenuation due to this particular liner configuration. If the values of \( R \) and \( X \) for liner 1 are varied systematically, whilst the other liner impedances are held constant, and a b-induct prediction for the attenuation is made each time, it is possible to generate a set of tables (see figure 9.2) that show how (at each 1/3 octave frequency) the attenuation for the whole bypass configuration varies with the impedance of liner 1.

![Figure 9.1: Bypass geometry with 4 liners whose positions have been fixed.](image)
Figure 9.2: Generation of b-induct attenuation tables for a given frequency by varying impedance of a single liner, $Z^1$, within a given bypass configuration, whilst the impedances of all other liners ($i = 2 - 4$) are kept constant at their initial values, $Z_{initial}^i$.

Figure 9.3: Contour plot representation of b-induct attenuation tables. Contours show how total bypass attenuation ($\Delta PWL$) varies with $R$ and $X$ of liner 1. Frequency: 1kHz, inlet Mach number: 0.3.

An illustration of such a b-induct table is given in figure 9.3 which is represented as a contour plot with simple linear interpolation to show how, at a frequency of 1kHz and an in-duct Mach number of 0.3, the total bypass attenuation varies with $R$ and $X$ of liner 1. For this case the plot suggests that maximum bypass attenuation at this frequency could be achieved by setting the resistance and reactance of liner 1 to $R = 1$ and $X = -1$ respectively.
Having generated a set of b-induct attenuation tables in this way, they can then be used as reference tables to optimise the impedance of liner 1 with respect to whole aircraft EPNL. This optimisation procedure is described in the following section.

### 9.2.2 Whole engine optimisation process

Whilst it is possible to optimise a liner in terms of its resistance, $R$ and reactance, $X$, at each frequency, in order to provide the required design specifications to a liner supplier it is necessary to know how to achieve the optimal impedances of the liner in terms of its physical properties. To do this it is necessary to use an impedance model to relate the physical properties of a liner to its impedance ($Z = R + iX$). The industry optimisation process used here has several in-built impedance models and the user can select an appropriate model based on the required construction of the liner (for example, whether a wire mesh or perforate facing sheet is to be used). For the optimisations presented here, the construction considered is a single layer liner consisting of a honeycomb cell structure of a given depth with a perforate facing sheet of a given porosity, or “percentage open area” (figure 9.4).

Figure 9.5 gives a schematic overview of the optimisation process: Once the liner type (and hence impedance model) has been selected, for any value of cell depth and of facing sheet porosity the impedance of the liner can be calculated at each frequency (step 1). The noise attenuation achieved by the given duct configuration with the liner set to this impedance is then extracted from the b-induct tables previously generated (step 2), and this attenuation is then applied within a whole engine prediction model to obtain a value for the aircraft EPNL (step 3). By repeating this calculation for many combinations of depth and porosity, a contour plot that shows how EPNL varies with respect to liner depth and porosity is obtained (step 4).
There are numerous complexities associated with each step of the optimisation process. In step 1, the specific construction of liner needs to be considered since the physical properties of the liner will determine its’ impedance. For instance, a single degree of freedom liner, consisting of one layer of honeycomb and facing sheet, will produce a large attenuation over a narrow frequency range, whereas a double degree of freedom liner, consisting of two layers of honeycomb, separated by a porous “septum” sheet, will improve the bandwidth but reduce the peak attenuation, and will increase both cost and weight. The choice of wire mesh or perforate material for the resistive facing sheet is also very important since the facing sheet construction determines the main mechanism by which energy is dissipated at the surface. Wire mesh liners dissipate energy predominantly by friction whilst the dominant loss mechanism for perforates is microvortex shedding at the orifice edge. Since dissipation associated with friction is linear with velocity and that associated with vortex shedding is non-linear, the variation of resistance with SPL will depend on facing sheet construction. When flow is introduced, a third dissipation mechanism arises due to interaction of the propagating sound with vortices shed as a consequence of lateral flow across the facing sheet. An ideal model for facing sheet resistance should therefore include both in-duct SPL and grazing flow Mach number. Most current liner impedance models only account for the dependence of resistance on grazing flow Mach number since this is the dominant effect at typical operating conditions [119]. These models are based on empirical data and, being fairly rudimentary, are in need of improvement [120].

When generating the attenuation tables in step 2, as always, computational costs act as a constraint on the optimisation process. Firstly, a matrix of R and X values at which
to run the b-induct predictions must be chosen. Whilst larger matrices will provide better resolution, the resulting attenuation table will take longer to generate. Likewise, the frequencies at which the predictions are to be run must also be selected, bearing in mind that predictions at higher frequencies incur larger computational costs.

In step 3, in order to achieve an accurate calculation of EPNL, it is important to use the correct source content within the whole engine model, as the choice of optimum impedance will depend strongly on the source to be minimised, and how it contributes to overall aircraft EPNL. The final stage of the optimisation process involves the selection of the range of values of liner depth and facing sheet porosity at which this EPNL is to be calculated. This will largely be influenced by practical constraints, such as the specific porosities that can be realistically achieved in the manufacturing process, and the values of cell depth that are feasible, given weight considerations and the space available for liner installation within the engine.

For the optimisation of a given liner, by producing b-induct attenuation tables and EPNL contour plots at each of the Approach, Cutback and Sideline conditions, an analysis of the acoustic benefits of various combinations of cell depth and facing sheet porosity can be made using engineering judgement. In this way, the cell depth and facing sheet porosity can be chosen to ensure a robust optimum for this particular liner, that provides good EPNL reductions over all operating conditions.

Referring back to the bypass duct in figure 9.1, once the optimum impedance for liner 1 has been chosen, the impedance of this liner is set to its optimum value $Z_{opt}^1$ and a new set of b-induct attenuation tables can be generated by varying the impedance of liner 2 (see figure 9.6). The optimisation of all liners within the bypass duct is thus an iterative process whereby each liner is optimised in turn.
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Figure 9.6: Generation of b-induct attenuation tables for a given frequency for liner 2 by varying impedance $Z^2$, having set the impedance of liner 1 to the chosen optimum $Z_1^{opt}$, and whilst the impedances of all other liners ($i = 3 - 4$) are kept constant at their initial values, $Z_i^{initial}$.

9.3 Impedance optimisations for realistic bypass configurations

9.3.1 Highly Curved duct

The process detailed above was used to optimise the liner impedances for a realistic configuration of liners within the Highly Curved geometry, with positions specified as they would be in the engine. Figure 9.7 shows this liner configuration, which consists of two Inner Fixed Structure (IFS) liners situated on the inner wall, and one Outer Fixed Structure (OFS) liner positioned on the outer wall. The initial liner specifications for this configuration are listed in table 9.1.

<table>
<thead>
<tr>
<th>Liner number</th>
<th>Wall</th>
<th>liner area (in^2)</th>
<th>SDOF/ DDOF</th>
<th>FCD (in)</th>
<th>FPO (%)</th>
<th>SCD (in)</th>
<th>SR (cgs rayl)</th>
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Figure 9.8: B-induct predictions of Bypass Liner Insertion Loss, ΔPWL, for Highly Curved initial liner configuration.

For this initial duct configuration, b-induct predictions of bypass liner insertion loss were made at each of the three flight conditions: Approach, Cutback and Sideline. These predictions are shown against frequency in figure 9.8.

The liner impedances for the Highly Curved duct were then optimised one by one. Starting from the initial liner configuration (figure 9.7 and table 9.1), the impedances of the IFS1 liner, the OFS liner and the IFS2 liner were each optimised in turn, as single degree of freedom liners. Figure 9.9 shows the EPNL contour plots at Approach and
Contour plots of EPNL Delta for Highly Curved IFS1 liner.

Contour plots of EPNL Delta for Highly Curved OFS liner.

Setting the IFS1 liner impedance to this optimum, a similar optimisation was performed for the OFS liner, the EPNL contour plots for which are shown in figure 9.10. From these an optimum depth, $d = 2\text{in}$ and porosity, $POA = 10\%$ were chosen for the OFS liner.

Lastly, with the IFS1 and OFS liner impedances set to their optimum values, EPNL contours for the IFS2 liner were produced (see figure 9.11). The optimum depth and porosity for the IFS2 liner were chosen as $d = 2\text{in}$ and $POA = 12\%$ respectively, such that the overall optimum configuration for the Highly Curved duct (with all three liner impedances optimised) is given in table 9.2.

In each of figures 9.9, 9.10 and 9.11, sharp contour characteristics are observed at certain values of cell depth. This is an antiresonance phenomenon due to the calculation of attenuations at the 1/3 octave centre frequencies, and shall be discussed further in section 9.4.

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1The contour plot for the Sideline condition is not shown in figure 9.9, as the flow parameters for this condition were undefined at this stage.
Having optimised the liner impedances within the Highly Curved duct, b-induct predictions of the bypass liner insertion loss for the proposed optimum configuration were made at each of the Approach, Cutback and Sideline conditions. These are shown against those for the initial liner configuration in figure 9.12. It can be seen from tables 9.1 and 9.2 that for the particular source used, the optimisation has resulted in larger cell depths (2 in for the optimised liners vs. less than 1 in for the initial liners). This shift from shallower to deeper liners has caused a shift in peak insertion loss frequency from about 1600 Hz to 1000 Hz, and the peak insertion loss achieved at Approach has increased by about 6 dB. Also, a reduction in the insertion losses at about 3150 Hz indicates the shift of the antiresonance from a higher frequency, due to the deeper liners used within the optimum configuration. In this particular case, the shift towards deeper liners is due to the presence of a low frequency tone in the source used for the optimisations. For different source characteristics the optimisations would obviously produce different results.

(a) Approach  
(b) Cutback  
(c) Sideline

Figure 9.11: Contour plots of EPNL Delta for Highly Curved IFS2 liner.

<table>
<thead>
<tr>
<th>Liner number</th>
<th>Wall</th>
<th>Liner area (in^2)</th>
<th>SDOF/DDOF</th>
<th>FCD (in)</th>
<th>FPO (%)</th>
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<td>12</td>
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</table>

This method was also used to optimise the liner impedances for the B2 geometry introduced in chapter 5 (section 5.4). Starting from the initial liner configuration (see figure 9.13 and table 9.3), the impedances of both RFC liners were kept at their baseline values and then the impedances of the IFS liner, the first OFS liner and the second OFS liner, which is tapered, were each optimised in turn, as single degree of freedom liners.

The b-induct prediction for the initial liner configuration is shown in figure 9.14.

* = average depth of tapered liner between start and end.

<table>
<thead>
<tr>
<th>Liner number</th>
<th>Wall</th>
<th>Liner area (in^2)</th>
<th>SDOF/ DDOF</th>
<th>FCD (in)</th>
<th>FPO (%)</th>
<th>SCD (in)</th>
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<td></td>
</tr>
</tbody>
</table>

Figure 9.14: B-induct predictions of Bypass Liner Insertion Loss, $\Delta PWL$, for B2 initial liner configuration.

The EPNL contour plots at Approach, Cutback and Sideline for the IFS liner are all shown in figure 9.15. Again, the sharp contour characteristics observed at certain cell depths indicate an antiresonance phenomenon, which is discussed further in section 9.4. Given the small variation of EPNL Delta at Approach, the choice of optimum is influenced more by the results at Cutback and Sideline, and analysis of this data indicates an optimum in the region of $d = 1.8$in. Independent analysis performed elsewhere for
Therefore optimum liner characteristics of $d = 2\text{in}$ and $POA = 12\%$ were selected.

Setting the IFS liner impedance to this optimum, a similar optimisation was performed for the OFS liner (the EPNL contours for which are shown in figure 9.16), for which an optimum depth, $d = 2\text{in}$ and porosity, $POA = 10\%$ were chosen.

For the OFS tapered liner the end depth is fixed due to physical constraints ($d_{\text{end}} = 0.55\text{in}$), if the start depth was set to the depth of the preceding optimised OFS liner, ($d_{\text{start}} = 2\text{in}$), the average depth of the tapered liner would be $d = 1.275\text{in}$. In order to investigate whether this resultant tapered liner also gives a low noise configuration, the tapered liner was optimised as a single liner of constant average depth, and EPNL contours were produced (see figure 9.17). As there is very small variation in EPNL delta across each of these contour plots, and as the EPNL delta at $d = 1.275\text{in}$ and $POA = 10\%$ on each plot is near the maximum, it was decided that the optimum average depth $d = 1.275\text{in}$, and facing sheet porosity $POA = 10\%$ were satisfactory for the OFS tapered liner.
Therefore, the chosen optimum liner configuration for the B2 geometry is that specified in table 9.4. B-induct predictions of liner insertion loss were made for this optimum configuration, at each of the Approach, Cutback and Sideline conditions, and these are compared to the predictions of liner insertion loss for the initial liner configuration in figure 9.18. Because the specifications for liner location evolved during the design process, the liner areas for the optimum configuration are different from those for the initial configuration, and this difference needs to be taken into consideration when comparing the spectra.

As a result of the optimisation, the liners for the optimum configuration are deeper than those for the initial liner configuration. This difference can be seen in the b-induct predictions, in that the deeper liners have caused both the peak liner insertion loss and the antiresonance to shift to lower frequencies. This shift towards deeper liners is again due to the presence of a low frequency tone in the source used for the optimisations.

<table>
<thead>
<tr>
<th>Liner number</th>
<th>Wall</th>
<th>liner area (in^2)</th>
<th>SDOF/DDOF</th>
<th>FCD (in)</th>
<th>FPO (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Inner</td>
<td>3124</td>
<td>SDOF</td>
<td>0.76</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>Outer</td>
<td>4410</td>
<td>SDOF</td>
<td>0.55</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>Inner</td>
<td>7274</td>
<td>SDOF</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>Outer</td>
<td>7261</td>
<td>SDOF</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>Outer</td>
<td>3877</td>
<td>SDOF</td>
<td>1.275</td>
<td>10</td>
</tr>
</tbody>
</table>

* = average depth of tapered liner between start and end.
Figure 9.18: B-induct predictions of Bypass Liner Insertion Loss, $\Delta PWL$, for B2 initial and optimum liner configurations.

9.4 Suggested improvements

As mentioned previously, in many of the EPNL contour plots obtained there is a liner antiresonance characteristic observed, whereby at certain cell depths the EPNL Delta appears to drop steeply. This phenomenon is due to the calculation of the bypass attenuations only at 1/3 octave centre frequencies. If the centre frequency and the cell depth are such that the particle velocity at the facing sheet is minimal, an attenuation minimum will be achieved. However, as this centre frequency attenuation is applied across the whole 1/3 octave frequency band this causes the calculation of EPNL Delta to be lower than it should be, and hence causes the sharp contour characteristic seen.

This effect is not representative of the true behaviour of the liner, and obviously renders a judgement of optimum cell depth and facing sheet porosity more difficult. A suggestion for improvement in future optimisations would be to perform calculations of attenuation at a larger number of frequencies than just the 1/3 octave centre frequencies, using smaller frequency steps, which would enable the smoothing of the EPNL contour plots and thus facilitate their interpretation.

A further suggestion for improvement is to obtain b-induct predictions of bypass attenuation at an increased number of values of $R$ and $X$. The attenuation tables generated
for the optimisations performed within this chapter have used combinations of $R$ and $X$ with values: $R = 0, 1, 2, 3, 4, 5,$ and $X = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5.$ In order to obtain better resolution for the region of peak attenuation and capture the location of the peak more accurately the number of points at which predictions are made in this area should be increased. As an example of this resolution improvement figure 9.19 compares a contour plot obtained using the initial impedance values with that obtained using a set of additional values: $R = 0, 0.5, 1, 1.25, 1.5, 1.75, 2, 2.5, 3, 4, 5,$ and $X = -5, -4, -3, -2, -1, -0.5, 0, 1, 2, 3, 4, 5.$

As can be seen, the use of an increased number of impedances causes the location and value of the peak attenuation to change from $Z = 1 - i$ and 20.1dB to $Z = 0.5 - 0.5i$ and 23.6dB. By using extended impedance tables such as this in the future, the location and value of the peak attenuation can be more accurately captured and hence provide an increased degree of accuracy in the calculations of whole aircraft EPNL from which the optimum is selected.

On a related note, an improvement over the simple linear interpolation between data points in the attenuation tables would be the use of a more sophisticated method such as polynomial curve fitting or quadratic interpolation.

### 9.5 Summary

A liner impedance optimisation process that incorporates b-induct, and hence takes account of the bypass geometry has been developed. This process can be used to optimise the liner impedances for a specific duct geometry and set of liner positions. The impedance of each liner in the bypass duct is optimised iteratively by examining aircraft EPNL contour plots for a given source definition at each of three aircraft flight conditions (Approach, Cutback and Sideline), and selecting a liner cell depth and facing sheet porosity to provide good EPNL reductions at all conditions.

The use of b-induct within the optimisation process has the advantage of being able to account for the specific bypass geometry and liner arrangement. This ensures a more realistic incident modal energy distribution on each liner than with previous methods, and allows the impedance of each liner to be optimised whilst also taking into account the attenuations due to the other liners within the duct.

The method presented here has produced sensible results for the impedance optimisation of a given set of liners within two different realistic bypass geometries. This method has also been used to aid the selection of bypass liner impedances for a new engine which is currently being developed, results for which cannot be shown here for proprietary reasons. Overall, the b-induct optimisation methodology has been shown to be of use as part of the engine design process, allowing the selection of impedances for an entire
bypass configuration in an industrially acceptable timescale (typically 1-2 weeks for a full-scale engine). The method presented here is therefore a promising tool to aid the choice of liners for future low noise engines.

Figure 9.19: Contour plots to show how bypass liner attenuation ($\Delta PWL$) varies with $R$ and $X$. Frequency: 1kHz, inlet Mach number: 0.3, $L/D = 3.49$
Chapter 10

Conclusions and future work

10.1 Overall conclusions

This work was motivated by the need for a realistic model for fan noise propagating rearwards through the bypass duct. Such a model must be capable of making predictions in industrially acceptable timescales whilst still capturing the important physical effects. Typical industry models for bypass acoustic liner performance are for idealised geometries such as a uniform duct with uniform flow. These methods are computationally inexpensive, and are capable of meeting industrial time constraints. However, the use of simplified duct geometries results in important physical effects, such as scattering and reflection by the non-uniform duct geometry and flow field, not being considered. The Noise Engineering department at Rolls-Royce wish to apply the most up-to-date numerical and theoretical approaches for predicting bypass noise, within the engine design process without compromising computational time.

This thesis assesses two recently developed numerical and theoretical approaches for propagation and radiation of rear fan noise: B-induct and GXMunt. These are an improvement on current industry standard models because they capture the important physical effects of the problem, and although they take longer computation time than analytic uniform duct models, they still allow predictions to be made using a reasonable amount of computational power (such as a normal desktop or laptop machine) and within industrially acceptable timescales.

As well as being computationally efficient, for a prediction method to be of use in the engine design process it must fulfil two important criteria: Firstly it must be capable of stand-alone predictions for rearwards propagating fan noise, in order to make fast and reliable assessments of potential low noise bypass configurations within the initial design stages of the engine. However, it must also be usable within a whole engine noise model such that the impact of rear fan noise on the total engine noise can be assessed.
Predictions for bypass liner attenuation made using the model b-induct have been incorporated in an industry standard whole engine model, in order to obtain far-field predictions of engine noise for a typical modern high bypass ratio engine. These predictions, alongside similar whole engine predictions made using an industry standard uniform rectangular duct model for the bypass attenuations, were compared to measured data obtained from full-scale static engine tests.

Both predictions show reasonable agreement with the measured data at low frequencies, where the liner is less effective, although there is a notable discrepancy in SPL at frequencies closer to that of the peak attenuation by the bypass liner. Due to the complexity of the whole engine prediction model, and the way in which source correction factors are calculated and applied, it is difficult to judge whether these differences are due to inaccuracies in the bypass liner prediction model, or in the source model itself. In spite of these discrepancies, the predictions made using b-induct are in closer agreement with the measurements than those made using the uniform rectangular duct industry model. This is not unexpected since b-induct is based on a more physically realistic geometry, and should therefore provide more accurate predictions for the bypass liner attenuation. The improvement of the b-induct predictions over the industry model predictions is an encouraging result for the future use of b-induct as a tool within the engine design process. However, the discrepancy between the predictions and the measured data, especially at peak attenuation frequencies, still requires explanation, which would involve exploring the whole engine prediction process in more detail.

As a stand-alone prediction model for assessing potential low noise bypass configurations, b-induct has been used extensively within this research. Parametric studies on the effect on noise propagation in which the curvature of an axisymmetric duct is systematically varied have suggested that for a given liner area, increased attenuation can be achieved for ducts with higher curvature. Two industrially feasible bypass geometries of interest have been examined in detail: a “Baseline” design based on a typical modern turbofan bypass duct, and a novel “Highly Curved” design which is shorter and therefore benefits from low weight and drag. Both of these geometries were manufactured on a 1:4.2 rig-scale and tested in the ISVR zero-flow facility using both broadband and tone sources, and for both axisymmetric and 3D builds.

For axisymmetric duct configurations with a broadband source, liner insertion loss predictions made using b-induct capture well the shape of the measured data, although there is a discrepancy in the level at peak attenuation which requires further investigation. The importance of examining narrowband data has been highlighted since use of 1/3 octave band data results in loss of information, especially in the region of peak attenuation. Measured fieldshapes compare well to predictions made using the analytic GXMunt radiation model, with some differences observed that are likely due to the fact that the GXMunt model is not an exact representation of the rig geometry. For a single azimuthal mode source, measured data from an in-duct radial microphone array
has been used to calculate the relative amplitudes of the incident radial modes, which have then been used to predict far-field directivity patterns, which capture the general fieldshape, especially the peak radiation angle, quite well. At certain angles, notably on axis, there is a difference between predicted and measured SPL. It is possible that at these angles there is contamination due to other azimuthal modes within the measured data.

For non-axisymmetric duct configurations with 3D features such as a pylon and lower bifurcation, for a broadband source, the incorporation of liner on the bifurcations has been shown to provide an increase in PWL attenuation compared to the axisymmetric lined build. Predictions of the attenuation due to the liner on the bifurcations made using 3D b-induct compare well with measured results. For a single azimuthal mode source, 3D b-induct predictions capture the effect of scattering by the bifurcation into other azimuthal modes, and the effect of reflection at the bifurcation causes the generation of a standing wave pattern within the duct which is also visible in the far-field.

The incorporation of b-induct attenuation tables into a current industry procedure for optimising liner impedances has allowed for the specific bypass duct geometry and liner positions to be taken into account when performing liner impedance optimisations. This is an improvement to the previous process, where the effects of reflection and scattering by the non-uniform geometry and flow field were not considered. The use of b-induct also permits a more accurate modal energy distribution incident on each liner than previous methods, and allows the impedance of each liner to be optimised whilst taking into account the attenuations due to the other liners within the duct. The b-induct optimisation methodology allows the selection of impedances for an entire bypass configuration with rapid iterations, compatible with industrially acceptable time-frames, and has been used to aid the selection of bypass liner impedances for a new engine which is currently being developed.

In conclusion, b-induct results have successfully been integrated into industry procedures for predicting whole engine noise and for optimising liner impedance, and b-induct has also been used as a stand-alone tool for assessing bypass propagation. The studies presented here have shown b-induct to be a useful tool for the engine design process, for both analysis of the impact of rear fan noise on whole engine noise, and assessment of potential low noise bypass configurations for future engines.
10.2 Future work

In the current study, whole engine predictions have been made using an empirical directivity model for bypass attenuations, which does not consider specifically the effect of refraction through the bypass shear layer. The next step is to make whole engine predictions using b-induct bypass attenuations but with a directivity pattern predicted using GXMunt, such that the effect of the shear layer is taken into account. In this way it will be possible to evaluate whole engine predictions made using b-induct + GXMunt attenuations against measured data, and so investigate the impact of using a more physically accurate directivity pattern on predicted whole engine noise.

From this work it is clear that the Highly Curved duct suffers from no significant acoustic penalty over the Baseline duct despite its reduced liner area. This result, coupled with the fact that the Highly Curved duct is shorter and therefore has associated weight and drag benefits suggests that the Highly Curved geometry is a promising new design for a low noise aeroengine. Further rig tests are to be performed for this geometry using a real fan as the noise source, providing more realistic flow conditions.

The no-flow rig tests undertaken have provided some good test data that has been used to confirm the initial suggestion of the acoustic benefit of a Highly Curved geometry. Future rig tests should have a larger number of measurement points in the far-field at more azimuthal angles. Also, the azimuthal mode generator designed for the tests involving single mode sources produced reasonably pure azimuthal modes but the microphones in the circumferential array were used only to detect mode purity, and not calibrated in order to measure individual mode amplitude. Therefore in future rig tests these microphones should be properly calibrated in order to obtain individual mode amplitude readings.

Finally, the optimisation of liner impedances for a given bypass configuration using b-induct attenuation tables could be further improved by generating attenuation tables that contain predicted attenuations at more values of $R$ and $X$, and at more frequencies than are currently used. This would allow for smoother interpolation of the attenuation within the optimisation procedure.
Appendix A

ANDANTE Tests: Matrix of Tones

For the tone tests, to reduce the number of test points necessary, mode sources at 2 or 3 different individual frequencies were combined into one multiple-tone signal. By ensuring that the frequencies used within this tone signal are not multiples, sums or differences of each other, the resulting sound field can be decomposed into the individual sound fields at each frequency.
<table>
<thead>
<tr>
<th>Tone No.</th>
<th>Frequency (Hz)</th>
<th>Azimuthal mode no. (m)</th>
<th>Cut-on radial mode no’s (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1250</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1600</td>
<td>0</td>
<td>1</td>
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<td></td>
<td>2000</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1250</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1600</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2000</td>
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<td>1</td>
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<tr>
<td></td>
<td>1600</td>
<td>4</td>
<td>1</td>
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<tr>
<td></td>
<td>2000</td>
<td>5</td>
<td>1</td>
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<tr>
<td>4</td>
<td>1250</td>
<td>3</td>
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<tr>
<td></td>
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<td>1</td>
</tr>
<tr>
<td>5</td>
<td>2500</td>
<td>0</td>
<td>1, 2</td>
</tr>
<tr>
<td></td>
<td>3150</td>
<td>0</td>
<td>1, 2</td>
</tr>
<tr>
<td></td>
<td>4000</td>
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<td>1, 2</td>
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<tr>
<td>6</td>
<td>2500</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
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<td>3150</td>
<td>5</td>
<td>1, 2</td>
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<tr>
<td></td>
<td>4000</td>
<td>6</td>
<td>1, 2</td>
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<td>7</td>
<td>2500</td>
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<td>1</td>
</tr>
<tr>
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<td>3150</td>
<td>7</td>
<td>1</td>
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<tr>
<td></td>
<td>4000</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>3150</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>4000</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>10000</td>
<td>14</td>
<td>1, 2, 3, 4</td>
</tr>
<tr>
<td>9</td>
<td>4000</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>5000</td>
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<td>1</td>
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<td>1, 2, 3, 4</td>
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<tr>
<td>12</td>
<td>5000</td>
<td>13</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>6000</td>
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<td>1, 2</td>
</tr>
<tr>
<td></td>
<td>8000</td>
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</tr>
<tr>
<td>13</td>
<td>10000</td>
<td>0</td>
<td>1, 2, 3, 4, 5</td>
</tr>
<tr>
<td>14</td>
<td>10000</td>
<td>4</td>
<td>1, 2, 3, 4, 5</td>
</tr>
</tbody>
</table>

Table A.1: Matrix of tones used for tone tests
Appendix B

ANDANTE Tests: Build Number List

The following tables identify the particular configuration of liners present for each build number. Builds 1-15 are the Baseline rig geometry, and builds 16-26 are the Highly Curved rig geometry. For each build, section numbers 1-4 correspond to the bypass duct sections as labelled in chapter 6, figure 6.10, and section 5 corresponds to the afterbody. The letter H signifies a section where both the inner and outer walls are hardwall, and the letter L signifies a section where both walls are lined. For diagrams that show the particular build configurations refer to figures B.1 to B.26 (where B.1 refers to build number 1, B.2 refers to build number 2 etc). For each diagram, the basic duct sections are omitted and the figures show only the rig sections that were added to the basic duct. Sections in bold indicate the presence of an acoustic liner. A side-on view is presented, along with an end-on view looking down the duct from the anechoic chamber to the reverberation chamber.
### BASELINE:

<table>
<thead>
<tr>
<th>Build No.</th>
<th>Section No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
<td>Hard-Walled Datum (HWD) + Hard Afterbody (HAB)</td>
</tr>
<tr>
<td>2</td>
<td>L L</td>
<td>Lined Datum (LD) + Lined Afterbody (LAB)</td>
</tr>
<tr>
<td>3</td>
<td>L L</td>
<td>LD + HAB</td>
</tr>
<tr>
<td>4</td>
<td>L L</td>
<td>LD + HW bifurcations @ 0° and 180° + HAB</td>
</tr>
<tr>
<td>5</td>
<td>L L</td>
<td>LD + HW bifurcations @ 90° and 270° + HAB</td>
</tr>
<tr>
<td>6</td>
<td>L L</td>
<td>LD + L bifurcations @ 0° and 180° + HAB</td>
</tr>
<tr>
<td>7</td>
<td>L L</td>
<td>LD + L bifurcations @ 90° and 270° + HAB</td>
</tr>
<tr>
<td>8</td>
<td>L L</td>
<td>LD + HW patch on section 1 outer wall @ 0° + HAB</td>
</tr>
<tr>
<td>9</td>
<td>L L</td>
<td>LD + HW patch on section 1 outer wall @ 30° + HAB</td>
</tr>
<tr>
<td>10</td>
<td>L L</td>
<td>LD + HW patch on section 1 outer wall @ 60° + HAB</td>
</tr>
<tr>
<td>11</td>
<td>L L</td>
<td>LD + HW patch on section 1 outer wall @ 120° + HAB</td>
</tr>
<tr>
<td>12</td>
<td>L L</td>
<td>LD + HW patch on section 1 outer wall @ 150° + HAB</td>
</tr>
<tr>
<td>13</td>
<td>L L</td>
<td>LD + HW patch on section 1 outer wall @ 180° + HAB</td>
</tr>
<tr>
<td>14</td>
<td>L L</td>
<td>LD + HW patch on section 1 outer wall @ 210° + HAB</td>
</tr>
<tr>
<td>15</td>
<td>L L L</td>
<td>Fully-Lined build + HAB</td>
</tr>
</tbody>
</table>

**Table B.1:** Build configurations tested for Baseline geometry.

### HIGHLY CURVED:

<table>
<thead>
<tr>
<th>Build No.</th>
<th>Section No.</th>
<th>Description</th>
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<tbody>
<tr>
<td>16</td>
<td>H H H H H</td>
<td>Hard-Walled Datum (HWD) + Hard Afterbody (HAB)</td>
</tr>
<tr>
<td>17</td>
<td>H L L H L</td>
<td>Lined Datum (LD) + Lined Afterbody (LAB)</td>
</tr>
<tr>
<td>18</td>
<td>H L L H H</td>
<td>LD + HAB</td>
</tr>
<tr>
<td>19</td>
<td>H L L H H</td>
<td>LD + HW bifurcation @ 180° + HAB</td>
</tr>
<tr>
<td>20</td>
<td>H L L H H</td>
<td>LD + HW bifurcation @ 90° + HAB</td>
</tr>
<tr>
<td>21</td>
<td>H L L H H</td>
<td>LD + HW bifurcation @ 270° + HAB</td>
</tr>
<tr>
<td>22</td>
<td>H L L H H</td>
<td>LD + L bifurcation @ 180° + HAB</td>
</tr>
<tr>
<td>23</td>
<td>H L L H H</td>
<td>LD + L bifurcation @ 90° + HAB</td>
</tr>
<tr>
<td>24</td>
<td>H L L H H</td>
<td>LD + L bifurcation @ 270° + HAB</td>
</tr>
<tr>
<td>25</td>
<td>L L L H H</td>
<td>LD + asymmetric HW patch (inner and outer walls) at beginning of section 2 with area equal to half area of section 1, and asymmetric HW patch (inner and outer walls) at beginning of section 3 with area equal to half area of section 1.</td>
</tr>
<tr>
<td>26</td>
<td>L L L L H</td>
<td>Fully-Lined build + HAB</td>
</tr>
</tbody>
</table>

**Table B.2:** Build configurations tested for Highly Curved geometry.
Figure B.1: Baseline Hard-Walled Datum (HWD) + Hard Afterbody (HAB)

Figure B.2: Baseline Lined Datum (LD) + Lined Afterbody (LAB)

Figure B.3: Baseline LD + HAB
Figure B.4: Baseline LD + HW bifurcations @ 0° and 180° + HAB

Figure B.5: Baseline LD + HW bifurcations @ 90° and 270° + HAB

Figure B.6: Baseline LD + Lined (L) bifurcations @ 0° and 180° + HAB
Figure B.7: Baseline LD + L bifurcations @ 90° and 270° + HAB

Figure B.8: Baseline LD + HW patch @ 0° + HAB

Figure B.9: Baseline LD + HW patch @ 30° + HAB
Figure B.10: Baseline LD + HW patch @ 60° + HAB

Figure B.11: Baseline LD + HW patch @ 120° + HAB

Figure B.12: Baseline LD + HW patch @ 150° + HAB
Appendix B ANDANTE Tests: Build Number List

Figure B.13: Baseline LD + HW patch @ 180° + HAB

Figure B.14: Baseline LD + HW patch @ 210° + HAB

Figure B.15: Baseline Fully-Lined build + HAB
Appendix B ANDANTE Tests: Build Number List

Figure B.16: Highly Curved Hard-Walled Datum (HWD) + Hard Afterbody (HAB)

Figure B.17: Highly Curved Lined Datum (LD) + Lined Afterbody (LAB)

Figure B.18: Highly Curved LD + HAB
Figure B.19: Highly Curved LD + HW bifurcation @ 180° + HAB

Figure B.20: Highly Curved LD + HW bifurcation @ 90° + HAB

Figure B.21: Highly Curved LD + HW bifurcation @ 270° + HAB
Figure B.22: Highly Curved LD + L bifurcation @ 180° + HAB

Figure B.23: Highly Curved LD + L bifurcation @ 90° + HAB

Figure B.24: Highly Curved LD + L bifurcation @ 270° + HAB
Figure B.25: Highly Curved LD + axisymmetric HW patch at beginning of section 2 with area equal to half area of section 1, and axisymmetric HW patch at beginning of section 3 with area equal to half area of section 1 + HAB

Figure B.26: Highly Curved Fully-Lined build + HAB
Appendix C

ANDANTE Tests: Matrix of Test Points for Tone Tests

<table>
<thead>
<tr>
<th>Build No.</th>
<th>Tone No.</th>
<th>Test Point No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 to 14</td>
<td>155 to 168</td>
</tr>
<tr>
<td>2</td>
<td>1 to 14</td>
<td>169 to 182</td>
</tr>
<tr>
<td>3</td>
<td>1 to 14</td>
<td>183 to 196</td>
</tr>
<tr>
<td>4</td>
<td>1 to 14</td>
<td>197 to 210</td>
</tr>
<tr>
<td>5</td>
<td>1 to 14</td>
<td>211 to 224</td>
</tr>
<tr>
<td>6</td>
<td>1 to 14</td>
<td>225 to 238</td>
</tr>
<tr>
<td>7</td>
<td>1 to 14</td>
<td>239 to 252</td>
</tr>
<tr>
<td>8</td>
<td>1 to 14</td>
<td>253 to 266</td>
</tr>
<tr>
<td>9</td>
<td>1 to 14</td>
<td>267 to 280</td>
</tr>
<tr>
<td>10</td>
<td>1 to 14</td>
<td>281 to 294</td>
</tr>
<tr>
<td>11</td>
<td>1 to 14</td>
<td>295 to 308</td>
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<td>309 to 322</td>
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<td>401 to 414</td>
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<td>463 to 476</td>
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<td>489 to 502</td>
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<td>517 to 530</td>
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<td>549 to 562</td>
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<td>25</td>
<td>1 to 14</td>
<td>577 to 590</td>
</tr>
<tr>
<td>26</td>
<td>1 to 14</td>
<td>609 to 622</td>
</tr>
</tbody>
</table>

Table C.1: Matrix of test points for tone tests, where each test point corresponds to a particular build number and tone number.
Appendix D

Calculation of relative mode amplitudes using in-duct radial microphone array measurements

For a bypass in-duct domain as shown in figure D.1, containing a radial array of eight microphones, an expression for the total calculated pressure $P_i^c$, at each radial microphone $i$ can be written:

$$P_i^c = \alpha_1 p_{in1} + \alpha_2 p_{in2} + \ldots + \alpha_N p_{inN} = \sum_{j=1}^{N} \alpha_j p_{inj}, \quad (D.1)$$

where $p_{inj}$ is the value for the predicted pressure at each radial microphone, $i$, due to each incident radial mode, $n_j$, present at the inlet with an amplitude of $1 + 0i$, $N$ is the total number of radial modes cut-on, and $\alpha_j$ gives the amount of each radial mode present.

![Figure D.1: Bypass model domain for tone calculations, using 8 field points located at radial microphone locations.](image-url)
Appendix D Calculation of relative mode amplitudes using in-duct radial microphone array measurements

This expression for the total calculated pressure, $P_c^i$, can then be used along with the value for the total measured pressure, $P_m^i$, at each radial microphone $i$, such that a least squares fitting can be performed in order to calculate the value of $\alpha_j$ for each radial mode and determine the relative amounts of each radial mode present in the rig test.

Defining the error, $E$, as the sum of the squares of the differences between the calculated pressure, $P_c^i$, and the measured pressure, $P_m^i$, over all eight microphones:

$$E = \sum_{i=1}^{8} |P_c^i - P_m^i|^2 = \sum_{i=1}^{8} |\alpha_1 p_{in1} + \alpha_2 p_{in2} + \cdots + \alpha_N p_{inN} - P_m^i|^2,$$

(D.2)

and then minimising $E$ with respect to the real and imaginary components of the unknown coefficients $\alpha_j$ gives:

$$\begin{bmatrix}
\sum_{i=1}^{8} p_{in1}^* p_{in1} & \sum_{i=1}^{8} p_{in1}^* p_{in2} & \cdots & \sum_{i=1}^{8} p_{in1}^* p_{inN} \\
\sum_{i=1}^{8} p_{in2}^* p_{in1} & \sum_{i=1}^{8} p_{in2}^* p_{in2} & \cdots & \sum_{i=1}^{8} p_{in2}^* p_{inN} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{i=1}^{8} p_{inN}^* p_{in1} & \sum_{i=1}^{8} p_{inN}^* p_{in2} & \cdots & \sum_{i=1}^{8} p_{inN}^* p_{inN}
\end{bmatrix} \begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_N
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{8} p_{in1}^* P_m^i \\
\sum_{i=1}^{8} p_{in2}^* P_m^i \\
\vdots \\
\sum_{i=1}^{8} p_{inN}^* P_m^i
\end{bmatrix},$$

(D.3)

where $^*$ denotes the complex conjugate. This matrix equation can then be solved to obtain values for $\alpha_j$, and hence determine the relative amounts of each radial mode present.
Appendix E

Calculation of transmitted sound power level from a set of pressure measurements recorded at a single polar microphone array.

For an acoustic source enclosed by a surface, $S$, the total power output, $W$ is given by integrating the acoustic intensity, $I$, travelling across the surface:

$$ W = \int_S I \cdot \mathbf{n} dS, \quad (E.1) $$

where $\mathbf{n}$ is the unit normal to the surface. Thus, if $\mathbf{n}$ is taken to be in the radial direction, $W$ can be written:

$$ W = \int_S I_R dS, \quad (E.2) $$

where $I_R$ is the radial component of intensity.

**Figure E.1:** Surface, $S$, enclosing an acoustic source, where $\mathbf{n}$ is the unit normal to $S$. 
Appendix E Calculation of transmitted sound power level from a set of pressure measurements recorded at a single polar microphone array.

For an accurate value of the total power output it would therefore be necessary to obtain far-field measurements of pressure from microphones located at all points over a spherical surface enclosing the source. However, in reality this is difficult to achieve, and it is common to have only one set of microphones arranged in a polar array, as shown in figure E.2.

In this case, the assumption is made that the pressure measured at a given polar angle, $\psi$, would be the same at all points on the circumference of the circle mapped out by the angle $\psi$, (i.e. at all values of azimuthal angle, $\theta$) and hence that the pressure field is completely axisymmetric. (This is of course quite a large assumption, especially for non-axisymmetric duct configurations, but the best choice given the amount of data available.) Thus, for a set of measured pressures from microphones within a far-field polar array of radius $R_0$ (figure E.3) an element of surface, $dS$ can be written:

$$dS = 2\pi R_0^2 \sin \psi d\psi,$$

and the total power output, $W$ becomes:

$$W = \int_{\psi=0}^{\pi} I_R(\psi) 2\pi R_0^2 \sin \psi d\psi,$$

where $I_R(\psi)$ is the radial component of intensity evaluated at angle $\psi$. 

Figure E.2: Polar microphone array.
In the absence of flow, for a spherical wave, the time-averaged acoustic intensity in the radial direction can be written:

\[ I_R(\psi) = \frac{|p(\psi)|^2}{2\rho_0 c_0}, \quad (E.5) \]

where \( \rho_0 \) and \( c_0 \) are the density of air and speed of sound in air respectively, and thus:

\[ W = \frac{\pi R_0^2}{\rho_0 c_0} \int_{0}^{\pi} |p(\psi)|^2 \sin \psi d\psi. \quad (E.6) \]

Therefore, for a series of pressure measurements from an array with microphones spaced every \( \Delta \psi \) from \( \psi = 0 \rightarrow \pi \):

\[ W = \frac{\pi R_0^2}{\rho_0 c_0} \sum_{\psi=0}^{\pi} |p(\psi)|^2 \sin \psi \Delta \psi, \quad (E.7) \]

and the sound power level, \( PWL \), can be found using:

\[ PWL = 10 \log \left( \frac{W}{W_{ref}} \right) = 10 \log \left( \frac{\pi R_0^2 \Delta \psi}{\rho_0 c_0 W_{ref}} \sum_{\psi=0}^{\pi} |p(\psi)|^2 \sin \psi \right), \quad (E.8) \]

where the reference power, \( W_{ref} \), is usually taken to be \( 10^{-12} \) Watts.

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1For a set of pressure measurements where the polar microphone array does not extend all the way from 0 to \( \pi \), the power is calculated using the data from the region of arc available, with the assumption that most of the power is radiated into this region.
Bibliography


