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UNIVERSITY OF SOUTHAMPTON

FACULTY OF ENGINEERING, SCIENCE AND MATHEMATICS

Institute of Sound and Vibration Research

The Development of Rail-head Acoustic Roughness

by

Briony Elizabeth Croft

Thesis for the degree of Doctor of Philosophy

October 2009

UNIVERSITY OF SOUTHAMPTON

ABSTRACT FACULTY OF ENGINEERING, SCIENCE AND MATHEMATICS INSTITUTE OF SOUND AND VIBRATION RESEARCH

Doctor of Philosophy

THE DEVELOPMENT OF RAIL-HEAD ACOUSTIC ROUGHNESS by Briony Elizabeth Croft

A model of the development of rail-head acoustic roughness on tangent track has been formulated. The model consists of a two-dimensional time domain wheel-rail interaction force calculation, with the normal force used as the input to a two-dimensional rolling contact and wear model. The possibility of multiple wear mechanisms arising from stress concentrations is considered by using a wear coefficient that can vary with the conditions at each point in the contact. The contact model is based on a variational technique, taking account of non-Hertzian and transient effects. A novel feature of the rolling contact model is the introduction of a velocity-dependent friction coefficient. In rolling contact this leads to a high frequency stick-slip oscillation in the slip zone at the trailing edge.

Roughness development depends on the dynamics of the track. Roughness growth has often been linked to the pinned-pinned frequency and other resonances of the coupled track and vehicle system. Here the effect of different vehicle and track parameters on track dynamics, wear and roughness development has been examined. Rail dampers are studied as they change the dynamic response of the track. Results are presented in the form of roughness growth rate functions both for individual vehicle types and for mixed traffic. The model parameters match those at a site used for measurements of roughness development taken by Deutsche Bahn AG as part of the EU project Silence.

The study shows that it is important to include non-Hertzian effects when studying roughness with wavelengths shorter than 100 mm. With a non-Hertzian contact model, no mechanism has been found for consistently increasing roughness levels. The model predicts that roughness wavelengths shorter than the contact length will wear away. Rail dampers are shown to reduce the pinned-pinned frequency and smooth the peaks and troughs in the track receptance. Rail dampers also reduce the dynamic wheel-rail interaction forces, especially around the pinned-pinned resonance, and shift the force spectrum to lower frequencies or longer wavelengths. However, rail dampers are not predicted to affect roughness growth rates significantly.

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DECLARATION OF AUTHORSHIP

I, Briony Elizabeth Croft, declare that the thesis entitled 'The development of rail-head acoustic roughness' and the work presented in the thesis are both my own, and have been generated by me as the result of my own original research. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- No part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.
- Parts of this work have been published as Croft *et al.* [2009].

Signed:

Date:....

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1 INTRODUCTION

1.1 Background

Demand for rail transport services is increasing with growing populations and environmental awareness. High-speed rail networks are expanding in many countries and conventional speed railways are being operated at higher capacities, driven by the political objectives of expanding economies and promoting growth. Populations are becoming increasingly mobile, demanding improved opportunities for travel and tourism, as well as access to increasingly diverse products. For example, in Europe the demand for freight transport volumes increased by 35% in the ten years up to 2006 (for all transport modes combined). The average number of kilometres travelled by people increased in the same time period by 1.3% each year [European Environment Agency, 2009].

In addition to the increasing demand for rail services caused by economic and population growth, increasing awareness of the need to reduce carbon emissions means that a shift in transport mode towards rail from the more energy intensive modes of road and air is desirable. Rail is an environmentally friendly means of transport when compared with the alternatives of road and air traffic. The average carbon dioxide emissions for rail transport are around 18 to 35 g/tkm^{*}, compared with those for road transport of 62 to 110 g/tkm and for air of over 665 g/tkm. [European Environment Agency, 2009].

As with other modes of transport, the increased noise levels that result from expanding rail networks are a significant restriction to development especially in areas with dense populations. Noise is not simply an annoyance; it is recognised by the World Health Organisation as being harmful to health, as well as interfering with performance of daily activities [Berglund *et al.*, 1999]. Noise and vibration concerns are often used to oppose new rail developments. In an interview for a UIC newsletter, Dr. Matthias Mather (Head of Environmental Protection in the Environment Centre of Deutsche Bahn AG) states that: "Residents are afraid that traffic, and hence noise, will increase. So they object to the expansion of rail freight traffic without simultaneous, effective measures for noise abatement. In other words, the residents will only accept the growth in rail freight traffic if the noise pollution falls at the same time." [UIC, 2008].

g/tkm is grams of CO₂ emitted when 1 tonne is transported 1 kilometre

In Europe, the Environmental Noise Directive [European Commission, 2002] requires the mapping of noise sources and the development of action plans to reduce noise. Noise legislation limiting the noise emitted by rail vehicles has been introduced via the Technical Specifications for Interoperability or TSIs [European Commission, 2005 & 2006]. Since noise limits can restrict rail vehicle speeds or the amount of rail traffic, the cost of noise control must be balanced with the need for capacity increases.

1.1.1 Railway noise and roughness

Railway noise has been studied extensively since the 1970s and models of rolling noise are now well established. Thompson's book [2009] is a comprehensive reference covering the overall topic of railway noise and vibration, including analysis of rolling noise mechanisms, modelling and control.

The dominant source of railway noise is the rolling of the wheels over the rails, except at very high speeds where aerodynamic noise can become the most significant source. This rolling noise has a broad spectrum in the frequency range up to about 5000 Hz, and its level and frequency content increase with train speed. The inherent roughness of the wheels and rails in the contact zone induces a relative motion which results in dynamic interaction forces, vibration of the wheel and track structures, and noise radiation [Thompson, 2009]. It is therefore the combined roughness of the wheel and rail that provides the fundamental excitation source for railway rolling noise.

Increasing roughness of the wheel and rail leads to higher noise levels and also to higher contact forces. Ultimately high roughness levels require maintenance attention. Wheels can be removed from service for re-profiling, but grinding the track to control roughness and corrugation is time-consuming and expensive. It is important to understand mechanisms of rail wear and roughness development in order to minimise rail roughness growth rates and hence the costs of noise control and maintenance. The roughness of interest is not limited to corrugation; it includes broadband roughness with wavelengths between 5 and 500 mm, referred to as 'acoustic roughness'.

Before developing the themes of this thesis on research into wheel-rail interaction and wear in the contact patch, this chapter provides an introduction to the topics of railway rolling noise and roughness. The objectives of this thesis are summarised, as are the contributions made to the understanding of rail roughness development. Some of the literature in the field is discussed in this chapter as background material, but most of the existing work on roughness and corrugation modelling is reviewed separately in Chapter 2.

1.2 Objectives

Understanding the mechanisms of the development of railway acoustic roughness is clearly useful for long-term noise control. In this thesis, a model is presented that predicts the change in roughness profile of a railhead as a wheel or wheels rolls over the rail surface. This work aims to further the understanding of the mechanisms of the development of railway roughness, and the factors that affect it. Results for different input scenarios are presented in the form of a roughness growth rate spectrum, which indicates the wavelengths at which roughness is expected to grow or decrease over time. Using this approach, it may be possible to optimise railway tracks to minimise long-term roughness growth rates.

Noise prediction models such as TWINS [Thompson, 1993a-e] give a clear understanding of the sources of railway rolling noise, but these models assume the roughness levels as an input and do not consider the propensity of a system to develop roughness or corrugation over time. The aim of the work of this thesis is to develop a model for the development of broadband acoustic roughness, and to use it to assess the effect of different track parameters on the rates of rail-head roughness development. Because the focus is on track components and parameters, rail roughness is examined rather than wheel roughness (although it could be modelled in the same way).

The development of roughness is a rolling contact mechanics problem. The first step towards modelling the change in roughness of the railhead over time is to develop a model of the dynamic interaction between vehicle and track. This is used to calculate the response of the track to the excitation arising from the movement of the wheels over the combined roughness of the surfaces. The resulting dynamic interaction forces between the wheels and the rail are input into a contact model with a wear calculation to predict the resulting change in the railhead roughness profile after a number of wheel passages.

A great deal of work has been published on the subjects of wheel-rail interaction, rolling contact mechanics and wear. In Chapter 2, some of the significant work in these complex

fields is reviewed in more detail. The conclusions drawn by other authors have been used to determine a suitable approach to take in modelling the development of railhead acoustic roughness. Following a similar procedure to that used by others to model corrugation development, the model of roughness development described in this work may be divided into three sections:

- 1. A wheel-rail interaction model (presented in Chapters 3 and 4)
- 2. A rolling contact mechanics model (Chapters 5, 6 and 8)
- 3. A wear model (Chapter 7)

At each stage of the model the effect of different vehicle parameters and track components is assessed.

1.2.1 The effect of rail dampers on roughness

Rail dampers are an example of a change in the track parameters that could affect roughness development. Rail damping devices have been shown to be highly effective for noise control, both in theoretical simulations and in installations on operating lines [Thompson *et al.*, 2007]. Adding rail damping components to the track to control noise will change the track dynamics, and a variation in the track dynamics may result in a change in the propensity of the system to develop roughness or corrugation. Therefore the long term effectiveness of rail dampers for noise control may be affected by the propensity of the upgraded system to develop roughness or corrugation over time.

In work linked to this research, measurements of roughness have been carried out by Deutsche Bahn AG over several years at a test site near Gersthofen in Germany as part of the EU Silence project [Stiebel, 2005; Asmussen *et al.*, 2008]. Rail dampers are installed at two locations at this site, with different rail pad stiffnesses. The measured roughness spectra are presented in Chapter 9 for comparison with the predictions of the roughness growth model. Measured track decay rates from the site have also been used to calibrate the finite element model of the track in Chapter 3, and the parameters for the vehicle model introduced in Chapter 4 are taken from typical train types at Gersthofen. In this work a 'typical' freight train refers to a typical train at this site, not a general freight train, and similarly for regional trains and high-speed trains.

1.3 Railway rolling noise

This work concentrates on the development of acoustic roughness rather than rolling noise explicitly. A detailed review of literature on roughness growth is given in Chapter 2. However, models of rolling noise and of roughness development share a basis in modelling the dynamic interaction between track and vehicle in a similar frequency range. Therefore a brief review of railway rolling noise models is given here. Methods of railway noise control are also mentioned, since part of this project includes an analysis of the effect of rail dampers on the development of railhead acoustic roughness.

1.3.1 Models for the prediction of railway rolling noise

Rolling noise models can be used to compare different wheel and track designs and to optimise the components selected for the best acoustic performance [Vincent *et al.*, 1996]. Remington [1976a,b] developed one of the first analytical models of the interaction between wheel and rail in order to predict the radiated noise. In this model the combined roughness of wheel and rail acts as a vertical excitation to the system. The vibration response of the wheel and rail to this excitation is calculated and used to predict the resulting noise. Thompson [1993a-e] extended Remington's theory and implemented the model as the computer program TWINS (Track-Wheel Interaction Noise Software).

The TWINS software has been validated by a series of comparisons with measurements, both for conventional wheel/track systems [Thompson *et al.*, 1996a,b] and for various noise-reducing track and wheel designs [Jones & Thompson, 2003]. These experiments have found the TWINS model to be capable of predicting noise levels from wheel and track components to within about 2 dB. A number of intermediate stages of the calculation have also been examined, including the track response to excitation by the combined wheel-rail roughness. Thus the wheel-rail dynamic interaction model has also been validated, and the linear relationship between roughness, wheel and rail vibration and the resulting noise radiation has been clearly established.

1.3.2 Rolling noise sources and reduction techniques

The sources of rolling noise are well understood, thanks largely to the development and validation of TWINS. For typical ballasted track, the wheels and the rails are both significant contributors to the overall rolling noise level. Figure 1.1 shows an example of

the sound power level calculated using TWINS for a freight train on track with soft rail pads. In this case the rail is the major contributor to the overall noise in the frequency range from 500 to 2000 Hz. At lower frequencies, noise from the sleepers dominates, and at higher frequencies wheel noise takes over from the rail. The sleeper noise is increasingly significant in cases with stiffer rail pads [Thompson & Jones, 2000].



Figure 1.1 Example sound power level contributions calculated using TWINS for a typical tread-braked freight train wheel on ballasted track with soft rail pads.

Rolling noise reduction techniques have been summarised by Thompson and Gautier [2006]. At the source, a noise reduction may be achieved by reducing the roughness of the wheels and rails. The next option is to deal with the vibration response to the roughness excitation, for example by using wheel or rail dampers. Finally, noise barriers may be used to limit propagation.

To reduce the roughness of the rails, the track may be ground. Grinding is usually applied as part of track maintenance rather than purely to control noise, although in Germany a programme of acoustic rail grinding is in place [Asmussen *et al.*, 2006]. For the wheels, using disc-braked vehicles or composite brake blocks rather than cast-iron blocks in tread-braking systems gives a significant reduction in wheel roughness [Oertli, 2008]. The railways are phasing out the use of cast-iron brake blocks mainly due to legislation in the form of the TSIs; however, it is expected to take many years before they are eliminated.

Rail damping devices have been designed in order to reduce at source the component of railway rolling noise radiated by the track [Maes & Sol, 2003; Thompson *et al.*, 2007;

Thompson, 2008]. The rail dampers proposed by Thompson *et al.* [2007] are tuned, damped mass-spring absorber systems, with either a single mass or two masses enclosed in an elastomeric material. In their current form these dampers are attached to the rail in pairs in the middle of each sleeper bay. The dampers reduce the effective radiating length of the rail by increasing the track decay rate. To control the response of the wheel, wheel dampers can be used. Wheel dampers are most useful in controlling curve squeal, but they also have some effect on rolling noise [Thompson and Gautier, 2006]. Wheel shape optimisation can also give some benefit.

Noise barriers restricting propagation should be seen as a last resort for noise control, erected at specific problem sites. They are expensive, are not effective at all locations, and are often visually intrusive.

For a given roughness level, the noise from the system can be reduced by increasing the damping of the track, by acoustic optimisation of wheels and track components or by barriers. In the long term, however, the noise level is dependent both on the noise produced for a given roughness and on the rate of roughness development.

1.4 Railhead roughness

The roughness responsible for rolling noise has wavelengths between 5 and 500 mm [Thompson, 2009] and amplitudes from less than 1 μ m up to about 50 μ m [Thompson & Jones, 2000]. For a train speed of 100 km/h, this wavelength range corresponds to a frequency range of around 50 to 5000 Hz. Roughness is usually expressed as decibel levels with a reference value of 1 μ m, so a 1 μ m (rms) roughness amplitude is equivalent to 0 dB.

Linear noise prediction models are based on the assumption that the roughness from the wheel and the rail can be combined by simple incoherent addition. This was found to be valid by Thompson [1996] for typical roughness levels. However if either the wheel or the rail is significantly rougher than the other component, the resulting noise spectrum will be dominated by the rougher surface. For example, the type of braking system employed by the rolling stock has a significant effect on the roughness of the wheels. If cast-iron block brakes are used then the wheel roughness will tend to dominate unless corrugation is present on the rail [Dings & Dittrich, 1996].

Figure 1.2 shows some example roughness spectra reproduced from Dings and Dittrich [1996]. The 'average' rail roughness shown was determined by measuring roughness at 30 sites on the Dutch rail network, while the wheel data was obtained from at least 30 samples for each type of braking system. The frequency axis on this figure corresponds to a train speed of 100 km/h.



The TSI roughness spectrum [European Commission, 2005 & 2006] is also shown as a reference in Figure 1.2. This roughness spectrum is intended as a limit for the rail roughness on track that is to be used for pass-by noise measurements. It is considered to be a realistic rail roughness spectrum, and it is seen in Figure 1.2 to have a similar level to the average rail roughness reported by Dings and Dittrich [1996]. The TSI spectrum is used as a reference roughness spectrum throughout this work.

1.4.1 Measurements of railhead roughness

The standard ISO 3095 [2005] specifies the conditions required to achieve reliable, reproducible measurements of the noise emitted by railbound vehicles. Its appendix gives a

description of rail roughness measurement and processing techniques. The introduction of the Technical Specifications for Interoperability [European Commission, 2005 & 2006] has led the European Committee for Standardisation (CEN) to develop a dedicated standard for roughness measurements, EN 15610:2009 [CEN, 2009]. In its development, this standard was tested in a 'road test' described by Jones *et al.* [2008a,b]. This test used a number of different measuring instruments and examined the consistency of approach and measured spectra produced by a number of teams independently following the standard. Roughness measurements were found to have approximately a ± 2 dB variation when different measurements were taken of the same line of roughness.

Several different manufacturers produce instruments for measuring roughness but they generally fall into two categories. One type of measurement device uses a fixed straight edge as a reference with a displacement transducer moving along the rail. This type of instrument measures roughness in short segments of around 1.2 m, and as a minimum, five separate measurements are then required to determine the roughness of a test section. The other type of device uses an accelerometer mounted on a trolley which is moved along an unlimited length of rail, with the signal integrated twice to give a displacement output. Both systems have been found in the 'road test' to be capable of measuring roughness accurately [Jones *et al.*, 2008a,b].

Although roughness can be measured in a standardised way, the time required for a change in roughness to be measurable at a particular site means that little data has been published on the development of broadband acoustic roughness over time. Cox and Wang [1999] describe roughness measurements taken on two nearly new, recently ground, sections of a high-speed line in Belgium and repeated one year later. The rail pads in one section were very soft, around 80 MN/m, at the other section the rail pad stiffness was around 370 MN/m which is a medium stiffness for typical ballasted track. The initial roughness spectrum at both sites had a peak in the 20 mm wavelength band, attributed to the grinding process, which was not apparent in the measurement taken a year later. The site with softer rail pads had initially higher roughness levels at wavelengths longer than 20 mm. After a year of traffic the roughness spectrum at both sites was very similar. Roughness had decreased slightly for wavelengths longer than 2 mm at the softer rail pad site, and roughness increased for wavelengths longer than 20 mm at the stiffer rail pad site. It was concluded from this study was that roughness grows more on track with stiffer rail pads, even though the final roughness levels measured were similar at both locations. Bracciali [2004] made repeated measurements throughout a year over a 120 m section of tangent track on a high-speed line in Italy. The track had been ground prior to the measurements, and had visible grinding marks that were worn away over several months. Both the left and right rails were observed. On the left rail, roughness wavelengths shorter than 31.5 mm were found to remain stable or decrease slightly, while longer wavelength roughness increased. On the right rail, roughness wavelengths longer than 125 mm remained stable while an increase of 2 to 3 dB was observed at shorter wavelengths. Roughness increased more slowly in wavelength bands where the initial roughness level was lower. In general roughness growth was found to be 'slow but significant'.

The change in roughness over time observed by Bracciali [2004] or Cox and Wang [1999] does not give a definitive indication of the long-term development of roughness when the approximate ± 2 dB variability in repeatability of results found in the roughness road test [Jones *et al.*, 2008a,b] is considered. A longer time frame is needed to observe the phenomenon. Measurements at a corrugated site and at an adjacent smooth site have taken over about four years in the Netherlands [Hiensch *et al.*, 2002; Nielsen, 2003]. Here the corrugated site displayed a growth rate of about 2 dB per year at a wavelength of 40 mm, with roughness at other wavelengths also growing consistently for wavelengths longer than about 20 mm. In comparison, roughness at the adjacent smooth site remained almost constant, growing slightly in some one-third octave wavelength bands and decreasing slightly in others.

Verheijen [2006] carried out a survey of roughness measurements on the Dutch network. This survey did not explicitly carry out repeated measurements at a single site; however some overall conclusions were drawn on the development of roughness over time and the effects of rail grinding. Grinding introduces peaks into the roughness spectrum, which are worn away over time. When tracks are not ground regularly, in most cases roughness levels were found to increase at a rate of 1 to 2 dB per year. Roughness growth was not observed uniformly – the smoothest track measured had not been ground for 18 years.

The most comprehensive monitoring of long term roughness growth is that carried out on the German railway network on the 'Specially Monitored Track' sections [Asmussen *et al.*, 2006]. Roughness has been monitored since 1998 over almost 1000 km of track using a dedicated car with smooth wheels which uses its own rolling noise as an indicator of the track roughness. The technique used means that only average results are available, with broadband roughness found to increase by about 0.7 dB per year on average.

1.4.2 Corrugation – a special case of roughness

Corrugation is a periodic irregularity on the rail surface, which in severe cases can be seen with the naked eye. A corrugated rail has a roughness spectrum with a significant peak at one particular wavelength. Although there is little data in the literature on the development of broadband roughness over time, a great deal of work has been published on the subject of corrugation. Sato *et al.* [2002] estimated that over 1500 papers had been written on the subject, and these have been added to since.

Instances of rail corrugation are commonly classified by a damage mechanism and a wavelength-fixing mechanism, as described by Grassie and Kalousek [1993]. In some instances the reasons for the corrugation formation are clear, for example in cases with plastic deformation and heavy haul traffic. One type of corrugation, known as 'short-pitch corrugation' or 'roaring rail' is less well understood. This type of corrugation has wavelengths in the range from 25 to 80 mm. Studies of this form of corrugation are of interest when considering broadband roughness development since corrugation can develop from an initial situation of broadband roughness via a process of differential wear [Grassie, 1996].

Measurements of the development of corrugation have shown that the material properties of the rail could be a factor in corrugation development. At adjacent sites 250 m apart in the Netherlands subject to the same traffic, corrugation developed at only one of the locations [Hiensch *et al.*, 2002]. The only obvious difference between the sites was the rail manufacturer. In this case the different tendency to corrugate was explained by different wear resistance of the rails, although the corrugated rail displayed a higher wear resistance than the smooth rail. A work-hardened, white etching layer was observed on both rails but was more pronounced on the corrugated rail.

Hiensch *et al.* [2002] associated the wavelength of corrugation with a low track receptance near the sleepers around 1200 Hz, caused by the pinned-pinned resonance. This is a constant-frequency rather than a constant-wavelength phenomenon, and the wavelength of the expected corrugation is then proportional to the vehicle speed. Many other authors have also linked the pinned-pinned resonance to corrugation, including Hempelmann *et al.* [1991]; Hempelmann & Knothe [1996]; Nielsen [2003]; Sheng *et al.* [2006] and Croft *et al.* [2009]. Measurements on the London Underground [Grassie *et al.*, 2007] show that corrugation at a particular wavelength corresponding to the pinned-pinned resonance was shifted to a different wavelength by a change in the operating speed of the vehicles at the site. The pinned-pinned resonance is considered to be a likely wavelength-fixing mechanism of short-pitch corrugation [Grassie, 2005].

The pinned-pinned resonance may act to determine the wavelength of short-pitch corrugations in some cases, but alone it is insufficient to explain all corrugation formation. Corrugation does not appear on all discretely supported track types, and also appears on some continuously supported systems. Corrugation therefore remains an area of intensive research. Theories of corrugation formation are discussed in more detail in Chapter 2 in the context of models to predict the differential wear of the railhead.

1.4.3 Contact filter effects

The contact between the wheel and the rail has a finite area, meaning that roughness wavelengths shorter than the size of the contact patch tend to be absorbed by the contact spring and do not excite the wheel-rail system as effectively as longer wavelength roughness. The area of the contact patch is possibly the reason that short wavelength roughness such as marks left after rail grinding are observed to decrease over time, and why corrugation does not develop at wavelengths shorter than around 25 mm.

To account for the effect of the contact patch size, a 'contact filter' is required when predicting the interaction force between wheel and rail resulting from the combined roughness. Remington [1976a] used a frequency domain filter to account for the contact patch size. This filter required an assumption of the extent of correlation of roughness in the direction across the rail head. More recently, Remington and Webb [1996] developed a three-dimensional 'Distributed Point Reacting Spring' (DPRS) model of the contact patch. A layer of independent, non-linear springs is assumed to lie between the contacting surfaces. This DPRS method is more computationally efficient than the alternatives of finite element analysis or using analytical results for the stresses and displacements caused by a point force on the surface of a half-space. The results from the DPRS model were compared with results from a more complete model of the deformation between contacting bodies using a Boussinesq procedure. The models showed good agreement. The DPRS contact model is also available in the TWINS software.

Thompson [2003] used the three-dimensional DPRS model with many parallel measured lines of roughness data 2 mm apart, and compared the resulting contact filter characteristic with that calculated using Remington's analytical model [1976a] for different wheel radii and loads. He found that the analytical model gives excessive attenuation at high frequencies.

Often in practice only a single line of roughness data is measured along the railhead and for this case Ford and Thompson [2006] produced a simplified two-dimensional DPRS model for calculation of the filtering effect of the contact patch. This particular model of the contact filter is employed in this work, as it is the most appropriate method when taking a two-dimensional approach to the wheel-rail interaction force analysis. It is described further in Section 4.4.4.

1.5 Contributions made in this thesis to understanding the development of acoustic roughness

This work concentrates on mechanisms for broadband roughness development, whereas much of the existing work in the field of wheel-rail wear in rolling contact (reviewed in Chapter 2) has concentrated on corrugation. Most authors studying corrugation have employed Hertzian contact theory. However, recent work has shown that the choice of contact model can affect the wear prediction (see Section 2.5.4). This work examines in detail the differences arising from Hertzian and non-Hertzian assumptions.

Studies have shown that friction between sliding surfaces decreases as the velocity of sliding increases. Velocity-dependent friction coefficients have been used in models of wheel squeal [Xie *et al.*, 2006; Huang *et al.*, 2008], but only in conjunction with simplified models of the tangential stress distribution. In this work, a velocity-dependent friction coefficient is introduced to solve the rolling contact tangential stress distribution. The effect of velocity-dependent friction on stress distribution and rail wear is examined. A stick-slip oscillation is identified in partially slipping rolling contact when a velocity-dependent friction law is used.

Many existing models of corrugation growth assume frictional abrasive wear to be the sole damage mechanism. These models predict 'infinite' corrugation growth. But in many practical cases corrugation reaches a saturation point and steady-state, and at many sites corrugation never forms at all. To model broadband roughness growth there is a need to consider a combination of potential wear mechanisms. Two wear mechanisms have been shown in the literature to be important for roughness development; frictional abrasive wear and ratchetting (see Section 2.4). In this work the wear coefficient throughout the contact is determined by the local stresses, and is not limited to mild wear. The contact conditions that are required for the transition between frictional abrasive and ratchetting wear are examined.

The effect of rail dampers on railway rolling noise when the wheel/rail roughness is presumed to be constant is well known; however their effect on track dynamics other than the track decay rate has not been examined. This work includes rail dampers in a finite element model of the track. The features and complexity of the rail damper model that are required in order to simulate their behaviour accurately are determined. The change in track receptance due to the rail dampers is then examined. It is of particular interest to examine the effect of the rail dampers on the anti-resonance above a sleeper at the pinned-pinned frequency of the track (see also Croft *et al.* [2009]).

The wheel-rail interaction forces are modified by the application of rail dampers, in particular at the pinned-pinned frequency and corresponding wavelengths. The pinned-pinned mode has been linked to railhead wear and corrugation. This work demonstrates that rail dampers reduce the wheel-rail interaction forces at wavelengths corresponding to the pinned-pinned frequency (Section 4.9). The resulting roughness development predictions for situations with and without rail dampers are presented in Chapter 9, along with the roughness spectra measured by Deutsche Bahn AG at the Gersthofen test site as part of the Silence Project.

2 REVIEW OF ROUGHNESS AND CORRUGATION MODELLING

2.1 Introduction

The aim of this work is to develop a model of the growth of broadband rail roughness. As described in Section 1.2, this requires the implementation of a model of the dynamic interaction between the wheel and track, as well as a model of the stress distribution in rolling contact and the resulting wear of the rail. In this chapter, existing work in these fields is reviewed with the aim of determining appropriate methods to use in the current analysis.

This review of the literature has been divided into three main sections corresponding to the three stages of modelling roughness development that have been identified in Section 1.2. Relevant literature on the subject of wheel-rail interaction is further divided into models in the frequency domain and models in the time domain. Work on wheel-rail contact mechanics begins with Hertzian contact theory and introduces subsequent developments in the field. The review of wear models examines different wear mechanisms and experimental work to characterise the different types of wear.

This chapter also reviews models of corrugation initiation and growth, in Section 2.5. These models are generally based on the theories and models reviewed in Sections 2.2, 2.3 and 2.4, and some of those theories and models were developed specifically to examine corrugation. Reviewing these corrugation models separately clarifies the appropriate methods required to model roughness development in this work.

Finally the approach used to model broadband roughness development in this work is summarised in Section 2.6.

2.2 Wheel-rail interaction force models

A model of the interaction between the vehicle wheels and the track is needed as the first step towards modelling the change in roughness of the railhead over time. The interaction between wheels and rails has been modelled extensively by many researchers for various purposes. Models are used to predict the forces on different track components, to predict the noise radiated from the system, and to predict the wear of the railhead and wheels. In this section, different wheel-rail interaction force models are discussed with the aim of determining the necessary model characteristics to include in this work. Most interaction force models may be categorised as either time domain or frequency domain models. Historically, time domain models have required a large computational capacity whereas frequency domain models are usually more efficient, albeit less flexible. Within these two broad categories, models for different applications employ different representations of the various track and vehicle components.

2.2.1 Early examples of frequency domain models

The TWINS model of wheel-rail noise generation developed by Thompson [1993a-e] is based on a frequency domain model of the wheel-rail interaction following the work of Remington [1976a]. Remington represents the rail as an infinite beam, coupled to a static wheel, with a moving roughness excitation passing between the two. This means that the receptances of wheel and rail are taken for a stationary force and do not include the splitting of resonance frequencies resulting from the vehicle motion [Thompson, 1993a]. This deficiency is corrected for the wheel in Thompson's model [1993e], where it is important for noise, and is implemented in the TWINS software.

Grassie *et al.* [1982] proposed two frequency domain models of the track, one with an infinite track on continuous supports and the other a periodically supported track (similar models of the track are included in TWINS). They calculated the contact forces between a moving wheel mass and a sinusoidal corrugated rail. At around the same time, Clark *et al.* [1982] developed a time domain model using similar parameters. Grassie noted qualitative agreement between the predictions from the two models. Some differences in magnitude were evident but could be attributed to differences between the damping models. At this stage of development, none of the models took account of the effect of the presence of multiple vehicle wheels on the wheel-rail interaction.

2.2.2 Frequency domain models with multiple wheels

Wu and Thompson [2001, 2002] developed a frequency domain model with multiple wheels on the rail, by using the superposition principle. In this method, one wheel is assumed to be active (roughness is present) while the others are passive. Each wheel is treated in turn as the active wheel. The principle of superposition may then be applied to determine the resulting forces and vibration. Coupling of the response to each wheel

through the vehicle body is neglected as the suspension will isolate the vehicle body at the frequencies of interest of above 50 Hz. Wu and Thompson [2001, 2002] found that for frequencies up to about 550 Hz (and for the track parameters used) the interaction forces are virtually unaffected by the presence in the model of multiple wheels on the rail. Between 550 Hz and 1200 Hz there is noticeable difference between the single and multiple wheel models. Wave reflections between the wheels are responsible for peaks in the interaction force spectrum. With softer rail pads, this effect is more noticeable than with stiffer rail pads; in the latter case the interaction force around the pinned-pinned frequency is dominant over the reflected wave effects.

Since the spectrum of the railhead wear is thought to be proportional to the interaction force spectrum, corrugation is expected to be associated with a peak in the interaction force spectrum. Consequently these authors recommend that effects from multiple wheels should be included in any model of interaction force and the resulting wear or roughness development. Wheels more than 10 m away may be neglected because of the decay of vibrations along the track.

2.2.3 Improving the representation of the rail in frequency domain models

For vertical vibration, a single Timoshenko beam model may be used only for frequencies up to about 2000 Hz. Above this frequency the foot response may be considerably higher than the head response. Figure 2.1 shows different possible representations of the rail cross-section of increasing complexity. Wu and Thompson [1999a] modelled the rail using a double Timoshenko beam model to give a better representation of the rail profile without the increase in computational effort required by a finite element model. One beam represents the rail head and web, connected by continuously distributed springs to another beam representing the rail foot (shown in Figure 2.1(b), symmetry means the two rail feet shown can be treated mathematically as a single beam). Wu and Thompson [1999a] also point out that a discretely supported rail model is important for frequencies above 1000 Hz.

A multiple beam model can also be used to examine the lateral behaviour of the track [Wu and Thompson, 1999b]. Compared with modelling vertical rail vibration, it is more difficult to model lateral rail vibration because cross-sectional deformation of the rail should be taken into account. Finite element methods may be used but these result in a large number of degrees of freedom. Wu and Thompson proposed a model where infinite

Timoshenko beams with torsion represent the head and foot of the rail, connected by an array of finite beams to represent the bending of the web.

Alternatively, the rail cross-section can be represented by finite elements as shown in Figure 2.1(c). An example is the frequency domain interaction model used by Müller [1999, 2000] in his studies of corrugation growth. This model uses finite element matrices to describe the rail so that the rail profile can be considered more accurately than by using a simple beam model. The track is assumed to be infinite, and the rail is discretely supported by rail pads, sleepers and ballast. Müller takes account of the elasticity of the wheelset, six degrees of freedom at each node, transient creep, shift of the contact point (due to the geometry of the initial roughness) and filter effects due to the size of the contact patch. This is a comprehensive model, building on the work of Hempelmann *et al.* [1991, 1996], within the limitations of assumed linearity about small levels of roughness and without including the effects of multiple wheels. As with all the frequency domain models reviewed so far, a further limitation is that the load does not move along the rail, rather a 'roughness strip' is moved between the wheel and the rail, and therefore parametric excitation due to the discrete supports is also not considered.



Figure 2.1 Increasing complexity of representation of rail cross-section: (a) single beam model; (b) multiple beam model; (c) finite elements [Wu & Thompson, 1999a]

2.2.4 Parametric excitation effects in frequency domain models

Parametric excitation occurs as a result of variation in the dynamic stiffness of a track arising from supporting the rail with sleepers at discrete locations. This variation in stiffness can be seen at the pinned-pinned resonance frequency in the rail receptance (the inverse of stiffness) above a sleeper and in the middle of a sleeper bay, shown in Figure 2.2. The wheel-rail interaction force therefore varies periodically with the sleeperpassing frequency. Wu and Thompson [2004] employed a single wheel frequency domain model to demonstrate the magnitude of this parametric excitation on the system, and to compare it with the excitation arising from normal wheel and rail roughness. The interaction force due to parametric excitation was found to increase with increasing train speed. The interaction force spectrum shows the sleeper passing frequency and its harmonics, with high force components seen around the pinned-pinned frequency. However if major discontinuities or large roughness levels are present, the parametric excitation effect becomes less significant compared with the higher excitation levels from other sources.



Figure 2.2 Vertical point receptance of rail from a single Timoshenko beam model with discrete supports: ——— mid-span; — — — above sleeper. Obtained using frequency domain model described in Chapter 3.

Parametric effects are most noticeable when the sleeper spacing is exactly periodic. Wu and Thompson [2000] examined the influence of random sleeper spacing and ballast stiffness on the dynamics of a track, using a method introduced by Heckl [1995]. The results show that with random parameters, the point receptance and track decay rate become distributed rather than having a fixed value. The random ballast stiffness mainly affects track vibration at low frequencies below 300 Hz, while the random sleeper spacing influences the response in the whole frequency range considered, 50 to 1500 Hz. In particular the pinned-pinned resonance becomes less sharp and can be suppressed

completely if there is enough variability in the sleeper spacing. The effects of random sleeper spacing are more noticeable on track with stiff pads than track with soft pads. Despite these variations, the regular and irregular track models do not predict a significant difference in the resulting rolling noise.

2.2.5 Recent developments in frequency domain models

Sheng *et al.* [2005] developed a frequency domain model based on spatial harmonics that is very efficient computationally, and allows the problem to be addressed by using the force-time history for a single sleeper bay. The dynamic force and thence the roughness growth are modelled as periodic with the sleeper spacing. This model has the disadvantage that the track structure must be exactly periodic; however it can include multiple moving wheels on the rail. A single Timoshenko beam was used to represent the vertical dynamics up to 3000 Hz. This wavenumber-based approach has been used to compare results with moving and stationary loads. The load speed is found to have a significant effect on the vibration response of the track at the pinned-pinned frequency. The height/depth of the peak/dip in the track vibration response spectrum at the pinned-pinned frequency is split into two.

Some improvements to this model have been made and are described by Sheng *et al.* [2007] incorporating a Fourier-series approach. This work removes the limitation that force and roughness must be periodic with the sleeper bay length, allowing the roughness excitation to be periodic over a greater number of sleeper bays.

Frequency domain wheel-rail interaction models can take account of multiple wheels and parametric excitation, and are more computationally efficient than many time domain models. The remaining limitation is the assumption of linearity inherent in a frequency domain model. This is valid as long as the roughness excitation of the system is low, so the frequency domain is often used for noise prediction models. In comparison, time domain models have been developed mostly to study cases where non-linear effects are significant, for example in modelling rail corrugation or forces arising from wheel flats or polygonisation.

2.2.6 Early time domain models

Clark *et al.* [1982] developed one of the earliest time domain models of the dynamic effects generated by a wheel running over a corrugated rail, and carried out an experiment to confirm the predicted dynamic responses. This model used a 20 sleeper bay length of rail, represented by Euler beam elements with fixed boundary conditions at the ends. The track must be a finite length because the time domain model uses a modal summation technique. The sleepers were also included in the track model as flexible beams, to incorporate their bending modes in the frequency range of interest which was from 0 to 1300 Hz.

Nielsen and Abrahamsson [1992] developed a general method for the analysis of problems involving moving non-linear systems on continuous damped beam structures. The method was based on earlier work by Lundén and Åkesson [1983], Abrahamsson [1988] and by Nielsen [1991] on the natural frequencies and modes of beam structures, and is expanded in later papers for the specific purpose of modelling wheel-rail interactions. This original Nielsen model entailed a track modelled by Timoshenko beam elements, fixed at each end, characterised by exact stiffness matrices and solved via the Wittrick-Williams algorithm [Wittrick & Williams, 1971] to obtain the modal parameters. The vehicle model was of half a bogie, *i.e.* two wheels, coupled to the track via constraints on contact force and the displacements and accelerations of the point of contact. These constraints allowed for the inclusion of a roughness function along the rail surface. Nielsen [1991] used interpolating polynomials to distribute the forces and displacement of the point of contact onto the adjacent nodes of the track finite element model as the wheel moves along the track with time. Another feature of Nielsen's model is the use of a state-space formulation of the problem to enable a solution by time-stepping using a standard Adams integration routine.

Nielsen [1994] used this model to examine the influence of various track parameters on the dynamic wheel-rail interaction. The effect on the system of changes in sleeper cross-sectional area, rail cross-sectional area, sleeper spacing, pad stiffness and damping and ballast stiffness under the end and middle of the sleeper was assessed. For this study, rail and wheel roughness were neglected and responses were found to be nearly quasi-static.

Further work by Fermér and Nielsen [1995] examined the influence of soft rail pads compared with stiff rail pads. The vertical dynamic interaction forces were calculated using Nielsen's model for a range of different loading cases and compared with measurements taken using an instrumented wagon. The standard deviation of the results was found to increase with vehicle speed, but in general the model calculated frequency response functions that matched well with measured values. The calculated interaction forces also agreed reasonably well with measurements for train speeds up to about 80 km/h. The sleeper responses were overestimated - it was concluded that the rail pad and ballast model used was overly simplistic to model the frequency dependent behaviour of the track throughout the full frequency range examined, up to 1000 Hz.

Nielsen and Igeland [1995] revised this model, which is known as 'DIFF', this time using standard polynomial finite elements and improving the efficiency of the model by moving away from the Wittrick-Williams algorithm. The sleeper model was extended from simple masses to beam elements. They used the model to investigate the effect of wheel and track imperfections on interaction forces. A finding was that a model with a single unsprung mass as the vehicle model may underestimate the dynamic response of the system.

Two wheels can interact through the vibration of the track structure. Igeland [1996] investigated this phenomenon in a study that combined the interaction force model with wear theory in an attempt to explain rail corrugation growth. She found that resonance in the coupled bogie/track system has a significant effect on the interaction forces, especially if the wheelbase is equal to an integer number of sleeper spacings, as this appears to exaggerate the effect of the pinned-pinned resonance. Consequently, including more than one wheel is important in an interaction force model. Igeland [1999] also introduced variable sleeper spacings into the model in order to compare theoretical results with full scale measurements.

Igeland [1997] modified the model further by improving the ballast model to include a ballast mass as well as stiffness and damping. However, she reverted to a single wheel vehicle model and a simplified rigid sleeper model.

2.2.7 Time domain models with more complex track support representation

The interaction model developed initially by Nielsen has continued to be updated. Two areas of improvement have been (1) modelling the vehicle using flexible components [Andersson & Oscarsson, 2000] and (2) the modification of the linear track model to

include state-dependent (non-linear) properties of the ballast and rail pads [Andersson & Oscarsson, 2000; Nielsen & Oscarsson, 2004].

Andersson and Oscarsson [2000] considered vertical track dynamics only. They used statedependent ballast and pad parameters, *i.e.* the parameters depend on the load, and are valid for a dynamic frequency range rather than just for static loadings. A three-parameter viscoelastic model of the rail pad was employed consisting of a spring in parallel with a Maxwell element. The model was found to represent both low and high frequency train/track interaction well. The vehicle representation was also extended, by including one wheel, half an axle (modelled by finite elements) and a quarter of a bogie frame represented by a lumped mass. Previously a lumped mass wheel model had been used. The findings from this study were that only small differences exist between the results for a state-dependent track model with a flexible wheel and a linear track model with a lumped mass wheel. A simple vehicle model was found to be sufficient if only the vertical motion of the system is to be considered.

Nielsen and Oscarsson [2004] used complex modal superposition in the presence of statedependent track properties to model the wheel-rail interaction. They included the stiffness and damping of rail pads and ballast by adding a linear contribution from the unloaded track to increments determined by the time-varying state of the simulation. The timevarying component was added in the form of external forces on the corresponding nodes of the model. This model has been validated by experimental measurements, and using statedependent track properties was found to improve the agreement of the model results with experimental data.

2.2.8 Recent developments in time domain models

Nielsen's model DIFF, developed at Chalmers University over many years, was extended to allow for general motion by Andersson and Abrahamsson [2002], who added lateral and longitudinal dynamics to the existing vertical track dynamics. They modelled both rails and two wheelsets to include the flexibility of the vehicle. This model (known as DIFF3D) was developed to form a basis for wear and corrugation studies for a frequency range of up to 1000 Hz. General vehicle dynamics results from the model were verified by comparison with a commercial multibody program with good agreement. Vertical dynamics results from DIFF3D were compared with the output from the two-dimensional DIFF. The normal forces calculated by the two and three-dimensional models are almost identical.

The two-dimensional DIFF model has been validated for the frequency range of 20 to 2000 Hz by a campaign of field tests using an instrumented wheelset and a wheel impact load detector [Nielsen, 2006]. Several different versions of the model were examined, considering different visco-elastic representations of the rail pad and either a single wheel or two-wheel model of the vehicle with the wheel itself included either as a rigid mass or a more detailed finite-element representation. All the vehicle models were found to give similar results although with some differences. Including more than one wheel in the model allows the capture of standing waves in the rail between successive wheels. A lumped mass wheel model neglects the influence of wheel modes in the frequency range of interest. These give several distinct peaks and troughs in the interaction force spectrum. The choice of track model was found to have a negligible effect on the calculated contact forces, indicating that a simple spring and damper representation of the rail pad is adequate to predict the force in the frequency range of the model, *i.e.* from 20 to 2000 Hz. Nielsen [2006] concludes: "Based on the good and consistent agreement between measured and simulated vertical contact forces, both with respect to magnitude and frequency content, it is argued that the computer program DIFF is a useful tool in investigations of vertical dynamic train-track interaction at high frequencies."

Other recent work in this area has concentrated on improving the computational efficiency of time domain wheel-rail interaction models. Baeza *et al.* [2006] proposed a modal substructuring approach, where the rail and sleeper, which display linear behaviour, are modelled using modal coordinates and the other components, such as the rail pads and ballast, are introduced by means of the forces in connecting elements. DIFF was used as a reference to validate the model, but the difference in computational cost of the new approach is not quantified in the paper.

Pieringer *et al.* [2008] represent the wheel and rail by sets of impulse response functions or Green's functions, a technique that has its origins in work on tyre/road contact. Since the Green's functions are calculated in advance for a particular wheel and track model, the calculation of interaction forces is extremely fast for a perfectly periodic track. The DIFF model has again been used as a reference for model validation, with good agreement found for the normal force calculation. An advantage of Pieringer's technique over DIFF is that

the rail is not divided into elements, so no discretisation effects occur. Also, it has the potential to include more complex representations of the wheel-rail contact, including tangential effects. Most time domain interaction force models simplify the contact to a Hertzian spring acting at a single point, and pre-process the roughness input to take account of the contact filter effect. Any tangential analysis of the contact follows as a separate calculation step. In Pieringer's model, the size of the contact and the distribution of normal and tangential stress taking account of the surface roughness may be determined at each time-step without the need for pre- or post-processing.

This review of time domain wheel-rail interaction models has concentrated largely on work done at Chalmers University of Technology. Other authors to use time domain modelling techniques include Zhai *et al.* [1996, 2001], Ilias [1999], Wu and Thompson [2004], and Xie and Iwnicki [2008b,c].

2.2.9 Summary of wheel-rail interaction models

Models of the interaction force between wheel and rail have been developed and refined over the last thirty years. The vertical interaction problem in particular has been studied in great detail, including the experimental validation of a two-dimensional time domain model by Nielsen [2006]. Depending on the frequency range and parameters of interest, models of varying complexity are available in both the time and frequency domains. The general statement that time domain models tend to be more flexible whereas frequency-domain models are more computationally efficient still holds, although advances in computational capabilities mean that time domain analyses are becoming possible for increasingly complex models. An example of this is the three-dimensional analysis of Andersson and Abrahamsson [2002]. In addition, the impulse response function approach of Pieringer *et al.* [2008] has the potential to make a significant reduction in calculation times compared with more established time domain techniques.

2.3 Wheel-rail contact mechanics

In most wheel-rail interaction force models, the effect of the discrete size of the contact patch is limited to filtering the roughness excitation to the system. The contact area is commonly replaced by a Hertzian spring acting at a single point. This is adequate for the determination of the overall normal forces. However to predict the wear of the railhead, a more detailed analysis of the wheel-rail contact is required. In this section, some of the
significant contact mechanics theories are reviewed, beginning with frictionless Hertzian contact, which considers normal forces only. Techniques for including tangential loadings are also discussed. Alternatives to Hertzian contact theory include Kalker's numerical methods and elastic foundation models.

2.3.1 Background

Johnson's book *Contact Mechanics* [2001] is a comprehensive text on the subject. It begins with stationary contact problems and describes theories of increasing complexity, including elastic and inelastic materials, normal and tangential loading, rolling and sliding contact and treatment of rough surfaces.

The aim of the contact analysis in the context of this current work is to determine the size and shape of the contact between the wheel and the rail, and the distribution of normal and tangential stresses throughout the contact area.

In the case of rolling contact, tangential loads can result in a relative displacement between parts of the contacting surfaces. In terms of the contacting bodies as a whole, this relative displacement is known as creep or creepage. Creep is relieved inside the contact area by a small relative motion over part of the interface known as 'slip'. The rest of the interface 'sticks' or deforms without relative motion. The magnitude of creep between contacting bodies is often defined as a ratio, for example in terms of the difference between the translational velocity of the wheel and its circumferential velocity at the rail (see Section 6.2).

The division of the contact into stick and slip zones is not known in advance and must be determined by trial, for example by initially assuming that no slip occurs anywhere and then examining the solution for stress distribution iteratively. With steady rolling, it may be assumed that there is no change in either the forces or contact geometry over time. Many models of corrugation initiation and growth assume steady rolling. But if the interaction forces fluctuate with time, as in the rolling of a wheel over a rough rail, any calculation of the stress distribution in the contact zone and the areas of stick and slip should proceed step by step from a set of initial conditions.

2.3.2 Hertzian theory

Hertzian theory (developed in 1880 by Heinrich Rudolf Hertz) shows that the contact area between parabolic surfaces is elliptical, describes how it grows in size with increasing load, and gives the magnitude and distribution of surface tractions transmitted across the interface. The restrictions or assumptions of Hertz theory are summarised by Johnson [2001]. The profiles must be parabolic, and any higher terms are neglected. Surfaces must be smooth, non-conforming and frictionless. Elastic half-space theory must be valid, that is the contact dimensions must be small compared with radii of curvature of the undeformed surfaces and the contact stresses must not depend on the shape of the bodies away from the contact patch.

Based on these restrictions, Hertzian theory is not strictly applicable for the wheel-rail contact. The roughness of the surface in practice is likely to contain wavelengths that are of comparable length to the dimensions of the contact patch, so the assumption of non-conforming surfaces may be incorrect on the scale of the roughness. Conforming surfaces may also arise across the width of the contact if the wheel and rail profiles are worn. In addition, the assumption of frictionless contact is inconsistent with a model to predict the wear of a rail due to frictional work.

Despite its limitations, Hertz theory gives a complete three-dimensional description of the normal stress distribution for a steady-state without oscillating forces. It has the advantage that the size of the elliptical contact and the ratio of the axes can be calculated directly from a simple set of equations with a low computational effort.

2.3.3 Including tangential effects

The tangential force that can be supported by the wheel-rail contact increases with increasing creep, until the friction limit is reached and the full contact area slips. Figure 2.3 shows a simple linear representation of the creep-force relationship. The first model of wheel-rail contact developed specifically for a railway application was that published by Carter [1926], a two-dimensional solution to the problem of frictional losses in driving or braked locomotive wheels. In this case, large tangential forces are transmitted. Carter developed a creep-force law connecting the driving-braking couple and the creep ratio (see Section 6.2). Carter's theory is considered insufficient for vehicle motion simulation as only longitudinal, and not lateral, forces are included [Kalker, 1991]. Despite this

limitation it has all the necessary elements for a basic prediction of the stress in the contact patch based on Hertz theory.



Figure 2.3 Example of a linear creep-force relationship

Vermeulen and Johnson [1964] generalized Carter's theory from the two-dimensional model, firstly, to include circular contacts and longitudinal and lateral creep (but no spin), and then to include elliptical contact areas. They assumed a linear relationship between tangential force and creep. Shen, Hedrick and Elkins [1983] then developed a non-linear creep law based on Vermeulen and Johnson's theory. They used more accurate creep coefficients than the approximate values used previously, and incorporated spin (although not entirely successfully according to Kalker [1991]). Their theory is valid for unrestricted creep but only for small spin, and is therefore useful for vehicle dynamics simulation on straight track where no flanging occurs. Again its use is confined to elliptical Hertzian contacts.

Kalker's linear creep contact theory makes use of three-dimensional Hertz theory [Kalker, 1991]. This linear theory is valid for cases with low creep or vanishing slip, as is the case for example if the coefficient of friction is high. Rolling takes place in the direction of one of the axes of the contact ellipse. Creep coefficients are defined depending only on Poisson's ratio and the ratio of the axes of the contact ellipse. The latter in turn depends only on the curvatures of wheel and rail.

Kalker provided a computer program called 'FASTSIM' for calculating the total force in rolling contact from a given creep and spin, assuming an elliptical Hertzian contact patch [Kalker, 1982]. The main advantage of FASTSIM is its speed and relative accuracy for cases with Hertzian contact, but it is less accurate if the Hertzian assumptions do not apply.

The algorithm divides the contact area into strips, treating each strip as a two-dimensional problem. This method neglects interaction between the strips, and works best if the contact patch width is much greater than its length in the rolling direction. The surface displacement at a point is determined only by the surface traction at the same point, whereas in reality the displacement at a point depends on the traction at all points on the surface.

2.3.4 Non-Hertzian contact

Kalker also developed an 'exact' method for all contact problems of bodies that can be described by half-spaces, as described in his book *Three-Dimensional Elastic Bodies in Rolling Contact* [1990]. This is implemented in the computer program 'CONTACT'. CONTACT works by a variational method, minimising a strain energy function subject to the constraint that the contact pressure is positive everywhere (and presumably zero at the edges of the contact). It can be used for both Hertzian and non-Hertzian contact problems, and takes account of transient effects by calculating step by step from given initial conditions, following the loading history of the particular problem.

The main limitation of CONTACT is the computation time [Kalker, 1991]. An extremely fine discretization of the potential contact area is required in order to deal with micro-roughness of the surface. Also, CONTACT is limited to elastic problems and does not include plastic deformation of any asperities.

If micro-roughness is included, it is predicted using CONTACT that the maximum contact stresses will actually be several times higher than those predicted by Hertzian theory, and correspondingly the real contact length will be less than predicted by Hertz. The high stresses are limited to a layer next to the surface which should experience some plastic deformation, which may be responsible for the formation of white etching layers [Kalker, 1991].

Although CONTACT has historically been prohibitively slow for analysing rolling over the distance required for railway wear analysis, since its inception it has remained the standard against which most other models have been validated. In Knothe's [2008] review of the history of wheel-rail contact mechanics he states, "Nowadays, most problems of rolling contact mechanics can be solved using Kalker's programs".

2.3.5 Elastic foundation models

An alternative to the elastic half-space assumption is to replace the contact between the two bodies by a Winkler elastic foundation or 'mattress'. This avoids the difficulty in elastic contact stress theory that the displacement at any point on the contact surface depends on the distribution of pressure throughout the whole contact [Johnson, 2001]. If there is no interaction between the springs of the model, then the contact pressure at any point depends only on the displacement at that point. Kalker's FASTSIM code uses this principle. The shape and size of the contact patch is determined from the elastic foundation model, then stick and slip conditions can be applied (as in other models) to find the distribution of the tangential traction.

Remington and Webb's [1996] three-dimensional 'Distributed Point Reacting Springs' model used non-linear independent springs to simulate the normal contact, but does not solve the tangential problem.

2.3.6 Other developments in contact mechanics

In Kalker's [1991] review of wheel-rail rolling contact theory he draws conclusions about the appropriate models to use for various purposes. For wear calculations where the profile is developing over time without particular corrugation, Kalker is of the opinion that only a complete, exact theory such as CONTACT is appropriate. Ten years later Knothe *et al.* [2001] reviewed the state of the art in techniques for modelling wheel-rail contact mechanics. CONTACT remained the most encompassing model available, although Knothe notes that simpler models have uses in steady-state cases.

Jin and Zhang [2001] have produced a rolling contact theory similar to Kalker's model but valid for general three-dimensional rolling contact, avoiding the half-space assumption. This allows the influence of the geometry of the contacting surfaces and any other boundary conditions, such as elastic deformation of the rail structure, to be included. However it is not clear how much difference this makes to the solution obtained.

Surface adhesion effects in rolling contact have been analysed by Hao and Keer [2007]. These effectively contribute to friction in the contact, but, while friction forces are commonly modelled as being proportional to the normal load, surface adhesion can occur in the absence of overall forces as a result of attraction between surfaces on an atomic level. This is an interesting tribological problem, but for railway applications the high normal forces mean that surface adhesion effects become less significant.

Recent work on rolling contact in many cases has been driven by vehicle motion simulation problems. Particular attention has been paid to improving on a Hertzian representation of the contact without increasing the computational cost of the problem, and also to the creep-force relationship, and predicting adhesion limits for traction control purposes.

Ayasse and Chollet [2005] allow for non-Hertzian conditions in the lateral direction (*e.g.* the transition from an arc to a flat) but assume Hertzian conditions in the direction of travel. This technique does not account for any effects of the surface roughness or possible two-point contacts, but has applications for improving on a purely Hertzian model in vehicle motion simulations. The size and shape of the contact patches are no longer elliptical due to the correction in the lateral direction. The calculation of stress in the patch is carried out in strips in a similar manner to Kalker's FASTSIM algorithm.

The effect of surface roughness on the contact problem has been studied by Bucher *et al.* [2002], to determine if the gradient of the creep-force relationship is changed by the presence of surface roughness. They show that the roughness can be considered as a boundary layer, and that increasing roughness levels reduce the tangential force that can be supported in the contact. Pauk and Zastrau [2003] used a similar technique to include roughness in a rolling contact analysis by means of dimensionless roughness parameters. This is an analytical model that effectively modifies the shape of the normal and tangential stress distributions depending on the overall roughness level rather than the particular roughness profile, giving more creep between rough rollers than between smooth rollers. These models are used in the low creep range leading up to the adhesion limit.

The negative gradient part of the creep-force curve at high creepages is another subject of study. This has been observed in measurements, for example by Zhang *et al.* [2002] and by Polach [2005], and is attributed to velocity-dependent friction, itself probably the result of thermal effects [Kragelskii, 1965]. Bucher *et al.* [2006] have modelled dry friction in the wheel-rail contact using the technique of movable cellular automata. They investigated different friction laws, temperature-velocity dependence and the effect of roughness at

three levels of consideration – macro, micro and nano. They found that the decrease in creep-force curves at high creep values is well explained by temperature, velocity and friction effects. The friction law was found to be highly dependent on the material properties, and on dynamic processes at the nano-scale.

The effect of temperature and velocity on the coefficient of friction has also been studied using finite element methods. Daves and Fischer [2002] solved the contact problem taking account of non-linearities, contact stress distribution, temperature effects and plastic deformation. Their study shows that the material properties at the contact may be highly variable over time. They conclude that assuming elasticity in the wheel-rail contact may be overly simplistic.

Giménez *et al.* [2005] introduced a velocity-dependent friction coefficient into Kalker's FASTSIM algorithm (see also Alonso and Giménez [2008a]). This model has been used by Xie *et al.* [2006] to study wheel squeal but until now, velocity-dependent friction has not been considered in studies of roughness growth or corrugation.

Historically it has been impossible to carry out experiments to characterise the contact between rough wheels and rails due to the difficulty in measuring stress distributions inside the contact area. Marshall *et al.* [2006] have used ultrasound measurements to quantify the stress distribution in new, worn and damaged wheel-rail contacts. The roughness was found to influence the stress distribution significantly compared with a Hertzian analysis. Short wavelength roughness on the wheel was also observed to become smoother over time, repeating, in the laboratory, a result that has been observed on rails in the field, where short wavelength grinding marks wear away after many wheel passages.

2.4 Wear mechanisms

In a general sense, wear may be divided into four main types [Rabinowicz, 1965]. (1) Adhesive wear occurs when material fragments are pulled off one, initially smooth, surface and adhere to the other surface in sliding contact. Material is transferred rather than lost. (2) Abrasive wear occurs when rough surfaces slide over one another, displacing material which forms loose wear particles. (3) Surface fatigue wear is observed during repeated sliding or rolling, causing the formation of cracks which eventually result in the break up of the surface. (4) Finally, corrosive wear occurs when sliding wears away the protective film formed by corrosion, allowing further corrosion to take place. Of these four wear mechanisms, abrasive wear and surface fatigue are the most relevant for railway roughness development.

Nearly all the discussion of railhead wear or damage mechanisms in the literature has centred on the mechanisms of corrugation formation and rolling contact fatigue, rather than on mechanisms for general roughness growth over time. This is because corrugation and rolling contact fatigue are more immediate problems for railways than the noise arising from the normal broad spectrum of rail roughness levels. The wear mechanisms involved in corrugation development, rolling contact fatigue and general roughness growth are not necessarily the same, and it is likely that a combination of the various wear mechanisms occurs in many cases.

For corrugation, Grassie and Kalousek [1993] summarised the knowledge of the time on its characteristics, causes and treatments. They classified six different types of corrugation (heavy haul, light rail, booted sleeper, contact fatigue, rutting and roaring rails). They attribute corrugation development to three damage mechanisms: plastic deformation, rolling contact fatigue and frictional or abrasive wear.

2.4.1 Abrasive wear

Abrasive wear occurs as a small amount of material is removed with every wheel passage. This is often assumed to be proportional to the frictional work done in the contact area. The wear relationship is commonly represented by Archard's wear equation [Archard & Hirst, 1956].

In the time and frequency domain interaction models discussed in Section 2.2, most authors that have gone on to examine roughness or corrugation have done so by assuming an abrasive wear mechanism. Models assuming abrasive wear tend to differ in the complexity of their representation of the contact mechanics, rather than in their representation of the wear, although there is some variation in the constant of proportionality used in the wear equation.

2.4.2 Rolling contact fatigue and 'ratchetting' wear

Rolling contact fatigue is generally thought of as micro-cracks in the rail material growing as the result of repeated loadings, and causing damage to the rail when they become large enough and reach the surface. Rolling contact fatigue can result in general wear of the railhead, as well as cracking. 'Ratchetting' refers to the wear mechanism that is a result of high stresses exceeding the elastic limit of the material and leading to some form of plastic flow [Kapoor, 1997]. Depending on the loading there may be some strain hardening so that on subsequent applications of the load, there is no further plastic flow. High loading can lead to varying degrees of plastic strain which may result in failure.

Alwahdi *et al.* [2005] considered ratchetting wear of a ductile material to be a possible mechanism for the wear of rails. They developed a model that divides the material into layers of 'bricks'. After failure, a brick will be removed when its layer reaches the surface of the material. This paper builds on earlier work by Kapoor and Franklin [2000] and Franklin *et al.* [2001, 2003].

The orthogonal shear stress used by Alwahdi *et al.* [2005] was calculated for a Hertzian elastic contact with an elliptical pressure distribution. The model allowed for partial slip in the contact region and the effects of partial slip and work hardening on the wear rate were examined and compared with the full slip case. The model predicted that, after an initial period with no wear, wear rates would increase sharply before settling down to a steady-state. Work hardening was shown to increase the duration of the initial period of no wear, and to reduce the wear rate in the steady-state period. Increased traction coefficients resulted in increasing wear rates. Increasing the creep increased the wear rate up to a maximum value. Overall, the wear rates calculated using this model were found to be in good agreement with measured values for total height of rail removed (about 0.5 mm over 16 months, or 376,000 cycles).

Recent work by Franklin and Kapoor [2007] and by Franklin *et al.* [2008] has developed the 'brick' model further to consider roughness effects and to include the effects of rail steel microstructure on wear and crack initiation.

2.4.3 Plastic deformation

Plastic deformation occurs when the stresses in the surface of the rail exceed the elastic limit of the material. This may occur in the wheel-rail contact area as a whole if corrugation of the rail leads to high wheel-rail interaction forces. It may also occur at individual peaks in the roughness profile at a microscopic level.

Clark *et al.* [1988] mentioned plastic flow as a possible mechanism counteracting abrasive wear at the corrugation troughs, whereby rails with softer steels might experience higher forces and more plastic flow at the peaks of corrugation. Bohmer and Klimpel [2002] looked at plastic deformation in conjunction with frictional wear. They found that plastic deformation tends to counteract wear until a steady-state is reached. This is because the effect of plastic deformation is predicted to be greatest at the peaks in the normal force which correspond roughly to the peaks in the corrugation. The results of the model show the surface of corrugation peaks being depressed while the slopes either side of the peaks are lifted. This effect, on its own, results in a decrease in corrugation amplitude, with the effects of plastic deformation being more pronounced over the sleepers than at mid-span. In conjunction with a frictional wear model, an approximate sort of steady-state corrugation is predicted.

Plastic deformation as a result of normal surface roughness has been investigated by Kapoor *et al.* [2002]. They examined the causes of plastic deformation in a 15 to 20 μ m thick sub-surface layer of a Shinkansen rail, where the operating conditions are such that the rail was not expected to deform plastically. They simulated the wheel-rail contact in a twin-disc experiment and also calculated the contact pressures and stresses arising from a measured roughness profile. They conclude that roughness causes contact pressures that can be a factor of eight higher than the stresses for a smooth surface, causing plastic flow within a few microns of the rail surface.

Daves and Fischer [2002] developed a finite element model of the contact between wheel and rail with realistic surface roughness. The time history of the force for the wheel rolling over the rail, in the area considered, is taken from the calculated contact pressure distribution along a straight line on the rail for a stationary contact. The model was used to show the plastic zone that is formed during a contact at the surface of the rail due to the surface roughness. The depth of this plastic zone is increased if longitudinal creep is applied simultaneously with the normal load. Surface asperities of height less than 5 μ m are found to lead to a plasticized depth of 1 mm after only three normal loadings with 0.1% creep. The deformation rate decreases with repeated loading and it is thought that a steadystate must be achieved in order to continue repeated loading of the rails. A possible reaction of the material is the development of a white etching surface layer which is much harder than the new rail steel. If this is the case, plastic deformation may be disregarded as a mechanism for general roughness growth with low initial roughness levels, in the absence of corrugations. Wen and Jin [2006] also observed plastic deformation effects stabilising under repeated wheel passages using a finite element analysis.

2.4.4 Experimental characterisation of wear

Recognising that several different wear mechanisms exist, an alternative approach to modelling wear by a single specific mechanism is to use data from experiments linking the severity of the contact conditions to the amount of material removed. Bolton and Clayton [1984] identified three regimes of wear of increasing severity from mild through severe to catastrophic. The wear rate was described as a linear function of the total tangential force, the creep and the contact area. Clayton [1996] presented a summary of experimental research on tribological aspects of wheel-rail contact. He noted that the most relevant results are achieved when the experimental test specimens match the field application, when the relative performance of specimens match equivalent field experience, and when overall patterns of behaviour are investigated.

Lewis and Dwyer-Joyce [2004] examined wear mechanisms and transitions for R8T railway wheel steel and compared its wear resistance to other wheel steels. Lewis and Olofsson [2004] mapped more general rail wear regimes and the transitions between them. This work followed the approach of Bolton and Clayton [1984], using data collected from small and large scale laboratory tests and field data. Lewis and Olofsson identified the same three wear regimes as Bolton and Clayton, as well as transitions between the regimes. They did not link the wear regimes to particular wear mechanisms, but found that trends in wear rate are similar for a range of rail steels. In the mild wear regime, there is little difference in the wear rate for different wheel/rail material combinations. For more severe wear rates, a clear difference between different material combinations was found.

A mathematical model to predict the wear of railway wheels has been developed by Braghin *et al.* [2006] also using the same approach. They divide the problem into a fourpart iterative procedure. The global contact parameters are determined from a multi-body model of the railway vehicle. A local contact analysis then calculates the slip and tangential stress in the contact patch. A wear model predicts the material removed, depending on the wear regime, using a table relating wear rate to tangential force, creep and contact area. Finally, the wheel profile is smoothed and updated and the loop begins again. Their experimental tests showed that in the first mild wear regime, the mechanism for material removal was abrasive removal of an oxidised surface layer. As the severity of the contact increased, the wear mechanism changed and material was removed by a delamination process. In this case (matching the ratchetting wear mechanism) plastic deformation below the surface led to cracking parallel with the surface and the separation of slivers of material from the bulk. In the final, catastrophic, wear regime these cracks changed direction to run into the material allowing larger chunks to break away. In normal wheel-rail contact this final wear regime is not expected to occur [Lewis & Olofsson, 2004].

Vuong and Meehan [2009] have suggested an analytical model based on fundamental contact mechanics and heat transfer to calculate the wear coefficients and transitions between the different wear regimes. However this approach, aimed at research into wear coefficients, is over-complicated for the purposes of modelling the wear in the current project.

2.5 Models of corrugation initiation and growth

A number of reviews of the field of modelling wheel-track interaction and corrugation growth have been published. Knothe and Grassie [1993] reviewed track dynamics models and vehicle/track interaction. Popp *et al.* [1999] examined vehicle-track dynamics with an overview of wear models. Sato *et al.* [2002] have reviewed rail corrugation studies, including an extensive summary of Japanese research as well as the better known European efforts. Nielsen *et al.* [2003] also reviewed train-track interaction and mechanisms of irregular wear on wheel and rail surfaces. In this section, selected papers are grouped by date, summarising significant developments in corrugation growth modelling over the last twenty-five years.

2.5.1 Early studies of corrugation

Grassie and Johnson [1985] and Clark *et al.* [1988] were among the first to study corrugation formation as a result of frictional wear. The interaction force component of the model of Clark *et al.* [1988] was derived from the original time domain model of Clark *et al.* [1982], but included vertical and lateral dynamics. The lateral dynamics were included to account for a wheelset having an angle of attack relative to the rail, as lateral creep was thought to be more significant in the development of corrugations than longitudinal creep. Previous work by Clark and Foster [1983] on a lateral and longitudinal model of the track showed the potential to develop corrugation patterns, but no wavelength fixing mechanism was found until vertical track dynamics were included.

An important feature of the model of Clark *et al.* [1988] is that the wear prediction was carried out for a series of wheel pass-bys at randomly varying speeds, rather than just for one speed, and also for varying lateral creep. Using the model it was suggested that short wavelength corrugations are initiated by lateral stick-slip vibration, and are likely to grow significantly if the vertical track dynamic stiffness is high at a frequency which corresponds to a wavelength of the order of twice the contact patch length at typical train speeds. Possible approaches to reduce corrugation formation were discussed including addressing wheelset yaw misalignment, and also modifying the track vertical receptance to minimise the vertical dynamic stiffness at the critical frequency perhaps by adding damping to the pinned–pinned mode. Running a variety of trains with different wheels, loadings and speeds was also recommended.

Frederick [1986] discussed the causes of rail corrugation, mentioning many aspects of the problem that are still being debated in the literature. Rail wear, plastic deformation, the effect of an initial surface roughness and wear resistance of rail steel were all considered. A frequency domain model of wheel-rail interaction forces and lateral and longitudinal creep was developed, and phase relationships were discussed. Among other things, it was pointed out that the pinned-pinned frequency has undesirable effects and that continuous support to eliminate this would be an improvement, also that damping lateral rail motion is desirable. In Frederick's work, short pitch corrugation was linked to high vertical impedance over sleepers and low lateral impedance of rails, combined with lateral creep of wheels. Longitudinal creep was thought to oppose the effect of lateral creep in the frequency range 600 to 1000 Hz.

Since the work of Clark *et al.* [1988] and Frederick [1986], many roughness growth and corrugation models have been developed that, for simplicity and computational efficiency, limit the degrees of freedom of the system, including only vertical and longitudinal track dynamics and neglecting the effects of lateral creep.

2.5.2 Efforts during the 1990s

Kalker's book [1990, Section 5.2.2.5] includes some remarks on corrugation. He applied both CONTACT and FASTSIM to predict the development of corrugation over time with a constant longitudinal creepage. Rather than examining corrugation initiation, Kalker included the effect of the force fluctuation as a result of the corrugation itself. The purpose of the analysis was to compare the steady-state FASTSIM predictions with the transient analysis of CONTACT. Little difference was found between the two. The analysis predicted that corrugation ridges would be ground down over time.

Building on the model of Hempelmann *et al.* [1991], Hempelmann and Knothe [1996] presented a linear model for the prediction of short pitch corrugation, applying Hertzian contact mechanics with constant lateral creepage. This model is valid only for the initial stage of corrugation initiation, with very small profile irregularities and linearization about a reference state. Longitudinal and spin creep were neglected. Rather than calculating the actual wear for each position as the wheel moves along the rail (although their model does do this too), they obtained an exponential growth law for corrugation formation. Corrugation growth was then expressed in terms of local corrugation growth rates at certain positions in a sleeper bay for certain frequencies. These corrugation growth occurs corresponding to the anti-resonance of the pinned-pinned mode at a sleeper. The local corrugation growth rates are independent of the number of wheelset passages and of the initial profile irregularity.

Higher corrugation growth rates are predicted by Hempelmann and Knothe [1996] in situations with stiff vertical structural dynamics of the track. This occurs where the rail has an anti-resonance, or when the structure is excited at the node of a modeshape. They also predict smoothing of short wavelengths as the contact filter provides a mechanism to suppress very short wavelength corrugation.

Corrugation growth has also been linked to standing waves in the track between the successive wheels of a bogie [Igeland, 1996]. The bogie wheelbase was found to be an important parameter and, if the wheelbase is equal to an integer number of sleeper spacings, can lead to amplification of pinned-pinned resonance effects. Separate work by Manabe [2000] using an analytical model without discrete rail supports also determined that standing waves between multiple wheels are a possible wavelength fixing mechanism for corrugation. This mechanism may explain why observed corrugation wavelengths do not always vary linearly with vehicle speed.

Igeland and Ilias [1997] compared corrugation development predictions developed using different wheel-rail interaction models, achieving similar results from both models. They found that non-linear and linear models give different predictions for roughness growth. Ilias [1999] linked stiffer rail pads with higher corrugation growth, and also noted that parametric excitation of the system is important for corrugation growth.

In an attempt to explain the lack of velocity dependence of corrugation wavelengths observed in practice, Müller [1999, 2000] investigated short pitch corrugation using a linear model. He found that, as well as the pinned-pinned resonance, other resonances and anti-resonances can dominate the development of the rail profile. The dominant wavelength that arises where there are multiple resonances is determined by a contact mechanical filtering function that limits corrugation growth to a particular wavelength band.

J.B. Nielsen^{*} [1999] neglected the dynamics of the track and vehicle, reducing the system to a cylinder rolling over a periodically varying surface. The development of corrugation then becomes purely a contact mechanics problem, treated using an analytical model including some non-Hertzian effects arising from the surface corrugation. J.B. Nielsen's model predicts a minimum wavelength at which corrugation will occur, corresponding to the filtering effect of the size of the contact patch. A characteristic wavelength at which corrugation is more likely to develop was also identified, determined by the magnitude of the creep and the length of the contact. This is a constant wavelength effect, not a constant frequency effect like the pinned-pinned resonance. For wheel-rail contact the characteristic

^{*} N.B. in general, references to 'Nielsen' mean 'J.C.O. Nielsen'

wavelength is likely to be between 25 and 100 mm, which corresponds to observed lengths for short-pitch corrugation.

2.5.3 Corrugation research 2001-2005

One of very few models of roughness growth that have been compared with measurements on track is that of Nielsen [2003]. He applied his time-stepping interaction force model to a tangent track, and predicted the rail roughness growth over time. Model parameters were determined by mobility measurements on the track. An initial realistic (but low level) roughness profile was assumed, filtered at each instant in time to account for contact patch effects. The change in the roughness spectrum due to frictional wear was calculated as an average from seven different initial roughness profiles.

The wear calculation employed an elliptical contact patch, but wear was only calculated along the centre-line of contact. Nielsen [2003] found it was acceptable to multiply wear after a single wheel passage by 200,000 to simulate the traffic over a year (since similar results were achieved using 8 intermediate steps of 25,000). The wear constant was determined from measured data to be 2.5×10^{-9} kg/Nm. The model predicted corrugation growth at wavelengths of about 30 to 40 mm, which corresponds well with measured corrugation on a line where 85% of the traffic is passenger trains at 130 km/h (the site was the same as that used by Hiensch *et al.* [2002]). Slower and less frequent freight trains were not predicted to have as much impact on roughness growth as the passenger trains.

Nielsen [2003] comments that adding damping to the rail would be one way to increase track receptance above sleepers around the pinned-pinned frequency, thus smoothing peaks and troughs in the receptance. He also comments that wear is possibly not the only damage mechanism, as 'identical' tracks do not always develop similar roughness patterns, referring to Hiensch *et al.* [2002]. Assessing the relative importance of different mechanisms is considered important for future work.

Andersson and Johansson [2004] predicted rail corrugation growth, applying a threedimensional interaction force model similar to that of Andersson and Abrahamsson [2002]. Hertz theory was used to deal with the normal contact; the tangential contact was modelled using the theory of Shen *et al.* [1983]. The spatial variations of creep and traction were evaluated using FASTSIM for the wear calculations. As the wear from a single wheel passage is very small, it was assumed that any resulting change in interaction forces is also very small. The wear after each bogie passage was therefore multiplied to simulate thousands of wheel passages, without recalculating the contact forces.

The wear prediction showed high wear rates around two wavelengths corresponding to resonant behaviour of the train-track system. The wheelset spacing was found to have a significant effect, in accordance with the results of Igeland [1996]. Increasing the vehicle speed shifted the growth of corrugation to longer wavelengths. Andersson and Johansson [2004] concluded that, for the generation of rail corrugation on tangent tracks, vertical track dynamics is of major importance and that lateral motion plays a minor role.

The simplest roughness development model is a two-dimensional Hertzian approach, applied originally by Grassie and Johnson [1985] and also used by Wu and Thompson [2005] in their study of multiple wheel effects on corrugation. The width of the contact is assumed constant for all loads and at each instant in time the contact patch is divided into a stick zone and a slip zone, with wear proportional to frictional work done in the slip zone. With soft pads, the interaction force has several peaks in the frequency range due to reflections between the wheels and consequently there are also several peaks in the corrugation growth rate. With stiff pads there is less difference between results with single and multiple wheels. Stiff pads emphasise the pinned-pinned effects and lead to a higher corrugation growth rate at that frequency than with soft rail pads.

A different approach was taken by Meehan *et al.* [2005]. They developed an analytical model to investigate corrugation growth, using a time domain model to confirm the analytical results. The analytical model considers the effect of dynamic components on corrugation growth rate, condensing a large number of parameters into two terms describing the contribution to corrugation growth from the vehicle/track dynamics and from the contact and wear properties. A sensitivity analysis was then performed to assess the influence of various parameters on corrugation. The parameters that had the most effect on corrugation growth were the wear coefficient, the ratio of the tangential load to the friction limit, the friction coefficient and the damping of the vehicle and track.

2.5.4 Recent contributions to corrugation and roughness modelling

Sheng *et al.* [2006] combined the interaction force model of Sheng *et al.* [2005] with a two-dimensional Hertzian-based wear model similar to that of Wu and Thompson [2005]. The results show that the discrete supports have a significant effect on the initiation of rail roughness beginning with a smooth rail. Maximum roughness growth was predicted around the pinned-pinned frequency. Low rail pad stiffness was found to reduce corrugation growth although broadband roughness still developed. The inclusion of multiple wheels in the calculation, instead of a single wheel, resulted in a lower predicted roughness growth.

The effect of wheel tread irregularities on roughness growth rate was examined by Johansson and Nielsen [2007]. They concluded that the averaging effect of many wheel passages means that wheel corrugations have a negligible effect on rail corrugation growth. The standing waves in the rail between the two wheels of a bogie were found to be more important for corrugation growth.

Meehan and Daniel [2008] and Bellette *et al.* [2008] used the analytical model of Meehan *et al.* [2005] to examine the effect of speed and the frequency of successive wheels on corrugation formation. They proposed increasing the spread of vehicle velocities at a particular site as a means of reducing corrugation growth rates. As with other multiple wheel corrugation models, increasing the wheel passing frequency (by decreasing the wheelbase) was also found to affect corrugation growth.

Another analytical model of wear pattern generation or corrugation is that of Hoffmann and Misol [2007]. They apply simple one or two degree-of-freedom models with a moving point contact to the problem of wavelength selection in cases of uneven wear. This work addresses the reason wear patterns are not always proportional to the velocity, by using stability analysis to show that dominant wavelengths can appear as a result of randomly distributed velocities.

An alternative mechanism for corrugation formation has been identified by Ciavarella and Barber [2008] and by Afferante and Ciavarella [2009]. Afferrante and Ciavarella "disagree that pinned-pinned resonance is so clearly uniquely responsible for short pitch corrugation". Their reasoning is that the wavelengths of a great deal of measured instances of corrugation cannot be explained by a constant-frequency mechanism. Using an analytical model neglecting discrete supports of the track but including partial slip in the contact based on Hertzian contact mechanics, they find that corrugation may appear in two regimes, firstly at low frequencies around 500 Hz and secondly at frequencies above 1500 Hz. At low frequencies the corrugation wavelength is linearly related to the vehicle speed, but in the high frequency regime the wavelength remains almost constant. The corrugation growth depends purely on the geometry and load conditions.

On the other hand, Grassie and Edwards [2008] explain corrugation growth in terms of varying normal forces. The principle is that if the tangential load is essentially constant, then fluctuations in the normal force lead to more or less slip in the contact and hence more or less wear. Decreasing the normal force gives more slip for the same tangential load. This work, using a simplified analytical model, gives some explanation for corrugation formation after track features such as welds and joints, which can lead to significant fluctuations in the normal force.

The importance of including transient effects in models of rolling contact has been investigated by Baeza *et al.* [2007, 2008]. They conclude that, although transient models converge rapidly to the results from stationary models when the applied forces are constant, when forces vary rapidly, significant differences occur between stationary and transient models. This has consequences when estimating wear at the wheel-rail interface, so a transient model of the rolling contact should be used. One conclusion reached is that "the type of contact model determines the corrugation pattern, mainly in the cases of short wavelength defects" (Baeza *et al.* [2008]).

Non-steady-state or transient contact mechanics effects have also been studied by Alonso and Giménez [2008b] and by Knothe and Groß-Thebing [2008]. Most corrugation models assume quasi-steady contact, where the stress distribution between the wheel and rail at each point of interest does not depend on the distribution at previous locations. Exceptions include, for example, Kalker's CONTACT program and the work of Sheng *et al.* [2006]. Knothe and Groß-Thebing [2008] find that non-steady-state effects should be included in contact models for corrugation initiation and growth studies. They act as a filter on the wavelengths of corrugation growth, limiting it to the range between 20 and 100 mm, corresponding to the typical wavelength range of short-pitch corrugation. This effect can combine with system resonances, such as the pinned-pinned resonance, to give a sort of 'worst case scenario' for corrugation growth at wavelengths of around 30 to 40 mm. Xie and Iwnicki [2008a] have programmed Kalker's variational method [Kalker, 1990] in Matlab and carried out a study of wear. They examined, firstly, constant normal forces and then, sinusoidal normal forces, with corrugations of various wavelengths. Non-linear effects, transient effects and non-Hertzian effects were all included in a three-dimensional analysis. They found that, with a non-Hertzian contact model, the maximum wear always occurs at the peaks of corrugations, and consequently roughness does not grow. Extending their model to include time domain wheel-rail interaction, Xie and Iwnicki [2008b] also calculated time-varying forces and creep and the wear for driven and undriven wheels for various train speeds and wavelengths of sinusoidal roughness. Maximum wear was again found to occur at the crest of the roughness, resulting in roughness decreasing rather than growing. Roughness was also found to decrease with a broadband roughness input. Similar results were obtained from a two-dimensional non-Hertzian contact model [Xie & Iwnicki, 2008c].

The finding of Xie and Iwnicki [2008a,b,c] that roughness tends to decrease supports the results of Jin *et al.* [2005, 2006]. They have studied initiation and evolution of corrugation on curved tracks, using a modified version of Kalker's non-Hertzian theory (CONTACT). They also concluded that corrugation should decrease with more wheel passages.

2.6 Approach for this work

The existing work in the fields of wheel-rail interaction force modelling, rolling contact mechanics, wear modelling and corrugation development has been reviewed. On the basis of this review, in this section the appropriate modelling approaches to take in order to achieve the aims of this work are discussed.

To summarise the existing work in the field, it can be said that wheel-rail interaction force models are well developed and established. Models are available with a wide range of complexity, and an appropriate model can be chosen and adapted as required.

Rolling contact models are also reasonably well established, although historically the implementation of the most comprehensive models has been limited by their computational cost. Even the most sophisticated contact models tend to rely on simplified friction laws to determine the tangential stress distribution.

Most existing models of corrugation development assume a simple frictional wear model. More comprehensive models of wear exist, taking account of different wear mechanisms and wear rates, but these have not been used to predict roughness or corrugation development.

2.6.1 Wheel-rail interaction force modelling approach

In this work the frequency range of interest is limited to between 100 and 2000 Hz. This includes the range of action of the rail dampers, and it permits the rail to be modelled as a single Timoshenko beam (see Section 2.2.3). At a train speed of 100 km/h this frequency range corresponds to wavelengths between 14 and 280 mm, while for a train at 150 km/h the wavelength range is around 20 to 420 mm. Interaction forces at shorter wavelengths will in any case be filtered by the size of the contact patch.

The wheel-track interaction force model used here is based on the two-dimensional time domain formulation published by Nielsen and Igeland [1995]. This model, in the form of DIFF, is well established and has been validated by measurements [Nielsen, 2006]. Although this model has been extended to general motion by Andersson and Abrahamsson [2002], the subsequent corrugation growth modelling of Andersson and Johansson [2004] indicated that for tangent track the vertical dynamics is most important with lateral dynamics having only a minor effect (see Section 2.2.8). Many other authors agree that vertical resonances of the track are important in determining the wavelengths at which corrugation develops, regardless of the direction of creep in the tangential stress analysis. For this reason, to maintain an appropriate level of complexity, only vertical interaction is considered. As motion is limited to the vertical plane, the vehicle's unsprung mass is the only significant component in the vehicle model, since the vehicle suspension isolates the bogie and body from the wheelset. Bending modes of the axle are not considered. Wheel radial resonances occur at and above 2000 Hz [Thompson, 2009] and are also neglected here. The roughness is pre-processed to account for contact filter effects, using the twodimensional method of Ford and Thompson [2006].

The track is represented by a finite element model of a finite length of track. The rails are represented by Timoshenko beams. The rail pads and ballast are represented as in DIFF as simple spring and viscous damper systems. More sophisticated three-parameter

viscoelastic models for the rail pads exist, as used by Andersson and Oscarsson [2000], but it becomes difficult to determine each parameter and the benefits do not necessarily compensate for the additional computational cost. Similarly fractional derivative models for the rail pad have been developed [Fenander, 1997, 1998] but the added complexity is unnecessary to calculate the interaction forces in the frequency range of interest. Some models introduce an independent ballast mass below the sleepers, or more complicated mass-spring-damper combinations, but these are mainly relevant below 100 Hz and it is difficult to obtain satisfactory parameter values for the ballast models. Therefore in this work a simple spring and viscous damper ballast model is used.

A model of the rail damper elastomer is developed using parallel Maxwell elements [Lockett, 1972] in combination with an additional spring. This technique gives an improved representation of the elastomer loss factor over a simple spring-damper system. More sophisticated methods for time domain modelling of viscoelastic elements have been developed, for example by Golla and Hughes [1985] and Lesieutre [1992]; however the computational cost of implementing these techniques in this model is prohibitive.

Track receptances and decay rates are calculated from the finite element model of the system and compared with measured values and with results from simple frequency domain track models in Chapter 3. Since the depths of the anti-resonances in the track receptance have been linked to rail corrugation, an assessment is made of the effects of rail dampers on the track receptance and on the dynamic wheel-rail interaction forces. The time history of the vertical interaction force obtained in Chapter 4 is used as an input to the rolling contact model.

2.6.2 *Rolling contact model approach*

It has been shown in models of corrugation development that the type of contact model selected can have a large influence on the corrugation pattern predicted, particularly for short wavelength roughness. Historically most models have used quasi-steady Hertzian models to determine the distribution of stresses throughout the contact patch, assuming that roughness wavelengths shorter than the length of the contact can be neglected in a corrugation analysis. Recent work has shown the importance of including transient effects, *i.e.* the tangential stress calculation should take account of the loading history in rolling

contact. In addition, models accounting for non-Hertzian effects give very different results to Hertz-based models (see Section 2.3.6 and 2.5.4).

Kalker's solution of the contact problem, as implemented in CONTACT, is considered the most complete numerical theory of elastic contact available. It is considered to be 'exact' in that within the elastic half-space assumption, the accuracy of the output is limited only by computational capacity and the size of the elements used to solve the problem. In this work a two-dimensional representation of transient, non-Hertzian rolling contact is implemented based on Kalker's method. This method is presented and discussed in Chapters 5 and 6.

A limitation of Kalker's method is the simple representation of friction by a constant friction coefficient. In Chapter 8, a velocity-dependent friction law is introduced for the solution of the rolling contact tangential stress distribution.

2.6.3 Wear modelling

Wear mechanisms and models have been discussed in Section 2.4. It is apparent that roughness growth and corrugation models assuming a single wear mechanism are limited, and that an ideal model of rail wear would account for factors such as elastic and plastic deformation, high temperature effects, work hardening, and local variation in material properties and wear resistance.

For this prediction of roughness growth development, alternative wear mechanisms are considered by using the experimentally determined wear rates of Braghin *et al.* [2006]. In this way the wear coefficient is determined at each element of the contact based on the severity of the conditions at that location. The advantage of the approach of Braghin *et al.* [2006] is that the wear relationship has been validated using laboratory tests under controlled conditions. It is a more comprehensive model than the single wear coefficient approach, which can only consider mild wear for all contact conditions.

Plastic deformation is not included in this study, as it is not thought to be a significant mechanism for general roughness growth, as discussed in Section 2.4.3. In any case the initial surface roughness levels used in the model have been chosen to prevent excessively high wheel-rail interaction forces that might lead to loss of elasticity.

2.6.4 Measurements of roughness development

Measurements of the development of railhead roughness have been discussed in Section 1.4.1. Because of the long time scales involved in roughness development, there is very little measurement data available that examines the change in roughness spectrum at a single site over time. For this work, roughness measurements made at Gersthofen as part of the Silence project have been made available by Deutsche Bahn AG (see Section 1.2.1). The test site includes track with two different rail pad stiffnesses, and the aim of the measurements was to examine the effect of rail dampers on roughness growth rates on typical ballasted track. The measured roughness spectra are presented in Chapter 9, and discussed in the context of the results of the roughness model.

2.7 Summary

Extensive studies of short-pitch corrugation development over a period of almost 30 years have not resulted in a clear consensus on the causes of corrugation initiation and the wavelength fixing mechanisms. Corrugation might be a constant frequency phenomenon, or a constant wavelength phenomenon, or neither or both. This confusion also applies to broadband roughness growth phenomena, if corrugation can develop by a process of differential wear from an initially broadband roughness level. It is clear from recent work in this field that the model used to predict the stress distribution in rolling contact can have a significant effect on the predicted wear of the rail.

In this work, a model of broadband roughness development is presented using the best available, non-Hertzian, transient rolling contact theory, alongside a wear model that takes account of multiple wear mechanisms. In addition, the rolling contact theory is extended to include the velocity dependence of the friction coefficient. This model is used to examine the effect of track and vehicle parameters on roughness development. The main parameters considered are the rail pad stiffness, the vehicle type and speed, and the effect of rail dampers. The effect of rail dampers on roughness growth is of particular interest, as they are known to affect the pinned-pinned resonance of the track which has been linked to corrugation growth.

3 MODELS OF TRACK DYNAMICS

3.1 Introduction

A mathematical representation of the track dynamic behaviour is required as the first step in modelling the dynamic interaction between the vehicle and the track. A great deal of work exists on the subject and many models of track have been developed in both the frequency and time domains. These models range in complexity from simple beams on continuous elastic support, to elaborate three-dimensional finite element models including several support layers under each rail.

One particular well-established modelling approach in the time domain using finite elements is used for the work of this project and is therefore described in greater detail. This model is based on the work of Nielsen *et al.* at Chalmers University of Technology [Nielsen, 1991; Nielsen & Abrahamsson, 1992; Nielsen & Igeland, 1995]. It is used throughout this thesis to determine the normal interaction force between the wheels and a single rail, required as the input to the wear model.

In this chapter some frequency domain track models developed by other authors are also described and compared. The frequency domain models are used to check the suitability of the time domain model of the track and to determine the parameters used to represent different track components in the finite element model. As an additional check, the finite element model is used to predict the track decay rate for a typical track configuration for which measured decay rates are available for comparison. Measurements have been taken by Deutsche Bahn AG and Corus as part of the EU project Silence at a site near Gersthofen in Germany [Asmussen *et al.*, 2008]. In this work the model parameters are chosen to match the track and vehicles at this site.

Rail dampers have not previously been included in a track model of this type. A model of the rail dampers is developed for inclusion in the finite element model. The effect of the rail dampers on the track receptance and decay rate is studied and compared with measurements taken at Gersthofen.

3.2 Frequency domain track models

The level of complexity required in a model of the track depends on the problem to be investigated. For some applications, if only low frequency behaviour is of interest and determination of the deformation of the rail cross-section is not necessary, it is adequate to represent the rail by a continuously supported simple beam [Knothe & Grassie, 1993]. More detailed models include higher frequency behaviour and more features of the track are included, such as different support layers or discrete supports.

3.2.1 Euler beam on continuous support

It is instructive to begin a description of frequency domain track models with the simplest model, an infinite Euler beam resting on a continuous elastic foundation as shown in Figure 3.1. This model provides the basis for more advanced models described later.



Figure 3.1 Infinite Euler beam on continuous elastic support

The Euler beam in Figure 3.1 may be characterised by its Young's modulus *E*, second moment of area of the section *I*, cross-sectional area *A* and density ρ . The support is characterised by the stiffness per unit length *k'* and a damping loss factor η . Damping of the support is included in the following equations by replacing *k'* with a complex stiffness *k'* (1+ η i).

Without damping, the equation of motion of the unloaded beam shown in Figure 3.1 is

$$EI\frac{\partial^4 u}{\partial x^4} + k'u + \rho A\frac{\partial^2 u}{\partial t^2} = 0$$
(3.1)

where x is the position along the beam, u is the vertical displacement of the beam in the z direction and t is time.

For harmonic motion of frequency ω , the equation of motion may be rewritten as a dispersion relation between ω and the wavenumber *h*:

$$EIh^4 + k' - \rho A \omega^2 = 0 \tag{3.2}$$

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The natural frequency ω_0 of the beam on the support stiffness is defined by

$$\omega_0 = \sqrt{\frac{k'}{\rho A}} \tag{3.3}$$

From Equation (3.2), the wavenumber may be written as

$$h^{2} = \pm \sqrt{\frac{\rho A \omega^{2} - k'}{EI}} = \pm \sqrt{\frac{\rho A (\omega^{2} - \omega_{0}^{2})}{EI}}$$
(3.4)

 ω_0 is also known as the cut-on frequency, as below this frequency the wavenumber cannot be real and no free wave propagation can occur [Thompson, 2009].

For this work the point receptance (displacement amplitude at a point on the rail per unit force applied at the same point) and track decay rate (the rate of attenuation of vibration along the length of the rail) from the model are of interest for comparison with measured results and results from alternative track models. The point receptance of the track at frequency ω (derived from the mobility given by Thompson [2009]) is

$$\alpha(\omega) = \frac{-(1+i)}{4EIh^3} \tag{3.5}$$

The track decay rate Δ in dB/m is given by

$$\Delta = -20\log_{10}(\exp(h_{im})) = -8.686h_{im}$$
(3.6)

where h_{im} is the imaginary part of the wavenumber [Jones *et al.*, 2006].

3.2.2 Timoshenko beam on continuous support

As the frequency range of interest increases it becomes necessary to include more features of the track. Using a Timoshenko beam, rather than an Euler beam, extends the frequency range of validity by including shear and rotational inertia effects. The dispersion relation between ω and the wavenumber *h* is then [Thompson, 2009]:

$$h^{4} + C_{2}(\omega)h^{2} + C_{3}(\omega) = 0$$
(3.7)

where

$$C_{2}(\omega) = \left(\frac{k' - \rho\omega^{2}}{G\kappa}\right) - \left(\frac{\rho I\omega^{2}}{EI}\right)$$
(3.8)

and

$$C_{3}(\omega) = \left(\frac{k' - \rho A \omega^{2}}{EI}\right) \left(1 - \frac{\rho I \omega^{2}}{GA \kappa}\right)$$
(3.9)

in which G is the shear modulus and κ is the shear factor.

The point receptance of the Timoshenko beam on continuous support is given by

$$\alpha(\omega) = i \sum_{\substack{n \text{ with} \\ \text{Im}(h_n) < 0}} \frac{1}{GA\kappa} \left(\frac{h_n^2 + C_1(\omega)}{4h_n^3 + 2h_n C_2(\omega)} \right)$$
(3.10)

where

$$C_1(\omega) = \left(\frac{GA\kappa}{EI}\right) - \left(\frac{\rho I \omega^2}{EI}\right)$$
(3.11)

Equation (3.10) has been adapted from the transfer receptance given by Thompson [2009] using the contour integration method of Grassie *et al.* [1982]. The summation is of the residues of the poles lying inside the appropriate contour, 2 in this case.

The receptance and decay rate for Euler and Timoshenko beam types are plotted in Figure 3.2 with parameters as listed in Table 3.1. The receptance curve has a single peak corresponding to the resonance of the rail mass on the support stiffness, at frequency ω_0 in this case occurring at 440 Hz. This is also the frequency at which the track decay rate begins to drop off. Above ω_0 the waves in the rail are uncoupled from the support, so the track decay rate becomes low.



Figure 3.2 (a) Vertical point receptance and (b) decay rate of track modelled as a beam on a single layer of continuous support: ——Euler; — — Timoshenko.

Description	Notation	Value
Youngs modulus of elasticity	Ε	$2.1 \times 10^{11} \text{N/m}^2$
Second moment of area of rail	Ι	$30.55 \times 10^{-6} \mathrm{m}^4$
Density of rail	ρ	7850 kg/m ³
Cross-sectional area of rail	Α	$7.69 \times 10^{-3} \mathrm{m}^2$
Shear modulus of rail	G	$0.77 \times 10^{11} \text{N/m}^2$
Shear factor for rail	к	0.4
Support layer stiffness per unit length	k'	$4.6 \times 10^8 \text{N/m}^2$
Support layer loss factor	η	0.20

Table 3.1 Input parameters for model of track as a beam on an elastic support.

The Timoshenko beam model is often applied rather than an Euler beam model in cases where higher frequencies are of interest. It might therefore be assumed that the results from the two models would converge at low frequencies. In fact, as seen in Figure 3.2, the inclusion of shear effects in the model leads to a small difference across the whole frequency range. The Euler beam may be viewed as a special case of the Timoshenko beam, with the effects of shear deformation and rotational inertia neglected. Although these effects are counteracting, the shear deformation effect is around three times larger than the rotational inertia effect [Timoshenko *et al.*, 1974]. Therefore, including these effects allows a Timoshenko beam to deform more under the same static load than an Euler beam. It appears to be 'softer' under static load and the receptance of the Timoshenko beam is slightly higher than that of the Euler beam throughout the frequency range.

Modelling the track as a beam on continuous elastic support is sufficient if the frequency range of interest for the track is below 500 Hz [Knothe & Grassie, 1993]. To represent the track response at higher frequencies more information about the track structure needs to be included in the model. The first improvement to make to the model is the inclusion of distinct support layers to represent the rail pads, sleepers and ballast.

3.2.3 Beam on two-layer continuous support

A two layer continuous support as shown in Figure 3.3 allows the resonances of the rail and sleeper masses on the rail pads and ballast to be simulated. In this model the rail pads and ballast are represented by distributed elastic layers with no mass, separated by a distributed mass layer representing the sleepers.



Figure 3.3 Infinite beam on two-layer continuous elastic support.

The support layers are now characterised by two stiffnesses per unit length, k_b' for the ballast layer and k_p' for the rail pad layer, with each layer incorporating a damping loss factor η_b and η_p respectively. The sleeper mass per unit length in the *x* direction is m_s' . The sleeper layer has no bending stiffness in this model. The resulting support stiffness as seen by the rail is frequency dependent [Thompson, 2009]:

$$k'(\omega) = \frac{k'_{p}(k'_{b} - \omega^{2}m'_{s})}{(k'_{p} + k'_{b} - \omega^{2}m'_{s})}$$
(3.12)

For this system the receptance and track decay rate may be calculated as for the beam on a single layer of support in Equations (3.5) and (3.6), for wavenumbers given by:

$$h^{2} = \pm \sqrt{\frac{\rho A \omega^{2} - \frac{k'_{p} (k'_{b} - \omega^{2} m'_{s})}{(k'_{p} + k'_{b} - \omega^{2} m'_{s})}}{EI}}$$
(3.13)

The corresponding results for a Timoshenko beam may be obtained by substituting the two-layer frequency dependent stiffness $k'(\omega)$ from Equation (3.12) into Equations (3.7) to (3.11) for the beam on a single support layer.

The receptances and decay rates for both beam types for this system are shown in Figure 3.4 for parameters as listed below in Table 3.2. The first resonance of the support system is the rail and sleeper mass on the ballast layer at 93 Hz for these parameters. The anti-resonance of the track at around 210 Hz corresponds to the resonance of the sleepers on the combined ballast and rail pad stiffnesses (with the rail fixed). Finally comes the resonance of the rail mass on the rail pad stiffness at around 400 Hz. Above this frequency the rail becomes decoupled from the supports. The track decay rate then decreases as waves propagate freely along the undamped rail.

The Timoshenko beam receptance and track decay rate diverge from the Euler beam results at high frequency; however in the frequency range shown here the results for both beams are similar.



Figure 3.4 (a) Receptance and (b) track decay rate comparison between beams on two layer continuous support: ————*Euler;* ————*Timoshenko.*

Description	Notation	Value
Youngs modulus of elasticity	E	$2.1 \times 10^{11} \text{N/m}^2$
Second moment of area of rail	Ι	$30.55 \text{ x} 10^{-6} \text{ m}^4$
Density of rail	ρ	7850 kg/m ³
Cross-sectional area of rail	Α	$7.69 \times 10^{-3} \mathrm{m}^2$
Shear modulus of rail	G	$0.77 \times 10^{11} \text{N/m}^2$
Shear factor for rail	к	0.4
Half sleeper mass per unit length	m_s'	245 kg/m
Rail pad layer stiffness	k_p'	$3.33 \times 10^8 \text{N/m}^2$
Rail pad layer loss factor	η_p	0.20
Ballast layer stiffness	k_b'	$8.33 \times 10^7 \text{N/m}^2$
Ballast layer loss factor	η_b	1.0

Table 3.2 Inputs for frequency domain model of track on two layer elastic support

3.2.4 Timoshenko beam on discrete supports

A continuously supported model is valid up to around 500 Hz [Knothe & Grassie, 1993]. Above this, the variation in point receptance between a position above a sleeper and a position at mid-span becomes significant and it is no longer adequate to spread the track characteristics into a continuous support model. A discretely supported track model as shown in Figure 3.5 is able to represent the pinned-pinned resonance of the track.



Figure 3.5 Infinite Timoshenko beam on discrete supports.

In order to calculate the response of this system, the discrete supports may be modelled as point forces applied to an infinite free beam. Supports at a large distance from the point of interest may be neglected due to the decay of vibrations along the rail. The number of supports therefore need not be infinite, but must be large enough to give an acceptable approximation to the behaviour of an infinitely supported track.

The receptance $\alpha(x,x_F)$ of a free Timoshenko beam is the response at a point *x* to a unit point force acting at x_F , and is calculated here using the equations detailed in Heckl [1995]. The displacement response of the beam u(x) to many different point forces of strengths F_j due to the discrete supports may be obtained from the principle of superposition, so that

$$u(x) = \sum_{j=1}^{J} F_{j} \alpha(x, x_{j}) + F_{0} \alpha(x, x_{0})$$
(3.14)

where F_0 is the external force applied by the wheel at x_0 .

The force at each support point F_j is related to the displacement $u(x_j)$ by

$$F_j = -Ku(x_j) \tag{3.15}$$

where, for a two-layer support with complex stiffnesses k_b for the ballast and k_p for the rail pad and the sleeper mass m_s , the dynamic stiffness K is given by

$$K = \frac{m_s \omega^2 k_p - k_p k_b}{m_s \omega^2 - (k_p + k_b)}$$
(3.16)

This has a similar form to the stiffness in Equation (3.12), but the sleeper mass and the stiffness of the rail pad and ballast are no longer distributed along the length of the rail.

From Heckl [1995], the receptance of the system is then the displacement response of the beam u(x) at a general point x to a unit force of frequency ω applied at x_0 :

$$\alpha(\omega) = u(x) = -\sum_{j=1}^{J} Ku(x_j)\alpha(x, x_j) + \alpha(x, x_0)$$
(3.17)

and the point receptance is the displacement response of the beam at x_0 to the unit force also acting at x_0 . Equation (3.17) is evaluated firstly for the point receptance at each of the *J* support points, giving a set of equations from which expressions for $u(x_j)$ may be extracted. These can then be used in Equation (3.17) to determine the receptance at any general point.

The average decay rate along the track may be calculated from the calculated transfer receptances in the same way that it is calculated from measured frequency response functions over many sleeper spans. The decay rate calculated in this way is not strictly identical to that calculated from the imaginary part of the propagating wavenumber as in Equation (3.6), as it includes all wave types, both propagating and near-field. However it is similar, especially at high frequencies, and is a good indication of the track properties. The calculation and the differences between the two methods are described in detail by Jones *et al.* [2006].

For measurements on a real track, the required frequency response measurements (mobility or accelerance) are taken at a grid of points, beginning with the point response in the middle of a sleeper bay and then moving the excitation point away. The points are closely spaced initially with four in each sleeper bay to capture the higher decay rates which are typically around 10 dB/m. The points may be spaced further apart to measure the response at larger distances in order to obtain lower decay rates.

With responses calculated from a track model it is possible to obtain the transfer receptance between any number of points, so the decay rate may be calculated in the same manner. From Jones *et al.* [2006], the track decay rate Δ in dB/m is given by

$$\Delta \approx \frac{4.343 |Y(0)|^2}{\sum_{x_j=0}^{x_{\text{max}}} |Y(x_j)|^2 \Delta x_j}$$
(3.18)

where Y(0) is the point mobility at $x_j = 0$, the first point in the grid, x_{max} is the maximum measurement or calculation distance, and Δx_j is the distance between the midpoints of each grid interval to the locations either side. The grid need not be evenly spaced.

Figure 3.6(a) shows the comparison of the track receptance at two points in a sleeper bay. All parameters are as for the two-layer distributed support model, but divided between the discrete supports at 0.6 m intervals. At a location in the middle of the sleeper bay, the pinned-pinned resonance results in a peak in the track receptance. Above a sleeper, there is a corresponding anti-resonance. The inverse of receptance is stiffness, so above a sleeper the track appears to be stiff, while at mid-span the track appears to be soft. For these parameters, the pinned-pinned frequency is around 1070 Hz. The pinned-pinned resonance also adds a peak to the track decay rate when using a discretely supported track model, as shown in Figure 3.6(b).



Figure 3.6 (a) Receptance and (b) decay rate for Timoshenko beam on two layer discrete supports: ———*Receptance mid-span;* ———*Receptance above sleeper.*

3.2.5 Frequency domain model with viscous damping

In the above frequency domain models, damping has been included in the form of a constant loss factor (a hysteretic or structural damping model). This is realistic for many materials, in particular the rail pads. However hysteretic damping can only be used in

frequency domain models as it leads to causality problems if applied to time domain models. The conventional damping model used in the time domain is the viscous damping model as shown in Figure 3.7.



Figure 3.7 Infinite Timoshenko beam on discrete supports (viscous damping).

For this damping model, the complex stiffness applied previously to the rail pad and ballast in the form $k(1+i\eta)$ is replaced by $k+i\omega c$ where c is the viscous damping. The constant term $k\eta$ has been replaced by the frequency-dependent term ωc . The response of this system may also be calculated using the technique of Heckl [1995], with the force at each support point F_j related to the displacement $u(x_j)$ by K as in Equation (3.15). K may be determined from the equations of motion of a single support as shown in Figure 3.8.



Figure 3.8 Single support with viscous damping

The equations of motion of this two-degree-of-freedom system are

$$m_{s}\ddot{u}_{1} + k_{b}u_{1} - k_{p}(u_{2} - u_{1}) + c_{b}\dot{u}_{1} - c_{p}(\dot{u}_{2} - \dot{u}_{1}) = 0$$

$$k_{p}(u_{2} - u_{1}) + c_{p}(\dot{u}_{2} - \dot{u}_{1}) = F_{i}$$
(3.19)

Writing the forces and displacements in the form $F_j = Fe^{i\omega t}$, $u_j = U_j e^{i\omega t}$ and eliminating the exponential term yields

$$-m_{s}\omega^{2}U_{1} + k_{b}U_{1} - k_{p}(U_{2} - U_{1}) + i\omega c_{b}U_{1} - i\omega c_{p}(U_{2} - U_{1}) = 0$$

$$k_{p}(U_{2} - U_{1}) + i\omega c_{p}(U_{2} - U_{1}) = F_{j}$$
(3.20)

Eliminating U_1 and rearranging into the form of Equation (3.15) gives

$$K = \frac{F_j}{U_2} = \frac{\left(-m_s\omega^2 + k_b + i\omega c_b\right)\left(k_p + i\omega c_p\right)}{-m_s\omega^2 + k_b + k_p + i\omega c_b + i\omega c_p}$$
(3.21)

This is the same as *K* in Equation (3.16), if c_p and c_b are set to zero and the pad and ballast stiffnesses are complex.

For comparison with the structural damping model, the track receptance and decay rate from a model with viscous damping are shown in Figure 3.9.


The rail pad and ballast stiffnesses are the same in each case shown in Figure 3.9. The rail pad damping for the viscous damping model is 12,000 Ns/m and the ballast damping is 130,000 Ns/m. With these values, the resulting rail pad damping is equivalent for both damping models at a frequency of 530 Hz. The ballast damping is equal for both damping models at a frequency of 180 Hz. The viscous damping model parameters have been chosen to match the decay rates with measured values on typical UIC60 track with relatively soft rail pads of stiffness 200 MN/m, from the Silence project test site at Gersthofen (see Section 3.5).

3.3 Finite element track model

A finite element model is an alternative to the above infinite beam models. The disadvantage of a finite element track model is that it requires the track length to be truncated, which could introduce errors if the modelled length is insufficient and reflections from the ends of the structure become significant. However, the advantages of a finite element model are the capability to simulate non-linear effects, for example in the wheel-rail contact and in the rail roughness. Irregular sleeper spacings or support stiffnesses may also be considered. In addition, a finite element model allows rail dampers to be included in a straight-forward manner, and variations in the damper design and placement to be considered. Finally a finite element model allows the calculation of wheel-rail interaction forces in the time domain with a moving vehicle.

The track model used in the remainder of this work is the finite element model described here. For the sake of efficiency and control, the finite element track model has been implemented in dedicated software. The method follows that of Nielsen and Igeland [1995].

A finite length of track is modelled by specifying a number of sleeper bays to include and the sleeper spacing. The rail is modelled using Timoshenko beam elements on discrete supports. Half the track only is considered, *i.e.* a single rail on half sleepers. The sleepers are modelled as lumped masses, while rail pads and ballast are modelled as spring-damper sets with a simple viscous damping model, shown in Figure 3.10.



Figure 3.10 Finite element two-dimensional track model.

3.3.1 Equations of motion for track

The track is modelled as Timoshenko beam elements on supports to simulate the rail pads, sleepers and ballast. The track is truncated to a finite length, with sleepers spaced periodically. In each sleeper bay the rail is divided into a number of elements. Each support/sleeper consists of two spring-damper elements representing the pad and ballast, with a lumped mass for the sleeper itself. The degrees of freedom of the model allow displacement *u* in the vertical direction and rotation θ in the vertical plane. Lateral effects are not included. The track ends are constrained in displacement and rotation depending on the chosen boundary conditions. The displacement vector $\{u_{ij}\}$ for a general element of the model linking nodes *i* and *j* is then

$$\left\{\mathbf{u}_{ij}\right\} = \left\{\begin{array}{c} u_i \\ \theta_i \\ u_j \\ \theta_j \end{array}\right\}$$
(3.22)

The mass and stiffness matrices of the beam elements are determined using Timoshenko beam theory as described by Petyt [1990]. The following equations are equivalent to those for an Euler beam if β is set to zero, thus neglecting the effects of shear and rotational inertia. The mass matrix for each rail beam element is

$$\mathbf{m}_{r} = \frac{\rho A a}{210(1+3\beta)^{2}} \begin{bmatrix} m_{1} & m_{2} & m_{3} & m_{4} \\ m_{2} & m_{5} & -m_{4} & m_{6} \\ m_{3} & -m_{4} & m_{1} & -m_{2} \\ m_{4} & m_{6} & -m_{2} & m_{5} \end{bmatrix} + \frac{\rho I}{30a(1+3\beta)^{2}} \begin{bmatrix} m_{7} & m_{8} & -m_{7} & m_{8} \\ m_{8} & m_{9} & -m_{8} & m_{10} \\ -m_{7} & -m_{8} & m_{7} & -m_{8} \\ m_{8} & m_{10} & -m_{8} & m_{9} \end{bmatrix}$$
(3.23)

where *a* is half the element length,

$$\beta = \frac{EI}{\kappa A G a^2} \tag{3.24}$$

and

$$m_{1} = 156 + 882\beta + 1260\beta^{2}$$

$$m_{2} = (44 + 231\beta + 315\beta^{2})a$$

$$m_{3} = 54 + 378\beta + 630\beta^{2}$$

$$m_{4} = (-26 - 189\beta - 315\beta^{2})a$$

$$m_{5} = (16 + 84\beta + 126\beta^{2})a^{2}$$

$$m_{6} = (-12 - 84\beta - 126\beta^{2})a^{2}$$

$$m_{7} = 18$$

$$m_{8} = (3 - 45\beta)a$$

$$m_{9} = (8 + 30\beta + 180\beta^{2})a^{2}$$

$$m_{10} = (-2 - 30\beta + 90\beta^{2})a^{2}$$
(3.25)

The stiffness matrix for each rail element is given by

$$\mathbf{k}_{r} = \frac{EI}{2a^{3}(1+3\beta)} \begin{bmatrix} 3 & 3a & -3 & 3a \\ 3a & (4+3\beta)a^{2} & -m_{4} & (2-3\beta)a^{2} \\ -3 & -3a & 3 & -3a \\ 3a & (2-3\beta)a^{2} & -3a & (4+3\beta)a^{2} \end{bmatrix}$$
(3.26)

The damping in the rail itself is known to be very low. The loss factor of steel is about 10^{-4} . A hysteretic loss factor of 0.02 is typically used in TWINS models, to make up for cross-section deformation effects at high frequency that are not explicitly included [Thompson *et al.*, 1996a]. Other frequency domain models have used rail loss factors of 0.01 [Sheng *et al.*, 2005], and 0.004 [Nordborg, 2002] if rail damping is included at all. Often it is neglected, as in the work of Nielsen and Igeland [1995]. In a finite-length model, the inclusion of a realistically low level of damping in the rail elements is beneficial in order to allow the length of the model to be minimised without increasing the effect of reflections from the track ends, particularly at higher frequencies where the rail vibration decouples from the supports which have higher damping levels. Therefore in this work damping of the rail elements is included in the form of stiffness-proportional Rayleigh-type damping.

For Rayleigh damping, the element damping matrix is proportional to a linear combination of the stiffness and mass matrices [Petyt, 1990]:

$$\mathbf{c} = a_1 \mathbf{m} + a_2 \mathbf{k} \tag{3.27}$$

For stiffness-proportional Rayleigh damping the coefficient a_1 is zero. Stiffnessproportional damping is chosen here rather than mass-proportional damping, or a combination of the two, as its effects increase with frequency, and it has been found to be reasonably representative of internal material damping [Petyt, 1990]. The coefficient a_2 is estimated and the damping ratio ζ at the frequency ω of any mode of the system may be calculated from:

$$a_2 = \frac{2\zeta}{\omega} \tag{3.28}$$

For this work a value of $a_2 = 1 \times 10^{-6}$ s is assumed. This corresponds to a damping ratio at modes at the upper end of the frequency range of interest (2000 Hz) of 0.006, or a loss factor of approximately 0.012. At lower frequencies the loss factor is correspondingly lower. The damping matrix for a rail element is then

$$\mathbf{c}_{\mathrm{r}} = a_2 \mathbf{k}_{\mathrm{r}} \tag{3.29}$$

The mass, stiffness and damping matrices for a rail pad element are listed below (including the sleeper mass). Each rail pad element connects a rail node and a sleeper node. Again the degrees of freedom of the model allow displacement of the sleeper in the vertical direction. Rotation of the sleeper mass in the vertical plane is neglected. m_s is the mass of half a sleeper (since one rail only is considered).

$$\mathbf{k}_{p} = \begin{bmatrix} k_{p} & 0 & -k_{p} & 0\\ 0 & \frac{k_{p}L_{p}^{2}}{12} & 0 & -\frac{k_{p}L_{p}^{2}}{12}\\ -k_{p} & 0 & k_{p} & 0\\ 0 & -\frac{k_{p}L_{p}^{2}}{12} & 0 & \frac{k_{p}L_{p}^{2}}{12} \end{bmatrix}$$
(3.31)

$$\mathbf{c}_{p} = \begin{bmatrix} c_{p} & 0 & -c_{p} & 0\\ 0 & \frac{c_{p}L_{p}^{2}}{12} & 0 & -\frac{c_{p}L_{p}^{2}}{12}\\ -c_{p} & 0 & c_{p} & 0\\ 0 & -\frac{c_{p}L_{p}^{2}}{12} & 0 & \frac{c_{p}L_{p}^{2}}{12} \end{bmatrix}$$
(3.32)

Here k_p is the vertical rail pad stiffness and c_p is the vertical rail pad damping. Rotational stiffness and damping due to the rail pad are included in the terms involving L_p , the length of the rail pad along the rail. The inclusion of these terms has a small effect on the pinned-pinned resonance of the track, particularly if stiff rail pads are modelled. Similarly for the ballast elements:

For the ballast element, the mass matrix is null as the sleeper mass has been included in the pad element. k_b is the vertical ballast stiffness, c_b is the vertical ballast damping. Any damping in the rotational degree of freedom due to the ballast layer is neglected, as is rotation of the sleeper mass. In addition, the ground nodes are constrained, that is, terms for the displacement and rotation of the ground nodes are set to zero after assembling the global matrices of the system. The global matrices are assembled from the element matrices in the usual way [Petyt, 1990].

The equations of motion for the track finite element model may be written as

$$\mathbf{M}\{\mathbf{\ddot{u}}\} + \mathbf{C}\{\mathbf{\dot{u}}\} + \mathbf{K}\{\mathbf{u}\} = \{\mathbf{f}\}$$
(3.34)

where **M**, **C** and **K** are the global mass, damping and stiffness matrices for the finite element model and {**u**} is the vector of displacements of the degrees of freedom in the model. {**f**} is a vector of forces and moments acting on the nodes of the model.

Equation (3.34) may be rearranged in order to represent the problem as a first order system:

$$\mathbf{A}^{track} \dot{\mathbf{y}} + \mathbf{B}^{track} \mathbf{y} = \begin{cases} \mathbf{f} \\ \mathbf{0} \end{cases}$$
(3.35)

where \mathbf{A}^{track} and \mathbf{B}^{track} are assembled from the global mass, stiffness and damping matrices and \mathbf{y} is a vector of the displacements and velocities of the degrees of freedom in the model:

$$\mathbf{A}^{track} = \begin{bmatrix} \mathbf{C} & \mathbf{M} \\ \mathbf{M} & \mathbf{0} \end{bmatrix}; \qquad \mathbf{B}^{track} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & -\mathbf{M} \end{bmatrix}; \qquad \mathbf{y} = \begin{cases} \mathbf{u} \\ \dot{\mathbf{u}} \end{cases}; \quad \dot{\mathbf{y}} = i\omega\mathbf{y}$$
(3.36)

Harmonic solutions are sought at frequency ω . With the right hand side of Equation (3.35) set to zero, solving the eigenvalue problem results in a system of complex eigenvalues $i\omega_n$ and a complex modal matrix **P** with the eigenvectors $\phi^{(n)}$ as columns [Nielsen & Igeland, 1995].

$$\mathbf{P} = \begin{bmatrix} \boldsymbol{\varphi}^{(1)} & \boldsymbol{\varphi}^{(2N)} \\ i\boldsymbol{\omega}_{1}\boldsymbol{\varphi}^{(1)} \dots & i\boldsymbol{\omega}_{2N}\boldsymbol{\varphi}^{(2N)} \end{bmatrix}$$
(3.37)

The size of **P** is $2N \times 2N$ where *N* is the number of degrees of freedom in the track model. The number of columns used can be reduced if not all modes are to be included in the modal summation. For this work, modes with natural frequencies up to 3000 Hz are included covering approximately twice the frequency range of interest.

The eigenvalues and eigenvectors in \mathbf{P} are complex conjugate pairs; hence the number of columns is twice the number of degrees of freedom in the model. The lower half of \mathbf{P} is equal to the upper half multiplied by the corresponding eigenvalues.

The modal matrix \mathbf{P} is used to transform the equations of motion of the track to modal coordinates, where \mathbf{q} is the modal displacement vector and \mathbf{Q} is the modal load vector:

$$\mathbf{y} = \mathbf{P}\mathbf{q}, \quad \mathbf{Q} = \mathbf{P}^T \begin{cases} \mathbf{f} \\ \mathbf{0} \end{cases}$$
(3.38)

The uncoupled equations of motion of the track are then [Nielsen & Igeland, 1995]

$$\operatorname{diag}(\mathbf{a})\dot{\mathbf{q}} + \operatorname{diag}(\mathbf{b})\mathbf{q} = \mathbf{Q} \tag{3.39}$$

where

diag(**a**) =
$$\mathbf{P}^T \mathbf{A}^{track} \mathbf{P}$$
; diag(**b**) = $\mathbf{P}^T \mathbf{B}^{track} \mathbf{P}$ (3.40)

The elements of the diagonal matrices **a** and **b** are functions of the modal mass, stiffness and damping (see [Abrahamsson, 1988; Nielsen & Abrahamsson, 1992]).

The uncoupled equations of motion of the track (Equation (3.39)), with the equations of motion of the vehicle and the equations governing the wheel/track interaction (see Chapter 4), are in a form suitable for solving using state-space formulation and a time-stepping routine.

3.3.2 Receptance and decay rate from finite element track model

The track receptance and decay rate may be calculated from the finite element track model for comparison with that from the frequency domain track models. The receptance, $\alpha_{jk}(\omega)$, is the response in degree of freedom *j* due to a harmonic force of unit magnitude and frequency ω applied to degree of freedom *k*. For the formulation used here *n* modes are included in the summation with ω_r being the natural frequency of mode *r*:

$$\alpha_{jk}(\omega) = \sum_{r=1}^{n} \frac{\mathbf{P}_{j,r} \mathbf{P}_{k,r}}{(\mathbf{a})_{r,r} (i\omega - i\omega_r)}$$
(3.41)

The calculation of track decay rates from frequency response functions is as in Equation (3.18) following the method described by Jones *et al.* [2006].

The finite element model parameters are listed in Table 3.3. They have been tuned to those of the Gersthofen test site by comparing calculated decay rates with measured track decay rates. These parameters are the same (where equivalent) as those used previously in the infinite track models.

Notation	Value	Description
L	0.6 m	Sleeper spacing
E	$2.1 \times 10^{11} \text{ N/m}^2$	Young's modulus of elasticity
I_r	$30.55 \times 10^{-6} \text{ m}^4$	Second moment of area of rail
ρ	7850 kg/m^3	Density of rail
к	0.4	Shear factor for rail
G	$0.77 \times 10^{11} \text{ N/m}^2$	Shear modulus of rail
A_r	$7.69 \times 10^{-3} \text{ m}^2$	Cross-sectional area of rail
m_s	147 kg	Half mass of sleeper
k_p	2×10 ⁸ N/m	Rail pad stiffness ('soft')
k_p	8×10 ⁸ N/m	Rail pad stiffness ('stiff')
c_p	12×10^3 Ns/m	Pad damping
k_b	150×10 ⁶ N/m	Ballast stiffness
Cb	130×10^3 Ns/m	Ballast damping
L_p	0.15 m	Rail pad width

Table 3.3 Input parameters for time-stepping interaction force model.

3.3.3 Model element size, truncated track length and boundary conditions

In this section the basic parameters selected for the finite element model are determined and validated. In representing the rail over each sleeper bay as a number of finite elements, it is important that the number of elements is high enough to give accurate results when calculating the wheel-rail interaction force in the frequency range of interest. Also, there is a need to be able to determine transfer receptances at quarter-sleeper intervals for the calculation of track decay rates. To maintain flexibility in the model when including rail dampers, rail nodes are also required for 'attaching' the dampers to the track model at particular locations. For geometric convenience then, the minimum number of elements per sleeper bay that can be used in the model is four. However, the number of elements used to represent the rail in each sleeper bay is set to eight, for reasons described in Section 4.4.1 where the effect of the rail element length on the interaction force is discussed further.

The number of sleeper bays included in the finite element model must be sufficiently large to allow track vibrations to decay significantly before being reflected from the ends. The necessary number of sleeper bays can therefore be estimated from the expected minimum track decay rate. The minimum decay rate from measurements and predictions from a simple beam track model described by Jones *et al.* [2006] for a similar track is about 0.3 dB/m. A similar minimum decay rate is predicted by the discretely supported track models (Figure 3.9). Vibration amplitudes will therefore decrease by 10 dB over a distance of at most 33 m. Assuming this reduction is sufficient to prevent reflections having a significant effect in the central sleeper bays, a track length of at least 16.5 m either side of the central sleeper bays of interest is required. With a sleeper spacing of 0.6 m the required length of the model is approximately 60 sleeper bays or 36 m.

The rail point receptance above a sleeper and at mid-span are shown in Figure 3.11(a) and (b), calculated from this 60 sleeper bay model with fixed end conditions. These are plotted against the corresponding results from the frequency domain model with periodic supports and viscous damping. Other end conditions have been considered but the fixed ends give the best match with the infinite frequency domain model results at the pinned-pinned frequency. The decay rates from these models are shown in Figure 3.11(c) and again the finite element model and frequency domain model give similar results. The decay rate for the finite element model has been calculated over the middle 25 sleeper bays of the model.



Figure 3.11 Track vertical receptance (a) above sleeper, (b) mid-span and (c) decay rate. ______finite element model; _____ discretely supported frequency domain model with viscous damping.

To minimise the effect of reflections from the ends of the track, the damping of the five rail pad elements at each extreme of the track model has been increased by a factor of five. A similar method was used by Andersson [2003]. Above 700 Hz the receptance calculated from the finite element model begins to show some very slight oscillations due to the discrete length of the model, however these are insignificant. This additional damping does not significantly affect the magnitude of the receptance. The small differences in the receptance seen at high frequencies in Figure 3.11(a) and (b) are due to limiting the number of modes included in the finite element modal summation.

The small differences in decay rate at high frequencies between the finite element and frequency domain models seen in Figure 3.11(c) are due mostly to the addition of Rayleigh damping to the rail in the finite element model. The additional damping applied to the rail pads at the track ends has a smaller effect on the decay rate than the Rayleigh damping. Limiting the number of modes included in the finite element model also affects the decay rate, but again this is less than the influence of the Rayleigh damping.

The effect of the Rayleigh damping that has been included in this model is shown in Figure 3.12. The small amount of Rayleigh damping used gives a slightly smaller dip at the pinned-pinned resonance above a sleeper compared with setting the coefficient a_2 to zero (Figure 3.12(a)). Using a much higher damping coefficient in the rail however has a significant effect on the pinned-pinned resonance, although it also smoothes out the minor undulations arising from reflections from the ends of the model. The track decay rates in each case shown in Figure 3.12(b) show the effect of the rail damping more clearly. The small amount of stiffness-proportional Rayleigh damping used here is reasonable, whereas a higher amount of rail damping would eliminate reflection effects entirely but have adverse effects on the model particularly by artificially reducing the pinned-pinned resonance.



Figure 3.12 (a) Track vertical receptance above a sleeper and (b) decay rate with different Rayleigh damping coefficients: $a_2=1\times10^{-6}$ *s*; $---a_2=0$ *s*; $\cdots a_2=10\times10^{-6}$ *s*.

Although it is frequency dependent, the effect of the Rayleigh damping in this work is similar at 2 kHz, the upper end of the frequency range, to the effect of setting a constant rail loss factor in TWINS. The intention in TWINS is to mimic the extra damping that arises at high frequencies due to cross-sectional deformation of the rail interacting with the rail pads [Thompson & Jones, 2000].

3.3.4 Effect of rail pad stiffness on track receptance and decay rate

The parameter that is subject to the greatest variation for typical ballasted railway tracks is the pad stiffness. Figure 3.13 presents the receptance and decay rate for track with stiffer rail pads, 8×10^8 N/m, but with all other parameters as in Table 3.3.



Figure 3.13 Track vertical receptance (a) above sleeper, (b) mid-span and (c) decay rate. ______stiff rail pads 8×10⁸ N/m; ______soft rail pads 2×10⁸ N/m.

With stiffer rail pads, the anti-resonance in the point receptance of the rail due to the sleeper mass vibrating on the ballast and pad stiffnesses is shifted to around 400 Hz, from around 200 Hz for the soft pads. The resonance of the rail mass on the support stiffness is also shifted up, from 410 Hz to 760 Hz. The pinned-pinned resonance frequency at the middle of a sleeper span is not greatly affected by the increase in pad stiffness although its amplitude is increased. The anti-resonance due to this mode at locations above sleepers is also much deeper with stiff pads than with soft pads and is increased in frequency. The slight oscillations in receptance caused by the truncated track seen with the soft pad model are no longer visible with the stiffer rail pads due to the higher decay rate.

The track decay rate rolls off at a higher frequency with the stiffer rail pads. This is the frequency at which the rail becomes decoupled from the supports. The decay rate is higher with stiffer rail pads in the frequency range between around 300 Hz and 2000 Hz.

3.4 Models of rail dampers

Rail dampers are available commercially with various designs produced by different manufacturers. The rail dampers modelled in this work are of the design proposed by Thompson *et al.* [2007] *i.e.* tuned, damped mass-spring absorber systems, with either one or two masses enclosed in an elastomeric material. Early versions of these dampers were applied in a continuous strip along each side of the rail. In their current form, as tested in the Silence project [Asmussen *et al.*, 2008], discrete dampers are fastened to the rail in pairs, either side of the rail in the middle of each sleeper bay.

The inclusion of rail dampers is achieved in the model by adding additional elements to the finite element track model. In order to understand the effect that they have on the track dynamics, models of the rail dampers with increasing complexity are examined here.

3.4.1 Additional mass added to rail without additional damping

The mass that a rail damper adds to the track is significant. The dampers used in the Silence project [Asmussen *et al.*, 2008] add 28 kg/m to the track, compared with a rail mass per unit length of 60 kg/m. Even in the absence of additional stiffness and damping, this extra mass changes the track dynamics. This is investigated first using the model shown in Figure 3.14. Figure 3.15 shows the receptance and decay rate results from the model of the track with soft rail pads, but with an additional 17 kg added at the rail node in

the middle of each sleeper bay, the mass of a pair of rail dampers for one sleeper bay of length 0.6 m.



Figure 3.14 Addition of lumped mass to a sleeper bay in the finite element model.



Figure 3.15 Effect of added mass at mid-span on track vertical receptance and decay rate (a) receptance above sleeper, (b) receptance mid-span, (c) decay rate. — 17 kg added mid-span; — — no added mass.

With the additional mass, the pinned-pinned frequency is shifted from 1070 Hz down to 760 Hz, and the magnitudes of the resonance and the anti-resonance at the respective locations are increased (see Figure 3.15(a) and Figure 3.15(b)). In terms of stiffness, the track will appear even stiffer above sleepers and even softer at mid-span than in the normal case without rail dampers or extra mass.

The additional mass introduces a peak into the track decay rate shown in Figure 3.15(c) corresponding to the shifted pinned-pinned frequency. This is a result of increasing the usual pinned-pinned effects by adding the masses at mid-span. In practice, measured decay rates on tracks without rail dampers can display features of a similar magnitude. These are an effect of the measurement technique which samples predominantly at mid-span (see Jones *et al.* [2006]). Here, however, the peak is due to the addition of the 17 kg mass to the rail at mid-span.

3.4.2 Lumped mass rail damper model with viscous damping

A simple model of a rail damper represents it as a single lumped mass attached to the rail in the centre of each sleeper bay via a spring and viscous damper system, as shown in Figure 3.16.



Figure 3.16 Addition of rail damper into a sleeper bay in the finite element model- simple viscous damping model.

This representation of the rail dampers exhibits a single resonance, of the damper mass m_d on the spring stiffness k_d . This 'tuning frequency' can therefore be chosen by setting k_d for a particular mass. The parameters for this model of the rail damper are given in Table 3.4. The three values of stiffness k_d have been chosen to give tuning frequencies below, at and above the pinned-pinned resonance. The damping coefficient c_d gives the same damping at

800 Hz for the damper tuned to 1050 Hz as the nominal loss factor of the damper elastomer, 0.35.

Table 3.4 Parameters for lumped mass representation of rail dampers with viscous damping, for various damper tuning frequencies.

Description	800 Hz	1050 Hz	1300 Hz
Mass of damper pair m_d	17 kg	17 kg	17 kg
Elastomer stiffness k_d	4.3×10 ⁸ N/m	7.4×10 ⁸ N/m	11.3×10 ⁸ N/m
Elastomer damping c_d	3.0×10^4 Ns/m	3.0×10^4 Ns/m	3.0×10^4 Ns/m
Equivalent loss factor at tuning frequency	0.35	0.46	0.57

The addition of this rail damper model to the track again results in a shift in the pinnedpinned anti-resonance above a sleeper to a lower frequency, as seen with the addition of pure mass to the system. In this case, shown in Figure 3.17(a) and (b), the pinned-pinned resonance peak at mid-span seen in the model without rail dampers has been smoothed out especially if the rail damper is tuned to 1050 Hz near the pinned-pinned frequency. A smaller peak appears instead at the shifted pinned-pinned frequency. This occurs at 690 Hz for the stiffer rail damper and slightly lower at around 660 Hz for the damper tuned near the original pinned-pinned resonance. The softer rail damper does not give a very distinct resonance at mid-span, as it is damping controlled at the new pinned-pinned frequency.

An extra resonance above the sleepers has been introduced with the addition of the rail dampers to the model. This corresponds to modes of the rail with the dampers as nodes in the middle of each sleeper bay, and the greatest rail displacements occurring above the sleepers. It is the same modeshape as the pinned-pinned mode of the rail without rail dampers, but shifted half a sleeper bay along the track. This resonance occurs at around 1180 Hz, higher than the pinned-pinned resonance without the rail dampers as it is affected by the stiffness of the rail pads. It is also more damped than the original pinned-pinned resonance as a result of the rail pad damping.

For this damper model, the tuning frequency of 800 Hz has the greatest effect on the pinned-pinned resonance, resulting in the largest shift to lower frequency and also in the greatest reduction in the depth of the anti-resonance above the sleepers. This may be partly because the rail damper tuning frequency corresponds to the new pinned-pinned frequency of the system with the rail dampers (and their additional mass).



Figure 3.17 Effect of rail damper (lumped mass on spring and viscous damper) on track vertical receptance and decay rate for various damper tuning frequencies: (a) above sleeper, (b) mid-span, (c) decay rate; —— no rail dampers; — — —1050 Hz; …… 1300 Hz; — 800 Hz.

The track decay rate from the model is shown in Figure 3.17(c). The decay rate rolls off as for the model without rail dampers at the frequency of decoupling of the rail from the supports. The decay rate then increases again at the damper tuning frequency, and remains high throughout the frequency range of interest despite there being no further resonances of the rail damper. Much of the effect on the decay rate above the damper tuning frequency is due to the viscous damping model giving increasing effective loss factor with increasing frequency. This effect may unintentionally mimic the behaviour of the actual rail dampers, which have two tuning frequencies and additional beam bending modes and therefore

increase the track decay rate across a broad frequency range. However a more comprehensive model of the rail damper than this simple single-degree-of-freedom system is desirable to give a better representation of the properties of the elastomer in the finite element model.

3.4.3 Measured rail damper properties

In order to model the effect of the rail dampers on the track dynamics, it is particularly important to represent the properties of the elastomer accurately. The stiffness and damping properties of the elastomer used in the rail dampers tested in the Silence project have been measured in the form of shear modulus and loss factor across the frequency range of interest and are shown in Figure 3.18 (supplied by Corus).



Figure 3.18 Measured shear modulus and loss factor vs. frequency for Silence project rail damper elastomer (supplied by Corus).

The elastomer stiffness (shear modulus) increases slightly with frequency, while the loss factor is relatively constant. The apparent decrease in loss factor measured above 1000 Hz is a function of the measurement procedure, as is the increasing gradient of the shear modulus. Resonances in the measurement apparatus result in an erroneous measurement of loss factor above about 1000 Hz, in practice the loss factor is not expected to decrease [Ahmad, 2009].

The stiffness parameter of the simple viscous damping model of a rail damper presented in the previous section can be set to give the correct damper tuning frequency. However, the ability to fit the damping characteristic is limited as the equivalent loss factor of the rail damper model is linearly dependent on frequency. The loss factor from the simple viscous damping model (using parameters from Table 3.4 for the damper tuned to 1050 Hz) is compared in Figure 3.19 with the measured loss factor and the nominal target loss factor for the damper design of 0.35 [Thompson *et al.*, 2007].



Figure 3.19 Equivalent frequency dependent loss factor of finite element model of rail dampers tuned to a natural frequency of 1050 Hz: ——— measured; $\cdots \cdots \cdot v$ is cous damping model; — — nominal constant value.

The loss factor of the simple viscous damping has been set to be equal to the nominal loss factor in the middle of the frequency range of interest, around 800 Hz in this case, however at lower frequencies the loss factor as modelled will be too low, and at higher frequencies the loss factor is over-estimated by this model. In order to model accurately the effect of the rail dampers on the track dynamics, an improved model of the damping characteristics is needed for inclusion in the finite element model of the track.

3.4.4 Lumped mass rail damper model with improved damping representation

In order to represent the properties of the rail damper elastomer better, a combination of 'Maxwell elements' [Lockett, 1972] in parallel with a spring is used as shown in Figure 3.20. The damper mass is again represented as a single lumped mass attached to the rail in the centre of each sleeper bay via the spring/damper system.



Figure 3.20 Addition of rail damper (improved damping model) the finite element model.

In practice, implementing Maxwell elements in a finite element code creates a difficulty in that the absence of any mass at the node required between each spring and damping element results in a singularity. To circumvent this difficulty, small additional masses are included at each of the intermediate nodes as shown in Figure 3.21. The natural frequencies of the added masses m_{f1} and m_{f2} on the damper springs k_{d1} and k_{d2} are set to be well above the frequency range of interest for the model. Modes of vibration of these small masses are then not included in the modal summation.



Figure 3.21 Damping element with small added masses m_{f1} and m_{f2} to remove singularities.

In order to calibrate the damper model to the actual damper, the equivalent stiffness and loss factor of the system are calculated and compared with measured values.

Equation (3.42) gives the equations of motion of the damping element in matrix form. These are obtained by neglecting all the masses in the system and using the degrees of freedom shown in Figure 3.22.



Figure 3.22 Degrees of freedom of damping element

$$\begin{bmatrix} c_{d1} + c_{d2} & -c_{d1} & -c_{d2} \\ -c_{d1} & c_{d1} & 0 \\ -c_{d2} & 0 & c_{d2} \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \end{bmatrix} + \begin{bmatrix} k_d & 0 & 0 \\ 0 & k_{d1} & 0 \\ 0 & 0 & k_{d2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ 0 \\ 0 \end{bmatrix}$$
(3.42)

The equations of motion may be rearranged to eliminate the internal degrees of freedom into the form $f_1 = ku_1$, where *k* is both complex and frequency dependent:

$$f_{1} = \left(\frac{c_{d1}^{2}\omega^{2}}{c_{d1}i\omega + k_{d1}} + \frac{c_{d2}^{2}\omega^{2}}{c_{d2}i\omega + k_{d2}} + c_{d1}i\omega + c_{d2}i\omega + k_{d}\right)u_{1}$$
(3.43)

For hysteretic damping $f = k(1+i\eta)u$ where η is the loss factor. Therefore the imaginary part of the bracketed term in Equation (3.43) may be normalised by dividing by the equivalent stiffness of the spring, *i.e.* the real part of the bracketed term, and compared with the nominal design loss factor of the elastomer of 0.35. This is shown in Figure 3.23 for the damper parameters listed in Table 3.5 for the damper tuning frequency of 1050 Hz. The damper parameters are scaled to give the same loss factor characteristic in each case for the three different tuning frequencies.

A good model of the damping characteristics of the elastomer is achieved by modelling the damping elements in this way over the frequency range of interest between 100 and 2000 Hz.



Figure 3.23 Equivalent frequency dependent loss factor of finite element model of rail damper elastomer: — measured; · · · · · · improved damping model (1050 Hz tuning frequency); — — nominal constant value.

Table 3.5 Input parameters for single lumped mass model of rail damper, for various damper tuning frequencies.

Description	800 Hz	1050 Hz	1300 Hz
Mass of pair of dampers m_d	17 kg	17 kg	17 kg
Elastomer stiffness k_d	2.58×10 ⁸ N/m	4.3×10 ⁸ N/m	6.45×10 ⁸ N/m
Stiffness 1 st Maxwell element k_{d1}	1.56×10 ⁸ N/m	2.6×10 ⁸ N/m	3.9×10 ⁸ N/m
Stiffness 2^{nd} Maxwell element k_{d2}	1.56×10 ⁸ N/m	2.6×10 ⁸ N/m	3.9×10 ⁸ N/m
Damping 1^{st} Maxwell element c_{d1}	1.2×10^5 Ns/m	2.0×10^5 Ns/m	3.0×10 ⁵ Ns/m
Damping 2^{nd} Maxwell element c_{d2}	1.2×10^4 Ns/m	2.0×10^4 Ns/m	3.0×10^4 Ns/m

The real part of the bracketed term in Equation (3.43) may be compared with the stiffness of the spring k_d across the frequency range, shown in Figure 3.24. In practice the elastomer stiffness increases with frequency as does the equivalent stiffness of the damper system. Consequently the Maxwell element model is a better representation of stiffness than that achieved by using a single frequency-independent value.



Figure 3.24 Equivalent frequency dependent stiffness of finite element model of rail damper elastomer: $\cdots \cdots$ as modelled (1050 Hz tuning frequency); — — — nominal stiffness of spring k_d ; — — measured shear modulus (right hand axis).

The rail dampers as described are included in the finite element model of the track, adding into the global mass, stiffness and damping matrices of Equation (3.34). Figure 3.25(a) and (b) show the predicted vertical track receptance with the addition of the rail dampers. The pinned-pinned frequency is shifted to a lower frequency and its resonances are smoothed as before (see Figure 3.17). The effect of this damper model on the sharpness of the anti-resonance above the sleepers is more pronounced than the effect of the simple viscous damping model. In general though, the effect on the receptance of this damper model is similar to the viscous damping model.

Figure 3.25(c) shows the decay rate characteristic of the track for this model of the rail dampers for several tuning frequencies. These are mostly similar to the decay rates predicted by the simple viscous damping model (Figure 3.17(c)). For the damper with the lowest tuning frequency (800 Hz), the decay rate drops off above the damper tuning frequency, whereas for the corresponding viscous damper model, the decay rate remains high throughout the frequency range of interest. A slight drop in the decay rate above the tuning frequency is also seen for the 1050 Hz damper in Figure 3.25(c) when compared with the viscous model decay rate.



Figure 3.25 Predicted change in track vertical receptance and decay rate with the addition of rail dampers (lumped mass on improved spring and viscous damper system) for various damper tuning frequencies. (a) Receptance above sleeper; (b) receptance mid-span; (c) decay rate: ——— no rail dampers; — — — 1050 Hz; …… 1300 Hz; — …… 800 Hz.

3.5 Comparison of modelled and measured track decay rates

Figure 3.26 shows the vertical track decay rates calculated by the model compared with measurements taken by Corus at the Silence project test site. The comparison is made for track both with and without rail dampers. Without rail dampers, the finite element model predicts slightly higher decay rates at high frequencies than seen in the measurements. This difference is due mostly to the use of a viscous damping model to represent the rail pad. The difference could be reduced by decreasing the damping in the rail pad model, but then a longer track length would be required to eliminate end effects from the results. The roll off in track decay rate above 300 Hz as the rail decouples from the sleepers and ballast is

represented well by the model. Overall the finite element model of the track without the rail dampers gives a good match for the decay rate compared with measurements.



Figure 3.26 Comparison of track vertical decay rates. Measurements taken by Corus at the Gersthofen test site with soft rail pads: ——measured without rail dampers; — — measured with rail dampers; ····· calculated without rail dampers; $- \cdot - \cdot - \cdot$ calculated with rail dampers tuned to 1050 Hz.

The effect of the rail dampers as modelled here on the track decay rate is clear but the model used does not fully represent the effect of the actual rail damper at all frequencies (Figure 3.26). This is to be expected as each rail damper is made up of two beams (with two tuning frequencies) rather than a single lumped mass as modelled. Consequently the model with a single lumped mass does not result in the same uniformly high decay rate as that measured at the Gersthofen site. An additional resonance above the sleepers and anti-resonance at mid-span above 1000 Hz has been introduced to the system by the rail dampers as modelled, resulting in an over-estimation of the track decay rate at this frequency. This corresponds to a bending mode of the rail with the dampers acting as nodes, which would not be expected to be as significant for a real damper attached at more than one point in the sleeper bay.

The damper modelled here has a single lumped mass and therefore a single tuning frequency. To make detailed studies of realistic commercial damping devices more detail should be included, taking into account the attachment to the rail (over approximately half the sleeper bay length) and multiple masses and tuning frequencies. More detailed modelling has been carried out to study the Corus rail damper design and is reported in Croft *et al.* [2009]. The modelling of specific designs of damper in this fashion requires

larger finite element models and greatly extended computing times. For the purpose of this thesis however, the presentation and discussion is limited to a generic lumped mass damper model. Overall the effect of the dampers on the track dynamics is represented by the model satisfactorily for the time being.

3.6 Summary of track modelling

In this chapter the finite element model of the track to be used in this work has been described. The parameters required to represent a typical ballasted track have been determined. The point receptance and decay rate of the track as modelled have been examined and compared with simple frequency domain models in the frequency range of interest up to 2000 Hz. A limitation of the finite element track model is the requirement to truncate the track. However comparison of finite element model results with results from an infinite beam model have shown that end effects have been minimised and that a track length of 60 sleeper bays is adequate.

The rail dampers have been included in the track model and their effect on the track receptance and decay rate has been investigated. The model shows that the mass of the rail dampers leads to a shift in the pinned-pinned frequency from 1050 Hz to around 760 Hz. The rail dampers also smooth the peaks and dips in the track receptance. The modelled track decay rates have been compared with measurements from Gersthofen with and without the rail dampers. Overall the track model as described here is a good representation of the typical ballasted track at this site.

4 WHEEL-RAIL INTERACTION FORCE

4.1 Introduction

The purpose of this chapter is to calculate the normal force between the wheel and rail to use as an input to the contact and wear model that follows. The normal force may be calculated by the simultaneous solution of equations describing the motion of the track and vehicle. A representation of the vehicle and the coupling between the vehicle and the rail must therefore be added to the model of the track.

The finite element model of the track has been described in the preceding chapter. In this chapter, a simple model of the vehicle is added. The coupling between the vehicle and track takes place through the stiffness of the contact spring between the wheel and the rail. Hertzian contact is assumed for this part of the calculation, and the roughness of the track is processed to account for the filtering effect of the size of the contact patch before entering the time-stepping routine.

The equations of motion of the track and vehicle system are solved using a state-space approach in a time-stepping routine. The variable of interest is the interaction force between the wheel or wheels and the rail. A set of initial values is assumed for the analysis and the calculation proceeds step by step from this initial point, converging to the required solution over a distance of several sleeper bays. The force results are then extracted from the middle sleeper bays of the track model. The interaction force model used here is not original; it is based on the work of Nielsen and Igeland [1995] with only minor changes. Sample results from the force model are compared with the output of Nielsen's model DIFF and also with output from the model of Pieringer *et al.* [2008] as a check on the implementation and coding.

The results from the interaction force model are of interest in their own right, to assess the effect of different track and vehicle parameters on the interaction forces. Different vehicle types and speeds result in different interaction force spectra. The effect of rail dampers on the track dynamics and interaction forces has not previously been studied. In particular the effect of the rail dampers on the interaction forces at and around the pinned-pinned resonance of the track is of interest. Some interaction force results with the rail dampers are therefore included as examples at the end of this chapter, with the full set of results presented in Appendix A.

4.2 Model of vehicle

The vehicle is modelled by either one or more uncoupled wheel masses each linked to the rail by a non-linear Hertzian contact spring, with an external static force applied to represent the sprung vehicle mass. Figure 4.1 is a representation of the elements of the vehicle and track models.



Figure 4.1 Finite element track and vehicle model.

In order to calculate the contact forces between the wheels and the rail as the wheels move along the track at constant speed v, the equations of motion of the system are solved by a time-stepping routine using a state-space formulation.

4.2.1 Equations of motion for vehicle

Each wheel (including its contact spring) has two degrees of freedom, the vertical translation of the contact point u_{ai} and the vertical translation of the wheel centre u_{bi} in the vertical direction (positive downwards) where *i* is the wheel number, as shown in Figure 4.2. The external static load on the wheel is F_{ei} , and the contact force at the interface with the rail is F_{ai} . The wheels need not be connected to each other as above around 10 Hz the vehicle suspension isolates the vehicle body from the unsprung mass of the vehicle and the track [Knothe & Grassie, 1993].



Figure 4.2 Wheels and contact springs.

The contact stiffnesses k_{Hi} for the wheels are non-linear and are determined by

$$k_{Hi} = \begin{cases} C_H \sqrt{u_{bi} - u_{ai}} & \text{N/m} & \text{for } u_{bi} - u_{ai} > 0\\ 0 & \text{else} \end{cases}$$
(4.1)

where C_H is a constant calculated from the Hertzian equations for an elliptical point contact. For this approximation of the contact between the wheel and the rail, the relation between the contact force F_{ai} and the approach of distant points δ in the two bodies is given in this form by Johnson [2001]:

$$\delta = \left(\frac{9}{16} \frac{F_{ai}^{2}}{RE^{*2}}\right)^{\frac{1}{3}}$$
(4.2)

where *R* is the equivalent radius of curvature and δ is equivalent to u_b - u_a . Assuming the wheels and rail are perpendicular cylinders, *R* is calculated from the radius of the wheel R_w and the radius of the rail R_r as

$$R = \sqrt{R_w R_r} \tag{4.3}$$

 E^* is determined from the Young's modulus E and Poisson's ratio v of the wheel and rail (here both steel) by

~ /

$$E^* = \left(\frac{1 - {v_w}^2}{E_w} + \frac{1 - {v_r}^2}{E_r}\right)^{-1}$$
(4.4)

Equation (4.2) may be rearranged into the form

$$F_{ai} = C_H \delta^{3/2} \tag{4.5}$$

so that the constant C_H is given by

$$C_{H} = \frac{4}{3}\sqrt{RE^{*2}}$$
(4.6)

The wheel radius for each vehicle type examined here is 0.46 m, while the material properties of both the wheel and rail are $E = 2.1 \times 10^{11} \text{ N/m}^2$ and v = 0.3. For a new rail profile the radius of the rail head across the rail is 0.3 m. These parameters result in a value of the Hertzian constant C_H of 93.8×10⁹ N/m^{3/2}.

Typical parameters for three vehicle types are listed in Table 4.1 below. The velocities are the averages for each vehicle type measured by Deutsche Bahn AG at the Gersthofen test site, as part of the Silence project.

Train Type	Notation	Value	Description
Freight	v	29.44 m/s	Train wheel velocity
TAMNS 895	$M_{\scriptscriptstyle W}$	488.5 kg	Unsprung wheel mass
(wheel BA04)	F_{e}	$100 \times 10^3 \mathrm{N}$	Static load on wheel
	l	0 m, 1.8 m, 5.04 m, 6.84 m	Axle spacing
Regional	v	37.78 m/s	Train wheel velocity
Doppelstockwagen	$M_{\scriptscriptstyle W}$	702.5 kg	Unsprung wheel mass
DBz751	F_{e}	$60 \times 10^3 \mathrm{N}$	Static load on wheel
(wheel BA220)	l	0 m, 2.5 m, 6.4 m, 8.9 m	Axle spacing
High-speed	v	43.06 m/s	Train wheel velocity
ICE1 coach	$M_{\scriptscriptstyle W}$	782 kg	Unsprung wheel mass
(wheel BA014)	F_e	$60 \times 10^3 \mathrm{N}$	Static load on wheel
	l	0 m, 2.5 m, 7.4 m, 9.9 m	Axle spacing

Table 4.1 Vehicle model parameters.

In matrix form, the resulting equations of motion for a single wheel vehicle model are

$$\begin{bmatrix} 0 & 0 \\ 0 & M_w \end{bmatrix} \begin{bmatrix} \ddot{u}_a \\ \ddot{u}_b \end{bmatrix} + \begin{bmatrix} k_H & -k_H \\ -k_H & k_H \end{bmatrix} \begin{bmatrix} u_a \\ u_b \end{bmatrix} + \begin{bmatrix} F_a \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ F_e \end{bmatrix}$$
(4.7)

where M_w is the unsprung mass of the wheel. The force at the contact point F_a is the variable of interest, to be determined from the model of the wheel coupled with the model of the rail.

For a two wheel model, the equations of motion of the system may be written as

4.2.2 Coupling of wheel and track models

In order to couple the physical model of the wheel with the modal track model, the force at the contact point F_a between each wheel and the rail needs to be distributed between the nodes of the track model on either side of the actual wheel location at each point in time.

A local coordinate $\zeta_j(t)$ is defined for $j = 1, ..., J_e$. J_e is the total number of finite element model track elements. Its value ranges from 0 to L_j for each finite element between two track nodes, where L_j is the distance between two nodes of the track model as shown in Figure 4.3. As the wheel moves with time *t*, at each time-step the distance is recalculated. This local coordinate is the same as that introduced by Nielsen and Abrahamsson [1992].



Figure 4.3 Element j of track model.

The load from a wheel at position $\xi_j(t)$ is distributed onto the adjacent nodes by use of Hermite interpolating polynomials [Martin & Carey, 1973; Nielsen & Abrahamsson, 1992]:

$$H_{1,j} = 1 - \frac{3\xi_j^2}{L_j^2} + \frac{2\xi_j^3}{L_j^3}$$
(4.9)

$$H_{2,j} = \xi_j - \frac{2\xi_j^2}{L_j} + \frac{\xi_j^3}{L_j^2}$$
(4.10)

$$H_{3,j} = \frac{3\xi_j^2}{L_j^2} - \frac{2\xi_j^3}{L_j^3}$$
(4.11)

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$$H_{4,j} = -\frac{\xi_j^2}{L_j} + \frac{\xi_j^3}{L_j^2}$$
(4.12)

The interpolating polynomials are assembled in a matrix **H** in the same order as the degrees of freedom of the finite element model: a vertical term followed by a rotation term for each node.

$$\mathbf{H} = \begin{bmatrix} H_1 & H_2 & H_3 & H_4 \end{bmatrix} \tag{4.13}$$

The rail deflection, velocity and acceleration at positions between the nodal degrees of freedom are estimated using the same interpolating polynomials, and equated to the wheel translation taking account of any initial track roughness *r*, which is a function of distance *x* or the local coordinate ξ_j for the each rail finite element.

These interpolating polynomials are chosen because they provide continuity in derivative values across element boundaries. They are cubic polynomials which can therefore represent Euler beam bending (*e.g.* as in the derivation of a two-node Euler beam bending finite element [Petyt, 1990]). The interpolation functions correspond to the deformed shape of an Euler beam element; however they do not provide a good approximation of the deformed shape of Timoshenko beam elements where shear effects are significant. This leads to discretisation effects in the calculation of the wheel-rail interaction force. An alternative to these interpolation functions would be to use the corresponding shape functions for a Timoshenko beam. This approach was taken by Nielsen and Igeland [1995]. However, the resulting derivatives are discontinuous across element boundaries. This again results in discretisation effects of significant magnitude. The Timoshenko beam functions have been trialled in this model but the discretisation effects were found to be worse than those with the standard interpolating polynomials. Discretisation effects are discussed further in Section 4.4.1.

The interaction forces are generated by the roughness function along the rail and also by parametric excitation due to the variation in track stiffness caused by the discrete supports. If the initial track roughness is set to zero then the interaction forces resulting from purely parametric excitation will be determined. The initial roughness along the rail may be set to zero, to a regular sinusoidal function, to a measured roughness profile, or to a roughness profile generated from a spectrum using randomly distributed phase for each wavelength component.

Applying the interpolation polynomials and any initial roughness, the displacement of the wheel u_{ai} at a point between two nodes of the track is given by the displacement of the rail and the relative distance between the track and wheel, *i.e.* the roughness function *r*:

_

$$u_{ai}(t) = \mathbf{H} \begin{bmatrix} u_{t,2j-1} \\ \theta_{t,2j} \\ u_{t,2j+1} \\ \theta_{t,2j+2} \end{bmatrix} + r(\xi_j)$$
(4.14)

In terms of modal coordinates, the four degrees of freedom of the adjacent track nodes correspond to \mathbf{P}^{int} , which is a partition of the relevant four rows of the modal matrix \mathbf{P} (from Equation (3.37)).

The displacement of the wheel and track at the interface can be written in modal coordinates (*i.e.* using the track modal analysis described in Chapter 3) as:

$$u_{ai}(t) = \mathbf{H} \mathbf{P}^{\text{int}} \mathbf{q}(t) + r(\boldsymbol{\xi}_{j})$$
(4.15)

The velocity and acceleration of the interfacial degree of freedom are given by time derivatives of the displacement:

$$\dot{u}_{ai}(t) = \mathbf{T}(t)\dot{\mathbf{q}}(t) + \mathbf{U}(t)\mathbf{q}(t) + \dot{r}$$
(4.16)

$$\ddot{u}_{ai}(t) = \mathbf{R}(t)\dot{\mathbf{q}}(t) + \mathbf{S}(t)\mathbf{q}(t) + \ddot{r}$$
(4.17)

where **T**, **U**, **R** and **S** are defined by:

$$\mathbf{T}(t) = \mathbf{H}\mathbf{P}^{\text{int}} \tag{4.18}$$

$$\mathbf{U}(t) = \frac{d\mathbf{H}}{d\xi} v \mathbf{P}^{\text{int}}$$
(4.19)

$$\mathbf{R}(t) = 2\frac{d\mathbf{H}}{d\xi} v \mathbf{P}^{\text{int}} + \mathbf{H} \mathbf{P}^{\text{int}} \operatorname{diag}(i\omega_n)$$
(4.20)

$$\mathbf{S}(t) = \frac{d^2 \mathbf{H}}{d\xi^2} v^2 \mathbf{P}^{\text{int}}$$
(4.21)

The derivatives with respect to time of the initial roughness function r are as defined by [Nielsen & Abrahamsson, 1992]:

$$\dot{r} = \frac{dr}{d\xi}v(t) \tag{4.22}$$

$$\ddot{r} = \frac{d^2 r}{d\xi^2} v^2(t) + \frac{dr}{d\xi} \dot{v}(t)$$
(4.23)

The contact forces can also be written in the modal form, making use of the interpolating polynomials to distribute the loads between the nodes either side of the contact point.

$$\mathbf{Q} = \mathbf{P}^{\text{int}\,T} \mathbf{H}^T \mathbf{F}_a(t) \tag{4.24}$$

4.3 State-space solution to equations of motion of system

The modal forms of the equations of motion for the track, vehicle and their interaction are to be solved simultaneously for the interaction forces between the wheels and the rail. These Equations (3.39), (4.8) and (4.24) are arranged in a standard matrix form as in Nielsen and Igeland [1995].

$$\mathbf{A}(\mathbf{g},t)\dot{\mathbf{g}} + \mathbf{B}(\mathbf{g},t)\mathbf{g} = \mathbf{F}(\mathbf{g},t)$$
(4.25)

A(g,t) and B(g,t) are as defined by Nielsen and Igeland [1995].

$$\mathbf{A}(\mathbf{g},t) = \begin{bmatrix} \frac{\mathrm{diag}(\mathbf{a}) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{P}^{\mathrm{int}\,\mathrm{T}}\mathbf{H}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{R} & \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{T} & -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$
(4.26)
$$\mathbf{B}(\mathbf{g},t) = \begin{bmatrix} \mathrm{diag}(\mathbf{b}) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{\mathrm{H}} & -\mathbf{K}_{\mathrm{H}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{K}_{\mathrm{H}} & \mathbf{K}_{\mathrm{H}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{0} \\ \mathbf{S} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$
(4.27)

The wheel damping C_w is assumed to be zero in this work. Extra wheels add more entries to the last 5 rows and columns in **A** and **B**.

 $\mathbf{g}(t)$ is a vector constructed from the modal coordinates \mathbf{q} and the parameters of interest to be solved for, namely the interaction force in the form of the impulse $\hat{\mathbf{F}}_a$, and the displacement and velocity of both the wheel centre and the contact point. The force

impulse $\hat{\mathbf{F}}_a$ at each location is the integral of the normal interaction force \mathbf{F}_a over the length of the time-step.

$$\mathbf{g}(t) = \left\{ \mathbf{q} \ \mathbf{u}_{\mathbf{a}} \ \mathbf{u}_{\mathbf{b}} \ \dot{\mathbf{u}}_{\mathbf{a}} \ \dot{\mathbf{u}}_{\mathbf{b}} \ \hat{\mathbf{F}}_{\mathbf{a}} \right\}$$
(4.28)

The forcing term $\mathbf{F}(\mathbf{g},t)$ in Equation (4.25) is given by

$$\mathbf{F}(\mathbf{g},t) = \left\{ \mathbf{0}^{\mathrm{T}} \quad \mathbf{0}^{\mathrm{T}} \quad \mathbf{F}_{b}^{ext^{\mathrm{T}}} \quad \mathbf{0}^{\mathrm{T}} \quad - \ddot{\mathbf{r}}^{\mathrm{T}} \right\}^{\mathrm{T}}$$
(4.29)

Equation (4.25) may be rearranged into the form of an ordinary differential equation:

$$\dot{\mathbf{g}} = \mathbf{A}^{-1} (\mathbf{F} - \mathbf{B}\mathbf{g}), \quad \mathbf{g}(t=0) = g_0$$

$$(4.30)$$

In order to improve computational efficiency, the matrix A(g,t) is divided into submatrices:

$$\mathbf{A}(\mathbf{g},t) = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}$$
(4.31)

This allows the diagonal submatrix A_{11} to be inverted separately before entering the timestepping function, and removes the need to invert the large matrix A(g,t) at every timestep, as the other submatrices are significantly smaller than A_{11} . Instead, the inverse of A(g,t) is then calculated using a method described by Barnett [1979]:

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A}_{11}^{-1} + \mathbf{A}_{11}^{-1} \mathbf{A}_{12} \mathbf{G}^{-1} \mathbf{A}_{21} \mathbf{A}_{11}^{-1} & - \mathbf{A}_{11}^{-1} \mathbf{A}_{12} \mathbf{G}^{-1} \\ - \mathbf{G}^{-1} \mathbf{A}_{21} \mathbf{A}_{11}^{-1} & \mathbf{G}^{-1} \end{bmatrix}$$
(4.32)

where

$$\mathbf{G} = \left(\mathbf{A}_{22} - \mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\right) \tag{4.33}$$

To solve the state-space system, a set of initial values are given to \mathbf{g} at time t = 0. The time is then incremented and new values for \mathbf{g} are calculated using a standard variable step size Adams-Bashforth-Moulton routine. The Matlab routine ODE113 [Shampine & Reichelt, 1997] has been used in this case. This routine returns results at the requested times, in this case those corresponding to regular 1 mm distance intervals. The wheel-rail interaction forces are thus determined as the wheels move along the model of the track.

In order to minimise the calculation time, the time-stepping routine is begun with the wheel or wheels already part way along the track. The interaction forces are calculated

over 5 m of the track only, or more if more than one wheel is included. This 5 m length includes just over five bays to allow the routine to settle down from the initial transient effects, followed by the three sleeper bays of interest for the calculation. Minimising the calculation length in this way significantly reduces the time to run the interaction force time-stepping routine, and examination of the results as shown in Figure 4.4 indicates that the routine is able to converge to a solution before reaching the middle three sleeper bays. Three bays (1.8 m) has been chosen as a suitable sample length to balance the computational cost of the wear model with the roughness wavelength range of interest, discussed further in Section 7.6.



Figure 4.4 Example of interaction force converging to solution for a single wheel freight model on a smooth rail. Model length 60 sleeper bays; calculation begins with wheel 13.6 m along the track and ends after wheel traverses the central three sleeper bays.

4.4 Effect of variations in the track model

The interaction force is sensitive to variations in the track model. In order to develop an understanding of the model sensitivity, the interaction force analysis has been completed for a series of baseline cases. The model variations considered are the number of beam elements in each bay, the number of vehicle wheels included, the effect of contact filtering, the initial roughness profile and the effect of variable sleeper spacings.

The track model input parameters are as listed in Table 3.3. The vehicle modelled in this section is a typical freight train at this site as described in Table 4.1.

4.4.1 Effect of number of beam elements per sleeper bay

Figure 4.5 shows the dynamic interaction force spectrum of the middle three sleeper bays of the model, calculated by discrete Fourier transform from the time history. The static load on the wheel has first been subtracted. Results are shown in one-third octave bands, plotted against roughness wavelength as well as frequency. The roughness wavelength is of interest as force results can then be compared later in this work with roughness spectra and roughness growth rates, independent of the vehicle velocity. The vehicle model for this example is a freight vehicle with a single wheel, and the rail is smooth with no initial roughness. As noted previously the Hermite interpolation functions used to determine the interaction force between two nodes of the model introduce some discretisation effects. The exact shape functions of a Timoshenko beam are not used for the interpolation of the results as they result in discontinuous derivatives at the element boundaries.

To illustrate these discretisation effects, the rail has been modelled using 2, 4, 8, 12, 24 or 48 elements in each sleeper bay as shown in Figure 4.5. The rail is smooth so the dynamic force is due to the parametric excitation and the element discretisation effects only. The parametric effects are evident in all these cases especially at 0.3 m which is half the sleeper spacing (to see the actual sleeper spacing of 0.6 m, more sleeper bays would need to be included in the analysis). Parametric effects are independent of the element length, although if less than eight elements are used in each sleeper bay the discretisation effects and parametric effects coincide, as shown in Figure 4.5(a) in the wavelength bands corresponding to half and quarter sleeper bay lengths. When more elements are used, shown in Figure 4.5(b), parametric effects dominate the force spectrum for wavelengths longer than about 0.125 m. Discretisation effects can be clearly seen at wavelengths shorter than 0.1 m, with each case showing a peak in the force-wavelength spectrum corresponding to the element length.

If it were possible to use even more elements, the discretisation effects would shift to even shorter wavelengths and eventually out of the wavelength range of interest. However this would require an unfeasible number of elements in a track model of this type. In practice, using more than eight or twelve elements per bay results in extremely high computer memory requirements and long calculation times.


4.4.2 Discretisation effects with realistic rail roughness

Now the effect of introducing a realistic rail roughness profile is examined for two example roughness levels. One roughness profile has been generated from the TSI limit spectrum [European Commission, 2005 & 2006], the other is a low-level broadband roughness generated from a smooth spectrum of similar level to measured roughness at the Silence project test site at Gersthofen (see also Section 4.4.4). These roughness spectra are shown in Figure 4.6. The roughness levels measured at Gersthofen are low, compared with

the TSI limit spectrum. Figure 4.7 shows the resulting dynamic force spectrum over three sleeper bays for track with these roughness levels.





At most wavelengths shown in Figure 4.7, the dynamic forces generated by the TSI level roughness are at least 10 dB higher than the dynamic forces due to parametric excitation or element discretisation effects. For the low level broadband roughness, the short wavelength dynamic forces are almost all well above the level of the discretisation effects. However in the 0.08 m one-third octave wavelength band and at longer wavelengths, the dynamic force due to the low level roughness is not significantly higher than the force due to the

discretisation effects with 8 elements per sleeper bay. Care must therefore be taken in interpreting results from the model in the 0.08 m one-third octave wavelength band in cases with low level roughness.

For this work eight elements are chosen to represent the rail in each sleeper bay. The effects of the discretisation remain. These are not significant in cases with a realistic TSI level roughness spectrum present on the railhead because they are masked by the dynamic forces due to the roughness as shown in Figure 4.7. With a lower rail roughness level, the element discretisation effects are insignificant in most of the wavelength range, but need to be kept in mind in analysing results in the 0.08 m one-third octave wavelength band.

4.4.3 Effect of initial sinusoidal roughness profile

In order to examine the effect of the railhead roughness on the wheel-rail interaction forces, results have been calculated for a series of cases with sinusoidal rail profiles of wavelengths 20, 40 and 80 mm. The amplitude of the sinusoidal roughness is 1×10^{-5} m in each case, corresponding to a roughness level of 17 dB re 1µm at that wavelength. All the following cases have eight beam elements per bay, regular sleeper spacing, soft rail pads and a single freight wheel model of the vehicle.

The spatial history (time history plotted against location) of the interaction force in the middle sleeper bay is shown in Figure 4.8 for the smooth rail case and for each of the cases with a sinusoidal initial profile. With an initial roughness, the periodicity of the roughness dominates the interaction force. Some variation throughout the sleeper bay, due to parametric excitation and resonances of the track, can also be seen.

In spectral terms, the interaction forces in Figure 4.9 show peaks corresponding to the original roughness wavelength in each case. The peaks in the smooth rail case arising from parametric excitation are much smaller than those where an initial sinusoidal profile is present. Harmonics of the initial profile wavelength can also be seen in the force spectrum. The magnitude of the dynamic force is much higher for the shorter wavelengths of harmonic roughness than for the smooth rail or for the harmonic roughness of wavelength 0.08 m.



Figure 4.8 Dynamic interaction force history of middle sleeper bay for (a) smooth rail; (b) 0.02 m wavelength profile; (c) 0.04 m wavelength profile; (d) 0.08 m wavelength profile.



Figure 4.10 shows the variation in the force history alongside the roughness profile. In all these sinusoidal roughness cases, a peak in the interaction force occurs shortly before each crest in the roughness profile.



Figure 4.10 Dynamic interaction force history compared to sinusoidal roughness profile: (a) 0.02 m wavelength; (b) 0.04 m wavelength; (c) 0.08 m wavelength; ——force; — — roughness profile (not to scale).

4.4.4 Realistic broadband roughness and contact filter effects

For this work, a broadband roughness profile based on the low level spectrum shown in Figure 4.6 is generated along the length of the finite element track model, as in Nielsen [2003]. Thirty harmonic roughness components with random phase have been included in each one-third octave wavelength band, to create roughness with the desired spectral content. Since this roughness, based on the measured spectrum, is low, and the wheel is assumed to be smooth, selected calculations are repeated for a case with a roughness profile generated to match the TSI limit spectrum. This is still a relatively low roughness, especially since in practice the combined roughness of the system will be a combination of the wheel and rail roughness.

With a broadband roughness profile, the dynamic interaction forces (as shown previously in Figure 4.7) are much higher at short wavelengths than the dynamic force with a smooth rail. However as the contact patch between the wheel and the rail has a finite area, roughness wavelengths shorter than the size of the contact patch do not excite the wheel-rail system as effectively as longer wavelength roughness. The interaction force therefore decreases with shortening wavelengths. To account for the effect of the contact patch size a 'contact filter' is required when predicting the interaction force resulting from a surface roughness. Here, the two-dimensional distributed point reacting spring (DPRS) model developed by Ford and Thompson [2006] is applied to all cases where the initial roughness has a realistic profile *i.e.* not zero or purely sinusoidal roughness.

For a roughness profile r(x), the total contact force *P* for a wheel centred at *x* is given by Ford and Thompson [2006] as

$$P(x) = \int_{-a}^{a} f(x') dx'$$
(4.34)

where

$$f(x') = \begin{cases} k(\delta - z(x') + r(x+x')) & \text{if } \delta - z(x') + r(x+x') > 0\\ 0 & \text{if } \delta - z(x') + r(x+x') \le 0 \end{cases}$$
(4.35)

and z(x') is the circular profile of the wheel as a function of x', the position in the contact patch relative to the centre at x. The integration of the contact force is performed over a range -a < x' < a such that all potential points of contact are included, and a value of the deflection δ is determined such that the total force P is equal to the static wheel load F_e . The equivalent roughness of the system including the filtering effect of the contact may then be determined from the difference between the calculated deflection δ and the nominal deflection

$$\delta_{nom} = \left(\frac{F_e}{\frac{4\sqrt{2}k}{3}\sqrt{R_m}}\right)^{\frac{2}{3}}$$
(4.36)

where

$$k = \frac{1}{2} \frac{E}{(1 - \nu^2)} \tag{4.37}$$

and the wheel radius R_m (adjusted to give the correct length assuming circular contact in a two-dimensional DPRS model) is

$$R_m = \frac{1}{2}R_w \tag{4.38}$$

This contact filter can be used in a quasi-static sense, by assuming the contact patch length and the interaction force are constant along the length of the track. In this case the equivalent roughness calculated along the length of the track model replaces the input roughness profile before calculation of the interaction forces in the time-stepping model. Alternatively the contact filter may be applied dynamically inside the time-stepping routine at every position *x* along the rail with the force at each position able to vary. This method increases the calculation time but is more realistic, especially if the interaction force is varying significantly, for example with high levels of roughness. Ford and Thompson [2006] tested both methods and found that quasi-static filtering was adequate for low levels of roughness. This finding is replicated here, with Figure 4.11 showing very little difference in the results obtained by quasi-static and dynamic filtering. In the remainder of this work, quasi-static filtering is used and the roughness profile is processed before being used to calculate the interaction forces.



4.4.5 Validation by comparison with established force models

The interaction force model used here has been validated by comparing results with the output from two established wheel-track interaction models developed at Chalmers University of Technology. The first of these is Nielsen's model DIFF, which has been validated by measurements [Nielsen, 2006]. The second is the model of Pieringer *et al.* [2008] which uses a different approach based on impulse response functions, and does not require the rail to be divided into elements. The author acknowledges the assistance of Astrid Pieringer in calculating the results shown in this section from the Chalmers models.

Results are presented in Figure 4.12 for a single freight wheel on smooth track with soft rail pads, from an 80 sleeper bay track model with eight rail elements per sleeper bay.



Figure 4.12 Force comparison for smooth rail case (a) Dynamic interaction force history and (b) dynamic interaction force spectrum: -- Croft; -- DIFF; $\cdots \cdots$ Pieringer.

The force history in the sleeper bay (Figure 4.12(a)) is similar from all three models, although there are differences at short wavelengths that can be seen more clearly in the spectrum (Figure 4.12(b)). The only excitation to the system in this case is parametric, so minor differences between the models lead to significant differences in the spectra.

The dynamic force histories (Figure 4.13(a)) for a corrugated case with a sinusoidal profile of wavelength 40 mm again show that the results from all three models are very similar. The short wavelength differences in the force spectrum seen in the smooth rail case are also apparent in the spectrum results shown in Figure 4.13(b). At the wavelength corresponding to the corrugation, though, the force results are the same from each model.



Finally a case with broadband roughness has been calculated using the three different models and is shown in Figure 4.14. The roughness was pre-processed to take account of the contact filter independently of the models being compared. In this case both the force history in a sleeper bay and the spectrum are very similar for the three models. A very small difference can be seen in the one-third octave band corresponding to the element length of 0.075 m, and small differences are also seen for wavelengths shorter than 0.008 m, but overall the agreement between the three models is excellent for this case with a realistic roughness excitation. It is therefore concluded that the force model used in this work is valid.



4.4.6 Effect of variable sleeper spacing

With a finite element model of the track it is possible to include sleeper spacings that are non-uniform. The actual sleeper spacings at the Silence test site were measured and found to have a mean spacing of 0.605 m with a standard deviation of 0.007 m. These are extremely regularly spaced sleepers. For comparison, the sleeper spacing at the Chilworth test track in Southampton has a mean of 0.628 m and a standard deviation of 0.039 m, while two Swedish sites have mean spacings of 0.652 and 0.650 m with standard deviations of 0.017 and 0.020 m respectively [Thompson, 2009].

The interaction force spectrum calculated from a model using the measured sleeper spacings from Gersthofen and from Chilworth is compared to the interaction force with regular sleeper spacings below. Results in Figure 4.15 are calculated for the same low level broadband roughness profile in all cases. Results shown are from five successive sets of three sleepers as listed in Table 4.2. A non-constant sleeper spacing results in small differences in the force spectrum at longer wavelengths, which are more noticeable with the more variable Chilworth sleeper spacings than with the Gersthofen spacings. However, the effect overall is small and in the presence of a realistic roughness, the interaction forces are dominated by the effects of the surface profile. Therefore, in the remainder of this work, the sleeper spacing will be assumed to be constant at 0.6 m intervals in all cases.

Site	Set 1	Set 2	Set 3	Set 4	Set 5
Gersthofen	612	605	601	598	607
	605	611	602	615	605
	612	605	606	605	612
Chilworth	600	625	630	663	635
	606	650	629	630	613
	660	642	656	636	631

Table 4.2 Sleeper spacings (mm) for variable spacing results shown in Figure 4.15.



4.4.7 Effect of number of vehicle wheels

Igeland [1996] concluded that the interaction force is different at successive wheels, and that it is therefore important to include more than just one wheel in any model of wheel-track interaction and also roughness growth. Other authors have also developed models to calculate the interaction forces between multiple wheels and the track and reached similar conclusions [Wu & Thompson, 2001; Sheng *et al.*, 2006].

All the force results presented in this section have been calculated using eight beam elements per sleeper bay and a realistic low level of rail roughness as shown in Figure 4.6.

Two rail pad stiffnesses are considered: 200 MN/m, which is relatively soft (Figure 4.16) and 800 MN/m which is relatively stiff (Figure 4.17). The interaction force between each wheel and the rail is calculated over the middle three sleeper bays of the model as before. The results from a model with a single wheel are compared with models with two and four wheels. The two wheels are the most closely spaced, *i.e.* those sharing a bogie. The four wheels are those from the bogies at the end and start of successive carriages. More distant wheels are omitted as they are unlikely to have a significant effect on the model results due to the decay of vibrations along the track length.



Figure 4.16 Effect of number of wheels on interaction force spectrum with soft rail pads and low-level broadband roughness: (a) ______single wheel model; _____two wheel model; (b) _____single wheel model; _____ four wheel model.

With soft rail pads (Figure 4.16) there are clear differences in the interaction force seen at two successive wheels as they pass over the middle three sleeper bays, compared with the force calculated with a single wheel on the track. These results support the findings of Igeland [1996] that it is important to include more than one wheel. The results from a four wheel model are similar to those from a two wheel model, but some differences remain in the spectrum for each wheel. Therefore four wheels should be included in the force model in all cases with soft rail pads to capture the interactions between wheels.



Figure 4.17 Effect of number of wheels on interaction force spectrum with stiff rail pads and low-level broadband roughness: (a) ——single wheel model; — — two wheel model; (b) ——single wheel model; — — four wheel model.

With stiff rail pads (Figure 4.17), the interaction force shows less sensitivity to the number of wheels included in the model. Results are very similar for all the models with different

numbers of wheels. With stiff rail pads it is therefore less necessary to include multiple wheels in the system than it is with soft rail pads. Four wheels will be considered, however, in order to provide uniformity when comparing the results from tracks with the two different rail pad stiffnesses.

An examination of the track decay rates plotted against roughness wavelength for the tracks with soft and stiff rail pads explains the reason for including more than one wheel in the model. Figure 4.18 shows that the track decay rates become low with soft rail pads at frequencies that correspond to roughness wavelengths shorter than around 0.075 m. By comparison, with stiff rail pads, the track decay rate remains high for wavelengths down to half this length, at around 0.0315 m. Differences in the interaction force between single and multiple wheel models are expected for wavelengths where the decay rate is low. Therefore, there is little difference between single and multiple wheel models if the track has softer rail pads it is important to include more than one wheel in the interaction model.



4.4.8 Summary of sensitivity of force to variations in input parameters

The number of elements required to represent the rail in each sleeper bay must be at least eight. If fewer elements are used, discretisation effects coincide with the parametric excitation effects and are noticeable particularly if the rail is assumed to be smooth. However, if the rail has a realistic level of broadband roughness, the discretisation effects are less significant as the interaction force is dominated by the rail roughness and discretisation effects therefore tend to be masked. The rail roughness is the main contributor to variation in the wheel-rail interaction force in the frequency range examined here.

It is important to include more than one wheel in the model if the rail pads are relatively soft, since the vibration decay rate along the rail is low in part of the frequency range of interest. Different interaction forces are seen if multiple wheels are included in the model on track with soft rail pads. With stiff rail pads, the effects of multiple wheels are less noticeable as the track decay rates are higher and waves do not travel as far between the wheels as on track with soft rail pads.

The sharpness of the pinned-pinned resonance is affected by the precision of periodicity of sleeper spacing. Therefore variable sleeper spacing has been examined to assess the effects on the dynamic interaction force. However, the cases considered here show only a small effect on the calculated interaction force spectrum. Therefore in the remainder of this work effects of variable sleeper spacing will be neglected.

4.5 Effect of variations in the vehicle parameters

4.5.1 Effect of vehicle speed on wheel-rail interaction force

The speed of the vehicle is a significant factor in the model to calculate the wheel-rail interaction forces. Figure 4.19 shows the mean interaction forces of a four wheel model of a freight train at three different speeds. Wavelength is shown, rather than frequency, to allow comparison with later results for different train types and also to allow comparison with the initial roughness profile. At the average freight train speed of 29.44 m/s measured in the Silence project, the wavelength of roughness and force that corresponds to the pinned-pinned frequency is 0.027 m. At a speed one-third higher than this, the wavelength shifts to 0.036 m. At two-thirds of this speed the wavelength is 0.018 m. The shift in the pinned-pinned effects can be seen in the force results; a peak appears in each case at the corresponding wavelength.

The interaction forces are generally higher for a faster vehicle, in particular at wavelengths between about 0.02 m and 0.08 m.



Figure 4.19 Effect of vehicle speed on freight vehicle dynamic interaction force spectrum with low level broadband roughness: -29.44 m/s; --20 m/s; $\cdots \cdot 40 \text{ m/s}$.

4.5.2 Effect of vehicle wheel spacing on wheel-rail interaction force

In order to assess the effect of the wheel spacing, results have been calculated for the freight train with more closely spaced wheels and wheels spaced further apart than usual. Figure 4.20 shows the mean interaction forces of a four wheel model of a freight train with three different bogie wheel spacings.



Figure 4.20 Effect of bogie wheel spacing on freight vehicle dynamic interaction force spectrum with low level broadband roughness and soft rail pads: ---1.8 m nominal; ---1.5 m; $\cdots \cdots 2.1$ m.

Varying the wheel spacing by up to 0.3 m has only a small effect on the force, in the wavelength range from 0.016 m to 0.063 m. The effect will be even less if the rail pads are stiffer, as the track decay rates are higher across the spectrum. Therefore, different wheel spacings will not be considered in the remainder of this work, except in that they are different anyway for the different vehicle types.

4.5.3 Effect of vehicle unsprung mass on wheel-rail interaction force

The unsprung mass of the vehicle includes the mass of the wheel, axle and any other components located below the primary suspension. Increasing or decreasing the unsprung mass by one third has very little effect on the predicted interaction force between the wheel and the rail in the wavelength range of interest (Figure 4.21). The force spectrum begins to show variation only at the longest wavelengths shown.



Figure 4.21 Effect of unsprung mass on freight vehicle dynamic interaction force spectrum with low level broadband roughness and soft rail pads: ---488.5 kg nominal; ---651 kg; $\cdots 326$ kg.

4.5.4 Effect of vehicle static load on wheel-rail interaction force

The static load on each wheel is a function of the total mass of each carriage. Combined with the dynamic interaction force, it gives the total force at the wheel-rail contact which will be used as the input to the roughness growth model. A one-third increase or decrease in the static load has an effect on the dynamic interaction force as shown in Figure 4.22. The effect is not uniform across the wavelength range, *i.e.* the dynamic force is not always higher with a higher static load; for some wavelengths there is little difference.

Wavelengths between 0.25 m and 0.063 m show the most significant differences. The static load affects the interaction force because it changes the stiffness of the contact spring between the wheel and rail. Also, parametric effects are increased by increasing the static load.



Figure 4.22 Effect of static load on freight vehicle dynamic interaction force spectrum with broadband roughness: — 100 kN nominal; — $-67 \text{ kN}; \cdots 133 \text{ kN}.$

4.6 **Results for different vehicle types**

The average interaction force spectra over four wheels for each train type are presented in Figure 4.23 and Figure 4.24. These results are for track with soft rail pads and for the two realistic roughness profiles generated to match the low level roughness and TSI limit roughness spectra shown in Figure 4.6. Different wavelengths are dominant for each of the train types due to their differing average speeds and wheelbase. For the low-level roughness, the forces for the regional and high-speed trains are quite similar throughout the wavelength range. This is because the static load is the same for these two types, as is the bogie wheel spacing; the only major difference, therefore, between the two vehicles is their speeds. The unsprung mass of the vehicle has been shown to have less effect on the calculated force.

With the higher TSI limit spectrum roughness, more differences become apparent especially in the wavelength range around 0.063 m. As expected, the faster high-speed train shows higher interaction forces than the regional train around this wavelength range.

For both roughness profiles, the slower freight vehicle shows lower interaction forces in this range.



Figure 4.23 Dynamic interaction force spectrum (soft pads) with low level broadband roughness for different vehicle types: _____freight; _____ regional; high-speed.



Figure 4.24 Dynamic interaction force spectrum (soft pads) with TSI limit spectrum roughness for different vehicle types: ——freight; — — regional; · · · · · · high-speed.

4.7 Effect of rail pad stiffness

In order to assess the effect of rail pad stiffness on the interaction forces, the average interaction force spectra from four wheels are presented in Figure 4.25 for the freight vehicle, with soft and with stiff rail pads. Stiff rail pads lead to higher impact loads on the

sleepers and ballast, leading many railways to use softer rail pads to minimise track damage. The freight vehicle sees higher forces between the wheels and rail at wavelengths longer than 0.063 m with stiffer rail pads. At short wavelengths the interaction forces are very similar for both rail pad stiffnesses, because the track receptance at high frequencies is independent of the rail pad stiffness.



4.8 Effect of initial rail profile

In this work the rail roughness profile along the 60 sleeper bays of the track model is generated to match a particular spectrum, by adding sinusoidal components with random phase. This means, firstly, that the roughness spectrum from the middle three sleeper bays (a relatively small sample) can show some variation between different generated profiles, and also that the actual spatial profile can be different. It is necessary to check that the low level roughness profile used as an input to the model in this work is representative of other roughness profiles of similar spectrum.

In Figure 4.26 the interaction force has been calculated using five different generated roughness profiles of similar spectral content. The interaction forces are similar so it can be concluded that a single roughness profile is adequate to examine the differences that arise due to other model parameters. Some variation is seen in the spectra particularly at longer wavelengths. This is as expected, since results are only shown for the middle three sleeper bays of the track model and over three bays the random phase used to generate the

roughness profile allows some variation between different profiles. The peak in the force spectrum at a wavelength corresponding to half a sleeper bay (0.3 m) is present in all five results.



Figure 4.26 Dynamic interaction force spectrum from five different rail profiles with similar low level broadband roughness for freight train.

4.9 Effect of rail dampers on wheel-rail interaction force

In Chapter 3, rail dampers have been shown to shift the pinned-pinned frequency of the track and to smooth the peaks and troughs in the track receptance. The effect of the rail dampers on wheel-rail interaction forces for a typical freight train is shown in Figure 4.27 for the low level broadband roughness. Similar results are seen for the TSI level roughness in Figure 4.28. Of particular interest is the force at the roughness wavelength corresponding to the pinned-pinned frequency, which for the typical freight vehicle speed is 0.027 m for the track without rail dampers but shifts to 0.043 m when the rail dampers are included. Results are shown for the 'improved model' of the rail damper elastomer (Section 3.4.4) with a more constant loss factor in the frequency range of interest. The freight train is used here as an example, the full set of interaction force results with rail dampers for all vehicle types are included in Appendix A.

With soft rail pads, the peaks in the one-third octave wavelength bands around 0.02 m and 0.0315 m have been smoothed out or shifted to longer wavelengths by the addition of rail dampers. The dip in the spectrum at around 0.063 m has been filled at least for some of the

wheels. The rail dampers have very little effect on the interaction force at wavelengths less than about 0.016 m or greater than 0.08 m.

With stiff rail pads, the peak in the interaction force at the roughness wavelength corresponding to the pinned-pinned frequency is more pronounced than with soft rail pads. The rail dampers again act to shift this peak to a longer wavelength. The force spectrum is unchanged by the dampers outside the wavelength range of 0.016 m up to 0.063 m.



Figure 4.27 Dynamic wheel-rail interaction force for a freight train on (a) soft and (b) stiff rail pads with low broadband roughness: ——— with rail dampers; ····· without rail dampers.



Figure 4.28 Dynamic wheel-rail interaction force for a freight train on (a) soft and (b) stiff rail pads with TSI roughness: — *with rail dampers; · · · · · without rail dampers.*

4.10 Summary of wheel-rail interaction force results

The finite element track model described in Chapter 3 has been combined with a simple vehicle model. The equations of motion of the system are solved in the time domain using a state-space approach following the method of Nielsen and Igeland [1995]. The model has been validated by comparison of a set of sample results with output from DIFF and the model of Pieringer *et al.* [2008]. The interaction force between the wheels and rail from the middle three sleeper bays of interest is required as an input to the contact and wear model to follow.

The effect of different track and vehicle parameters on the dynamic interaction force between a wheel or wheels and the rail has been investigated. It is important to divide the rail into at least eight elements in each sleeper bay. If the rail pads are relatively soft it is important to include more than one wheel in the model, as the wheels are coupled by the track and can interact to give a different interaction force spectrum to that if a single wheel is considered. Including more than one wheel is less important if the track has stiff rail pads or rail dampers, since the track decay rate is higher, at higher frequencies, leading to less interaction between the wheels in the model. However, for uniformity, four wheels are included in the interaction force model as standard.

The roughness of the rail is the single parameter that has the most effect on the interaction force spectrum. Differences are also seen between different vehicle types passing over the same track and rail roughness profile, as a result of their different speeds and static loads. The stiffness of the rail pads also has a significant effect on the interaction force between wheels and rail. Variations in the wheel spacing and unsprung mass of the wheel have less effect on the force.

Adding rail dampers to the track shifts the pinned-pinned frequency of the track and consequently shifts the peak in the force spectrum corresponding to this resonance to longer wavelengths.

5 STRESS DISTRIBUTION IN THE WHEEL-RAIL CONTACT

5.1 Introduction

The previous chapters have concentrated on describing a model of the overall normal interaction force between a wheel or wheels and a rail of the track. For this purpose it has been sufficient to model the wheel-rail contact as a Hertzian spring. However, to predict the wear of the rail surface resulting from the passage of the wheels, a more detailed model of the contact patch is required. The size and shape of the contact area and the distribution of normal and tangential stresses throughout the wheel-rail interface is needed.

In this chapter, a summary of existing analytical and numerical contact mechanics theory is presented. The work of Johnson [2001] has been used as a source for much of the background material to describe the contact problem. The variational method developed and proven by Kalker [1990] is used for the analysis of wheel-rail contact. Kalker implemented this theory in his CONTACT program, which remains recognised as the benchmark solution to the rolling contact problem [Knothe, 2008]. However, in practice the application of CONTACT to determine the distribution of stresses in three dimensions between railway wheels and rails has been limited by the calculation times required for the analysis. This chapter describes the implementation in Matlab of a contact theory based on that of Kalker [1990], firstly in three-dimensional form and then in a two-dimensional version. The two-dimensional version with a suitable assumed contact width gives a good approximation to the results from the centre-line of the three-dimensional model when lateral and spin effects are neglected.

This chapter deals with static cases only, *i.e.* the application of constant normal and tangential forces in stationary contact. Rolling contact is treated in Chapter 6.

5.2 Contact geometry and definitions

The minimum inputs to a contact model are the overall normal force between the contacting bodies, and the initial profile of the bodies in their undeformed state before coming into contact. In this work, the overall normal force between the wheel and the rail-head is determined by the interaction force model described in Chapters 3 and 4. Although both the wheel and rail are rough in practice, here the wheel is, for simplicity, assumed to be smooth and the combined roughness of the system is attributed to the rail.

Johnson's book [2001] contains the necessary background to the contact problem. The two three-dimensional bodies in contact shown in Figure 5.1 are defined by their undeformed surface profiles $z_1(x,y)$ and $z_2(x,y)$. The undeformed distance between the two surfaces at the point of first contact is then given by h(x,y).



$$h(x, y) = z_1(x, y) - z_2(x, y)$$
(5.1)

Figure 5.1 Geometry of bodies in contact. Upper body is body 1, lower body is body 2.

When the two bodies are pressed into contact under a normal load, the surfaces deform resulting in a discrete contact area. The normal load results in a distributed normal pressure across the contact area.

The relative motion of two surfaces in contact may be defined as in Johnson [2001] in terms of sliding, rolling and spin. Sliding occurs when a relative linear velocity Δv is present between the two surfaces at the contact point.

$$\Delta v = v_1 - v_2 \tag{5.2}$$

where v_1 and v_2 are the linear velocities of each body relative to the origin O. The sliding velocity Δv may have components in the x and y directions (but not in the z direction as the bodies are assumed to remain in contact and not overlapping). In this work, however, lateral motion is neglected so any sliding is purely in the x direction.

Rolling is a relative angular velocity $\Delta \omega$ between two bodies about an axis lying in the tangent plane. The angular velocities of the bodies relative to the origin are ω_1 and ω_2 . In this case rolling is purely about the *y* axis.

$$\Delta \omega = \omega_1 - \omega_2 \tag{5.3}$$

Rolling contact is treated in more detail in Chapter 6.

Spin is a relative angular velocity about the common normal, here the z axis. Spin may be reasonably neglected for modelling the wheel-rail contact for tangent track with no flange contact.

5.2.1 Forces in the contact patch

The overall forces that may be transmitted through the contact area *S* are the compressive normal force *P* and the tangential force *Q* due to friction. Neglecting lateral motion, the tangential force *Q* is in a direction along the *x* axis to oppose any sliding velocity. The forces *P* and *Q* are related by the coefficient of friction μ such that the magnitude of *Q* is less than or equal to the friction limit.

$$Q| \le \mu P \tag{5.4}$$

The normal force P and the tangential force Q are distributed across the interface area S which lies in the *x*-*y* plane. This distribution leads to a normal pressure p and a tangential stress q across the surface area such that

$$P = \int_{S} p \, dS \tag{5.5}$$

$$Q = \int_{S} q \, dS \tag{5.6}$$

In fully sliding contact, Q acts in the direction opposing the sliding velocity and the maximum value of Q is given by

$$Q_{\max} = -\frac{\Delta v}{|\Delta v|} \mu P \tag{5.7}$$

5.2.2 Stick and slip zones

If the magnitude of the tangential force is less than that of Q_{max} , then the contact is not purely sliding. A relative movement or slip will occur between the surfaces in part of the interface, and another part of the interface will stick, or deform without relative motion between the two surfaces. The contact patch may then be divided into stick and slip zones. At points in stick zones the tangential stress must be less than the limiting value due to friction, that is

$$|q(x,y)| < \mu p(x,y) \tag{5.8}$$

In a slip zone the tangential stress is at its maximum and is equal to the friction limit

$$q(x, y) = \mu p(x, y) \tag{5.9}$$

The slip s is defined as the relative displacement between two initially coincident points in the contacting bodies. Slip is taken to be positive when the upper body moves in the positive x direction relative to the lower body. The tangential stress is in a direction opposing the direction of slip, so that

$$\frac{q(x, y)}{|q(x, y)|} = -\frac{s(x, y)}{|s(x, y)|}$$
(5.10)

In a stick region the slip is zero.

5.2.3 Contact between bodies of quasi-identical materials

The solution to the contact problem is the distribution of p and q and hence the location of the stick and slip zones in the contact area. In a general case, the normal and tangential stresses are coupled since, in the presence of friction, a normal force leads to tangential displacements at the interface as well as normal displacements. If the two materials in contact are different, the resulting tangential displacements of the two bodies will be different and slip may occur, although this is opposed by friction. Therefore in a central region of the contact the surfaces may stick, while at the edges they may slip, even in the absence of overall sliding or rolling motion, unless the friction is high enough to prevent all relative motion. However a simplification may be made if the materials in contact are the same, the tangential displacements resulting from a normal contact force are the same in both bodies, and therefore no slip arises as a result of the normal contact force in the absence of overall sliding or rolling. The normal stress distribution can therefore be developed independently of the tangential stress.

5.3 Hertz theory for calculation of normal stress distribution

Hertz developed his analytical theory describing the contact between parabolic surfaces in 1880. The theory describes the normal stress distribution throughout the contact area. The equations are summarised by Johnson [2001] for general profiles within the limitations of

the theory, and are replicated here for simple circular contacts. The material properties are represented by the combined Young's modulus E^* , as defined in Equation (4.4).

The equivalent radius of the system *R* is defined from the radii of the two spherical bodies R_1 and R_2 as

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$
(5.11)

The size of the contact area for a normal load of magnitude P is then given by the radius of the contact circle a

$$a = \left(\frac{3PR}{4E^*}\right)^{\frac{1}{3}}$$
(5.12)

The maximum contact pressure p_0 at the centre of the contact circle is given by

$$p_0 = \left(\frac{3P}{2\pi a^2}\right) \tag{5.13}$$

and the approach of distant points in the two bodies δ is

$$\delta = \frac{a^2}{R} \tag{5.14}$$

The overall stress distribution p(x,y) is

$$p(x, y) = p_0 \left(1 - \frac{r^2}{a^2}\right)^{\frac{1}{2}}$$
 for $r = \sqrt{x^2 + y^2}$ (5.15)

Where radii of curvature differ in perpendicular directions the contact is elliptical rather than circular and can be described by a modified version of the above equations. For rough and possibly conforming surfaces, *i.e.* when the surfaces are not parabolic, a different approach is required using numerical rather than analytical methods.

5.4 Numerical methods for calculation of stress distributions

For cases where Hertzian contact does not apply, other descriptions must be developed for the stress distributions p and q, and the contact area S. If the undeformed shapes of the two surfaces $z_1(x,y)$ and $z_2(x,y)$ are known, and the overall force P over the contact area is known, it is possible to evaluate numerically the size and shape of the contact area and the distribution of the normal contact stresses. When both bodies have the same material properties, the normal force does not lead to any tangential stresses and the tangential stress distribution may be determined subsequently from the known tangential loading and the friction limit on maximum tangential stress.

Numerical methods for the evaluation of the stress distributions are usually either direct methods, where boundary conditions are satisfied exactly at specified matching points, or variational methods such as that developed by Kalker [1990], where the values of traction at elements are chosen to minimise an appropriate energy functional. For both these methods a potential area of contact is defined in the *x*-*y* plane that is greater than the actual contact area.

The direct method is also known as the matrix inversion method and is described in Johnson [2001]. It is not suitable for calculating the contact stress distribution in great detail at many positions due to the computational cost of the requirement for the inversion of large matrices at each position of interest.

In the variational approach the potential contact surface is divided into *N* elements, each of length Δx and width Δy [Xuefeng & Bhushan, 1996]. The normal elastic displacement u_z of the centre of each of these elements satisfies the equation

$$u_{z} + h(x, y) - \delta \begin{cases} = 0 & \text{in contact} \\ > 0 & \text{outside contact} \end{cases}$$
(5.16)

where δ is the approach of distant points in the two bodies as in the Hertzian contact equations and *h* is the undeformed distance between the bodies. The centre of each element may also undergo a tangential elastic displacement u_x . To determine the elastic displacements u_z and u_x , normal and tangential 'influence coefficient' matrices C_{ij} and D_{ij} are required. These matrices give the displacement of the centre of an element *i* on the contact surface due to a unit pressure applied at another element *j*. The total displacement of each element can then be determined from the sum of the displacements due to the normal pressure *p* or tangential stress *q* on all of the elements in the potential contact surface:

$$u_{zi} = \sum_{j=1}^{N} C_{ij} p_j$$
(5.17)

$$u_{xi} = \sum_{j=1}^{N} D_{ij} q_j \tag{5.18}$$

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5.4.1 Influence coefficients C_{ij} and D_{ij}

The displacement of a general point on the surface of a half-space resulting from uniform pressure acting on a rectangular area was analysed by Love [1929]. Kalker [1990] derived the influence coefficient matrices in a form suitable for use in his variational method algorithm (CONTACT). The following expressions describe the influence coefficients in the form used in this work, based on the method of Kalker.

Figure 5.2 shows the relevant geometry of the potential contact area. Define *a* as half the length of each element in the *x* direction and *b* as half the length of each element in the *y* direction. For each of the $N_x \times N_y$ possible combinations of elements, influence coefficients C_{ij} and D_{ij} are calculated as follows [Kalker, 1990]. The distance in the *x* and *y* directions between the centres of elements *i* and *j* is denoted x_{ij} and y_{ij} . The distances in the *x* and *y* directions between the centre of one element and the four corners of another element are then given by:

$$x_{1} = x_{ij} + a; \quad y_{1} = y_{ij} + b$$

$$x_{2} = x_{ij} - a; \quad y_{2} = y_{ij} - b$$

$$x_{3} = x_{ij} - a; \quad y_{3} = y_{ij} + b$$

$$x_{4} = x_{ij} + a; \quad y_{4} = y_{ij} - b$$
(5.19)



Figure 5.2 Top view of geometry of potential contact area, showing representative elements i and j.

The straight line distances between the centre of one element and four corners of another element are expressed as:

$$L_{1} = \sqrt{x_{1}^{2} + y_{1}^{2}}$$

$$L_{2} = \sqrt{x_{2}^{2} + y_{2}^{2}}$$

$$L_{3} = \sqrt{x_{3}^{2} + y_{3}^{2}}$$

$$L_{4} = \sqrt{x_{4}^{2} + y_{4}^{2}}$$
(5.20)

The normal influence coefficient C_{ij} between any two elements is given by:

$$C_{ij} = \frac{1}{\pi G} (1 - \nu) \left[(f_1 + g_1) + (f_2 + g_2) - (f_3 + g_3) - (f_4 + g_4) \right]$$
(5.21)

and the tangential influence coefficient D_{ij} between two elements is given by

$$D_{ij} = \frac{1}{\pi G} \left[(f_1 + g_1) + (f_2 + g_2) - (f_3 + g_3) - (f_4 + g_4) - \nu (f_1 + f_2 - f_3 - f_4) \right]$$
(5.22)

Here G is the shear modulus of the material, given by

$$G = \frac{E}{2(1+\nu)} \tag{5.23}$$

and the functions f_{1-4} and g_{1-4} represent terms involving the geometrical distances derived in Equations (5.19) and (5.20). Functions f_{1-4} and g_{1-4} are defined as

$$f_{1} = x_{1} \log_{e}(L_{1} + y_{1}); \quad g_{1} = y_{1} \log_{e}(L_{1} + x_{1})$$

$$f_{2} = x_{2} \log_{e}(L_{2} + y_{2}); \quad g_{2} = y_{2} \log_{e}(L_{2} + x_{2})$$

$$f_{3} = x_{3} \log_{e}(L_{3} + y_{3}); \quad g_{3} = y_{3} \log_{e}(L_{3} + x_{3})$$

$$f_{4} = x_{4} \log_{e}(L_{4} + y_{4}); \quad g_{4} = y_{4} \log_{e}(L_{4} + x_{4})$$
(5.24)

These influence coefficients are valid for cases with no tangential stress in the *y* direction and for contact between identical materials. With different materials, or cases involving spin, combined lateral and longitudinal forces occur as well as normal forces. The element displacement in each direction is then affected by forces acting in other directions. The influence coefficients are then more complicated as described by Kalker [1990].

5.4.2 Variational method for calculation of stress distribution in the contact

To find the values of the normal stress p_j and the tangential stress q_j for each element j in the potential contact area, a variational method may be used. Kalker's CONTACT algorithm is an example of a variational method. In this technique, a solution is found that minimises an appropriate energy functional. Kalker [1990] has shown that the contact area and stress distribution may be determined by solving a quadratic minimisation problem involving the total complementary energy, V^* . The internal complementary energy U_E^* is also known as the complementary strain energy or the stress energy. It is a function of the internal force or stress and is represented by the area above the stress-strain curve (see Figure 5.3). The area below the stress-strain curve represents the elastic strain energy U_E , which is a function of the elongation of the body [Richards, 1977].



Figure 5.3 Equivalence of complementary energy U_E^* and strain energy U_E in linear elastic range of material [Xuefang & Bhushan, 1996].

For linear elastic materials the internal complementary energy U_E^* is numerically equivalent to the elastic strain energy U_E . This is because the area above and below the linear stress-strain curve is the same for these materials, as shown in Figure 5.3.

The total complementary energy V^* in the absence of tangential loading can be written in terms of the internal complementary energy of the two stressed bodies U_E^* as [Johnson, 2001]

$$V^* = U_E^* + \int_S p(h - \delta) \mathrm{d}S \tag{5.25}$$

The strain energy of the system is expressed in terms of the normal stresses and displacements of the elements in the contact surface as

$$U_{E}^{*} = U_{E} = \frac{1}{2} \int_{S} p u_{z} \,\mathrm{d}S \tag{5.26}$$

Substituting the expression for the displacements from Equation (5.17) into the strain energy Equation (5.26) and the resulting expression for U_E^* into (5.25) yields the total complementary energy function to be minimised for values of *p* throughout the contact area *S*. The tangential stresses are neglected here, since for similar materials they have no effect on the normal stress distribution.

$$V^* = \frac{1}{2} \int_{S} p \, u_z \, \mathrm{d}S + \int_{S} p(h - \delta) \mathrm{d}S \tag{5.27}$$

The numerical evaluation of the total complementary energy due to the normal stresses over the elements of the potential contact surface is then

$$V^* = \frac{1}{2} \sum_{i=1}^{N} A_i p_i \sum_{j=1}^{N} C_{ij} p_j + \sum_{i=1}^{N} A_i p_i (h_i - \delta)$$
(5.28)

In Equation (5.28) the approach of the two bodies δ is a constant, as is the area of each of the elements A_i . These, therefore, do not affect the minimisation problem and the function to be minimised for the distribution of the normal stress p in the contact patch may be written as

$$\min F_{\text{norm}} = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} p_i C_{ij} p_j + \sum_{i=1}^{N} h_i p_i$$
(5.29)

For the tangential stress, the function to be minimised has a similar form to that of the normal stress distribution. The term for the undeformed distance between the surfaces h_i in Equation (5.29) is replaced by a term representing relative tangential displacement between the two surfaces. This results from the application of the tangential force, consisting of a rigid tangential shift $W_{i\tau}$ and the prior displacement difference between the surfaces due to elastic deformation, $u'_{i\tau}$.

$$\min F_{\text{tang}} = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} q_i D_{ij} q_j + \sum_{i=1}^{N} (W_{i\tau} - u'_{i\tau}) q_i$$
(5.30)

In a case examining the direct application of an overall tangential force Q less than the friction limit, both the rigid shift and the slip between the surfaces prior to the application of the tangential load are zero.

Equation (5.30) is valid for cases with no tangential stress in the *y* direction and for contact between identical materials, and when the normal stress distribution is known already and is not affected by the tangential stress distribution or vice versa.

Note that the actual overall complementary energy function for combined normal and tangential load includes the terms corresponding to the normal displacements as in Equation (5.28). However these may be omitted in the minimisation function of

Equation (5.30) to calculate the tangential stress distribution as they are known and therefore do not affect the minimisation.

These functions for the normal and tangential problems are quadratic functions of the stress, and as such correspond to the standard form of quadratic minimisation problems. They may therefore be solved using standard quadratic programming algorithms. Kalker [1990] developed the algorithms 'NORM' and 'TANG' as part of CONTACT to find the solution. In this work the Matlab routine 'quadprog' is used.

Two constraints apply to the solution of the minimisation problem for the normal case. The first is that the normal contact pressure p must be positive everywhere, or compressive, unless it is equal to zero outside the contact area. Also, the sum of the normal stresses p_i on all the elements multiplied by the element area A_i must be equal to the total overall normal force P.

$$P = \sum_{i=1}^{N} A_i p_i \quad \text{and} \quad p_i \ge 0 \tag{5.31}$$

In the tangential direction the constraint on the solution is that the magnitude of the tangential stress in the contact patch must be everywhere less than the friction limit

$$|q_i| \le \mu p_i \tag{5.32}$$

In addition, if the overall tangential load Q is known it must be equal to the sum of the stress on the individual elements multiplied by the element area A_i .

$$Q = \sum_{i=1}^{N} A_i q_i$$
 (5.33)

5.4.3 Comparison of normal stress from variational method with Hertz theory

The normal stress distribution obtained from the variational method is compared with the analytical solution calculated from the Hertz equations for the contact between a smooth sphere of radius 0.46 m and a flat plane. The overall normal force P in the contact is taken as 100 kN. As expected, the stress distribution is circular and symmetrical. Figure 5.4 shows the three-dimensional stress distribution in the contact patch, which has a radius of around 7 mm.


Figure 5.4 Normal stress distribution in contact patch from variational method.

Figure 5.5 shows a cross-section of the same result, compared with the Hertzian stress distribution which has a radius of 6.7 mm. The agreement between the two methods is very good. The noticeable difference between them is at the edges of the contact patch; this is due to the discretisation of the contact area for the variational method. In this case, the elements in the contact have dimensions of 1 mm by 1 mm, so it is only possible to determine the radius of the contact to the nearest millimetre. Reducing the element size leads to greater accuracy.



Figure 5.5 Cross-section of normal stress in contact patch: ——— Hertz equations; — — — variational method.

5.5 Combined normal and tangential loading

A simple case involving tangential loading occurs when two bodies, pressed into contact by a normal force *P*, experience a tangential force that is initially zero but increases steadily up to the friction limit μP .

If the two bodies have the same material properties, then the tangential force has no effect on the normal stresses and displacements and the overall size and shape of the contact is determined purely by the normal force and the undeformed profiles of the bodies. Before the application of the tangential force, the whole contact area is in a state of stick. The application of a tangential force causes a shear elastic displacement of each of the bodies. As long as *Q* remains less than the friction limit, there is no overall sliding but slip occurs over part of the interface. The problem is to determine the location of the stick and slip zones.

5.5.1 Analytical solution using Hertz theory

This problem was treated analytically for the contact of two cylinders by Cattaneo in 1938 and also by Mindlin in 1949, and is summarised by Johnson [2001] who also describes the extension of the theory to spherical contacts. As the bodies are assumed to be smooth, Hertz theory may be used. For a circular contact with a tangential load less than the friction limit applied along the *x* axis, the tangential stress q(x) acts in the same direction, and is symmetrical with a central stick area of radius *c* concentric with the overall contact area of radius *a*. In the slip zone (the annulus around the stick zone) the tangential stress is at its maximum, given by the friction limit

$$q_{slip}(x, y) = \mu p(x, y) \tag{5.34}$$

where p is the Hertzian normal stress from Equation (5.15). The radius of the central stick region c is determined from the magnitude of the tangential force as [Johnson, 2001]

$$c = a \left(1 - \frac{Q}{\mu P} \right)^{\frac{1}{3}} \tag{5.35}$$

In the stick zone the tangential stress distribution is given by

$$q_{stick}(x, y) = q_{slip} - \frac{c}{a} \mu p_0 \left(1 - \frac{r^2}{c^2}\right)^{1/2}$$
(5.36)

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The tangential stress distribution along the *x* axis of the contact patch for increasing tangential loads is shown in Figure 5.6. With no tangential load the entire contact patch is a stick zone, then as the load increases the stick zone shrinks as slip zones appear around the circumference of the contact. In the slip zones, the tangential stress is equal to the friction limit, *i.e.* the coefficient of friction multiplied by the normal stress at that point, here taken from the cross-section of the Hertzian normal stress shown in Figure 5.5. At the moment when the stick zone vanishes when sliding is imminent, the maximum tangential stress is equal to μ times the maximum normal stress p_0 .



Figure 5.6 Cross-section of tangential stress in the contact between a sphere and a flat plane: (a) $Q = 0.25 \times \mu P$; (b) $Q = 0.5 \times \mu P$; (c) $Q = 0.75 \times \mu P$; (d) $Q = 1.0 \times \mu P$ imminent sliding. P and Q respectively are the overall normal and tangential forces in the contact, $\mu = 0.3$ is the coefficient of friction.

5.5.2 Tangential stress solution using variational method

In the simplest case of combined normal and tangential loading, first the normal stress distribution is determined independently of the tangential load. Then the tangential load is applied 'instantly'. The prior tangential displacement difference between the surfaces $u'_{i\tau}$ is zero. The division into stick and slip zones is not calculated explicitly but can be derived from the tangential stress distribution, using the definition of Equation (5.9) that the tangential stress is equal to the friction limit in a slip zone. Figure 5.7 shows the results obtained by the variational method for this case compared with the analytical solution. Differences are due to the discretisation of the contact patch into 1 mm square elements. The accuracy of the division into stick and slip zones depends on the size of the elements employed.



Figure 5.8 shows an example of the three-dimensional distribution of the tangential stress in the contact patch calculated using the variational method.



Figure 5.8 Tangential stress distribution in contact patch calculated using variational method for $Q = 0.5 \times \mu P$ with the coefficient of friction $\mu = 0.3$.

5.6 Two-dimensional representation of the contact problem

The three-dimensional determination of the stress in the contact patch converges towards the exact solution as the element discretisation becomes finer. The numerical method therefore works well as long as the elasticity and friction assumptions are correct [Kalker, 1990].

The disadvantage of the technique when it is to be used in a wear calculation is the high computational cost of the minimisation over a grid of elements. The stresses at a single position may be determined relatively quickly (in a few seconds), but the time to calculate the stress at many wheel positions along a rail adds up to a significant computation time.

For this work the roughness is known and required only along a single line of the wheelrail contact. Lateral forces, lateral creep and spin are assumed to be insignificant. There is therefore a large computation time benefit in simplifying the contact problem to two dimensions. Also, in the absence of detailed rail and wheel profile data the threedimensional model is itself an approximation as it assumes a constant rail-head curvature. In two dimensions, the problem is that of a cylinder in contact with a plane, with constant stress across the width of the contact (in the *y* direction). Kalker [1970; 1971a; 1971b] analysed this problem using the variational method. The solution process is the same as for the three-dimensional analysis except that a different matrix of influence coefficients is required, and the overall normal force in the contact P is expressed as a force per unit width. The same influence coefficients are used for the normal and tangential calculation. The width of the contact in the *y* direction is assumed to be a constant.

5.6.1 Influence coefficients for two-dimensional analysis

To determine the normal and tangential elastic displacements in the two-dimensional case, a 'piecewise linear' representation of stress distribution is commonly used [Bentall & Johnson, 1967; Kalker, 1970, 1971a, 1971b & 1972; Sheng *et al.*, 2004]. This representation differs from the 'piecewise constant' elements used in the three-dimensional case. In a piecewise constant representation, the stress p or q is constant across the surface of each element as shown in Figure 5.9, with a step change in the stress at the element boundaries. In piecewise linear representation the stress is built up as the sum of a series of overlapping triangles, removing the step change at the element boundaries as in Figure 5.10 [Johnson, 2001].



Figure 5.9 Piecewise constant stress elements



Figure 5.10 Piecewise linear stress elements

The two-dimensional matrix of influence coefficients B_{ij} is given by Kalker [1971a] following the method of Bentall and Johnson [1967]:

$$B_{ij} = -\frac{1}{\pi E * \Delta x} \{ (k - \Delta x)^2 \log_e |k - \Delta x| - 2k^2 \log_e |k| + (k + \Delta x)^2 \log_e |k + \Delta x| \}$$
(5.37)

with $k = x_i - x_j$

 B_{ij} gives the displacement of the centre of an element *i* on the contact surface due to a unit peak stress applied at another element *j*. The total displacement of each element is again determined from the sum of the displacements due to the stress on all of the elements in the potential contact surface:

$$u_{zi} = \sum_{j=1}^{N} B_{ij} p_j$$
(5.38)

$$u_{xi} = \sum_{j=1}^{N} B_{ij} q_{j}$$
(5.39)

In the two-dimensional case, the normal and tangential displacements may be calculated using the same influence matrix, as long as the datum used for the displacements is the same in each case [Johnson, 2001].

5.6.2 Two-dimensional analytical solution for tangential contact

For a cylinder in contact with a plane and a tangential load less than the friction limit applied along the *x* axis, the tangential stress q(x) is again symmetrical with a central stick area of half-length *c* centred in the overall contact area of half-length *a*. The Hertzian analysis is similar to that for the three-dimensional analysis in Section 5.5.1, except that the half-length of the central stick region *c* is slightly different:

$$c = a \left(1 - \frac{Q}{\mu P} \right)^{1/2} \tag{5.40}$$

The tangential stress distribution in the stick zone is given by Equation (5.36) as before.

5.7 Comparison of results from two and three-dimensional models

The normal stress distribution calculated using the two-dimensional variational method may be compared with the results for the Hertzian contact between a cylinder and a plane and also with the results for Hertzian contact between a sphere and a plane. Figure 5.11 shows the same case as calculated in Section 5.4.3 using the Hertz equations for a 0.46 m radius sphere. The radius of curvature of the cylinder is also 0.46 m. For the two-dimensional variational method and for the Hertz equations for a cylinder, the width of the contact in the *y* direction is set to 13.4 mm which is the diameter of the resulting contact area in three dimensions.



It can be seen firstly that the agreement between the variational method and the analytical results for a cylinder on a flat plane is very good, and as before the differences are due to the element discretisation. The length of the contact patch is different between the two-dimensional and three-dimensional representations, as is the magnitude of the stress. This is to be expected as the two-dimensional representation assumes that the overall force is distributed equally along the cylinder in the *y* direction, whereas the three-dimensional

representation allows for a higher value of stress along the centre-line of the contact which then falls to zero at the edges of the contact in the *y* direction. The cylindrical stresses are therefore lower and the contact patch length is slightly shorter.

A contact width for the two-dimensional case of 11 mm results in a closer match in the contact length compared with the three-dimensional case, shown in Figure 5.12. The magnitude of the normal stress for a cylinder remains less than that from the centre-line of the sphere calculated in three dimensions.



Figure 5.13 shows the tangential stresses, for a load equal to half the friction limit, calculated by the variational method in two dimensions compared with the analytical solutions using Hertz theory for cylindrical and spherical contact. A three-dimensional spherical contact gives higher tangential stresses along the centre-line of the contact than the two-dimensional cylindrical contact. This means that even though the width of the cylinder has been adjusted to give the same overall contact length as the spherical case, a greater amount of the contact is found to be in a state of slip if a cylindrical model is used.



5.8 Summary of modelling stress distribution in the contact

A three-dimensional contact model based on Kalker's variational method has been implemented in Matlab. The model calculates the distribution of normal and tangential stresses throughout the contact area. For the simple case of contact between a smooth sphere and a flat plane, results from the model have been compared with analytical results using Hertzian contact theory. The accuracy of this numerical model is limited only by the size of the elements used to represent the potential contact area, but the three-dimensional model is computationally expensive.

A two-dimensional representation of the contact model has also been developed. This is required in order to reduce the calculation times to carry out analyses of rolling contact in the following chapters. The two-dimensional model requires the assumption of a constant width of the contact patch in the lateral direction. This width can be chosen to give the correct length of the contact patch in the longitudinal direction.

The normal and tangential stress distributions from the two-dimensional model have been compared with analytical results as well as with the results from the centre-line of the three-dimensional model. The agreement with the analytical results from Hertzian contact theory is again limited only by the size of the elements used in the longitudinal direction. Compared with the results from the centre-line of a three-dimensional model, the twodimensional model underestimates the maximum normal force in the contact and can exaggerate the length of slip zones slightly. These differences arise from the assumption that the stress distribution is constant in the lateral direction in the two-dimensional model. In the absence of detailed wheel and rail profile information in the *y* direction, for this work the two-dimensional contact model is adequate. It is also necessary, in order to reduce the calculation time when analysing rolling contact with very small element sizes.

6 ROLLING CONTACT

6.1 Introduction

In this chapter, the contact model described in Chapter 5 for static contact problems is extended to rolling contact. The extension to rolling contact is straightforward, as the variational method inherently includes transient effects. For rolling the model is applied in a time-stepping fashion and the contacting surface is stepped along the wheel and rail-head surfaces. The stresses and displacements calculated along the railhead (and wheel tread) at each time-step in rolling depend on the values at the previous position. These were previously assumed to be zero to calculate the static results presented in Chapter 5.

The variables of interest to be determined by the model are the distribution of normal and tangential stress in the contact as the wheel rolls along the rail. The size and shape of the contact and the division into stick and slip zones may also be derived from the model. In the event that parts of the contacting surfaces slip, the relative sliding velocity of the two bodies in the slip zone is important for the calculation of rail wear. The size of the elements used to define the potential contact area is important as it affects the accuracy of the division into stick and slip zones and the slip velocity calculation.

In this chapter the calculation of slip velocity at each location in the contact area is described and the effect of the rolling speed on the slip velocity is investigated.

Rolling can occur without any tangential force being transmitted, *i.e.* with Q equal to zero. This is known as 'free rolling'. Free rolling is not of interest in this work as it does not result in wear. Free rolling occurs for idealised unpowered or unbraked wheels only. Cases with non-zero tangential force Q are known as 'tractive rolling' [Johnson, 2001]. Tangential loading of the wheel-rail contact along the rail head arises in acceleration or braking and in overcoming frictional losses. Powered or braked wheels experience a sizeable value of Q. Unpowered trailer wheels may also experience small values of Q as a result of minor misalignments in rolling leading to creep. This chapter includes a description of creep in Section 6.2. Lateral forces can also occur but, as in the rest of this thesis, these are neglected here.

A simple example of rolling contact is steady rolling. In steady tractive rolling with friction, the normal force P is constant and a constant tangential force Q is transmitted.

Carter [1926] developed a two-dimensional wheel-rail contact model which is an analytical solution to the steady rolling contact problem of a smooth wheel on a smooth rail. When using the variational method, however, steady rolling may be considered to be a special case that develops over a period of time in transient rolling from a set of initial conditions with unchanging external loads. In this chapter, the transient variational method is used to develop steady tractive rolling contact from an initial stationary position. The resulting tangential stress distributions are compared with the analytical solutions for the initial position and once steady rolling has been achieved.

Some examples of rolling contact are examined using the variational method for situations involving roughness of the rail head. These illustrate the importance of considering non-Hertzian effects in the form of the rail roughness when determining the distribution of normal and tangential stresses. Very different results are calculated for the stress distributions when the effects of surface roughness are included.

6.2 Creep, creep ratio and creep-force relationship

In tractive rolling contact, tangential loading can lead to a difference in elastic deformation of the two bodies in the stick zone. The difference in elastic deformation or strain in the stick zone is relieved by slip elsewhere in the contact area, which is known as 'creep'. As described by Johnson [2001], creep arises when elastic deformation causes the surface of a wheel and rail to stretch in tractive rolling contact. Creep can occur in the longitudinal and lateral directions and also in the form of spin creep, where the relative slip between wheel and rail is rotational. Lateral creep arises for example if the plane of the wheel is rotated away from parallel with the rail during rolling. In this work only longitudinal creep is examined.

For longitudinal creep under tangential tensile load, the wheel will move forward a distance in one revolution that is greater than its undeformed circumference. This means the effective circumference of the wheel under load is longer than it is under no load. If the load on the wheel is compressive then the wheel will move forward a shorter distance in a complete revolution. This description leads to the definition of the longitudinal creep ratio ξ as used by Johnson [2001]. The creep ratio is the fraction given by the difference in the distance travelled in one revolution by the deformed and undeformed wheels, divided by the undeformed circumference. It may also be expressed in terms of the velocity of the

surfaces of the contacting bodies. If the velocity of a point on the surface of the wheel in the contact is v_1 and the wheel moves along the rail with overall velocity v_2 then the longitudinal creep ratio is given by [Johnson, 2001]

$$\xi = \frac{v_1 - v_2}{v_2} \tag{6.1}$$

The creep ratio is often expressed as a percentage. For example it is common in modelling trailer wheels of railway vehicles to assume a small level of constant longitudinal creep of the order of 0.1%. This arises when the conicity of the wheels and assumed small lateral displacements of the wheel on the rail head cause the two wheels of a wheelset to run on different radii [Johnson, 2001]. The resulting difference in rolling radius of the wheels causes slip as the wheels travel a different distance to each other with each full axle rotation.

The maximum tangential load that can be supported in the contact area in rolling before the onset of full sliding is of interest for the acceleration and braking of railway vehicles. This adhesion limit determines the maximum torque that can be applied to the wheel. In the absence of a detailed knowledge of the stress distribution in the contact patch, the relationship between longitudinal creep and overall tangential force is used to simulate adhesion limits for accelerating or braking vehicles. The creep-force relationship is also often used in vehicle motion simulations in place of a detailed model of the stress distribution, as it is much faster to calculate.

In many cases a constant friction coefficient is assumed throughout the contact, which results in steadily increasing tangential force with increasing creep up to the adhesion limit. However, measurements of creep-force relationships show that for large creep ratios there is an optimum adhesion, with a decreasing section beyond this maximum [Polach, 2005]. The reduced capacity of the wheel-rail contact to support tangential loads without slipping for high creep is due to the dependence of the friction coefficient on sliding velocity. It is widely known that friction coefficients are different in static and dynamic situations. The dynamic friction coefficient depends on the sliding velocity between the surfaces – higher slip velocities result in lower friction coefficients. For high creep the falling friction coefficient causes the slope of the creep-force relationship to become negative. Figure 6.1 shows the shape of a creep-force curve with constant friction coefficient and one with a velocity-dependent friction coefficient.



Figure 6.1 Creep-force relationship (simplified representation): ——*constant friction coefficient;* — — *friction coefficient decreasing with increasing slip velocity.*

In this chapter the rolling contact analysis is carried out under the assumption of constant friction. A velocity-dependent friction law is introduced into the model in Chapter 8.

6.3 Analytical solution for steady rolling contact

The two-dimensional wheel-rail rolling contact model developed by Carter [1926] and the three-dimensional extension described by Johnson [2001] are analytical solutions to the steady rolling problem.

As with the case for contact with a tangential force transmitted but no rolling, as long as the magnitude of Q is less than the friction limit, the contact is divided into stick and slip zones. However, with rolling, the condition that the direction of the slip must oppose the direction of the tangential stress means that the slip zone must be located at the trailing edge of the contact with a stick zone at the leading edge.

For the static case described in Chapter 5, the distribution of the tangential stress in the stick zone was obtained analytically by the subtraction of a component from the maximum tangential stress which occurs in the slip zone. The resulting stick zone is symmetrical about the centre of the contact. In the case of steady rolling contact, the stick zone is shifted by a distance d corresponding to the difference between the half-length of the contact a and the half-length of the stick zone c. The following equations are from Johnson [2001] but the sign of d has been reversed to match the rolling direction convention used here. As in the static case, the width of the stick zone is determined by the magnitude of the tangential load Q.

$$d = c - a \tag{6.2}$$

$$q_{stick}(x, y) = q_{slip} - \frac{c}{a} \mu p_0 \left(1 - \frac{(r+d)^2}{c^2} \right)^{\frac{1}{2}}$$
(6.3)

This simple shift in the location of the stick zone is valid along the centre-line of a spherical contact for c as defined in Equation (5.35). However, away from the centre-line, a simple circular stick zone does not follow the leading edge of the contact area (as shown in Figure 6.2). Therefore this three-dimensional approximation does not meet the expected condition of stick along the leading edge. No analytical solution has been developed for the three-dimensional problem that meets this condition fully [Johnson, 2001].



Figure 6.2 Shape of stick and slip zones viewed from above in three-dimensional approximation of steady rolling contact of a sphere on a flat plane. The circular approximation of the stick zone is valid along the centre-line only.

For the two-dimensional case of cylinders contacting along parallel *y* axes as studied by Carter [1926], the same expression for q_{stick} applies but *c* is slightly different, as defined in Equation (5.40). The resulting contact area and slip and stick zones are effectively rectangular as shown in Figure 6.3.



Figure 6.3 Approximation of shape of stick and slip zones viewed from above in twodimensional steady rolling contact of a cylinder on a flat plane.

For the two-dimensional case, Figure 6.4 shows an example of the tangential stress distribution in steady-state rolling contact calculated analytically for a cylindrical 0.46 m radius wheel rolling over a smooth rail. The width of the contact is assumed to be 8 mm. The total normal load *P* is 100 kN and the tangential stress is half of the friction limit μP . The friction coefficient μ is 0.3. The trailing edge is slipping while the leading edge sticks.



6.4 Transient rolling contact by the variational method

Analytical methods are limited to steady rolling contact problems, as described in Section 6.3. However a wheel rolling over a rough rail is an unsteady or transient rolling contact problem, because the forces and stresses in the contact vary with time. Some models apply a quasi-static (or stationary) method to determine the stress distribution in rolling. For this method the normal force is allowed to vary, but the resulting tangential stress distribution is calculated as if in steady rolling at each time-step. This approach was used, for example, by Nielsen [2003] and by Wu and Thompson [2005].

By choosing the variational method to determine the stress distribution in rolling contact, true transient rolling is accounted for. For the variational method, the stress distribution in the contact patch at each time-step depends on the stresses and displacements at the previous position. Steady rolling contact may then be considered as an extension or special case of the transient rolling contact theory [Kalker, 1990].

The frame of reference is assumed to move with the contact patch. For a wheel rolling about the *y* axis along a rail, with no overall sliding and no spin, the surfaces then move through the potential contact area with rolling velocity v in the negative *x* direction (Figure 6.5).



Figure 6.5 Moving frame of reference in rolling.

Although the wheel moves along the rail in the positive *x* direction, the flow of material through the contact patch is in the negative *x* direction, from right to left. Rolling is assumed to take place in increments of time Δt with the current time given by *t* and the previous time by *t'*. In each time-step the wheel rolls forward a distance $\Delta x = v\Delta t$ which is chosen to correspond to the length of an element in the potential contact area.

The tangential loading on a rolling system may be either an imposed overall tangential force Q, or an imposed rigid shift $W_{i\tau}$. Accelerating or braking a wheel corresponds to imposing an overall tangential force Q, while imposed longitudinal creep may be described as a rigid shift. For example, if two wheels in a pair joined by an axle have different radii, when running on tangent track one wheel will have to slip or shift longitudinally to keep up with the rotation of the other wheel, while the other will slip backwards, leading to a resultant tangential force in the contact patch.

The presence of a tangential force in rolling leads to elastic deformation along the *x* axis $u_x(x,t)$ of the material on the contacting surfaces. Point *i* of body 1 in the potential contact area at time *t* has an undeformed position defined as $x_{1i}(t)$. In its deformed state, its

position is given by $x_{1i}(t) + u_{x1i}(x,t)$. The corresponding point on body 2 has an undeformed position of $x_{2i}(t)$ and a deformed position $x_{2i}(t) + u_{x2i}(x,t)$.

In general, if corresponding points on the rail and wheel are in contact in their deformed state at time $t' = t - \Delta t$, then the slip between their positions that occurs in a time-step Δt is the sum of the 'rigid' shift and the 'deformation' shift given by

$$s_{i}(t-t') = (x_{1i} - x'_{1i}) - (x_{2i} - x'_{2i}) + \{(u_{x1i} - u'_{x1i}) - (u_{x2i} - u'_{x2i})\}$$
(6.4)

where the *x* terms represent the 'rigid' shift and the u_x terms represent the 'deformation' shift [Kalker, 1990]. The deformation shift is the difference in the elastic displacement occurring in time Δt of the corresponding particles on the wheel and rail. The displacement difference u_{xi} is defined as the difference between the points on the two bodies at a particular time,

$$u_{xi} = u_{x1i} - u_{x2i} \tag{6.5}$$

The displacement difference is taken to be positive when body 1 moves in the positive x direction relative to body 2. The displacement difference at time t may be calculated from the tangential stress distribution and the influence coefficients using Equation (5.18). The tangential elastic displacement difference u_{xi} is necessarily zero by definition in the stick zones, although the tangential stress q and the deformations of each body u_{x1i} and u_{x2i} are not zero.

To minimise the function given in Equation (5.30) at time *t*, the rigid shift $W_{i\tau}$ between the bodies in a time-step and the tangential displacement difference $u'_{i\tau}$ at the previous time *t'* are required. If the tangential loading considered is purely in the form of an overall imposed tangential load *Q*, then the rigid shift is zero.

At time t'=0, the stress distribution q in the contact area is determined from the initial conditions of overall tangential loading or rigid shift. The elastic displacement difference in each element may then be determined, along with the division into stick and slip zones. The wheel then rolls forward a distance Δx in time Δt . The frame of reference moves along with the wheel. Now the displacement differences calculated at time t' correspond to the previous set of potential contact elements. A slightly modified influence coefficient matrix D_{ij}' is required to relate the tangential stress at time t' to the displacement of the elements in the potential contact at time t. D_{ij}' is calculated by adding the distance Δx to the distance between each element combination x_{ij} in Equation (5.19) before following the steps up to Equation (5.24). In fact the matrix D_{ij}' is identical to D_{ij} but with the first row dropped and an additional row added. Figure 6.6 shows this shift in terms of the potential contact area for a three-dimensional analysis.

In the two-dimensional case with transient rolling contact, a modified influence coefficient matrix B_{ij} ' is used to determine the displacements in the current time-step due to the stresses in the previous time-step. This corresponds to the matrix D_{ij} ' in the three-dimensional case and is calculated similarly by adding the distance Δx to the distance between each element combination *k* in Equation (5.37).

In each time-step, an element i of the N elements in the potential contact area is either in the contact zone or outside it. In the contact zone, each element is also in either a stick zone or a slip zone. The corresponding stresses satisfy

$$p_{i} > 0 Inside contact$$

$$p_{i} = 0 Outside contact$$

$$|q_{i}| < \mu p_{i} Stick zone (6.6)$$

$$|q_{i}| = \mu p_{i} Slip zone$$

This division is implicit in the stress on each element, but the calculation of the stress distributions can proceed without explicitly dividing the contact into stick and slip zones.



Figure 6.6 Shift in potential contact area in each time-step.

6.5 Example: development of steady-state rolling from rest

As an example of the calculation of the tangential stress distribution in transient rolling contact, consider a cylindrical wheel of radius 0.46 m initially at rest on a smooth rail of transverse radius 0.46 m. These parameters are equivalent to a sphere of radius 0.46 m rolling on a flat surface, and result in a circular contact patch. A normal load *P* of 100 kN is applied to the wheel. A tangential force $Q = 0.75 \times \mu P$ is applied to the wheel initially and this remains constant throughout rolling. The rigid shift is zero. The friction coefficient μ is 0.3. The wheel is then permitted to roll along the rail with a constant velocity V = 1 m/s.

This example corresponds to a transient rolling contact problem known as 'Cattaneo to Carter' and is used by Kalker [1990] to verify the variational method. The initial tangential stress distribution is that determined analytically by Cattaneo in 1938 as described in Section 5.5.1 (Cattaneo simplified the problem to two dimensions, in this example the extension to spherical contacts described by Johnson [2001] is used). The final steady-state stress distribution may be compared with Carter's [1926] analytical results again in the three-dimensional version as described in Section 6.3. The 'Cattaneo to Carter' problem thus describes the evolution of the contact stress distribution as rolling begins from a stationary contact until steady rolling is achieved.

The potential contact area is defined as a 20 mm \times 20 mm square. This gives 400 elements in total, of length 1 mm in each direction, and the wheel will roll 1 mm in each time-step. At time t = 0 before rolling, the tangential stress distribution along the centre-line of the contact is identical that shown in Figure 5.6(c). The development of the tangential stress distribution shown over ten positions as rolling proceeds is shown in Figure 6.7. The stress distribution after rolling 10 mm is approaching a steady-state, and is shown in three dimensions in Figure 6.8. Figure 6.9 shows the comparison of the steady-state tangential stress distribution obtained by the variational method and by the analytical method based on the Hertz equations. The agreement is satisfactory given the spatial resolution used here.



Figure 6.7 Development of steady-state tractive rolling contact from rest: —— tangential stress q; — — — friction limit. Above each figure is the distance rolled from the initial position. Centre-line of contact shown from three-dimensional model. This corresponds to the case 'From Cattaneo to Carter' used by Kalker [1990] to verify the variational method. $Q = 0.75 \times \mu P$, $\mu = 0.3$.

This case, with a constant tangential force Q, corresponds to a driven wheel with a constant torque supplied to the wheel and constant normal loads. The creep is zero initially when the wheel is at rest and increases to a constant value in the steady-state. For driven wheels with varying normal loads, the creep is not constant but varies in each time-step.



Figure 6.8 Three-dimensional tangential stress distribution after 10 mm of rolling.



Figure 6.9 Cross-section of steady-state tangential stress in rolling contact between a sphere and a flat plane: — — analytical solution based on Hertz equations; — variational method. $Q=0.75 \times \mu P$.

6.6 Example: steady-state rolling with constant longitudinal creep

It is common in modelling non-driven wheels of railway vehicles to assume a small level of constant longitudinal creep. This creep arises from the conicity of the wheel and assumed small lateral displacements of the wheel on the rail head, and is typically of the order of 0.1% [Xie & Iwnicki, 2008b; Sheng *et al.*, 2006; Wu & Thompson, 2005]. With a constant assumed creep in a time-step of duration Δt , the rigid shift of the system W_{it} (the same for all elements in the potential contact area) is given by

$$W_{i\tau} = \Delta x \xi \tag{6.7}$$

Note that the assumed creep in this case is negative, that is body 1 (the wheel) shifts in the negative x direction compared to body 2 (the rail) in the contact zone.

Figure 6.10 shows the development of steady-state rolling contact from rest for a case with constant longitudinal creep of 0.1%. Again the centre-line of the contact calculated from the three-dimensional model is shown.



Figure 6.10 Development of steady-state rolling contact from rest with constant longitudinal creep: ——— tangential stress; — — — friction limit. Above each figure is the distance rolled from initial position. Centre-line of contact shown (from three-dimensional model).

The tangential stress shown in Figure 6.10 is entirely due to the rigid shift arising from the constant longitudinal creep, there is no overall tangential stress Q applied and no constraint on the sum of the tangential stress over all the elements of the potential contact. The overall stress Q increases in each time-step until the steady-state is reached. All other parameters are the same as in the previous example (Section 6.5). In the initial time-step the tangential stress is below the friction limit throughout the contact, meaning there is no slip until a distance of just over 5 mm has been rolled.

With constant creep imposed, the system requires more time to reach a steady-state than for the previous case with a constant Q imposed. In each step shown in Figure 6.10 the wheel has rolled 5 mm (the calculation step size is the same, but not all calculated steps are shown). The steady-state is reached after a distance of around 2 to 2.5 times the overall contact length. A similar distance for convergence was found by Kalker [1990].

6.7 Calculation of slip and slip velocity in the contact patch

At each position as the wheel rolls along the rail, the relative slip *s* between corresponding elements on the two bodies is given by the creep across the element $W_{i\tau}$ added to the change in the elastic displacement difference occurring in each time-step, given by the elastic displacement in the current time-step u_{xi} minus that from the previous time-step $u_{xi'}$

$$s_i = W_{i\tau} + u_{xi} - u'_{xi} \tag{6.8}$$

The slip velocity \dot{s} is determined by dividing the slip by the time-step duration Δt :

$$\dot{s}_i = \frac{s_i}{\Delta t} \tag{6.9}$$

In the stick zone there is no slip and the slip velocity is zero automatically. In the slip zone, the slip velocity increases up to its maximum at the trailing edge. Outside the contact zone the slip velocity falls away again, although this is meaningless. Those elements in the potential contact zone but outside the actual contact experience elastic deformation as a result of the stresses on the elements in the contact zone, but this cannot result in friction or wear. Therefore the slip velocity in elements outside the contact patch is set to zero before calculating the material removed from the rail in each time-step. This is not necessary if the stresses outside the contact are exactly zero, but the numerical minimisation technique can result in very small non-zero stresses outside the contact patch.

The two-dimensional model is used here in order to investigate the effect of element size in the numerical model results. Figure 6.11 shows the slip velocity and corresponding tangential stress distribution calculated using the two-dimensional variational method after reaching a steady-state under constant normal and tangential loading. The parameters are the same as used previously with a normal load of 100 kN, a constant longitudinal creep of 0.1%, a cylindrical wheel of radius 0.46 m, a contact width of 11 mm and a smooth flat rail. The coefficient of friction μ is 0.3 and the rolling velocity v = 1 m/s. It can be seen that the slip occurs only at the trailing edge of the contact and is zero elsewhere. With the 1 mm element size used, there are three elements in a state of slip, with the maximum slip velocity of around 3 mm/s.



Figure 6.11 Slip and stress distribution in steady-state rolling with 0.1% creep and 1 mm element size. (a) Slip velocity; (b) tangential stress distribution: ——variational method with 1 mm elements; · · · · · · friction limit.

6.7.1 Effect of element size on slip velocity

If the stick-slip division is required in more detail, more elements of smaller size may be used to define the potential contact zone. Figure 6.12 shows the results from the same model but with four times as many elements in the potential contact. There are now twelve elements making up the slip zone with non-zero slip velocity. The tangential stress distribution is also much smoother when calculated using smaller elements. The peak slip velocity at the trailing edge of the contact is now 4 mm/s which is significantly higher than the peak value calculated with 1 mm elements. The slip velocity increases up to a maximum at the trailing edge, so smaller elements give a better approximation of the peak slip velocities.



Figure 6.12 Slip and stress distribution in steady-state rolling with 0.1% creep and 0.25 mm element size. (a) Slip velocity; (b) tangential stress distribution: ——— variational method with 1mm elements; · · · · · · · friction limit.

Figure 6.13 shows the slip velocity in a steady-state calculated with various different element sizes. There is very little difference in the slip velocity between an element size of 0.1 mm and 0.25 mm, suggesting that at least 0.25 mm elements are required to represent accurately the slip velocity in the contact patch with a constant friction coefficient. In Chapter 8, however, it is found that even smaller elements are required if a velocity-dependent friction coefficient is used. Therefore in the remainder of this work 0.1 mm elements are used.



6.7.2 Effect of rolling speed on slip and slip velocity

If the longitudinal creep is constant, then the amount of slip in each time-step in rolling is independent of the rolling speed. This is because the distance rolled in a time-step is unchanged, since this distance is determined by the element size in the potential contact. The length of the slip zone is unaffected by the rolling speed for steady rolling and constant creep, as is the tangential stress distribution. However the slip velocity is directly proportional to the rolling speed, as shown in Figure 6.14. Doubling the rolling speed from 1 m/s to 2 m/s doubles the slip velocity throughout the slip zone.



Figure 6.14 Effect of rolling speed on steady-state slip velocity: ——— *speed 1 m/s;* — — — 2 *m/s;* … … 10 *m/s.*

6.8 Comparison with results from 2-D and 3-D analytical solutions

An example of the tangential stress in steady-state rolling contact is shown in Figure 6.15 to compare the results from the two-dimensional variational model with analytical solutions in two and three dimensions. The parameters are the same as used previously but again with an assumed contact width of 11 mm. As with the normal stress distribution (Figure 5.12), the magnitude of the tangential stress is under-estimated when using a two-dimensional representation. The location of the stick-slip boundary is similar. With the small 0.1 mm elements used in the variational method, the precision of determination of the location of the stick-slip boundary makes the two-dimensional analytical and numerical methods almost indistinguishable.



The rolling contact case 'From Cattaneo to Carter' has been calculated again using the two-dimensional variational model and is shown in Figure 6.16. The parameters are the same as used for the three-dimensional case presented in Figure 6.7, but with an assumed contact width of 11 mm and smaller 0.1 mm elements. As with the three-dimensional model, before the initiation of rolling, there is a central stick zone with a slip zone at each end of the contact patch. With rolling, the slip zone becomes restricted to the trailing edge and a steady-state is achieved after around 10 mm of rolling.



Xie and Iwnicki [2008b,c] have obtained similar results from two- and three-dimensional models. Here it is clear that the two-dimensional version of the model can give an accurate representation of the length of the contact. If it is used to simplify spherical contact to two dimensions, the magnitude of the actual normal stress and hence of the tangential stress is under-estimated at the centre-line. The location of the stick-slip division may also be affected, which could influence the wear results in a frictional wear model. However in the cases considered here only the roughness profile of the centre-line is known. This profile can be assumed to extend over the width of the rail head with the rail head curvature, but the three-dimensional model is then itself an approximation of the actual rail profile. The true lateral roughness distribution is not known and for a worn rail, the rail head curvature

in the contact zone could be flattened out. In this situation the contact would be better represented by a cylinder on a plane than by the three-dimensional model.

6.8.1 Contact width for two-dimensional model of actual wheel and rail

In the case of contact between a cylindrical wheel of radius 0.46 m and a transversely curved rail of radius 0.3 m, the three-dimensional contact is elliptical rather than circular, and shorter in the lateral direction. An assumed width of the contact in the *y* direction of 8 mm for the two-dimensional model gives a good approximation of the length of the contact in the *x* direction for the vehicle static loads modelled in this work.

6.9 Rolling over a corrugated rail

If the rail is not perfectly smooth, the contact length and the distribution of the stresses and slip in the contact patch varies with the contacting profile. For example, if a short wavelength corrugation is present on the rail, the contact may occur at more than one location and there may be several zones of stick and slip. Figure 6.17 shows an example with a 5 mm wavelength sinusoidal corrugation of amplitude 1×10^{-5} m. The normal load is 60 kN and is assumed constant, *i.e.* does not fluctuate as a result of rolling over the corrugations. The wheel radius is 0.46 m and the width of the contact is assumed to be 8 mm. This case could therefore represent the regional or high-speed train type, but with a constant normal load. Note that this amplitude of corrugation is unrealistic in practice at this short wavelength, so the example is hypothetical until wavelengths longer than the length of the contact are considered. The element length is 0.1 mm.

This 5 mm corrugation wavelength is much less than the length of the contact, so when the wheel is centred over a corrugation trough, there are four corrugation peaks in contact with the wheel (Figure 6.17(a)). Note that the friction limit shown in this figure is proportional to the normal stress and therefore indicates areas in contact. Although four of the corrugation peaks are in contact, there are two corrugation troughs that are not in contact, so rather than a single contact patch, there are three contact patches when the wheel is centred over a corrugation trough. The leading contact patch is a stick zone, the middle contact patch has two stick zones and two slip zones, while the trailing contact patch is fully slipping. When the wheel moves on to be centred over a corrugation peak (Figure 6.17(c)), only the two peaks either side of the central peak are in contact and there

is a single contact patch, although stresses are very low in the corrugation troughs. At this position there are two slip zones and two stick zones.



Figure 6.17 Effect of 0.005 m wavelength sinusoidal roughness on tangential stress and contact length. (a) Roughness trough; (b) roughness upwards slope; (c) roughness crest; (d) roughness downwards slope: —— tangential stress; — — — friction limit.

Increasing the wavelength of the corrugation for the same amplitude reduces the effects of the roughness. The contact is confined to a single contact patch for a wavelength of 10 mm, shown in Figure 6.18, although the corrugation peaks still distort the stress distribution considerably. Here a full corrugation wavelength fits within the length of the contact.

In Figure 6.19, with a corrugation wavelength slightly longer than the contact length, the effect of discrete peaks is less visible although the contact length overall is affected by the location of the wheel centre relative to the corrugation peaks and troughs. In a trough, the contact is longer and the normal stresses are lower and more evenly spread throughout the contact than is the case with the wheel centred over a corrugation peak. As the corrugation wavelength increases, the stress distribution and maximum stresses approach the values for a smooth rail.



Figure 6.18 Effect of 0.01 m wavelength sinusoidal roughness on tangential stress and contact length. (a) Roughness trough; (b) roughness upwards slope; (c) roughness crest; (d) roughness downwards slope: —— tangential stress; — — — friction limit.



Figure 6.19 Effect of 0.02 m wavelength sinusoidal roughness on tangential stress and contact length. (a) Roughness trough; (b) roughness upwards slope; (c) roughness crest; (d) roughness downwards slope: —— tangential stress; — — — friction limit.

Overall, the peak normal stresses are much higher with a corrugated rail than those between a smooth wheel and rail, even in these examples where the normal load is constant, neglecting any dynamic effects arising from rolling over the corrugation. In fact the maximum normal contact stress predicted for the 5 mm wavelength case is approximately 1500 MPa (although as noted this example of corrugation is unrealistic at this wavelength), compared with a maximum of around 800 MPa for the same case with a smooth rail. For the 10 mm corrugation the maximum normal stress is around 1170 MPa, and for the 20 mm corrugation case it is 920 MPa. It is possible then that the normal stress in parts of the contact can exceed the yield strength of the rail steel. The contact model used here does not account for plastic effects. It is known that plastic deformation does occur in normal operation; an example is studied by Kapoor et al. [2002] for rail steel with a nominal yield stress of 480 MPa. However, under repeated loading the surface layer of the rail steel is known to harden, allowing it to remain elastic under higher stresses than the nominal yield limit. Also if plastic deformation does occur initially, the process of shakedown means that subsequent load cycles will give decreasing amounts of plastic deformation [Olofsson & Lewis, 2006; see also Daves & Fischer, 2002; Wen & Jin, 2006].

6.10 Normal and tangential stress distribution in rolling with a rough rail

In Chapter 4 it was seen that the roughness of the system is the factor that has the greatest effect on the normal interaction force. In transient rolling contact, the normal force is allowed to vary in each time-step and here is taken from the interaction force model described in Chapter 4. The time-stepping model returns the normal force at 1 mm intervals as the wheel rolls along the rail. This force history is interpolated to give the force at intervals determined by the element length in the potential contact area, here at 0.1 mm spacings.

The effect of including the roughness of the contacting profiles on the magnitude of the stress in the contact patch may be assessed by comparing the maximum stress in the contact patch along a length of rough track with the stress calculated using a Hertzian contact model that neglects the roughness. Figure 6.20 shows an example with a freight wheel (parameters are listed in Table 4.1) where a Hertzian contact model would predict a maximum normal stress of around 1000 MPa. If the combined roughness of the wheel and rail is of the order of the TSI rail roughness limit the actual maximum normal stress might

be around 1200 MPa, and if the combined roughness is 10 dB higher the maximum might be 1700 MPa.



This analysis assumes purely elastic contact. The roughness asperities act as stress concentrators and can have a significant effect on the normal and hence tangential stress in rolling contact. With realistic combined wheel and rail roughness levels, the normal stress in rolling contact will exceed the nominal yield stress for rail steel leading to work hardening and shakedown. In the remainder of this work the analysis is limited to low-level broadband roughness of a similar level to the measured spectrum at the Gersthofen test site (see Section 4.4.4). In this way stress levels remain within the yield limit, since with work hardening and shakedown effects it is difficult to be sure exactly where the elastic limit lies.

Figure 6.21 is an example of a tangential stress distribution for a freight wheel on a rail with low level broadband roughness. With low level roughness there is typically only one slip zone at the trailing edge of the rolling contact as seen here. Multiple stick and slip zones are only seen with higher levels of roughness or corrugation.



Figure 6.21 Example of tangential stress in the contact area: _____stress; _____ friction limit.

6.11 Discussion - transient rolling contact

Transient rolling contact is the default calculation state when using the variational method. Results for steady-state rolling have been obtained by allowing the system to converge under constant normal and tangential forces. The tangential forces can be input to the model either in the form of a constant overall tangential force, or in the form of creep. The steady-state results from the variational method have been compared with those calculated using analytical formulae based on Hertz theory.

The derivation of the slip velocity in the rolling contact has been described. Although the stress distribution and the slip in rolling are not affected by the rolling speed, the slip velocity is directly related to rolling speed. It is also important to use sufficiently small elements in the potential contact area to capture the slip velocity correctly. This favours using the two-dimensional model for the rolling contact analysis.

The variational method has been used to calculate example stress distributions in transient rolling. It is clear that the roughness of the surfaces has a significant effect on the normal and tangential stress distributions.
7 RAIL-HEAD WEAR MECHANISMS

7.1 Introduction

A wheel rolling over a rail results in wear of the two surfaces in contact. The wear can be gradual, with a small amount of material removed with every wheel passage. In some cases the wear may be more severe, for example fatigue may lead to ratchetting or surface cracking and the removal of material at a higher rate. As the contacting surfaces are not perfectly smooth initially and the forces between the wheel and rail vary with time, the amount of material removed varies along the rail. This uneven wear results in a change in the roughness profile of the rail (and wheel) after many wheel passages.

Rail-head wear can occur by several different mechanisms, depending on the contact conditions [Lewis & Olofsson, 2004]. Frictional abrasive wear, or mild wear, is generally thought to be the mechanism for roughness growth on tangent track in the absence of corrugation or other defects in the contacting surfaces [Nielsen *et al.*, 2003]. It is therefore common in roughness growth predictions to assume a single wear coefficient at all locations representing mild wear. Implicit in this assumption is the idea that wear only occurs in tractive rolling, in the slip region of the contact patch.

However, the inclusion of non-Hertzian effects in the contact means that stress concentrations may arise from the roughness profile of the surfaces in contact. It is possible that these stress concentrations might lead to higher wear rates in some parts of the contact area. This in turn might affect the prediction of roughness growth rates and is therefore included in this model.

This chapter begins with a description of the frictional wear and ratchetting wear mechanisms and mathematical models used to simulate them. A common feature of these wear models is that the amount of material removed is determined by the severity of the normal and tangential stresses in the contact zone, and the slip velocity. Previous chapters have described techniques to calculate the forces distributed through the contact patch and the resulting normal and tangential stresses and slip velocities. Here the stress distribution and slip velocity is linked to the removal of material from the rail. The work of Braghin *et al.* [2006] on wheel wear rates is used to predict the wear of rails taking account of three wear regimes of increasing severity. The contact conditions that are required to develop wear in the more severe wear regimes are investigated.

This chapter also includes an analysis of the differences in wear predictions that can arise from making an assumption of Hertzian contact conditions in the presence of even lowlevel surface roughness.

7.2 Frictional abrasive wear

Frictional or abrasive wear occurs as a small amount of material is removed with every wheel passage, proportional to the frictional work done in the slip zone of the contact area. The relationship between the work done and the material removed is given by Archard's wear equation [Rabinowicz, 1965]. In each time-step as the wheel moves along the rail, the slip in each element of the potential contact can be calculated from the elastic displacements that arise from the tangential stress in the contact patch.

The depth of material dz removed across the area of an element *i* in the slip zone in each time-step is given by

$$dz_i = \frac{K\Delta t |\dot{s}_i q_i|}{\rho} = \frac{K |s_i q_i|}{\rho}$$
(7.1)

where ρ is the density of the material and *K* is a wear constant that is determined by experiment or is taken from the literature. This equation is a form of Archard's wear equation [Archard & Hirst, 1956], which relates the amount of material removed to the work done by the tangential stress in the slip zone. A commonly used value of the wear constant *K* for railway applications is 2.5×10^{-9} kg/Nm which was obtained by Nielsen [2003] by tuning a wear model to measured growth rates at a corrugated site in the Netherlands. More attention is given to the choice of wear constant in Section 7.4.

The material depth removed is accumulated in each element as the wheel or wheels pass over the track and finally is subtracted from the original rail roughness profile. In a steadystate case the material removed is independent of the velocity of the wheel. This is because the effect of the rolling speed on the slip velocity (shown in Section 6.7.2) is cancelled out by the dependence of the work done in the slip zone on the time-step Δt . Higher tangential loads in the form of higher creep or higher torque applied to the wheel result in more material removed, as these cases lead to more slip.

Figure 7.1 shows that when a single wheel rolling over a smooth rail under constant normal and tangential loadings reaches a steady-state, the material depth removed by a passage of

the wheel is a constant in each time-step. With smaller elements and hence shorter timesteps, the material removed in a single time-step is less than if bigger elements are used. In Figure 7.1 the results have been normalised by the length of the element Δx in each case, to allow the results to be compared for different element sizes. It is clear that in this case, if the element size is 1 mm giving three elements in the slip zone, the representation of the material removed in a time-step is relatively coarse compared with that calculated using a higher number of smaller elements. The results converge with decreasing element size. The two smallest element sizes give very similar results and therefore an element length of 0.1 mm is chosen for this work.



Figure 7.1 Effect of element size on depth of material removed at steady-state. Wear depth in a single time-step has been divided by the length of the element to give comparable results: -0.05 mm elements; $-0.05 \text$

The distribution of the material removed in a time-step in steady-state rolling (shown in Figure 7.1) is controlled by the tangential stress and by the slip. At the border between the stick and slip zones the tangential stress is high but the slip velocity is zero, so the material removed is zero. At the trailing edge of the contact the slip velocity is at its maximum but the tangential stress is zero so again no material is removed. The peak in the material removed therefore occurs in the middle of the slip zone for steady rolling. In transient rolling with rough surfaces, the smooth distribution of material removed will be disrupted.

Figure 7.2 shows the accumulated material removed as a wheel rolls along a rail for element sizes and rolling increments from 1 mm down to 0.05 mm. A steady-state for wear

is attained after around 0.15 m of rolling under constant tangential load. This is a longer distance than found in Section 6.6, where a steady-state for tangential stress distribution appears after 2 to 2.5 times the contact length, or about 0.03 m. This indicates that although the tangential stress and slip in transient rolling develop rapidly to a value close to their final value, true convergence in terms of the material removed from the rail under steady-state conditions requires a larger rolling distance.

From Figure 7.2 it can also be seen that the element size and rolling increment have an effect on the predicted wear. Higher peak slip velocities arise from a finer discretisation and result in more work in the slip zone. Consequently more material is removed in the steady-state as shown in Figure 7.2. The wear constant *K* is 2.5×10^{-9} kg/Nm and the normal load is 100 kN.



Figure 7.2 Effect of element size on accumulated depth of material removed under wheel rolling from rest along rail with constant normal force: ---0.05 mm elements; ---0.5 mm elements; --0.5 mm elements; ---0.5 mm elements; ---0.5

7.3 Wear by ratchetting of a ductile material under repeated loading

The mechanism of wear by plastic ratchetting is described by Kapoor [1997]. It occurs when a material is loaded above its yield strength in repeated sliding contact cycles. The plastic deformation accumulates over many cycles, causing small slivers of material to be compressed and gradually extruded from thin sub-surface layers. Even if the plastically deformed material does not extrude out or break off, wear may occur by fatigue with micro-cracks occurring in the material after repeated loading. Damage may occur in layers

below the surface, but particles are only removed from the surface as previously subsurface layers are exposed.

The high stresses required for plastic deformation can occur in the contact of rough surfaces even if the average stress in the contact is below the yield stress. The peaks of roughness lead to higher stresses at the asperities. In tractive rolling with even partial slip, the sliding of the surfaces across one another in the slip zone ensures that high contact pressures due to asperities are spread across the surfaces.

Kapoor and Franklin [2000] developed a model of the ratchetting wear mechanism by simulating the sliding of a two-dimensional cylindrical asperity over a smooth surface. In this model the surface is described as a number of thin layers, each of which accumulates an increment of shear strain if the stress in the layer is sufficiently high. As well as a critical strain to failure of the material, each layer has an associated effective or current shear strength which takes account of any strain hardening. The stress distribution is assumed to be Hertzian with a constant friction coefficient used to determine the maximum shear stress in the fully sliding contact. With each pass of the cylindrical asperity, the shear strain increment resulting from the peak shear stress is calculated at each layer and added to the accumulated strain at that depth. If the total accumulated strain in the top layer exceeds the critical value, then it fails and is removed allowing an estimation of the wear rate from the depth of material removed over many cycles.

This model of wear was expanded by Franklin *et al.* [2001] by modifying the surface representation into a 'brick' form rather than the layers used previously. The use of 'brick' elements allows the consideration of roughness development rather than even wear, since bricks may fail and be removed while adjacent bricks remain in place. The failure of a brick is taken to depend not only on its accumulated total shear strain, but also on the status of the surrounding elements. A brick that has failed in isolation is not removed from the surface until at least one of the adjacent bricks has also failed. Additionally, a 'healthy' brick may be removed if it is not supported on either side by other bricks.

The effect of surface roughness on the contact pressure has been modelled by Kapoor *et al.* [2002]. The resulting predicted wear rates and contact stresses were compared with experimental results from a twin-disc machine used to simulate the operating conditions of the Japanese Shinkansen train. The nominal operating conditions and contact stresses were

expected to be below the critical yield stress and the shakedown limit, but cross-sections of the rail showed sub-surface plastic deformation. The model and the experiment confirm that roughness leads to plastic deformation of thin layers of the surface. A limit of the model was noted; the contact stresses are calculated assuming elastic deformation, although the resulting stresses are high enough to lead to plastic flow. However, it is clear that roughness can cause the elastic limit of the material to be exceeded.

The brick model has also been used to study the effect of random variations in the material properties of the surface on the wear rate [Franklin *et al.*, 2003; Alwahdi *et al.*, 2005]. More recently this variation has been used to model the microstructure of steels, assigning different material properties to pearlite and ferrite grains [Franklin & Kapoor, 2007]. The effect of partial slip in rolling contact has also been considered by Alwahdi *et al.* [2005] using a Hertzian contact stress distribution between a rolling cylinder and a half-space, with the tractive force less than the friction limit. The results of this work show that wear rates increase as traction increases, and are heavily dependent on the friction coefficient. Overall, the predicted wear rates correspond well with full scale experimental results. The roughness growth rate of the rail was not examined, although it is noted by Franklin and Kapoor [2007] that this is possible. However, a great number of iterations of the contact model would be required.

7.4 Wear by multiple mechanisms

An alternative to simulating ratchetting wear directly is to make use of experimentally determined wear rates including several different wear regimes. The work of Braghin *et al.* [2006] on wheel wear rates may be adapted to predict the wear of rails taking account of three wear regimes of increasing severity. Twin disc laboratory tests have been used to determine, under controlled conditions, the relationship between the tangential stress distribution in the slip zone of a contact patch and the rate of material removal. The wear rate is a linear function of the wear index $T\gamma/A$ where *T* is the tangential force, γ is the non-dimensional slip and *A* is the contact area of an element. The variables *T* and γ correspond to the tangential force *Q* and the slip *s* used in this thesis, but with different units (see Table 7.1 and Equation (7.2)). This wear index model builds on the work of several authors in particular Bolton and Clayton [1984], Clayton [1996] and Lewis and Olofsson [2004]. It is assumed here, as in Braghin *et al.* [2006], that the wear relationship can be applied to individual elements of the contact area, although the wear index was originally developed

for the contact area as a whole. Figure 7.3 and Table 7.1 show the variation in wear rate with different wear regimes. The first of these regimes is equivalent to the frictional abrasive wear model described in Section 7.2. Here the wear is directly proportional to the wear index (although the constant of proportionality or wear coefficient typically used for pure frictional abrasive wear is different). In the second regime the wear rate is constant for a range of $T\gamma/A$ values. This second regime corresponds to the ratchetting wear mechanism. The wear rate in the third regime is again proportional to the wear index but with a much higher gradient indicating the rapid removal of material that occurs under the most severe contact conditions.



Figure 7.3 Change in wear rate with different wear regimes: 1 'mild wear'; 2 'severe wear'; 3 'catastrophic wear'.

Table 7.1 Wear regimes and coefficients [Braghin et al, 2006]. The equivalent wear coefficient is in the form used in Equation (7.1).

	Τγ/Α	Wear rate ($\mu g/m/mm^2$)	Equivalent wear coefficient <i>K</i> (kg/Nm)
1	<i>T</i> γ/ <i>A</i> <10.4	5.3 <i>T</i> γ/A	5.3×10 ⁻⁹
2	10.4< <i>T</i> γ/A<77.2	55.0	55.0×10 ⁻⁹ /(<i>T</i> γ/A)
3	77.2< <i>T</i> γ/A	61.9 (<i>T</i> γ/A-77.2)+55.0	[61.9 (<i>Tγ/A</i> -77.2)+55.0] /(<i>Tγ/A</i>) ×10 ⁻⁹

The wear index $T\gamma/A$ can be evaluated for each element of the potential contact area and used to predict the amount of material removed from the rail in each time-step. In terms of the notation used in this work, the wear index for each element of the contact area (with an assumed constant contact width) in units of N/mm² may be written as

$$\frac{T\gamma}{A} = \frac{|s_i q_i|}{\Delta x} \times 1 \times 10^{-6} \tag{7.2}$$

The material depth removed over the surface of each element i is then calculated from Equation (7.1) but instead of the single wear coefficient K used previously the equivalent value of K for the corresponding wear regime is used from Table 7.1.

In the frictional abrasive wear model described in Section 7.2, the wear coefficient used was 2.5×10^{-9} kg/Nm which is less than half the equivalent wear coefficient for the mild wear regime indicated in the literature of the multiple wear mechanisms approach. However Nielsen's value was obtained by tuning the wear model to field measurements of roughness on operating track [Nielsen, 2003], and is therefore an average wear rate which could include more than one wear mechanism. Braghin *et al.* [2006] note that normal wheel-rail contact leads to wear in only the first two regimes in normal operation, and that the third regime of catastrophic wear would only be reached in severe curves. The single wear coefficient used in Section 7.2 is compared with the wear relationship with multiple mechanisms in Figure 7.4.



Figure 7.4 Wear rate comparison: ——— three wear regimes with different wear coefficients; — — a single proportional wear coefficient.

7.5 Factors that affect the wear rate

The wear rate as modelled in this work is a function of the tangential stress and the slip. As noted in Section 7.2, in steady rolling with constant normal and tangential forces, the rolling speed alone does not affect the amount of material removed from the rail head. In transient rolling however, the normal force can fluctuate and its magnitude is higher for vehicles moving at higher speeds (Section 4.5.1). Higher initial roughness levels also result

in higher normal forces. It is of interest to investigate and understand the effect of the normal force and roughness profile on the wear rate in isolation before examining the material removed from the rail with simultaneously varying forces and roughness. In all the following examples the element size used in the potential contact area is 0.1 mm.

7.5.1 Effect of magnitude of normal force with constant tangential loading

Figure 7.5 shows that reducing the magnitude of the normal force with the same longitudinal creep of 0.1% results in less wear in steady-state rolling. If the normal force is reduced, the contact length is smaller, meaning an element of the rail is in contact with the wheel for a shorter time. Also the friction limit, tangential stress and slip velocity are all lower with a reduced normal force. The only factor which opposes the reduced wear is that more of the contact patch is in a state of slip if the normal force is lower for the same creep (Figure 7.6), but this does not result in higher wear.



Figure 7.5 Effect of normal force magnitude on accumulated material removed for wheel rolling from rest with constant 0.1% creep: ---100 kN; $\cdots \cdots \cdot 60 \text{ kN}$; ----40 kN; $---- \cdot 10 \text{ kN}$.



Figure 7.6 Effect of normal force magnitude on contact length and tangential stress distribution with constant 0.1% creep: —_____tangential stress; ______friction limit.

In these simple steady-state rolling cases with a smooth wheel and rail, the wear regime is purely mild wear. For a 0.46 m radius wheel and a 100 kN normal force, the maximum wear index in steady rolling is around 0.5 N/mm² which is well below the transition to severe wear at 10.4 N/mm². Therefore using the mixed-mechanism model, the pattern of material removed (shown in Figure 7.5) is similar to that found in Section 7.2. The only difference is that more material is removed as a result of the higher wear coefficient that is applied to the mild wear regime in the mixed mechanism wear model.

7.5.2 Effect of varying rail profile with constant normal force

If the rail has a sinusoidal profile, the depth of material removed in a steady-state from the rail is also periodic for a constant normal force and constant 0.1% longitudinal creep. Figure 7.7 shows the material depth removed after a single wheel passage for different sinusoidal profile wavelengths all with amplitude 1×10^{-5} m. The variation in the wear depth depends on the wavelength of the initial profile. Short wavelengths create the highest stress concentrations and hence the greatest variation in the material removed. Increasing the wavelength of the periodic profile causes the material removed to approach the steady-state constant value seen for the smooth wheel case (note, however, that the shortest

wavelength rail profile is not realistic, as corrugation does not form at such wavelengths shorter than the length of the contact patch).



Figure 7.7 Effect of sinusoidal roughness profile on depth of material removed at steadystate: — material removed; $\cdots \cdots$ roughness profile (amplitude 1×10^{-5} m, not shown to scale). Roughness wavelengths top to bottom 5 mm, 10 mm, 20 mm, 40 mm, 80 mm.

The maximum material removed occurs near the crests in the rail profile, indicating that the ridges tend to be worn down. With a 5 mm wavelength initial profile there is almost no material removed from the corrugation troughs, suggesting that the troughs do not see significant normal or tangential stress levels and the load is taken by the profile peaks. This finding corresponds to the stress distributions shown in Figure 6.17.

7.5.3 Effect of varying normal force

Now the wheel moving along the smooth rail with constant longitudinal creep of 0.1% is subject to a sinusoidal normal force. Five force wavelengths are considered ranging from 5 mm to 80 mm. The force varies with amplitude 10 kN around the 100 kN static load in Figure 7.8 and by +/- 50 kN around a 60 kN static load in Figure 7.9. The maximum force is therefore the same in both sets of results.



Figure 7.8 Effect of small amplitude sinusoidal normal force on accumulated depth of material removed: — material removed; · · · · · · force (not to scale). Force wavelengths top to bottom 5 mm, 10 mm, 20 mm, 40 mm, 80 mm.

With the small fluctuation in force (Figure 7.8), the size of the contact and the length and location of the slip zone do not vary much. The variation in the overall contact length over

the force range is only around 1.5 mm, with an average contact length of around 14 mm. The slip zone is typically around 2 to 3 mm long and is centred around 6 mm behind the centre-line of the contact. So if the force does not fluctuate enough to change the contact characteristics significantly, the pattern of material removed from the rail follows the sinusoidal pattern of application of the force.



Figure 7.9 Effect of large amplitude sinusoidal normal force on accumulated depth of material removed: — material removed; · · · · · · force (not to scale). Force wavelengths top to bottom 5 mm, 10 mm, 20 mm, 40 mm, 80 mm.

With the larger amplitude force variation shown in Figure 7.9 the wear pattern is no longer clearly sinusoidal, although all cases show periodic wear. The maximum material removed increases with the wavelength of application of the normal force, *i.e.* with a 5 mm force wavelength the average material removed overall is similar to the minimum material depth

removed with an 80 mm force wavelength. For the cases with force varying at wavelengths of 10 mm or longer, the wear pattern consists of a region of low wear followed by a shorter distance with high wear, before dropping quickly back to low wear. The peak material removed occurs near the force minima.

The variability in the material depth removed may be understood by examining the variation in the stress distribution and location of the slip zones as the wheel moves along the rail.

When the force fluctuates quickly with a 5 mm wavelength (Figure 7.10), there are regions where there is little or no slip zone, including the time when the force is at its maximum. When the force is at its minimum, the contact is in full slip but the overall contact length is very short, the tangential stresses are very low and the resulting wear is correspondingly low. The normal force increases rapidly while rolling only a short distance, insufficient for a slip zone to develop before the force decreases again. Consequently the overall material removed with this quickly fluctuating force is low although a short wavelength variation in the rail profile develops.



Figure 7.10 Effect of 0.005 m wavelength large amplitude normal force on tangential stress and contact length. (a) Force minimum; (b) force increasing; (c) force maximum; (d) force decreasing: ——— tangential stress; — — — friction limit.

When the force fluctuates more slowly, as shown in Figure 7.11, there is more time for the stress distribution to adapt to the changing forces. With a 10 mm force wavelength, the slip zone vanishes as the force increases from its minimum value. At the maximum force location, a short slip zone is present at the trailing edge, and this trailing edge of the contact remains a slip zone which extends as the force decreases again until at the minimum force location the entire contact slips. In steady-state rolling, the slip zone is confined to the trailing edge, but here a small slip zone also appears at the leading edge of the contact as the normal force decreases (Figure 7.11(d)).



Increasing the force wavelength further to 20 mm (Figure 7.12), the stress and slip distributions at each location begin to resemble the typical steady-state distribution, although the contact remains in full slip when the normal force is at its lowest.



7.5.4 Discussion on factors that affect the wear rate

The preceding examples illustrate that the wear rate at a series of locations along the rail head is affected by the rail profile and by the normal force. Examining these parameters in isolation has shown that more wear occurs in steady-state rolling for higher normal contact forces. This would suggest, for example, that more material is removed from the rail head by the passing of a typical loaded freight wheel than other wheel types, as a result of the higher load on each axle of a freight vehicle. However, wear of an initially smooth system with steady-state rolling is uniform along the track and does not result in a change in roughness profile. A varying wear rate along the rail is seen when either the force or the rail profile is not constant over time.

Looking at a single sinusoidal component of roughness with a constant normal force, more material is removed from the peaks of the profile than from the troughs, suggesting that the profile will be worn down over time. This effect arises because the profile peaks act as stress concentrators, and the amount of material removed is proportional to the tangential stress. This is seen for all the single wavelength roughness profiles considered, from 5 mm to 80 mm.

If the rail is smooth but the normal force is fluctuating, the effect on the wear rate depends on the rate and magnitude of the fluctuation. A small magnitude force fluctuation leads to variable wear, but the size of the contact and the length of the slip zone do not vary much so the pattern of material removed follows the pattern of the normal force variation. The wear rate and the force are not necessarily in phase. If the force fluctuates rapidly and with large amplitude, the contact stress does not have time to settle into a distribution resembling the steady-state distribution. The resulting pattern of wear along the rail is no longer sinusoidal. There are periodic 'bursts' where more material is removed, corresponding to the force minima (see Figure 7.9).

It is expected from these results that if the force does not vary significantly, the existing rail profile will have more effect on the location of wear than the varying force. If the force magnitude varies rapidly and significantly, for example if the rail were corrugated, it would dominate the location of wear.

In Chapter 4 it was found that with a sinusoidally corrugated rail profile, the peak normal interaction force occurs near the peak in the roughness (see Figure 4.10). The minimum normal force occurs near the roughness troughs. If the stress concentrating effects of the rail profile are neglected, this combination has the potential for roughness growth. Roughness may grow if the force variation is high enough, as then more material would be removed from the troughs than from the peaks. However, the force fluctuation arising from these sinusoidal rail profiles with amplitude 1×10^{-5} m is around +/- 10 kN so the pattern of material removed is likely to be dominated by the stress concentrating effects of the rail profile.

7.6 Wear with normal forces determined from interaction model

The normal interaction force between a wheel and a rough rail has been calculated in Chapter 4 for cases with a smooth rail, sinusoidal corrugations and also for broadband roughness. The wear of the rail can then be determined using the actual rail profile and normal force at each location of interest. In calculating the wheel-rail interaction force it was necessary to include many sleeper bays in the track model to represent the dynamic characteristics of an infinite track accurately. In calculating the wear, however, the effect of the track dynamics and vehicle speed are included in the varying normal force. Therefore the force and rail profile in the middle sleeper bays may be extracted from the rest and used as inputs to the wear model.

Here a sample length of three sleeper bays or 1.8 m has been chosen. This is longer than the 1.2 m of data that can be measured by fixed-edge roughness measurement devices (see Section 1.4.1). This length is therefore sufficient to allow the change in roughness profile to be assessed, but is short enough to keep the calculation time for a single case reasonable. Results can be averaged for a number of cases if required (as described by the roughness measurement standard [CEN, 2009]). To allow the transient model of the rolling contact to converge, an additional 0.3 m of track is included in the wear analysis before the wheel in question reaches the sleeper bays of interest.

7.6.1 Wear of initially smooth or sinusoidal rail profiles

Figure 7.13 shows that the material removed from a perfectly smooth rail is not constant in a sleeper bay, but varies slightly as a result of the variation in force due to parametric excitation, *i.e.* the varying stiffness of the track caused by the discrete supports. When a sinusoidal profile is present on a rail and the force is allowed to vary, the material removed takes an overall sinusoidal shape. For each wavelength considered, more material is removed from the peaks and down-slopes of the profile than from the troughs or up-slopes. This suggests that a corrugation will shift longitudinally along the rail and also tend to be worn down over time.

Figure 7.14 shows a 'zoomed in' view of the material removed along a single wavelength of the 40 mm wavelength sinusoidal profile with the normal force determined using the interaction force model. Also shown is the corresponding result when the force is taken to be constant, from Figure 7.7. If the force is constant, the maximum material removed occurs closer to the peak in the roughness profile than if the force is taken from the interaction model. So in both cases the profile is worn down and shifted along the rail over time, but the longitudinal shift is less if the normal force between wheel and rail is constant as it passes over the sinusoidal rail profile. More material is removed from the rail head in this case if the normal force can fluctuate.



Figure 7.13 Material removed along rail with varying rail profile and normal force from interaction model: — *material removed;* — *material profile (not to scale).*



The wear rates for all the cases shown in Figure 7.13 remain low and in the mild wear range. The highest wear index of 0.6 N/mm² is seen for the shortest wavelength rail profile examined, 20 mm. This is, however, only slightly higher than the 0.5 N/mm² wear index for a perfectly smooth wheel and rail with only parametric excitation. Clearly with low longitudinal creepage of 0.1%, no lateral and spin creep and no very short wavelength or broadband roughness to act as stress concentration factors, mild wear is the only mechanism present.

7.6.2 Wear of rail with initial broadband roughness

Figure 7.15 shows the material removed from a short 40 mm length of rail with broadband roughness. Note that although the element size used in the contact model is 0.1 mm, results are presented here with 1 mm spacing as the original roughness profile information is normally available at 1 mm intervals. More material is removed from locations with peaks in the roughness profile than from locations with troughs in the roughness profile, suggesting that roughness levels at least at short wavelengths will reduce after many wheel passages. Although the difference in material removed along the rail shows a great deal of variation, the actual change in the rail profile at each location resulting from a single wheel passage is very small.

The material removed from each location in each time-step is a function of the slip velocity and the tangential stress. Both these parameters vary throughout the slip zone. The slip velocity is at a maximum at the trailing edge of the contact, while the tangential stress is highest for a smooth contact at either the front edge of the slip zone or at the centre of the contact. For a smooth rail this results in a parabolic distribution of material removed in the slip zone, as shown in Figure 7.1, where the most material is removed from the centre of the slip zone. When the rail is rough, the asperities in the contact result in higher normal stresses and consequently higher tangential stresses at the roughness peaks. The distribution of material removed then depends on the surface profile in the contact zone, and more material tends to be removed at roughness peaks where the tangential stresses are higher. This means that short wavelengths of roughness will tend to be worn away when using a non-Hertzian contact model to calculate the wear.



Figure 7.15 Material removed by a single wheel pass on rail with broadband roughness and normal force from interaction model: — material removed; — — rail profile (not to scale).

The maximum wear index seen in the three sleeper bays examined for this low-level broadband roughness case is 0.75 N/mm^2 . This is higher than the maximum wear index with a perfectly sinusoidal rail profile, but still well below the change to severe wear at a wear index of 10.4 N/mm^2 (Table 7.1). This result again confirms that with low creep and low roughness levels, mild frictional abrasion is the dominant wear mechanism.

Increasing the roughness levels but for the same constant 0.1% longitudinal creep results in an increase in the maximum wear index. For example, TSI level roughness with a freight

wheel and the same longitudinal creep leads to a wear index typically around 0.77 N/mm². Roughness that is 10 dB higher than the TSI level leads to a wear index typically around 1.4 N/mm². Even with 0.3% creep and roughness 10 dB above the TSI limit curve, the wear index is below 3 N/mm² and only mild wear is predicted.

Clearly for purely longitudinal creep even with higher level roughness, the dominant wear mechanism remains mild wear even when a non-Hertzian model is used to determine the stress distribution and slip velocity in the contact patch.

7.6.3 Wear assuming Hertzian contact

The following results in Figure 7.16 and Figure 7.17 have been determined using the same variational method, model and input forces as those presented in Figure 7.14 and in Figure 7.15. However, to represent the contact as Hertzian, the roughness of the rail has been disregarded in calculating the stress distribution. At each position along the rail, the length of the contact patch varies depending on the variation in the normal force, but the rail profile is assumed to be smooth and unvarying. The Hertzian assumption of smooth surfaces leads to very different results in terms of the predicted material removed from the rail. For the rail with a 20 mm wavelength sinusoidal profile (Figure 7.16(a)), the Hertzian model predicts the maximum material is removed from the troughs of the initial profile, so the corrugation worsens under many wheel passages. However, with the non-Hertzian model more material is removed from the peaks than from the troughs. For the 40 mm wavelength case (Figure 7.16(b)) the maximum material removed predicted by the Hertzian model occurs near the middle of the down-slope of the peak, whereas with the non-Hertzian model, the maximum material is removed is, again, closer to the peak in the profile. When the profile wavelength increases to 80 mm there is less difference between the predictions from the two model types, although there is more variation in the material removed along the rail with the non-Hertzian model, so changes in the rail profile would occur more rapidly.

When the roughness profile of the rail is low-level but broadband (Figure 7.17), the Hertz model does not allow the roughness to produce any stress concentrations throughout the contact, so in effect the variation in the material removed along the rail is averaged out throughout the length of the contact. With a Hertzian model, the rapid variations in the amount of material removed along the rail shown in Figure 7.15 are not present, suggesting

that short wavelength roughness will not be predicted to be worn away as much with a Hertzian model as with a non-Hertzian model.



Figure 7.16 Material removed by a single wheel along rail with sinusoidal profile, (a) 20 mm wavelength, (b) 40 mm wavelength, (c) 80 mm wavelength: —— material removed assuming Hertzian contact; · · · · · · material removed with non-Hertzian model; — — rail profile (not to scale).



Figure 7.17 Material removed by a single wheel along rail with broadband roughness assuming Hertzian contact: ______material removed; _____ rail profile (not to scale).

7.7 Discussion on wear rates and regimes

In typical wheel-rail rolling contact for tangent track, the interaction force between wheel and rail does not reach high enough levels to move the contact into the severe wear regime. Mild wear only is predicted by this model for cases with low-level broadband roughness and small amounts of longitudinal creep. Although this model allows for the possibility of multiple wear mechanisms in different parts of the contact patch, in practice only mild wear is present and the same results could be obtained from using a single wear coefficient.

The most significant finding in this chapter (confirming the conclusions of Xie and Iwnicki [2008a,b,c]) is the difference in the prediction of material removed along the rail if a Hertzian contact representation is used in place of a more comprehensive non-Hertzian one. The variation in the stress distribution in the contact between the two models is significant even when the roughness has a very low level. Roughness asperities act as stress concentrators. This means that more material tends to be removed from roughness peaks than dips. In particular, short wavelength roughness is predicted to wear away when a non-Hertzian model is used, but not if a Hertzian model is used. Clearly it is important to consider the effects of surface roughness on the contact conditions when predicting the development of acoustic roughness over time.

8 VELOCITY-DEPENDENT FRICTION MODEL

8.1 Introduction

In Chapters 5 and 6 the variational method used by Kalker [1990] for the determination of the normal and tangential stress distribution in the contact patch has been described and investigated. This method allows the determination of the stick and slip zones and the slip velocity in rolling contact between rough surfaces. This can then be used to predict the wear of the surfaces. The method used to calculate the wear and to consider different wear mechanisms has been described in Chapter 7.

A limitation of the method used in the previous chapters to calculate the tangential stress distribution is the assumption of a constant coefficient of friction. In fact the friction coefficient is not constant, but depends on contact parameters such as temperature and slip velocity. In this chapter, a velocity-dependent friction law is introduced into the non-Hertzian contact model. This has not been done previously. The resulting tangential stress distribution and slip velocity in rolling contact are examined and compared with the results calculated using a constant coefficient of friction.

In Chapter 7 it has been seen that, although there are several different wear mechanisms that could contribute to the development of acoustic roughness, when a constant coefficient of friction is used, only mild wear is seen, despite the stress-concentrating effect of the roughness asperities at some locations. The effect of the velocity-dependent friction law on the wear index and likely wear mechanisms is also discussed in this chapter. The roughness growth rate function initially developed by Hempelmann and Knothe [1996] is used here as a means of comparing the wear predictions from the middle three sleeper bays of the track model for different contact models and input parameters.

8.2 Background studies in force-friction relationships

A simple description of the friction-force relationship is contained in the theory known as Amonton's law of friction [see Bowden, 1974]. This theory states that under sliding the friction force is proportional to the normal load, and that friction between bodies is independent of their size. The constant of proportionality is known as the friction coefficient μ . The consideration of different static and dynamic friction coefficients μ_s and μ_d is known as Coulomb's theory [see Bowden, 1974]. This theory explains the difference in force required to start an object sliding compared with the force required to maintain sliding. Coulomb believed that for metal-on-metal contacts, the velocity of sliding makes very little difference to the dynamic friction coefficient [Kragelskii, 1965].

In more recent times, tribological research has examined the mechanisms behind friction and obtaining understanding of the effect of elastic deformation, plastic deformation, adhesion of bodies and lubrication. A large amount of work has been carried out in the field, both experimentally and theoretically. The work of Bowden [1974] provides a good introduction to the field. Kragelskii's book 'Friction and Wear' [Kragelskii, 1965] describes in more detail the historical development of different theories and in particular the work carried out by Russian researchers. Martins *et al.* [1990] have extensively reviewed studies into static and dynamic friction. Most experimental work on dynamic friction coefficients and stick-slip mechanisms involves pure sliding rather than rolling contact, and measurements involve the tangential force in an overall sense rather than the force distributed throughout the contact, as the distributed force is extremely difficult to measure.

For very low sliding speeds there is some evidence to suggest that the friction coefficient increases with increasing speed, particularly for contact between different materials. This has been explained by Kragelskii and others [Martins *et al.*, 1990] as resulting from the deformation of interface asperities, which would require more force to deform at higher speeds. For hard metal pairs, however, this effect is thought to be negligible. For sliding speeds greater than about 10^{-5} m/s, the friction coefficient tends to decrease with increasing sliding velocity. This phenomenon is commonly attributed to thermal effects, for example by Kragelskii [1965].

Martins *et al.* [1990] conclude that even so-called steady sliding is inherently unstable, and can lead to self-excited stick-slip oscillations of very high frequency. The apparent decreases in dynamic friction with increasing sliding speed may then be the result of increasing amplitudes of these high-frequency stick-slip motions as the sliding speed increases.

8.3 Force-friction relationships in railway contact mechanics

In the field of railway contact mechanics, friction research has been motivated largely by a need to understand the adhesion limit, or the maximum tangential load that can be supported in the contact area in rolling before the onset of full sliding. The adhesion limit is of interest for the acceleration and braking of railway vehicles as it determines the maximum torque that can be applied to the wheel.

In the absence of a detailed knowledge of the stress distribution in the contact patch, the relationship between longitudinal creep and overall tangential force is used to simulate adhesion limits for accelerating or braking vehicles. This creep-force relationship is also often used in vehicle motion simulations in place of a detailed model of the stress distribution, as it is much faster to calculate. The creep-force theory of Shen *et al.* [1983] has been used extensively for this purpose.

Measurements of creep-force relationships for railway rolling applications, *e.g.* Zhang *et al.* [2002] and Polach [2005], demonstrate that with high creep, the capacity of the wheel-rail contact to support tangential loads without fully slipping is reduced. As discussed briefly in Section 6.2, this falling creep-force relationship is attributed to variations in the friction coefficient. The dynamic friction coefficient is dependent on the sliding velocity between the surfaces – higher slip velocities result in increased temperatures in the contact and lower friction coefficients. For high creep, a velocity-dependent friction law causes the slope of the creep-force curve to become negative (see Figure 6.1).

8.4 Velocity-dependent friction law

In this work the velocity-dependent friction law to be implemented is given by

$$\mu = \mu_s \left[\frac{50}{100 + \left| \dot{s}^2 \right|} + \frac{0.1}{0.2 + \left| \dot{s} \right|} \right]$$
(8.1)

This is taken from Xie *et al.* [2006] with a static coefficient of friction $\mu_s = 0.3$. To reduce computational effort, Xie *et al.* [2006] used a simplified version of this friction law in their work, with the same static coefficient of friction μ_s but replacing the friction curve with a linear relationship for slip velocities up to 0.38 m/s. At greater slip speeds, they applied a constant dynamic friction coefficient of $\mu_d = 0.181$. This simplification is not made here, instead the relationship between velocity and friction is variable at all speeds. The resulting relationship between friction coefficient and slip velocity is shown in Figure 8.1 for $\mu_s = 0.3$.



Figure 8.1 Velocity-dependent friction curve from Equation (8.1).

The work of Xie *et al.* [2006] is based on a modified version of FASTSIM to solve the contact problem with a velocity-dependent friction law, following the work of Giménez *et al.* [2005]. The implementation of the velocity-dependent friction law in FASTSIM required the elimination of a derivative term to ensure mathematical stability of the resulting stress distribution. In this work the variational method described by Kalker [1990] is applied to the contact problem and no such mathematical difficulty is experienced.

The implementation of this velocity-dependent friction law in the variational method requires an iterative loop, shown in Figure 8.2. With a constant friction coefficient, the slip velocity does not affect the normal or tangential stress distribution. With a variable friction coefficient, the tangential stress distribution depends on the slip velocity and vice versa, so iteration is required at each location along the rail. Firstly the normal stress distribution is calculated as before. To begin a rolling contact analysis, the friction limit is initially set using the static friction coefficient. Once rolling is underway, the friction limit is calculated based on the slip velocity distribution at the previous time-step. The tangential stress distribution and slip velocity are then calculated as preliminary estimates for the current time-step. The friction limit throughout the contact area is then re-calculated and this revised friction limit is used to obtain the tangential stress and slip velocity. It has been

found that ten iterations of the tangential stress calculation are required at each time-step to ensure that the system has converged to a solution at each location along the rail.



Figure 8.2 Iteration loop for inclusion of velocity-dependent friction coefficient.

8.5 Effect of velocity-dependent friction on tangential stress and slip velocity

The variational method requires the wheel to travel over the rail for some distance before it will converge to a steady-state, even if the normal force is constant and the wheel and rail are assumed to be perfectly smooth. Figure 8.3 shows the total tangential force Q summed over all the elements in the contact patch for 'steady-state' rolling at a speed of 30 m/s with an assumed constant creepage of 0.1%. This has been calculated using the two-dimensional rolling contact model. It is clear that with a velocity-dependent friction coefficient, the system does not converge to a single constant value of Q, but instead develops a high-frequency stick-slip oscillation shown in more detail in Figure 8.4. In general, a higher total tangential force is transmitted in the contact in steady-state rolling if the friction coefficient is assumed to be constant rather than velocity-dependent.

An element length of 0.1 mm has been used in the potential contact area as in Chapter 7. Very small elements are needed to model the high-frequency stick-slip behaviour. The wavelength of this stick-slip oscillation is around 1.2 mm which is much shorter than the length of the contact. It is also shorter than the typical wavelengths of interest for acoustic roughness, since variations in roughness on this scale are filtered by the contact patch and do not excite the wheel-rail system causing noise. However, the instability in otherwise steady rolling is of interest as it may also lead to variations in the roughness at longer wavelengths.



Figure 8.3 Total tangential force in the contact with 0.1% creep: — — constant friction coefficient; — velocity-dependent friction coefficient.



Figure 8.4 Detail of stick-slip instability in otherwise steady rolling contact with a velocity-dependent friction coefficient.

The saw-tooth shape of the variation in the total tangential force supported in the contact area is typical of stick-slip vibration. The constant slip zone seen in the results for steady rolling with a constant friction coefficient is no longer present. To visualise this, start from a point immediately after a peak in the slip velocity, with some of the contact area in a state of stick and some in a state of slip, but with the slip velocity decreasing. This corresponds to Position 1 in Figure 8.5. As the slip velocity decreases, the friction limit that defines the maximum value of tangential stress in the contact increases. Eventually the slip velocity becomes almost zero and the entire contact approaches a state of stick (not just the leading edge, which is in a state of stick throughout). The total tangential stress

builds up until it is relieved by a sudden slip at the trailing edge of the contact, shown in Position 6 in Figure 8.5.



Figure 8.5 Effect of velocity-dependent friction coefficient on slip velocity in the contact patch, shown at six locations along the rail 0.2 mm apart.

Figure 8.6 shows the resulting range of variation in the tangential stress distribution throughout the contact area. Moving through positions 1 to 5, the slip zone at the trailing edge shrinks steadily as the slip velocity decreases and the friction limit increases. The slip phase is initiated when the slip zone is vanishing, as at this position the friction limit is at its maximum and the total tangential stress can build up to its maximum before being relieved. The tangential stress is then at its minimum in Position 6, as, at this position, the stress has been relieved by the sudden slip at the trailing edge of the rolling contact. The slip zone is longest at this position.

It can be seen in Figure 8.6 that including a velocity-dependent friction coefficient results in a periodically jagged variation in the tangential stress in the stick zone of the contact. The reason for this variation is not known. It does not appear to be directly related to the size of the elements used or the distance rolled in each time-step, as it repeats regularly along the length of the stick zone with a period of 1.1 mm. Figure 8.6 shows the stress distribution at successive locations 0.2 mm apart, and the peaks are 0.2 mm apart,

suggesting the locations of higher stresses in the stick zone are stationary as the wheel rolls along. It is possible that the tangential stress in the stick zone does not fall smoothly to zero at the leading edge of the contact. However, in the stick zone, the slip velocity is zero so the friction coefficient should be constant in those elements, within the numerical tolerance of the results. Another possibility is that the stick-slip oscillation in the trailing edge of the contact could be affecting the stress distribution in the leading stick zone in some way. In any case, for this work, the stress distribution in the stick zone is not of interest as it does not affect the wear of the rail. Therefore the reason for the periodically jagged variation in the tangential stress in the stick zone seen in Figure 8.6 remains an open question^{*}.



Figure 8.6 Effect of velocity-dependent friction coefficient on tangential stress in the contact patch, shown at six locations along the rail, 0.2mm apart.

Figure 8.7 and Figure 8.8 compare the slip velocity and tangential stress distribution in the contact patch from a velocity-dependent friction model with a constant friction model. The inclusion of a velocity-dependent friction coefficient results in lower slip velocities at most locations along the rail, when the stick-slip mechanism is sticking more than slipping. At locations where slip occurs suddenly, the slip velocity is much higher and a greater

^{*} N.B. An apparently similar effect is discussed by E.A.H. Vollebregt in his paper: Refinement of Kalker's rolling contact model. In *Proceedings of the 8th International Conference on Contact Mechanics and Wear in Rail/Wheel Systems*, pages 149-156, 2009.

proportion of the contact zone slips than is the case if a constant friction coefficient is used. The tangential stress (Figure 8.8) is generally lower in the slip zone of the contact area if a velocity-dependent friction coefficient is used.



Figure 8.7 Slip velocity in the contact patch: ——*steady-state with a constant friction coefficient;* — — *range of results with a velocity-dependent friction coefficient.*



Figure 8.8 Tangential stress in the contact patch: ——steady-state with a constant friction coefficient; — — range of results with a velocity-dependent friction coefficient.

8.6 Effect on creep-force relationship for increasing longitudinal creep

The stick-slip oscillation arising in otherwise steady rolling contact means that the overall tangential force in the contact is not necessarily a smooth function of the overall creep. Instead, the total tangential force has a range of possible values for a particular level of creep. The analysis of Section 8.5 has been repeated for a series of creep values from 0.1% up to 0.5%, which is expected to cover the peak in the creep-force relationship with a falling friction coefficient. The rolling velocity is again 30 m/s. Figure 8.9 shows the mean total tangential force transmitted by the contact, as well as the maximum and minimum values calculated from 50 mm (or 500 calculation points) of 'steady-state' rolling.

Note that when the creep is low, ten iterations of the velocity-dependent tangential stress calculation loop is sufficient to reach convergence and the resulting stick-slip oscillation in otherwise steady rolling contact might be described as a 'stable' instability, as it repeats predictably along the rail with a reasonably steady saw-tooth shape. However, as the longitudinal creep increases, the stick-slip oscillation becomes more unstable. This is reflected in the increasing variation in the range of values of total tangential force calculated for the different creep values and shown in Figure 8.9. Although the calculation can be performed for these higher creep levels, it is difficult to assess how realistically the results resemble the real behaviour in the time domain of such an unstable system. Despite this, the average creep-force relationship shown in Figure 8.9 seems to be realistic and to match the expected shape for tractive rolling contact with velocity-dependent friction. The minimum velocity-dependent creep-force relationship converges to the creep-force line corresponding to the velocity-dependent friction law (as shown in Figure 8.1) at the rolling speed of 30 m/s.

The mean tangential force shows an optimum or maximum adhesion at a creep of around 0.25%, although for higher values of creep the range of values calculated is quite large. In general, for increasing creep values less than 0.25%, the creep-force relationship increases steadily, albeit with regular oscillations as a result of the stick-slip mechanism. The part of the contact at the leading edge remains in a state of stick at all times. For higher creep values the overall force transmitted by the contact is much more unstable, with some periods of full slip predicted in the contact. Compared with a model with a constant friction coefficient, the optimum adhesion with velocity-dependent friction occurs on average at a

much lower creep value, and the average total tangential stress transmitted is around 60% of the optimum adhesion for constant friction.

For the low value of creep of 0.1% often assumed in roughness growth prediction calculations, there is very little difference in the creep-force relationship between the model with a constant friction coefficient and that with a velocity-dependent friction coefficient. This confirms that the simple model is adequate for vehicle motion simulation in cases with low creep, where the total tangential force is of interest, rather than its distribution throughout the contact patch. However, for predictions of wear, the distribution of tangential stress and slip velocity are crucial. It has been shown here that the stress distribution and slip velocity are significantly modified by the inclusion of a velocity-dependent falling friction coefficient, even for low creep.



Figure 8.9 Creep-force relationship: $\cdots \cdots \cdots$ constant friction; ——average with velocity-dependent friction; — — maximum and minimum with velocity-dependent friction; $-\cdot - \cdot - \cdot$ from Equation (8.1) for speed 30 m/s and 100 kN normal load.

8.7 Effect of velocity-dependent friction on wear prediction

The inclusion of velocity-dependent friction clearly affects the tangential stress distribution and the slip velocity in the contact patch. It is therefore likely that the pattern of wear of the system will also be different with velocity-dependent friction. For the otherwise steadystate rolling cases examined so far, the variable friction model has resulted in shortwavelength effects, which are not necessarily important for roughness growth at acoustic wavelengths. In transient rolling with rough surfaces differences may arise in the wear prediction from constant and velocity-dependent friction models.

Rail roughness profile information is normally available at 1 mm intervals, and the interaction force model returns data at 1 mm intervals. These results are interpolated before applying the rolling contact model with an element size of 0.1 mm (see Section 6.7.1 for discussion on the element size required). The material removed along the rail can therefore be examined using results from every 0.1 mm along the rail or alternatively with results at 1 mm spacings. If the friction coefficient is constant, there is very little difference between these results, but with a velocity-dependent friction coefficient, short wavelength effects are very noticeable. Examples are shown in Figure 8.10 for the material removed from a corrugated rail and in Figure 8.11 for the material removed from a rail with low-level broadband roughness. The rapid variation in the amount of material removed is caused by the stick-slip oscillation in the contact patch. Although it is interesting theoretically to model this instability in a partially slipping contact, the very short wavelength variation is outside the range of interest for acoustic broadband roughness growth. The rail profiles used in this work do not include any information on roughness variation at these short wavelengths. Therefore results in subsequent figures show the wear depth sampled at 1 mm intervals only.




Figure 8.11 Material removed by a single wheel along rail with broadband roughness. Velocity dependent friction model: ——— results at 1 mm intervals to match initial roughness data; — — – raw results at 0.1 mm spacing.

8.7.1 Wear of initially rough rail with various contact models

The pattern of material removed from a corrugated rail was examined in Section 7.6 using the non-Hertzian model with constant friction, compared with the material removed if Hertzian contact is assumed. Figure 8.12 shows the same results along with those from the velocity-dependent friction model. The wear depth with velocity-dependent friction is less than that with the other models since the overall tangential force in the contact patch at each location is less. The wear depth is no longer smooth and sinusoidal, although the overall shape of the curve over one wavelength of the sinusoidal rail profile is similar for the constant and velocity-dependent friction models.

An example wear pattern for a system with broadband roughness is shown in Figure 8.13. Here the results from the constant friction model and the velocity-dependent friction model are again similar although the amount of material removed in an overall sense is less with the velocity-dependent friction law. The pattern of material removed remains dominated by the initial roughness profile, as was seen in Figure 7.15 where the peaks in the wear depth correspond to the asperities in the initial roughness. To examine the differences between the two models more quantitatively across the whole three sleeper bays included in the analysis, the results need to be examined in the frequency (or wavelength) domain in terms of roughness growth rate spectra.



Figure 8.12 Material removed by a single wheel along rail with a sinusoidal profile: —_____non-Hertzian model with velocity dependent friction; — — ____non-Hertzian model constant friction; … Hertzian model.



8.8 Rail roughness growth rates

It is difficult to draw any general conclusions about the development of acoustic roughness from the spatial history of wear along the rail for different contact models. In order to examine the effect of the contact model (and also the vehicle type and track parameters) on roughness development, it is of interest to present the results in the form of a roughness growth rate spectrum (strictly a transfer function rather than a spectrum). This roughness growth rate can be compared with the dynamic interaction force spectrum (described in Chapter 4) and also the spectrum of the initial roughness profile.

In the remainder of this work and, in particular, in the case studies presented in the following chapter, roughness growth results are presented as a global rate. This rate is independent of the number of wheel passages and the initial roughness level. The concept of a 'global growth rate' is described by Hempelmann and Knothe [1996]. It allows the comparison of results for roughness or corrugation predictions that are calculated for different input parameters and different reference states. A global growth rate is a single transfer function providing a mean value for the prediction in each one-third octave wavelength band for the full length of track included in the investigation, in this case, three sleeper bays. The global growth rate ψ is calculated from the initial roughness amplitude A_0 and the final roughness amplitude A_n in each wavelength band k after n wheel passages as [Hempelmann & Knothe,1996]:

$$\Psi_{k} = \frac{1}{n} \ln \left(\frac{A_{k,n}}{A_{k,0}} \right) \tag{8.2}$$

If the roughness level in a particular wavelength band has increased after the passage of a wheel or wheels, the roughness growth rate is positive in that wavelength band. A negative roughness growth rate in a wavelength band indicates that the roughness level tends to decrease in that wavelength band.

The roughness growth rate concept can be used to compare different contact models by carrying out the calculation for each model type using the same input parameters. Figure 8.14 is an example of the roughness growth rate calculated using three different contact models for the same inputs, in this case a single freight wheel rolling over a track with low-level broadband roughness.

If a Hertzian contact model is used, as shown in Figure 8.14, the roughness growth rate is slightly negative or around zero for wavelengths longer than 40 mm. The maximum roughness growth rate occurs in the one-third octave wavelength bands centred around 20 to 25 mm. For a freight train at 29.44 m/s, this wavelength range includes the pinned-pinned resonance at 1050 Hz. So if Hertzian contact is assumed in this case, roughness is predicted to grow at wavelengths around that corresponding to the pinned-pinned resonance.

However, if a non-Hertzian contact model is used, the roughness growth rate is negative throughout the whole wavelength range. Short wavelength roughness is predicted to decrease dramatically. The roughness growth rate function with a constant friction coefficient is relatively smooth. The roughness growth rate function with a velocity-dependent friction model is highly variable and suggests roughness may grow slightly in the 32 mm and 125 mm wavelength bands in this example, but will otherwise decrease.



Figure 8.14 Example roughness growth rate for a single freight wheel on a track with lowlevel broadband roughness: ———— non-Hertzian model with velocity dependent friction; — — — non-Hertzian model constant friction; · · · · · · · Hertzian model.

The initial roughness profile affects the stress distribution significantly when a non-Hertzian contact model is used; therefore general conclusions cannot be drawn from a single case viewed in isolation such as shown in Figure 8.14. Figure 8.15 shows the roughness spectra and corresponding dynamic interaction force spectra for five different initial roughness profiles of similar spectral level. The roughness profiles have been generated over the full length of the track included in the interaction force model. However, the wear is only calculated along a 1.8 m length, or three sleeper bays. Therefore, the roughness, force and roughness growth rate spectra shown calculated over these three sleeper bays display higher variability at longer wavelengths, as there is less information on the long-wavelength behaviour held in a short sample length.



Figure 8.15 (a) Initial roughness spectra and (b) dynamic normal interaction force over middle three sleeper bays for five different initial rail profiles corresponding to the results shown in Figure 8.16. TSI limit spectrum.

Figure 8.16 presents the resulting roughness growth rates calculated using the two non-Hertzian contact models for these five different initial roughness profiles. Figure 8.16(a) suggests that if a constant friction model is used, similar roughness growth rates are obtained from different initial roughness profiles across most of the wavelength range. At longer wavelengths there is more variation in the roughness growth rate results than at short wavelengths, reflecting the fact that the initial roughness spectra and dynamic interaction force spectra (Figure 8.15(a) and (b)) also display more variation at longer wavelengths. Figure 8.16(b), calculated using velocity-dependent friction, shows much more variability across the whole wavelength range in the roughness growth rate results for the different initial roughness profiles. In general, roughness levels at wavelengths shorter than around 25 mm are predicted to decrease rapidly with the non-Hertzian, velocity-dependent friction model. Longer wavelength roughness may grow slightly in some cases at some wavelengths, or decrease slightly, or remain relatively steady at around zero on the roughness growth rate spectrum.



Figure 8.16 Roughness growth rates for a single freight wheel on track with five different initial low-level broadband roughness profiles of similar spectral level. (a) Constant friction model; (b) velocity dependent friction model.

8.9 Wear index and mechanisms with velocity-dependent friction

The maximum wear index seen over the three sleeper bays with low-level broadband roughness and a constant friction model was 0.75 N/mm² (see Section 7.6.2), well below the transition from mild to severe wear. The corresponding results with the same inputs but with velocity-dependent friction lead to a much higher maximum wear index of 4.9 N/mm² (still in the mild range). Higher initial roughness levels lead to higher wear indices. A test case calculated with 0.1% longitudinal creep, roughness 10 dB above the TSI limit and velocity-dependent friction gives a maximum wear index of 9.6 N/mm² which is approaching the transition to severe wear at a wear index of 10.4 N/mm².

Due to the stick-slip instability of the system with velocity-dependent friction at high creep levels, no effort is made here to predict the wear index in cases with higher roughness levels and high longitudinal creep. However it is likely that, with combined roughness levels (including both the wheel and rail roughness) more than 10 dB above the TSI limit, at some locations, the wear regime is likely to be in the severe range. In this range the wear mechanism is not purely frictional wear and may include plastic deformation or ratcheting effects. An example of the normal stress distribution at one location along a rail with this roughness level was presented in Figure 6.20, and the peak normal stress of 1700 MPa indicates that some plastic deformation is likely.

When the combined roughness of the wheel-rail system is low, even with a velocitydependent friction model, the wear of the system is attributed solely to mild frictional wear. If a constant friction model is used to predict the wear of the system, only mild frictional wear is predicted even for higher roughness levels. When velocity-dependent friction is included in the model, higher roughness levels and higher creep are likely to result in more severe wear rates at some locations. The combined wheel and rail roughness level that might cause this transition to severe wear is a realistic level around 10 dB above the TSI limit, which is present on many tracks in service. For example the roughest rails recorded by Verheijen [2006] are at least 20 dB above the TSI limit at some wavelengths, as are some of the roughest wheels.

8.10 Discussion

It has already been seen in Chapter 7 that the type of contact model used can have a significant effect on the prediction of the development of rail-head acoustic roughness.

Hertzian and non-Hertzian contact models give very different results. Other authors studying non-Hertzian contact have reported similar findings, in particular, the recent efforts by Jin *et al.* [2006] and Xie and Iwnicki [2008a,b,c].

From this chapter, it is clear that the friction law used in a non-Hertzian contact model can also affect the prediction of the development of roughness. In the examples shown in Figure 8.16, assuming constant friction throughout the contact leads to the prediction that low level roughness consistently does not grow in any wavelength band, and that rails therefore become smoother with the passing of many wheels. However, when a velocitydependent friction law is included in the model, the roughness growth rate is more variable. It is positive in some instances in some wavelength bands, mostly at longer wavelengths. Roughness wavelengths shorter than around 25 mm are predicted to decrease in roughness in the examples shown here.

The roughness growth rate function allows the comparison of results with different input parameters. In the next chapter, a series of case studies are presented to examine the development of acoustic roughness for different input parameters, including different track components, vehicle types and mixed traffic.

9 APPLICATIONS AND CASE STUDIES

9.1 Introduction

In this chapter, the model described in the preceding chapters is applied to a series of case studies to examine the development of rail-head acoustic roughness. Results are presented in the form of roughness growth rate functions. This allows comparison between the different contact model theories, as well as different vehicle and track scenarios. Roughness growth rates are presented for freight, regional and high-speed vehicle types on tracks with soft and stiff rail pads.

When using this model, the material removed from the wheel-rail interface can be calculated for a particular vehicle type on a particular track with a particular roughness profile. The model does not directly consider the effects of mixed traffic on the roughness development. However, if the approximate proportion of each vehicle type is known, a mixed traffic roughness growth rate can be derived from the results for each individual vehicle type. Roughness growth rate results calculated over three sleeper bays for various different initial rail profiles can be averaged to get an improved approximation of predicted roughness development over longer sections of track. For conciseness, where roughness growth rates are presented in this chapter as averages over five different initial roughness profiles, the individual results are included in Appendix B. Table B.1 lists all the roughness growth rate cases considered.

The cases studied here examine the roughness development predicted by different contact theories and friction laws. The three contact theories considered are Hertzian contact, non-Hertzian contact with a constant friction coefficient and non-Hertzian contact with a velocity-dependent friction law. Of these three, Hertzian theory has historically been most widely used to model rail wear (including for the purpose of predicting the development of corrugation). However, for broadband roughness development, including the stressconcentrating effects of surface roughness in a non-Hertzian wear model is essential. Finally the effect of including velocity-dependent friction in a non-Hertzian contact model is examined and the findings compared with those from the other contact theories. This contact theory has not been used previously to predict wheel or rail roughness development. For most of the cases examined here, a constant longitudinal creep of 0.1% is assumed. Some examples are also presented examining the roughness growth rate for accelerating or braking wheels with constant torque, using the non-Hertzian model with constant friction coefficient.

Rail dampers affect the wheel-rail interaction force spectrum, as shown in Chapter 4. Previous work by Croft *et al.* [2009] using a Hertzian contact model has also predicted that the rail dampers lead to reduced roughness development at wavelengths around the pinnedpinned resonance. Here the non-Hertzian model with velocity-dependent friction is used to examine the effect of rail dampers on the roughness growth rate.

Finally, some measured roughness spectra from the Silence project test site near Gersthofen are presented. These measurements have been repeated over several years by Deutsche Bahn AG on track with two different rail pad stiffnesses, with and without rail dampers installed. It is not possible to compare directly the predicted and measured results, as the amount of material removed from the rails at the site cannot be measured. Nevertheless some comparisons between the model and measurements can be made.

Throughout this chapter, 'soft' rail pads have a stiffness of 200 MN/m, while 'stiff' rail pads have a stiffness of 800 MN/m. The model parameters, as listed in Table 3.3 for the track and in Table 4.1 for the vehicle, have been chosen to match as closely as possible the track and traffic at the Gersthofen site.

9.2 Roughness growth rates with a Hertzian contact model

Results for the Hertzian contact model have been calculated using the variational method, but neglecting the roughness of the rail in the contact model used for the wear calculation. This means that the normal force calculated by the interaction force model is the only varying input to the contact and wear model. The friction coefficient is constant, $\mu = 0.3$.

The importance of including more than one wheel in the interaction force model has been examined in Section 4.4.7. If the rail pads are soft, the track decay rate is low across part of the frequency range of interest. Successive wheels are coupled via the rail, leading to a different force spectrum for a single wheel model when compared with the results from the four wheels of the multiple-wheel model. This effect is less significant if the rail pads are

stiff. However, when calculating the interaction force for track with soft rail pads it is important to include more than one wheel in the model of the vehicle.

The calculation times for the wear module of the model are very long. Using a 2.2 GHz processor, a calculation time of up to 20 hours is required for each case with a single wheel if the friction coefficient is assumed to be constant. To calculate a set of results for two track types, three vehicle types and five different initial roughness profiles requires around 600 hours of CPU time. Using a four-wheel vehicle model quadruples this calculation time. It is therefore of interest to check the validity of a single wheel model compared with a multiple wheel model in terms of the wear calculation. Results are presented here firstly from a single wheel model for both rail pad stiffnesses, then from a four-wheel model for the soft rail pad case.

9.2.1 Single wheel vehicle model results

Figure 9.1 shows the roughness growth rates calculated for each vehicle and track type using a single wheel vehicle model if Hertzian contact is assumed. These are the average roughness growth rates calculated over five different initial roughness profiles. Results for the individual roughness profiles are included in Appendix B.

Figure 9.1(a) shows results for track with soft rail pads. The roughness growth rate for all three vehicle types is positive in the 40 mm one-third octave wavelength band and for shorter wavelengths. For longer wavelengths, the roughness growth rate is around zero or very slightly negative. This implies that short wavelength roughness will increase over time, while wavelengths longer than 40 mm will remain stable or decrease slightly. Roughness is predicted to grow fastest for the freight and regional vehicle types at wavelengths of around 20 to 25 mm. For the high-speed train, roughness is predicted to grow fastest in the 20 to 31.5 mm wavelength bands.

The roughness growth rates for track with stiff rail pads (Figure 9.1(b)) show a similar trend with high roughness growth predicted for wavelengths shorter than 40 mm. Again, roughness levels at wavelengths longer than 40 mm are not predicted to grow or decrease significantly, although the freight and regional trains have a slightly positive roughness growth rate in the wavelength range from 40 mm to 80 mm that is not seen in the results with soft rail pads. There are other small differences in the results for each vehicle type,

with the high-speed vehicle showing high roughness growth rates at slightly longer wavelengths than the regional and freight vehicle types, but then negative roughness growth in the 40 to 80 mm wavelength bands.



Figure 9.1 Average roughness growth rate from a single wheel model, Hertzian contact: (a) soft rail pads; (b) stiff rail pads. ——freight; — — regional; · · · · · · high-speed. Lines 1,2,3 correspond to pinned-pinned resonance for freight, regional and high-speed trains respectively.

Effects at various wavelengths are expected as the result of the different speeds of the vehicles. However all the vehicle types show a drop in roughness growth rates for wavelengths shorter than around 12 mm. If the wavelengths of high roughness growth were purely determined by the pinned-pinned resonance and the speed of the vehicle, then they would appear in the 25 mm wavelength band for the freight vehicle, in the 31.5 to 40 mm bands for the regional vehicle and in the 40 mm band for the high-speed vehicle, shown by vertical lines in Figure 9.1(b). These results are not so clearly staggered by wavelength. In fact, negative roughness growth is predicted for the high-speed train type

on stiff rail pads in the wavelength band corresponding to the pinned-pinned resonance. These results suggest that a constant-wavelength mechanism for roughness or corrugation growth may be present as well as the constant-frequency mechanism associated with the pinned-pinned resonance of the track. Other authors have concluded (also using Hertzian contact models) that constant wavelength effects are important for corrugation growth, including J.B. Nielsen [1999], Hoffmann and Misol [2007], Ciavarella and Barber [2008], Afferante and Ciavarella [2009] and Knothe and Groß-Thebing [2008].

9.2.2 Four-wheel model results for soft rail pads and Hertzian contact

The average roughness growth rates over five roughness profiles obtained from a model with four wheels are presented in Figure 9.2 along with the single-wheel model results. The differences between the two sets of results occur in the wavelength bands between 16 mm and 63 mm, including the wavelengths at which roughness is predicted to grow significantly. This is the same range in which a difference is apparent in the force spectrum with a multiple-wheel model compared with a single-wheel model (see Section 4.4.7). The results for the freight train, Figure 9.2(a), show two peaks in the roughness growth rate function at 25 mm and at 40 mm whereas the single-wheel result has just one smoother peak, spread between the 20 mm and 25 mm wavelength bands. The differences in the results for the other vehicle types have less effect on the shape of the results in that no new peaks or dips are introduced by the inclusion of the extra wheels. This may be explained by the difference in the wheel spacing on a bogie (Table 4.1). The freight wheels are closer together, 1.8 m apart in a bogie compared with 2.5 m for the regional and high-speed trains, and therefore including more wheels in the freight case has more effect on the results than for the other vehicle types.

Despite the differences, the same observations can be made about the four-wheel model results as have been made for the single-wheel model roughness growth rates. The wavelengths of roughness growth or removal are more or less unaffected by the number of wheels included in the model.



Figure 9.2 Average roughness growth rate, Hertzian contact: (a) freight vehicle; (b) regional vehicle; (c) high-speed vehicle. — single wheel; — four wheels.

9.3 Roughness growth rates with non-Hertzian contact and constant friction

Roughness growth rates are now presented that have been calculated using the non-Hertzian contact model. These use the same initial rail profiles and interaction force histories as the Hertzian cases, but include the rail profile in the determination of the normal and tangential stress distribution as the wheel rolls along the rail. The friction coefficient remains constant, $\mu = 0.3$. It has been seen in Section 9.2 that similar roughness growth rates are obtained from a model with a single wheel and a model with four wheels in the Hertzian contact case. The same test is repeated here for the non-Hertzian contact case.

9.3.1 Roughness growth rates with non-Hertzian contact and a single wheel

Figure 9.3 shows the average roughness growth rates for the individual vehicle types for track with soft rail pads and stiff rail pads in Figure 9.3(a) and (b) respectively. Again the roughness growth rates for each different initial rail profile before averaging are included in Appendix B. The most noticeable difference between these non-Hertzian results and the Hertzian results in Figure 9.1 is that when the roughness of the surfaces is taken into account, the roughness growth rate is negative across the entire wavelength range for both soft and stiff rail pads. There is some variation between the results for different vehicle types, mostly in the 31.5 mm one-third octave wavelength band.



Figure 9.3 Average roughness growth rate for a single wheel, non-Hertzian contact with constant friction: (a) soft rail pads; (b) stiff rail pads. — freight; — — regional; $\cdots \cdots \cdot$ high-speed.

In general, long wavelength roughness is predicted to decrease slightly, while roughness wavelengths shorter than 25 mm are predicted to decrease more rapidly. This finding agrees with the work of Jin *et al.* [2005, 2006] who conclude using a non-Hertzian model

that corrugation will decrease over time. Similarly Xie and Iwnicki [2008a,b,c] predict that both roughness and corrugation will decrease under many wheel passages.

9.3.2 Non-Hertzian contact with multiple wheels

The average roughness growth rates from the four-wheel vehicle running over five different initial roughness profiles are presented in Figure 9.4 for each vehicle type on track with soft rail pads. These demonstrate the effect that including multiple vehicle wheels has on the roughness growth rates, and allow an assessment of the importance of including multiple wheels in the wear model.



Figure 9.4 Average roughness growth rate, non-Hertzian contact with constant friction: (a) freight vehicle; (b) regional vehicle; (c) high-speed vehicle. ——*single wheel;* — — *four wheels.*

Including four wheels leads to a variation in the roughness growth rates in the wavelength bands from 16 mm to 63 mm; the same range for which variation is seen with the Hertzian model. For the freight vehicle type, shown in Figure 9.4(a), the results calculated with a four-wheel model show peaks in the 25 mm and 40 mm one-third octave wavelength bands. The roughness growth rate function is otherwise smooth. For the other vehicle types the differences are less marked, because the wheels are spaced further apart.

Despite these observations, including more than one wheel in the vehicle model does not affect the overall trend of the roughness growth rate function. For each vehicle type examined with this contact model, roughness is not predicted to grow at any wavelength; instead the model suggests roughness should decrease over time, especially at short wavelengths.

Given the finding that the inclusion of multiple wheels does not significantly affect the roughness growth rate, remaining results in this chapter are presented for a single-wheel model only.

9.3.3 Constant creep versus constant tangential stress

In the results presented so far, a constant low value of longitudinal creep of 0.1% has been assumed. The sign of the creep (positive or negative) makes no difference to the calculated roughness development. This is intended to represent wheels that are not accelerating or braking, in normal operation on tangent track. The model can also be used to simulate a powered (or braked) wheel by applying a constant overall tangential force to the contact area rather than assuming constant creepage. This section presents roughness growth rates calculated from an initial assumption of constant torque applied to a wheelset, rather than constant longitudinal creep. The objective is to examine cases where powered wheels are providing tractive effort leading to higher tangential loads in the contact area than those seen without acceleration or braking forces, but still below the adhesion limit so that full slip does not occur.

If a velocity-dependent friction coefficient is used, enforcing a constant tangential force constraint on the solution to the minimization problem removes the stick-slip oscillation effects seen with this contact model. The results obtained are then the same as those calculated without the falling friction coefficient. Therefore this analysis is carried out only for the non-Hertzian model with constant friction coefficient.

In the first example, shown in Figure 9.5 for a single initial roughness profile, the constant tangential force applied as the constraint in the minimisation problem has been chosen to correspond approximately with the constant creep value of 0.1%. The total tangential force corresponding to this creepage is around 10.5 kN for the freight vehicle and around 7.8 kN for the regional and high-speed vehicle types modelled. With a wheel radius of 0.46 m in each case, the resulting torque for the freight train is taken to be 9660 Nm per wheelset and 7176 Nm per wheelset for the regional and high-speed trains. As the level of tangential loading is approximately the same in each case, differences in the roughness growth rates between Figure 9.5(a) and (b) are due to the different assumptions of either constant creep or constant tangential force.



Figure 9.5 Roughness growth rate for a single wheel on low-level broadband roughness, non-Hertzian contact, constant friction: (a) constant creep 0.1%; (b) constant tangential stress for similar creep. — freight; — — regional; … high-speed.

The roughness growth rates shown in Figure 9.5(b) indicate that very short wavelength roughness will be reduced even more rapidly by powered wheels than by the non-powered and un-braked wheels. Roughness wavelengths shorter than 125 mm all display negative growth rates when the constraint is applied as a constant tangential force. Wavelengths longer than 125 mm have roughness growth rates around zero or above, with some positive growth possible.

The freight train exhibits the lowest roughness growth rate (or highest roughness reduction rate) of the three vehicle types examined with a powered or braked wheel. Results for the other vehicle types are similar to each other. This difference is due to the higher tangential load limit set as the constraint for the freight case, which has a higher friction limit because the normal static load is higher.

These tangential loads are well below the friction limit obtained by multiplying the friction coefficient of 0.3 by the normal static load, *i.e.* 100 kN for the freight vehicle and 60 kN for the other vehicle types. Figure 9.6 shows the effect of the magnitude of the tangential load on the roughness growth rates for a single freight wheel. The tangential load constraint is applied in the form of constant creep in Figure 9.6(a) and as constant torque/tangential force in Figure 9.6(b). The two sets of results are approximately equivalent in terms of creep (Figure 8.9 shows the creep-force relationship for the freight case with constant friction coefficient of 0.3).

The highest tangential load applied, 20 kN, corresponds to a torque of 18400 Nm per wheelset, a creep of about 0.22%, and is two-thirds of the friction limit. In general, increasing the tangential force in the contact in this model increases the rate at which roughness is predicted to be worn away. If the tangential load is applied in the form of a constant creep, this occurs across the full wavelength range (Figure 9.6(a)). However, if the tangential load is applied as a constant force (simulating braking or accelerating, Figure 9.6(b)), then increasing the tangential load increases the rate at which roughness wavelengths shorter than 0.125 m are worn away. Longer wavelength roughness is not greatly affected by the tangential load when it is applied as a constant force rather than as a constant creep.



Figure 9.6 Roughness growth rate for a single freight wheel on low-level broadband roughness, non-Hertzian contact with constant friction: (a) constant creep -0.05%; --0.1%; $\cdots 0.22\%$; (b) constant tangential force -5.25 kN; --1.05 kN; $\cdots 20$ kN.

9.4 Roughness growth rates with velocity-dependent friction

Including a velocity-dependent friction coefficient in the non-Hertzian contact and wear model leads to more variation in the roughness growth rates predicted across the wavelength range for a particular initial rail profile. An example of this has been shown in Figure 8.16. There it was seen that there is much more variation possible in the roughness growth rate if the friction coefficient is velocity-dependent than if the friction is constant. With a constant friction coefficient the roughness growth rates are all negative in the onethird octave wavelength bands shorter than 250 mm, but with a velocity-dependent friction coefficient some positive roughness growth rates are seen in some cases (see Figure 8.16). However, although there is variation in the results for different initial rail profiles (see Appendix B), the average roughness growth rates over five initial roughness profiles presented here are relatively smooth across the wavelength range. The roughness growth rates calculated using the non-Hertzian contact model with velocitydependent friction are shown in Figure 9.7(a) and (b) for soft and stiff rail pads respectively for the different vehicle types. A single-wheel vehicle model only is considered, as up to 60 hours is required for each wheel analysis for every case when a velocity-dependent friction coefficient is used. The overall pattern of the average roughness growth rates is similar to that when a constant friction coefficient is used. Roughness is predicted to remain stable or decrease slightly at wavelengths longer than 25 mm. Roughness levels are predicted to decrease significantly at wavelengths shorter than 25 mm. Some differences are evident in the results for the different vehicle types, similar to the differences seen in the roughness growth rates calculated using the non-Hertzian contact model but with a constant friction coefficient (Figure 9.3).



Figure 9.7 Average roughness growth rate for a single wheel, non-Hertzian contact with velocity-dependent friction: (a) soft rail pads; (b) stiff rail pads. ——freight; — — — regional; · · · · · · high-speed.

9.4.1 Magnitude of initial roughness profile

The roughness growth rate is intended to give results that are independent of the initial roughness spectrum to allow the comparison of results between different vehicle and track types. To confirm that it is not dependent on the initial roughness level, example roughness profiles have been generated according to target spectra with three different levels. Figure 9.8 shows the three roughness spectra and the resulting interaction force between a freight wheel and rail with soft rail pads over three sleeper bays. The low level roughness is similar to the rail roughness at the Gersthofen test site, and has been used throughout this work. A second profile has been generated to have a similar level to the TSI spectrum, and finally the third roughness profile has a level approximately 10 dB higher than the TSI spectrum. The TSI spectrum is also shown as a reference in Figure 9.8(a).



In Figure 9.8(b) the dynamic interaction force spectra reflect the differences in magnitude seen in the roughness spectra. The low level roughness is well below the TSI reference at longer wavelengths, but approaches the TSI reference at short wavelengths. The force spectrum for these cases shows a similar trend. The highest roughness level leads to a dynamic interaction force spectrum that is around 10 dB above the force spectrum for the TSI level case across the wavelength range.

Figure 9.9 shows the roughness growth rates calculated for the freight vehicle on track with soft rail pads and the three initial roughness profiles with different spectral magnitudes. These are not average roughness growth rates, each has been calculated for a single initial profile and for a single vehicle type. Both non-Hertzian contact models are examined; the results for constant friction are shown in Figure 9.9(a) and those for velocity-dependent friction in Figure 9.9(b).



If the friction coefficient is assumed to be constant, then the roughness growth rate is independent of the magnitude of the initial roughness spectrum. The small differences in the roughness growth rates seen in Figure 9.9(a) may be attributed to variability in the different generated profiles, and are no larger than the differences between roughness growth rates calculated for many different initial profiles with similar spectral levels shown in Figure 8.16(a).

If the friction coefficient is dependent on the slip velocity (Figure 9.9(b)), the roughness growth rates are no longer entirely independent of the magnitude of the initial roughness spectrum. Distinct peaks are visible in the result for the low-level initial roughness. These peaks do not appear at the same wavelength for different initial roughnesses with the same spectral level, as the roughness growth rate calculated using velocity-dependent friction is highly variable even when the initial roughness profiles have the same spectral level (see Figure 8.16(b)). The TSI-level roughness prediction also displays some variability in the roughness growth rate across the wavelength range, but not as much as seen in the low-level roughness case. As the initial roughness level increases the roughness growth rate begins to resemble that calculated with a constant coefficient of friction.

In summary, if roughness levels are very low initially, including a velocity-dependent friction coefficient can have a significant effect on the predicted wear pattern compared with a constant friction model. But if the initial combined roughness level of the wheel and rail is higher than the TSI limit, there is little difference in the results obtained from the two friction models for 0.1% creep.

9.5 Roughness growth rates for mixed traffic

In a situation with a variety of traffic on a particular track, a mixed roughness growth rate can be estimated if the proportions and the typical length of each vehicle type are known. At the Gersthofen test site, approximately 40% of the wheel passages are freight wheels, 27% are associated with regional vehicles and around 33% are high-speed vehicles. Figure 9.10 shows the resulting mixed traffic roughness growth rates for soft and stiff rail pads calculated using the three different contact models: Hertzian, non-Hertzian and non-Hertzian with velocity-dependent friction.



Figure 9.10 Roughness growth rate for mixed traffic: (a) soft rail pads; (b) stiff rail pads. —— velocity-dependent friction model; — — — constant friction model; · · · · · · *Hertzian contact model.*

In line with the previous findings, the Hertzian contact model is the only one of the three that predicts roughness growth. For track with soft rail pads, a Hertzian model results in roughness growth at all wavelengths shorter than 40 mm, and a peak in the roughness growth function at around 20 to 25 mm. Roughness is predicted to decrease slightly at wavelengths longer than 40 mm. With stiff rail pads and Hertzian contact, the predicted roughness growth rates are similar to those with soft pads, with the exception that stable or slightly increasing roughness is predicted in the wavelength range from 40 to 80 mm.

Combining the roughness growth rates calculated for each vehicle type into a mixed traffic result removes many of the differences between the two non-Hertzian contact models. The variability between each individual case when a velocity-dependent friction coefficient is used averages out to give a similar mixed traffic result to that calculated with a non-Hertzian, constant friction coefficient model. The mixed traffic roughness growth rates do

not show roughness increasing at any wavelengths. With non-Hertzian contact and mixed traffic, roughness is predicted to be worn away in all one-third octave wavelength bands considered, and to be worn away most rapidly at wavelengths shorter than around 25 mm.

The Hertzian and non-Hertzian contact models give very different roughness growth rate predictions for all wavelengths shorter than 125 mm. At longer wavelengths, however, the roughness growth rates are converging. This suggests that if only roughness of wavelengths longer than 125 mm is of interest then a Hertzian model can be used in the wear calculation. For shorter wavelengths, non-Hertzian effects are significant and should be taken into account in a wear model.

Several authors have linked high rail pad stiffness to increased roughness or corrugation growth, for example Ilias [1999], Cox and Wang [1999], Wu and Thompson [2005], Sheng *et al.* [2006]. The connection between pad stiffness and high roughness growth rates is thought to be that stiff pads emphasise the pinned-pinned effects, leading to a higher corrugation growth rate at that frequency than with soft rail pads. Here, if Hertzian contact is assumed, roughness is predicted to increase across a wider range of wavelengths if the track has stiff rail pads than if the track has soft rail pads (Figure 9.10). However, the rate of roughness growth for wavelengths shorter than 40 mm with mixed traffic is not predicted to be affected much by the rail pad stiffness of the track.

With the non-Hertzian contact models and mixed traffic, there are some differences in the predicted roughness growth rates for track with different rail pad stiffnesses. However, these differences are not consistent across the wavelength spectrum. Soft pads are better at some wavelengths, while hard pads are better at others. In general, this model does not suggest any clear connection between the rail pad stiffness and the roughness growth rates.

9.6 Roughness growth rates with rail dampers

It has been shown in Chapters 3 and 4 that rail dampers affect the pinned-pinned frequency and the interaction force between wheel and rail. Previous work by Croft *et al.* [2009] has examined the effect of rail dampers on roughness growth rates using a Hertzian contact model. In this section, roughness growth rates with rail dampers are presented using the non-Hertzian contact model with velocity-dependent friction. The rail damper model used is that described in Section 3.4.4, with a tuning frequency of 1050 Hz and parameters as listed in Table 3.5.

The roughness growth rates for the various vehicle and track types with rail dampers included are shown in the following figures. Figure 9.11 presents the results for freight vehicle cases, Figure 9.12 for the regional vehicle and Figure 9.13 for the high-speed vehicle. All are average results from five different low-level broadband roughness profiles using a model with a single wheel. The roughness growth rates for the individual cases are included in Appendix B.



Figure 9.11 Roughness growth rate for freight vehicle: (a) soft rail pads with rail dampers; (b) stiff rail pads with rail dampers. ——— with out rail dampers; — — — with rail dampers. Vertical lines indicate shift in wavelength corresponding to the pinned-pinned resonance with the addition of rail dampers.



Figure 9.12 Roughness growth rate for regional vehicle: (a) soft rail pads with rail dampers; (b) stiff rail pads with rail dampers. ——— without rail dampers; — — — with rail dampers. Vertical lines indicate shift in wavelength corresponding to the pinned-pinned resonance with the addition of rail dampers.

For all the vehicle types and rail pad stiffnesses, the rail dampers make a clear difference to the roughness growth rate. At some wavelengths the roughness growth rate with rail dampers is higher than without rail dampers, and at others it is lower. In general, the addition of the rail dampers tends to shift or add peaks in the roughness growth function, in particular in the 31.5 mm wavelength band for the freight case (Figure 9.11) and in the 40 mm wavelength band for the regional (Figure 9.12) and high-speed cases (Figure 9.13). This effect is more pronounced in the cases with stiff rail pads. The roughness growth rate for the high-speed case with rail dampers and stiff rail pads (Figure 9.13(b)) is positive in two wavelength bands, whereas it is negative at all wavelengths without the rail dampers and for all the soft rail pad results.

Rail dampers shift the pinned-pinned frequency of the rail from 1050 Hz to 760 Hz, largely as a result of the additional mass they add to the system (see Section 3.4). Using a Hertzian contact and wear model, Croft *et al.* [2009] found a corresponding shift in the roughness growth rates of track with rail dampers compared with track without rail dampers. High roughness growth rates at wavelengths corresponding to the pinned-pinned resonance were shifted to longer wavelengths with the addition of rail dampers. In the present results, with a non-Hertzian contact model, this effect is not discernable. Without the rail dampers, there is no peak in the roughness growth function corresponding to the pinned-pinned resonance. The peaks introduced by the rail dampers do not correspond to the new pinned-pinned resonance. Vertical lines in Figures 9.11 to 9.13 indicate the wavelength shift that might be expected from the shift in the pinned-pinned resonance.



Figure 9.13 Roughness growth rate for high-speed vehicle: (a) soft rail pads with rail dampers; (b) stiff rail pads with rail dampers. ——without rail dampers; — — with rail dampers. Vertical lines indicate shift in wavelength corresponding to the pinned-pinned resonance with the addition of rail dampers.

The effect of rail dampers on the roughness growth rates with mixed traffic are shown in Figure 9.14. These results have been calculated using the velocity-dependent, non-Hertzian contact model. With soft rail pads, shown in Figure 9.14(a), including rail dampers in the model makes almost no difference to the predicted mixed traffic roughness growth rates. With stiff rail pads, a greater difference is seen between the results with and without rail dampers. At some wavelengths the roughness growth rate is lower with the rail dampers and at some wavelengths it is higher. The general trend remains: roughness levels are predicted to decrease slightly over time at wavelengths longer than 25 mm, and to decrease more rapidly at wavelengths shorter than 25 mm. From these results, rail dampers do not result in either a clear-cut benefit or a disadvantage in terms of rail roughness growth rates.



Figure 9.14 Roughness growth rate for mixed traffic from velocity-dependent friction model:(a) soft rail pads; (b) stiff rail pads. — without rail dampers; — — with rail dampers.

9.7 Measured roughness spectra

Measurements of roughness have been carried out by Deutsche Bahn AG at a test site near Gersthofen in Germany as part of the Silence project [Asmussen *et al.*, 2008]. Corus rail dampers were installed at locations with two pad stiffnesses, corresponding to the soft and stiff pads modelled in this study. At the location with soft rail pads the dampers are installed on both rails of the track. An adjacent untreated section of track with the same rail pads provides a reference. At the location with stiff rail pads the dampers have been applied to a single rail only, with the other rail acting as the reference. All measurement locations are on tangent track well away from any curves.

Roughness measurements are available from the time of the damper installation in 2006, after the dampers had been in place for one year and also after two years.

9.7.1 Measured roughness development examining the effect of rail dampers

For the track with soft rail pads, results are shown in Figure 9.15 without rail dampers and in Figure 9.16 with rail dampers. In the initial measurements a peak is found at 10 mm corresponding to grinding marks. This and other short wavelength roughness has been worn away after one year. Most noticeably, the 10 mm wavelength band shown in Figure 9.16 shows a change in roughness level of around 7 dB in the first year, which is well above the normal measurement uncertainty. The two most recent measurements shown in both figures are very similar, the differences could be entirely due to the variance in the measurement. In fact, aside from some short wavelength bands, all the measurements are close enough to each other for it to be difficult to determine whether the changes recorded are systematic or an artefact of roughness measurement variance, which is around +/- 2 dB [Jones *et al.*, 2008b]. The change in roughness over time at this site is low, so measurements over a longer time period would be needed to determine the systematic roughness development.

There is no clear difference between the results for the locations with and without rail dampers. Despite this, for track with soft pads, there is some evidence to suggest that long wavelength roughness remains relatively stable while short wavelength roughness tends to wear away over time, by up to 4 dB in some wavelength bands over the two years. Longer wavelengths might grow slightly or be worn down but really there is not a significant

change in the spectrum over this time period. The rail roughness at the site was very low to begin with, well below the TSI limit.



Figure 9.15 Measured roughness spectrum for track with soft rail pads without rail dampers: $\cdots \cdots 2006$; ---2007; ---2008; ----TSI roughness limit.



Figure 9.16 Measured roughness spectrum for track with soft rail pads and rail dampers: $\dots \dots 2006; \dots \dots 2007; \dots \dots 2008; \dots \dots \dots \dots TSI$ roughness limit.

For the location with stiff rail pads the results are shown in Figure 9.17 and Figure 9.18. Here the trends are even less clear. Without the rail dampers, the roughness over the two years has reduced across the wavelength range. On the rail with rail dampers installed, the roughness is slightly higher after two years at shorter wavelengths (where it was initially very smooth), and has reduced at longer wavelengths. This pattern is not consistent however; at some wavelengths roughness decreased in the first year and then increased the next year. This result emphasises the difficulty in measuring small changes in roughness accurately, even over a two year period.

The effect of the rail dampers on track with stiff rail pads is also difficult to assess because the initial roughness of the rail with dampers is very different from the roughness of the reference rail. At short wavelengths, the initial roughness shown in Figure 9.18 is the lowest measured at any location. The measured increase of between 1 and 3 dB in roughness in wavelength bands shorter than 25 mm could be a real increase, or an artefact of measurement uncertainty. In any case the final roughness level measured at this location with the rail dampers is very similar to the final roughness level on the rail without dampers shown in Figure 9.17, and remains well below the TSI limit spectrum.



Figure 9.17 Measured roughness spectrum for track with stiff rail pads without rail dampers: $\cdots 2006$; ---2007; ---2008; ----TSI roughness limit.



Figure 9.18 Measured roughness spectrum for track with stiff rail pads and rail dampers: $\dots \dots 2006; \dots \dots 2007; \dots \dots 2008; \dots \dots \dots \dots TSI$ roughness limit.

Examining these measurements, no evidence has been found to suggest that the rail dampers make a significant difference, either positive or negative, to the growth of acoustic roughness.

9.7.2 General comments on measured roughness development

In general, roughness levels at the site have not increased significantly or consistently in any wavelength bands. Figure 9.19 shows the mean of all the measured spectra for each year across all locations, with and without rail dampers. In this representation, roughness has decreased by around 2 dB across the wavelength range, except for the wavelengths between 12.5 and 25 mm where roughness grew slightly in the first year but then decreased in the second year, overall remaining relatively stable.

A difference of 2 dB is not enough to give confidence that the measured decrease in roughness levels is real and not an artefact of normal measurement variation. However, the measurement also does not show any clear increase in acoustic roughness levels over time. In this broad sense therefore the measurements agree with the results of the non-Hertzian contact and wear model.

It is often assumed that roughness must consistently and monotonically increase over time. This is supported by data from the Dutch and German railway networks [Verheijen, 2006; Asmussen *et al.*, 2006], whereas reports in the literature on measurements at other specific sites are somewhat mixed in terms of roughness decreasing or increasing, as discussed in Section 1.4.1, [Cox and Wang, 1999; Bracciali, 2004; Hiensch *et al.*, 2002; Nielsen, 2003]. Here, the measured roughness spectra do not show any evidence of consistently increasing roughness levels over time. Moreover, for this situation on tangent track with initially very low roughness levels, no mechanism is shown in the model for consistently increasing roughness levels in the long term.



Figure 9.19 Mean measured roughness spectrum over all locations, with and without rail dampers: $\cdots \cdots 2006; - - 2007; - 2008; - - \cdot - \cdot TSI$ roughness.

9.8 Summary

A series of roughness growth rate calculations have been made to examine the development of broadband acoustic roughness over time. The parameters used in these calculations have been chosen to match the track at the Silence project measurement site near Gersthofen, to enable some comparison between the model results and measured roughness development over a long period.

The roughness growth rate has therefore been examined for two different track forms, one with relatively soft rail pads of 200 MN/m stiffness and the other with relatively stiff rail pads of 800 MN/m stiffness. The addition of rail dampers to the track has also been included in both the modelling work and the measurements. Three different vehicle types (listed in Table 4.1) are considered as representatives of the type of traffic operating at the Gersthofen site. The approximate proportions of wheel passages due to each vehicle type is known, and has been used to estimate a 'mixed traffic' roughness growth rate from the model results.

When calculating the interaction force between the wheel and rail for track with soft rail pads, it is important to include more than one wheel in the vehicle model since the track decay rate is low and allows interaction between successive wheels coupled by the rail. However, when using a non-Hertzian contact model to predict the wear of the rail and the roughness growth rates, similar results were obtained from a model with a single wheel and a model with four wheels. Including multiple wheels therefore does not significantly affect the predicted roughness growth rates.

Roughness growth rate results have been compared from three different versions of the model. If a Hertzian contact model is used, roughness is predicted to grow significantly in the one-third octave wavelength bands between 12.5 mm and 40 mm. Outside this range, roughness is predicted to grow slightly at shorter wavelengths and decrease slightly or remain stable at longer wavelengths. However, if a non-Hertzian contact model is used, the results are very different. Roughness is then predicted to decrease in all wavelength bands, particularly at wavelengths shorter than 25 mm. The inclusion of a velocity-dependent coefficient of friction causes some variation in the wear of each particular case, and in some cases at some wavelengths roughness growth is positive. When roughness growth rates are averaged over many initial rail profiles however there is very little difference in the predicted results compared with when a constant coefficient of friction is used with the non-Hertzian contact model.

The non-Hertzian contact models, therefore, do not support any tendency of the rails to become rougher over time. In fact, the model predicts that, as long as only longitudinal forces and creep are significant, tangent track with an initially low level of broadband roughness should become smoother under the passage of many wheels. This also assumes
that the wheels are smooth, since the combined roughness of the wheels and rails provides the excitation of the system.

Roughness measurements over two years at the Gersthofen test site do not show any significant or consistent tendency of the rails to become rougher over time at any wavelength. There is some evidence to suggest that rails may become smoother over time, as predicted by the model. However, the differences observed over the two-year period are not large enough in most wavelength bands to give confidence that they are not an artefact of normal measurement variance.

Neither the modelled nor the measured roughness growth rates indicate that the installation of the rail dampers will have a significant effect on the development of broadband acoustic roughness over time, either positive or negative. Also, the roughness growth rate results presented in this chapter do not suggest that roughness should develop differently on track with different rail pad stiffnesses.

The roughness development predicted by a non-Hertzian contact model is generally independent of the rail pad stiffness, the wheel spacing, the pinned-pinned resonance and the addition of rail dampers. It is therefore largely independent of the track dynamics (the effect of vehicle speed on roughness growth rates has not been explicitly examined, although different vehicle types have different speeds). So, although the wheel-rail interaction force is highly dependent on the dynamics of the track and vehicle system, it appears that the resulting roughness growth rate is not.

10 CONCLUSIONS

The aim of this work was to improve the understanding of roughness development on tangent track, which has potential benefits in the areas of long-term noise control and track maintenance. In this thesis a model of wheel-rail interaction forces, rolling contact and wear has been used to study the development of broadband rail-head acoustic roughness in the wavelength range from 5 mm to 250 mm. The effect of different track components on the roughness growth rate of the track-vehicle system has been examined. Using this model, the impact of track design changes on the roughness growth rates has been assessed. In particular, rail damping devices designed to reduce rolling noise have been examined to determine if the addition of rail dampers to a track will lead to a change in the tendency of the track to develop rail-head roughness.

10.1 Conclusions from the literature

Rail-head wear is a complex problem. An ideal model would account for factors such as elastic and plastic deformation, high temperature effects, work hardening, and local variation in material properties and wear resistance. It would also include a threedimensional analysis of vehicle dynamics, with torsional modes of the wheelsets and varying lateral and spin creep as well as the longitudinal creep considered here. However, the computational cost of such a model would be enormous.

It is clearly necessary to simplify the problem in order to gain an understanding of the relative impact of different effects on roughness growth rates. In the literature various relationships between the resonances of track and vehicle have been proposed to suggest why roughness might grow more rapidly at some wavelengths than at others, and indeed why roughness should grow at all. The pinned-pinned resonance of the track has been identified as a possible wavelength-fixing mechanism in some cases of corrugation. The geometry of the contact between wheel and rail has been identified as promoting roughness or corrugation growth at particular wavelengths. Some models of corrugation attribute uneven railhead wear to lateral creep due to wheelset misalignment. Recent studies have tended to concentrate on longitudinal effects. The fundamental mechanisms of roughness growth and corrugation formation are clearly not well understood.

Based on a review of the literature, it was decided to examine motion in the vertical plane only, enabling the simplification of the problem to two dimensions. Such a model is a suitable starting point for studying the development of general acoustic roughness rather than corrugation phenomena. Efforts have then been concentrated on the contact mechanics calculation, including non-Hertzian and transient effects. A non-constant, velocity-dependent friction law has been investigated. This has not been implemented before in a non-Hertzian tangential contact stress distribution calculation.

10.2 Conclusions on roughness growth mechanisms

It has been confirmed that it is essential in roughness growth predictions to take account of the low-level, broadband roughness profile of the contacting surfaces within the contact model. The surface roughness, however small compared with the overall dimensions of the wheel and rail, acts as a stress concentrator that must be considered when determining the distribution of normal and tangential stress and the slip velocity in the contact. Assuming Hertzian contact, and thereby neglecting the roughness profile, leads to a completely different prediction of the tendency of a track to develop roughness. With a Hertzian model, roughness is predicted to grow or worsen under many wheel passages, and roughness growth is more significant at some wavelengths than at others. The non-Hertzian model has been found to predict roughness decreasing over time.

The effect of the initial roughness on the stress distribution is not limited to roughness wavelengths shorter than the length of the contact patch. The roughness growth rates predicted from Hertzian and non-Hertzian models are clearly different for all wavelengths shorter than around 100 mm, only converging at wavelengths longer than this. If the wavelength range of interest for short-pitch corrugations is 25 to 100 mm then this result suggests that non-Hertzian effects should also be considered in studies of the mechanisms of corrugation formation and growth.

The results presented in Chapter 9 from the non-Hertzian contact models predict mostly negative roughness growth rates. Some combinations of initial roughness and vehicle parameters can lead to positive roughness growth at some wavelengths, but the average roughness growth developed over five different initial rail profiles is negative in all cases considered. Short-wavelength roughness in particular is predicted to become smoother under the passage of many wheels. This is expected as the result of the filtering effect of the contact patch, and has been observed in practice, most noticeably in the wearing down

of short wavelength grinding marks. However, the observation that roughness at longer wavelengths does grow at many sites has not been explained by the model predictions.

The wear calculation employed here includes the possibility that different wear mechanisms might occur in different parts of the contact area, as a result of stress concentrations arising from the roughness profile. However this situation was found not to occur for the mostly low-level roughness profiles examined in this work. It has been found that roughness development may therefore be modelled adequately using a single wear coefficient representing the mild wear regime.

Roughness measurements taken over a two year period at a location with mixed traffic and initially low level, broadband roughness show that it is possible that roughness might have reduced slightly over time at this site. However, it is difficult to be sure that the small changes seen in the roughness spectra are real, because of the variance of roughness measurements. It might be more correct to say that roughness levels at the test site have not increased significantly in the time period of the measurements, than to claim a decrease in measured roughness levels.

10.3 Conclusions on the velocity-dependent friction law

Including a velocity-dependent friction coefficient in the tangential contact analysis has been found to introduce a stick-slip oscillation into the slip zone at the trailing edge of an otherwise steady rolling contact. This is turn creates an uneven pattern of wear on an otherwise smooth surface. This is a very short wavelength effect, much shorter than the contact length, and as such is not a significant feature in the resulting roughness growth rates in transient rolling contact cases with realistic initial roughness levels. With increasing longitudinal creep and a velocity-dependent friction coefficient, the stick-slip oscillation increases until the creep-force relationship becomes increasingly unstable.

If roughness levels are very low initially, the inclusion of a velocity-dependent friction coefficient can have a significant effect on the predicted wear pattern for a particular initial profile. However, if the combined roughness level of the wheel and rail is initially higher than the TSI limit, there is little difference in the results obtained by the two friction models. Moreover, if roughness growth rate results are averaged over a number of different initial rail profiles, the results from models with the different friction laws are not

significantly different, even for very low initial roughness levels. In this average sense roughness is not predicted to grow at any wavelengths regardless of the friction law used in the model.

10.4 Conclusions pertaining to rail dampers

Rail dampers change the dynamic response of the rail, reducing the pinned-pinned frequency and smoothing the peaks and troughs in the track receptance. Rail dampers have been found to reduce the dynamic interaction forces between wheels and the rail, especially around the pinned-pinned resonance. Rail dampers shift the wheel-rail interaction force spectrum to lower frequencies or longer wavelengths.

Although the rail dampers have a significant effect on the dynamic interaction force, the addition of rail dampers to a track is not predicted to make a significant difference to the growth of broadband roughness levels, at least for tangent track where the initial roughness levels are low. The roughness growth is found to be neither greater nor less with rail dampers fitted. The measurements taken over two years at the Gersthofen test site also show no clear evidence of a change in roughness growth rates as a result of the installation of rail dampers. The measurement results are therefore consistent with the findings from the model. However, measurements should be continued over a longer time period to give increased confidence in this conclusion.

10.5 Recommendations for future work

Several questions have been raised by this work about the accepted understanding of the mechanisms of roughness development and corrugation formation. With the model including non-Hertzian and transient effects, no mechanism has been found for consistent roughness growth. Moreover, no connection has been found between the pinned-pinned resonance of the track and high roughness growth rates at the corresponding wavelengths. This does not mean that the pinned-pinned resonance is not a wavelength fixing mechanism for short-pitch corrugation. However, in practice, discretely supported tangent tracks do not all develop corrugation at wavelengths corresponding to the pinned-pinned frequency, even after many years of traffic. This evidence agrees with the results from the model developed here, for tangent track with purely longitudinal creep and where the roughness levels are sufficiently low to keep deformation in the contact within the elastic region.

Future work on roughness development and corrugation should consider that if Hertzian contact is assumed, any change in the wheel-rail interaction forces leads to a change in the predicted wear. The Hertzian assumption therefore results in an exaggerated relationship between the resonances of the track and vehicle system and the wear of the rail.

The measurements taken by Deutsche Bahn AG as part of the Silence project are valuable because there are very few documented cases of roughness measurements being repeated at the same location over several years. Even over two years, the roughness at the site has not changed enough to be sure that the observed decreases in roughness are not due to measurement variability. These measurements should be continued. It is commonly accepted that broadband roughness on tangent track does increase, but roughness growth is not necessarily linear with time. Smooth track (possibly in conjunction with smooth wheels) may remain in good condition over many years, while track with initially high roughness levels may worsen more rapidly. At the Gersthofen test site initial roughness levels were uniformly low so it has not been possible to investigate differences in roughness development caused by the initial level. It would be useful to prove that roughness level and roughness development.

More measurements of roughness development over time are required to validate the roughness growth model and to understand the mechanisms of roughness growth. A useful long term experiment would be to take roughness measurements at six-monthly intervals over many years at locations with initially low roughness levels. It would also be useful to compare measurements from track with predominantly disc-braked vehicles, with roughness levels on track where the traffic is mostly tread braked. This would assess whether higher wheel roughness leads to higher roughness growth on the rails, possibly as a result of plastic deformation. Other factors that might be monitored in addition to the roughness itself include traffic axle loads, lateral forces and/or wheelset alignment, and traction and braking forces. However, these would be more difficult (and perhaps impossible) to collect on the time scales of roughness development.

In terms of short-pitch corrugation research, the absence of any mechanism for corrugation formation found here suggests that the search for causes of corrugation should concentrate on lateral effects. Corrugation is more common on curves than on tangent track, where

lateral dynamics are more important. Discrete longitudinal effects such as rail joints or wheel-rail defects could also be investigated further as possible initiators of corrugation. In addition, metallurgical factors that have not been considered in this work could be important, including hardness and variations or defects in the crystalline structure of the rail steel.

In the rolling contact analysis used here, the rail and the wheel have been treated as halfspaces. Track and wheel dynamic effects have been included by means of their effect on the normal wheel-rail interaction forces. The stick-slip effect identified with the velocitydependent friction law is a very high-frequency phenomenon. It has a wavelength of the order of 1.2 mm in steady rolling, which corresponds to about 25 kHz at typical train speeds. It could conceivably interact with resonances of the track or wheel system. Further investigation is needed including the track and wheel dynamics in the rolling contact analysis to examine if the stick-slip oscillation might lock into another resonance of the system.

APPENDIX A RAIL DAMPER EFFECT ON FORCE

This appendix contains the wheel-rail interaction force spectra showing the effect of the rail dampers for all three vehicle types (the freight vehicle results only have been shown in Section 4.9).



Figure A.1 Dynamic wheel-rail interaction force for a freight train on (a) soft and (b) stiff rail pads with low broadband roughness: ——— with rail dampers; · · · · · · without rail dampers.



Figure A.2 Dynamic wheel-rail interaction force for a freight train on (a) soft and (b) stiff rail pads with TSI roughness: ——with rail dampers; · · · · · · without rail dampers.



Figure A.3 Dynamic wheel-rail interaction force for a regional train on (a) soft and (b) stiff rail pads with low broadband roughness: ——— with rail dampers; · · · · · · without rail dampers.



Figure A.4 Dynamic wheel-rail interaction force for a regional train on (a) soft and (b) stiff rail pads with TSI roughness: ——— with rail dampers; · · · · · · without rail dampers.



Figure A.5 Dynamic wheel-rail interaction force for a high-speed train on (a) soft and (b) stiff rail pads with low broadband roughness: ——— with rail dampers; · · · · · · without rail dampers.



Figure A.6 Dynamic wheel-rail interaction force for a high-speed train on (a) soft and (b) stiff rail pads with TSI roughness: ——— with rail dampers; · · · · · · without rail dampers.

APPENDIX B ROUGHNESS GROWTH RATES

The following figures present the individual roughness growth rates calculated for five different initial roughness profiles for all the cases where average results have been shown in Chapter 9. Table B.1 is a summary of the cases contained in this appendix.

Contact model	Vehicle model	Rail pad stiffness	Figure number
Hertzian	Freight 1 wheel	Soft & stiff	B.1
	Regional 1 wheel	Soft & stiff	B.2
	High-speed 1 wheel	Soft & stiff	B.3
	Freight 4 wheels	Soft	B.13(a)
	Regional 4 wheels	Soft	B.13(b)
	High-speed 4 wheels	Soft	B.13(c)
Non-Hertzian, constant	Freight 1 wheel	Soft & stiff	B.4
friction	Regional 1 wheel	Soft & stiff	B.5
	High-speed 1 wheel	Soft & stiff	B.6
	Freight 4 wheels	Soft	B.14(a)
	Regional 4 wheels	Soft	B.14(b)
	High-speed 4 wheels	Soft	B.14(c)
Non-Hertzian, velocity-	Freight 1 wheel	Soft & stiff	B.7
dependent friction	Regional 1 wheel	Soft & stiff	B.8
	High-speed 1 wheel	Soft & stiff	B.9
Non-Hertzian, velocity-	Freight 1 wheel	Soft & stiff	B.10
dependent friction with rail	Regional 1 wheel	Soft & stiff	B.11
dampers	High-speed 1 wheel	Soft & stiff	B.12

Table B.1 Roughness growth rate figure list.

B.1 Hertzian contact model



Figure B.1 Roughness growth rate for a single freight wheel for five different low-level broadband roughness profiles: (a) soft rail pads; (b) stiff rail pads.



Figure B.2 Roughness growth rate for a single regional wheel for five different low-level broadband roughness profiles: (a) soft rail pads; (b) stiff rail pads.



Figure B.3 Roughness growth rate for a single high-speed wheel for five different low-level broadband roughness profiles: (a) soft rail pads; (b) stiff rail pads.

B.2 Non-Hertzian constant friction contact model



Figure B.4 Roughness growth rate for a single freight wheel for five different low-level broadband roughness profiles: (a) soft rail pads; (b) stiff rail pads.



Figure B.5 Roughness growth rate for a single regional wheel for five different low-level broadband roughness profiles: (a) soft rail pads; (b) stiff rail pads.



Figure B.6 Roughness growth rate for a single high-speed wheel for five different low-level broadband roughness profiles: (a) soft rail pads; (b) stiff rail pads.

B.3 Non-Hertzian velocity-dependent friction contact model



Figure B.7 Roughness growth rate for a single freight wheel for five different low-level broadband roughness profiles: (a) soft rail pads; (b) stiff rail pads.



Figure B.8 Roughness growth rate for a single regional wheel for five different low-level broadband roughness profiles: (a) soft rail pads; (b) stiff rail pads.



Figure B.9 Roughness growth rate for a single high-speed wheel for five different low-level broadband roughness profiles: (a) soft rail pads; (b) stiff rail pads.





Figure B.10 Roughness growth rate for a single freight wheel for five different low-level broadband roughness profiles: (a) soft rail pads; (b) stiff rail pads.



Figure B.11 Roughness growth rate for a single regional wheel for five different low-level broadband roughness profiles: (a) soft rail pads; (b) stiff rail pads.



Figure B.12 Roughness growth rate for a single high-speed wheel for five different lowlevel broadband roughness profiles: (a) soft rail pads; (b) stiff rail pads.



Figure B.13 Roughness growth rate for four wheel vehicle models on track with soft rail pads and five different low-level broadband roughness profiles: (a)freight; (b) regional; (c) high-speed.



Figure B.14 Roughness growth rate for four wheel vehicle models on track with soft rail pads and five different low-level broadband roughness profiles: (a)freight; (b) regional; (c) high-speed.

APPENDIX C LIST OF SYMBOLS

Α	Cross-sectional area of beam (Chapter 3)	
A	Area	
A_0	Initial roughness amplitude in 1/3 octave wavelength band (Chapter 8)	
A_n	Final roughness amplitude in 1/3 octave wavelength band (Chapter 8)	
Α	System matrix for state-space solution (Chapter 4)	
\mathbf{A}^{track}	Assembled from the global mass and damping matrices (Chapter 3)	
B_{ij}	Two-dimensional influence coefficient matrix (Chapter 5)	
B_{ij}'	Two-dimensional influence coefficient matrix for previous time-step (Chapter 6)	
В	System matrix for state-space solution (Chapter 4)	
\mathbf{B}^{track}	Assembled from the global mass and stiffness matrices (Chapter 3)	
<i>C</i> ₁₋₃	Timoshenko beam coefficients (Chapter 3)	
C_H	Hertzian contact constant (Chapter 4)	
C_{ij}	Normal influence coefficient matrix (Chapter 5)	
С	Global damping matrix for track (Chapter 3)	
Cw	Global damping matrix for vehicle/wheels (Chapter 4)	
D_{ij}	Tangential influence coefficient matrix (Chapter 5)	
D_{ij}'	Tangential influence coefficient for previous time-step (Chapter 6)	
Ε	Young's modulus	
E^*	Combined Young's modulus in contact	
F	Force (Chapter 3)	
F_a	Contact force between wheel and rail (Chapter 4)	
F_{e}	External static load on wheel (Chapter 4)	
Fnorm	Function to minimise for the normal contact problem (Chapter 5)	
F _{tang}	Function to minimise for the tangential contact problem (Chapter 5)	
\mathbf{F}_a	Wheel rail interaction force matrix (Chapter 4)	
$\hat{\mathbf{F}}_{a}$	Wheel rail interaction force impulse matrix (Chapter 4)	
G	Shear modulus	
G	Schur complement of matrix A (Chapter 4)	
H_{1-4}	Hermite interpolating polynomials (Chapter 4)	
Н	Matrix of Hermite interpolating polynomials (Chapter 4)	
Ι	Second moment of area of the cross-section	
Ι	Identity matrix (Chapter 4)	
J	Number of discrete supports (Chapter 3)	

J_e	Number of rail elements in track model (Chapter 4)	
Κ	Combined system stiffness (Chapter 3)	
Κ	Wear constant (Chapter 7)	
K	Global stiffness matrix for track (Chapter 3)	
K _H	Global contact stiffness matrix (Chapter 4)	
L	Sleeper spacing (Chapter 3)	
L_j	Length of rail element <i>j</i> (Chapter 4)	
L_p	Rail pad length along rail (Chapter 3)	
L_{1-4}	Distance from element centre to corners of other element (Chapter 5)	
M_w	Unsprung mass of wheel (Chapter 4)	
Μ	Global mass matrix for track (Chapter 3)	
$\mathbf{M}_{\mathbf{w}}$	Global mass matrix for vehicle/wheels (Chapter 4)	
Ν	Number of degrees of freedom in track model (Chapter 3)	
Ν	Number of elements in potential contact area (Chapter 5)	
0	Origin of axes	
Р	Total normal force in wheel-rail contact	
Р	Modal matrix (Chapter 3)	
Q	Total tangential force in wheel-rail contact	
Q	Modal load vector (Chapter 3)	
R	Equivalent radius of curvature for Hertz equations (Chapters 4, 5)	
$R_{1,2}$	Radius of spherical bodies 1, 2 (Chapter 5)	
R_m	Modified wheel radius (Chapter 4)	
R_r	Radius of rail in y-z plane (Chapter 4)	
R_w	Radius of wheel in x-z plane (Chapter 4)	
R	Component of acceleration of wheel-rail interface (Chapter 4)	
S	Total contact area (Chapter 5)	
S	Component of acceleration of wheel-rail interface (Chapter 4)	
Т	Tangential force (Chapter 7)	
Т	Component of velocity of wheel-rail interface (Chapter 4)	
U	Displacement amplitude (Chapter 3)	
U_E	Elastic strain energy (Chapter 5)	
U_E^*	Internal complementary energy (Chapter 5)	
U	Component of velocity of wheel-rail interface (Chapter 4)	
V^*	Total complementary energy of contact system (Chapter 5)	
$W_{ au}$	Rigid tangential shift (Chapters 5, 6)	

Y	Point mobility (Chapter 3)	
а	Half-length of element in <i>x</i> direction (Chapters 3, 5)	
a	Half-length of contact patch (radius if circular) (Chapters 4, 5, 6)	
$a_{1,}a_{2}$	Rayleigh damping coefficients (Chapter 3)	
a	Diagonal matrix (Chapter 3)	
b	Half the length of element in y direction (Chapter 5)	
b	Diagonal matrix (Chapter 3)	
С	Viscous damping (Chapters 3, 4)	
С	Half-length of stick region (Chapters 5, 6)	
C_d	Damper damping (Chapter 3)	
c	Damping matrix for an element (Chapter 3)	
dz.	Depth of material removed from railhead (Chapter 7)	
f	Force on a rail damper element (Chapter 3)	
f_{1-4}	Functions of geometric distances between elements (Chapter 5)	
f	Vector of forces and moments for an element (Chapter 3)	
<i>8</i> 1-4	Functions of geometric distances between elements (Chapter 5)	
g	Vector of modal coordinates and parameters of interest (Chapter 4)	
h	Wavenumber (Chapter 3)	
h	Undeformed distance between two surfaces (Chapter 5)	
i	Imaginary unit, $\sqrt{-1}$ (Chapter 3)	
k	Stiffness (Chapters 3, 4)	
k	Distance between element centres in two-dimensional contact (Chapter 5)	
k'	Stiffness per unit length (Chapter 3)	
k_H	Hertzian contact stiffness between wheel and rail (Chapter 4)	
k	Stiffness matrix for an element (Chapter 3)	
l	Vehicle axle spacing (Chapter 4)	
т	Mass (Chapter 3)	
m'	Mass per unit length (Chapter 3)	
m_d	Damper mass (Chapter 3)	
m_s	Half sleeper mass in finite element model (Chapter 3)	
m	Mass matrix for an element (Chapter 3)	
р	Normal pressure on element of contact	
p_0	Maximum normal pressure at centre of Hertzian contact (Chapter 5)	
q	Tangential stress on element of contact	
q	Modal displacement vector (Chapter 3)	

r	Rail roughness profile (Chapter 4)	
r	Radial distance to centre of Hertzian contact (Chapters 5, 6)	
S	Slip	
t	Time	
ť	Time at previous time-step	
<i>u</i> , <i>u</i> _z	Displacement in vertical direction	
u_x	Displacement in longitudinal direction	
$u'_{i\tau}$	Tangential displacement difference at previous time (Chapter 5)	
u	Displacement vector for an element (Chapters 3, 4)	
V	Velocity	
x	Longitudinal direction	
<i>x'</i>	Position in the contact relative to the centre at x (Chapter 4)	
у	Lateral direction	
У	Matrix assembled from \mathbf{u} and $\dot{\mathbf{u}}$ (Chapter 3)	
Z,	Vertical direction	
z(x')	Circular profile of wheel for contact filter (Chapter 4)	
<i>z</i> _{1,2}	Undeformed surface profile of body 1, 2 (Chapter 5)	
Δ	Decay rate (Chapter 3)	
Δt	Time increment	
Δv	Relative velocity between two surfaces	
Δx	Distance increment	
Δx	Length of element in potential contact area (Chapter 5)	
Δy	Width of element in potential contact area (Chapter 5)	
$\Delta \omega$	Relative angular velocity between two surfaces in contact (Chapter 5)	
α	Receptance (Chapter 3)	
β	Parameter used in Timoshenko beam finite element theory (Chapter 3)	
γ	Non-dimensional slip in wear model (Chapter 7)	
δ	Approach of distant points in two contacting bodies	
ζ	Damping ratio (Chapter 3)	
η	Damping loss factor (Chapter 3)	
θ	Rotation in <i>x</i> - <i>z</i> plane	
κ	Timoshenko shear factor (Chapter 3)	
μ	Coefficient of friction	
μ_s	Static coefficient of friction (Chapter 7)	
μ_d	Dynamic coefficient of friction (Chapter 7)	

V	Poisson's ratio
ξ	Local coordinate in <i>x</i> direction along rail element (Chapter 4)
ξ	Longitudinal creep ratio (Chapter 6)
ρ	Density
φ	Eigenvector (Chapter 4)
Ψ	Global roughness growth rate (Chapter 8)
ω	Circular frequency, angular velocity
ω_0	Natural frequency (Chapter 3)

C.1 Common Subscripts

r	Rail
р	Rail pad
S	Sleeper
b	Ballast

d Rail damper

REFERENCES

- Abrahamsson, T. (1988). Modal-parameter extraction for nonproportionally damped linear structures. *International Journal of Analytical and Experimental Modal Analysis* 3(2): 62-68.
- Afferrante, L. and M. Ciavarella (2009). Short-pitch rail corrugation: A possible resonance-free regime as a step forward to explain the "enigma"? *Wear* 266(9-10): 934-944.
- Ahmad, N. (2009) A methodology for developing high damping materials with application to noise reduction of railway track. *Doctoral Thesis*, Institute of Sound and Vibration Research, University of Southampton.
- Alonso, A. and J. G. Giménez (2008a). Non-steady state contact with falling friction coefficient. *Vehicle System Dynamics* 46(SUPPL1): 779-789.
- Alonso, A. and J. G. Giménez (2008b). Non-steady state modelling of wheel-rail contact problem for the dynamic simulation of railway vehicles. *Vehicle System Dynamics* 46(3): 179-196.
- Alwahdi, F., F. J. Franklin and A. Kapoor (2005). The effect of partial slip on the wear rate of rails. *Wear* 258(7-8): 1031-1037.
- Andersson, C. (2003). Modelling and Simulation of Train-Track Interaction including Wear Prediction. *Doctoral dissertation*, Department of Applied Mechanics, Chalmers University of Technology, Gothenburg.
- Andersson, C. and J. Oscarsson (2000). Dynamic train/track interaction including statedependent track properties and flexible vehicle components. *Vehicle System Dynamics* 33(Suppl): 47-58.
- Andersson, C. and T. Abrahamsson (2002). Simulation of interaction between a train in general motion and a track. *Vehicle System Dynamics* 38(6): 433-455.
- Andersson, C. and A. Johansson (2004). Prediction of rail corrugation generated by threedimensional wheel-rail interaction. *Wear* 257(3-4): 423-434.
- Archard, J. F. and W. Hirst (1956). The wear of materials under unlubricated conditions. *Proceedings, Royal Society* A236: 397-410.
- Asmussen, B., H. Onnich, R. Strube, L. M. Greven, S. Schroder, K. Jager and K. G. Degen (2006). Status and perspectives of the "Specially Monitored Track". *Journal of Sound and Vibration* 293(3-5): 1070-1077.
- Asmussen, B., D. Stiebel, P. Kitson, D. Farrington and D. Benton (2008). Reducing the noise emission by increasing the damping of the rail: results of a field test. In *Noise and Vibration Mitigation for Rail Transportation Systems Proceedings of the 9th*

International Workshop on Railway Noise, Munich, Germany, 4 - 8 September 2007: 229-235, Springer: Berlin

- Ayasse, J. B. and H. Chollet (2005). Determination of the wheel rail contact patch in semi-Hertzian conditions. *Vehicle System Dynamics* 43(3): 161-172.
- Baeza, L., A. Roda and J. C. O. Nielsen (2006). Railway vehicle/track interaction analysis using a modal substructuring approach. *Journal of Sound and Vibration* 293(1-2): 112-124.
- Baeza, L., F. J. Fuenmayor, J. Carballeira and A. Roda (2007). Influence of the wheel-rail contact instationary process on contact parameters. *Journal of Strain Analysis for Engineering Design* 42(5): 377-387.
- Baeza, L., P. Vila, A. Roda and J. Fayos (2008). Prediction of corrugation in rails using a nonstationary wheel-rail contact model. *Wear* 265(9-10): 1156-1162.
- Barnett, S. (1979). Matrix Methods for Engineers and Scientists. London, McGraw-Hill.
- Bellette, P. A., P. A. Meehan and W. J. T. Daniel (2008). Effects of variable pass speed on wear-type corrugation growth. *Journal of Sound and Vibration* 314(3-5): 616-634.
- Bentall, R. H. and K. L. Johnson (1967). Slip in the rolling contact of two dissimilar elastic rollers. *International Journal of Mechanical Sciences* 9(6): 389-404.
- Berglund, B., T. Lindvall and D. H. Schwela (Eds.) (1999). *Guidelines for community noise*.World Health Organisation, Geneva.
- Bohmer, A. and T. Klimpel (2002). Plastic deformation of corrugated rails A numerical approach using material data of rail steel. *Wear* 253(1-2): 150-161.
- Bolton, P. J. and P. Clayton (1984). Rolling-sliding wear damage in rail and tyre steels. *Wear* 93(2): 145-165.
- Bowden, F. P. and D. Tabor (1974). *Friction : an introduction to tribology*. London, Heinemann Educational Books Ltd.
- Bracciali, A. (2004). Roughness Growth A practical case. In Proceedings of the Eighth International Workshop on Railway Noise, Buxton, England, 8–11 September 2004(2): 625-633.
- Braghin, F., R. Lewis, R. S. Dwyer-Joyce and S. Bruni (2006). A mathematical model to predict railway wheel profile evolution due to wear. *Wear* 261(11-12): 1253-1264.
- Bucher, F., K. Knothe and A. Theiler (2002). Normal and tangential contact problem of surfaces with measured roughness. *Wear* 253(1-2): 204-218.
- Bucher, F., A. I. Dmitriev, M. Ertz, K. Knothe, V. L. Popov, S. G. Psakhie and E. V. Shilko (2006). Multiscale simulation of dry friction in wheel/rail contact. *Wear* 261(7-8): 874-884.

- Carter, F. W. (1926). On the action of a locomotive driving wheel. *Proceedings, Royal Society* A112: 151.
- CEN (2009). Railway Applications Noise emission Rail roughness measurement related to rolling noise generation, EN 15610:2009.
- Ciavarella, M. and J. Barber (2008). Influence of longitudinal creepage and wheel inertia on short-pitch corrugation: a resonance-free mechanism to explain the roaring rail phenomenon. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal* of Engineering Tribology 222(3): 171-181.
- Clark, R. A., P. A. Dean, J. A. Elkins and S. G. Newton (1982). Investigation into the dynamic effects of railway vehicles running on corrugated rails. *Journal of Mechanical Engineering Science* 24(2): 65-76.
- Clark, R. A. and P. Foster (1983). On the mechanics of rail corrugation formation. *Vehicle System Dynamics* 12(1-3): 35-33.
- Clark, R. A., G. A. Scott and W. Poole (1988). Short wave corrugations an explanation based on slip-stick vibrations. In *Proceedings of Applied Mechanics Rail Transportation Symposium*, Chicago November 27- December 2, 1988, Published by the American Society of Mechanical Engineers, New York.
- Clayton, P. (1996). Tribological aspects of wheel-rail contact: a review of recent experimental research. *Wear* 191(1-2): 170-183.
- Cox, S. J., A. Wang (1999). Effects of pad stiffness on rail roughness growth and wayside noise levels for high speed track. In *Proceedings of the Sixth International Congress on Sound and Vibration*, Copenhagen, Denmark, 5 - 8 July, 1999: 2679-2686.
- Croft, B. E., C. J. C. Jones and D. J. Thompson (2009). Modelling the effect of rail dampers on wheel-rail interaction forces and rail roughness growth rates. *Journal of Sound and Vibration* 323(1-2): 17-32.
- Daves, W. and F. D. Fischer (2002). Modelling of the plastification near the rough surface of a rail by the wheel-rail contact. *Wear* 253(1-2): 241-246.
- Dings, P. C. and M. G. Dittrich (1996). Roughness on Dutch railway wheels and rails. *Journal* of Sound and Vibration 193(1): 103-112.
- European Commission (2002). Directive 2002/49/EC of the European Parliament and of the Council of 25 June 2002 relating to the assessment and management of environmental noise. In *Official Journal of the European Union*. L189/12-25.
- European Commission (2005). Commission decision of 23 December 2005, concerning the technical specification for interoperability relating to the subsystem 'rolling stock—

noise' of the trans-European conventional rail system. In *Official Journal of the European Union*. L037/1-49.

- European Commission (2006). Commission communication in the framework of the implementation of Council Directive 96/48/EC of 23 July 1996 on the interoperability of the trans-European high-speed rail system. In *Official Journal of the European Union*. C332/30-34.
- European Environment Agency (2009). Transport at a crossroads TERM 2008: indicators tracking transport and environment in the European Union. *EEA Report* No 3/2009.
- Fenander, A. (1997). Frequency dependent stiffness and damping of railpads. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit 211(1): 51-62.
- Fenander, A. (1998). A fractional derivative railpad model included in a railway track model. *Journal of Sound and Vibration* 212(5): 889-903.
- Fermér, M. and J. C. O. Nielsen (1995). Vertical interaction between train and track with soft and stiff railpads - full-scale experiments and theory. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 209(1): 39-47.
- Ford, R. A. J. and D. J. Thompson (2006). Simplified contact filters in wheel/rail noise prediction. *Journal of Sound and Vibration* 293(3-5): 807-818.
- Franklin, F. J., I. Widiyarta and A. Kapoor (2001). Computer simulation of wear and rolling contact fatigue. Wear 250-251(2): 949-955.
- Franklin, F. J., T. Chung and A. Kapoor (2003). Ratcheting and fatigue-led wear in rail-wheel contact. *Fatigue and Fracture of Engineering Materials and Structures* 26(10): 949-955.
- Franklin, F. J. and A. Kapoor (2007). Modelling wear and crack initiation in rails. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit 221(1): 23-33.
- Franklin, F. J., J. E. Garnham, D. I. Fletcher, C. L. Davis and A. Kapoor (2008). Modelling rail steel microstructure and its effect on crack initiation. *Wear* 265(9-10): 1332-1341.
- Frederick, C. O. (1986). A rail corrugation theory. In Proceedings of the International Symposium on Contact Mechanics and Wear of Rail/Wheel Systems 2, Kingston, Rhode Island.
- Giménez, J. G., A. Alonso and E. Gomez (2005). Introduction of a friction coefficient dependent on the slip in the FastSim algorithm. *Vehicle System Dynamics* 43(4): 233-244.

- Golla, D. F. and P. C. Hughes (1985). Dynamics of viscoelastic structures a time domain, finite element formulation. *Transactions of the ASME. Journal of Applied Mechanics* 52(4): 897-906.
- Grassie, S. L., R. W. Gregory, D. Harrison and K. L. Johnson (1982). Dynamic response of railway track to high frequency vertical excitation. *Journal of Mechanical Engineering Science* 24: 77-90.
- Grassie, S. L. and K. L. Johnson (1985). Periodic microslip between a rolling wheel and a corrugated rail. *Wear* 101(4): 291-309.
- Grassie, S. L. and J. Kalousek (1993). Rail corrugation: characteristics, causes and treatments. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit 207(1): 57-68.
- Grassie, S. L. (1996). Short wavelength rail corrugation: field trials and measuring technology. *Wear* 191(1-2): 149-160.
- Grassie, S. L. (2005). Rail corrugation: advances in measurement, understanding and treatment. *Wear* 258(7-8): 1224-1234.
- Grassie, S. L., J. Edwards and J. Shepherd (2007). Roaring rails an enigma largely explained. *International Railway Journal*. XLVII July: 31-33.
- Grassie, S. L. and J. W. Edwards (2008). Development of corrugation as a result of varying normal load. *Wear* 265(9-10): 1150-1155.
- Hao, S. and L. M. Keer (2007). Rolling contact between rigid cylinder and semi-infinite elastic body with sliding and adhesion. *Transactions of the ASME. Journal of Tribology* 129(3): 481-494.
- Heckl, M. A. (1995). Railway noise can random sleeper spacings help? *Acustica* 81(6): 559-564.
- Hempelmann, K., F. Hiss, K. Knothe and B. Ripke (1991). Formation of wear patterns on rail tread. Wear 144(1-2): 179-195.
- Hempelmann, K. and K. Knothe (1996). An extended linear model for the prediction of short pitch corrugation. *Wear* 191(1-2): 161-169.
- Hiensch, M., J. C. O. Nielsen and E. Verheijen (2002). Rail corrugation in the Netherlands measurements and simulations. *Wear* 253(1-2): 140-149.
- Hoffmann, N. P. and M. Misol (2007). On the role of varying normal load and of randomly distributed relative velocities in the wavelength selection process of wear-pattern generation. *International Journal of Solids and Structures* 44(25-26): 8718-8734.
- Huang, Z. Y., D.J. Thompson, C.J.C Jones (2008). Squeal prediction for a bogied vehicle in a curve. In Noise and Vibration Mitigation for Rail Transportation Systems Proceedings

of the 9th International Workshop on Railway Noise, Munich, Germany, 4 - 8 September 2007: 313-319, Springer: Berlin.

- Igeland, A. (1996). Railhead corrugation growth explained by dynamic interaction between track and bogie wheelsets. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 210(1): 11-20.
- Igeland, A. (1997). Dynamic train/track interaction: simulation of railhead corrugation growth under a moving bogie using mathematical models combined with full-scale measurements, *Doctoral Dissertation*, Chalmers Solid Mechanics, Gothenburg: 1-79.
- Igeland, A. and H. Ilias (1997). Rail head corrugation growth predictions based on non-linear high frequency vehicle/track interaction. *Wear* 213(1-2): 90-97.
- Igeland, A. (1999). Dynamic train/track interaction: parametric study and comparisons with full-scale measurements. *Heavy Vehicle Systems* 6(1): 221-237.
- Ilias, H. (1999). The influence of railpad stiffness on wheelset/track interaction and corrugation growth. *Journal of Sound and Vibration* 227(5): 935-948.
- ISO3095 (2005). Railway applications Acoustics Measurement of noise emitted by railbound vehicles.
- Jin, X. and J. Zhang (2001). A complementary principle of elastic bodies of arbitrary geometry in rolling contact. *Computers and Structures* 79(29-30): 2635-2644.
- Jin, X. S., Z. F. Wen and K. Y. Wang (2005). Effect of track irregularities on initiation and evolution of rail corrugation. *Journal of Sound and Vibration* 285(1-2): 121-148.
- Jin, X. S., Z. F. Wen, K. Y. Wang, Z. R. Zhou, Q. Y. Liu and C. H. Li (2006). Threedimensional train-track model for study of rail corrugation. *Journal of Sound and Vibration* 293(3-5): 830-855.
- Johansson, A. and J. C. O. Nielsen (2007). Rail corrugation growth influence of powered wheelsets with wheel tread irregularities. *Wear* 262(11-12): 1296-1307.
- Johnson, K. L. (2001). Contact Mechanics, Cambridge University Press.
- Jones, C. J. C. and D. J. Thompson (2003). Extended validation of a theoretical model for railway rolling noise using novel wheel and track designs. *Journal of Sound and Vibration* 267(3): 509-522.
- Jones, C. J. C., D. J. Thompson and R. J. Diehl (2006). The use of decay rates to analyse the performance of railway track in rolling noise generation. *Journal of Sound and Vibration* 293(3-5): 485-495.
- Jones, C., P. Fodiman, F. Letourneaux and B. Croft (2008a). Testing the new acoustic rail roughness measurement standard. In *Noise and Vibration Mitigation for Rail*

Transportation Systems Proceedings of the 9th International Workshop on Railway Noise, Munich, Germany, 4 - 8 September 2007: 363-369, Springer: Berlin.

- Jones, C. J. C., F. Letourneaux and P. Fodiman (2008b). Testing a new rail roughness measurement standard. *Proceedings of Acoustics 08*. Paris: 2649 2654.
- Kalker, J. J. (1967). On the rolling contact of two elastic bodies in the presence of dry friction. Delft, University of Technology. PhD.
- Kalker, J. J. (1970). Transient phenomena in two elastic cylinders rolling over each other with dry friction. *Transactions of the ASME. Series E, Journal of Applied Mechanics* 37(3): 677-688.
- Kalker, J. J. (1971a). A minimum principle for the law of dry friction, with application to elastic cylinders in rolling contact. I. Fundamentals-application to steady rolling.
 Transactions of the ASME. Series E, Journal of Applied Mechanics 38(4): 875-880.
- Kalker, J. J. (1971b). A minimum principle for the law of dry friction. II. Application to nonsteadily rolling elastic cylinders. *Transactions of the ASME. Series E, Journal of Applied Mechanics* 38(4): 881-887.
- Kalker, J. J. (1972). On elastic line contact. *Transactions of the ASME*. Series E, Journal of *Applied Mechanics* 39(4): 1125-1132.
- Kalker, J. J. (1982). Fast algorithm for the simplified theory of rolling contact. *Vehicle System Dynamics* 11(1): 1-13.
- Kalker, J. J. (1990). *Three-Dimensional Elastic Bodies in Rolling Contact*. Dordrecht, Kluwer Academic Publishers.
- Kalker, J. J. (1991). Wheel-rail rolling contact theory. Wear 144(1-2): 243-261.
- Kapoor, A. (1997). Wear by plastic ratchetting. Wear 212(1): 119-130.
- Kapoor, A. and F. J. Franklin (2000). Tribological Layers and the wear of ductile materials. *Wear* 245(1): 204-215.
- Kapoor, A., F. J. Franklin, S. K. Wong and M. Ishida (2002). Surface roughness and plastic flow in rail wheel contact. *Wear* 253(1-2): 257-264.
- Knothe, K. and S. L. Grassie (1993). Modelling of railway track and vehicle/track interaction at high frequencies. *Vehicle System Dynamics* 22(3-4): 209-262.
- Knothe, K., R. Wille and B. W. Zastrau (2001). Advanced contact mechanics road and rail. *Vehicle System Dynamics* 35(4-5): 361-407.
- Knothe, K. (2008). History of wheel/rail contact mechanics: from Redtenbacher to Kalker. *Vehicle System Dynamics* 46(1): 9 - 26.
- Knothe, K. and A. Gross-Thebing (2008). Short wavelength rail corrugation and non-steadystate contact mechanics. *Vehicle System Dynamics* 46(1-2): 49-66.
Kragelskii, I. V. (1965). Friction and Wear. London, Butterworths.

- Lesieutre, G. A. (1992). Finite elements for dynamic modeling of uniaxial rods with frequencydependent material properties. *International Journal of Solids and Structures* 29(12): 1567-1579.
- Lewis, R. and R. S. Dwyer-Joyce (2004). Wear mechanisms and transitions in railway wheel steels. *Proceedings of the Institution of Mechanical Engineers, Part J (Journal of Engineering Tribology)* 218(J6): 467.
- Lewis, R. and U. Olofsson (2004). Mapping rail wear regimes and transitions. *Wear* 257(7-8): 721-729.
- Lockett, F. J. (1972). Nonlinear Viscoelastic Solids. London, Academic Press.
- Love, A. E. H. (1929). The stress produced in a semi-infinite solid by pressure on part of the boundary. *Philosophical Transactions of the Royal Society of London* A228: 377-420.
- Lundén, R. and B. Åkesson (1983). Damped second-order Rayleigh-Timoshenko beam vibration in space an exact complex dynamic member stiffness matrix. *International Journal for Numerical Methods in Engineering* 19(3): 431-449.
- Maes, J. and H. Sol (2003). A double tuned rail damper increased damping at the two first pinned-pinned frequencies. *Journal of Sound and Vibration* 267(3): 721-737.
- Manabe, K. (2000). A hypothesis on a wavelength fixing mechanism of rail corrugation. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit 214(1): 21-26.
- Marshall, M. B., R. Lewis, R. S. Dwyer-Joyce, U. Olofsson and S. Bjorklund (2006). Experimental characterization of wheel-rail contact patch evolution. *Transactions of the* ASME. Journal of Tribology 128(3): 493-504.
- Martin, H. and G. Carey (1973). Introduction to Finite Element Analysis, McGraw-Hill.
- Martins, J. A. C., J. T. Oden and F. M. F. Simões (1990). A study of static and kinetic friction. *International Journal of Engineering Science* 28(1): 29-92.
- Meehan, P. A., W. J. T. Daniel and T. Campey (2005). Prediction of the growth of wear-type rail corrugation. *Wear* 258(7-8): 1001-1013.
- Meehan, P. A. and W. J. T. Daniel (2008). Effects of wheel passing frequency on wear-type corrugations. *Wear* 265(9-10): 1202-1211.
- Müller, S. (1999). A linear wheel-track model to predict instability and short pitch corrugation. *Journal of Sound and Vibration* 227(5): 899-913.
- Müller, S. (2000). A linear wheel-rail model to investigate stability and corrugation on straight track. *Wear* 243(1-2): 122-132.

- Nielsen, J. B. (1999). Evolution of rail corrugation predicted with a non-linear wear model. *Journal of Sound and Vibration* 227(5): 915-933.
- Nielsen, J. C. O. (1991). Eigenfrequencies and eigenmodes of beam structures on an elastic foundation. *Journal of Sound and Vibration* 145(3): 479-487.
- Nielsen, J. C. O. and T. J. S. Abrahamsson (1992). Coupling of physical and modal components for analysis of moving nonlinear dynamic-systems on general beam structures. *International Journal for Numerical Methods in Engineering* 33(9): 1843-1859.
- Nielsen, J. C. O. (1994). Dynamic interaction between wheel and track a parametric search towards an optimal design of rail structures. *Vehicle System Dynamics* 23(2): 115-132.
- Nielsen, J. C. O. and A. Igeland (1995). Vertical dynamic interaction between train and track influence of wheel and track imperfections. *Journal of Sound and Vibration* 187(5): 825-839.
- Nielsen, J. C. O. (2003). Numerical prediction of rail roughness growth on tangent railway tracks. *Journal of Sound and Vibration* 267(3): 537-548.
- Nielsen, J. C. O., R. Lunden, A. Johansson and T. Vernersson (2003). Train-track interaction and mechanisms of irregular wear on wheel and rail surfaces. *Vehicle System Dynamics* 40(1-3): 3-54.
- Nielsen, J. C. O. and J. Oscarsson (2004). Simulation of dynamic train-track interaction with state-dependent track properties. *Journal of Sound and Vibration* 275(3-5): 515-532.
- Nielsen, J. C. O. (2006). *High-frequency vertical wheel-rail contact forces validation of a prediction model by field testing*. 7th International Conference on Contact Mechanics and Wear of Rail/Wheel Systems, Brisbane, Australia.
- Nordborg, A. (2002). Wheel/rail noise generation due to nonlinear effects and parametric excitation. *Journal of the Acoustical Society of America* 111(4): 1772-1781.
- Oertli, J. (2008). Railway noise abatement: the case for retrofitting freight vehicles with composite brake blocks. In Noise and Vibration Mitigation for Rail Transportation Systems Proceedings of the 9th International Workshop on Railway Noise, Munich, Germany, 4 - 8 September 2007: 236-242, Springer: Berlin.
- Olofsson, U. and R. Lewis (2006). Tribology of the Wheel-Rail Contact. *Handbook of Railway Vehicle Dynamics*. S. D. Iwnicki (ed.), Taylor & Francis: 121-141.
- Pauk, V. and B. Zastrau (2003). Rolling contact problem involving surface roughness. Mechanics Research Communications 30(1): 45-51.
- Petyt, M. (1990). *Introduction to finite element vibration analysis*. Cambridge, Cambridge University Press.

- Pieringer, A., W. Kropp and J. C. O. Nielsen (2008). A time-domain model for wheel/rail interation aiming to include non-linear contact stiffness and tangential friction. In *Noise and Vibration Mitigation for Rail Transportation Systems Proceedings of the 9th International Workshop on Railway Noise*, Munich, Germany, 4 8 September 2007: 285-291, Springer: Berlin.
- Polach, O. (2005). Creep forces in simulations of traction vehicles running on adhesion limit. *Wear* 258(7-8): 992-1000.
- Popp, K., H. Kruse and I. Kaiser (1999). Vehicle-track dynamics in the mid-frequency range. *Vehicle System Dynamics* 31(5-6): 423-464.
- Rabinowicz, E. (1965). Friction and wear of materials. New York, Wiley.
- Remington, P. J. (1976a). Wheel/rail noise part I: characterization of the wheel/rail dynamic system. *Journal of Sound and Vibration* 46(3): 359-379.
- Remington, P. J. (1976b). Wheel/rail noise part IV: rolling noise. *Journal of Sound and Vibration* 46(3): 419-436.
- Remington, P. and J. Webb (1996). Estimation of wheel/rail interaction forces in the contact area due to roughness. *Journal of Sound and Vibration* 193(1): 83-102.
- Richards, T. H. (1977). Energy Methods in Stress Analysis: with an introduction to finite element techniques. Chichester, Ellis Horwood Limited.
- Sato, Y., A. Matsumoto and K. Knothe (2002). Review on rail corrugation studies. *Wear* 253(1-2): 130-139.
- Shampine, L. F. and M. W. Reichelt (1997). The MATLAB ODE suite. *SIAM Journal on Scientific Computing* 18(1): 1-22.
- Shen, Z. Y., J. K. Hedrick and J. A. Elkins (1983). Comparison of alternative creep force models for rail vehicle dynamic analysis. *Vehicle System Dynamics* 12(1-3): 79-78.
- Sheng, X., D. J. Thompson and C. J. C. Jones (2004). *Modelling rail roughness growth on tangent tracks*, Institute of Sound and Vibration Research Technical Memorandum 929.
- Sheng, X., C. J. C. Jones and D. J. Thompson (2005). Responses of infinite periodic structures to moving or stationary harmonic loads. *Journal of Sound and Vibration* 282(1-2): 125-149.
- Sheng, X., D. J. Thompson, C. J. C. Jones, G. Xie, S. D. Iwnicki, P. Allen and S. S. Hsu (2006). Simulations of roughness initiation and growth on railway rails. *Journal of Sound and Vibration* 293(3-5): 819-829.
- Sheng, X., M. Li, C. J. C. Jones and D. J. Thompson (2007). Using the Fourier-series approach to study interactions between moving wheels and a periodically supported rail. *Journal* of Sound and Vibration 303(3-5): 873-894.

- Stiebel, D. (2005). G.D5 Identification of Adequate Test Tracks, SILENCE project deliverable, Deutsche Bahn.
- Thompson, D. J. (1993a). Wheel-rail noise generation, Part I: Introduction and interaction model. *Journal of Sound and Vibration* 161(3): 387-400.
- Thompson, D. J. (1993b). Wheel-rail noise generation, Part II: Wheel vibration. *Journal of Sound and Vibration* 161(3): 401-419.
- Thompson, D. J. (1993c). Wheel-rail noise generation, Part III: Rail vibration. *Journal of Sound and Vibration* 161(3): 421-446.
- Thompson, D. J. (1993d). Wheel-rail noise generation, Part IV: Contact zone and results. *Journal of Sound and Vibration* 161(3): 447-466.
- Thompson, D. J. (1993e). Wheel-rail noise generation, Part V: Inclusion of wheel rotation. *Journal of Sound and Vibration* 161(3): 467-482.
- Thompson, D. J. (1996). On the relationship between wheel and rail surface roughness and rolling noise. *Journal of Sound and Vibration* 193(1): 149-160.
- Thompson, D. J., P. Fodiman and H. Mahe (1996a). Experimental validation of the TWINS prediction program for rolling noise, Part 2: results. *Journal of Sound and Vibration* 193(1): 137-147.
- Thompson, D. J., B. Hemsworth and N. Vincent (1996b). Experimental validation of the TWINS prediction program for rolling noise, Part 1: description of the model and method. *Journal of Sound and Vibration* 193(1): 123-135.
- Thompson, D. J. and C. J. C. Jones (2000). Review of the modelling of wheel/rail noise generation. *Journal of Sound and Vibration* 231(3): 519-536.
- Thompson, D. J. (2003). The influence of the contact zone on the excitation of wheel/rail noise. *Journal of Sound and Vibration* 267(3): 523-535.
- Thompson, D. J. and P. E. Gautier (2006). Review of research into wheel/rail rolling noise reduction. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 220(4): 385-408.
- Thompson, D. J., C. J. C. Jones, T. P. Waters and D. Farrington (2007). A tuned damping device for reducing noise from railway track. *Applied Acoustics* 68(1): 43-57.
- Thompson, D. J. (2008). A continuous damped vibration absorber to reduce broad-band wave propagation in beams. *Journal of Sound and Vibration* 311(3-5): 824-842.
- Thompson, D. J. (2009). *Railway noise and vibration: mechanisms, modelling and means of control.* Oxford, Elsevier.
- Timoshenko, S., D.H. Young, W. Weaver (1974) *Vibration Problems in Engineering*. 4th ed., New York, John Wiley & Sons.

UIC Freight Noise Focus (March 2008) No.2: 1-5.

- Verheijen, E. (2006). A survey on roughness measurements. *Journal of Sound and Vibration* 293(3-5): 784-794.
- Vermuelen, P. J. and K. L. Johnson (1964). Contact of non-spherical bodies transmitting tangential forces. *Journal of Applied Mechanics* 31: 338-340.
- Vincent, N., P. Bouvet, D. J. Thompson and P. E. Gautier (1996). Theoretical optimization of track components to reduce rolling noise. *Journal of Sound and Vibration* 193(1): 161-171.
- Vuong, T. T. and P. A. Meehan (2009). Wear transitions in a wear coefficient model. *Wear* 266(9-10): 898-906.
- Wen, Z. F. and X. S. Jin (2006). Elastic-plastic finite-element analysis of repeated, twodimensional wheel-rail rolling contact under time-dependent load. *Proceedings of the Institution of Mechanical Engineers, Part C (Journal of Mechanical Engineering Science)* 220(C5): 603-613.
- Wittrick, W. H. and F. W. Williams (1971). A general algorithm for computing natural frequencies of elastic structures. *Quarterly Journal of Mechanics and Applied Mathematics* 24: 263-284.
- Wu, T. X. and D. J. Thompson (1999a). A double Timoshenko beam model for vertical vibration analysis of railway track at high frequencies. *Journal of Sound and Vibration* 224(2): 329-348.
- Wu, T. X. and D. J. Thompson (1999b). Analysis of lateral vibration behavior of railway track at high frequencies using a continuously supported multiple beam model. *Journal of the Acoustical Society of America* 106(3): 1369-1376.
- Wu, T. X. and D. J. Thompson (2000). Influence of random sleeper spacing and ballast stiffness on the vibration behaviour of railway track. *Acustica united with Acta Acustica* 86(2): 313-321.
- Wu, T. X. and D. J. Thompson (2001). Vibration analysis of railway track with multiple wheels on the rail. *Journal of Sound and Vibration* 239(1): 69-97.
- Wu, T. X. and D. J. Thompson (2002). Behaviour of the normal contact force under multiple wheel/rail interaction. *Vehicle System Dynamics* 37(3): 157-174.
- Wu, T. X. and D. J. Thompson (2004). On the parametric excitation of the wheel/track system. *Journal of Sound and Vibration* 278(4-5): 725-747.
- Wu, T. X. and D. J. Thompson (2005). An investigation into rail corrugation due to micro-slip under multiple wheel/rail interactions. *Wear* 258(7-8): 1115-1125.

- Xie, G., P. D. Allen, S. D. Iwnicki, A. Alonso, D. J. Thompson, C. J. C. Jones and Z. Y. Huang (2006). Introduction of falling friction coefficients into curving calculations for studying curve squeal noise. *Vehicle System Dynamics* 44: 261-271.
- Xie, G. and S. D. Iwnicki (2008a). Calculation of wear on a corrugated rail using a threedimensional contact model. *Wear* 265(9-10): 1238-1248.
- Xie, G. and S. D. Iwnicki (2008b). Simulation of wear on a rough rail using a time-domain wheel-track interaction model. *Wear* 265(11-12): 1572-1583.
- Xie, G. and S. D. Iwnicki (2008c). Simulations of roughness growth on rails Results from a 2D non-Hertzian, non-steady contact model. *Vehicle System Dynamics* 46(1-2): 117-128.
- Xuefeng, T. and B. Bhushan (1996). A numerical three-dimensional model for the contact of rough surfaces by variational principle. *Transactions of the ASME*. *Journal of Tribology* 118(1): 33-42.
- Zhang, W., J. Chen, X. Wu and X. Jin (2002). Wheel/rail adhesion and analysis by using full scale roller rig. *Wear* 253(1-2): 82-88.
- Zhai, W. M., C. B. Cai, S. Z. Guo (1996). Coupling model of vertical and lateral vehicle/track interactions. *Vehicle System Dynamics* 26(1): 61-79.