In this paper, we introduce the psychological concept of anxiety into agency theory. An important benchmark in the anxiety literature is the inverted-U hypothesis, which states that an increase in anxiety improves performance when anxiety is low, but reduces it when anxiety is high. We show that the inverted-U hypothesis is consistent with evidence that high-powered incentives can reduce the agent’s optimal effort and expected performance. In equilibrium, however, a profit-maximizing principal never offers such counterproductive incentives. We also show that the inverted-U hypothesis can explain empirical anomalies related to monitoring, the informativeness principle, and the risk–reward tradeoff.

1. Introduction

The economic concept of risk aversion expresses a primitive distaste for uncertainty. In psychology, however, attitudes toward risk are subsumed under the much broader concept of anxiety, which extends well beyond a preference for certainty to include cognitive and physiological components. For example, anxiety is often measured using cognitive self-report scales as well as physical indicators such as heart rate, blood pressure, muscle tension, and EEG frequency. An important benchmark in the anxiety literature is the inverted-U hypothesis (IUH) or
Yerkes-Dodson Law, which states that the relationship between anxiety and performance is an inverted-U—an increase in anxiety improves performance when anxiety is low, but reduces it when anxiety is high. The book “Just Enough Anxiety” by Rosen (2008) popularizes this idea for a broad business audience.

There is substantial evidence on the importance of anxiety, even for professionals. Singh (1998) finds that certain stressors reduce the performance of professional salespeople, whereas Jordet et al. (2007), Dohmen (2008), and Apesteguia and Palacios-Huerta (2008) find that anxiety and pressure are an important element of penalty kicks in professional soccer. Ariely et al. (2005) consider the effects of incentives on performance in six separate tasks. These experiments were conducted in rural India, which allowed the experimenters to provide extremely high incentives. Indeed, the payment for success in any one task in their high-incentive treatment was equivalent to about one month’s per capita consumption. In all six tasks, they found either a decreasing or inverted-U relationship between incentives and performance. To explain their findings, Ariely et al. explicitly invoke the IUH and suggest that many of their subjects “collapsed under pressure” similar to students suffering from test anxiety.2

To assess the relative importance of anxiety as compared with other psychological and personality factors, we consider the five factor model in personality psychology, where the five main traits are Neuroticism, Extraversion, Openness to Experience, Agreeableness, and Conscientiousness.

Neuroticism, often labeled by the positive pole of the trait Emotional Stability, is the tendency to show poor emotional adjustment in the form of stress, anxiety, and depression. Extraversion represents the tendency to be sociable, dominant, and positive . . . Individuals who score high on Openness to Experience are creative, flexible, curious, and unconventional . . . Agreeableness consists of tendencies to be kind, gentle, trusting and trustworthy, and warm. Finally, Conscientious individuals are achievement-oriented and dependable . . ., as well as orderly and deliberate.


According to the meta-analysis of Judge and Ilies (p. 803), the most important factor for motivation (e.g., an agent’s choice of effort) is Neuroticism (which includes anxiety) followed by Conscientiousness.

2. We thank Roland Bénabou for informing us about this paper.
Likewise, the meta-analysis in Hurtz and Donovan (2000, table 1) finds that the most important trait for job performance (including customer service, managerial, sales, and other jobs) is Conscientiousness, followed by Neuroticism. On this basis, we conclude that anxiety is one of the most important psychological factors in principal–agent relationships.

In this paper, we introduce the IUH into the linear principal–agent model of Holmström and Milgrom (1991). The specifics of our model are motivated by the processing efficiency theory (PET) of Eysenck and Calvo (1992) in cognitive psychology, where the initial impact of anxiety is to induce worry, a cognitive activity which uses up scarce attentional resources and “crowds out” the agent’s effort in the principal’s task. On the other hand, worry also stimulates various coping mechanisms which can offset its crowding-out effects, so an increase in anxiety can either increase or decrease the agent’s effort and performance.

Because worry is evidently the agent’s next best activity, we incorporate the IUH into the agent’s disutility of effort, which represents the opportunity cost of time spent on the principal’s task. Specifically, we assume that anxiety is proxied by income risk (the standard deviation of income) and the marginal cost of effort is U-shaped in the latter. As income risk increases, the agent’s anxiety grows, but initially his coping mechanisms more than suffice and the marginal cost of effort declines. Eventually, however, the marginal cost of effort is increasing in anxiety because of diminishing returns to coping. Given this assumption, there is an inverted-U relationship between the agent’s optimal effort and incentives, as well as between optimal effort and output risk (the standard deviation of the productivity shock) in accordance with the IUH. In particular, high-powered incentives can induce “choking under pressure” as in the Ariely et al. experiments.

In addition to Ariely et al., Gneezy and Rustichini (2000), Gneezy (2003), and the survey by Frey and Jegen (2001) provide evidence that incentives can in fact backfire (i.e., reduce effort and performance). The theoretical literature on the hidden costs of incentives includes Bénabou and Tirole (2003, 2006) and Guo and Ou-Yang (2006). At the same time, there is no doubt but that incentives can be an extremely powerful motivator. For example, Lazear (2000, p. 1347) reports that productivity increased by about 44% (some of it due to selection effects) when the Safelite Glass Corporation switched from hourly wages to piece rates:

Some conclusions are unambiguous. Workers respond to prices just as economic theory predicts. Claims by

3. Casadesus-Masanell (2004) (see the case of ethical standards) and Ramalingam and Rauh (2008) consider the work ethic, which is subsumed under Conscientiousness.
sociologists and others that monetizing incentives may actually reduce output are unambiguously refuted by the data.

As Prendergast (1999, p. 18) observes, “While this idea [counterproductive incentives] holds some intuitive appeal, it should be noted that there is little conclusive empirical evidence (particularly in workplace settings) of these influences.” A similar dichotomy occurs in our model—although they can indeed backfire, a profit-maximizing principal never offers such incentives. There is therefore nothing inherently contradictory about observing counterproductive incentives in experiments (where they are often exogenous) but not in the field (where they are endogenous). Our paper therefore formalizes aspects of the more general but informal critique in Glaeser (2003).

The IUH can also explain other empirical anomalies. According to the informativeness principle due to Holmström (1979), a given performance measure should be included in the contract iff it is informative (at the margin) with respect to the agent’s effort. In particular, the principal should never inject pure noise into the contract. As Prendergast (1999, p. 21) notes, however, “perhaps the most striking aspect of observed contracts is that the Informativeness Principle . . . seems to be violated in many occupations.” For example, Oyer and Schaefer (2005) document that many firms grant stock options to all employees even though they impose substantial risk with only trivial incentive effects. In this paper, we show that the principal wants to inject pure noise into the contract when the workplace environment is too static because in that case an increase in anxiety is motivational.

Another empirical anomaly is the mixed evidence for the classical risk–reward tradeoff (see Prendergast, 1999), which predicts that incentives should be decreasing in output risk. In this paper, we show that the risk–reward tradeoff is violated when anxiety is low and the agent’s optimal effort is increasing in both incentives and output risk. In that case, the latter are strategic complements for the principal and will therefore be positively related. Finally, we consider some implications of the IUH for optimal monitoring in light of the evidence in Barkema (1995).

Although Casadesus-Masanell (2004) and Oyer (2004) provide important alternative explanations for violations of the informativeness principle and Guo and Ou-Yang (2006) and Prendergast (2002a, b) for the risk–reward tradeoff, an advantage of our approach is that it addresses all of the above issues within a single theoretical framework. As Rosen (2008) emphasizes, the common intuition underlying all our results is

4. The experimental literature on endogenous incentives includes Fehr and Gächter (2000) and Fehr et al. (2007), among others.
that one of the central tasks of management is to foster an optimal amount of tension in the workplace.

The anxiety literature in economics was pioneered by Loewenstein (1987) and Caplin and Leahy (2001). The former paper introduces the concept of the utility of anticipation to explain why a rational agent might delay a pleasant experience in order to savor it, but hasten an unpleasant one to reduce the period of dread (or anxiety). The latter paper provides a comprehensive theoretical framework for thinking about anxiety and other anticipatory emotions. An important element in that framework is an abstract exogenous map from economic outcomes (e.g., income) to psychological states (e.g., anxiety), which is taken to be an economic primitive akin to preferences. In Rauh and Seccia (2006), we develop a more structured anxiety concept, albeit in a specific learning context, and show that the IUH obtains over a certain region of parameter space. In a closely related paper, Caplin (2003) considers the use of fear as a policy instrument (e.g., antismoking campaigns) and shows that its effects can be nonmonotonic, because moderate fear messages can induce an appropriate behavior by capturing the recipient’s attention, whereas extreme ones can backfire by provoking avoidance (i.e., defensive inattention).

The plan for the rest of the paper is as follows. In the next section, we briefly survey aspects of the anxiety literature in economics, psychology, and other disciplines. In Section 3, we present our model and results. Section 4 examines some of our modeling choices in more detail, including our reliance on the linear Holmström and Milgrom (1991) framework and our assumptions on preferences that incorporate the IUH. Section 5 concludes. All proofs are in the Appendix.

2. Anxiety and the Linear Model

For motivational purposes, we recall aspects of the linear agency model with a single task. Assume output is given by $q = e + \epsilon$, where $e$ is the agent’s effort and $\epsilon$ is a normally distributed productivity shock with mean zero and variance $\sigma^2$. In this context, the agent’s expected performance (i.e., expected output) is equivalent to effort. We restrict the class of feasible contracts to those that are linear in output $I = \alpha + \beta q$, where $I$ is income, $\alpha$ the fixed component, and $\beta$ the piece rate. We discuss the assumption of linear contracts in detail in Section 4 below.

Assume the agent’s utility function is given by

$$-\exp(-r[I - (1/2)ke^2]),$$

(1)

where $r$ is the coefficient of absolute risk aversion and the expression following income is the agent’s quadratic disutility of effort. Under
these conditions (see Bolton and Dewatripont, 2005, p. 137–139), the agent’s certainty equivalent is

$$\alpha + \beta e - (1/2)ke^2 - (1/2)r\beta^2\sigma^2,$$

where the first two terms represent expected income and the final term is the risk premium, which expresses the cost of risk in monetary terms.

In this context, the agent’s optimal effort $e = \beta/k$ is strictly increasing in $\beta$ and is therefore inconsistent with the evidence on counterproductive incentives (e.g., Ariely et al.). It is also completely independent of output risk $\sigma$, which is not generally the case but an artifact of the specific assumptions of negative exponential utility, linear contracts, and normality. The discrepancy with the anxiety literature is even more stark, where there is substantial experimental and field evidence that anxiety has important implications for behavior.

2.1 Definitions

The IUH has been expressed in terms of stress, arousal, and anxiety. Although definitions have not been standardized, the first is often defined as

a state in which some demand is placed on the individual, who is then required to react in some way to be able to cope with the situation . . . If one doubts one’s ability to cope with the stressor, then feelings of anxiety will likely ensue.

Woodman and Hardy (2001, p. 290)

Note that stress can be a negative or positive state depending on the agent’s expectations about coping with it. The concept of arousal has been defined in terms of alertness, intensity, motivation, and readiness.

Although controversy remains over the meaning of arousal, most contemporary researchers . . . operationalize arousal as a condition that ranges along the sleep-high excitation continuum and finds expression in physiological, psychological (cognitive and affective), and behavioral terms.

Zaichkowsky and Baltzell (2001, p. 320–322)

According to Woodman and Hardy (2001, p. 290–291),

Anxiety is generally accepted as being an unpleasant emotion . . . Researchers in mainstream psychology have suggested that anxiety might have at least two distinguishable components: a mental component normally termed cognitive
anxiety or worry, and a physiological component normally termed somatic anxiety or physiological arousal.

(italics in the original)

With respect to the first component,

Worry is a cognitive phenomenon, it is concerned with future events where there is uncertainty about the outcome, the future being thought about is a negative one, and this is accompanied by feelings of anxiety.

MacLeod et al. (1991, p. 478)

(as quoted in Caplin and Leahy, 2001)

The term physiological arousal refers to objective physiological manifestations of arousal (e.g., an elevated heart rate), whereas somatic anxiety is the agent’s subjective perception of them which can be assessed using self-report scales.

2.2 The Processing Efficiency Theory

The relationship between anxiety, effort, and expected performance is the subject of the PET of Eysenck and Calvo (1992) from cognitive psychology. According to the PET, anxiety has two potential effects. First, the initial impact of a negative stressor (e.g., income risk) is to induce worry, which consumes the agent’s cognitive resources and “crowds out” his on-task effort (e.g., effort on the principal’s task). The second potential effect distinguishes the PET from previous anxiety theories.

Subsequently, there are two major types of reaction to poor performance and threat of aversive consequences: (1) coping directly with the current level of threat and worry (e.g., repression or denial; calming down and self-revalorisation) . . . (2) reduction or elimination of the effects of worry on performance by applying additional resources (e.g., effort or time) or activities (e.g., rote learning, articulatory rehearsal, seeking external assistance) to the task.

Eysenck and Calvo (1992, p. 416)

That is, the agent can either cope directly, which reduces worry and its associated crowding-out effects, or he can increase effort or effort capacity. In either case, on-task effort increases. The overall effect of anxiety on effort and expected performance therefore depends on the
relative magnitudes of these two effects: the initial crowding-out effect and the agent’s subsequent response.

The IUH follows from the PET assuming diminishing returns with respect to the second effect. In that case, the agent’s coping mechanisms and/or re-allocation of effort resources prevail when anxiety is low, but eventually the crowding-out effect dominates when anxiety is high. We also emphasize that the above considerations extend well beyond a primitive dislike of risk. In the linear model, optimal effort does not depend on income risk and the only effect of the latter is to increase the amount of the fixed component $\alpha$ of the contract to compensate the agent for the increase in his risk premium. In more general moral hazard models such as Holmström (1979), a risk-averse agent chooses effort in part to reduce risk, but that characterization falls well short of the cognitive processes (e.g., worry and coping) described in the PET.

2.3 Evidence for the Inverted-U Hypothesis

Despite its status as the main benchmark in the anxiety literature, the evidence for the IUH is mixed. As Muse et al. (2003, p. 355) point out, the evidence is strongest in the case where the IUH is expressed in terms of arousal. For example, the Ariely et al. experiments fall into this category, because arousal is often characterized in terms of incentives and motivation. In their survey of the sports psychology literature, Zaichkowsky and Baltzell (2001) discuss several experimental and field studies that support the IUH, as well as some that do not. Anderson (1990) defends the usefulness of the arousal concept and the IUH against the criticisms in Neiss (1988) and notes that the evidence for the IUH is stronger for cognitive as opposed to motor tasks, where the former are often more important in workplace environments.

The pioneering study in the stress literature was Anderson (1976), who found an inverted-U relationship between recovery performance and perceived stress among 102 small business owners after Hurricane Agnes in 1972. Similar to the PET, high recovery performance was associated with moderate perceived stress and problem-solving coping mechanisms. Likewise, Baer and Oldham (2006) find an inverted-U relationship between creativity and creative time pressure among employees at a certain cereal manufacturer who scored high on creative support and Openness to Experience in the five factor model. The authors recommend (p. 969) that

if management is interested in boosting creativity, supervisors might first identify the objective conditions that produce the experience of creative time pressure and alter those
conditions so employees experience intermediate pressure with respect to creative pursuits.

Rosen’s (2008) “Just Enough Anxiety” makes the same point more broadly.

Overall, the evidence is weaker when the IUH is expressed in terms of stress. According to Muse et al., out of 52 studies 46% find a negative linear relationship, 13% a positive linear one, 4% support the IUH, and the rest report either mixed evidence or no relationship between stress and performance. As they point out, however, the bulk of these studies report linear correlations and therefore rule out the IUH a priori. Furthermore, a positive or negative monotonic relationship is not necessarily inconsistent with the IUH, because the data might correspond to either the left or right component of the inverted-U, respectively. Finally, Muse et al. argue that all but one of the 52 studies suffer from at least one of the following defects: (i) using stress measures that are not designed to register low levels of stress or (ii) that are exclusively associated with negative connotations of stress and (iii) over-sampling high-stress subjects.

### 2.4 Professionals

The subjects in the Ariely et al. experiments were relative novices. Do professionals choke under pressure? Palacios-Huerta (2003) and Palacios-Huerta and Volij (2007, 2008) find that chess and professional soccer players choose approximately equilibrium strategies in strategic settings that mimic their professional environments, whereas college students do not.5

Nevertheless, there is substantial evidence that professionals can and do choke under pressure. For example, Dohmen (2008) shows that professional soccer players are more likely to choke on penalty kicks at home. He explains his findings in terms of the social pressure hypothesis from social psychology, which states that even friendly observation can impair performance (note that Ariely et al. found similar results). Jordet et al. (2007) find that choking on penalty kicks is more likely in World Cup soccer matches than in the European Championships and Copa America because the stakes are higher in the former. Apesteguia and Palacios-Huerta (2008) find that the team who kicks first in a penalty shoot-out wins about 60% of the time. The source for this first-mover advantage is that the probability of scoring is worse when the other team is ahead. Indeed, about 98% of 242 players and coaches interviewed

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5. We thank an anonymous referee for referring us to this literature.
stated a preference for kicking first and 96% explained their preference
in terms of putting pressure on the other team.

Finally, many of the studies on performance and stress in the
previous subsection are based on field data from workplace settings.
For example, Baer and Oldham (2006) found evidence in support of the
IUH using field data on employees at a cereal manufacturer and Singh
(1998) found a negative relationship between sales performance and
certain stressors (role ambiguity and role conflict) using survey data
from actual salespeople.

3. The Model and Results

3.1 Incorporating the IUH

According to the PET, the initial impact of anxiety is to induce worry, a
cognitive activity that uses up scarce attentional resources and crowds-
out the agent’s effort on the principal’s task. Because worry is evidently
the agent’s next best activity (anxiety causes a substitution away from
effort toward worry) and his disutility of effort captures the opportunity
cost of time spent on the principal’s task, we incorporate the effects
of worry in the latter. In particular, we assume anxiety increases the
marginal cost of effort as the agent devotes more cognitive resources
toward worry. Likewise, the agent’s responses in terms of coping mech-
anisms and the re-allocation of effort resources also seem connected
to the marginal cost of effort. For example, positive thoughts (“self-
revalorisation”) which reduce worry and free up attentional resources
for the principal’s task reduce the marginal cost of effort. We therefore
incorporate the effects of anxiety into the linear model by assuming
the agent’s cost of effort depends on the standard deviation \( \sigma_I = \beta \sigma \) of
income:

\[
U = \alpha + \beta e - (1/2)\phi(\beta \sigma)e^2 - (1/2)r\beta^2\sigma^2. \tag{3}
\]

This is the same as (2) except the constant \( k \) has been replaced by \( \phi(\sigma_I) \).
In (3), the risk premium reflects the sense in which anxiety is generally
a “negative emotion” increasing in the level \( \sigma_I \) of stress, whereas the
motivational effects of anxiety corresponding to the three main elements
of the PET—worry, coping, and the re-allocation of effort resources—
are captured by the function \( \phi \). Assumptions 1 below operationalize the
IUH by assuming \( \phi \) is smooth, positive, and U-shaped.

**Assumptions 1:** (i) \( \phi \) is positive and twice continuously differentiable on
\([0, \infty), \text{ with } \phi'' > 0. \) (ii) There exists a constant \( \hat{\sigma}_I > 0 \) such that \( \phi' < 0 \) on
[0, \hat{\sigma}_I), \phi'(\hat{\sigma}_I) = 0, and \phi' > 0 on (\hat{\sigma}_I, \infty). We assume \phi'(0) is finite. (iii) Let 
\psi(\sigma_I) \equiv \phi(\sigma_I) - \sigma_I\phi'(\sigma_I). \tag{4}
There exists a constant \tilde{\sigma}_I > \hat{\sigma}_I such that \psi(\tilde{\sigma}_I) < 0.

As anxiety \sigma_I increases from a low level on [0, \hat{\sigma}_I), the crowding-out effects of worry are dominated by the agent’s adaptive responses and \phi and the marginal cost of effort fall. Assuming diminishing returns to coping, the two effects are evenly balanced at \hat{\sigma}_I, where \phi' = 0 and \phi achieves its minimum. After that, increases in anxiety \sigma_I on (\hat{\sigma}_I, \infty) increase \phi and the marginal cost of effort because the crowding-out effect dominates. We postpone our discussion of (iii) until after Proposition 1 below.

Remarks. In this paper, our purpose is not to explain the IUH but to explore some of its implications for incentive and monitoring mechanisms and to show that it can account for several empirical anomalies. Although one would prefer a model that does both, our approach via the reduced-form representation \phi does have advantages in terms of simplicity and generality, because the IUH lacks generally accepted formal theoretical foundations and \phi is noncommittal in that respect. Another issue is that the agent’s preferences are defined directly in terms of \textit{income risk} \sigma_I instead of more basic outcomes such as income. We discuss these and other related issues in detail in Section 4 below.

\section{3.2 Optimal Effort}

Given output risk \sigma and the contract (\alpha, \beta), the agent maximizes (3) subject to \( e \geq 0 \). Throughout the paper, partial derivatives are indicated by subscripts.

\textbf{Proposition 1:} (i) The agent’s optimal effort is given by \( e = \beta/\phi \) with 
\( e_\beta = \psi/\phi^2 \) and \( e_\sigma = -\beta^2 \phi'/\phi^2 \). (ii) The IUH obtains in the sense that optimal effort and expected performance have an inverted-U relationship with \beta when \sigma > 0 is held fixed and with \sigma when \beta > 0 is held fixed. (iii) The cross-partial is 
\[ e_{\beta\sigma} = \frac{\beta}{\phi^3} \left[ 2\sigma_1 \phi'^2 - \phi(2\phi' + \sigma_1 \phi'') \right]. \tag{5} \]

In the linear model \( e = \beta/k \), which is strictly increasing in incentives and independent of output risk. In contrast, in our model \( e = \beta/\phi \) where \phi is U-shaped in income risk. It follows that optimal effort has an inverted-U relationship with output risk holding \beta > 0 constant,
where the maximum occurs at $\sigma_I = \hat{\sigma}_I$. Assumption 1(iii) ensures that eventually $\psi < 0$ (the marginal cost of effort is sufficiently increasing in income risk) and $e_\beta < 0$. Incentives are therefore counterproductive when they generate too much anxiety as in the Ariely et al. experiments. We state (iii) for future purposes.

Figure 1 illustrates the two main forces at work when the principal increases incentives.

From (3), the marginal benefit of effort is the piece rate $\beta$, whereas the marginal cost of effort is $\phi e$. In Figure 1, optimal effort is $e_1$ when the piece rate is $\beta_1$ and the marginal cost of effort is $MC_1$. If the principal increases incentives from $\beta_1$ to $\beta_2$ then optimal effort would increase from $e_1$ to $e_2$. In the linear model, there are no further effects and the final level of effort is $e_2$.

In our model, however, an increase in incentives also increases income risk and anxiety. If the former was initially low then the subsequent increase in anxiety provides an additional source of motivation as per the IUH. In that case, $\phi$ falls and the marginal cost of effort shifts right from $MC_1$ to $MC_2$, causing a further increase in effort from $e_2$ to $e_3$ for an overall increase from $e_1$ to $e_3$. In this case, incentives are even more powerful than in the linear model because of the additional motivational effects of anxiety. In contrast, if income risk was initially high then the increase in anxiety is demotivational and the marginal cost of effort shifts up from $MC_1$ to $MC_3$, leading to a reduction in effort from $e_2$ to $e_4$. 

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**FIGURE 1. INCENTIVES, AND MARGINAL BENEFITS AND COSTS OF EFFORT**

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When the latter effect is strong enough, the overall impact of incentives is to reduce optimal effort from $e_1$ to $e_4$. In this case, high-powered incentives cause the agent to choke under pressure as in the Ariely et al. experiments. Note that changes in output risk $\sigma$ only shift the marginal cost curve.

Bénabou and Tirole (2003) show that incentives can have hidden costs when they signal to the agent that the principal’s task is difficult or distasteful. In Bénabou and Tirole (2006), incentives can discourage prosocial behavior (e.g., donating blood) when they spoil the reputational value of the latter by creating uncertainty about whether the agent was motivated by prosocial preferences or money. Guo and Ou-Yang (2006) show that optimal effort can be decreasing in incentives and the risk–reward tradeoff violated when the agent chooses both effort and output risk $\sigma$ and his utility function exhibits wealth effects. Because optimal effort is always increasing in incentives in Bénabou and Tirole (2003), none of these explanations seem to apply to the Ariely et al. and related experiments discussed in the previous section.

### 3.3 Example

Assume

$$\phi(\sigma_I) = \frac{1}{a \sigma_I^2 + b \sigma_I + c}. \quad (6)$$

If $a = -1$, $b = 2$, and $c = 1$ then $\phi$ is U-shaped as in Assumptions 1. In this example, optimal effort is

$$e = \beta[c + \beta \sigma(b + a \beta \sigma)]. \quad (7)$$

If we fix the above parameter values and set $\sigma = 2$ then a plot of (7) as a function of $\beta$ confirms the inverted-U relationship stated in Proposition 1(ii).

### 3.4 The Principal’s Problem and Ultra-Incentives

We assume the principal is risk neutral, the price of output is one, and the agent’s outside option is zero. The principal’s profit is therefore $q - I - M(\sigma)$ and her expected profit $e - \alpha - \beta e - M(\sigma)$. After substituting the agent’s participation constraint $U = 0$ we obtain

$$\Pi = e - (1/2)\phi e^2 - (1/2)r \beta^2 \sigma^2 - M(\sigma). \quad (8)$$
In the standard case where $\phi$ is constant and $\sigma$ is exogenous, we obtain the familiar expression

$$\beta_L = \frac{1}{1 + r\sigma^2\phi}$$

(9)

for the optimal incentive in the linear model (see Bolton and Dewatripont, 2005, p. 139).

In this paper, we follow Milgrom and Roberts (1992, Chapter 7) and assume the principal has access to a monitoring technology which can reduce output risk $\sigma$ at some cost, so output becomes a more informative signal of effort. For example, the principal could invest in better information and technology systems to more accurately track and record performance or redesign the agent’s job to increase its focus, make his responsibilities clearer, and his performance more easily measurable. This would reduce what Singh (1998) and the marketing literature refer to as “role ambiguity.” Formally, we assume the principal can achieve a desired level of $\sigma$ at cost $M(\sigma)$, where $M' \leq 0$, $M'' \geq 0$, and the marginal cost of monitoring is $|M'| = -M'$. Let $\sigma$ be the baseline level of output risk associated with zero monitoring expenditure, so $M(\sigma) = 0$.

After substituting optimal effort $e = \beta/\phi$, the principal’s problem is to choose $\beta$ and $\sigma$ to maximize (8) subject to $0 \leq \sigma \leq \sigma$.

**Proposition 2:** (i) The principal’s problem has at least one solution which entails $0 < \beta < 2$. (ii) The optimal incentive can exceed one (the price of the product).

Given our normalization of the price, the optimal incentive (9) in the linear model never exceeds one. As we will see, however, the optimal incentive in our model does exceed one in a neighborhood of $\sigma = 0$ where anxiety is low and $\phi$ is decreasing. This is because incentives increase the value of the firm by reducing the cost of effort more than the corresponding increase in the risk premium (the proof of Proposition 6 in the Appendix establishes this rigorously). Intuitively, arousal ranges along the “sleep-high excitation continuum” (see the previous section) and the agent perceives anxiety as excitement when anxiety is low and is therefore willing to pay more than the first best value of the firm. Indeed, Zivnuska et al. (2002) find an inverted-U relationship between job tension and job satisfaction using survey data on 270 hotel managers. Note that these ultra-incentives assume no wealth constraints such as $\alpha \geq 0$. 
3.5 Optimal Incentives

Although Proposition 1 shows that exogenous incentives can be demotivational in the sense that $e_\beta < 0$ at the agent’s optimum.

**Proposition 3:** A profit-maximizing principal never offers counterproductive incentives in equilibrium.

Suppose $e_\beta < 0$ in equilibrium. Because $\psi < 0$ and $\phi' > 0$ (see Assumptions 1 and Proposition 1), a reduction in incentives would lower the agent’s cost of effort, increase optimal effort, and reduce the risk premium. Such a contract could therefore never be optimal.

This simple result reconciles experimental findings like Ariely et al. with empirical work based on field data such as Lazear (2000), where incentives have strong and positive motivational effects. Although these two literatures appear contradictory, they may in fact be complementary. Indeed, there is nothing inherently contradictory about observing counterproductive incentives in experiments where incentives are exogenous (Proposition 1) but not in workplace settings where they are endogenous (Proposition 3). Furthermore, field evidence cannot be used to refute experimental evidence, despite Lazear’s claim to the contrary (see the full quotation in the Introduction).

Glaeser (2003) makes a similar point much more generally but informally:6

outside of the laboratory, emotionally-powerful situational factors . . . are almost always endogenous and often the result of self-interested entrepreneurs. As such, laboratory work and, indeed, psychology more generally, gives us little guidance as to market outcomes.

(from the abstract)

Although Glaeser’s insight is valid in our model, it may not hold generally. For example, Fehr and Gächter (2000) (see also their working paper cited in their references) conducted a series of experiments where principals in the trust treatment could only make lump-sum payments in discretionary amounts, whereas those in the incentive treatment could choose a fine for agents caught shirking. The fines implemented in the incentive treatment backfired in the sense that effort was significantly lower than in the trust treatment, although profits were higher. These experiments therefore suggest that profit-maximizing incentives can indeed be counterproductive (in terms of effort). Likewise, the majority of principals in the Gneezy and Rustichini (2000, Section III) experiments

6. We thank an anonymous referee for drawing our attention to this paper.
(87% in the IQ experiment and 76% in the donation experiment) also chose demotivational incentives.

Finally, we have assumed the agent’s type (e.g., the level of income risk that maximizes effort) is common knowledge. If there is incomplete information, or the principal has to offer the same contract to a group of heterogeneous agents, then incentives may only be motivational on average.

3.6 Optimal Monitoring

We now consider the principal’s optimal choice of output risk $\sigma$. Fix $\beta > 0$. With zero monitoring, $\sigma$ starts out at its maximum or baseline level $\bar{\sigma}$. Because optimal effort has an inverted-U relationship with $\sigma$, an increase in monitoring holding incentives fixed would reduce $\sigma$ and increase effort. On this region, there is therefore a positive relationship between monitoring and optimal effort. As $\sigma$ continues to decline, at some point income risk $\sigma_I = \beta \sigma$ reaches $\hat{\sigma}_I$ where effort is maximized, so further increases in monitoring will reduce effort and be demotivational in that sense. Note that this discussion has implicitly assumed $\beta \sigma > \hat{\sigma}_I$, so $\beta$ is large enough so the right-hand side of optimal effort which is decreasing in $\sigma$ is available. We formalize the above discussion as follows:

**Definition 1:** Monitoring is motivational in equilibrium if $e_\sigma < 0$ at the optimum and demotivational if $e_\sigma > 0$.

**Proposition 4:** Monitoring can be motivational or demotivational in equilibrium. In particular, (i) monitoring is necessarily demotivational in equilibrium when $M' \equiv 0$. (ii) Otherwise, a necessary and sufficient condition for motivational monitoring is $\beta_A < \beta_L$, where $\beta_A$ is the optimal incentive in our model and $\beta_L$ is the optimal incentive (9) in the standard linear model.

Unlike incentives, the relationship between monitoring and effort can be positive or negative in equilibrium. According to (i), monitoring is necessarily demotivational when the marginal cost $|M'|$ of monitoring is sufficiently low (zero in the extreme) because the principal chooses a relatively small $\sigma$ where anxiety is motivational. Condition (ii) provides a useful and intuitive necessary and sufficient condition: the relationship between effort and monitoring is positive (negative) when incentives are weaker (stronger) than the standard linear model predicts. For example, monitoring is also demotivational when the principal offers ultra-incentives (incentives that exceed one) as in Proposition 2(ii) above.
In the standard model, the risk–reward tradeoff holds—the principal chooses low $\sigma$ when incentives are high to reduce the risk premium and high $\sigma$ when incentives are low to conserve on monitoring costs (see Milgrom and Roberts, 1992, Chapter 7). In our model, however, $\sigma$ has a direct effect on effort and the risk–reward tradeoff is violated (see Proposition 6 below). In this context, the principal chooses high $\sigma$ ($\sigma_I > \hat{\sigma}_I$) when monitoring is expensive, which implies high anxiety and motivational monitoring ($\phi' < 0$ and $e_\sigma < 0$). To reduce anxiety, she chooses relatively weak incentives $\beta_A < \beta_L$. In contrast, the principal chooses low $\sigma$ when monitoring is inexpensive, which implies low anxiety, a low risk premium, and demotivational monitoring. In that case, $\beta_A > \beta_L$ because anxiety is motivational at low levels.

In our model, monitoring has a benign aspect because it reduces the agent’s anxiety. There are, however, other forms of monitoring that can have potentially negative effects. For example, Ariely et al. and Dohmen (2008) find evidence for the social pressure hypothesis, where performance suffers in the presence of an audience (even a sympathetic one). Indeed, Frey (1993) assumes monitoring reduces the agent’s marginal benefit of effort because it undermines trust, intrinsic motivation, self-determination, and/or self-evaluation. On the other hand, monitoring decreases the marginal cost of effort because it forms the basis for penalties and rewards. Frey suggests that monitoring tends to be demotivational in close personal relationships where the first effect dominates, but motivational in formal or distant agency relationships. Akerlof and Kranton (2008) consider a similar set of issues, but from the perspective of the economics of identity. In their model, monitoring is formalized as the probability of detecting shirking and they provide sufficient conditions (p. 214) such that the principal prefers not to monitor because it causes the agent to adopt an “outsider” status.

In terms of evidence, Barkema (1995) studies the relationship between monitoring and effort (hours worked) using survey data on 116 senior managers in medium-sized Dutch firms. He finds that the relationship is negative when the monitor is a fellow executive (the CEO) but positive in the case of a distant parent company. Although Barkema interprets his findings in terms of Frey (1993), they are also consistent with Proposition 4 above. In close agency relationships, both the marginal cost of monitoring and the baseline level $\bar{\sigma}$ of noise should be relatively low because frequent contact should provide substantial information about effort even with zero active monitoring. Both effects imply low $\sigma$, low anxiety, and demotivational monitoring. The opposite should hold in formal or distant relationships.

Falk and Kosfeld (2006) conduct a series of experiments where the principal can enforce a lower bound on effort and interpret this
in terms of “control or monitoring devices which restrain the agent from his most opportunistic choices” (p. 1611).\textsuperscript{7} When the lower bound is low, monitoring backfires in the sense that average effort is higher when the principal simply trusts the agent. Those who were controlled told the experimenters that it signaled distrust and undermined their autonomy. As a result, a majority of principals elected not to monitor. These findings are therefore consistent with Frey (1993) and Akerlof and Kranton (2008).

In our model, monitoring is defined in terms of risk reduction rather than control, so the literature on stress and performance discussed in the previous section is more directly relevant. For example, Singh (1998) finds that sales performance is negatively related to role ambiguity, so an increase in monitoring in the form of clearer procedural guidelines or job redesign to reduce the latter should be motivational. This is consistent with Proposition 4(ii) above when the baseline level $\sigma$ of noise and the marginal cost of monitoring are relatively high, which seems likely for salespeople.

### 3.7 The Informativeness Principle

As Prendergast (1999) notes, the informativeness principle is often violated in real-world contracts. For example, many firms grant stock options to all employees even though they impose significant idiosyncratic risk with only trivial incentive effects. Indeed, Oyer and Schaefer (2005) reject the incentive-provision explanation for their use based on calibrations of a version of the linear model. In our model, however, the principal is averse to a completely static risk-free environment (unlike the principal in the linear model, who achieves the first best with zero noise) because a moderate amount of risk is motivational in accordance with the IUH.

**Proposition 5:** In our model, the principal never sets $\sigma = 0$ even when the marginal cost of monitoring is zero.

The principal therefore injects pure noise into the contract when $\sigma$ is too low and the cost of doing so is not too large. This could be achieved through profit-sharing, stock option, or stock ownership plans which impose significant risk but provide few incentives because of the $1/n$ problem, where $n$ is the number of employees. Alternatively, the principal could increase role ambiguity through less transparent objectives, procedures, and guidelines, introduce more subjective evaluation, and/or change the composition of work groups to “shake things up.”

\textsuperscript{7} We thank an anonymous referee for alerting us to this paper.
In Casadesus-Masanell (2004), the informativeness principle fails because moderate risk environments favor the development of altruism, ethical standards, and norms. Oyer (2004) shows that it can be optimal to pay for noise that is positively correlated with the agent’s outside opportunities in a version of the linear model with no moral hazard. Our paper complements the latter in the sense that noise can also have positive effects via the incentive compatibility constraint as well as the participation constraint. Furthermore, our model shows why principals tend to prefer noise (e.g., own-firm stock options) which have at least some connection to the agent’s effort, whereas the optimal contract in Oyer’s model depends on all signals that are informative about the agent’s outside opportunities.

3.8 The Risk–Reward Tradeoff

In the linear model, incentives are decreasing in output risk when the latter is exogenous (see (9) with \( \phi \) constant). If output risk is endogenous, incentives and monitoring are strategic complements for the principal because an increase in incentives increases the risk premium and therefore calls for an increase in monitoring to reduce \( \sigma \). After surveying the relevant empirical literature, however, Prendergast (1999) concludes that the evidence for these predictions is “rather mixed.” Proposition 6 below shows that the risk–reward tradeoff is violated in our model in a neighborhood of \( \beta = 1 \) and \( \sigma = 0 \) (the first best for the linear model).

**Proposition 6:** (i) There exists an open neighborhood of the point \( \beta = 1 \) and \( \sigma = 0 \) where incentives and output risk are strategic complements for the principal, so incentives and monitoring are strategic substitutes. (ii) When \( \sigma \) is exogenous, the risk–reward tradeoff is violated in that neighborhood.

To explain (i), we substitute \( \beta = 1 \) and \( \sigma = 0 \) into (5) to obtain

\[
e_{\beta \sigma} = -\frac{2\phi'(0)}{\phi(0)^2} > 0.
\]  

(10)

It follows that incentives and output risk are complementary in motivating effort in an open neighborhood of that point and are therefore strategic complements for the principal. That is, incentives and monitoring are strategic substitutes. As for (ii), an exogenous increase in \( \sigma \) starting from \( \beta = 1 \) and \( \sigma = 0 \) will induce the principal to raise incentives above one because incentives and output risk are strategic complements. The latter are therefore locally positively related, so the risk–reward tradeoff is violated.

We illustrate (ii) in Figure 2, where we assume \( \phi \) has the functional form in (6) and \( a = -1, \ b = 2, \ c = 1, \) and \( r = 0.1. \)
The nonmonotonic curve is the optimal incentive $\beta_A$ in our model as a function of $\sigma$, whereas the monotonically decreasing curve is the optimal incentive $\beta_L$ in the linear model (9). We first note that $\beta_A$ increases from $\beta_A = 1$ to slightly above $\beta_A = 1.1$ as output risk increases from $\sigma = 0$ to about $\sigma = 0.35$. As in Proposition 6(ii), the risk–reward tradeoff is therefore violated. At some point, however, $\beta_A$ must be decreasing in output risk because income risk increases the risk premium and is also eventually demotivational. In Figure 2, the risk–reward tradeoff holds for all $\sigma \geq 0.35$, which suggests that in general the optimal incentive has an inverted-U relationship with output risk as depicted in Figure 2. Second, we observe that $\beta_A > \beta_L$ before $\sigma \approx 1$, and $\beta_A < \beta_L$ afterwards, which shows Proposition 4(ii) is not empty. This is because incentives are more powerful in our model when anxiety is low (and therefore motivational) but less powerful when anxiety is high.

In the above discussion, it appears that ultra-incentives (incentives above one) are necessary for violations of the risk–reward tradeoff as in Figure 2. In Appendix B, we add a wealth constraint $\alpha \geq 0$ and show that Proposition 6 above still holds, except incentives are always less than one.

Alternative explanations include Guo and Ou-Yang (2006) mentioned previously and Prendergast (2002a), who shows that the
principal prefers delegation and incentives in unstable environments where she is uncertain about which action the agent should choose. In contrast, the principal prefers authority and monitoring in stable environments where there is more certainty about the appropriate action, so there is a positive relationship between incentives and risk.

4. Discussion

4.1 Assumptions about Preferences

The theoretical literature on anxiety and the IUH includes the PET, the psychological expected utility theory in Caplin and Leahy (2001), and the endogenous learning-by-doing model in Rauh and Seccia (2006). Because (3) contains a standard deviation $\sigma_I$ inside a nonmonotonic function $\phi$, it appears that a simple expectation (i.e., integration) is unable to produce such an expression (in contrast, the risk premium is linear in the variance of income in the linear model). Our model therefore seems inconsistent with standard expected utility theory. Because the anxiety concept is much broader than simple risk aversion, we do not view this as a drawback of our approach. On the other hand, it can be shown that (3) is consistent with the Caplin and Leahy framework, albeit under restrictive assumptions about the main element in their model—the map from economic outcomes to psychological states—similar to the example in Section IV of their paper. The model in Rauh and Seccia (2006) can be viewed as a formalization of the PET in a specific learning context, but seems intractable for our purposes. Given the lack of generally accepted formal theoretical foundations for the IUH, we chose to assume it directly.

In our model $\phi$ is assumed to be a function of income risk $\sigma_I$, so the agent’s preferences are defined directly in terms of $\sigma_I$ as opposed to more basic outcomes such as income. A similar deviation from standard expected utility theory is used in mean-variance analysis in finance, the sharecropping model in Stiglitz (1974), and other literatures—and seems natural for comparison purposes within the linear framework of Holmström and Milgrom (1991). It does, however, raise questions about the generality of our results.

It may therefore be useful to consider a version of the two-outcome moral hazard model in Bolton and Dewatripont (2005, Chapter 4), where anxiety is defined as the difference in wages in the good and bad states and the agent’s marginal cost of effort is again U-shaped in anxiety as per the IUH. Because most of our results are driven directly by the

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IUH, we would expect them to generalize to that and other contexts. Although this two-outcome version of our model does not require the assumption of linear contracts, it cannot distinguish between incentives (also the difference in wages across the two states) and anxiety as can the model in this paper. It would be appropriate, however, for the concept of arousal (see Section 2) which is often defined in terms of incentives and motivation.

Finally, it is generally believed that emotions such as anxiety and fear evolved in humans because of their motivational advantages (e.g., “fight or flight”). The literature on endogenous preferences, including Rotemberg (1994) and Casadesus-Masanell (2004), offers one way to formalize this.

4.2 Suboptimality of Linear Contracts

We chose to work within the linear framework of Holmström and Milgrom (1991) because the notions of “incentive” and output “risk” have simple one-dimensional representations in terms of $\beta$ and $\sigma^2$. This allows us to incorporate the IUH in the form of a simple function $\phi$ of income risk $\sigma_I$ and to formalize monitoring as a costly technology that reduces $\sigma^2$.

As is well known, however, linear contracts are in general suboptimal. In particular, Mirrlees (1999) has shown (also see Bolton and Dewatripont, 2005, Chapter 4) that the first best can be approximated arbitrarily closely by a contract which entails a severe punishment if output falls below some lower threshold and a fixed payment slightly higher than the first best (to compensate for risk) otherwise. It is therefore natural to inquire how the addition of anxiety to the model affects the suboptimality of linear contracts.

A crucial difference is that in our model the first best level of income risk should be positive because zero anxiety is suboptimal. In that case, the principal should be able to set the three parameters in a Mirrlees-type contract (i.e., the threshold, punishment, and reward) to attain exactly both the first best level of effort and income risk. In contrast, in the general moral hazard model the principal can only approximate the first best because the agent is risk averse and the first best level of income risk is zero. Because linear contracts are dominated by Mirrlees-type contracts in both settings, with and without anxiety, the qualitative nature of their suboptimality is the same. In that sense, the addition of anxiety does not appear to further undermine the assumption of linear contracts.9

9. We thank an anonymous coeditor for suggesting this argument.
If such punishments are infeasible, either by law or because the agent is wealth-constrained, then Mirrlees-type contracts cannot be enforced. Bose et al. (2008) consider a version of the Holmström (1979) moral hazard model where the agent is risk averse and has zero wealth. Under specific functional forms for preferences and the stochastic production process, they show that linear contracts perform quite well relative to the unconstrained second best contract (in fact, the former secure at least 90% of the expected profits of the latter). This is because the optimal linear contract approximates the unconstrained optimal contract reasonably well for output levels in a neighborhood of the second best effort. Although the approximation may be poor elsewhere, those output levels are unlikely in equilibrium. In that case, linear contracts may be preferred because of their simplicity.

A similar argument may be valid in our model. In particular, Proposition 3 above suggests the principal is only interested in implementing effort levels which are increasing in incentives. In that case, the unconstrained second best contract should be increasing in output in a neighborhood of the second best effort level and the optimal linear contract may once again provide a good local approximation. Intuitively, however, to reduce anxiety the unconstrained second best contract may be an inverted-U in output, where the second best effort level occurs to the left of the maximizer. Nevertheless, the gains to implementing the unconstrained second best contract (or a piecewise-linear approximation) may be small relative to additional contractual complexity as in Bose et al. Furthermore, if the principal has access to a monitoring technology (e.g., informative signals) then there will be less need for an inverted-U wage schedule because the principal can reduce output risk directly, and output levels outside the relevant neighborhood of the second best effort level will become even less likely. Both of these effects should improve the approximation provided by the optimal linear contract.10

### 5. Conclusion

In this paper, we assembled evidence from multiple disciplines that anxiety, arousal, and stress collectively constitute one of the most important psychological factors in professional principal–agent relationships. In particular, the experiments in Ariely et al. (2005) suggest that extremely high-powered incentives can induce agents to “collapse under pressure” in accordance with the inverted-U hypothesis. Our purpose in the present paper was to show that their findings have

10. We have benefited from discussions with Debashis Pal on these issues.
broader implications for incentive and monitoring mechanisms, the informativeness principle, the classical risk–reward tradeoff, and other empirical anomalies. Although our model assumes (rather than explains) the Ariely et al. findings, it does show that all of these phenomena may be a direct consequence of an extremely simple idea—that one of the central tasks of management is to foster an optimal amount of tension in the workplace. For example, if output risk (and therefore income risk) is too small then the principal will either want to violate the informativeness principle and inject some noise into the contract or offer ultra-incentives which exceed the agent’s marginal revenue product (although wealth constraints may prohibit the latter).

In terms of specific managerial implications, an important conclusion of the paper is that moderate incentives may be even more powerful than standard theory predicts (see Figure 1) because moderate levels of anxiety are motivational. Given the stylized fact that incentives tend to be low powered in firms, this suggests that they may be under-utilized. The inverted-U hypothesis may also have unexplored consequences for organizational architecture that extend well beyond the issues considered in this paper. For example, Holmström and Milgrom (1991) argue that for incentive reasons it may be optimal to bundle tasks into jobs according to output risk, so that one agent performs exclusively low-risk tasks and another high-risk tasks. In contrast, in our model it may be optimal to mix them in order to achieve a medium (and perhaps optimal) amount of tension for each. In light of the aforementioned evidence on the importance of anxiety, we hope that the present paper stimulates further work on these issues.

**APPENDIX A**

*Proof of Proposition 1.* The expressions in (i) and (iii) follow from straightforward computations. To prove (ii), fix $\beta > 0$. Because $e = \beta / \phi$ and $\phi$ is U-shaped in $\sigma_I$, optimal effort is an inverted-U in $\sigma$. Now fix $\sigma > 0$. Because $\psi(0) = \phi(0) > 0$, $e_\beta > 0$ at $\beta = 0$. For all $\beta > 0$, $\psi' = -\sigma_I \phi'' < 0$. It follows that $\psi$ is initially positive, strictly decreasing in $\beta > 0$, and eventually negative at $\tilde{\sigma}_I$ by Assumption 1(iii). Because $e_\beta = \psi / \phi^2$ and $\phi > 0$, optimal effort is at first increasing in $\beta$, peaks at some unique $\beta > 0$, and is then decreasing in $\beta$. □

*Proof of Propositions 2 and 3.* Substituting optimal effort $e = \beta / \phi$ into (8),

$$\Pi = \frac{\beta \left( 2 - \beta - r \beta \sigma^2 \phi \right)}{2 \phi} - M(\sigma).$$

(A1)
It follows that $0 < \beta < 2$ at the optimum, so the principal effectively chooses $(\beta, \sigma)$ from the nonempty compact set $[0, 2] \times [0, \bar{\sigma}]$. Because (A1) is continuous, a solution to the principal’s problem exists. The first-order condition for $\beta$ is

\[
\Pi_\beta = \frac{2(1 - \beta)\phi - 2r\beta \sigma^2 \phi^2 - (2 - \beta)\beta \sigma \phi'}{2\phi^2} = 0 \tag{A2}
\]

or

\[
(2 - \beta)\psi - \beta \phi(1 + 2r \sigma^2 \phi) = 0. \tag{A3}
\]

Because $\beta < 2$, we must have $\psi \geq 0$ and $e_\beta \geq 0$. The example in connection with Figure 2 demonstrates that the optimal $\beta$ can exceed one. □

Proof of Propositions 4 and 5. We first prove (i). Differentiating (A1),

\[
\Pi_\sigma = -\frac{2\phi^2(r\beta^2 \sigma + M') - (2 - \beta)\beta^2 \phi'}{2\phi^2}. \tag{A4}
\]

If $M' \equiv 0$ the numerator becomes

\[-2\phi^2 r\beta^2 \sigma - (2 - \beta)\beta^2 \phi'. \tag{A5}\]

Because $0 < \beta < 2$ and $\phi'(0) < 0$, (A5) is positive when $\sigma = 0$ which therefore cannot be optimal. This proves Proposition 5. It follows that $\sigma$ is either interior or $\sigma = \bar{\sigma}$ so (A4) is nonnegative. Because $\phi > 0$ the first term must be negative. This implies $\phi' < 0$ and completes the proof of (i). To prove (ii), we re-arrange the numerator in (A2)

\[2\phi [1 - \beta(1 + r \sigma^2 \phi)] = (2 - \beta)\beta \sigma \phi'. \tag{A6}\]

Because $\sigma > 0$, the sign of $\phi'$ is determined by $1 - \beta(1 + r \sigma^2 \phi)$ and (ii) follows. To provide an example of motivational monitoring in equilibrium, we assume $|M'|$ is sufficiently large such that (A4) is positive for all $0 \leq \sigma \leq \bar{\sigma}$. In that case, $\sigma = \bar{\sigma}$ at the optimum and $\beta$ is determined by (A6). Although the example in Figure 2 assumes $\sigma$ is exogenous, it still applies in the current context where only (A6) is relevant and $\sigma = \bar{\sigma}$. From Figure 2, we observe that monitoring is motivational when $\bar{\sigma}$ is large enough so that $\beta_A < \beta_L$ and therefore $\phi' > 0$. □

Proof of Proposition 6. Differentiating (8) with respect to $\beta$,

\[ (1 - \phi e)e_\beta - (1/2)\sigma (\phi' e^2 + 2r \beta \sigma). \tag{A7}\]

Substituting the identity $\beta \equiv \phi e$,

\[ (1 - \beta)e_\beta - (1/2)\sigma (\phi' e^2 + 2r \beta \sigma). \tag{A8}\]
As an aside, note that if $\beta > 1$ then the reduction in the cost of effort must be at least as great as the increase in the risk premium because $e_\beta \geq 0$ (see the discussion following Proposition 2). To continue the proof, we now differentiate with respect to $\sigma$,

$$(1 - \beta)e_\beta e - (1/2)e^2 (\phi' + \beta \sigma \phi'') - \sigma (\phi' e e + 2r \beta). \quad (A9)$$

Substituting $\beta = 1$ and $\sigma = 0$, this reduces to $-(1/2)e^2\phi'(0)$. Because $e = \beta/\phi = 1/\phi(0)$, the latter becomes $-(1/2)[\phi'(0)/\phi(0)^2]$, which is strictly positive. By continuity, this also holds in an open neighborhood of $\beta = 1$ and $\sigma = 0$. □

**Appendix B**

In this Appendix, we verify the claim following Proposition 6 that a wealth constraint $\alpha \geq 0$ implies a violation of the risk–reward tradeoff in a neighborhood of $\sigma = 0$ where incentives are less than one. That is, violations of the risk–reward tradeoff are not necessarily tied to the use of ultra-incentives. Assuming $\sigma$ is exogenous, the principal maximizes expected profits $e - \alpha - \beta e$ subject to incentive compatibility $e = \beta/\phi$, the participation constraint

$$\alpha + \beta e - (1/2)\phi e^2 - (1/2)r \beta^2 \sigma^2 \geq 0 \quad (B1)$$

and $\alpha \geq 0$. Substituting optimal effort, the principal maximizes

$$\frac{(1 - \beta) \beta}{\phi} - \alpha \quad (B2)$$

subject to

$$\alpha + (1/2)\frac{\beta^2}{\phi} - (1/2)r \beta^2 \sigma^2 \geq 0 \quad (B3)$$

and $\alpha \geq 0$. Given the latter, (B3) is redundant when $\sigma^2 \leq 1/(r \phi)$, which holds in a neighborhood of $\sigma = 0$. For the rest of the proof, we assume $\sigma$ is sufficiently small such that (B3) is slack. The problem then becomes

$$\max_{\alpha, \beta} \frac{(1 - \beta) \beta}{\phi} - \alpha \quad (B4)$$

subject to $\alpha \geq 0$ or simply

$$\max_{\beta} \frac{(1 - \beta) \beta}{\phi}, \quad (B5)$$
where we denote the objective function in (B5) as $\hat{f}$. The first and second derivatives of $\hat{f}$ with respect to $\beta$ are

$$
\frac{(1 - 2\beta)\phi - (1 - \beta)\sigma_1 \phi'}{\phi^2} \quad \text{(B6)}
$$

and

$$
\frac{2(1 - \beta)\sigma_1 \phi^2 + \sigma \phi [(4\beta - 2)\phi' - (1 - \beta)\sigma_1 \phi''] - 2\phi^2}{\phi^3} \quad \text{(B7)}
$$

Setting (B6) equal to zero and solving,

$$
\phi' = \frac{(1 - 2\beta)\phi}{(1 - \beta)\sigma_1}. \quad \text{(B8)}
$$

Substituting (B8) into (B7),

$$
-\frac{2\phi + (1 - \beta)\sigma_1 \sigma \phi''}{\phi^2} < 0. \quad \text{(B9)}
$$

Because the second derivative is negative whenever the first is zero, $\hat{f}$ is strictly quasi-concave (see Vives, 1999). The problem therefore has a unique solution on $0 < \beta < 1$. In particular, the solution is $\beta = 1/2$ when $\sigma = 0$. The cross-partial is

$$
\hat{f}_{\beta\sigma} = \frac{\beta \left[ 2(1 - \beta)\sigma_1 \phi^2 + \phi [(3\beta - 2)\phi' - (1 - \beta)\sigma_1 \phi''] \right]}{\phi^3}. \quad \text{(B10)}
$$

At $\beta = 1/2$ and $\sigma = 0$, (B10) reduces to

$$
-\frac{\phi'(0)}{4\phi(0)^2} > 0. \quad \text{(A11)}
$$

It follows that an increase in $\sigma$ leads to an increase in $\beta$ at $\sigma = 0$ as claimed in the text (see Vives, 1999, p. 30). Furthermore, (B5) shows that the optimal incentive is always less than one.

\[\square\]

References


