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Tasks that support the development of geometric reasoning at KS3

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Students at Key Stage 3 (ie aged 11-14) in English schools are expected to learn the definitions of the properties of triangles, quadrilaterals and other polygons and to be able to use these definitions to solve problems (including being able to explain and justify their solutions). This paper focuses on a pair of Year 8 students (aged 12-13) working on a task using dynamic geomtry software. In the research, the children investigated triangles and quadrilaterals by dragging two lines within a shape (ie the diagonals of a quadrilateral, or base and height of a triangle) and noting the position and orientation of the lines which gave rise to specific shapes. Following this, the students were asked to use what they had found in order to construct specific triangles and quadrilaterals when starting with a blank screen. While the research is currently ongoing, and is using a design research methodology, the evidence to date is that the task has the potential to scaffold students' thinking around the properties of 2D shapes and hence support the development of geometric reasoning.

Keywords: dynamic geometry, task, design-based research

Introduction

The *Framework for Secondary Mathematics in England* (DCSF, 2008) indicates that Year 8 students (aged 12-13 years) are expected to know and understand the properties of triangles and quadrilaterals, to be able to solve problems using these properties and to classify quadrilaterals according to geometric properties. Yet simply expecting students to memorise such shapes and their properties is likely to be insufficient support for students developing their own meaningful concepts in geometry (Battista, 2002). This paper describes an attempt to devise a task which would encourage students to develop a deeper understanding of how shapes can be defined by considering the properties of two internal perpendicular lines. These lines are the diagonals in the case of certain quadrilaterals and the height and base in the case of triangles. The students explored these shapes in a dynamic geometry environment, specifically *The Geometers Sketchpad* (GSP) version 4 (Jackiw, 2001).

Using tasks to support learning in geometry

Open problems in geometry have been shown to encourage children to develop meaningful concepts (eg: Mogetta *et al* 1999 a). Open problems usually consist of a short statement where students are asked to explore connections between elements of a figure. Open problems do not lend themselves to solution solely through the use of learned procedures; students have to decide *how* to explore the problem and there may be a number of results that could be reasonable solutions to the problem. The benefit to the students of working through open problems is that the outcomes are meaningful to them and the opportunity to explain their results may be a pre-cursor to being able to prove in geometry (eg: Jones, 2000).

Working on a problem in a computer environment also has benefits. Papert (1993) argued that computers encourage concrete thinking and ways of solving problems that involve finding approximate solutions and then tweaking them until the optimal solution is found. This approach seems to suit children.

It is helpful to provide problems which the students find engaging. Ainley, Pratt and Hansen (2006) consider that tasks which involve programming computers can provide purpose and utility, which enriches students' learning of mathematical concepts; de Villiers (1994) said something similar when he described 'functional understanding' ie the understanding of the usefulness or value of doing a specific bit of mathematics, as being just as important as relational and logical understanding which was described by Skemp (1973).

The task, described in this paper, was designed to take all of the above into account. It was intended that the task would stimulate the students into thinking more deeply about the methods of constructing specific triangles and quadrilaterals and that they would be able to explain why their methods worked. As Mogetta *et al* (1999 b) say, switching between what the student notices on the screen (empirical evidence) and the geometrical theory may stimulate the students' capability to prove as they have to explain the reasons behind what they are observing. Problems can only really be solved when what is observed on the screen is explained geometrically.

Dragging and measuring in a dynamic geometry environment

With DGS software (such as *GSP*, *Cabri*, *GeoGebra*), an important function is the dragging mode which allows the user to drag geometric objects on the computer screen. The computer interface allows direct manipulation of the drawing on the screen via the drag mode whilst, at the same time, preserving all the geometric properties used to construct the figure (Laborde, 1993).

Another function in DGS is the *measure* menu. Students can measure lines and angles on the diagram and, as the diagram is dragged, the measurements given on the screen are updated continuously. Olivero and Robutti (2007) describe students using the drag mode to adjust a sketch on the screen until the measurements indicate that they have obtained a particular figure from a generic one and they called this an example of 'guided measuring'. When students check their constructions through measuring and dragging this is called 'validation measuring'.

Design-based research

The methodology used in this study was that of *design-based research* (Brown, 1992; Design-based research collective, 2003). In a design-based research experiment, the researchers aim to study *how* learners learn by *designing tasks and learning situations* through which they hope to see improved learning outcomes. Design-based research uses the design experiment to study learning and develop theories about learning in a specific context but which can be extrapolated to theorise about learning in a broader context (Barab and Squires, 2004).

In a learning situation, even a simple one where there are two students and one instructor / researcher as in this case, cognition is not separate from the thinker, the task or the environment - these all need to be treated as one complex system (Design-based research collective, 2003). The learning environment is a complex system of inter-relating aspects where one aspect cannot be changed without it affecting all other aspects. Design-based research methodology accepts this as the case and works with it rather than against it (Brown, 1992). Design-based research experiments thus need to take account of all the aspects of the learning situation and how these all work together. Testing, scrutinising and revising a design results in an iterative process over several cycles of the research (Cobb et al, 2003).

Clearly there are some challenges in design-based research which need to be considered. The role of the researcher(s) in a design-based experiment means that they have a large influence in how the experiment proceeds (Barab and Squire, 2004). Design-based research experiments can also generate a large amount of data and the researcher must choose which data to focus on in the analysis. The researcher must be aware of the need to be objective and not select data that backs up their own preconceived ideas (Brown, 1992). This shows how important it is for the researcher to try to be objective and for all their activities to be transparent.

The experiment

Two boys and two girls worked in pairs in the summer term of Year 8 (students aged 13) for two 50 minute sessions. Data was collected from these sessions in the form of an audio tape and a recording of the computer screen using image capture software. The students were given a task which was loosely related to a 'real world' situation in that they were asked to imagine a toy kite made of two 'sticks' which provide the scaffolding for the kite. The fabric which makes up the kite is imagined to be elastic so that the 'sticks' can be moved around to create different shaped 'kites'. The students were asked to investigate the different shapes that can be made in this way and to describe the orientation of the 'sticks' inside each shape. In a later session the students were asked to construct 'drag proof' shapes starting with a blank screen.

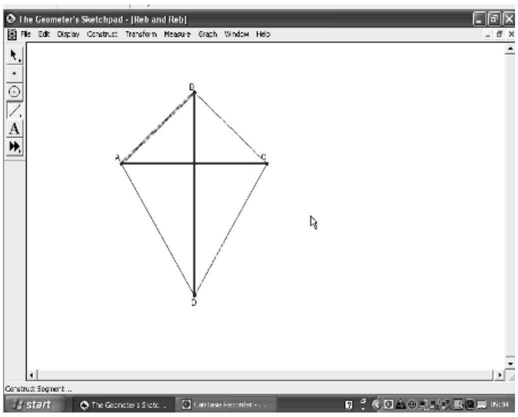
The findings

The first session allowed the researcher to assess the students' prior knowledge of shapes and their properties. The students were presented with a GSP file which contained a 6 cm horizontal bar and 8 cm vertical bar. The students completed the shape by joining the ends of the bars (see figure 1) and then constructed the interior of the shape. Even though the bars could be dragged anywhere on the screen, the students preferred shapes that had vertical symmetry. By dragging the bars they were able to make kites, a rhombus, an isosceles triangle, right angled triangles in different orientations, and concave kites. The students were then asked to measure objects in the shape in order to be absolutely sure that they had made the shape. They did this using the *measure* facility of GSP, which shows the measurements as text on the screen.

Next the students were provided with a GSP file with perpendicular bars of adjustable lengths. With two equal perpendicular bars the students were able to make a square as well as various kites, an isosceles triangle and a right angled isosceles triangle. Each time the students made a shape they were asked to describe the orientation of the bars inside the shape. The students realised that the measures changed as they dragged the bars inside the shapes. The students moved the bars inside the shape whilst checking the measurement given on the screen until the measurements were as close as they could get them. This is an example of 'guided measuring' as described by Olivero and Robutti (2007).

The students were happy to accept measurements which were close, but not exact, in order to feel confident that they had generated a particular shape. This mirrors the use of measurement in the pencil and paper environment. They also noticed, when prompted, the orientation of the bars inside the shape which would be useful in order to carry out the task in the second session when they would be asked to construct shapes starting with a blank screen.

Figure 1: the Kite task in GSP



For the second session the students were provided with ‘instruction cards’ which explained how to do various operations from the Construct and Transform menus (such as mid-point of a line, perpendicular to a line). These cards were placed on the desk so that the students could choose which ones they needed. The instruction cards served to give hints to the students as to what might be a useful construction.

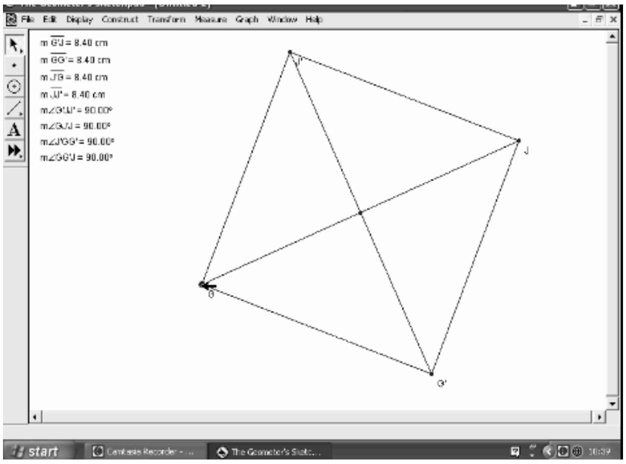
For the first part of the session, the students were asked to make an isosceles triangle. It was clear from observing their attempts that they had remembered the orientation of the bars from the previous session. At first they drew lines to represent the bars ‘by eye’ and then completed the outside edges of the shape. When they discovered that the result could be dragged out of shape they realised they needed to make shapes that were ‘drag proof.’ It was at this point that they looked at the help cards on the desk.

The best constructions that the students made were when they chose what they would like to do to their diagram and found ways to get the GSP to achieve that. For example when making the square they first drew a line, then constructed a mid-point and then constructed a perpendicular line through this mid-point. They were perplexed when the perpendicular was an infinite line and they did not know where to place the opposite corners of the square along the infinite line. The following conversation ensued with the girls.

> **Researcher**: What would you like to do? If you don’t know how to do it, what would you like to do? And I might be able to tell you how to do it.
>
> **Girl 1**: Find a point here which is, like, the same distance and will make a right angle.
>
> **Researcher**: Well we know if it’s on that line it’ll be a right angle. So what do you think you could do?
>
> **Girl 2**: Is there some way you could, almost, spin it round?
>
> **Researcher**: If you spin it round, what’s that called?
>
> **Girl 2**: Rotate.

The students then rotated the first line 90 degrees onto the infinite perpendicular line to find out where the corners of the square would go. As they did in session 1, the students used the *measure* facility to check that they had made the shapes correctly. An example of a square with measurements is shown in figure 2.

Figure 2: constructing a square in GSP



Discussion

The levels of geometrical reasoning devised by van Hiele (1986) are often used to characterise the level of students' development in geometrical reasoning. If the task used in this research is useful for developing the students' reasoning then we should see some progress within the levels. The students showed, in the first session, that they were able to understand shapes as being collections of properties, which is evidence of reasoning at van Hiele level 2 (ie shapes as being collections of properties). In the second session the students were able to construct specific shapes starting with a blank screen, namely isosceles triangle, kite, square, equilateral triangle. This indicates that they had learnt something about the properties of the diagonals of the quadrilaterals, or the base and height of triangles, in order to do this. Being able to solve problems using these properties, such as constructing the shapes in a dynamic geometry environment, would indicate progression towards van Hiele level 3. In addition, the provision of 'instruction cards' served to support the development of the students' geometrical vocabulary. The students started to talk about what they were doing *using the language on the cards*, which is also the language used by the software.

Conclusion

What has been described in this paper is the first iteration of the design experiment process. The evidence to date is that the task has the potential to scaffold students' thinking around the properties of 2D shapes but that the task needs to be developed further in order to consolidate and build on what the students have learned. For example, a third session would be useful, where the students might be asked to build a macro which would generate a shape such as a square. This would also encourage the students to find efficient ways to generate shapes - leading them, perhaps, to a realisation of the *minimum properties* required to render a specific shape such as a square. This activity could also suggest to students that they explain *why* their macro works - which may lead them onto more formal proof and would provide more solid evidence of development towards van Hiele level 3.

The use of geometrical language to support learning is an issue which has received only modest attention in this study so far. Future iterations of the work with students could usefully consider the potential that the task and the dynamic geometry software have to develop students' use of geometrical language.

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