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# THE DESIGN OF TEXTBOOKS AND THEIR INFLUENCE ON STUDENTS' UNDERSTANDING OF 'PROOF' IN LOWER SECONDARY SCHOOL

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*In this paper we report on our analysis of textbooks commonly used for teaching students about proof in geometry in lower secondary school in Japan. From our analysis we found that, as expected from the curriculum specification, deductive reasoning is prominent in Japanese textbooks. Yet the way that proof and proving is presented in these textbooks shows geometry as a very formal subject for study, one that omits to illustrate convincingly for students the difference between formal proof and experimental verification. As such, we argue that an improvement in textbook design is likely to involve providing students with more effective instructional activities so that they appreciate more fully the notion of 'generality of proof'.*

## INTRODUCTION

Given the widespread findings of international research that teaching the key ideas of proof and proving to all students is not an easy task (see, for example, Mariotti, 2007; Mariotti and Balacheff, 2008), this paper focuses on the design of curriculum materials that are used by teachers in lower secondary schools in Japan. In our paper we tackle the issue of how students' natural cognitive needs for conviction and verification might be changed and developed through instructional activity that is mediated by school textbooks in common use in Japan. Our premise, as Yackel & Hanna (2003, P234) emphasize, is that one of the most challenging undertakings for mathematics educators in their efforts to help students acquire competency in proof is to "design means to support teachers in developing forms of classroom mathematics practice that foster mathematics as reasoning".

This paper reports on part of our findings from one of our studies on proof and proving (for more details of our studies, see, for example, Fujita & Jones, 2002, 2003). In another study (Kunimune, Fujita & Jones, 2008), we report that lower secondary school students who can construct deductive proof do not necessarily understand *why* such deductive argumentations are necessary in geometry. In this paper, we consider what is likely to be a source of some of the influences on why such uncertain knowledge and belief in proof in geometry is constructed among students. We do this by identifying features of geometry and proof, and approaches to geometry learning, privileged in school textbooks, focusing on those in common use in Japan (Fujita & Jones, 2002; 2003).

## THE STUDY OF TEXTBOOK IN MATHEMATICS EDUCATION

Various studies, including the Third International Mathematics and Science Study (TIMSS), have demonstrated that textbooks continue to play an important role in classrooms around the world (see, for example, Stylianides, 2008; Valverde *et al*, 2002). In terms of the research reported in this paper, we argue that studying textbooks is important because such artifacts influence both teachers and learners. This is demonstrated by the following:

- The design of the curriculum in any particular country influences and 'shapes' students' knowledge in mathematical proof (Healy and Hoyles, 1999)
- The intended schqool mathematics curriculum (as specified in documents such as National Curriculum in England or the Japanese 'Course of Study') is experienced by pupils through the textbooks that are used in their classrooms (and for their homework) (Schmidt *et al*, 2001, p. 22; Valverde *et al*, 2002, p. 5.)

In these ways, textbooks constitute an important component of the potentially implemented curriculum - something which mediates between the intended and implemented curriculum. Recent relevant textbook research has examined how proof is presented in Swedish textbooks (Nordström & Löfwall, 2005) and Stylianides (2008) gives a detailed analysis of how proof and proving is prompted in US curriculum materials and guidance for teachers. One reason that, in our work, we focus on geometry is provided by Hanna and de Bruyn (1999) who comment that 'only in the topic of geometry do the textbooks do a reasonable job of providing opportunities to learn proof".

## ANALYSIS FRAMEWORK AND METHODOLOGY

To enable us to undertake a systematic examination of the design of textbooks, our analysis is framed by the following procedure, derived mainly from Valverde *et al* (2002):

- division of the geometry parts of textbooks into 'units' and 'blocks';
- coding of each 'block' in terms of content, performance expectations and perspectives (Table 1, see also Valverde *et al*, 2002, pp. 184-7);
- identifying features of geometry in the textbooks;
- reviewing the design of textbooks in terms of latest findings based on classroom-materials-based research in proof such as Nordström. & Löfwall (2005) and Stylianides (2008).

The detail of our analysis framework is provided in Table 1.

In the component of our study reported in this paper, the textbooks selected for analysis were *New Mathematics* (or *Atarashii Suugaku*) for lower secondary school (published by Tokyo Shoseki, one of the major Japanese publishers, in 2001). In previous analyses, we have examined textbooks published in Scotland

(Fujita and Jones, 2002; 2003) and we are currently analysing the latest editions of the textbooks published in Japan by Tokyo Shoseki, and the *SMP Interact* series of textbooks published in England by Cambridge University Press.

| Block type | Content | Performance Expectations | Perspective |
|---|---|---|---|
| 1 Central instructional narrative<br>2 Related instructional narrative<br>3 Unrelated instructional narrative<br>4 Graphic (those directly related narrative)<br>5 Graphic (those not directly related narrative)<br>6 Question<br>7 Exercise Set<br>8 Suggested activities<br>9 Worked examples<br>10 Others | 1.1. Geometry: Position, visualisation, and shape<br>1.1.1. 2-D geometry: Co-ordinate geometry<br>1.1.2. 2-D geometry: Basics (point, line, and angles)<br>1.1.3. 2-D geometry: Polygons and circles<br>1.1.4. 3-D geometry<br>1.1.5. Vectors<br>1.2. Geometry: Symmetry, congruence, and similarity<br>1.2.1. Transformation<br>1.2.2. Symmetry<br>1.2.3. Congruence<br>1.2.4. Similarity<br>1.2.5. Constructions using straightedge and compass<br>1.3. Measurement<br>1.3.1. Perimeter, area, and volume<br>1.3.2. Angle and bearing | 2.1. Knowing<br>2.1.1. Representing<br>2.1.2. Recognising equivalents<br>2.1.3. Recalling properties and theorems<br>2.1.4. Consolidating notation and vocabulary<br>2.1.5. Recognising aims of lessons<br>2.2. Using routine procedures<br>2.2.1. Using equipment<br>2.2.2. Performing routine procedures<br>2.2.3. Using more complex procedures<br>2.3. Investigating and problem solving<br>2.3.1. Formulating and clarifying problems<br>2.3.2. Developing strategy<br>2.3.3. Solving<br>2.3.4. Predicting<br>2.3.5. Verifying<br>2.4. Mathematical reasoning<br>2.4.1. Developing notation and vocabulary (proof)<br>2.4.2. Developing algorithms<br>2.4.3. Generalising<br>2.4.4. Conjecturing and discovering<br>2.4.5. Justifying and proving<br>2.4.6. Axiomatising<br>2.5. Communicating<br>2.5.1. Using vocabulary and notation<br>2.5.2. Relating representations<br>2.5.3. Describing/discussion<br>2.5.4. Critiquing | 3.1. Attitude toward science, mathematics, and technology<br>3.2. Careers involving in science, mathematics, and technology<br>3.2.1. Promoting careers in science, mathematics, and technology<br>3.2.2. Promoting the importance of science, mathematics, and technology in non-technical careers<br>3.3. Participation in science and mathematics by underrepresented groups<br>3.4. Science, mathematics and technology to increase interest<br>3.5. Scientific and mathematical habits of mind |

**Table 1: Codes used for the analysis**

Our intention in this paper is qualitative rather than quantitative. As such, the discussion that follows our analysis focuses on lessons containing the aspect ‘justifying and proving’ and how this is achieved. We then look at how these textbook designs might influence students in geometry proving in Japan.

## FINDINGS AND DISCUSSION

Our analysis suggest following:

- *The design of lessons in the textbooks*: Japanese textbooks start from a problem solving situation (for example, about 37% of lessons in Grade 8 begin with problem solving situations). A narrative block (which recalls some facts and theorems) comes later and is accompanied by some exercises. The principles of how to proceed with mathematical proof are explained in detail, including explanations of 'definitions' and 'mathematical proof'.
- *Performance expectations*: 'Justifying and proving' is very prominent in Japanese textbooks and mainly uses congruency to prove various geometrical facts and theorems.
- *Content of geometry*: Japanese textbooks concentrate on 1 or 2 topics in each unit (for example, 2-D basic geometry, 2-D polygons and circles, symmetry, construction, measurement of areas and angles, etc) rather than mixing various topics (as happens in some textbooks in, for example, Scotland).
- *Proof in geometry:* In Japanese textbooks, proof in geometry is described as 'Proof is to demonstrate that a statement is true by using already learnt properties as evidence'

From the national curriculum specifications in Japan, we expected to find that deductive reasoning would be prominent in Japanese textbooks and that is exactly what we did find. For example, in *New Mathematics 2* (for grade 8), 33 out of 37 geometry lessons focus on 'justifying and proving' geometrical facts. We found that the manner of mathematical proof is built up through proving various geometrical statements. Given the evidence about how curricula approaches influence students' views of geometry of students, this fits with what we report elsewhere (Kunimune, Fujita & Jones, 2008) that Japanese students tend to see geometry as a very formal subject for study and it is this issue that needs some attention.

The reason for this need for further attention is that, notwithstanding the design of Japanese textbooks, research indicates that Japanese students can have difficulties in fully understanding proof in geometry (see, for example, Kunimune, 1987; 2000). In a related paper, we capture students' understanding of proof in geometry in terms of two notions: 'Construction of proof' and 'Generality of proof' (Kunimune, Fujita and Jones, 2008). These two notions are related to students' understanding in definitions, assumptions, theorems, logical circularity and so on (the notion of 'Construction of proof'), and difference between formal proof and experimental verification (in the notion of 'Generality of proof'). Data collected in 1987, 2000, and 2005 show that while most 14-15 year-old students (Japanese secondary school third grade) can write down a proof (that is, they know about ‘Construction of proof’), around 70% cannot understand why proofs are necessary (that is, they do not necessarily know the ‘Generality of proof’).

Considering that textbooks remain one of the most influential artifacts, there are opportunities to improve the design of textbooks for the teaching of proof in geometry. In terms of Japanese textbooks, an improvement is likely to involve providing students with more effective instructional activities so that they appreciate more fully the notion of ‘generality of proof’ in geometry.

## CONCLUDING COMMENTS AND FUTURE RESEARCH

Our analysis of textbooks commonly used for teaching students about proof in geometry in lower secondary school in Japan indicates that deductive reasoning is prominent. Yet the way that proof and proving is presented in these textbooks shows geometry as a very formal subject for study, one that omits to illustrate convincingly for students the difference between formal proof and experimental verification. As such, we argue that an improvement in textbook design is likely to involve providing students with more effective instructional activities so that they appreciate more fully the notion of ‘generality of proof’.

As we stated above, we are currently undertaking an analysis of the latest editions of textbooks. While we continue to use our existing analytical approach, we are extending our analysis further by focusing ‘proof’ in Japanese textbooks, e.g. different types of proof, presentations of proof, and so on, which are suggested by the recent studies in textbooks and proof. For example, Hanna and de Bruyn (1999 p. 182) use the following sub-categories to classify direct proof; basic, analysis, existence or construction, induction, and miscellaneous. Our impression from our initial analysis is that ‘direct proof’ and ‘direct proof by analysis’ are dominant in Japanese textbooks, but we have yet to conduct an analysis to see if the other types of proof identified by Hanna and de Bruyn appear in Japanese or English textbooks.

*How* proof is presented in textbooks is also an interesting issue for investigation. Nordström & Löfwall (2005) examine how visible (or invisible) are the different aspects of proof (such as inductive/ deductive, conviction/ explanation, etc) in textbooks. They argue that it might be helpful for students if the various aspects of proof were more visible in textbooks, for example by showing the logical structure of proof. Further analysis from these points of view should provide us with further considerations how textbook designs influence and shape students’ knowledge and understanding of proof in geometry.

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