ROBUST REPETITIVE FEEDBACK CONTROL OF A THREE-PHASE GRID CONNECTED INVERTER

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Keywords: Distributed Generator (DG), Grid Connected Converters, Current Control, Repetitive Controller (RC)

Abstract

This paper discusses the design of a repetitive feedback controller for a grid-connected two-level three-phase voltage-source inverter connected between a DC source and the grid through an LCL filter. The controller incorporates a classical two loop feedback of the output current and the capacitor current in addition to a repetitive feedback loop. The results show that the proposed technique improves the steady state error and the total harmonic distortion of output current in presence of utility harmonics.

1 Introduction

Pulse width modulation (PWM) voltage source inverters (VSI) similar to that shown in Fig.1 are commonly used to connect distributed generator (DG) systems such as micro combined heat and power (CHP) and renewable energy sources to an AC grid or local loads. They convert DC from photovoltaic generators, batteries, fuel cells or variable frequency AC from wind and marine turbine into 50/60 Hz AC power. The output current of the converter should meet the total harmonic distortion (THD) standards in the presence of grid harmonics [1, 2]. This is commonly achieved using active feedback control of the current injected into the grid.

Alternative control strategies and structures have been used for grid-connected inverters such as deadbeat control [3], optimal control [4], state-feedback [5], sliding mode [6] and resonant controllers [7], in addition to PID and classical compensators. It is also common to use the d-q transformation [8]. The objective of these controllers is to increase the outer loop gain, and, hence improve disturbance rejection. But most of these controllers tend to suffer from relatively low loop gain at the fundamental frequency and its harmonics and hence tend to have poor disturbance rejection which results in poor output current THD if the grid voltage THD is relatively high. A better controller is required with high gain at the harmonic frequencies of interest.

Repetitive feedback based control techniques have the potential to improve the THD quality of the converter output by effectively increasing the loop gain at the fundamental frequency and its harmonics [9]. The effectiveness of repetitive control in terms of eliminating harmonic distortion in a voltage source inverter operating has been demonstrated in several publications [9-15].

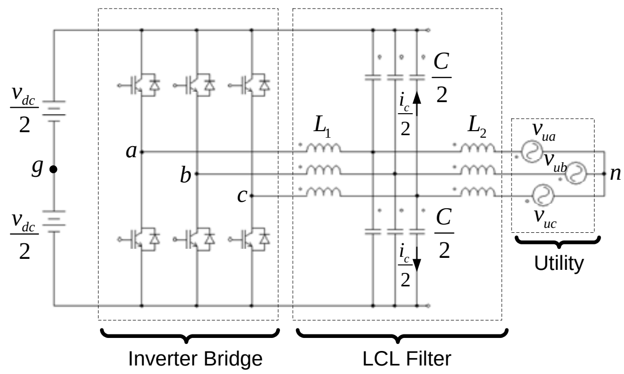

Fig. 1: Three Phase Grid Connected Inverter with LCL Filter

Parameter	Value
Utility Phase voltage	230 V (rms)
DC Link Voltage	800 V dc
Inductor L_1	350 µH
Inductor L_2	50 µH
Capacitance C	22.5 µF
Switching Frequency	10 KHz

Table 1: Electrical Parameters

This paper discusses the design of an alternative control system based on repetitive feedback for the 3-phase grid connected inverter shown in Fig.1. Stability constraints and trade-off between steady state error and system transient response are analysed. Table 1 shows the electrical parameters of the system.

2 System Modelling

The analysis and design of the control system for the voltage source grid connected inverter in Fig.1 is based upon the single phase equivalent circuit shown in Fig.2. The small resistances of inductors and the equivalent series resistance (ESR) of the capacitors are neglected.

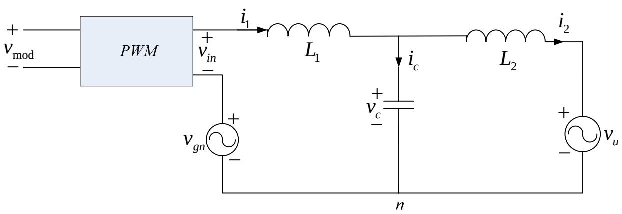

Fig. 2: Single Phase Equivalent Circuit

In Fig. 2, v_{gn} is the voltage difference between the neutral point and middle of the dc link. In control terms this may be viewed as a source of disturbance caused by phase interaction. The disturbance v_{gn} can be expressed by the following equation:

$$v_{gn} = \frac{v_{ag} + v_{bg} + v_{cg}}{3} \tag{1}$$

where v_{ag}, v_{bg} and v_{cg} are phase voltages of the phase with respect to the ground. Equation (1) shows that the phase interaction voltage v_{gn} depends on the switching states of all three phases. It can be shown that, when filter capacitors are connected to dc link as shown in Fig.1, the voltage $v_{gn} \approx 0$, showing only a very small switching frequency ripple component [16].

To derive the transfer function of the grid connected inverter we could write the following equations using Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL) based upon Fig. 2.

$$v_{in} - v_c = L_1 \frac{di_1}{dt} \tag{2}$$

$$i_c = i_1 - i_2 \tag{3}$$

$$i_c = C\frac{dv_c}{dt} \tag{4}$$

$$v_c - v_u = L_2 \frac{di_2}{dt} \tag{5}$$

Based on these equations, we can represent the system using the following block diagram.

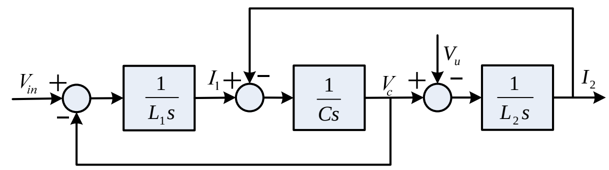

Fig. 3: Block Diagram of the Single-Phase Circuit

3 Proposed Control Scheme

The proposed digital controller comprises a conventional two loop feedback system and a repetitive controller, as shown in Fig. 4. The repetitive feedback controller (RC) requires the basic plant (i.e. the two loop system $G_p(s)$) to be stable. Since direct feedback of the output grid current of an LCL filter on its own is inherently unstable, it is necessary to have another feedback loop of the capacitor current or the current in the main inductor L_1 [16] to stabilize the system. The transfer function relating the output current I_2 to the reference current I_{ref} (assuming the PWM block is a unity gain block) can be shown to be,

$$I_2 = \frac{G_c(s)G_s(s)}{1+G_c(s)G_s(s)} I_{ref} - \frac{G_s(s)}{1+G_c(s)G_s(s)} D(s) \tag{6}$$

where, $G_s(s)$ is the transfer function of the two loop plant given by,

$$G_s(s) = \frac{1}{(L_1 L_2 C)s^3 + (K_c L_2 C)s^2 + (L_1 + L_2)s} \tag{7}$$

and $D(s)$ is the transfer function of input disturbance, which is given by,

$$D(s) = V_u (L_1 C s^2 + K_c C s + 1) \tag{8}$$

The system in Fig. 4 can be reduced to that in Fig. 5, with $G_P(z)$ given by,

$$G_p(z) = \frac{G_c(z)G_s(z)}{1+G_c(z)G_s(z)} \tag{9}$$

The simplified form of the overall control scheme has been represented by Fig. 5 which will be used for analysis later on.

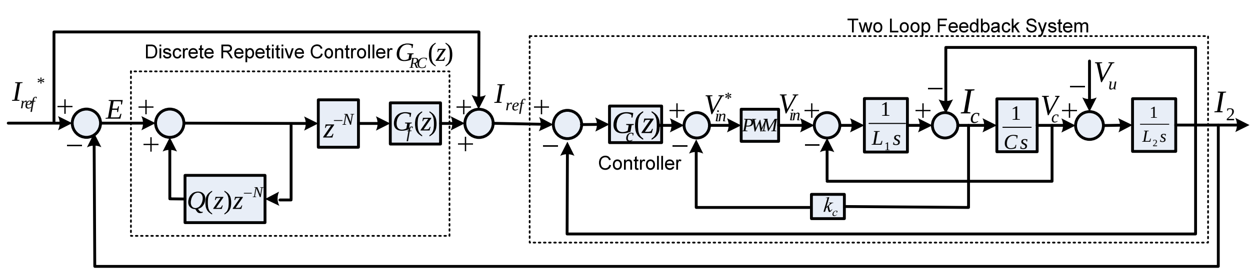

Fig 4: Overall Block Diagram of Proposed Control Scheme

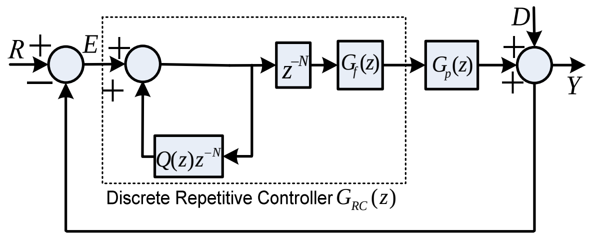

Fig 5: Simplified Block Diagram of Proposed Controller

3.1 The Conventional Two Loop Feedback System

We choose the controller to be a simple proportional controller, such that $G_c(z) = K_P$, since alternative classical controllers such as PID or one of its derivatives, were found to provide marginal improvements (if any) in comparison, at the expense of additional complexity. The values of $K_p = 6$ and $K_c = 13$ were selected to provide a compromise between stability, speed of response and disturbance rejection as discussed in [16]. The bode diagram of the two loop system is shown in Fig. 6; the system has a phase margin of 52.6^o and a gain margin of 7.88 dB. The loop gain at 50 Hz is 18 db and reduces further at higher frequencies. Hence the disturbance rejection at 50 Hz and its harmonics will be relatively poor. However, it is not possible to increase the loop gain any further as without compromising stability.

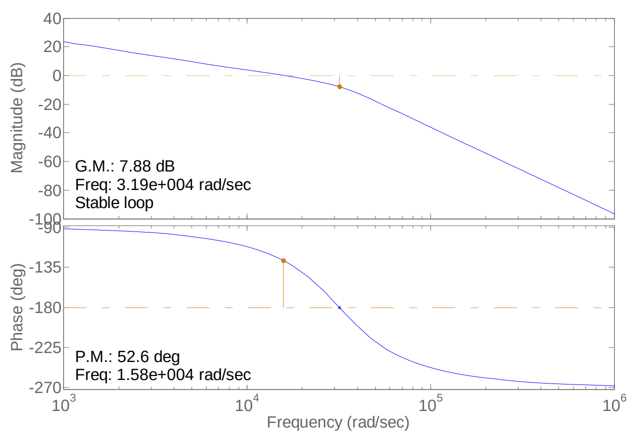

Fig. 6: Bode Plot of Conventional Two Loop Feedback System

3.2 The Digital Repetitive Controller

The theory of repetitive control (RC) is based on the internal model principal [17-19], whereby a model of the repetitive reference and disturbance signals is included in the controller. The RC tracks the error on a cycle by cycle basis and corrects the control effort on a periodic basis to compensate for the error.

In the discrete time domain, a periodic signal with a known period T can be generated by a time delay block z^{-N} with a positive feedback loop. Here, N is the number of samples in one period given by,

$$N = \frac{T}{T_s} \tag{10}$$

where, T is the time period of the any periodic input and T_s corresponds to sampling time. Normally N is a large number and hence a basic RC requires a large memory buffers which is one of its drawbacks [20]. The transfer function $G_{RB}(z)$ of a basic RC comprises a gain K_R multiplied by the transfer function of a periodic signal generator,

$$G_{RB}(z) = K_R \frac{z^{-N}}{1 - z^{-N}} = \frac{K_R}{z^N - 1} \tag{11}$$

The control objective is to find an appropriate optimal value of the repetitive controller gain K_R such that the tracking error converges to zero as the number of iterations approaches to infinity.

$$\lim_{k \to \infty} \| e(k) \| = 0 \tag{12}$$

The basic repetitive feedback control in (10) is most suitable for those applications where the period T is constant or accurately measureable [20]. This basic repetitive feedback does not ensure stability and error convergence criteria and is normally modified to overcome these problems.

To avoid pure integration, a filter $Q(z)$ is introduced in the basic repetitive control structure, followed by a compensator $G_f(z)$ such that

$$G_{RC}(z) = G_R(z) G_f(z) \tag{13}$$

where, $$G_R(z) = K_R \frac{z^{-N}}{1 - Q(z) z^{-N}} \tag{14}$$

The filter $Q(z)$ ensures the stability and robustness of the system. It can be either a low-pass filter or a constant less than 1.

The compensator $G_f(z)$ should ideally be designed to be a zero magnitude and phase compensator for the closed loop transfer function of the plant [9]. This however results in a complex compensator, which is computationally costly to implement. A reasonable approximation is achieved by selecting $G_f(z)$ to be equal to a gain K_R multiplied by the time advance unit z^k [15].

$$G_f(z) = z^k K_R \tag{15}$$

The time advance z^k compensates for the phase lag of the inverter to improve stability.

Using equations (13) and (15), we can rewrite the transfer function of RC as follows:

$$G_{RC}(z) = \frac{K_R z^{-N+k}}{1 - Q(z) z^{-N}} \qquad (16)$$

There are various schemes to design the *Q*-filter and the compensator $G_f(z)$ to improve the robustness of RC [21] . In this paper we select *Q* filter as a constant less than 1 and *k*=3 for the leading unit of the compensator. The value of K_R is adjusted after selecting the value of *Q*. The values of K_R and *Q* are tuned to ensure stability while achieving a good speed of response and steady state error.

From Fig. 5, the error *E(z)* in terms of the reference *R(z)* and the disturbance *D(z)* can be derived as:

$$E(z) = \frac{[1 - G_p(z)][z^N - Q(z)]}{z^N - [(Q(z) - G_f(z) G_p(z)]} R(z) + \frac{[Q(z) - z^N]}{z^N - [(Q(z) - G_f(z) G_p(z)]} D(z) \qquad (17)$$

Theorem: Assume two systems G_1 and G_2 are connected in a feedback loop, then the closed loop system is input-output stable if $\| G_1 \| . \| G_2 \| < 1$

According to the above gain theorem the overall stability conditions can be devised as:

a) The roots of characteristic equation, $1 + G_c(z) G_s(z) = 0$ of conventional two loop feedback system without RC should be inside the unit circle.

b) From equation (17),

$$\left| H(e^{j\omega T}) \right| < 1$$

where,

$$H(e^{j\omega T}) = Q(e^{j\omega T}) - e^{j\omega T} K_R G_P(e^{j\omega T}) \qquad (18)$$

and,

$$\omega \in [0, \frac{\Pi}{T}], \text{ and } T = \text{Sample Time.}$$

4 Selection of Controller Parameters for Robustness

The two parameters, K_R and Q, are closely related to the system stability. The critical value $Q_{critical}$ of Q at which the system becomes unstable, for a given value of K_R, was calculated using equation (18) and verified by simulation. The results are plotted in Fig. 8. Fig. 9 illustrates the relationship between the speed of response of the controller and the parameters *Q* and K_R. Basically, for a given value K_R the speed of response improves by increasing *Q*, at the expense of reducing stability. Increasing K_R improves the steady state error (SSE) for a given value of *Q*. Using Figs. 7 and 8, the values of *Q* and K_R can be selected to ensure stability and achieve a fast speed of response and small steady state error. *Q* = 0.9, and K_R = 0.4 were found to give a satisfactory performance.

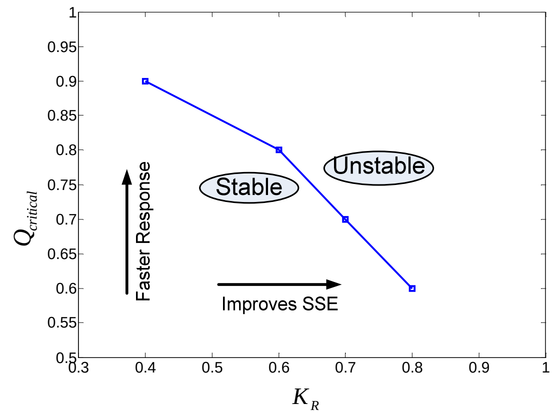

Fig. 7: Relationship between $Q_{critical}$ and K_R

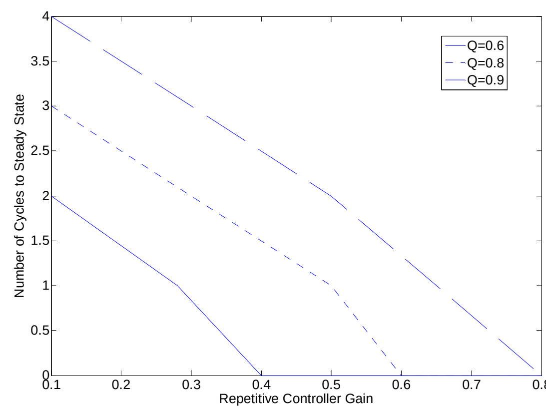

Fig 8: Graph between Number of Cycles to SSE and K_R

The value of the inductor L_2 , which is determined by the grid impedance, can vary significantly depending on the site where the inverter is installed. This uncertainty needs to be taken into account to ensure that the system remains stable under the worst condition. To assess the robustness of the system to the uncertainty in the value of L_2 the bode diagram of the system including the RC was plotted for different values of L_2 as shown in Fig. 9. As it can be seen, the effect of L_2 on system stability and gain at the fundamental frequency and its harmonics is negligible, thus illustrating the robustness of the system to variations in the value of L_2. This was also verified by simulation.

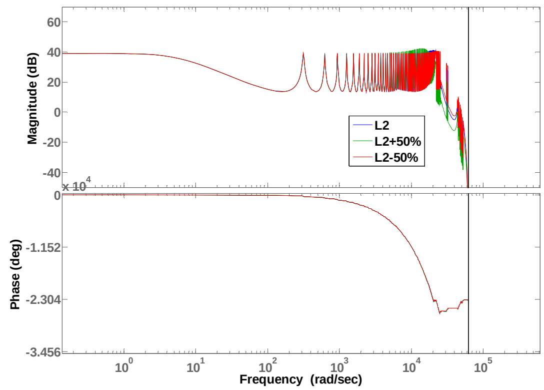

Fig 9: Bode diagram of the system including RC illustrating robustness to variation in L_2

5 Simulation Results

Detailed simulation has been carried out using the MATLAB Simpower Systems Toolbox. The system parameters are shown in Tables 1 and 2. Four cases with different utility THD values have been considered; the grid harmonic content when the THD was 14% is shown in Table 3. The reference current was 100 A (peak).

Parameter	Value
Outer Loop Controller Gain K_P	6
Inner Loop Capacitor Gain K_C	13
Repetitive Controller Gain K_R	0.4
Value of Q	0.9
N for delay term z^{-N}	400

Table 2: Controller Parameters

Harmonic Number	3^{rd}	5^{th}	7^{th}	9^{th}	11^{th}	13^{th}
Fundamental Component	35	25	15	5	2	1

Table 3: Grid voltage harmonics when the THD is 14%

Fig. 11 shows the output current without the use of repetitive controller, while Fig. 12 shows simulation results with the repetitive controller. The THD of the output current improves significantly in any case when the RC is used. For example, in the 2^{nd} case when the utility THD is 14 %, the output current THD with the repetitive controller improves from 9.5% to 4.4 %.

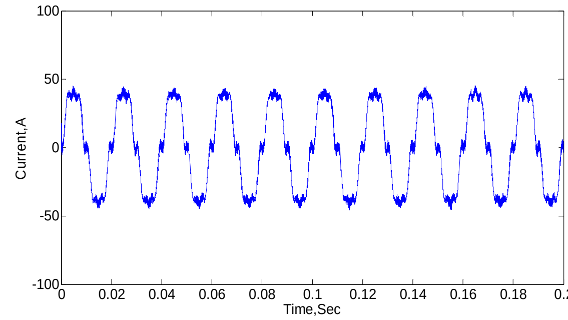

Fig 11: Output Current without RC

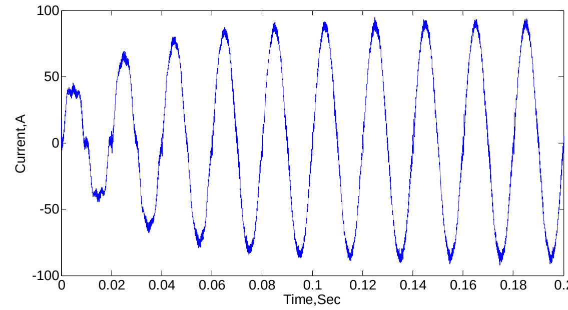

Fig 12: Output Current with RC When Utility THD=14 % and $Q = 0.9, K_R = 0.4$

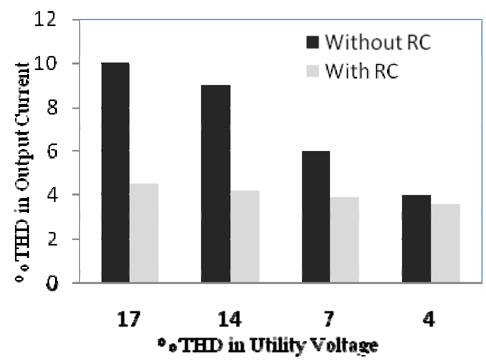

Fig 13: Comparison of % THD in Output Currents at Different Utility THD levels with and without RC

Conclusion

Simulations results show repetitive control can significantly improve the THD quality of the output current. The RC parameters need to be selected carefully to ensure stability despite uncertainty in grid impedance, while achieving a fast response and a small steady state error. The proposed controller was demonstrated to be robust to changes in grid impedance. Further work is needed to improve the steady state error.

Acknowledgements

Mr. M. Jamil is thankful to National University of Science and Technology (NUST), Pakistan for giving scholarship for his PhD studies. Also thanks to Dr. Wali Mohammed Trust for partial support.

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