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A Hamiltonian Approach to Propagation in Chirped and Non-Uniform Bragg Grating Structures

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Abstract

The frequency-dependent time delay in chirped Bragg gratings, and the trapping of light at defects in otherwise uniform gratings, are treated analytically using a Bloch wave Hamiltonian. The approach provides benefits in insight and an appealing physical picture.

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One current and evolving use of fibre gratings is for chirped grating dispersion compensation[1]. Another is in the formation of effective $\lambda/4$ lasing cavities for DFB fibre lasers by imposing a slow local perturbation on the grating parameters[2]. In both cases the essential analytical problem is the same: How to treat the behaviour of light in spatially heterogeneous Bragg gratings? In this paper an approach is developed based on the Hamiltonian optics elegantly summarised by a number of authors, including notably Arnaud in his 1976 book *Beam and Fibre Optics*[3]. The Hamiltonian approach can be applied where the dispersion relation in the homogeneous structure is known, and where, in the heterogeneous real structure, parameters like average index vary slowly in space. It is essentially an analytical method for stepping through a non-uniform structure, matching phase velocities normal to the gradient of the heterogeneity at each step, and propagating along the local group velocity to the next point. This process is described by solutions of Hamilton's equations, which take the general form:

$$\frac{d\mathbf{x}}{d\sigma} = \nabla_k H, \qquad \frac{d\mathbf{k}}{d\sigma} = \nabla H \tag{1}$$

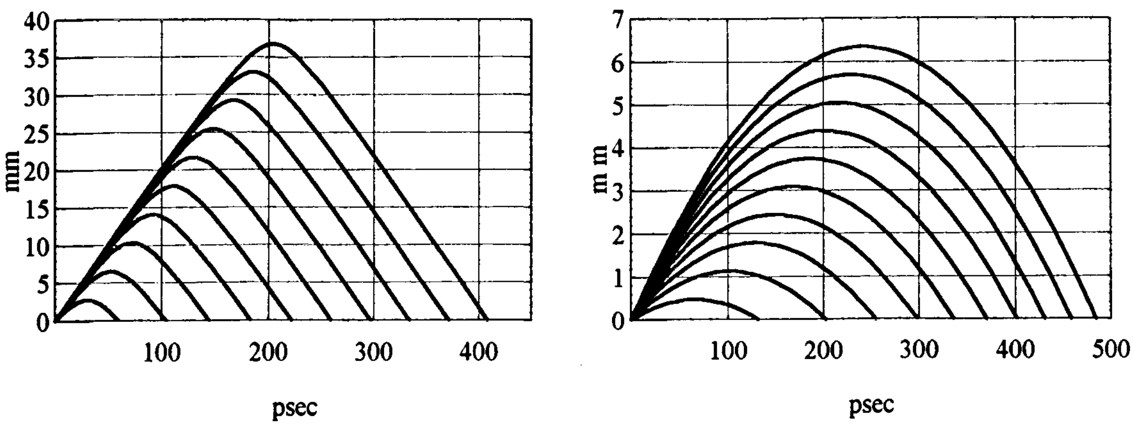
where $\mathbf{x} = \{x, y, z, -t\}$ is the four-vector for space-time, $\mathbf{k} = \{k_x, k_y, k_z, \omega\}$ the generalised wavevector, σ an arbitrary parameter, and $H(\mathbf{x}, \mathbf{k})$ the Hamiltonian, which may be expressed directly from the dispersion relation for the waves. Note that in general $\mathbf{k}$ depends on position. The Hamiltonian itself may be written in a number of equivalent ways, in all of which a phase front is given by the equation $H = 0$. In obtaining solutions to (1), it is important to distinguish total for partial differentiation. For a uniform weakly modulated 1-D grating H takes the special form[4]:

$$H \equiv \omega n_o/c - K/2 - \sqrt{(k - K/2)^2 + \kappa^2} = 0 \tag{2}$$

where κ is the usual grating coupling constant, $K=2\pi/\Lambda$ is the grating vector (Λ being the grating pitch), k the Bloch wavevector, and $k_0=\omega n_0/c$ the average wavevector in the grating. This Hamiltonian applies to any 1-D periodic structure (such as a fibre Bragg grating) whose effective index distribution is given by:

$$n^2(z) = n_o^2(z)[1 + M(z)\cos(K(z)z)] \tag{3}$$

where all the parameters are assumed to vary very slowly over many periods. Note that the Bloch



(a) (b)

Figure 1: Space-time plots of ray paths in (a) weak and (b) strong grating (see text for parameters). The rays are spaced by 0.1 GHz. In (a), because the velocity at input and exit is close to the average value c/n_o, there will be only weak boundary reflections, eliminating Fabry-Perot effects. In (b), because the velocity is significantly less than c/n_o, there will be strong reflections at the boundary and strong Fabry-Perot effects.

waves are the normal modes of electromagnetic propagation in periodic media [4-6], just as plane waves are the modes of isotropic space. Their group velocities describe faithfully the ray paths taken by the light, and permit accurate and detailed explanations for the complex and often beautiful phenomena that can be seen in, for example, periodic planar waveguides[5-7].

The solution of (1) for the Hamiltonian (2) is particularly simple. Without loss of generality, it is given by:

$$\int_{z_o}^{z} \frac{n_o(z)\,dz}{\sqrt{1-1/\delta^2(z)}} = ct \qquad (4)$$

where $\delta = \vartheta(z)/2\kappa(z)$ and $z = z_0$ at t = 0 (the dephasing parameter $\vartheta \equiv 2k_0 - K$). This expression relates time and position in the grating, allowing for example direct calculation of the time taken for light of a given wavelength to be reflected out again from a chirped grating, or the time taken for a complete cycle of oscillation in a heterogeneous DFB resonator. The integral in (4) may be evaluated analytically in a number of special cases. Let us now look at two examples.

The first is a linearly chirped grating. In this case, taking $\vartheta(z) = \vartheta_0 + az$, the solution is:

$$(\vartheta_0 + az)^2 = (2\kappa)^2 + \left(cta/n_o + \sqrt{\vartheta_0^2 - (2\kappa)^2}\right)^2 . \qquad (5)$$

Time/space plots of this solution for different incident conditions are given in Figure 1. The diagrams illustrate one of the limitations of the Hamiltonian approach as developed here; since it is classical, tunnelling effects are not included. Photons of course *do* tunnel through the potential barrier created by the grating stop-band; this process can be incorporated in the analysis by including an ad-hoc tunnelling probability near the stop-band edge. The case treated in Figure 1 corresponds to a pulse of bandwidth 10 GHz at 1.55 μm broadened by 350 psec in a fibre link. Notice that as the wavelength varies, the position where the Bloch waves are turned around shifts as expected. The time taken for light at a given frequency to be reflected is:

$$\tau = \frac{2n_0\sqrt{\vartheta_0^2 - (2\kappa)^2}}{ca}. \tag{6}$$

For a weak grating ($\kappa = 0.05$/mm and $\vartheta_0 = 0.15$/mm for the first-reflected ray, Figure 1a) at $a = 0.018/\text{mm}^2$, the compensation is very linear, although the reflection efficiency will be low (strong tunnelling); the time-of-flight in the grating corresponds closely to a velocity of c/n_0, i.e., $(\vartheta_0/2\kappa)^2 \gg 1$. For a strong grating ($\kappa = 10$/mm and $\vartheta_0 = 20.05$/mm for the first-reflected ray, Figure 1b) at $a = 0.1/\text{mm}^2$, on the other hand, the compensation is significantly nonlinear (owing to the proximity of the strong stop-band) although the efficiency will be much higher. This nonlinearity may be eliminated to a large degree by operating so that none of the frequencies in the pulse see Bragg reflection until they are already well into the grating; this of course implies the need for a longer grating. Notice however that, owing to the reduced group velocities near the stop-band edge, the grating length required is 7 times shorter than in the weak-grating case. The Hamiltonian solution as presented does not treat the reflection at the input boundary to the grating. This causes Fabry-Perot-like interference fringes in the cavity formed between itself and the turning point of the rays in the grating. If, however, the Bragg condition is not satisfied for any of the frequencies in the pulse in the initial few mm of the grating, the visibility of these fringes will be insignificant.

In the second example we consider a grating in which the square of the coupling constant varies quadratically with position:

$$\kappa^2 = \kappa_0^2(1 + az + bz^2). \tag{7}$$

The solution in this case takes the form:

$$\begin{aligned} z &= z_0\cos(\gamma t) + z_1\sin(\gamma t), \qquad \gamma = 2c\kappa_0\sqrt{b}/(n_0\vartheta), \\ z_1 &= \sqrt{(v_0/\gamma)^2 - z_0^2}, \qquad v_0 = (c/n_0)\sqrt{1-(2\kappa_0/\vartheta)^2} \end{aligned} \tag{8}$$

where z_0 is the initial position and γz_1 is the initial group velocity, equal to v_0 if $z_0 = 0$. These solutions are plotted in Figure 2 against t and $v_0 n_0/c$ for $z_0 = 0$, $a = 0$ and $b = 0.15/\text{mm}^2$. As the launch velocity decreases (moving closer to the stop-band edge), the amplitude (in mm) of the oscillation and the cavity round trip time both decrease as expected. The actual quantised frequencies of oscillation will be determined by the usual requirement that the round trip phase be a multiple of 2π, a matter we do not address here. Notice that if $b<0$, i.e., the curvature of κ^2 is reversed, and the behaviour is "deflective", by which we mean that a time delay is introduced, the light being transmitted, or the light is turned around and kicked out of the grating after a certain delay.

In conclusion, the Hamiltonian approach outlined in this paper provides an elegant alternative viewpoint from which to analyse and understand the behaviour of light in spatially heterogeneous 1-D grating structures. Its limitations are that the properties must vary slowly over many periods (well approximated in fibre Bragg gratings), and that tunnelling (not a classical concept) cannot be treated easily. More complicated spatial heterogeneities can be treated by numerical integration of (4).

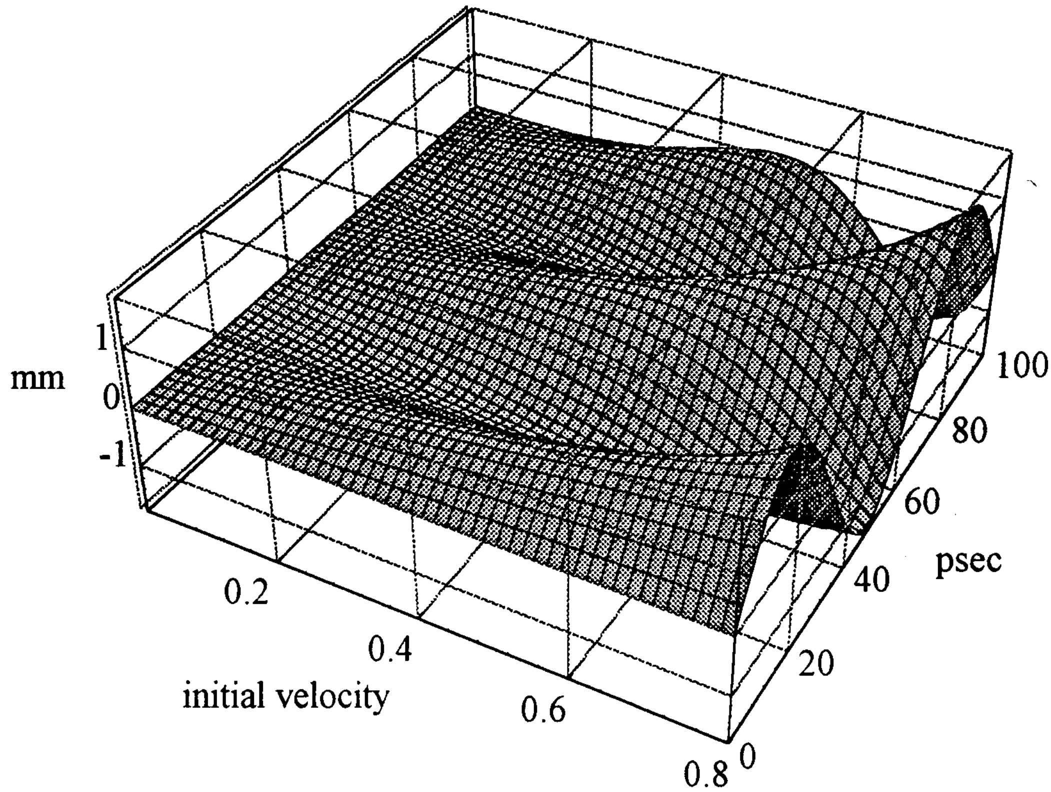


Figure 2: Plot of the motion of rays launched at different initial velocities at z=0 in a grating with a quadratic variation in grating strength with z. See the text for the parameters used.

References

[1] See, e.g., J. Lauzon, S. Thibault, J. Martin and F. Ouellette, "Implementation and characterisation of fibre Bragg gratings linearly chirped by a temperatire gradient," *Opt.Lett.* **19** (2027-2029) 1994.

[2] J. T. Kringlebotn, J.-L. Archambault, L. Reekie and D.N. Payne, "Er^{3+}:Yb^{3+} co-doped fiber distributed feedback laser," *Opt.Lett.* **24** (2101-2103) 1994.

[3] J. A. Arnaud, *Beam and Fiber Optics*, (Academic Press, New York, San Francisco, London, 1976)

[4] P.St.J. Russell, "Bloch wave analysis of dispersion and pulse propagation in pure distributed feedback structures", *J.Mod.Opt.*, **38** (1599-1619) 1991; Erratum: J. Capmany and P.St.J. Russell, *J.Mod.Opt.*, 1994.

[5] P.St.J. Russell, T.A. Birks and F.D. Lloyd-Lucas, "Photonic Bloch Waves and Photonic Band Gaps," in *Confined Electrons and Photons: New Physics and Applications*, E. Burstein & C. Weisbuch (editors), Plenum Press, 1995

[6] P.St.J. Russell, "Interference of integrated Floquet-Bloch waves," *Phys.Rev.*, **A33** (3232- 3242) 1986.

[7] R. Zengerle, "Light propagation in single and doubly periodic planar waveguides," *J. Mod. Opt.* **34** (1589-1617) 1987.