PHOTONICS WEST '95
Invited Talk
9-15

HARMONICALLY MODE-LOCKED FIBRE SOLITON LASERS AND THEIR APPLICATIONS

A.B.Grudinin and S.Gray

Optoelectronics Research Centre, University of Southampton, Southampton, SO17 1BJ, UK

ABSTRACT

We present a detailed description of a passive harmonically mode-locked laser. Experimental results are consistent with the suggestion of a passive self-stabilization effect driven by transverse acoustic wave excitation due to electrostriction. We also demonstrate some applications of the laser.

1. INTRODUCTION

Fibre lasers based on erbium-doped single mode optical fibres offer a unique possibility to engage simultaneously several key effects from different branches of modern physics: laser physics, nonlinear optics and fibre optics. Erbium-doped fibres are almost ideal as the basic component for lasers and amplifiers operating near 1550 nm which is the wavelength of lowest fibre loss and where the combined effect of negative group velocity dispersion and Kerr nonlinearity gives rise to the possibility of generating optical solitons i.e. pulses capable of propagating down the fibre without significant changes of their shape. That is why fibre lasers capable of producing ultra-short nonlinear optical pulses have been subject of considerable interest over the past several years.There are two main approaches in the development of short pulse fibre lasers. The first method is a traditional one and comes from "classical" laser physics and is based on the incorporation of an amplitude or phase modulator inside the laser cavity in order to achieve active mode-locking. Active modelocking employing intracavity modulators has been shown to be capable of generating very high repetition rates[1,2]. The second method based on specific properties of optical fibres exploits the Kerr nonlinearity and is fully passive. Passive modelocking, which is attractive because of its simplicity, has been demonstrated in several different configurations. The fundamental idea of all passive mode-locking techniques is to provide additional loss for the low-intensity component by incorporating of an intensity-dependent component into the laser cavity. Historically, the first example of a passively mode-locked fibre laser was the so called figure of eight laser[3-5] which uses the nonlinear switching characteristics of a nonlinear amplifying loop mirror (NALM)[6] to achieve modelocking. In such a laser waves travelling in opposite directions within the loop experience a different nonlinear Kerr phase shift due to the asymmetrically placed fibre amplifier.

The simplest type of passively mode-locked fibre laser is a ring configuration[7-9]. In this type of laser the effect of cross-phase modulation between eigen-polarized modes results in a different state of polarization for low- and high-intensity components. The incorporation of a polarizer into the laser cavity causes the required loss difference to suppress low-level radiation and to achieve mode-locking.

The two types of passively mode-locked fibre laser are in fact the sources producing optical solitons. Due to the soliton nature of the generated pulses there is a strong relationship between the temporal width of the pulse and its energy, while the pulsewidth is defined by the laser cavity parameters (primarily by the length and intracavity dispersion). This leads to the fundamental difference between conventional mode-locked lasers and soliton lasers: a change in the pump power of a conventional laser results in a change of the parameters of the generated pulses (peak intensity, pulsewidth etc) but any changes in the pump power of soliton lasers changes the number of propagating pulses.

This remarkable feature of the soliton laser is the result of the specific output characteristics of fibre soliton lasers: the output radiation consists of a number of soliton pulses and a non-soliton component. The number of circulating pulses is defined by the ratio of the stored intracavity energy to the soliton energy. Generally this ratio is not an integer and this results in the formation of the non-soliton component which plays the role of a buffer: any excess of stored intracavity energy (caused, for example, by small fluctuations of pump power) translates into the nonsoliton component leaving the parameters of the solitons largely unchanged.

The main problem associated with these lasers is instability in the repetition rate because the output of soliton lasers suffers from timing jitter and in many ways resembles a long distance soliton transmission line: in both systems a stream of solitons experiences periodic gain and loss and is subject to the same instabilities such as Gordon-Haus jitter[10] and soliton interactions with various fields.

One solution is to operate the laser with just a single pulse circulating inside the cavity, but this generally leads to low repetition rates, low average output powers (unless short cavities of a few meters are used) and actually does not prevent timing jitter but just makes it less noticeable. Other techniques involving additional sub-cavities[11], extra-cavity feed-back[12] or intracavity modulation[13] have been demonstrated enabling higher harmonic mode-locking, but these techniques would appear to be difficult to implement in a practical system.

Extensive studies of soliton transmission lines (see, for example[14] and references therein) have shown that to maintain the soliton stream over unlimited distance requires spectral filtering and synchronous modulation. Hence to achieve stable (in terms of repetition rate) mode-locking in soliton lasers one has to implement a spectral filter and a modulator inside the laser cavity.

However it has been recently demonstrated[8] that under certain conditions it is possible to achieve stable harmonic modelocking in a fully passive scheme. It was found[15] that in this regime of operation spectral filtering is provided by the finite bandwidth of the gain medium while acoustic wave generation by the pulses provides the phase modulation resulting in the passive stabilization of the repetition rate.

In this paper we present a detailed description of such a passive harmonically mode-locked fibre laser and give some applications of this very simple source of picosecond pulses.

2. THE EFFECT OF REPETITION RATE SELF-STABILIZATION

When a soliton propagates down an optical fibre the intense electric fields generated in the fibre core distort the fibre material and produce an acoustic wave. The acoustic wave causes density changes in the fibre which change the refractive index for following pulses imposing phase modulation.

Following the work of [16] the electric field of a pulse in the fibre can be written as

$$E(z,t,r)=\frac{1}{2}\mathbf{e}F(r)\ [E(t)\,e^{i(\beta z-\omega t)}+cc] \qquad \textbf{(1)}$$

where **e** is the electric field polarization vector, E(t) represents the pulse envelope and F(r) is the transverse field distribution which is taken to be gaussian.

The displacement vector u of a point in the fibre can be described by the equation

$$\frac{\partial^2 u}{\partial t^2} - v^2\nabla^2 u + 2\Gamma\frac{\partial u}{\partial t} = \frac{1}{2nc}\frac{\partial \epsilon}{\partial \rho}\nabla F(r)\, I(t) \qquad (2)$$

where v is the velocity of sound in the fibre, ρ is the density, ε is the dielectric constant, Γ is the acoustic damping coefficient and I(t) is the temporal pulse shape.

The solution of Eq.2 gives an expression for the refractive index change due to a single pulse[16]

$$\delta n(t) = -\frac{\rho_0}{4cn^2}\left(\frac{\partial \epsilon}{\partial \rho}\right)^2 I_0 t_p \frac{\sum_n \left[B_n C_n e^{-\Gamma t}\frac{\sin(\Omega_n t)}{\Omega_n}\right]}{\int F^2(r)\, dS} \qquad (3)$$

where B_n and C_n are constants describing overlapping of acoustic and light fields and Ω_n is n-th acoustic eigen-frequency.

Eq.3 shows that the magnitude of the acoustic response is proportional to the soliton energy. The refractive index change is plotted in Fig 1a for a 0.7 ps pulse. The lifetime of the response is of order 2 ns with a peak of $\delta n \approx 10^{-11}$.

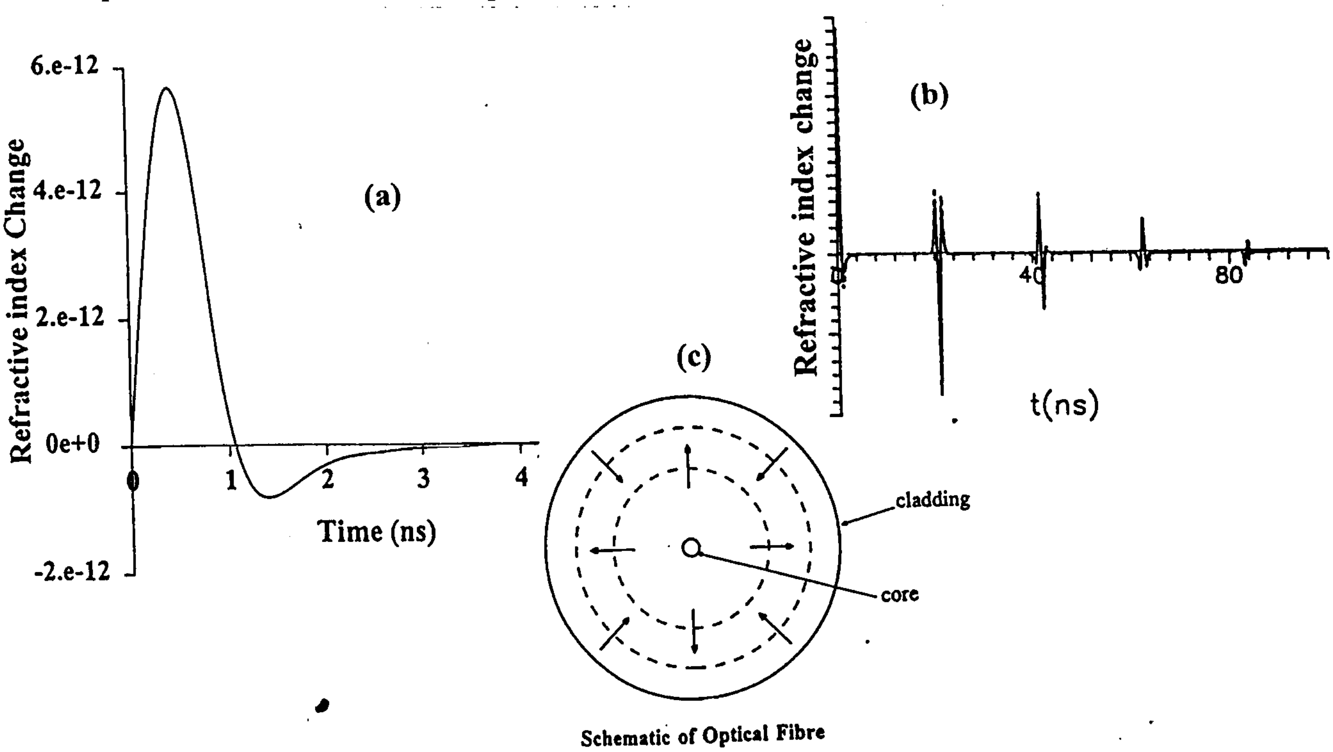

Fig.1 Acoustic response of an optical fibre to a 0.7 ps soliton

The wave travels radially outwards from the core and is reflected from the cladding boundary back towards the core (Fig.1c). The wave crosses the core region of the fibre again after a round trip time of ~20 ns as shown in Fig.1b. The reflections of the waves from the cladding boundary mean that the acoustic waves generated by one pulse can affect the propagation of many following pulses.

We can now consider the effect of propagating a soliton stream. In this situation the fibre acts as a damped oscillator driven by a periodic force. The equation for the refractive index change becomes

$$\delta n(t) = -\frac{\rho_0}{4cn^2}\left(\frac{\partial \epsilon}{\partial \rho}\right)^2 I_0 t_p \frac{\sum_n \sum_m \left[B_n C_n e^{-\Gamma(t+mT)} \frac{\sin(\Omega_n(t+mT))}{\Omega_n}\right]}{\int F^2(r)\, dS} \tag{4}$$

which for an infinite pulse train reduces to

$$\delta n(t) = -\frac{\rho_0}{4cn^2}\left(\frac{\partial \epsilon}{\partial \rho}\right)^2 \frac{I_0 t_p}{\int F^2(r)\, dS} \sum_n \left[\frac{B_n C_n}{\Omega_n} e^{-\Gamma t} \frac{[\sin(\Omega_n t) + e^{-\Gamma T}\sin(\Omega_n(T-t))]}{1+e^{-2\Gamma T}-2e^{-\Gamma T}\cos(\Omega_n T)}\right] \tag{5}$$

From Eq.5 we can see that whenever the pulse repetition frequency is close to one of the eigenfrequencies, or one of its harmonics, the argument of the cosine term in the denominator is close to 2π. The denominator is therefore small and the magnitude of the refractive index change increases. In this situation only one term dominates the summation and the refractive index change is approximately sinusoidal.

The spectrum of the acoustic response is shown in Fig.2[16].

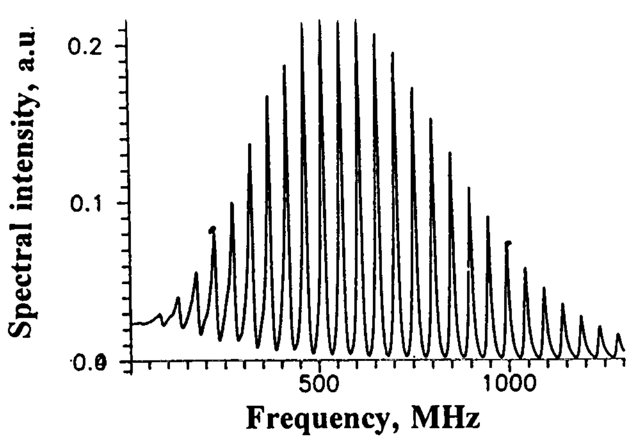

Fig.2 Spectrum of the acoustic response of optical fibre

This shows that the maxima of the response occur at the natural eigenfrequencies of the fibre with the peak response occurring at frequencies close to 500 Mhz.

Fig.3 shows the refractive index change generated by streams of solitons at 463 Mhz (10th eigenfrequency) with peak $\delta n \approx 10^{-9}$.

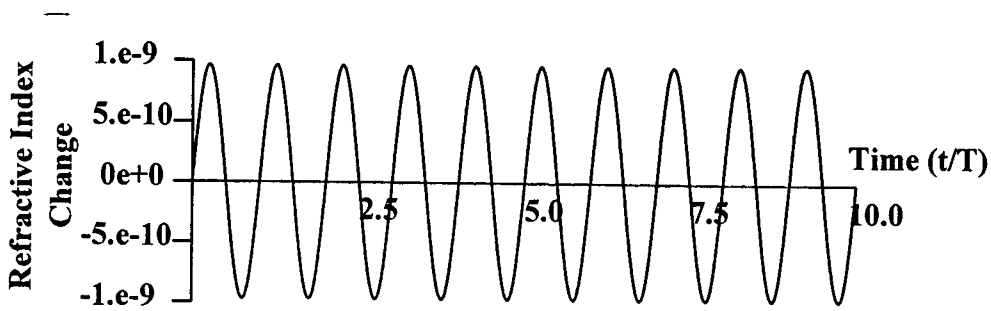

Fig.3 Temporal dependence of the refractive index when the laser repetition rate is equal to 10-th acoustic harmonic of the laser fibre

The pulses are positioned close to the peaks of the refractive index change. This imposes a phase modulation on the pulses which acts to stabilize the repetition rate.

This model of the refractive index change allows us to make a theoretical estimate of the time jitter by inserting the phase modulation as a perturbation to the Nonlinear Schrodinger Equation (NSE). For a passively modelocked fibre laser the NSE can be written in the form

$$i\frac{\partial\psi}{\partial z}+\frac{1}{2}\frac{\partial^2\psi}{\partial t^2}+|\psi|^2\psi=iG\psi+i\beta\frac{\partial^2\psi}{\partial t^2}+(\phi_{ac}-\gamma t^2)\psi+i\alpha|\psi|^2\psi+S(z) \quad (6)$$

where the left hand side is the unperturbed NSE. The first term on the right hand side describes the laser gain, the second term describes the intracavity filter formed by the amplifier bandwidth, the third term describes the action of the acoustic phase modulation with

$$\phi_{ac}=\frac{2\pi}{\lambda}\delta n_{ac}z_d \quad (7)$$

$$\gamma=\phi_{ac}\frac{2\pi^2}{T^2} \quad (8)$$

where δn_{ac} is the peak refractive index change, z_c is the cavity length, z_d is the dispersion length and T the pulse separation normalized to the soliton pulsewidth. The fourth term on the rhs of Eq.6 represents modelocking by a fast saturable absorber and S(z) is the noise from the laser amplifier. In our model we assume that the time jitter is small compared to the pulse period and therefore treat T as a constant.

Taking the solution to the unperturbed NSE as

$$\psi(z,t)=\eta sech(\eta(t-t_c)\exp(i(\Omega t-\phi)) \quad (9)$$

and applying perturbation theory[17] we obtain an equation for the soliton temporal position t_c where we have assumed that the noise impact on the soliton position is driven by fluctuations of the soliton carrier frequency.

The noise correlation function is given by

$$\frac{d^2t_c(z)}{dz^2}+\frac{4\beta}{3}\frac{dt_c(z)}{dz}+2\gamma t_c(z)=S_\Omega(z) \tag{10}$$

$$<S_\Omega(z)S^*{}_\Omega(z')>=\delta(z-z')\frac{(G^2-1)}{3N_0\xi_c}=\delta(z-z')N_\Omega \tag{11}$$

where N_0 is the number of photons per unit energy and ξ_c is the cavity length.

Taking the Fourier transform of Eq.10 and solving in the spatial frequency domain, k, gives us an expression for the variance of the time jitter[15]

$$\sigma_T^2=\int_{-\infty}^{+\infty}\left\langle|t_c(k)|^2\right\rangle dk=\frac{3N_\Omega}{2\beta\gamma}=\frac{(G^2-1)T^2\Omega_f^2}{32N_0\phi_{ac}\xi_c^2} \tag{12}$$

where $\beta=2/(\Omega_f^2\xi_c)$ and Ω_f is the dimensionless filter bandwidth.
Eq.12 gives us the dependence of the jitter on the filter bandwidth, the repetition rate and the refractive index change which is also dependent on the repetition rate due to the resonances mentioned above.

The strength of the phase modulation is proportional to the gradient of δn close to the peak. In Eq.10 γ corresponds to a restoring force on the pulses. Increasing the repetition rate decreases the value of T and increases γ. Therefore we expect the jitter to decrease with increasing repetition rate. The actual behaviour of the jitter is quite complicated due to the dependence on ϕ_{ac} on the repetition rate.

3. PASSIVE HARMONICALLY MODE-LOCKED FIBRE SOLITON LASER. EXPERIMENTAL STUDY

For passive stabilization of the laser we need to make the acoustically induced refractive index change as large as possible. Therefore we have to achieve the energy of the laser pulses as high as possible. To do so we have used a short cavity length to generate short pulses and standard telecom fibre with dispersion D=17 ps/nm·km.

As it was pointed out above, the number of circulating pulses within the cavity is proportional to the intracavity energy and to achieve the required repetition rate of ~500 Mhz we need a large power from the amplifier which was obtained by using Er^{3+}/Yb^{3+} codoped fibre pumped at 1064 nm by a Nd:YAG laser.

The laser configuration used is shown in Fig.4. It comprises 4 metres of Er^{3+}/Yb^{3+} fibre, a length of standard telecom fibre, a polarization sensitive isolator and two sets of polarization controllers. The pump laser was connected to an external feedback loop which kept pump power fluctuations to below 1%. Modelocking of the laser was achieved by adjustment of the polarization controllers and at certain positions of the controllers a harmonic modelocked regime was observed. By changing the pump power we were able to change the repetition rate of the laser.

The time jitter was measured by analyzing the RF spectrum of the laser output intensity[18].

The jitter measured as a function of pulse repetition rate is plotted in Fig.5 for a laser cavity with fundamental frequency of 5.78 Mhz and pulsewidth 0.8 ps.

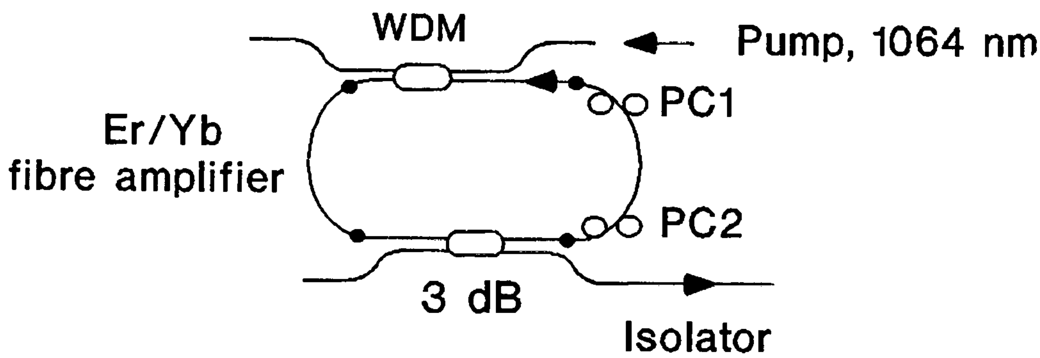

Fig.4 Laser cavity configuration

The theoretical curve from equation 7 is plotted for comparison.

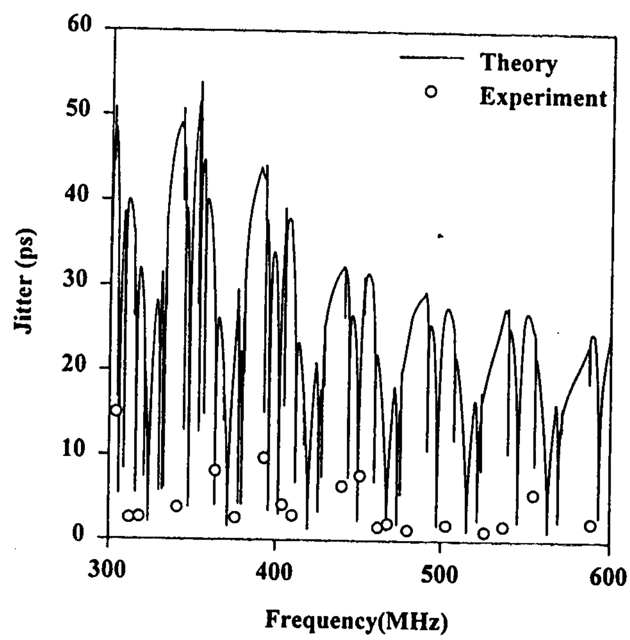

Fig.5 Experimental (O) and theoretical (solid line) dependance of the timing jitter

The experimental points show a strong oscillatory behaviour with a tendency to decrease at frequencies close to 500 Mhz. The theoretical curve shows a complicated dependence on frequency but demonstrates the same qualitative behaviour. The reason for this is the dependence of the refractive index change on the repetition rate. Resonant effects not only occur at the eigenfrequencies but also at fractions and harmonics of the eigenfrequencies.

efficiency dependence on gain.

From the presented results one can conclude that there is an optimal gain G_{opt}, corresponding to the maximum switching efficiency and a clean compressed output pulse with time-bandwidth product of 0.28 (which indicates a slight deviation of the switched pulse envelope from $sech^2$-shape). For gain exceeding G_{opt} at the NALM output we observed autocorrelations and spectra corresponding to multisoliton pulses. For $G<G_{opt}$ the time-bandwidth product of the output pulses was close to 0.31, while pulsewidths varied by over a factor of two.

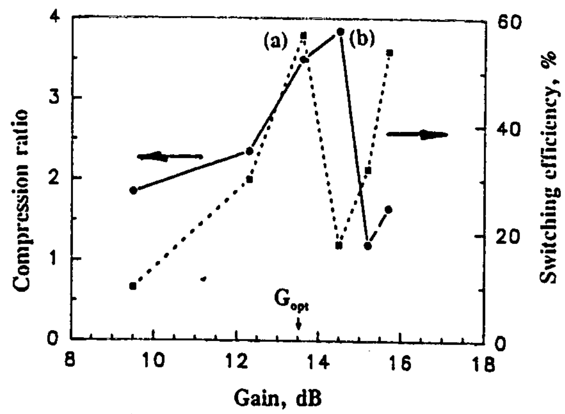

Fig.7 Gain dependance of compression ratio and switching efficiency

The gain dependence of the pulsewidth gives rise to the possibility of a developing tunable source of femtosecond pulses exploiting the effect of the soliton self-frequency shift. It is known that due to Raman gain the soliton central wavelength experiences a Stokes frequency shift[20]. Thus using an auxiliary fibre at the NALM output one can translate the pulsewidth variation into a controllable wavelength shift.

To demonstrate such a source we decreased the pulsewidth of the fibre laser output to 1.3 ps by shortening the laser cavity length. The length of the undoped fibre in the NALM was also reduced to 6 m. In this configuration by changing the NALM gain in the range of 13.5-15 dB we observed reduction of the pulsewidth from 660 fs to 250 fs, while the time-bandwidth product change was from 0.32 to 0.28.

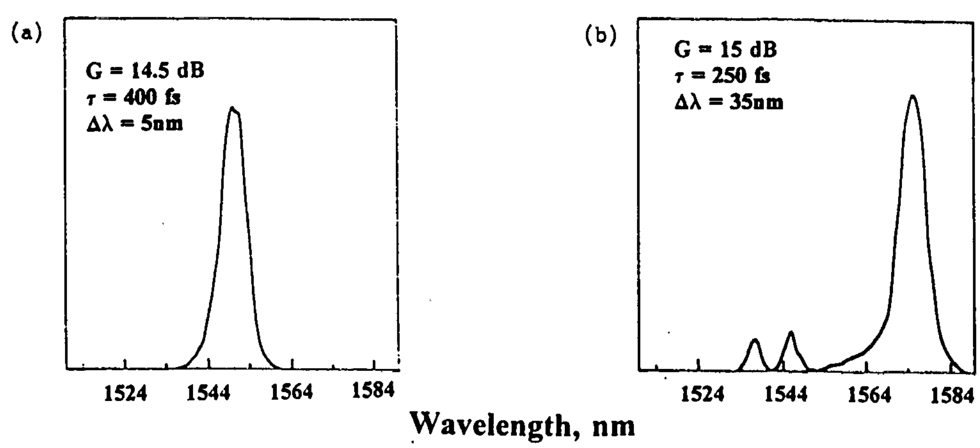

Fig.8 Spectra after 300 m of standard telecom fibre corresponding to
(a) gain 14.5 dB, pulsewidth 400 fs and spectral shift 5nm
(b) gain 15 dB, pulsewidth 250 fs and spectral shift 35 nm

For 250 fs solitons travelling in a lossless fibre with a dispersion of D=17 ps/nm·km the soliton self-frequency shift is expected to be $\delta\lambda \sim$ 10nm/100 m. Fig.8a shows the spectrum of the 400 fs pulse after 300 m of the auxiliary fibre for a NALM gain of 14.5 dB. The pulse spectral shift is 5 nm. For 15 dB gain the pulsewidth reduced to 250 fs that led to a spectral shift of 35 nm (Fig.8b). The residual signal at the original wavelength corresponds to the non-soliton component and can be suppressed by an appropriate non-linear technique, based, for example, on nonlinear polarisation rotation.

4.2 Detailed study of soliton-soliton interaction

In ultralong distance soliton transmission the limit to the single channel bit rate is defined by jitter in the pulse arrival time. The fundamental origin of the time jitter is the change of the soliton central frequency due to perturbations experienced by the propagating solitons. Such perturbations can arise from spontaneous emission noise in optical amplifiers (the Gordon-Haus effect)[10], partial overlapping of adjacent pulses ("classical" soliton-soliton interaction effect)[21] or excitation of a weak acoustic wave[16].

Another widespread source of perturbations is a weak cw component, accompanying almost any propagating soliton. Here we demonstrate the practical importance of such perturbations.

Pulses from the laser were first passed through a Michelson interferometer to convert them into pulse pairs. A micrometer screw adjustment of the length of one interferometer arm allowed the pulse separation to be varied continuously from zero to a few tens of picoseconds while piezoelectric transducer control of the other arm allowed for the precise adjustment of the relative optical phase. The pulses were amplified after the interferometer with an Er^{3+}/Yb^{3+} codoped fibre amplifier to maintain fundamental soliton propagation in the transmission fibre, which consisted of 2 km of standard telecom fibre (D=17 ps/nm km) making the propagation distance of the order of 20 dispersion lengths with negligible losses. The soliton pulse pairs were detected with an autocorrelator and optical spectrum analyzer. PC1 was first adjusted to set the laser mode-locked with a minimal fraction of non-soliton component.

Fig.9a shows the experimental result for soliton-soliton interaction in which output pulse separation τ_{out} is plotted as a function of input pulse separation τ_{in} for both in-phase (relative soliton phase θ=0) and opposite phase ($\theta=\pi$) cases, which fitted well with theoretical predictions (solid lines) based on the working[21].

However, by increasing the fraction of the non-soliton component (by changing the position of the polarization controller PC1, we were able to change the fraction of the non-soliton component energy from less than 1% to 20%) we observed significant changes in the soliton behaviour. Fig.9b shows the dependence of the output pulse separation from τ_{in} for the cw component having 20% of the average power respectively. Corresponding optical spectra are shown in the inset. From the presented data, one can see a well-defined oscillatory dependence of with a period T= 6±0.5 ps. The phase of the oscillation is dependent on θ, and a strong dependence of τ_{out} on θ was still observed for τ_{in} exceeding five soliton pulsewidths, where "classical" soliton interaction forces are expected to be very weak. In addition, the oscillations are more pronounced with increasing fraction of the cw component. Thus a frequency shifted non-soliton component results in an oscillatory behaviour of the output pulse separation even when "classical" soliton-soliton interaction forces are negligibly small.

Applying perturbation theory based on the inverse scattering transformation, we have solved the Nonlinear Shrodinger Equation and found that[22]

(i) the non-soliton component causes negligibly small perturbations to the soliton amplitude, while the change in the soliton central frequency results in a noticeable temporal shift of the soliton;

(ii) τ_{out} contains an oscillatory term with amplitude dependent on the frequency shift as well as the amplitude of the non-soliton component;

(iii) the oscillatory term vanishes for a frequency unshifted non-soliton component, and there is a frequency difference Ω_m resulting in a maximum value of the oscillatory term.

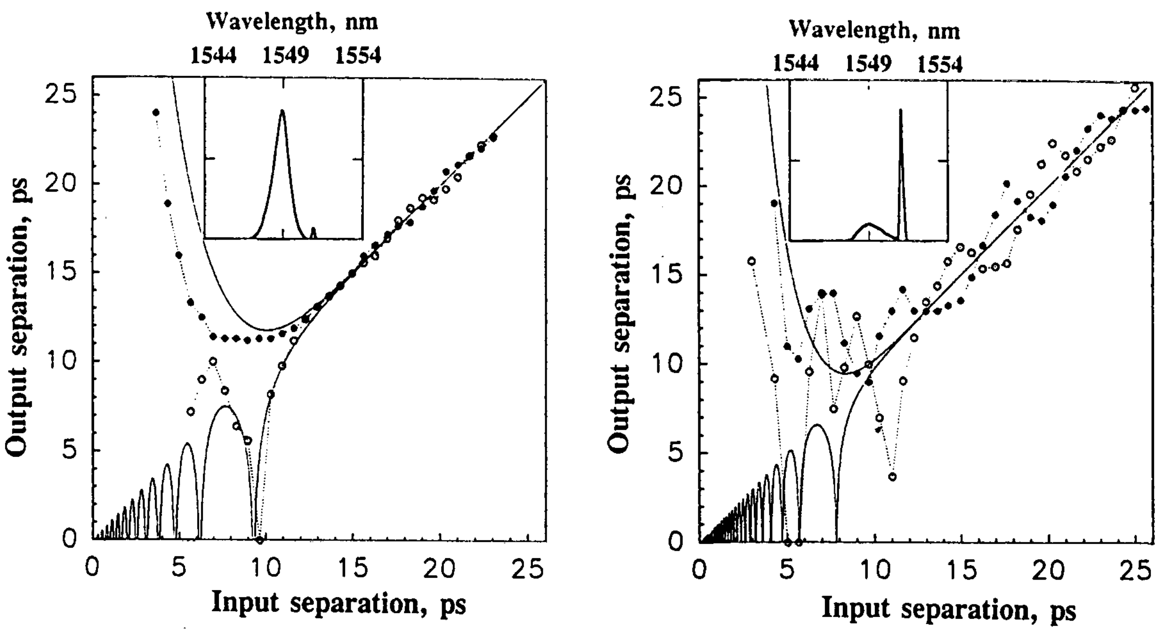

Fig.9 Pulse separation at the output of a 2 km fibre when interacting solitons are in phase (open circles) and in opposite phase (filled circles)
(a) Fraction of cw component is less than 1%
(b) Fraction of cw component 20%

5. CONCLUSION

We presented here a detailed description of the passive harmonically mode-locked fibre laser. The effect of acoustic wave generation induced by the propagating pulses results in repetition rate self-stabilization. Theoretical consideration of this effect revealed key effects responsible for timing jitter and indicated methods to diminish this inevitable effect. In particular incorporation of an intracavity spectral filter and the use of uncoated fibre (in order to increase life-time of acoustic waves) should further decrease the timing jitter of such a laser. Note that the effect of passive self-stabilization occurs not only in ring fibre lasers, but also in other types of passively mode-locked fibre lasers such as the figure eight laser or lasers with intracavity saturable absorbers that makes it a very attractive way to achieving rather high repetition rate in a simple cavity configuration.

We also demonstrated some applications of the laser. In particular a simple way to create a tunable source of femtosecond pulses has been shown.

6.REFERENCES

1. Takada, A., Miyazawa,H., "30 GHz picosecond pulse generation from actively mode-locked erbium-doped fibre laser", Electronics Letters, **26**, pp 216-217 (1990).
2. Davey, R.P., Smith, K., McGuire, A., "High speed, mode-locked, tunable, integrated erbium fibre laser", Electronics Letters, **28**, pp 482-484, (1992).
3. Duling, III, I.N., Electronics Letters, "Subpicosecond all-fibre soliton laser", **27**, 544, (1991).
4. Richardson, D.J., Laming ,R.I., Payne, D.N., Phillips, M.W., Matsas, V.J., Electronics Letters, "320 fs soliton generation witha passively mode-locked laser", **27**, 732, (1991).
5. Nakazawa, M., Yoshida, E., Kimura, Y., "Generation of 98 fs optical pulses directly from an erbium-doped fibre laser", Electronics Letters, **29**, pp 63-65, (1993).
6. Fermann, M. N., Haberl, F., Hofer, M. and Hochreiter, H., " Nonlinear amplifying loop mirror", Optics Letters, **15**, pp 752-754, 1990
7. Matsas, V.J, Richardson, D.J., Newson, T.P., Payne, D.N., "Self-starting passively mode-locked fibre ring laser exploiting nonlinear polarization rotation", Electronics Letters, **28**, pp 1391-1392, (1992).
8. Grudinin, A.B., Richardson, D.J., Payne, D.N., "Passive harmonic mode-locking of a fibre soliton ring laser", Electronics Letters, **29**, 1860 (1993).
9. Tamura, K., Doerr, C.R., Haus, H.A., Ippen, E.P., "Soliton fibre ring laser stabilization witha broad intracavity filter", IEEE Photonic Technoligy Letters, **6**, pp 697-699, (1994)
10. Gordon, J.P., Haus, H.A., "Random walk of coherently amplified solitons in optical fibre transmission", Optics Letters, **11**, 665 (1986).
11. Yoshida, E., Kimura, Y., and Nakazawa, M., "Laser diode-pumped femtosecond erbium-doped fibre laser with a sub-ring cavity for repetition rate control", Applied Physics Letters, **60**, pp 932-934, (1992)
12. Dennis, M.L., Duling, I.N. III, "High repetition rate figure eight laser with extracavity feedback", Electronics Letters, **28**, pp 1894-1895, (1993)
13. Khrushchev, I. Y., Richardson, D.J., Dianov, E.M., "Generation of 800 MHz stable train of sub-picosecond solitons from a figure-8 fibre laser", European Confrence on Optical Communication, Montreux, Switzerland, Paper MoC 2.2, (1993)
14. Mecozzi, A., Moores, J.D., Haus, H.A., Lai, Y., "Modulation and filtering control of soliton transmission", Journal of the Optical Society of America, B, **9**, pp 1350-1357, (1992).
15. Gray, S., Grudinin, A.B., Loh, W.H., Payne, D.N., "Femtosecond harmonically mode-locked fibre laser with time jitter below 1 ps", Optics Letters, **20**, (1995)
16. Dianov, E.M., Luchnikov, A.V., Pilipetskii, A.N., Starodumov, A.N., "Long-range interaction of soliton pulse trains in asingle-mode fibre", Sov. Lightwave Commun., **1**, pp 37-43, (1991)
17. Bondeson, A., Lisak, M., Anderson, D., "Soliton perturbations: a variotional principle for the soliton parameters", Physica Scripta, **20**, pp 479-485, (1979).
18. von der Linde, D.,"Characterization of the noise in continuosly operating mode-locked lasers", Applied Physics B, **39**, pp 201-217, (1986).
19. Gray, S., Loh, W.H., Grudinin, A.B., Afanasjev, V.V., Payne D.N., "Soliton compression in a nonlinear amplifying loop mirror", Proc. of Ultrafast Phenomena Topical Meeting, Paper WC 18, May 2-6, Dana Point, CA, USA, (1994)
20. Gordon, J.P. "Theory of the soliton self-frequency shift", Optics Letters, **11**, 662-664, (1986)
21. Gordon, J.P., " Interaction forces among solitons", Optics Letters, **8**, pp.596-598, (1983)
22. Loh, W.H., Grudinin, A.B., Afanasjev, V.V., Payne, D.N., "Soliton interaction in the presense of a weak cw component", Optics Letters, **19**, pp. 698-700, (1994)