

# Second Harmonic Generation in a Quasi-Phase Matched Structure Containing a Saturable Absorber

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Quasi-phase matching has achieved wide recognition as a powerful and versatile tool for efficient second harmonic generation (SHG) [1,2]. In such media, for imperfect phase matching, the process of frequency up-conversion evolves periodically with distance because the residual pump wave is parametrically amplified in regions of high conversion efficiency. This results in frequency down-conversion. Even when perfect phase-matching is met, any noise power at the fundamental frequency will be strongly amplified in regions of high harmonic power, causing down conversion [3,4]. In practice, these effects limit the SHG efficiencies that can be achieved under stable conditions.

We consider here the case of a nonlinear quasi-phase matched structure that contains a saturable absorber, in the form of a dopant with a resonant transition whose frequency equals the fundamental frequency, and whose saturation is relatively low. In this case, absorption does not play any essential role on the up-conversion process when the pump wave is strong enough to saturate the transition. It comes into the action, however, at the point where the pump is depleted such that its intensity becomes lower than the saturation threshold. This means that deleterious instabilities, caused by parametric down-conversion in regions of high second harmonic conversion efficiencies, are suppressed by elimination of the residual pump power.

In the plane wave approximation the process of SHG in a quasi-phase matched structure with saturable absorption obeys the following set of equations,

$$\begin{aligned}\frac{dA_p}{dx} &= -A_p A_s \sin \Psi - \frac{\sigma A_p}{1 + \gamma_s A_p^2}, \\ \frac{dA_s}{dx} &= A_p^2 \sin \Psi, \\ \frac{d\Psi}{dx} &= \vartheta - \left( 2A_s - \frac{A_p^2}{A_s} \right) \cos \Psi,\end{aligned}\tag{1}$$

where  $A_p(x)$  and  $A_s(x)$  are the normalized amplitudes of the pump wave and harmonic respectively,  $\Psi(x) = \phi_s(x) - 2\phi_p(x)$  is the relative phase between the second harmonic and the pump waves, and  $\vartheta$  is the parameter describing the dephasing from exact phase matching.

The dimensionless parameters  $\sigma$  and  $\gamma_s$  describe resonant absorption and its saturation respectively and can be written as,

$$\sigma = \frac{\sigma_0 N}{k^{(2)}}, \quad \gamma_s = \frac{\sigma_0 c n_p \tau I_0}{4 \pi \hbar \omega},$$

where  $\sigma_0$  is the absorption cross section,  $\tau$  is the lifetime of the upper excited level,  $N$  is the density of the resonant dopants,  $k^{(2)}$  is the normalized constant of the nonlinear interaction,  $I_0$  is the total input intensity, and  $n_p$  is the non-resonant refraction index at the fundamental frequency.

The full system of coupled equations (1) must be solved numerically, however it is possible to obtain some insight analytically. In the spatially homogeneous case when  $dA_{p,s}/dx = 0$ ,  $d\Psi/dx = 0$ , the solution of the system (1) is,

$$A_p(\infty) = 0, \quad A_s(\infty) = \text{const}, \quad \cos \Psi_\infty = \frac{\theta}{2 A_s(\infty)}, \quad (2)$$

with  $A_s(\infty)$  being a function of the parameters  $\theta$ ,  $\sigma$ ,  $\gamma_s$ , and of the boundary conditions  $A_{p,s}(0)$ ,  $\Psi(0)$ . From the form of the equation for  $A_p$  of the system (1) it follows that the homogeneous solution (2) remains stable at  $\sin \Psi(\infty)$  provided

$$A_p^2 \leq P_{tr} = \frac{\sigma I_s}{\sqrt{A_s^2(\infty) - \frac{\theta^2}{4}}} - I_s, \quad (3)$$

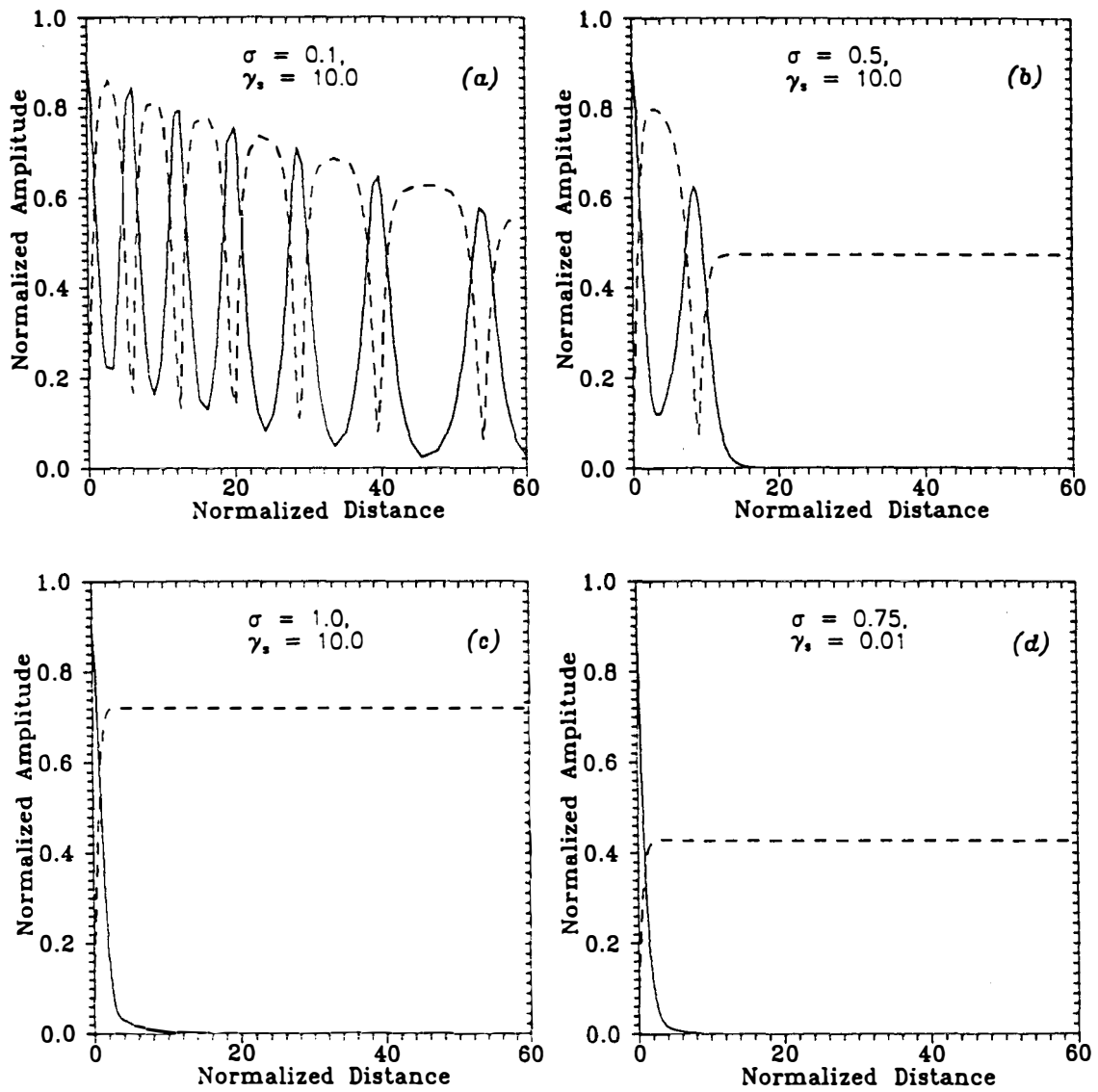
where  $I_s = 1/\gamma_s$  is the saturation intensity for the pump wave.

At small  $\sigma$  the processes of frequency up- and down-conversion vary periodically in space, but the absorption, even small, decreases the minimum value of  $A_p$  attainable at any given spatial cycle that eventually leads the condition of (3) to be satisfied. Afterwards the amplitudes  $A_p$  and  $A_s$  accept their homogeneous values, and the parametric instability appears to be suppressed.

The numerical integration of Eqs.(1) which results are depicted on Fig. 1 demonstrates interplay between parametric instability and saturating absorption. Furthermore these allow one to find such values of the parameters  $\sigma$  and  $\gamma_s$  when the non-reciprocal scenario of SHG can be provided on the very first cycle of the spatial beating. Assuming a fiber of 2 m long with nonlinear grating of  $\chi^{(2)} = 10 \text{ pm/V}$  doped with  $\text{Er}^{3+}$ , a refractive index of 1.45 and around 400 mW of incident pump, one has  $10^{19} \text{ cm}^{-3}$  the concentration of the dopant necessary to observe the promising behaviour predicted in Fig. 1(c).

## References

1. P. G. Kazansky *et al.*, Optics Letters **19**, 1221 (1994).
2. J. Webjörn *et al.*, Optics Commun. **106**, 91 (1994).
3. P. St. J. Russell, OSA Technical Digest **10**, 252 (1993).
4. P. St. J. Russell, IEEE Journ. Quant. Electron. **QE-27**, 830 (1991).



**Figure 1** Normalized pump and second harmonic against normalized distance for  $A_p(0) = 0.9$ ,  $A_s(0) = 0.1$ ,  $\Psi(0) = 0$  and  $\vartheta = 0.1$ .