Noise considerations in twin-core channel equalisers

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I. Introduction

Fibre optic channel equalisers are devices of prime importance in multi-channel telecommunication links and networks. They are used to compensate for the channel power imbalances accumulated along amplified long links and stabilise the channel optical powers. In multi-stage amplified optical links, channel power imbalances occur as a result two factors. Firstly, the emission and absorption cross-section variations across the erbium-doped fibre amplifier (EDFA) bandwidth result in channels experiencing different gains and acquiring unequal output power. Secondly, the output power imbalances are further deteriorated by the fact that the Er$^{3+}$ transition is predominantly homogeneously broadened at room temperature. Therefore, the strongest signal (channel) saturates the gain medium and compresses the gain uniformly at the expense of the power of the weaker signals.

To avoid the build-up of channel power imbalances and the subsequent detrimental effects on signal-to-noise ratio in multichannel optical links, various channel equalisation schemes have been proposed. They include the use of Mach-Zehnder optical filters [1] or acousto-optic tunable filters [2] to selectively attenuate the higher-gain channels using active servo-loops. Dynamic channel equalisation can also be achieved by controlling the gain, by adjusting the pump, in a two-stage amplifier with complimentary gain spectra in each stage [3]. Also, passive channel equalisation has been demonstrated by cooling the amplifiers down to ~77K so that they become predominantly inhomogeneously broadened [4].

![Diagram of twin-core erbium-doped channel equaliser](image_url)

**FIGURE 1:** Twin-core erbium-doped channel equaliser
Recently, a passive, all-optical channel equalising amplifier, using an erbium-doped twin-core fibre, has been demonstrated [5-7]. The basic device is shown schematically in Figure 1. Both cores are Er\(^{3+}\) doped. The signals \(\lambda_1\) and \(\lambda_2\) are initially launched into one core only and, therefore, are forced to cross-couple between the two cores periodically as they propagate along the fibre. The spatial period is wavelength dependent and, as a result, the two channels become spatially decoupled and, therefore, interact with different erbium ions. The device operation relies on this channel spatial separation and the power-dependent differential gain saturation given by Er\(^{3+}\). Initial experiments with a twin-core channel equaliser demonstrated a moderate equalisation rate of about -0.11dB per dB of channel power difference [5]. Further investigation has shown that, under optimum geometrical (e.g., core-to-core separation, fibre length) and optical (e.g., input pump and signal powers) conditions, it has been shown that equalisation rates in excess of \sim -0.35dB per dB of initial power imbalance are possible [8]. When used in a cascade, twin-core EDFA’s have been shown to provide adequate passive channel power stabilisation. They also protect the optical link against excess channel losses and they passively restore the optical powers to their initial level (link self-healing) [7-8].

In the previous analyses [7-8], generation and amplification of spontaneous emission was ignored and the effect of amplified spontaneous emission (ASE) on the equalisation rate and noise figure of the twin-core channel equaliser ignored. In this paper, we concentrate on these two issues and calculate the degradation of equalisation and noise figure as a function of the geometrical parameters (i.e., fibre length, core-to-core separation) and input optical powers.

II. Noise performance of twin-core fibre amplifiers

The noise figure (NF) in optical amplifiers, which is defined as the signal-to-noise ratio (SNR) at the input devided by the SNR at the output, is a measure of the signal degradation due to the amplification process. In multimode fibre amplifiers, the optical noise figure is given by [9]:

\[
NF(z) = \frac{1+2N(z)}{G(z)} + \frac{MN^2(z)}{G^2(z)\langle n(0) \rangle} + \frac{MN(z)}{G^2(z)\langle n(0) \rangle}
\]  

(1)

where \(M\) is the number of (polarisation) fiber modes, \(\langle n(0) \rangle\) is the average input photon number, \(G(z)\) the amplifier gain and \(N(z)\) is the average ASE photon number. This expression includes the signal shot noise and signal-ASE beat noise (first term), the ASE-ASE beat noise (second term) and the ASE shot noise. In the case of high input signal power, \(G \langle n(0) \rangle \gg N(z)\), the noise figure can be approximated by:

\[
NF(z) = \frac{1+2N(z)}{G(z)}
\]  

(2)

Equation (2) implies that in high input signal situations with \(G(z) \gg 1\) the amplifier noise figure is dominated by the signal-ASE beat component. It is also realised that in this case the noise figure of a multimode fibre amplifie
does not depend explicitly on the total number $M$ of spatial modes. This dependence occurs implicitly through the way that the ASE power build-up (differential increase) is affected by $M$. It is shown that the gain $G(z)$ and mean ASE photon number $N(z)$ evolution are given by [10]:

$$\frac{d}{dz} \ln G(z) = a(z) - b(z)$$  \hspace{1cm} (3)$$

$$\frac{d}{dz} N(z) = Ma(z)$$  \hspace{1cm} (4)$$

where $a(z) = \sigma_e N(z)$ and $b(z) = \sigma_a N(z)$. $\sigma_{e,a}$ are the emission and absorption cross-sections and $N_{e,2}$ are the populations of the ground and excited state, respectively. The noise power in a certain optical bandwidth $B$ is given by $P_{ASE} = hB N(z)$, where $h$ is Planck’s constant and $\nu$ the optical frequency. Combining Equations (3) and (4), it can be shown that the ASE power evolution in a multimode fibre amplifier is given by:

$$\frac{d}{dz} P_{ASE}(z) = [a(z) - b(z)] P_{ASE}(z) + M h B a(z)$$  \hspace{1cm} (5)$$

Twin-core fibres support four nondegenerate polarisation modes, i.e. the symmetric and antisymmetric x- and y-polarised eigenmodes. Therefore in this case, the ASE power evolution is given by Equation (5) with $M=4$.

The evolution of the various signals and pump along the twin-core EDFA can be calculated by using standard coupled-mode theory with distributed gain/loss [11]. In this case the distributed gain/loss is provided by the gain medium [7,8]. The gain depends on the relative values of $N_1(z)$ and $N_2(z)$ along the fibre, which in turn depend on the local values of pump, signals and ASE. The forward and backward ASE in each core is described as a quasi-monochromatic optical signal of equivalent bandwidth $B$.

III. Numerical results

In the case of two signals, pump, forward and backward ASE, the power evolution is described by 16 differential equations which are integrated by the Runge-Kutta-Merson method. The values used for the numerical calculations are summarised in Table I. The emission and absorption cross-sections of the two signals are supposed to be the same. All signals and pump are supposed to be launched in core#1 at the input end.

The solid lines in Figure 2(a) show the pump power evolution in core#1 and #2 as a function of the fibre length. The input pump power is 50mW while the input power of signal#1 and #2 are -20dBm and -30dBm, respectively, and the fibre length is 85m. Figures 2(b) and (c) (solid lines) show the forward and backward ASE power evolution in core#1 and #2, respectively. It is shown that the ASE build-up exhibits plateaus quasi-periodically at the positions along the fibre where the pump power falls below threshold leaving short fibre segments literally
unpumped. This is in contrast with the smooth ASE power variation in the case where both cores are equally excited at the input (dotted lines). It is realised that despite the differences in the local distribution of pump power and ASE evolution, on average the final levels of forward and backward ASE are nominally the same in both cases. This implies that any deterioration in the noise performance of the twin-core channel equaliser (see Figures (3)-(5)) cannot be attributed to the large pump-power variations along the fibre.

**TABLE I: Typical values used in the NF calculations**

<table>
<thead>
<tr>
<th></th>
<th>Pump</th>
<th>Signal1</th>
<th>Signal2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength:</td>
<td>980nm</td>
<td>1555nm</td>
<td>1556nm</td>
</tr>
<tr>
<td>Absorption cross-section (m^2):</td>
<td>2.55x10^{-25}</td>
<td>4.4x10^{-25}</td>
<td>4.4x10^{-25}</td>
</tr>
<tr>
<td>Emission cross-section (m^2):</td>
<td>4.7x10^{-25}</td>
<td>4.7x10^{-25}</td>
<td></td>
</tr>
<tr>
<td>Fibre NA:</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cut-off wavelength:</td>
<td>835nm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Norm. core-to-core separation:</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dopant concentration:</td>
<td>10^{24} m^3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equivalent ASE bandwidth (B):</td>
<td>10nm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3 shows the effect of the ASE on the gain, equalisation rate and noise figure of the twin-core channel equaliser as a function of the fibre length. The signal#1 and #2 input powers are -20dBm and -30dBm, respectively, and the pump power 50mW. The solid and dashed lines correspond to the cases with and without ASE, respectively. For small fibre lengths, the generated ASE is relatively low and, therefore, has no effect on the gains. The two signals hardly saturate the medium and the differential gain and equalisation rate are negligible (see Figures 3(a)&(b)). For lengths greater than ~45m and gains greater than ~25dB, the two signals and ASE grow strong enough and start saturating the medium. As a result, the twin-core amplifier provides channel equalisation (since $G_2 > G_1$). It is shown that in this regime the ASE suppresses both signal gains and reduces the equalisation rate. As shown in Figure 3(c), for high gain, the noise figure of channels#1 and #2 are always greater than 6dB. This is in contrast with the single core EDFA where the noise figure approaches and remains above 3dB. This 3dB difference in noise figure is a direct consequence of the increased number of nondegenerate modes ($M=4$ in Equation (5)) supported by the twin-core geometry. Single-core fibres, on the other hand, support only two polarisation modes ($M=2$ in Equation (5)). At optimum length (in this case, ~85m), $NF_1$ is about 2.2dB higher than $NF_2$. This is a direct result of the fact that $G_1$ is about 2.3dB lower than $G_2$. A further increase in the fibre length increases slightly the equalisation rate but at a
much more severe penalty in noise figure.

In Figures 4(a)-(c), the individual gains, equalisation rate and noise figures are plotted as a function of the signal#1 input power. Again, the case with ASE (solid lines) is contrasted with the case without ASE (dashed lines). The signal#2 input power is always kept 5dB lower than the one of signal#1. The pump power is 50mW and the fibre length 70m. Firstly, it is realised that the equalisation rate increases with the input power of the signal#1. For very small input signals (< ~ -35dBm), the equalisation rate is negligible due to inadequate signal saturation. In this range, the amplifier is self-saturated by the ASE with the result that both gains are suppressed by the same amount. The negative effect of the ASE on the gains and equalisation rate becomes gradually smaller with increasing the signal#1 input power. For input powers around -5dBm, the saturation of the medium is dominated by the signals and the effect of the ASE is negligible. For input powers greater than ~ -5dBm the equalisation rate starts decreasing due to oversaturation of the medium. From Figure 4(c), it is shown that the resulted noise figures are always in excess of 7dB and increase with the signal#1 input power.

Figure 5 compares the equalisation rates and noise figures for fibre lengths of 70m and 90m with ASE taken into account. The rest of the parameters are as in Figure 4. In Fig. 5(a), it is shown that, for relatively low input powers, longer fibre lengths result in higher equalisation rates due to higher degree of saturation and spatial decoupling. For input signal power higher than ~ -8dBm, the equalisation rate provided by the longer fibre deteriorates slightly faster than the one of the shortest fibre, since long fibres are driven into oversaturation earlier. However, as it was expected, the noise performance of the the longest fibre is the worst due to the larger amounts of generated ASE.

In Figures 6(a)-(c), the individual gains, equalisation rate and noise figures are plotted as a function of the signal#2 input power. The signal#1 input power is fixed to 0dBm and the input pump power is 50mW. The fibre length is 90m. It is shown that for large input signal differences, the equalisation rate is rather poor and noise figure of the strongest signal affected severely. The maximum equalisation rate (in excess of -0.25dB/dB) is achieved for a moderate input signal difference of about 5dB. As the signal2 input power increases and the gains $G_1, G_2$ become comparable, the noise figures of both channels converge at the same value.

IV. Conclusions

The effect of the forward and backward ASE on the equalisation rate and noise performance of a twin-core channel equaliser has been studied in detail. Firstly, it has been shown that the ASE increases the background saturation and, consequently, reduces the degree of channel equalisation. The reduction of channel equalisation rate depends on the input power of the optical channels. In the case of two imbalanced channels, the maximum equalisation rate (~ 0.28dB/dB) is achieved for a strong signal of input power of about -7dBm. In this regime, the gain medium saturation is dominated by the signal and, therefore, the effect of the ASE on the equalisation rate is
negligible. The optimum input power can be controlled to some degree by changing the saturation characteristics of the fibre and/or the signal wavelength.

On the other hand, under optimum conditions and maximum equalisation rates, the noise performance of the twin-core amplifier exhibits optical noise figures in excess of 7dB. The increased noise figure is an inherent feature of this device stemming from the fact that the twin-core geometry supports four nondegenerate polarisation modes.

To minimise the NF penalty and achieve maximum equalisation rate, the input power of the various channels should be kept in the range of -5dBm. This can be achieved by preceding the twin-core equaliser by a single-core EDFA [5] or an optical limiting amplifier.

REFERENCES


[10] *ibid*, Chapter 2.3

FIGURE 2: (a) Pump power evolution in cores#1 and #2 as a function of the fibre length. The input pump power is 50mW, the input power of signals#1 and #2 are -20dBm and -30dBm, respectively, and the fibre length is 85m. Forward and backward ASE power evolution (b) in core#1 and (c) in core#2. Solid and dotted lines correspond to the pump launched in one only and both cores, respectively.
FIGURE 3: (a) Gain, (b) equalisation rate and (c) noise figure of the twin-core channel equaliser as a function of the fibre length. The signal\#1 and \#2 input powers are -20dBm and -30dBm, respectively, and the pump power 50mW. The solid and dashed lines correspond to the cases with and without ASE, respectively.
FIGURE 4: (a) Gain, (b) equalisation rate and (c) noise figure as a function of the signal#1 input power. Solid and dashed lines correspond to the case with and without ASE, respectively. The signal#2 input power is always kept 5dB lower than the one of signal#1. The pump power is 50mW and the fibre length 70m.
FIGURE 5: (a) Equalisation rate and (b) noise figure for fibre lengths of 70m and 90m with ASE taken into account. The rest of the parameters are as in Figure 4.
FIGURE 6: (a) Gain, (b) equalisation rate and (c) noise figure as a function of the signal#2 input power. The signal#1 input power is fixed to 0dBm and the input pump power is 50mW. The fibre length is 90m.