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THE EFFECT OF ACOUSTIC NOISE ON OPTICAL FIBRE BASED LASER DOPPLER VELOCIMETRY SYSTEMS

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ABSTRACT

The potentially serious problem of acoustic noise pick-up in fibre-based LDV systems is addressed both theoretically and experimentally. A new broadening mechanism, called phase noise broadening, is discussed and the magnitude of this effect is estimated for typical LDV systems. This broadening leads to an overestimate of the turbulence index which is more serious for low speed flows. In a representative situation the magnitude of the error in the turbulence index is of the order of 5%.

1. INTRODUCTION

1.1 General

This work addresses the question of accuracy of a class of optical fibre based Laser Doppler velocimetry (LDV) systems when the systems are operated in conditions of high ambient acoustic noise and vibration, such as in or adjacent to the working section of a wind tunnel. Under such conditions random phase noise results in 'fringe jitter' in the measurement volume. The first moment of the velocity probability density function (pdf) will be unchanged, but the second moment will be affected under realistic operating conditions in such a way that an overestimate of turbulence index will be obtained.

Fibre optic based velocimetry systems fall broadly into two categories, Pannell, Tatam, Leilabady, Jones and Jackson (1986) depending on where the division of the transmitting beam is performed. In the first category fall the so-called fibre linked instruments in which a single transmitting fibre is used for each velocity component connecting the laser to the instrument head. The instrument head contains the necessary optical components to collimate the light emerging from the (single mode) fibre and to split the beam into two for the formation of a remote measurement volume.

In the second category, the so-called dual-fibre instruments, a separate optical fibre is used for each beam so that a single transverse component instrument would require two single mode transmitting fibres and one multimode receiving fibre. The head of such an instrument may then be extremely simple and compact, as no beamsplitting components are required. In order to maintain high visibility fringes in the measurement volume the polarisation states of the two interfering beams must be matched, this usually being achieved by the use of highly birefringent fibre, Payne, Barlow and Hausen (1982).

If the linking transmitting fibres of such LDV systems are exposed to ambient fluctuations of temperature, pressure and strain as they will be in a wind tunnel environment, instruments of the first type will not be affected because the single transmitting fibre offers excellent common mode rejection.

Thermal effects (to which the optical fibre is particularly sensitive), Jackson and Jones (1986), cause changes in the way the state of polarisation evolves along the single low birefringence transmitting fibre used in instruments of the first type with the result that the exact state emerging from the distal end cannot be predicted. However, the two interfering beams will always have matching states and provided the beamsplitter has low sensitivity to polarisation, the intensities of the beams will remain constant, these conditions ensuring constant fringe visibility.

Aside from the effect on polarisation, the effects of pressure, strain and temperature produce random fluctuations in the phase of the light field. In the fibre linked variety of instrument, again this does not matter as the instantaneous changes in the two beams are well correlated. In dual and multiple fibre systems this is not necessarily true and precautions are normally taken to insulate the fibres from their surroundings and to keep them in close mechanical contact for example by cabling them together. As we shall see this method does not always work as well as might be expected.

The problem of environmental noise in a dual fibre system was partially solved by Knuhtsen et al, Knuhtsen, Olldag and Buchhave (1982). Instead of two separate birefringent fibres, the two eigenmodes of a single birefringent fibre were used to convey light to the instrument head. The light in one eigenmode had been frequency shifted in order to remove directional ambiguity. The tight mechanical coupling between the modes resulted in a great improvement, but was not a prefect solution as some differential sensitivity to strain and pressure still exists. The differential sensitivity to strain turns out to be the most important in the present context, and has in fact been deliberately exploited in a number of sensor configurations, Jackson and Jones (1986). Typical figures are shown in table 1 showing that sensitivity to pressure or strain induced phase noise can be reduced by two orders of magnitude. It can never be eliminated completely in a birefringent fibre due to the inherently different sensitivities of the two modes of propagation to strain and pressure, however dual core fibre appears to offer a better solution in this case, Sabert, Dong and Russell (1991).

1.2 Purpose and Method of Work

The work reported here is an experimental and theoretical study of the effect of random noise on a dual fibre LDV system,modelled as a Mach-Zehnder interferometer. The Mach-Zehnder interferometer is essentially equivalent optically to the system used in an LDV for measurement volume formation. In the Mach-Zehnder interferometer two complimentary outputs are available, here a single output only is required, provided by a pointlike detector in the fringe system.

The experimental section gives brief details of acoustic noise spectra obtained with fibre interferometers in the 13'x9' tunnel at RAE, Bedford, UK, followed by results obtained in the laboratory using a fibre Mach-Zehnder interferometer, one or both arms of which were placed in a region of random acoustic noise of known spectral density.

This is followed by an analysis of the effect of a random acoustic noise field on the Mach-Zehnder interferometer and the derivation of the functional relation between the broadened detector current spectrum and the power spectral density of the noise field. In particular an analytic expression for the variance of the output current spectrum is obtained which allows noise-induced broadening to be incorporated into broadening calculations on the same basis as transit effects.

Finally the importance of this new broadening mechanism "phase noise broadening" in relation to the phenomenon of transit broadening is discussed, for some typical situations.

2. EXPERIMENTAL

In order to investigate the problem of induced phase noise on a Doppler difference laser velocimetry system, a number of fibre interferometers were introduced into the test section of the wind tunnel using the procedure described by us previously, Pannell, Tatam, Leilabady, Jones and Jackson (1986). The main conclusions were that: (i) The noise spectrum was confined to the audio range for subsonic velocities ($s(\omega)/s(0) < -60$dB for $\omega > 10$kHz) and (ii) cabling the fibres together in the same protective jacket did little to increase common-mode rejection indicating that the effects of differential longitudinal strain rather than hydrostatic pressures were responsible, and that substantial correlation of the strain was difficult to obtain.

In order to provide more quantitative data the apparatus of figure 1 was constructed. This consisted of an anechoic box of approximately $0.3m^3$ capacity into which optical fibres could be introduced and subjected to a random acoustic noise field having a known power spectral density. Figure 2 shows the spectra obtained from the microphone output of a DAWE sound level meter placed in the box. Figure 3 shows the amount of broadening appearing on the detector current in terms of the second moment, as a function of total sound intensity. Three cases are illustrated, the fibre or fibres comprising one or both arms of a Mach-Zehnder interferometer. A carrier was generated by a Bragg cell placed in one arm.

It can be seen that the sleeved fibre is less susceptible to noise than a "bare" fibre (only buffer coat) by an approximately constant value of 3dB up to a sound level of at least 115dB, and that approximately 10dB of common mode rejection could be obtained by cabling the two fibres together. An initially surprising contrast between the "real wind tunnel" experiment and the anechoic box emerged. Substantial common mode rejection was only obtained in the box because the large uncorrelated strains produced by wind buffeting in the tunnel were absent. This effect was present in the tunnel even if the fibres are cabled in the same jacket. Very careful design of the cabling and a particularly stiff jacketing would solve the problem but at the expense of manoeverability.

3. THEORETICAL TREATMENT OF PHASE NOISE INDUCED IN DUAL-FIBRE LDA SYSTEM BY AN ACOUSTIC NOISE FIELD

3.1 Origin of Phase Noise Broadening

This section presents a theoretical analysis of the anechoic box experiment of the previous section. One arm of a fibre interferometer is subject to random pressure variation and to random strain through being in contact with structures vibrating in sympathy with the acoustic noise field. The effect of thermally induced phase jitter on accuracy of the second moment may be discounted as it is slow (a few Hz) compared to phase changes produced by strain and pressure varying at acoustic frequencies. We may ask the following questions:

1) How large is the broadening effect produced by noise induced jitter on the measurement volume fringes and when is it important?
2) What is the functional form of the interferometer output and how does it depend on the acoustic noise spectrum?
3) Can a simple mathematical form for a measure of spread of the interferometer output spectrum (e.g. standard deviation) be found?

In general, if we can answer (2) we can also answer (3). It turns out however that (3) can be answered without having to find the explicit form of the output spectrum.

In order to find the spectrum of the interferometer detector current we begin with the autocorrelation function

$$\Gamma(\tau) = \langle i(t)\, i(t+\tau)\rangle \qquad (1)$$

where

$$i(t) = 1 + \cos(\omega_c t + \phi(t)) \qquad (2)$$

Here, ω_c is the carrier angular frequency and 100% visibility fringes are assumed. $\phi(t)$ is the random phase term associated with the noise field. If the fibres are suspended in air, ϕ depends only on hydrostatic pressure. If as is more usual the cabled fibres are in contact with a structure which is being agitated by the sound field, the resulting strain will also contribute to the random field ϕ. The high sensitivity of optical fibres to strain compared to hydrostatic pressure results in a "magnification" of the effect. It is assumed here that the magnification produces no change in the spectrum of ϕ, that is the mechanism of transmission of strain from a vibrating surface, via friction, to a fibre with which it is in contact, is independent of frequency. This assumption will be valid as long as the vibrational amplitude is small enough that slippage does not occur. As surface displacements of several nanometres are sufficient to account for the observed results this assumption would appear to be well founded.

The autocorrelation function of the detector current of equation (1) can be written, using the form of $i(t)$ given by (2) as

$$\Gamma(\tau) = 1 + \frac{1}{4}\left\langle e^{i[\omega_c\tau + \Delta\phi(t,\tau)]} + e^{-i[\omega_c\tau + \Delta\phi(t,\tau)]}\right\rangle \qquad (3)$$

i.e.

$$\Gamma(\tau) = 1 + \frac{1}{4}\langle I\rangle + \frac{1}{4}\langle I^{*}\rangle \qquad (4)$$

where

$$\Delta\phi(t,\tau) = \phi(t+\tau) - \phi(t) \qquad (5)$$

and

$$I = e^{i\omega_c\tau}\langle e^{i\Delta\phi(t,\tau)}\rangle \tag{6}$$

The random process ϕ is ergodic and has a Gaussian probability density function (pdf). In this case we can replace the time average with an ensemble average. We then find that the autocorrelation function of the detector current is given by

$$\Gamma(\tau) = 1 + \frac{1}{4}\left[\left(e^{i\omega_c\tau} + e^{-i\omega_c\tau}\right)e^{\gamma(\tau)-\gamma(0)}\right] \tag{7}$$

where γ is the phase autocorrelation function, given by

$$\gamma(\tau) = \langle\phi(t+\tau)\phi(t)\rangle \tag{8}$$

The moments of the frequency spectrum given by the Fourier transform of equation (7) may be found from the characteristic function, Papoulis (1989).

$$M(\alpha) = \int_{-\infty}^{+\infty} e^{-i\alpha v}\left\{\int_{-\infty}^{+\infty} e^{i2\pi v\tau}\, e^{G(\tau)}d\tau\right\}dv \tag{9}$$

where

$$G(\tau) = \gamma(\tau) - \gamma(0) \tag{10}$$

It can be shown that the explicit form of the function M is simply

$$M(\alpha) = e^{G\left(\frac{\alpha}{2\pi}\right)} \tag{11}$$

and the m'th order moment $\mu^{(m)}$ is given by

$$\mu^{(m)} = \lim_{\alpha\to 0}\left\{\frac{d^m M(\alpha)}{d\alpha^m}\right\}\Big/(-i)^m \tag{12}$$

The spectral broadening is given by the standard deviation, i.e. the square root of $\mu^{(2)}$.

In the experiments performed in the anechoic box it was found that the acoustic noise spectrum was well approximated by a function of the form

$$S_A(v) = I\delta e^{-\delta v} \tag{13}$$

with $\delta \approx 10^{-3} Hz^{-1}$

The pressure autocorrelation function is

$$\gamma_p(\tau) = \rho c\mathcal{F}[S_A(v)] \tag{14}$$

where ρc is the acoustic impendence of air, $\mathcal{F}$ denotes Fourier transform, and S_A is the power spectral density of the acoustic noise field.

The phase autocorrelation for a length ℓ of fibre is then found to be

$$\gamma(\tau) = \frac{\ell^2 s^2 \rho c I\delta^2}{2\pi^2}\;\frac{1}{\left(\frac{\delta}{2\pi}\right)^2 + \tau^2} \tag{15}$$

where s is the sensitivity factor, to be determined empirically.

Equations (11), (12) and (15) give for the standard deviation

$$\sigma = \frac{2\ell s}{\delta}\sqrt{I\rho c} \tag{16}$$

and for the 'C' weighted acoustic measurement scale

$$I = 10^{\left(\frac{dB-120.2}{10}\right)} \tag{17}$$

where dB = reading on meter
I = Intensity of sound field

$$\therefore\ \sigma = \frac{2\ell s}{\delta}\sqrt{\rho c}\ 10^{\left(\frac{dB-120.2}{20}\right)} \tag{18}$$

The fitted curves in figure 3 are obtained with this equation, with

(1)	Bare unsleeved fibre	$s = 0.170$ rad Pa^{-1} m^{-1}
(2)	Sleeved fibre	$s = 0.122$ rad Pa^{-1} m^{-1}
(3)	Dual fibres in sleeve	$s = 0.054$ rad Pa^{-1} m^{-1}

The sensitivity of an unsleeved fibre to hydrostatic pressure alone is approximately 50 μrad Pa^{-1} m^{-1} showing the enormous magnification of sensitivity of the fibre link due to the transmission of acoustic strain into the fibre. In fact for a sound level of 100 dB (C scale) the RMS pressure variation in air is approximately 2 Pa, and for a bare fibre (unsleeved) with a measured sensitivity of 0.170 rad^{-1} the resulting RMS phase variation is 0.34 rad. If the sensitivity to longitudinal strain is 10^7 rad m^{-1} (a typical figure) the RMS strain required to produce the observed value of phase variation is approximately 34 nm. Thus friction between the fibre and the surface with which it is in contact would only have to transmit sufficient force to producean RMS elongation of 34 nm over a 1 m length to account for the observed effects.

4. EFFECT OF PHASE NOISE BROADENING

It is instructive to compare the broadening produced by phase noise in a dual fibre LDV system to the broadening roduced for example by transit effects.

The measured probability density function (pdf) of Doppler frequencies is the convolution of the 'actual' pdf and an instrumental function

$$P_D(\omega_D) = P_u(\omega_D) \otimes P_B(\omega_D) \tag{19}$$

↑ measured function ↑ velocity function ↑ instrument function

and the variances add, assuming the statistical independence of broadening effects:

$$\sigma^2_{D(measured)} = \sigma^2_u + \sum \sigma^2_{(broadening)} \tag{20}$$

One fundamental mechanism of broadening is due to the finite extent of the measurement volume, and the variance associated with this is

$$\sigma^2_{transit} = \frac{U^2}{2R^2} \tag{21}$$

Where U is the mean velocity and R is the radius of the measurement volume. The mean Doppler frequency is related to the mean velocity U and fringe spacing s by

$$\nu_D = U/s \tag{22}$$

and the turbulence intensity is given by

$$T = \frac{\sigma_u}{\nu_D} = \frac{s\sigma_u}{U} \tag{23}$$

In the presence of phase noise broadening and transit time broadening the true turbulence intensity will be given by

$$T^2_{actual} = \left(\frac{s}{U}\right)^2 \left(\sigma_D^2 - \sigma^2_{noise} - \sigma^2_{transit}\right) \tag{24}$$

i.e.

$$T^2_{actual} = T^2_{measured} - s^2 \left\{ \left(\frac{\sigma_{noise}}{U}\right)^2 + \frac{1}{2R^2} \right\} \tag{25}$$

The contribution to error due to transit time effects is constant and will therefore be more important at low values of turbulence. The contribution due to phase noise broadening is proportional to (1/velocity) and therefore more important at low mean velocities.

The mean velocity U at which both broadening contributions are equal is given by

$$U_e = \sqrt{2}\ R\sigma_{noise} \tag{26}$$

For a dual fibre LDV probe system previously described by the authors, Pannell, Tatam, Leilabady, Jones and Jackson (1986).

$$s = 10\ \mu m$$
$$R = 350\ \mu m$$
$$\ell = 10\ m$$

and assuming a noise level of 100 dBm (C) we find

$$\sigma_{noise} = 2.2 \text{ kHz}$$
$$U_e \approx 1 \text{ ms}^{-1}$$

Thus at mean velocities below 1 ms^{-1}, whatever the turbulence intensity, the contribution to error due to phase noise broadening dominates. At a sound level of 90 dB (C) which was the maximum level encountered in wind tunnel experiments this velocity drops to 0.3 ms^{-1}. In the first case the correction to T at a mean velocity of 1 ms^{-1} is of the order of 5% for the combined effects of phase noise and transit time effects.

5. CONCLUSIONS

The potentially serious problem of acoustic noise pick-up on the optical fibre leads of a multiple fibre LDV system is discussed. Experimental results have been presented which have been performed with controlled noise power spectra to show the magnitude of the effect in typical fibre LDA systems. A theoretical treatment has been presented for the first time by which the quantative relationship between the power spectral densities of the random noise field and the detector current at the output of the system, modelled as a Mach-Zehnder interferometer, can be found.

A simple expression has been presented by which the various moments of the detector current can be expressed in terms of the autocorrelation function of the acoustic noise field. In particular the second moment about the mean (i.e. the variance) can be used as a measure of phase noise broadening and can be incorporated into error calculations along with other broadening mechanisms such as transit effect.

The fibre's sensitivity to longitudinal strain has been shown to dominate over hydrostatic pressure by 3-4 orders of magnitude as the mechanism for noise pick up, when the fibre is in contact with a surface in conditions of high ambient noise.

Finally the phenomenon of phase noise broadening has been shown to behave in a different manner to transit effects in that it increases as 1/(mean velocity) while the contribution due to transit effects is constant.

REFERENCES

Jackson D.A. and Jones J.D.C. 1986 Fibre Optic Sensors. *Optica Acta*, 12, 1469-1503.

Knuhtsen J., Olldag E. and Buchhave P. 1982. Fibre Optic Laser Doppler Anemometry with Bragg Frequency Shift Utilizing Polarization Preserving Single Mode Fibre. *J. Phys. E.* 15, 1188.

Pannell C.N., Tatam R.P., Leilabady P.A., Jones J.D.C. and Jackson D.A. July 7-9, Lisbon, 1986. Fibre Based Velocimetry : An Optical Link and Two Dimensional Probe. *Proc 3rd Int. Symp on the Applications of Laser Anemometry to Fluid Mechanics, Paper 8.3.*

Papoulis A. 1989. Stochastic Processes. *McGraw-Hill.*

Payne D.N., Barlow A.J. and Hausen J.J.R. 1982. Development of Low and High-Birefringence Optical Fibres. *IEEE J. Quant. Elect.* QE-18 4, 477-488.

Sabert H., Dong L. and Russell P.St.J. Dec 1991. Versatile Acousto-optic Flexure Wave Modulator and Frequency Shifter in Dual-Core Fibre. *To be published in Int J. Optoelectronics.*

	$\frac{1}{\ell}\frac{\partial\phi_f}{\partial j}$	$\frac{1}{\ell}\frac{\partial}{\partial j}(\phi_f - \phi_s)$
$j = T$	100	5 rad K^{-1} m^{-1}
$j = P$	50×10^{-5}	5×10^{-6} rad Pa^{-1} m^{-1}
$j = \Delta\ell/\ell$	0.65×10^{7}	0.65×10^{5} rad m^{-1}

Table 1 Typical figures for the absolute and differential sensitivities of a birefringent optical fibre to pressure temperature and longitudinal strain.

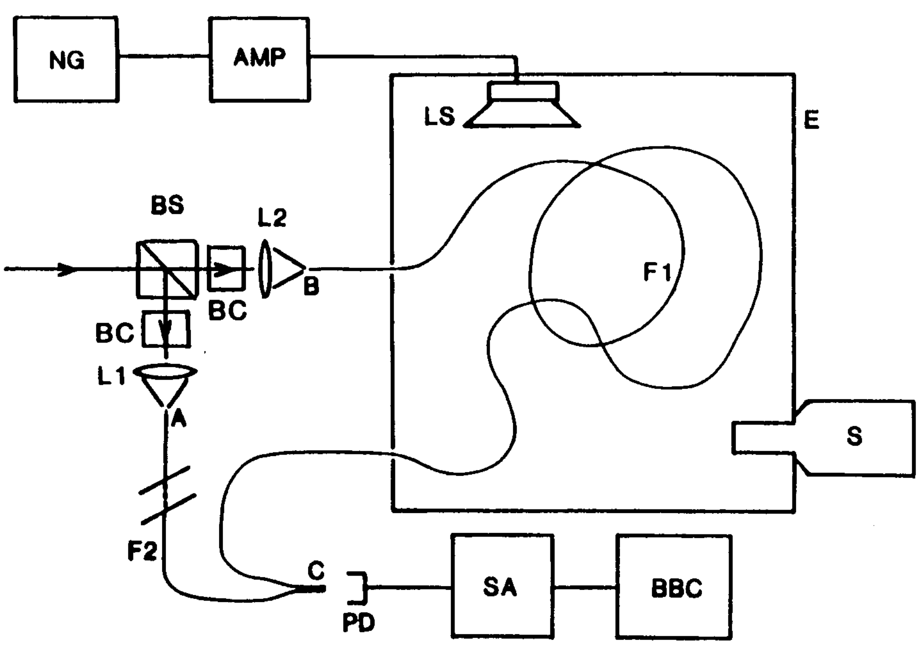

Figure 1 Apparatus used to perform experiments on the effect of random acoustic noise on a fibre optic LDV system.

Key: NG - noise generator; BC - Bragg cell; AMP - audio amplifier; LS - loudspeaker; S - sound level meter; E - enclosure; PD - photodiode; SA - spectrum analyser; BBC - microcomputer; L1,L2 - launching lenses; BS - beamsplitter. F1 and F2 are birefringent fibres length AC = length BC.

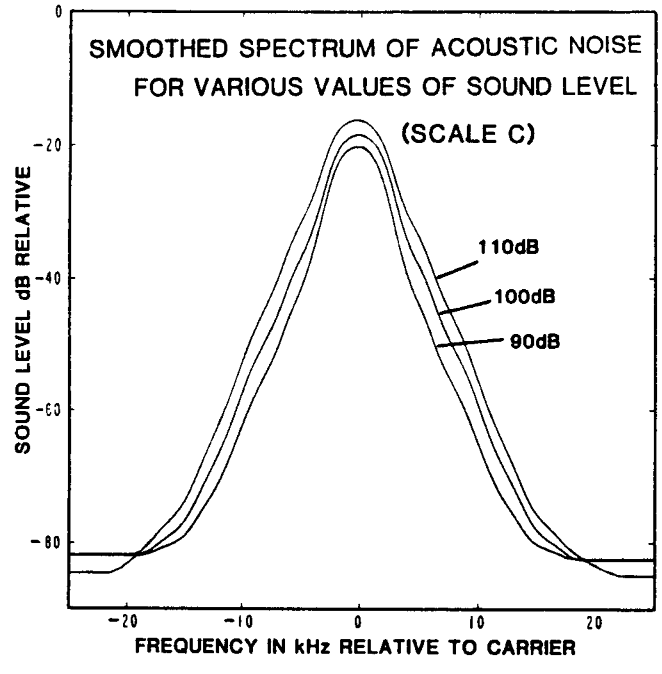

Figure 2 Effect of insonification of one or both arms of a fibre Mach-Zehnder interferometer on the spectrum of detector current.

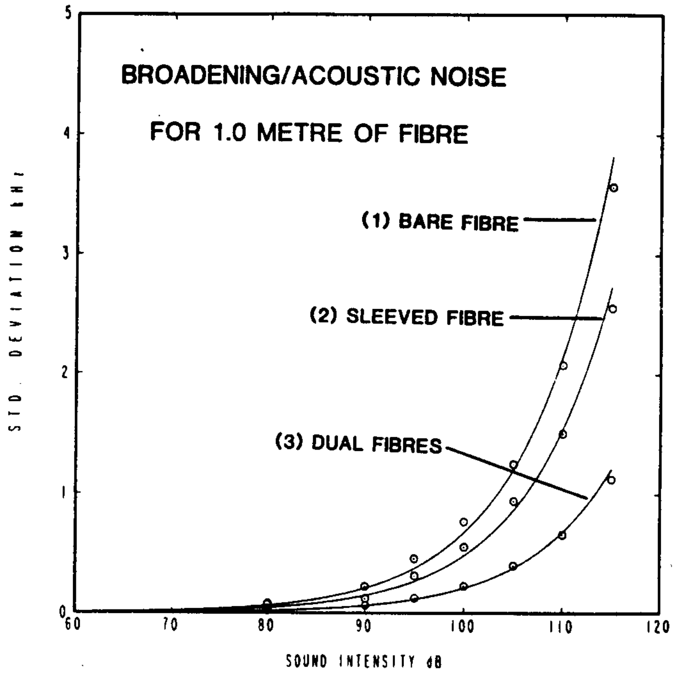

Figure 3 Effect of insonification on the spectral width (as measured by the second moment about the mean) of the Mach-Zehnder interferometer.