Modulational Instability Gain Spectrum of Nonlinear Photonic Bloch Waves in Periodic Media

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Abstract

The modulation instability gain spectrum of nonlinear photonic Bloch waves (the nonlinear normal modes of DFB gratings) is obtained analytically for the first time. The analysis is relevant to bistability, oscillation and gap-soliton formation in these structures.

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Summary

Photonic Bloch waves (PBW's), which are the normal optical modes of linear periodic media[1,2], exhibit rich and complex behaviour in the presence of optical nonlinearities[3,4]. To a good approximation, a PBW can usually be represented by a pair of backward and forward plane (or *partial*) waves, forming an entity that travels or evanesces at a constant group velocity or decay rate. The nonlinear dispersion relation of these waves has been presented elsewhere[4]. Here we use that analysis as a basis for examining the modulational instability (MI) gain of nonlinear PBW's and assessing their stability. The representation adopted for the fields thus includes a strong pump (the PBW whose stability is to be assessed) and two additional weak PBW's at side-bands spaced at frequency Ω from the pump frequency ω:

$$E(z,t) = \frac{1}{2}\left(V_f + f_1 e^{-j\phi_1} + f_2^* e^{j\phi_1^*}\right)e^{-j\phi_o} + \frac{1}{2}\left(V_b + b_1 e^{-j\phi_1} + b_2^* e^{j\phi_1^*}\right)e^{-j\psi_o} + \text{c.c.} \quad (1)$$

where V_f and V_b are the forward and backward partial waves of the pump, f_1, f_2, b_1 and b_2 are small constant side-band partial wave amplitudes and $\phi_o = (k_f z - \omega t) = (\psi_o + Kz)$, $\phi_1 = (qz - \Omega t)$ where $k_f \pm q$ are the wavevectors at the upper and lower

sidebands, k_f the wavevector of the forward wave and K the grating vector. It may be appreciated from (1) that phase matching is implicit in the PBW approach. In the absence of side-bands, it is straightforward[4] to relate the reflection efficiency η from a periodic half-space (extending from $z = 0$ to $z \to \infty$, see Figure 1) to the level of nonlinearity Δ ($\propto$ pump power), the coupling constant κ ($\propto$ grating strength) and the degree of dephasing ϑ from the linear Bragg condition:

$$\Delta/\kappa \quad = \quad \left[\{\vartheta/(\kappa[1+\eta])\} \pm \eta^{-1/2}\right]. \qquad (2)$$

Definitions of these parameters are available elsewhere[4]. Typical solutions of (2) are presented in Figure 2, for three different values of dephasing; notice that as ϑ becomes increasingly negative, the stop-band moves to higher values of Δ as expected.

The stability of solutions (2) is assessed by including the side-band terms in (1), entering it into the nonlinear wave equation, linearising the problem by neglecting terms of order f_1^2, obtaining a set of four homogeneous linear equations for f_1, f_2, b_1, b_2 and solving the ensuing eigen-value problem for q. Four values of q result. Instability occurs if the normalised Poynting vector of at least one of the four MI eigen-modes has the same sign as the gain (the imaginary part of q). Applying this condition to the solutions in Figure 2 at $\hat{\Omega} = 0.5$ ($\hat{\Omega}$ is the MI frequency shift normalised to half the stop-band width) shows that the dotted sections of curve are unstable; this permits positive identification of the parameter ranges where bistability is possible (at this normalised frequency). In Figure 3 the calculated MI gain and Poynting vector spectra at point A in Figure 2 are given. The general small-signal behaviour follows from superposition of the linearised MI eigenmodes.

In a long fibre grating with index modulation 10^{-5}, at a 1064 nm pump intensity of ~ 500 Wμm^{-2} (at -17 MHz from the Bragg condition), a weak pulse of bandwidth ~ 50 MHz injected at 330 MHz from the pump frequency would grow at 51 m^{-1}, with a group velocity $0.08 \times c$.

In conclusion, a simple analytical formalism exists for the MI gain of nonlinear PBW's assuming no pump depletion. It should prove useful for interpreting the results of global numerical simulations.

References

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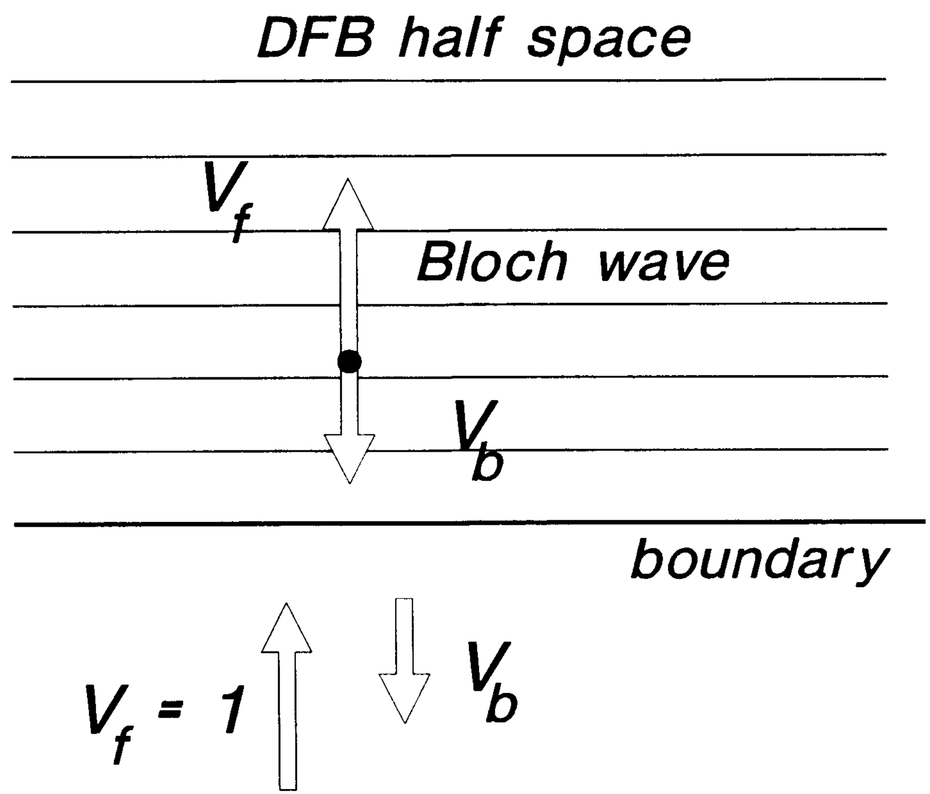
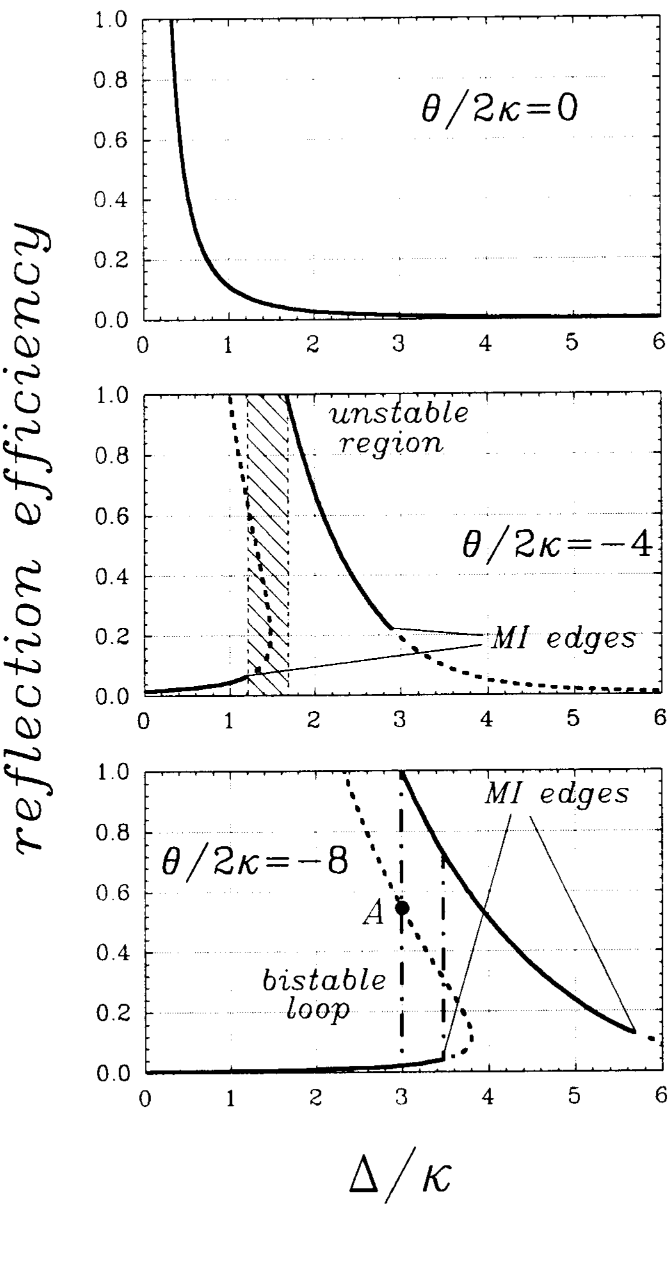
Figure Captions

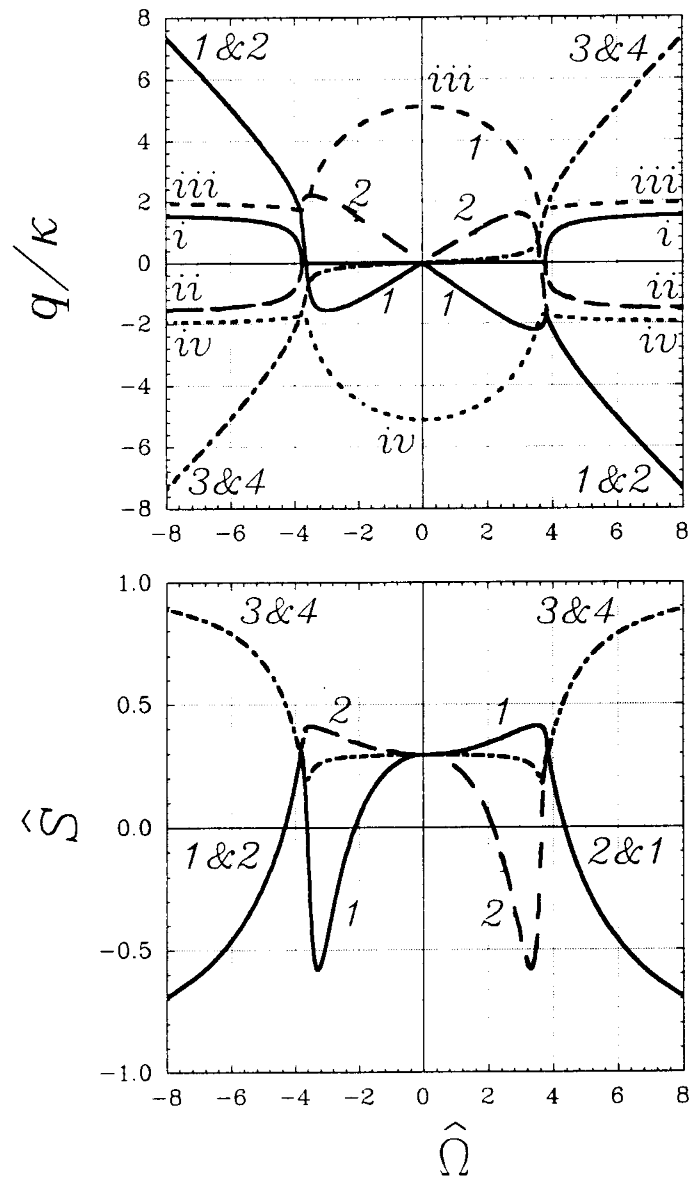
1. Boundary condition at DFB half-space. Incident and reflected waves appear in the isotropic medium. Inside the grating (assuming no discontinuity in average index) the backward and forward PBW amplitudes are identical; however they have slightly modified wavevectors and are "pinned together", sharing a common group velocity.
2. Reflection efficiency versus nonlinearity for three different values of Bragg condition dephasing. The dotted sections of curve are unstable at $\hat{\Omega} = 0.5$ (see text – the full MI spectrum at point A is plotted in Figure 3). Note that regions of large-signal instability (oscillation) and bistability exist.
3. Real and imaginary part of q together with the normalised Poynting vector against MI frequency shift for point A in Figure 2 ($\eta = 0.54$, $\Delta/\kappa = 3$ and $\vartheta/2\kappa = -8$). Four eigen-modes exist, labelled 1/i, 2/ii, 3/iii and 4/iv. The real parts of q are labelled with arabic numerals and the imaginary parts with roman numerals.



Russell & Archambault "Modulational . . .", Figure 1



Russell & Archambault "Modulational . . . ", Figure 2



Russell & Archambault "Modulational . . . ", Figure 3