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Detailed studies have been made of the magneto-optic effect in single-mode fibres exhibiting both linear and circular retardations. A novel technique enables the Verdet constant to be easily measured. Applications of the effect, together with the magnitude of electromagnetic interference in optical fibres likely to be encountered in practice, are discussed.

1. Introduction

An earlier study<sup>1</sup> of Faraday rotation in graded-index multi-mode fibres showed that the optical properties could be explained in terms of simple linear retardation. However we have shown<sup>2</sup> that single-mode fibres may, in addition, exhibit optical activity, depending on the fibre parameters, and that the local retardation, fast-axis rotation and circular rotation act cumulatively. All three factors strongly influence the magneto-optical interaction which can be of great importance in several fibre applications. In addition fibre attenuation may well be sensitive to the local polarization state at bends, imperfect splices and couplers. Thus varying magnetic fields would produce polarization changes with possible loss modulation resulting in a form of electromagnetic interference. Methods of minimizing such degradation should be sought and we have therefore carried out a detailed investigation of magnetic interactions with single-mode phosphosilicate fibres.

2. Theory

The analysis<sup>1</sup> for a graded-index fibre considers only retardance  $\Delta$  but, in general, rotation  $\phi$  of the fast and slow birefringent axes and circular rotation  $\Omega$  must also be included. A given length of fibre may be represented by one rotator and one retardation plate so that the output (input) electric field components  $E_x, E_y, (E_{x0}, E_{y0})$  are given<sup>3</sup> by:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \bar{M}_0 \begin{pmatrix} E_{x0} \\ E_{y0} \end{pmatrix} \quad \text{where } \bar{M}_0 = \begin{pmatrix} A_0 & -B_0^* \\ B_0 & A_0^* \end{pmatrix} \quad \dots (1)$$

and

$$\left. \begin{matrix} A_0 \\ B_0 \end{matrix} \right\} = \cos(\frac{1}{2}\Delta) \begin{matrix} \cos \\ \sin \end{matrix} \Omega + j \sin(\frac{1}{2}\Delta) \begin{matrix} \cos \\ \sin \end{matrix} (2\phi + \Omega) \quad \dots (2)$$

If the fibre comprises three consecutive lengths  $L_1, L_2, L_3$  as in Fig.1 and  $L_2$  is exposed to a uniform magnetic intensity  $H$  then

$$\bar{M}_0 = \bar{M}_1 \cdot \bar{M}_2 \cdot \bar{M}_3$$

and the Faraday rotation is  $\theta = VHL_2$  where  $V$  is the Verdet constant.

We assume  $\theta \ll \Delta_2$  which implies that  $L_2$  and hence also  $\phi_2, \Omega_2$  are small. Then it may be shown that<sup>4</sup>

$$\bar{M}_0 = \bar{M}_u + F\bar{M} \quad \text{where } F = 2\theta/\Delta_2 \quad \dots (3)$$

and

$$\bar{M} = \begin{pmatrix} A & -B^* \\ B & A^* \end{pmatrix} \quad \text{where } \begin{cases} A = a + jc \\ B = b + jd \end{cases} \quad \dots (4)$$

$\bar{M}_u$  is an unperturbed term representing the fibre properties when  $H = 0$ . Matrices  $\bar{M}_0$  and  $\bar{M}_u$  can be determined experimentally but  $\bar{M}$  is more difficult since  $a, b, c, d$  are complicated functions of the various  $\Delta, \phi$  and  $\Omega$ . However further analysis gives:

$$a^2 + b^2 + c^2 + d^2 = \sin^2(\frac{1}{2}\Delta_2) \quad \dots (5)$$

and, if the perturbations in  $\Delta, \phi$  and  $\Omega$  caused by the magnetic field are  $F\delta\Delta, F\delta\phi$  and  $F\delta\Omega$ , then additional simultaneous equations relating these quantities, which can be measured, and  $a, b, c, d$  may be obtained<sup>4</sup>.

When a linearly-polarized beam is aligned with the fast axis at the input end of the fibre with  $H = 0$  then the output beam is also linearly polarized along an axis we define as the x-axis, see Fig.1, so that  $E_y=0$ . When a magnetic field is now applied the output becomes elliptically polarized and  $|E_y|^2$ , when normalized with respect to the total output power, becomes:

$$|E_y|^2 = F^2 \left\{ [b \cos \Omega_u - a \sin \Omega_u]^2 + [d \cos(2\phi_u - \Omega_u) - c \sin(2\phi_u - \Omega_u)]^2 \right\} \quad \dots (6)$$

This is a completely general expression showing that even in the presence of optical activity  $|E_y|^2 \propto V^2 H^2 L_2^2$ .

$V$  may be determined by measuring the slopes of the curves of  $\Delta, \phi$  and  $\Omega$  as a function of  $H$  (see for example Figs.3, 4) and solving the set of simultaneous equations mentioned above.

In the simple case of fibres which exhibit only linear retardance<sup>1</sup> ( $\phi_u = \text{constant}; \Omega_u = 0$ ) then (6) reduces to:

$$|E_y|^2 = \theta^2 = V^2 (HL_2)^2 \quad \dots (7)$$

and  $V$  can be obtained from the slope of the curve of  $|E_y|^2$  versus  $H^2$ . Alternatively, knowing  $V$ , the polarized output power represented by  $|E_y|^2$  provides a simple measure of magnetic field. The presence of optical activity, while complicating somewhat the derivation of experimental quantities, nevertheless enables the sensitivity to magnetic field to be varied by appropriate selection of the factors in (6).

### 3. Experiment

Measurements have been carried out on single-mode fibres, of normalized frequency  $V=2.2$ , having phosphosilicate cores of diameter

4.3 $\mu$ m. The attenuation was below 5dB/km at 0.75 $\mu$ m. An axial magnetic field was applied over a fibre length of 4.3cm by an electro-magnet for a range of exciting currents. Polarized light from a He/Ne laser was launched into the fibre through a half-wave plate and a polarizer. Cladding modes were removed and the output was focussed onto a detector through an analyzer and a removable quarter-wave plate.

The linear dependence of  $|E_y|^2$  on  $H^2$  is shown in Fig.2 for the distances of the magnetic field from the input end ( $L_1$  cm) shown on the curves. Confirmation of equation (6) is thus provided and the fact that the lines are not coincident indicates the presence of optical activity. Fig.2 also verifies that the sensitivity to magnetic field can be changed by variation of  $L_1$ . These and other experiments indicate that the assumptions made (e.g.  $2\theta/\Delta_2 \ll 1$ ), and the theory, are applicable to our fibres. Figs. 3 and 4 show the measured retardation, fast axis rotation and circular rotation as a function of  $H$  for a fibre length of 1.4m. The slopes give (all in min of arc  $\text{cm}^{-1}$ ):

$$2VL_2(\delta\Delta/\Delta_2) = 0.042; 2VL_2(\delta\phi/\Delta_2) = 0.22; 2VL(\delta\Omega/\Delta_2) = -0.032$$

and solution of the simultaneous equations yields the Verdet constant

$$V = 0.0212 \text{ min of arc cm}^{-1} \text{ amp}^{-1} \\ = 0.0169 \text{ min of arc cm}^{-1} \text{ gauss}^{-1}$$

The effects of fluctuating magnetic fields and of the resulting electromagnetic interference are being studied and will be reported at the Conference.

#### 4. Conclusions

A theory has been developed to describe Faraday rotation in single-mode fibres exhibiting birefringence and optical activity which is in good agreement with experiment. The retardation, fast-axis rotation and circular rotation vary linearly with the applied magnetic field. Simple methods have been devised for determining the Verdet constant. For a particular plane of polarization the output optical power varies linearly with  $H^2$  even in the presence of optical activity. The interference caused to propagation in optical fibres by fluctuating magnetic fields is being studied.

#### 5. References

1. H.Harms, A.Papp and K.Kempter, Appl.Opt. 15, 799-801 (1976)
2. W.A.Gambling, D.N.Payne and H.Matsumura: 'Birefringence and optical activity in single-mode fibres', Topical Meeting on Optical Fibre Transmission II, 1977, Williamsburg, Paper TuD5
3. F.P.Kapron, N.F.Borrelli and D.B.Keck, IEEE JQE-8, 222-5 (1972)
4. H.Matsumura et al (to be published)

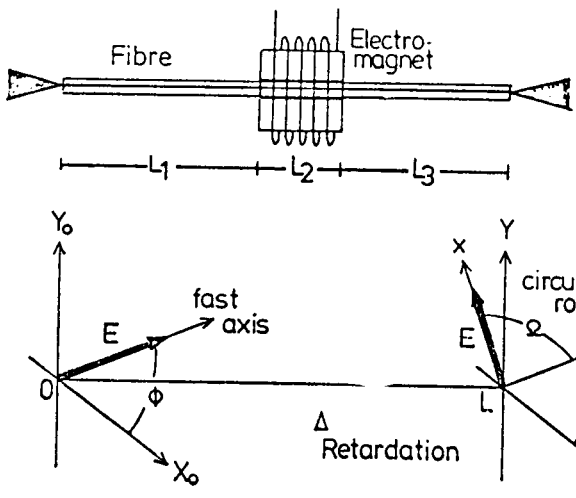


Figure 1

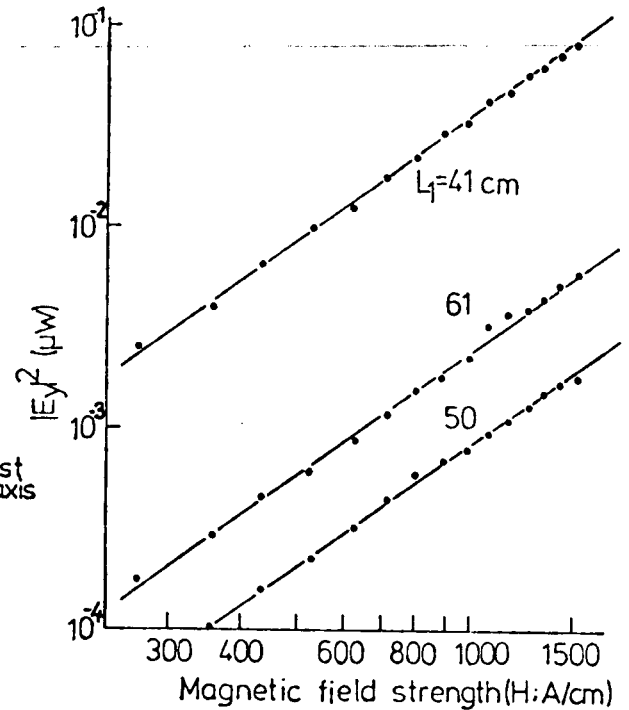


Figure 2

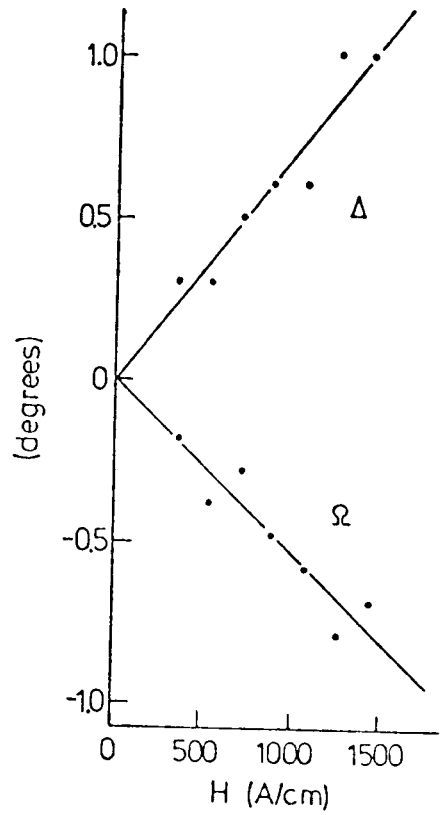


Figure 3

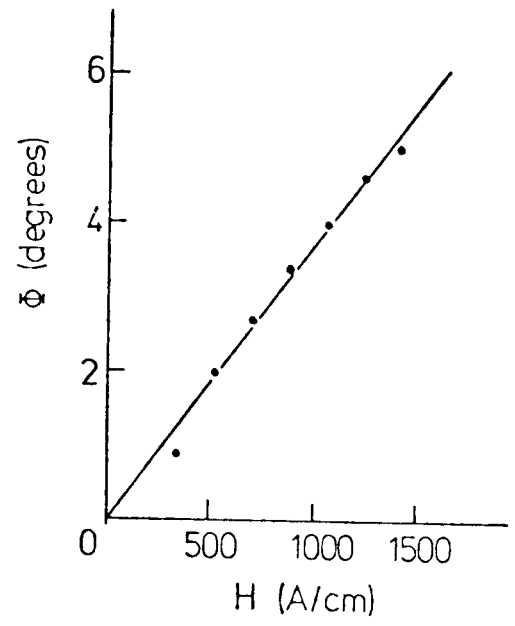


Figure 4