Propagation Characteristics of Curved Optical Fibers

W. A. GAMBLING, *Nonmember* and H. MATSUMURA, *Member*

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Summary Experimental and theoretical studies have been carried out on propagation in curved single-mode optical fibres. Mode conversion at the start of a bend produces a transition region which affects the bend loss, transverse energy distribution and gives rise to ray emission. It is shown that the modes of a curved multimode fibre can be conveniently and simply described using parabolic cylindrical co-ordinaces.

1. Introduction

Single-mode fibres have considerable potential as transmission lines in long-distance optical communication systems. Much work has been carried out on reducing the range of group velocities in multimode fibres by suitable gradation of the refractive-index profile but there are great difficulties in realising the bandwidths which are theoretically possible. Single-mode fibres, on the other hand, are limited only by material and mode dispersion and pulse dispersions of better than 0. 1ns km have already been observed(1). Bandwidths of tens of gigahertz over tens of kilometreso are available, in principle, at 0. 9 μm and much larger values(2), limited only by mode dispersion, at 1. 25 μm.

Despite these potential advantages(3) comparatively little effort has been expended on an investigation of single-mode fibres. A study was therefore initiated in these laboratories and a simple method(4) has been reported for their characterization. An important aspect of single-mode fibres about which little is known in any detail is the effect of bends on propagation. Several theories have been proposed, most of which assume(5,6) that the fields at a bend can be approximated by those in a straight fibre. However when a fibre is progressively curved the field distribution can change considerably(7). In addition to the pure bending loss a redistribution of energy must therefore take place at the beginning of a bend(8) and the loss due to mode conversion in this transition region must also be considered. The losses predicted by the various theories presentey available are not in good agreement with experiment and furthermore a recent observation(9) shows that the radiation emitted at a bend is not continuous but seems to occur preferentially in discrete rays. We present here further results on losses in curved single-mode fibres which confirm the periodicity in the field distribution and show that any theory must take account of mode conversion in the transition region.

During the course of the experiments on bend loss a coupling of energy from core to cladding modes was observed. It was further noted that the near-field and far-field radiation patterns are not circularly symmetric but have mode lines which are parabolic in shape. Taking this experimental pattern as a guide the cladding modes, which are essentially the same as those for a multimode fibre, have been analysed for the curved configuration by using parabolic cylindrical co-ordinates. We find that the modes and caustic of a curved multimode fibre can be accurately described in this way.

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The authors are with the Department of Electronics, University of Southersity, Southampton SO9 5NH, England.

2. Bend Transition Region in Single-Mode Fibres

2. 1 Ray emission

As noted above the radiation emitted in the transverse direction at a bend is not uniform but appears initially in the form of discrete divergent rays(9). In order to confirm this observation the intensity distribution of the radiation emitted perpendicular to the curved fibre, and in the plane of the bend, has been measured as a function of distance along the axis. As can be seen from Fig. 1 the radiation intensity distribution is oscillatory at the commencement of the bend, the peaks corresponding to ray radiation. With increasing distance along the curved fibre the beams broaden and decrease in amplitude, so that the radiation gradually becomes more uniform and characteristic of that of a stable curved mode. As illustrated in Fig. 1, we refer to the oscillatory section as the transition region, because here energy conversion occurs from the mode of the straight fibre to that in a curved fibre. The section following the transition region, where the radiation is non-oscillatory, is called the pure-bend, or stable-mode region.

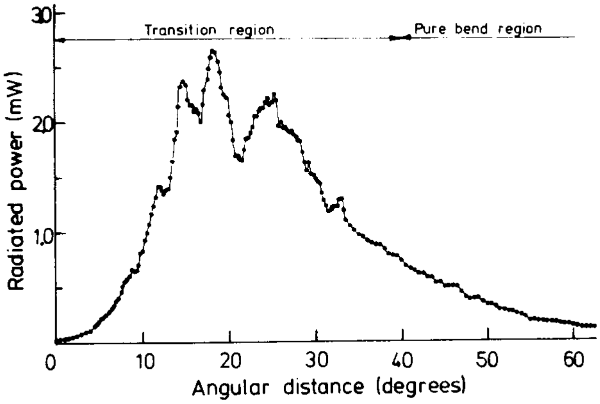


Fig. 1 Variation of radiated intensity in plane of bend, from a curved fibre of bend radius 10 mm, core radius 4. 1 μm, NA_s=0. 06 and V=2. 38.

2.2 Mode shift

Associated with the mode conversion there must be a change in the energy distribution. However it is difficult to observe such changes in a single-mode fibre as it is progressively curved since even very slight movements ($\sim 1\,\mu$m) of The core cannot be tolerated. However a very simple technique has been devised for avoiding this difficulty. The single-mode fibre investigated was made by homogeneous chemical vapour deposition and had a dip in refractive index at the centre of the core. The end of the fibre was carefully cleaved and cemented to one side of a semi-cylindrical plastic former of the desired radius. By rotating the former the length of bend, of fixed radius, was varied while the bend radius could be changed by using formers of different size. The output end of the fibre, when excited by a Gaussian beam from a helium neon laser, was imaged by a lens and the magnified nearfield pattern was recorded with a scanning diode array.

It was found that the dip in the core profile produced a small drop in intensity in the near-field pattern. As expected the depression was centrally situated in the field pattern when the fibre was straight but seemed to vary in its position as the end portion of the fibre was curved, at constant radius, over progressively longer lengths. In fact, of course, the profile dip remained stationary while the mode pattern in the core moved relative to the axis. This movement of the "mode" energy distribution is consistent with the ray emission.

It has been[10] shown that the effect of a dip in the refractive index on propagation of the HE_{11} mode is quite small in the fibres used in the experiments. Therefore the field deformation due to curvature was similar to that of a perfect step-index fibre. As expected it has been observed experimentally[11] that the modal field distribution in the pure bending region is constant along the fibre length.

2.3 Bend loss

Further evidence of a transition region is given by the variation of bend loss with bend radius which has been measured for two single-mode fibres, one of normalized frequency $V=2.10$ and numerical aperture $NA_s=0.056$ and the other of $V=2.18$, $NA_s=0.102$. It can be seen from Fig. 2 that the fibre of larger numerical aperture has an appreciably lower loss, amounting to less than 0.1dB/km at a bend radius of 2 cm. As a comparison with the theory which ignores the transition region the bend loss has been calculated using the results of reference 12 and is shown as the solid lines in Fig. 2. The agreement with the measured results is good for small bend radii but the measured loss greatly exceeds that calculated for more shallow curvatures.

The transmission losses in Fig. 2 were measured over two identical loops of fibre connected to the source and detector by straight sections of fibre. Thus the measured loss includes that due to uniform curvature

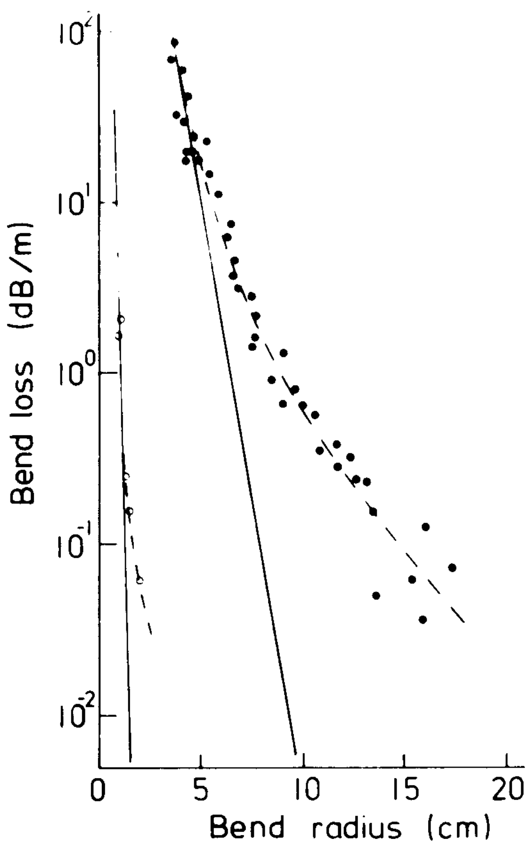


Fig. 2 Measured bend loss for single-mode fibres as a function of bend radius for fibres having $V=2.18$, $NA_s=0.102$ (o) and $V=2.10$, $NA_s=0.056$ (•). The solid lines are calculated using the results of reference 12.

as well as the mode conversion loss at the junctions between straight and curved portions. The bend and transition losses for the fibre of $NA_s=0.056$ have thus been calculated for these experimental conditions and are given in Fig. 3. Curve (a) gives the loss, calculated using the results of various authors for a uniform bend of length $4\pi R$ (see papers cited in reference12). Because of the low value of V, and consequent slight distortion of the field of the fundamental mode, it also approximates to Marcuse's result[7].

Curve (b) has been calculated using the approximate modes of the curved fibre found by Miyagi and Yip[8] and represents the contribution to the total loss due to coupling, at discontinuities in the fibre curvature, between guided modes of the straight fibre and the radiation modes of the curved fibre. Finally, curve (c) shows the power loss due to both uniform curvature and changes in curvature.

It can be seen that curve (c) provides a much better estimate of the measured loss at large radii of curvature than (a). Thus discontinuities due to changes in curvature play a significant role in determining fibre curvature losses, especially at large bend radii.

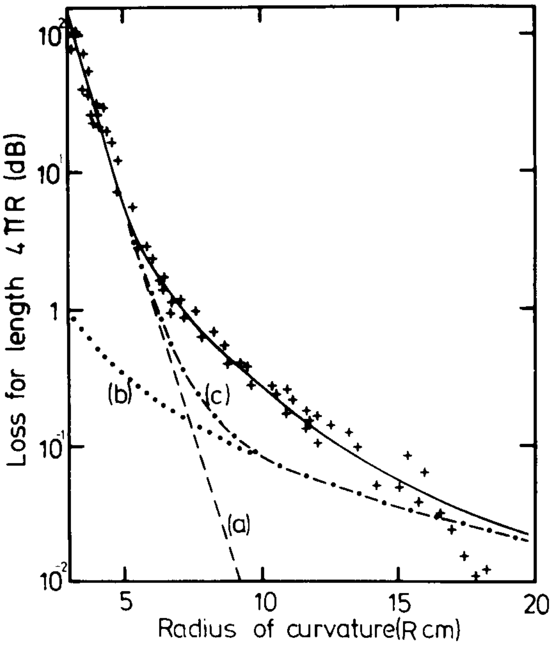


Fig. 3 Radiation loss over two complete loops of fibre as a function of radius of curvature R for V=2.1 and core radius 3.8 μm. The points denote experimental values. Curve (a) represents pure bend loss (12), curve (b) represents mode con-version loss (8) while (c) represents a sum-mation of (a) and (b).

2.4 Mode coupling

Another insight into the transition region may be obtained through the theory of mode coupling. In practice a single-mode core is surrounded by a finite cladding so that the HE_{11} mode at a bend couples with leaky core modes, and also with bound cladding modes of circumferential order 0 or 2. The number of these unwanted modes generated is limited since the mode-coupling coefficients depend strongly on the difference in propagation constants. In the transition region both forward and backward coupling occurs between the HE_{11} mode and the unwanted cladding modes until an equilibrium state is reached. Evidence for this mechanism is demonstrated by the curves in Fig. 4 which show the power (a) in the core and (b) in both core and cladding, as a function of distance from the beginning of the bend. The fibre had parameters V=2.38, NA_s=0.059 and cladding/core diameter ratio C=13.8. The bend radius was 1.5 cm.

It can be seen that the measured power in the core decreases with distance owing to radiation loss but in addition, there are regions where the power remains constant, or even increases, as a result of power returning to the core from the cladding modes. This interchange of energy between the HE_{11} mode and the unwanted modes causes the observed undulation in position of maximum power in the core described above as well as the apparent radiation of discrete beams.

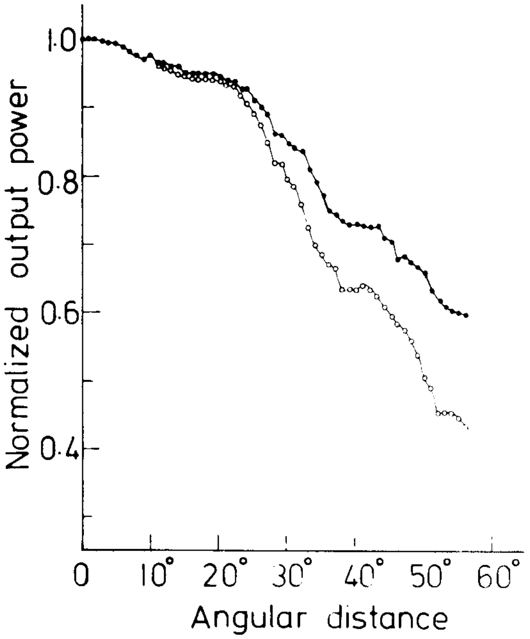


Fig. 4 Normalized total power in core and cladding (•) and power in core only (o) as a function of angular distance travelled around a bend of radius 15 mm. The fibre parameters are V=2.38, NA=0.06.

This mechanism has been observed and analyzed in detail in the case of multimode fibres[13] and the undulation of position of the beam locus has been observed and photographed.

Further confirmation of this mode coupling effect is provided by the theoretical work[14] of Sammut and the measurements[15] of Nelson et al.

3. Modes in Curved Fibres

The difference between the two curves in Fig. 2 represents the power being coupled from the core into the cladding in a curved single-mode fibre. During our experimental studies of single-mode fibres many observations were made of the near-field and far-field patterns of the cladding modes. It seemed desirable to analyse the modes of the curved fibre but existing theoretical studies are rather complex involving, for example, a double Fourier-Bessel series expansion[7] or a perturbation[8] method, in terms of circular cylindrical co-ordinates. However the observed mode patterns are not circular and the mode lines seem to be of parabolic shape. In the same way that one uses circular cylindrical, and not rectangular, co-ordinates to study modes in a uniformly circular system such as a straight fibre, it seemed worthwhile, in view of the observed mode patterns, to attempt to analyse a curved fibre in terms of a parabolic system. As a result we find that by invoking parabolic cylindrical coordinates the modes and the caustics of a curved fibre can be simply and accurately obtained. The method gives a clear insight into the mode structure and is in excellent agreement with experimentally observed field patterns.

3.1 Theory

From the weak-guidance approximation and the conformal mapping technique for the field of the guided modes Marcuse has shown[7] that the transverse field component E can be obtained from the scalar wave equation

$$\nabla^2 E + (kn)^2(1 + 2r\cos\theta/R)E = 0 \tag{1}$$

where n=refractive index, R=radius of curvature and k=free-space wave number. If, now, parabolic cylindrical co-ordinates (μ, λ, z) are used instead of the usual circular cylindrical co-ordinates (r, θ, z), together with the relations

$$\lambda = (2r)^{1/2}\cos(\theta/2) \text{ and } \mu = (2r)^{1/2}\sin(\theta/2) \tag{2}$$

then using the technique of separation of variables the scalar wave equation can be written

$$E = \eta(\lambda)\xi(\mu)\exp(-j\beta z) \tag{3}$$

where $\eta(\lambda)$ and $\xi(\mu)$ are the solutions of the differential equations

$$\frac{d^2\eta(\lambda)}{d\lambda^2} + \lambda^2 Q(\lambda)\eta(\lambda) = 0,$$
$$\frac{d^2\xi(\mu)}{d\mu^2} + \mu^2 p(\mu)\xi(\mu) = 0 \tag{4}$$

having the separation constant α, and

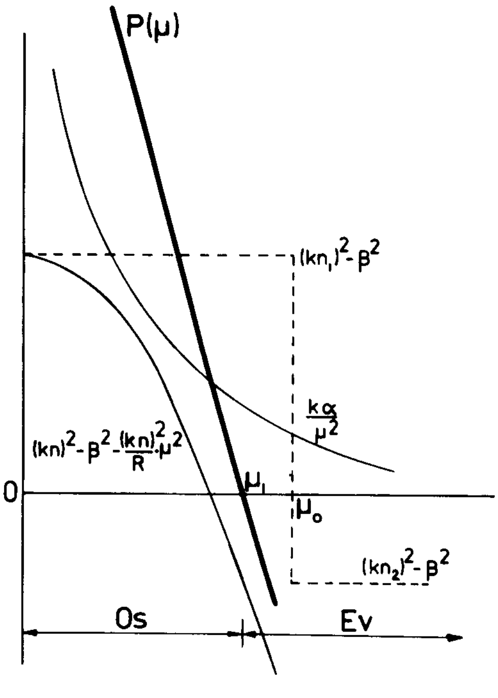


Fig. 5 Variation of $P(\mu)$ and its components with μ. O_S and E_V denote regions of oscillation and evanesent fields, respectively. The core/cladding interface is denoted by μ_0.

$$Q(\lambda) = (kn)^2 - \beta^2 + (kn)^2\lambda^2/R - k\alpha/\lambda^2$$
$$P(\eta) = (kn)^2 - \beta^2 - (kn)^2\mu^2/R + k\alpha/\mu^2 \tag{5}$$

These equations can be used to find the eigenfunctions (n, ξ) and the eigenvalues (α, β). Since the eigenfunctions are independent the field distribution of a guided mode can be completely described by the parabolic co-ordinates. We show here only the schematic of the mode pattern which is compared with those obtained experimentally; more detailed calculations will be presented elsewhere.

With the help of the WKB method it is found that there are regions of oscillating fields, corresponding to $Q(\lambda) > 0$; $P(\mu) > 0$, and of evanescent fields for which $Q(\lambda) < 0$; $P(\mu) < 0$, as indicated by the plots of $P(\mu)$

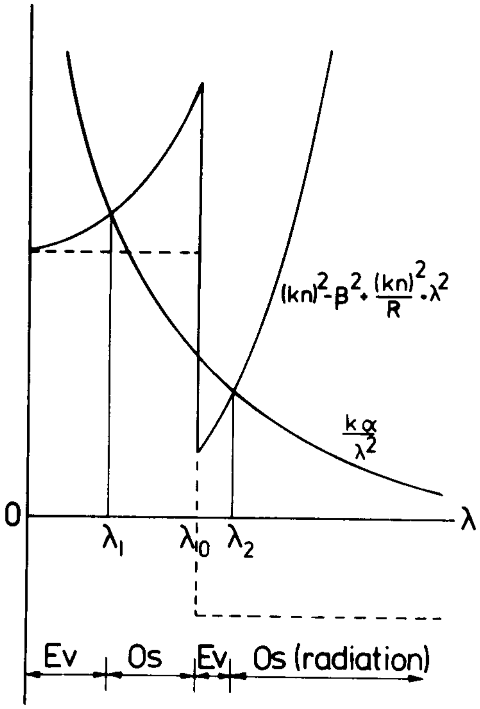


Fig. 6 Variation of components of $Q(\lambda)$. The core/cladding interface is denoted by λ_0.

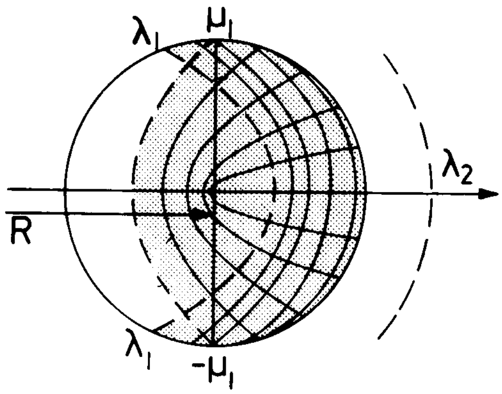


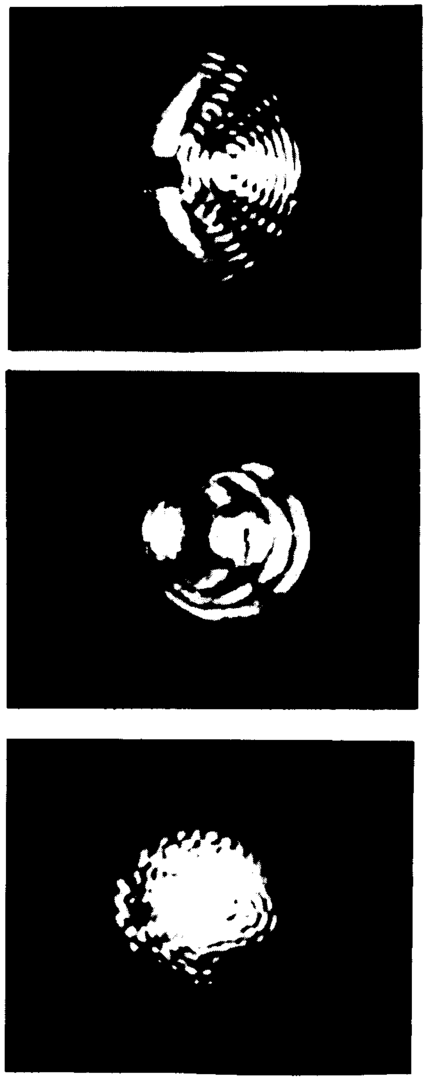
Fig. 7 Schematic of mode lines in curved fibre of bend radius R.

versus μ and $Q(\lambda)$ versus λ in Fig. 5 and 6. It can be seen that in the μ direction there is a caustic at $\mu=\pm\mu_1$ and the locations of the evanescent field and the oscillating field are shown schematically in Fig. 7, relative to the core of the fibre and the centre of curvature. Similarly in the λ direction a periodic field exists for $\lambda_1<\lambda<\lambda_0$ where $\lambda_0=(2a)^{1/2}\cos(\theta/2)$. There is a second caustic at $\lambda=\lambda_2$ beyond which the radiation corresponding to bend loss appears. As expected the radiation is emitted, and a shift of energy in the core occurs, in the direction away from the centre of curvature as illustrated in Fig. 7.

3.2 Experiment

Experimental confirmation of the above theory has been obtained from the near-field and far-field patterns of the cladding modes of the single-mode fibre. The fibres were given a silicone-rubber coating having a refractive index lower than that of the cladding which thus acted as the core of a multimode step-index fibre. The inner core of this three-layer structure was thus the excitation source since the bend loss studies have shown[12] that when the cladding of a single-mode fibre has a finite thickness the HE_{11} mode couples with the LP_{1m} cladding modes. As the volume of the single-mode core is small compared with that of the cladding, these cladding modes can be thought of as the core modes of the step-index, silicone-clad, multimode fibre.

Another factor to be borne in mind is that the cladding modes excited in a single-mode fibre depend strongly on the spot size of the HE_{11} mode, which in turn is directly related to the numerical aperture (NA_s) of the single-mode fibre, and on the cladding/core diameter ratio C. Two typical near-field patterns in a curved multimode fibre are shown in Fig. 8 (a) and (b) for $NA_s=0.094$, $C=26$ and $NA_s=0.063$, $C=18$, respectively. In each case the numerical aperture of the multimode fibre is 0.333, and the bend radius $R=15$ mm. It can be shown[16] that higher-order cladding modes are excited with higher values of NA_s and this is evident from the photographs. The mode in Fig. 8 (a) contains five lobes in the μ direction and seven in the λ direction and the parabolic shape of the former can be clearly seen. In this case the energy (rays) propagates without striking the core/cladding interface on the inside of the bend and the caustic separating the oscillation and evanescent regions is well inside the core. In contrast, because of the smaller diameter (83 μm compared with 133 μm in Fig. 8 (a)) the rays still strike the interface on the inside of the bend for the fibre in Fig. 8 (b). There are five lobes in the μ direction, starting near the inside bend wall. The parabolic shape is evident in the three λ lobes. Thus both Fig. 8 (a) and (b) comprise a combination of two parabolic mode patterns, although the λ lobes in Fig. 8 (a) look circular because of the higher value of λ there. It is interesting to note that the far-field pattern in Fig. 8 (c), which was obtained for the same fibre and conditions as Fig. 8 (a), is similar to those illustrated by Kapany[17].



(a)

(b)

(c)

Fig. 8 Near-field (a) and far-field (c) mode patterns of multimode fibre of core diameter 136 μm, NA=0.333 and bend radius R=15mm. The near-field pattern in (b) is for a fibre of core diameter 86 μm and $NA_s=0.063$.

4. Conclusions

Studies of propagation in curved single-mode fibres have shown the importance of the transition region at the beginning of the bend where mode conversion occurs as the stable mode of the straight fibre transforms to the stable mode of the curved fibre. This mode conversion gives rise to the ray radiation previously observed[9], to additional bend loss, to fluctuations in the transverse energy distribution and to a modulation in the variation of power density around the bend. Some of these features can be inferred from the simplified ray

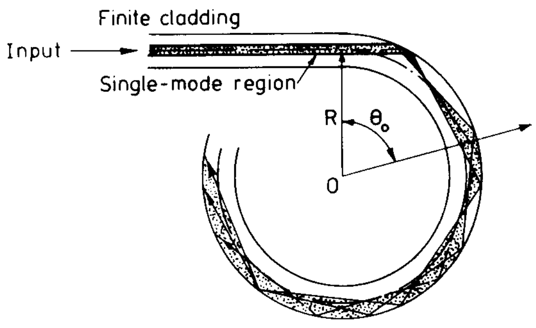


Fig. 9 Schematic diagram of ray propagation in a curved fibre.

schematic shown in Fig. 9.

The field distribution in a curved fibre can be simply and clearly analysed by using parabolic cylindrical co-ordinates. The theory presented is in good qualitative agreement with experiment. It should be possible to make a direct measurement of the axial propagation constant by observing the position of the two caustics μ_1 and μ_1 and solving eqn[(5)]. Detailed theoretical and experimental results will be reported elsewhere.

Acknowledgement

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