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Conditions for efficient build-up of power in photorefractive ring cavities

Małgosia Kaczmarek

Department of Physics, University of Exeter, Exeter EX4 4QL, United Kingdom
e-mail: M.Kaczmarek@exeter.ac.uk

Robert W. Eason

Optoelectronics Research Centre and Department of Physics and Astronomy, University of Southampton,
Southampton SO17 1BJ, United Kingdom

Abstract

We present results on modelling of amplification of light in a photorefractive ring resonator and on the optimum parameters required to achieve a high build-up of power. We show that an efficient resonator can be realised even with moderate coupling coefficients, of the order of 5 cm^{-1}, provided the value of absorption is low, namely below 0.1 cm^{-1}. These two conditions can be simultaneously fulfilled in a Rh:$BaTiO_3$ crystal, using a near-infrared (1.06 μm) laser as a pump beam. The low absorption coefficient condition also relaxes the dependence on the absolute value of transmission / reflection of the out-coupling element.

Key words

photorefractive ring-resonator, two-beam coupling, Rh:$BaTiO_3$, beam clean-up

Background

Numerous interaction geometries have been developed for photorefractive materials that rely on amplification of light via the two-beam coupling effect. One of the most elegant and simple configurations is a unidirectional ring resonator[1], shown in figure 1.

This scheme consists of a photorefractive crystal placed in a ring cavity and pumped by an external beam. The incident pump beam fans inside the crystal, and some of the fanned light is directed around the loop to re-enter the crystal, via the three mirror set-up $M_{1,2,3}$. The light in this ring is unidirectional because the two-beam coupling gain is itself directional[2], as determined by the crystal's symmetry, alignment and the charge transport properties, so any backward travelling light will experience loss. If the two-beam coupling gain is above threshold, the resonating beam will build up from the amplification of fanned light. The circulating power can be monitored by, for example, a beam splitter that couples light out of the cavity.

Typical resonator conditions require that the optical path length inside the cavity is equal to an integral number of the incident pump wavelengths. However, uniquely in photorefractive ring resonators, the oscillation builds up almost regardless of the optical cavity length, its frequency being determined by the round trip phase condition. The phenomenon of spontaneous occurrence of small frequency shift between the resonating beam and the external beam provides the right phase difference[3] to satisfy the resonator condition. If the ring-cavity length changes, the frequency of the resonating beam changes to compensate for the difference.

The theory describing this type of interaction is based on non-degenerate two-wave mixing, and shows that the coupling coefficient[2] is smaller than in the degenerate case, and depends on the frequency difference between the beams. This coupling coefficient also depends on the material's parameters and on the experimental geometry. Its magnitude determines the amplification of the resonating light.

All in all, changes in the ring resonator's length affect two independent quantities[3] - frequency and intensity of the resonating light. This is a unique feature of photorefractive ring resonators. However, a successful exploration of its potential relies on the resonator working most efficiently, namely achieving a high build-up of power in the oscillating beam.

Results and discussion

The most crucial parameters for effective resonator design are: the wavelength of the pump light and the type of photorefractive material. In order to investigate the resonating beam dependence on these parameters, we have performed intensity-dependent modelling. The resonating beam accumulates energy from successive amplification in a photorefractive material until saturation sets in, but loses energy from absorption and other losses such as Fresnel reflections from the crystal, and imperfect mirrors. The power conversion from the external pump beam into the resonating beam can be defined as a ratio of the resonating beam intensity to the intensity of the external pump beam: $\xi=I_{osc}/I_{pump}$. The dependence of the power conversion ξ on the absorption coefficient can be calculated in two ways. The first method is a simple, iterative modelling of two-beam coupling in a ring resonator. It enables one to monitor how quickly the steady-state gain is reached and provides flexibility in checking the effect of different losses or intensity-dependent phenomena, such as laser-induced change of absorption[4]. The second approach is to derive an expression for ξ analytically, which in the straightforward two-beam coupling case is[1]:

$$\xi = \frac{I_{osc}}{I_{pump}} = \frac{(1-R)-(e^{\alpha-\Gamma)L}}{e^{\alpha L}-(1-R)} \tag{1}$$

If, for simplicity, we assume that there are no passive optical losses from optical elements within the resonator other than from the intentional out-coupling via the beam splitter, then R is the beam splitter's reflectivity (figure 1), Γ is the coupling coefficient, L is the length of the photorefractive material, and α is the absorption coefficient. For simplicity, in all our modelling we assume L=0.5 cm - a typical length of a photorefractive crystal.

The oscillating beam builds up, providing the coupling efficiency exceeds the combined absorption and resonator losses. The threshold condition can be determined from the following expression[2]:

$$(\Gamma L)_{th} \geq \alpha L - \ln(1-R) \tag{2}$$

Figure 2a shows the resonator power conversion factor, ξ, versus absorption coefficient for different values of coupling coefficient, Γ, and assuming a very weak reflection from the beam splitter, namely 0.1%. The most interesting observation from this example is that a high intensity oscillating beam can build up, reaching the magnitude of 20 to 170 × the intensity of the pump beam, even for modest coupling coefficients (above $\Gamma=5$ cm^{-1}), provided that absorption is simultaneously low (from 0.1 to 0.01 cm^{-1}). The other effect that should be noted is that the intensities of the resultant oscillating beams in resonators with higher coupling coefficients (between 10-100 cm^{-1}) are approximately identical. This is an important conclusion which dictates the optimum conditions for achieving the highest accumulation of circulating energy inside a resonator. It is crucial to keep the value of *absorption* to a minimum, while ideally also maximising the magnitude of coupling coefficient (by cutting or otherwise orienting the crystal, for example). Since the coupling coefficient (and absorption) of a photorefractive material strongly depends on the wavelength of the external pump beam, the most efficient operation is achieved through careful attention to the wavelength of the external pump beam used. For example, these optimum conditions can be simultaneously fulfilled in a sample of Rh:$BaTiO_3$ crystal, which has low absorption (0.06 cm^{-1}) yet a relatively high coupling coefficient (11.2 cm^{-1}) at 1.06 μm[5].

If more power is extracted from the resonator on each round trip by using a beam splitter with a greater reflectivity, the intensity of the oscillating beam decreases, as expected. Figure 2b shows the change in the oscillating beam intensity from 170 to 20 × the pump beam intensity for the range of various beam splitter reflectivities, from 0.1% to 5%, assuming a coupling coefficient of 10 cm^{-1}.

Since the magnitude of absorption is the most crucial parameter for an efficient ring resonator, the effect of light induced changes in absorption[4] has to be taken into account. Photorefractive materials often show this effect, where the value of absorption coefficient is larger (or smaller) at high incident light intensities than at weak intensities, such as, for example, encountered in spectrometers measuring absorption spectra. In $BaTiO_3$ crystals the effect of laser-induced-absorption is typically observed. However, in Rh:$BaTiO_3$ crystals, we have observed earlier strong laser-induced transparency in the visible (up to $\Delta\alpha = -0.9$ cm^{-1}) and small, but not negligible, laser-induced-absorption in the near-infrared[5].

The other parameter that characterises photorefractive ring-resonators is the efficiency of energy extraction, namely the ratio of the intensity of light out-coupled from the resonator (I_{out}) to the intensity of the external pump beam[1] (I_{pump}). This parameter can also be calculated either via iterative modelling or analytically:

$$\eta = \frac{I_{out}}{I_{pump}} = \frac{R}{1-R}\,\frac{(1-R)-(e^{\alpha-\Gamma)L}}{e^{\alpha L}-(1-R)} \qquad (2)$$

Figure 3 presents the output coupling efficiency η of a ring resonator for different values of beam splitter reflectivity. Parameters for coupling coefficients and absorption are as measured in our Rh:$BaTiO_3$ and nominally undoped $BaTiO_3$ crystals[5]. As expected, the highest efficiency (up to 0.95) can be achieved using the wavelength of 1.06 μm in Rh:BaTiO$_3$ (curve a). Using other wavelengths in this crystal, such as 800 nm (curve b) or 647 nm (curve c), in spite of record high two-beam coupling coefficients, the losses due to absorption significantly reduce the intensity of the oscillating beam.

In some samples of nominally undoped BaTiO$_3$ relatively high two-beam coupling gains were observed at near-infrared wavelengths (800-850 nm). Coupling coefficients as high as 6.8 cm^{-1} at 800 nm and 7.6 cm^{-1} at 850 nm were reported[6,7]. In figure 3 (curve d) we have also included the output coupling efficiency expected for nominally undoped BaTiO$_3$ using the data provided by MacCormack[5]. Low absorption (0.09 cm^{-1}) at 800 nm ensures that high efficiency can be achieved, but it is not as high as the maximum efficiency predicted for Rh:BaTiO$_3$ at 1.06 μm.

Another point that is interesting to note concerns the shape of coupling efficiency curves. For 1.06 μm (curve a), the output coupling efficiency is high (close to 1) and constant for a wide range of beam splitter reflectivities. Analysing expression (2) for η one can look at its different limits. For example, for $R \rightarrow 1$ or $R \rightarrow 0$, $\eta \rightarrow 0$. The other important limit is when $e^{(\alpha-\Gamma)L} \rightarrow 0$ and $e^{\alpha L} \rightarrow 1$, which gives $\eta \rightarrow 1$. This is the case of very low absorption and finite coupling (such as for 1.06 μm radiation incident on a sample of Rh:BaTiO$_3$) and providing that the beam splitter reflectivity is far from 0 or 1, the output coupling efficiency remains constant with a value close to 1.

The curves presented in figure 3 show different 'width' or range of beam splitter reflectivites for which η remains approximately constant. This range can be a useful piece of information, particularly when setting up an optimal experimental configuration. In order to get a better insight into this effect, we have compared the range of 'constant' η predicted for different crystal samples. First of all we defined 'bandwidth' - $\Delta\eta$ - as the range of the output coupling efficiencies where their value remains within 10% of the maximum η. We have then determined $\Delta\eta$ from two-beam coupling experimental data and absorption spectra of different crystals. We have considered: 400 ppm Rh:BaTiO$_3$, original[8] 'blue' Rh:BaTiO$_3$ and nominally undoped BaTiO$_3$ samples[9] at the wavelengths of 647 nm, 800 nm, 850 nm, 1.0 μm and 1.06 μm; data provided by MacCormack[6], and the set of data published by Brignon et al.[10]. Table 1 lists all the samples and the important magnitudes.

Using ten sets of data we were able to show a strong $\Delta\eta$ dependence on absorption coefficient (figure 4a). The smaller the absorption coefficient is, the wider is the range of beam splitter reflectivities for which output coupling efficiency, η , remains approximately constant. It is also worth noting that broad bandwidth $\Delta\eta$ is related to high maximum value of output coupling efficiency η (figure 4b). Therefore, keeping absorption to a minimum ensures both high amplification of the oscillating beam, and the flexibility in choosing the output coupler.

Earlier experimental results by MacCormack[6] show that with little care paid to minimising passive losses a power conversion factor ξ of 3.25 can be achieved in a ring resonator with photorefractive, nominally undoped $BaTiO_3$, and a laser diode operating at 800 nm. Similar results have also been obtained by other groups[11].

Using the data supplied by MacCormack, the relevant coupling coefficient was $\Gamma = 6.8\ cm^{-1}$ and absorption coefficient, α, was $0.09\ cm^{-1}$. Using the data provided we deduced the value of output coupling efficiency η being equal to 0.05625. For simplicity, we can incorporate all the passive losses within the resonator used, such as Fresnel reflections from crystal and lens' faces, and imperfect mirrors, into an 'effective absorption coefficient'. In this case we calculated that the value of α effectively increases from $0.09\ cm^{-1}$ to $0.576\ cm^{-1} \pm 0.08\ cm^{-1}$.Figures 5a shows the result of power conversion factor ξ modelling versus absorption coefficient α, assuming $\Gamma=6.8\ cm^{-1}$ and the beam splitter reflectivity of 1.7% (as used by MacCormack[5]). The black dot represents the experimental result of $\xi=3.25$ for $\alpha=0.576\ cm^{-1}$. There is an excellent agreement between the predicted conversion factor and the observed one. Figure 5b presents the theoretical curve of η dependence on the beam splitter reflectivity assuming $\Gamma=6.8\ cm^{-1}$ and $\alpha=0.576\ cm^{-1}$. The black dot represents his experimentally determined value of $\eta=0.05625$ achieved with the 1.7 % reflecting beam splitter. The coupling efficiency predicted by the theory agrees very well with the experimental value. There are two important observations that can be made from this comparison. First of all, as expected, it is crucial to keep all the passive losses to a minimum as the intensity of the oscillating beam (figure 5a) can be up to 4 × higher. Secondly, even with the existing passive losses, a higher coupling efficiency, up to 10 × higher, could have been achieved if a different, more highly reflecting beam splitter was used (figure 5b).

Conclusions

The potential of photorefractive ring resonators for the *efficient* build-up of optical power has been little explored to date. Our modelling suggests that with attention given to values of absorption coefficients, the results of experiments with photorefractive ring resonators can be further improved if the optimum wavelength and optical components are chosen for a particular photorefractive material. Achieving high amplification of light experimentally, as predicted by the theory and our modelling, will be an interesting goal in itself, but will also be essential for applications of a photorefractive ring resonator in detection and sensing techniques. The important parameters are: low absorption coefficient (below 0.1 cm^{-1}) and moderate coupling coefficient (above 5 cm^{-1}) to ensure the efficient operation of a ring resonator. These conditions can be met with a Rh:$\mathrm{BaTiO_3}$, crystal and a near-infrared wavelength (above 1 μm) external pumping beam. Low absorption coefficient also provides a greater degree of flexibility for the choice of transmission/ reflection properties of the out-coupling element within the ring resonator.

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Figure captions

Figure 1	Unidirectional photorefractive ring resonator
Figure 2a	Power conversion factor ξ inside a ring resonator for different values of absorption coefficient α (assuming beam splitter reflectivity $R=0.1\%$).
Figure 2b	Power conversion factors ξ versus absorption coefficient for different reflectivities of an out-coupling beam splitter (assuming $\Gamma=10\mathrm{cm}^{-1}$).
Figure 3	Output coupling efficiency η in a resonator versus beam splitter reflectivity for different sets of coupling, Γ, and absorption, α, coefficients measured in: Rh:$BaTiO_3$ crystal at different wavelengths of external pump beam: (a) 1.06 μm - solid line ; (b) 800 nm - dashed line; (c) 647 nm - dash-dot line; (d) nominally undoped $BaTiO_3$ at 800 nm - dash-dot-dot line.
Figure 4a	The dependence of bandwidth $\Delta\eta$ on absorption coefficient. Dots represent ten sets of data measured at different wavelengths in different samples of Rh:$BaTiO_3$ and nominally undoped $BaTiO_3$ (see Table 1).
Figure 4b	The relation between the maximum output coupling efficiency η on the bandwidth $\Delta\eta$ for ten sets of data from Table 1.
Figure 5a	Power conversion factor ξ versus absorption coefficient for $\Gamma=6.8\ \mathrm{cm}^{-1}$ and beam splitter reflectivity of 1.7 %. Dot - experimentally measured $\xi=3.25$ with an 'effective absorption coefficient' $\alpha=0.576\ \mathrm{cm}^{-1}$ (ref. 6).
Figure 5b	Output coupling efficiency η for $\Gamma=6.8\ \mathrm{cm}^{-1}$ and an 'effective' $\alpha=0.576\ \mathrm{cm}^{-1}$ versus beam splitter reflectivity. Dot - experimentally measured $\eta=0.05625$ with $R=1.7$ % (ref. 6).

Sample and wavelength	Γ [cm^{-1}]	α [cm^{-1}]	$\Delta\eta$	Maximum η
400 ppm Rh:BaTiO$_3$ at 1 μm (ref. 8)	6.8	0.05	0.68	0.888
original 'blue' Rh:BaTiO$_3$ at 1.06 μm (ref. 5)	11.2	0.06	0.787	0.946
undoped BaTiO3 at 800 nm (ref. 6)	6.8	0.09	0.696	0.851
3200 ppm Rh:BaTiO$_3$ at 1.06 μm (ref.10)	9.35	0.13	0.656	0.882
400 ppm Rh:BaTiO$_3$ at 850 nm (ref. 8)	7.7	0.25	0.531	0.773
400 ppm Rh:BaTiO$_3$ at 800 nm (ref. 8)	10.2	0.36	0.48	0.773
undoped BaTiO$_3$ at 647 nm (ref. 8)	14.6	0.4	0.424	0.798
400 ppm Rh:BaTiO$_3$ at 514 nm (ref. 8)	14.9	1.32	0.24	0.493
original 'blue' Rh:BaTiO$_3$ at 800 nm (ref. 5)	17	1.5	0.209	0.458
original 'blue' Rh:BaTiO$_3$ at 647 nm (ref. 5)	33	4.8	0.109	0.09

Table 1

Coupling coefficients, absorption coefficients, bandwidth $\Delta\eta$ and the maximum η output coupling efficiency for different wavelengths and samples of Rh:BaTiO$_3$ and BaTiO$_3$ crystals.

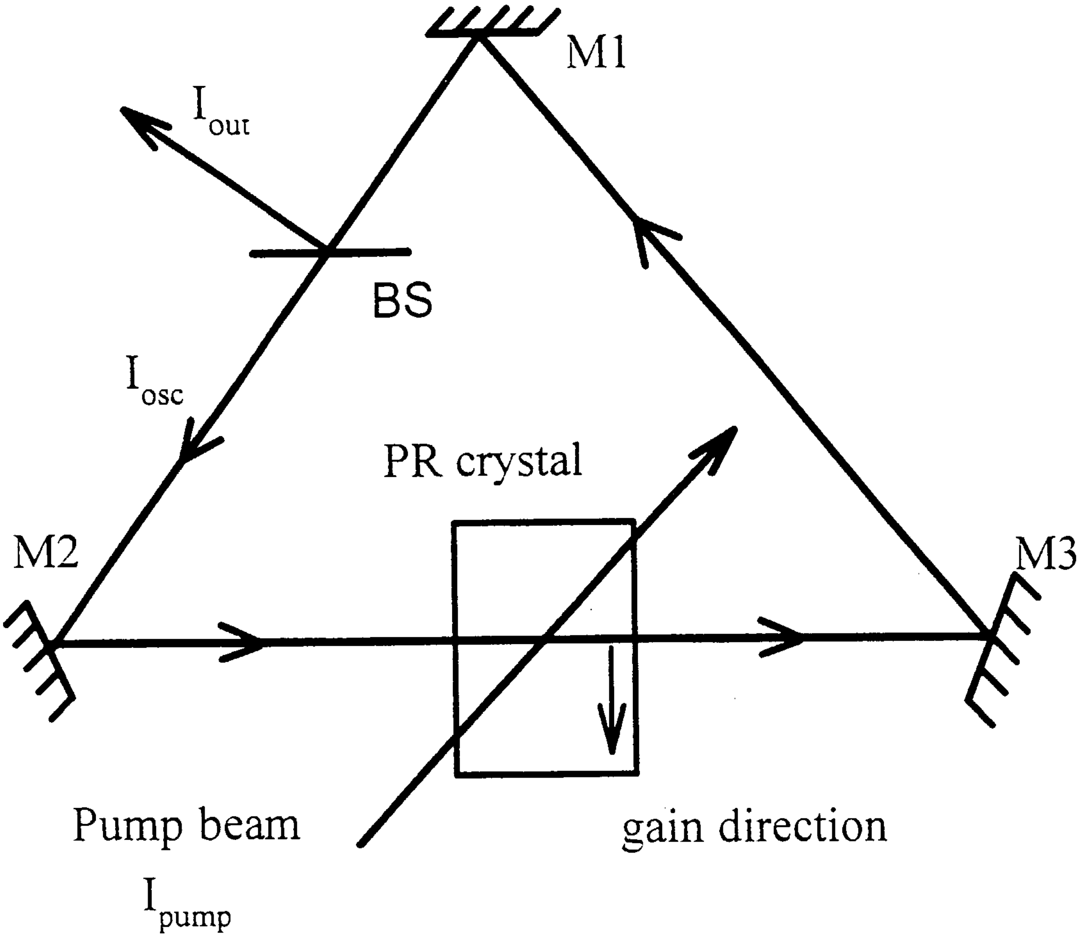

M1
I_{out}
BS
I_{osc}
PR crystal
M2
M3
Pump beam
I_{pump}
gain direction

Figure 1

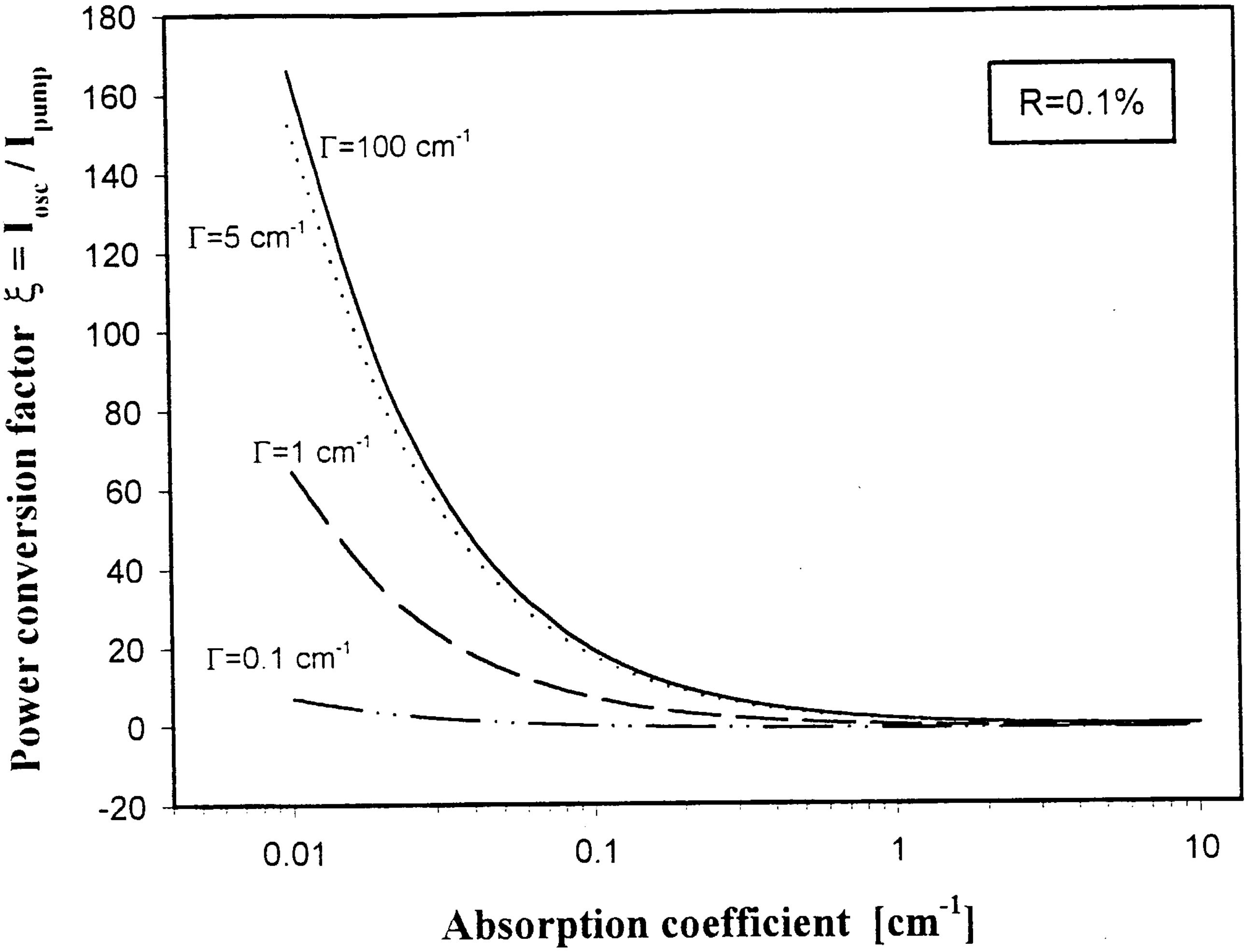

Power conversion factor $\xi = I_{osc} / I_{pump}$
180
160
140
120
100
80
60
40
20
0
-20
0.01
0.1
1
10
Absorption coefficient [cm^{-1}]
R=0.1%
Γ=100 cm^{-1}
Γ=5 cm^{-1}
Γ=1 cm^{-1}
Γ=0.1 cm^{-1}

Figure 2a

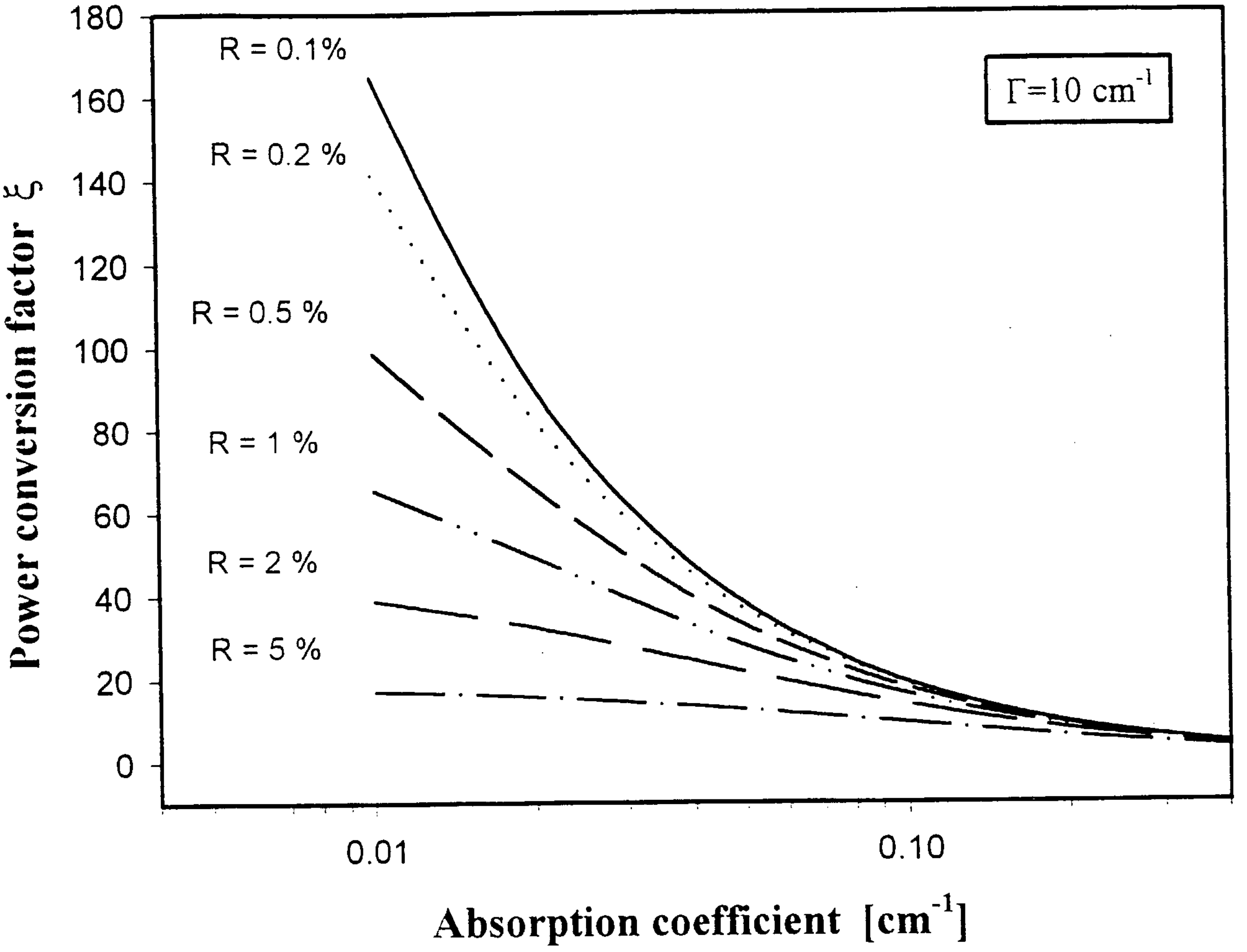

Γ=10 cm⁻¹
R = 0.1%
R = 0.2 %
R = 0.5 %
R = 1 %
R = 2 %
R = 5 %
180
160
140
120
100
80
60
40
20
0
0.01
0.10
Power conversion factor ξ
Absorption coefficient [cm⁻¹]

Figure 2b

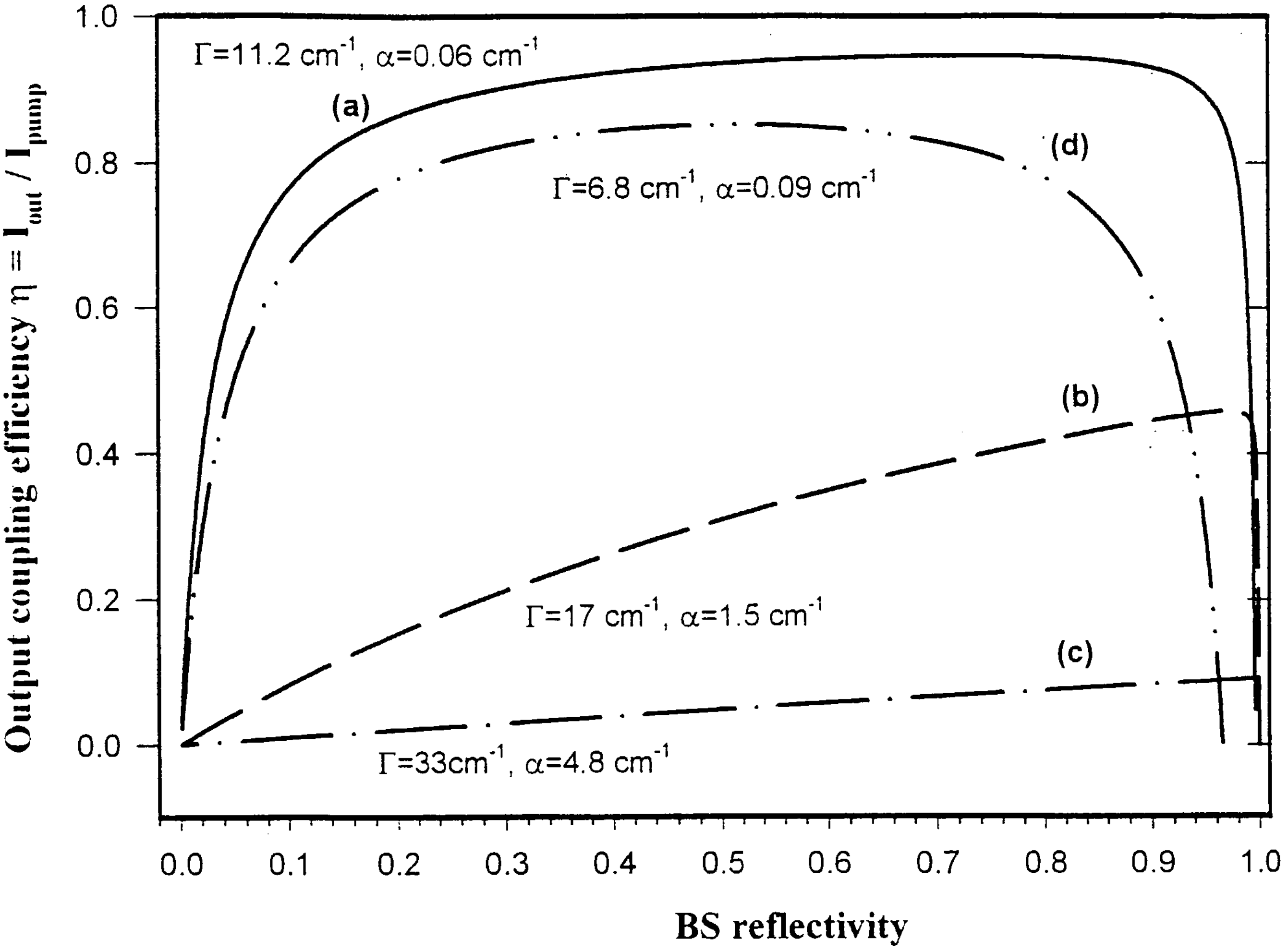

Output coupling efficiency η = I_out / I_pump
BS reflectivity
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Γ=11.2 cm⁻¹, α=0.06 cm⁻¹
(a)
(d)
Γ=6.8 cm⁻¹, α=0.09 cm⁻¹
(b)
Γ=17 cm⁻¹, α=1.5 cm⁻¹
(c)
Γ=33cm⁻¹, α=4.8 cm⁻¹

Figure 3

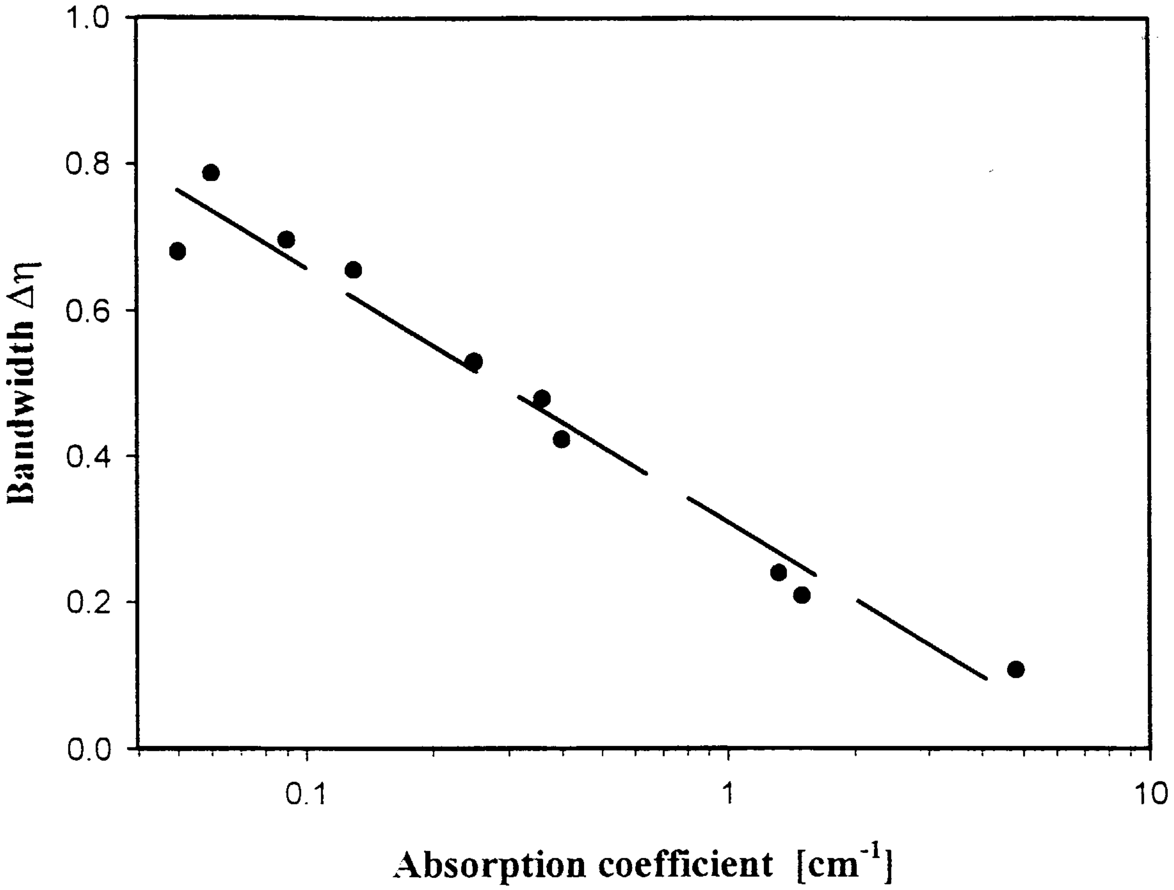
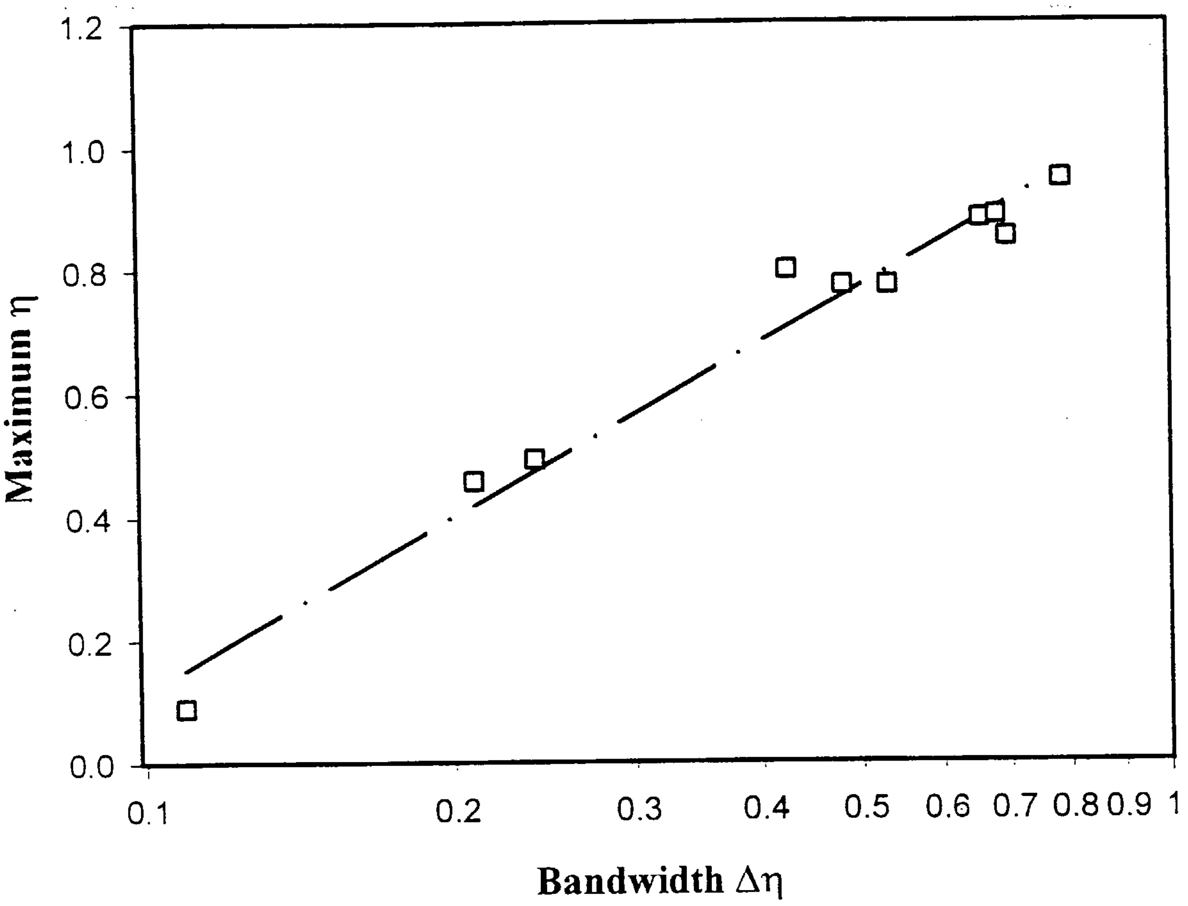

1.0
0.8
0.6
0.4
0.2
0.0
Bandwidth Δη
0.1
1
10
Absorption coefficient [cm⁻¹]

Figure 4a



Maximum η
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Bandwidth Δη

Figure 4b

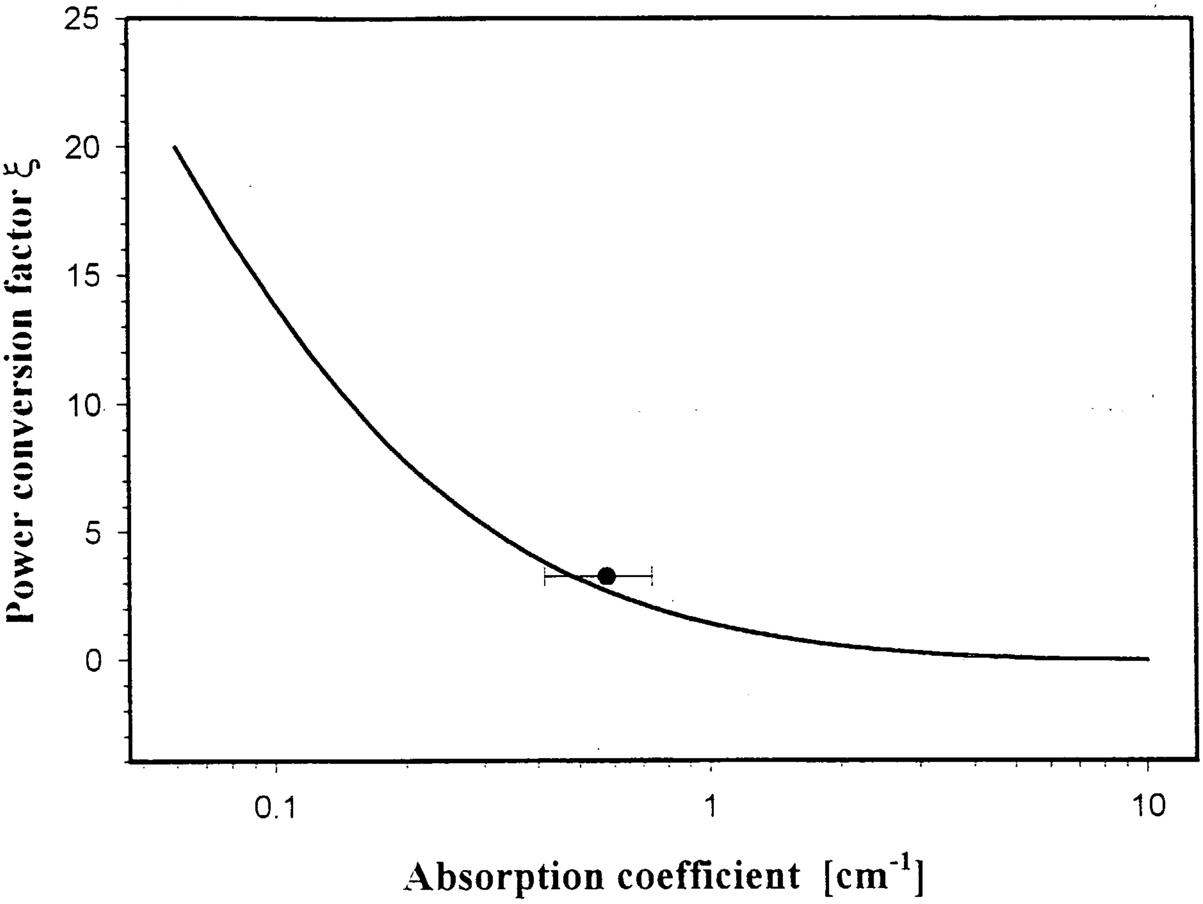

25
20
15
10
5
0
0.1
1
10
Power conversion factor ξ
Absorption coefficient [cm-1]

Figure 5a