

# Beat length measurement in directional couplers by thermo-optic modulation

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# Abstract

In integrated optical directional couplers formed by two parallel waveguides, we measure the difference  $\Delta\beta=\beta_{even}-\beta_{odd}$  between the propagation constants of the 'supermodes'. They are coupled locally by heating a fine spot on one of the guides. When the spot is scanned along the coupler the output power from one of the guides is found to vary periodically. The period of variation is the modal beat length  $\Lambda=2\pi/\Delta\beta$ . We demonstrate this technique with directional couplers fabricated by K<sup>+</sup>-exchange in glass. Beat lengths in the range of 0.6...2.2 mm are measured with an accuracy of  $\pm 0.3\%$ .

Key components of integrated optics rely on the distributed coupling of waves in two adjacent guides, and the practical measurement of this coupling is a fundamental task. The quantities of prime interest are the propagation constants  $\beta_e$  and  $\beta_o$  of the two lowest order 'supermodes'. Their difference  $\Delta\beta=\beta_e-\beta_o$  is the rate at which optical power is coupled back and forth between the guides. The spatial period of this exchange is the beat length  $\Lambda=2\pi/\Delta\beta$  of the supermodes. In a symmetric coupler, the beat length is related by  $\Lambda=\pi/C$  to the coupling coefficient C of the two guides.

Among the methods known for measuring  $\Lambda$  in directional couplers<sup>1-4</sup>, each has particular advantages as well as limitations, such as narrow wavelength range<sup>1,2</sup>, need for sophisticated equipment<sup>3</sup>, or limited  $\Lambda$  range<sup>4</sup>. Moreover, these methods are all based on probing the evanescent field and, therefore, cannot be applied to buried guides.

In this Letter we determine the beat length in directional couplers by a method of 'modulated interference' which avoids some of the problems mentioned. We analyze this method theoretically and demonstrate it with couplers fabricated by K<sup>+</sup>-exchange in glass, see Fig. 1.

The principle is closely related to the measurement of birefringence in optical guides<sup>5-7</sup>. There, the orthogonally polarized modes play the same role that the supermodes of the two coupled guides do here. Hence we can adopt here the method of thermo-optic birefringence measurement<sup>7</sup> which permits the analysis of 'buried' couplers.

We couple optical power  $P_1$  into port 1 of the directional coupler and detect the output power  $P_3$  from port 3, see Fig. 1. This power  $P_3$  changes by a small amount  $\delta P_3$  when we heat the coupler at a position  $z_o$ . We scan the heated spot along the guide and record  $\delta P_3(z_o)$ . As we will show below, this function varies periodically with  $z_o$ , and the period represents directly the desired beat length of coupling,  $\Lambda$ .

The theory underlying this measurement is considered first in the absence of the perturbation. We assume the coupler to be lossless, but possibly asymmetric, and we consider only one polarization. The waves in guides 1 and 2 have electric field distributions  $G_i(x,y)$ and amplitudes  $E_i(z)$ , with i = 1, 2, respectively. We normalize the  $G_i$  so that the power in guide i is  $P_i = |E_i|^2$ . In the central section II of the device these waves are coupled and exchange power. Their amplitudes  $E_i(z)$  vary with the spatial periodicity  $\Lambda$ .

An alternative description results after diagonalization of the coupled mode equations. The field in section II is represented now as the superposition of two 'supermodes' which have different propagation constants,  $\beta_e$  and  $\beta_o$ . These modes are distinguished as 'even' and 'odd' according to their field distributions,  $\mathbf{F}_e(x,y)$  and  $\mathbf{F}_o(x,y)$ , respectively<sup>8</sup>. Their amplitudes  $A_e$  and  $A_o$  are constant along the length L of section II. They are excited by the  $E_i$  at the transition I $\rightarrow$ II. To allow here for asymmetric amplitude splitting between the two supermodes we introduce a splitting parameter  $\sigma_1$  and express  $A_e = \cos \sigma_1 E_1 + \sin \sigma_1 E_2$  and  $A_o = \sin \sigma_1 E_1 - \cos \sigma_1 E_2$ . At the transition II $\rightarrow$ III the inverse transformation yields the output amplitudes,  $E_3 = \cos \sigma_2 A_e + \sin \sigma_2 A_o$  and  $E_4 = \cos \sigma_2 A_e - \sin \sigma_2 A_o$  employing a splitting parameter  $\sigma_2$ . We note that the coupler is symmetric if  $\sigma_{1,2} = \pi/4$ .

Following this description, optical transmission from port 1 to 3 may be understood as a two-beam interference, see Fig. 2(b). Light travels from port 1 to 3 along the two paths which are indicated by the full lines. At the output these waves interfere. Their amplitudes are  $V_e = \cos \sigma_1 \cos \sigma_2 \exp(+j\Delta\beta L/2)$  and  $V_o = \sin \sigma_1 \sin \sigma_2 \exp(-j\Delta\beta L/2)$  if we assume unit input power,  $P_1 = 1$ . The phases result from the delays  $\beta_e L$  and  $\beta_o L$  of the waves in section II. In specifying  $V_{e,o}$  here and below, we suppress the mean phase delay  $\overline{\phi} = \phi_1 + \phi_3 + (\beta_e + \beta_o)/2$  of these waves, where  $\phi_1$  and  $\phi_3$  refer to the input and output arms. The output signal is  $V_3 = V_e + V_o$ , the output power  $P_3 = |V_3|^2$ .

When a small volume of the sample is heated, the dielectric tensor  $\epsilon$  is modified directly by the temperature coefficients of the materials, and indirectly by thermal stress and the elasto-optic effect. It is not necessary here, however, to consider these effects in detail, as they only determine the magnitude of  $\delta P_3$ , not its periodicity. It suffices that there is a variation  $\Delta \epsilon(x,y,z)$  in the vicinity of  $z_o$ . As a consequence, the two supermodes become coupled as indicated in Fig. 2. At the output two additional signals  $\delta V_{eo}$  and  $\delta V_{oe}$  appear, represented by the dashed lines in Fig. 2(b). Moreover, the main output signals  $V_e$  and  $V_o$  will generally change slightly by  $\delta V_{ee}$  and  $\delta V_{oo}$ , respectively. Typically all these modifications

are small, of the order of  $|\delta V_{mn}/V_m| \leq 10^{-3}$ .

Standard perturbation analysis yields the distributed coupling functions  $\kappa_{mn}(z)$  of the supermodes (m,n = e,o). The coupled wave equations for  $A_{e,o}(z)$  are formally integrated over the the axial extent  $\Delta z$  of the heated spot, using a local variable  $\zeta = (z - z_0)$  inside the perturbed region. Thus the effects of the perturbation are lumped into four discrete coupling constants  $K_{mn}$  which specify the modifications of the output signals described above. Using a normalisation  $N_m = \iint |\mathbf{F}_m|^2 dxdy$  and the propagation constant  $k_o$  in free space, we obtain

$$\kappa_{mn}(z) = \frac{k_o^2}{2N_m \beta_m} \iint \mathbf{F}_m^* \, \Delta \epsilon(x, y, z) \, \mathbf{F}_n dx dy \tag{1}$$

$$K_{mn} = \int_{\Delta z} \kappa_{mn}(\zeta) \, \exp[j(\beta_n - \beta_m)\zeta] \, d\zeta \tag{2}$$

$$\delta V_{mn} = j K_{mn} V_n \exp[j(\beta_n - \beta_m) z_0]$$
(3)

$$V_e^{\odot} = V_e + \delta V_{ee} + \delta V_{oe} \tag{4}$$

$$V_o^{\odot} = V_o + \delta V_{eo} + \delta V_{oo} \tag{5}$$

The last two lines express the modified output signals, labelled by the superscript  $\odot$ . The four perturbations  $\delta V_{mn}$  modify the original output signals  $V_e$  and  $V_o$  as illustrated in Figs. 2(c,d). Signals  $\delta V_{ee}$  and  $\delta V_{oo}$  can be shown to be in phase quadrature to their main signals, thus representing pure phase modulations. This might have been expected when heating the guides. These signals do produce a variation  $\delta P_3$  of the output power, however the modulation due to these signals does not vary when  $z_o$  is moved along the guide because  $\delta V_{ee}$  and  $\delta V_{oo}$  are independent of the position  $z_o$  of the perturbation.

The  $z_o$ -dependent variation results from the cross-coupled signals  $\delta V_{eo}$  and  $\delta V_{oe}$ . As these signal components travel through a distance  $(L/2+z_o)$  in one of the supermodes and the remaining distance  $(L/2-z_o)$  in the other one, their phase delays at the output are  $\pm (\Delta \beta z_o)$  relative to  $\overline{\phi}$ . When  $z_o$  is varied, the modified output amplitudes  $V_{e,o}^{\odot}$  move in opposite senses around the circles indicated in Fig. 2(c). The sum of the 6 output signals is  $V_3^{\odot} = V_3 + \sum \delta V_{mn}$ . As indicated in Fig. 2(d) this  $V_3^{\odot}$  will generally trace out an ellipse

when  $z_o$  is changed, so that the output power  $P_3^{\oplus} = |V_3^{\oplus}|^2$  varies periodically. Here, as with the circles, one revolution corresponds to a variation of  $z_o$  through a distance  $\Lambda$  with  $\Delta\beta\Lambda = 2\pi$ . This variation is the basis of our measurement.

Some asymmetry of the directional coupler, introduced here formally by the parameters  $\sigma_1$  and  $\sigma_2$ , is necessary to obtain the variation  $\delta P_3$ . A detailed analysis shows that the semi-axes of the ellipse are proportional to  $\sin(\sigma_1 \pm \sigma_2)$ . In a symmetric coupler  $(\sigma_1 = \sigma_2)$ , therefore, the ellipse  $\delta V_3$  degenerates into a line, in phase quadrature to  $V_3$ , and  $\delta P_3 \to 0$ . Nevertheless, symmetric couplers can be measured if an asymmetry is introduced by feeding the input light simultaneously into ports 1 and 2 or by detecting a superposition of the outputs from ports 3 and 4.

Experimentally we demonstrate the method by measuring a number of directional couplers as shown in Fig. 1. Narrow single-mode stripe waveguides ( $w \approx 2.5 \,\mu\text{m}$ ) were fabricated in soda-lime glass by thermal K<sup>+</sup>-exchange. They were then 'buried' under a 0.3  $\mu$ m layer of Teflon AF, deposited by evaporation. Finally, to facilitate radiative heating of the transparent samples, a 1  $\mu$ m layer of black ink from a marker-pen was spin-coated on their surface.

The couplers were operated with light from a HeNe laser ( $\lambda=0.633\,\mu\mathrm{m}$ ), launched into port 1. Output light passed through a TM-oriented polarizer to a Si photodetector. Thermo-optic modulation was applied by irradiating the black layer with  $\approx 5\,\mathrm{mW}$  of 980 nm light, delivered from a semiconductor laser by a single-mode fibre to a point  $h\approx 20\,\mu\mathrm{m}$  above the surface. The resulting spot on the surface had  $\approx 10\,\mu\mathrm{m}$  diameter. To discriminate against fluctuations of the laser power, the heating light was square-wave modulated at 340 Hz. The modulation  $\delta P_3(z_o)$  was extracted from the detector signal using a lock-in amplifier.

The heated spot was scanned along the guide at a rate of a few  $\mu m/s$  giving a typical measured signal as shown in Figs. 3(a,b). From such data the beat length is evaluated by fitting a sinusoid. For the particular coupler in Fig. 3(b) we found  $\Lambda = (0.910 \pm 0.003) \,\mathrm{mm}$  giving a relative error of  $\delta \Lambda/\Lambda \approx 3 \cdot 10^{-3}$ . To illustrate the potential of this method, we show in Fig. 3(c) the beat lengths of 6 couplers fabricated from masks with different gaps g

between the guides. The observed exponential dependence  $\Lambda(g)$  is in agreement with theory<sup>9</sup> and may be viewed as a direct measurement of the decay rate of the evanescent fields.

Fig. 3(d) shows a scan of the modulated spot across the coupler, transverse to the direction of propagation. The S shape of this function reflects the coupling between an even and an odd mode. The separation  $\Delta y \approx 10 \,\mu\mathrm{m}$  of the two extrema characterizes the width of the heating beam rather than the gap between the guides.

In conclusion, we have demonstrated a method of measuring the beat lengths in directional couplers. It is simple, sensitive, nondestructive, and can yield good accuracy. It is especially attractive for buried guides which do not permit access to the evanescent field. As the signal  $\delta P_3(z)$  results from a 6-beam interference, the method requires some asymmetry, either in the coupler itself or in its connections at input or output. The method should be applicable to a wide variety of other directional couplers, including couplers with single or multiple phase reversals.

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## **FIGURES**

- Fig. 1. Experimental setup. Light from the laser diode heats a spot on one of the guides. LD laser diode, PD photodiode, POL linear polarizer.
- Fig. 2. (a) Directional coupler with modal amplitudes  $E_i$  and  $A_m$ ; (b) interpretation as an interferometer: (c)(d) phasor diagrams of output signals in the complex plane.
- Fig. 3. (a,b) Measured beat signals of directional couplers ((a):  $w=2.3 \,\mu m, g=1.5 \,\mu m$ ; (b):  $w=g=2.3 \,\mu m$ ; (c) measured dependence of beat length on gap width g; (d) modulation signal  $P_3(y_o)$  obtained by scanning the perturbation transversely across the coupler.

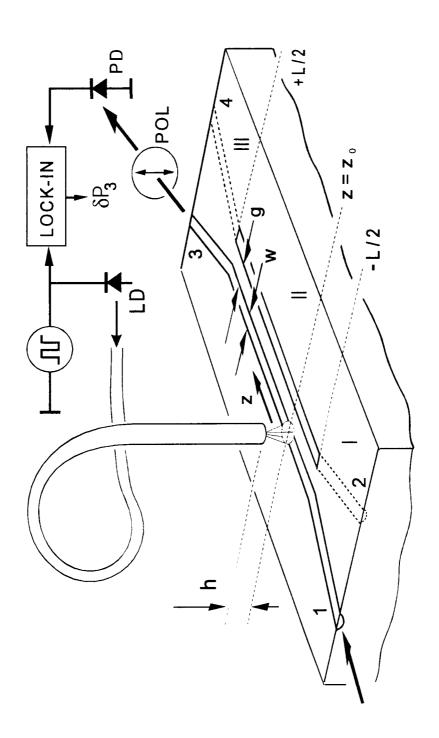


Fig. 1

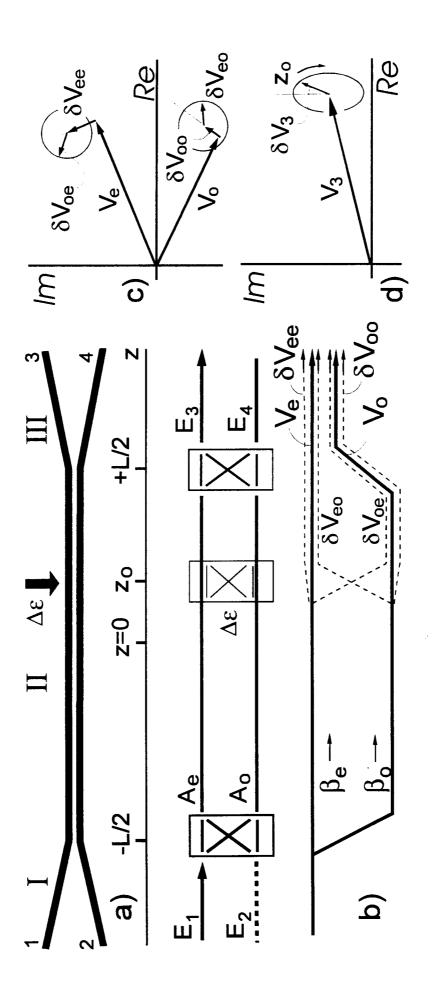


Fig. 2

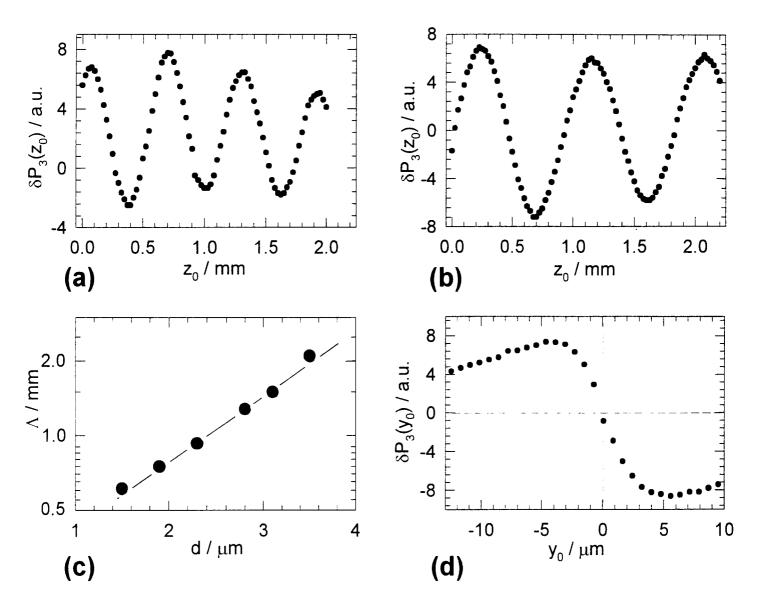


Fig. 3