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Eigenstates of Polarization of Elliptical Single-Mode Fibers

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Abstract—**The degree of polarization of radiation in an elliptical polarization-maintaining fiber and the polarization states of eigenpolarization modes of such fibers have been experimentally investigated in a wide spectral range. It has been shown that the elliptical fibers are uniform, homogeneously twisted fibers with elliptically polarized eigenpolarization modes. The ellipticity of the eigenpolarization mode is independent of fiber length and increases with wavelength, while the azimuth of the eigenpolarization mode is spectrally independent.**

I. Introduction

Exploitation of polarization-maintaining fibers (PMF) in coherent communication systems [1], in fiber-optic sensors and devices [2], [3], and in the investigations of different nonlinear processes [4], [5] requires information about the polarization states of the fiber eigenpolarization modes, also known as PMF polarization eigenstates, or eigenpolarizations [6]–[13].

It is known that in ideal polarization-maintaining fiber the eigenpolarization modes are linearly polarized along the symmetry axes of the PMF cross-section [14]. However, in practice, the azimuthally nonsymmetric perturbations of the fiber such as twist, bending, transverse pressure, and electric and magnetic fields perturb the dielectric constant tensor of ideanl PMF and change the polarization states of its eigenpolarization modes. These effects can arise either during the fiber manufacturing process or as a consequence of the action of external forces. Combined effects of various perturbations lead to elliptial polarization of the eigenpolarization modes in polarization-maintaining fiber. The polarization states of the eigenpolarization modes depend not only on the type of perturbation, but also on its distribution along the fiber. At this point there are two types of PMF; uniform and nonuniform polarization-maintaining fibers. In uniform PMF the different perturbations distribute regularly along the fiber, and the eigenpolarization modes of such fibers are independent of propagation length [6]–[10], [13]. In nonuniform polarization-maintaining fibers there are either the discrete mode-coupling centers [12] or the perturbations distributed randomly along the fiber [11], [13]. The eigenpolarization modes of nonuniform PMF have a local nature and are different at different lengths of the same fiber.

In this paper, we have experimentally investigated the polarization eigenstates of single-mode fibers with elliptical stress-induced and outer claddings, which have been made using the preform deformation technique [15]. Recently, their polarization characteristics have been examined, both theoretically and experimentally [16]–[20], while study of polarization eigenstates in such fibers in a wide spectral range has not yet been done.

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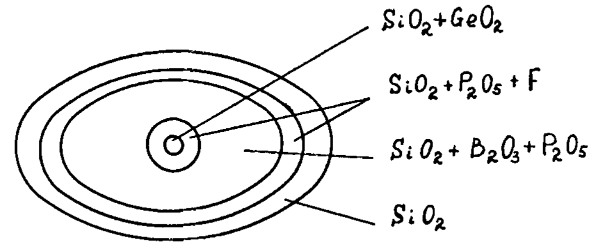

Fig. 1. Fiber cross sections.

II. Fiber Structure

The cross section of the test elliptical fibers consists of five regions: a circular core (SiO_2 + GeO_2), a circular buffer cladding (SiO_2 + P_2O_5 (1.5 mol.%) + F (1 mol.%)), an elliptical stress-induced cladding (SiO_2 + B_2O_3 (17 mol.%) + P_2O_5 (4 mol.%)), an additional elliptical cladding (SiO_2 + P_2O_5 (1.5 mol.%) + F (1 mol.%)) and an elliptical outer silica cladding (Fig. 1). The circular buffer cladding reduces excess loss related to an intrinsic absorption of B_2O_3 in stress-induced cladding, and the elliptical claddings cause thermal stress in the core [18]. In all test fibers the refractive indexes of the claddings are equal to the refractive index of SiO_2.

Parameters of test fibers are listed in Table I, where λ_c is the cutoff wavelength, Δn is difference between refractive indexes of the core and claddings, $2a_c$ is a diameter of the core, $\varepsilon_s = a - b/a + b$ is stress induced cladding ellipicity (a and b are the semimajor and semiminor geometrical axes, respectively, of the elliptical stress-induced cladding, $\varepsilon_o = A_x - A_y/A_x + A_y$ is an outer cladding ellipticity (A_x and A_y are semimajor and semiminor geometrical axes, respectively, of the elliptical outer cladding), and γ is a buffer cladding-core radii ratio. The studies of polarization characteristics of these fibers have been published recently [16], [18].

III. Experimental Apparatus and Procedure

The experimental setup is shown in Fig. 2. The radiation of a YAG laser 1 excites the Raman scattering in an auxiliary fiber 2 in 1.06–1.8-μm wavelength range. The

TABLE I
TEST FIBER PARAMETERS

Fibers Parameters	Fibers 1	2	3
λ_c (μm)	1.05	1.05	1.09
$\Delta n\ 10^3$	8.13	7.89	8.4
$2a_c$ (μm)	8.0	8.0	8.1
ε_S	0.45	0.41	0.4
ε_O	0.24	0.22	0.22
γ	3.43	3.91	3.54

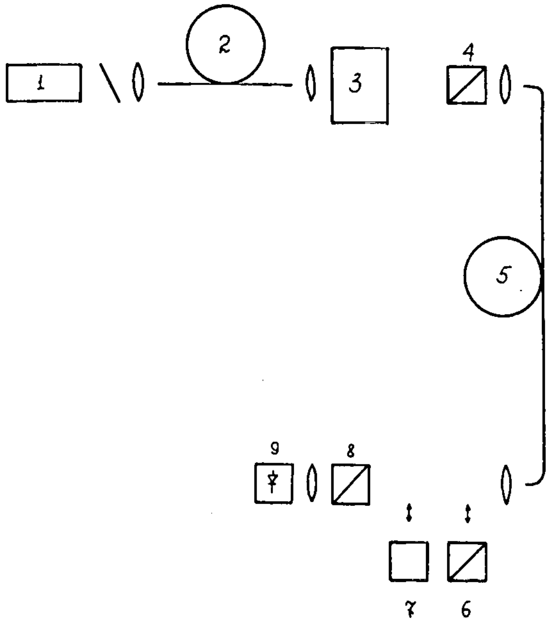

Fig. 2. Experimental setup.

output depolarized radiation passes through a monochromator 3 and Glan-Thomson polarizer 4 after which we have linearly polarized light, which is launched into the test fiber 5. To avoid undesirable external influences the elliptical fibers were accurately wound on the 10 cm radius drum. The Glan-Thompson polarizers 6 and 8 and phase plate 7 are used for the measurement of Stokes parameters of output light according to the technique described in [22]. All polarizers were adjusted in the same manner so that the aximuths of their transmission axes are equal to the angle between transmission axis and the positive direction of the OX axis of the laboratory reference frame.

The investigation of eigenstates of polarization is carred out by measurements of the state and degree of polarization of the partially polarized light at the fiber output.

It is known [21] that for complete specification of partially polarized light we have to determine total intensity I, the degree of polarization P, and characteristics of the totally polarized component of light—an azimuth φ, which is the angle between the major axes of polarization ellipse and the positive direction of the OX axis of the laboratory reference frame, and an ellipticity angle

$$\psi = \operatorname{arctg} \frac{b_p}{a_p} \tag{1}$$

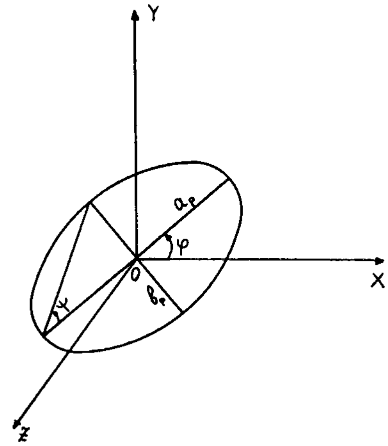

Fig. 3. Totally polarized component of light at elliptical fiber output.

where a_p and b_p are the semimajor and semiminor axes, respectively, of polarization ellipse of the totally polarized component (Fig. 3). In experiment the orthogonal, right-handed, Cartesian frame XYZ is chosen as a laboratory reference frame, in which light travels along the OZ axis in the direction of increasing z, the OX axis corresponds to horizontal direction, and the OY axis corresponds to the vertical one. Thus all required parameters (I, P, φ, and ψ) might be determined from the measurements of Stokes parameters of outgoing light. In terms of the Cartesian components of the transverse electric field of the light, the four Stokes parameters are defined as follows [21].

$$S_0 = \langle |E_x(t)|^2\rangle + \langle |E_y(t)|^2\rangle \tag{2}$$
$$S_1 = \langle E_x(t)|^2\rangle - \langle |E_y(t)|^2\rangle \tag{3}$$
$$S_2 = 2\langle \mathrm{Re}\,[E_x(t)E_y^*(t)]\rangle \tag{4}$$
$$S_3 = 2\langle \mathrm{Im}\,[E_x(t)E_y^*(t)]\rangle \tag{5}$$

where $E_x(t)$ and $E_y(t)$ represent the linear, simple-harmonic oscillations of the electric-field components along the OX and OY axes, * indicates complex conjugation, and $\langle\,\rangle$ means time average. Stokes parameters have the simple physical meaning. S_0 is the total intensity of light. S_1 is the excess of linear polarization along the OX axis over the linear polarization along the OY axis. S_2 is the excess of linear polarization at $+45°$ with respect to the OX axis over the linear polarization at $-45°$ with respect to the OX axis. S_3 is the excess of right circular polarization over left circular polarization. In experiments, therefore, all information about partially polarized light at elliptical fiber output may be extracted from the measurable Stokes parameters [21].

$$I = S_0 \tag{6}$$
$$P = \frac{(S_1^2 + S_2^2 + S_3^2)^{1/2}}{S_0} \tag{7}$$
$$\varphi = \frac{1}{2}\operatorname{arctg}\left(\frac{S_2}{S_1}\right) \tag{8}$$
$$\psi = \frac{1}{2}\arcsin\left(\frac{S_3}{(S_1^2 + S_2^2 + S_3^2)^{1/2}}\right). \tag{9}$$

IV. Degree of Polarization of Radiation Propagating Along the Elliptical PMF

We start our study from measurements of degree of polarization of light at the elliptical fiber output against fiber length z and the wavelength λ. These dependencies allow us to make conclusions about distribution of different types of perturbations along the fiber. The measurement of degree of polarization P has been done in the 1.2–1.6-μm wavelength range. The fiber was excited by linearly polarized quasi-monochromatic light with nearly rectangular-shape spectrum. The spectral width of incident light was adjusted by output slits of the monochromator, so the coherence time of incident light τ_c was a constant 6 ps throughout the experiments.

The degree of polarization at fiber output was determined from measurements of the Stokes parameters of the output light and the experimental values have been substituted into (7).

Dependencies $P(z)$ and $P(\lambda)$ were investigated for the different azimuths φ_i of transmission axis of the incident polarizer 4 in Fig. 2. For this purpose the slow axis of the fiber coincides with the OX axis and the fast coincides with the OY axis of the laboratory reference frame. Thus the azimuth φ_i is equal to the angle between the slow axis of PMF and the direction of oscillation of the electric-field vector of incident linearly polarized light. Note that the term "principal axis" is usually used when the eigenpolarization mode of PMF is linearly polarized. If the eigenpolarization mode is elliptically polarized, the axis is the major axis of the polarization ellipse of eigenpolarization mode.

Since it is known that for the elliptical fibers the principal axes do not coincide with the geometric axes of fiber cross-sections [19], the principal axes of investigated PMF were aligned to the laboratory reference frame in the following way. First, the major and minor geometric axes of the fiber input surface were roughly aligned to the horizontal and vertical lines, respectively, with a microscope. Then, at the aximuth of the transmission axis of the incident polarizer 4 $\varphi_i = 45°$ we twisted the input surface in the fiber cross-section plane with the step equals 0.3°. measuring the degree of polarization of the outgoing light after each twist. With the minimal degree of polarization at the output, both uniform [10], [13] and nonuniform [11], [12] PMF, corresponds to equal excitation of eigenpolarization modes at fiber input, i.e., the principal axes of PMF are aligned to the laboratory reference frame.

Since dependencies $P(z)$ and $P(\lambda)$ are analogous for all investigated elliptical fibers, structural parameters of which are listed in Table I, the measurement results are represented only for fiber 3.

Fig. 4 shows the dependence of the degree of polarization of the outgoing light against length z of the elliptical fiber 3. The fiber was excited by quasi-monochromatic linearly polarized light with central wavelength $\lambda = 1.3\,\mu$m and azimuths $0°, 15°, 30°$, and $45°$.

One can see that the degree of polarization depends on the incident polarization state and coherence time. For each fixed φ_i the degree of polarization decreases, monotonically to a certain stationary level. The reason for this effect is

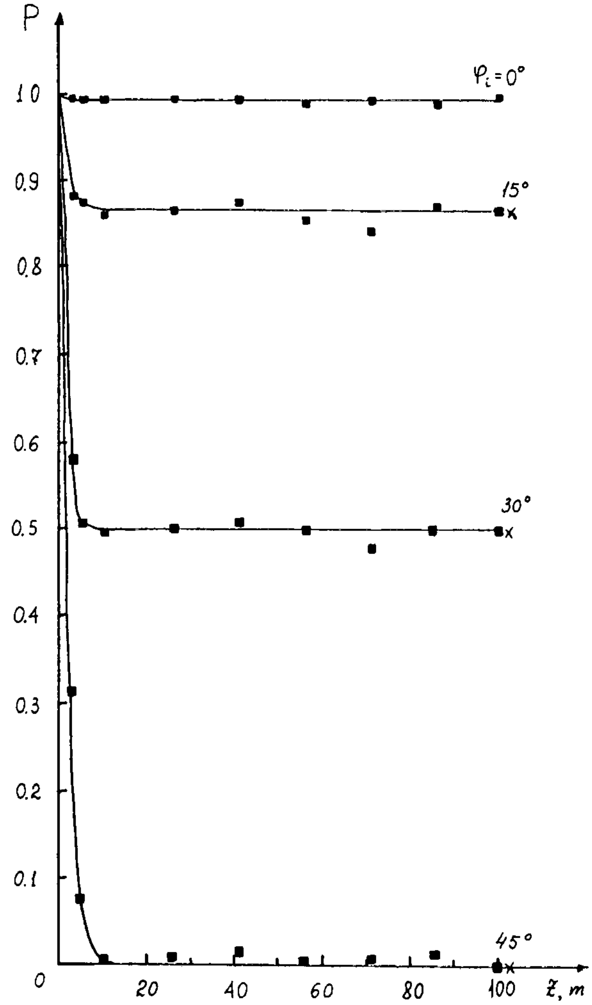

Fig. 4. Dependencies of degree of polarization of light at output of elliptical fiber 3 on fiber length of $\lambda = 1.3\,\mu$m.

a gradual loss of the mutual coherency of eigenpolarization modes. Polarization dispersion of fiber 3 is $\tau_p = 0.66$ ns/km at $\lambda = 1.3\,\mu$m and a delay $\Delta\tau = \tau_p z$ between eigenpolarization modes becomes equal to coherency time of light at fiber length of $z_d = \tau_c/\tau_p \simeq 9$ m. z_d is depolarization length.

It is clear from Fig. 4 that when the fiber length z becomes greater than z_d, i.e., $\Delta\tau > \tau_c$, the eigenpolarization modes of investigated PMF become mutually incoherent and the degree of polarization is constant and determined only by incident condition.

Obviously, the closer the incident state of polarization to the eigenpolarization state, the higher degree of polarization P. Therefore, since the principal axes of investigated PMF are adjusted along the azimuths 0° and 90°, respectively, the higher value of degree of polarization corresponds to $\varphi_i = 0°$ and the lower one corresponds to $\varphi_i = 45°$.

Behavior of degree of polarization is similar at another wavelength λ. Figs. 5 and 6 show the spectral dependencies of P for fiber 3, when fiber length is equal to 3.15 m and 0.86 m, respectively. It is seen from Fig. 5 that at $z = 3.15$ m the degree of polarization differs at different wavelengths and the shape of the spectral dependence of P is the same for different azimuths of incident linearly polarized light.

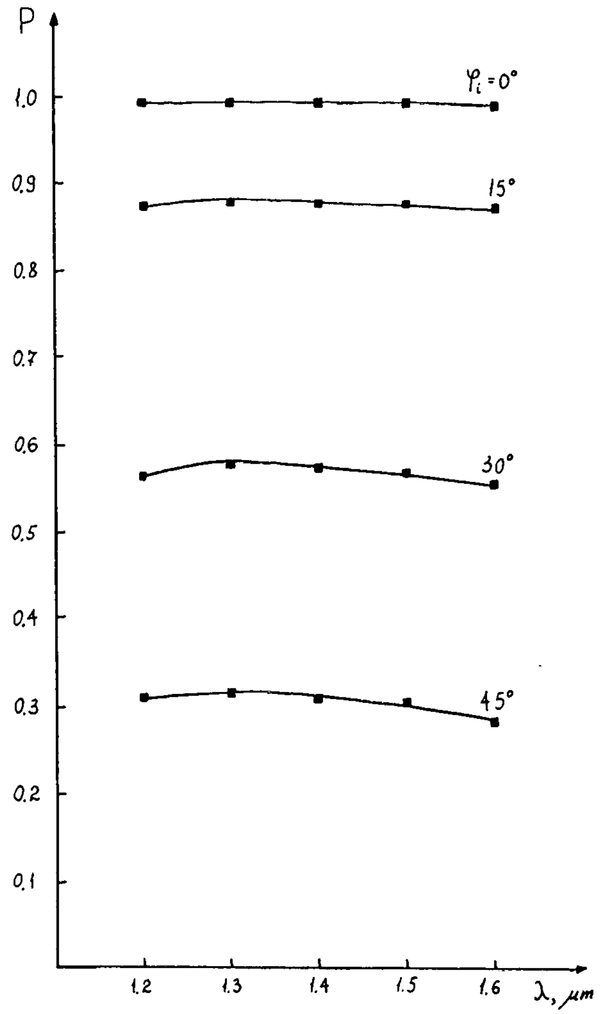

Fig. 5. Spectral dependencies of degree of polarization of light at output of fiber 3 having length $z = 3.15$ m.

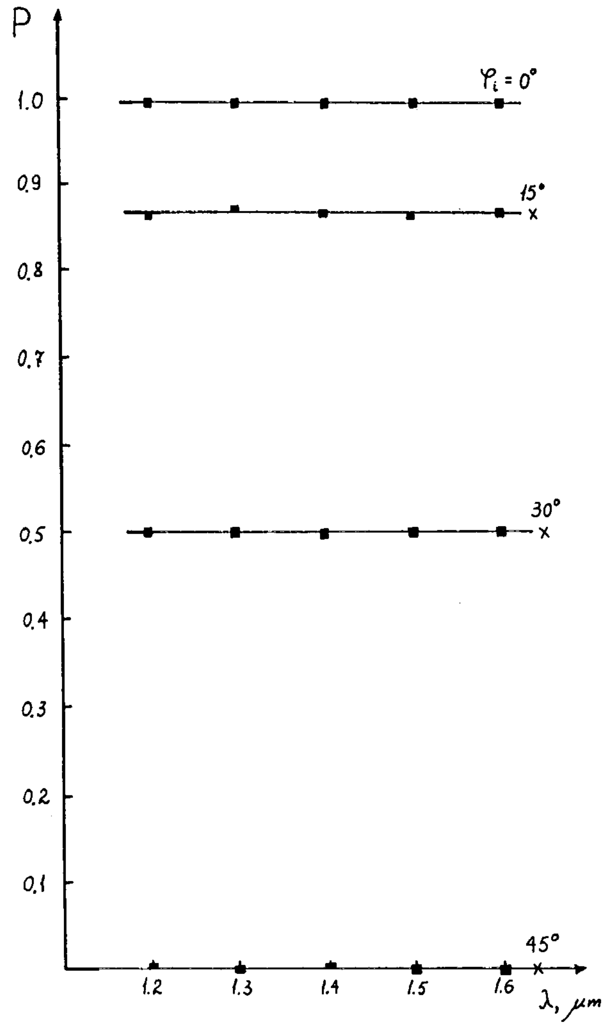

Fig. 6. Spectral dependencies of degree of polarization of light at output of fiber 3 having length $z = 0.86$ m.

So at any φ_i, the degree of polarization increases with a wavelength of $\lambda = 1.3\,\mu$m and monotonically decreases beyond this wavelength. The main reason is the behavior of polarization dispersion: It decreases monotonically with a wavelength in the range of 1.2–1.3 μm and increases in the range of 1.3–1.6 μm (Fig. 7). Therefore, the degree of polarization at $\lambda = 1.3\,\mu$m is greater than at $\lambda = 1.2\,\mu$m and $\lambda = 1.6\,\mu$m. Note also that as one can see from Fig. 6, at the length of 0.86 m the eigenpolarization modes are incoherent at fiber output in the whole spectral range ($P = 0$ at $\varphi_i = 45°$) and P does not depend on wavelength and is determined only by the state of polarization of the incident light.

The dependencies shown in Figs. 4–6 allow us to conclude that the tested fibers are uniform ones, since it is known [11] that in nonuniform polarization-maintaining fibers, eigenpolarization modes are always mutually coherent and the depolarization length of such fibers z_d is determined not only by coherent properties of light, but also by the polarization-holding parameter. Therefore, complete depolarization of light at $\varphi_i = 45°$ and $z_d = \tau_c/\tau_p$ is evidence of uniformity of the test fiber.

For confirmation of this conclusion we have calculated the degree of polarization at $z > z_d$ using the expression that has been derived for uniform polarization-maintaining fibers [10], [13]:

$$P = \left[1 - 4(1 - |f|^2)\frac{|a_1|^2|a_2|^2}{(|a_1|^2 + |a_2|^2)^2}\right]^{1/2} \tag{10}$$

where f is a mutual correlation function between two eigenpolarization modes, and $|a_1|$ and $|a_2|$ are the amplitudes of the eigenpolarization modes at a fiber input.

Since investigated PMF is excited by linearly polarized light, we can write

$$a_1 = I_i^{1/2} \cos\varphi_i \tag{11}$$

$$a_2 = I_i^{1/2} \sin\varphi_i \tag{12}$$

where I_i is the total intensity of incident light. For fiber length greater than the depolarization length, eigenpolarization modes are incoherent, and consequently $f = 0$. Then

$$P(z > z_d) = \cos 2\varphi_i. \tag{13}$$

The calculated values at $\varphi_i = 15°, 30°$, and 45° are presented in Figs. 4 and 6 by the x's. The calculated and

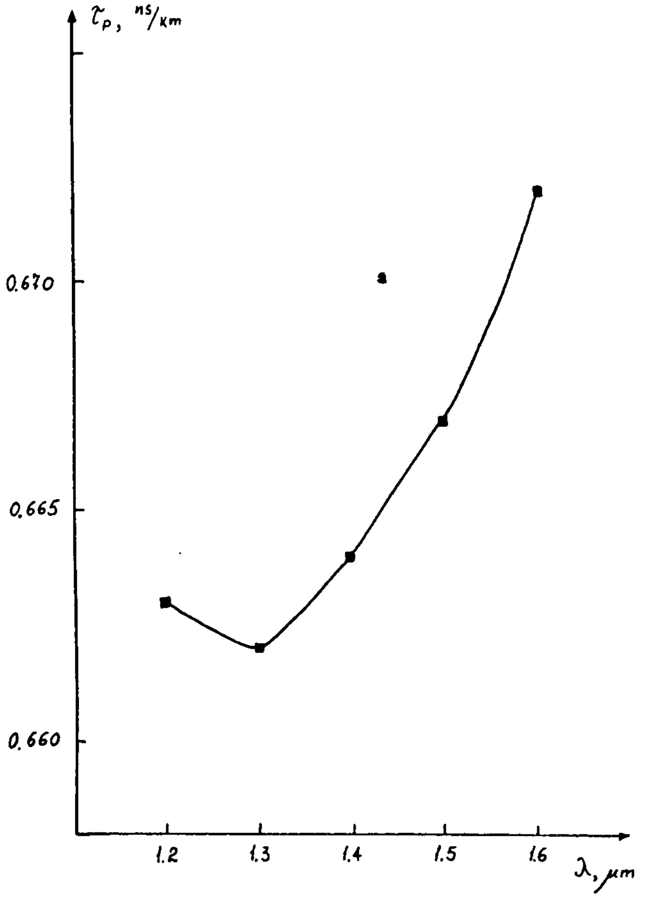

Fig. 7. Spectral dependence of polarization dispersion of fiber 3.

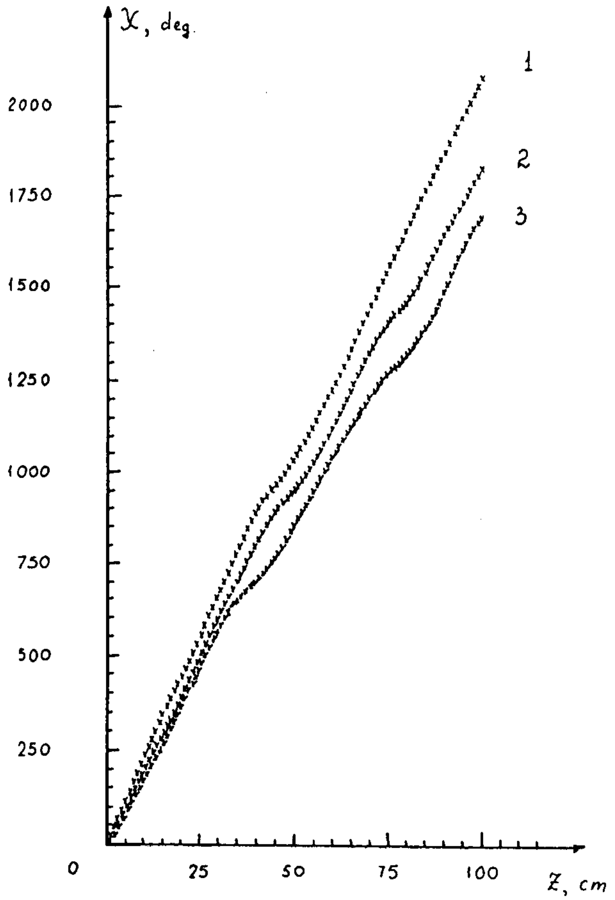

Fig. 8. Dependencies of rotation angle of geometrical axes of the cross-section ellipse of elliptical fibers 1, 2, and 3 versus fiber length.

measured values are in good agreement with each other in all spectral range, which is the proof of the uniformity of investigated fiber.

At the same time, as one can see from $P(z)$ and $P(\lambda)$ dependencies at $\varphi_i = 0°$, light propagating along the investigated uniform elliptical fiber 3 is partially depolarized, when PMF is excited by the linearly polarized light, the direction of oscillation of the electric-field vector of which coincides with the principal axis of the fiber. Consequently, the eigenpolarization modes of this fiber are elliptically polarized. It is known [8], [9] that the eigenpolarization modes of PMF are elliptically polarized, when either the longitudinal magnetic field or the fiber twist act upon the fiber. Since the investigated fibers have been accurately wound on the spool to avoid external influences, and the presence of magnetic field is excluded, the single reason resulting in an elliptical state of polarization might be an internal rotation of the principal axes of PMF occurring during the drawing process. Keeping in the mind uniformity of the investigated fiber, we can anticipate that internal rotation of the principal axes is distributed regularly along the PMF.

The existence of the rotation has been confirmed by the results of measurement of rotation angle χ of geometric axes of the cross-section ellipse of the elliptical fibers 1, 2, and 3 versus fiber length z, which is shown in Fig. 8.

The measurements were carried out with the backscattering technique [23]. A He-Ne laser was used as a light source. It can be seen from Fig. 8 that the internal rotation of principal axes of investigated fibers is distributed nearly regularly along the PMF, with the average rate of rotation of 21.6°/cm for fiber 1, 19.1°/cm for fiber 2 and 17.46°/cm for fiber 3. The exceptions are the small pieces of about $\simeq$5 cm, which appeared at intervals of $\simeq$35 cm, where the rates of rotation slightly decreased. A possible reason for that is an additional periodic mechanical perturbation during the drawing process.

Thus the analysis of $P(z)$ and $P(\lambda)$ dependencies at $\varphi_i = 0°, 15°, 30°$, and 45° shows that the investigated elliptical fibers are the uniform regularly twisted PMF.

V. Methods of Determination of the Eigenstates of Polarization

The uniformity of the investigated elliptical fibers allows us to propose two simple methods to find states of polarization of eigenpolarization modes of these fibers.

A. First Method

The Fourier's components of the transverse part of the electrical field in real polarization-maintaining fiber $\vec{E}(z,\omega)$ can be expanded in terms of the eigenpolarization modes of some ideal fiber $\vec{e}_x$ and $\vec{e}_y$, which are linearly polarized along the cross-section symmetry axes OX and OY, respectively, of an ideal fiber

$$E(z,\omega) = [E_x(z,\omega)\vec{e}_x + E_y(z,\omega)\vec{e}_y]e^{i\omega t}. \quad (14)$$

The expansion coefficients $E_x(z,\omega)$ and $E_y(z,\omega)$ satisfy the simple coupled-mode equations [13]

$$\frac{dE(z,\omega)}{dz} = TE(z,\omega) \quad (15)$$

where

$$E(z,\omega) = \begin{vmatrix} E_x(z,\omega) \\ E_y(z,\omega) \end{vmatrix} \quad (16)$$

$$T = \begin{vmatrix} -i\beta_{xi} & h \\ -h^* & -i\beta_{yi} \end{vmatrix}. \quad (17)$$

Here β_{xi} and β_{yi} are the propagation constants of eigenpolarization modes of an ideal fiber (we assume that the fiber has no attenuation, and then β_{xi} and β_{yi} are the real), h is the coupling coefficient

$$h = |h|e^{i\sigma}. \quad (18)$$

In uniform fiber the parameters $\beta_{xi}.\beta_{yi},|h|$, and σ are independent of the fiber length z, and then from (17) we can find out propagation constants of polarization-maintaining fiber

$$\beta_{1,2} = \beta \pm m \quad (19)$$

and corresponding to them orthogonal eigenpolarization modes of real fiber

$$\vec{E}_{1,2} = \frac{1}{\sqrt{2}}\begin{vmatrix} (1 \mp n)^{1/2} \\ e^{-i(\sigma \pm \pi/2)} \quad (1 \pm n)^{1/2} \end{vmatrix}. \quad (20)$$

Here upper and lower signs correspond to $\vec{E}_1$ and $\vec{E}_2$, respectively, and

$$m = [|h|^2 + (\Delta\beta_i/2)^2]^{1/2} \quad (21)$$

$$\beta = (\beta_{xi} + \beta_{yi})/2 \quad (22)$$

$$\Delta\beta_i = \beta_{xi} - \beta_{yi} \quad (23)$$

$$n = \frac{\Delta\beta_i}{2m}. \quad (24)$$

It follows from (19) that the difference between propagation constants of the eigenpolarization modes of real fiber is

$$\Delta\beta = \beta_1 - \beta_2 = 2m. \quad (25)$$

By substitution of (20) into (3)–(5) one can find Stokes parameters of eigenpolarization modes of the real fiber

$$S_{1e} = \mp n \quad (26)$$

$$S_{2e} = \pm(1-n^2)^{1/2}\sin\sigma \quad (27)$$

$$S_{3e} = \mp(1-n^2)^{1/2}\cos\sigma. \quad (28)$$

The azimuth φ_e and ellipticity angle ψ_e of eigenpolarization modes according to (8) and (9) are equal to

$$\varphi_e = \frac{1}{2}\,\mathrm{arctg}\left[-\frac{(1-n^2)^{1/2}}{n}\sin\sigma\right] + \begin{cases} 0 & \text{for } \vec{E}_1 \\ \pi/2 & \text{for } \vec{E}_2 \end{cases} \quad (29)$$

$$\psi_e = \frac{1}{2}\arcsin[\mp(1-n^2)^{1/2}\cos\sigma]. \quad (30)$$

Consequently, in order to find states of polarization of eigenpolarization modes one needs to know n and σ. As one can conclude from (18), and (21)–(24), parameters n and σ belong to an ideal fiber and therefore cannot be measured directly. We show how the parameters n and σ can be determined from results of measurement of Stokes parameters at the output of the real uniform PMF.

It is known [13] that in the case of fully polarized incident light the normalized Stokes parameters at the fiber output are

$$\frac{S_1(z)}{S_0(z)} = -nA + (1-n^2)^{1/2}B \cdot \cos(\Delta\beta z) + (1-n^2)^{1/2}C\sin(\Delta\beta z) \quad (31)$$

$$\frac{S_2(z)}{S_0(z)} = (1-n^2)^{1/2}A\sin\sigma + (nB\sin\sigma + C\cos\sigma)\cos(\Delta\beta z) + (nC\sin\sigma - B\cos\sigma)\sin(\Delta\beta z) \quad (32)$$

$$\frac{S_3(z)}{S_0(z)} = -(1-n^2)^{1/2}A\cos\sigma - (nB\cos\sigma - C\sin\sigma)\cos(\Delta\beta z) - (nC\cos\sigma + B\sin\sigma)\sin(\Delta\beta z) \quad (33)$$

Here we denote:

$$A = -\left(\boldsymbol{w}, \frac{\vec{S}(O)}{S_O(O)}\right) \quad (34)$$

$$B = \mathrm{Re}\,M \quad (35)$$

$$C = -\mathrm{Im}\,M \quad (36)$$

$$M = \left(\vec{\boldsymbol{u}} - i\vec{\boldsymbol{v}}, \frac{\vec{S}(O)}{S_O(O)}f\right) \quad (37)$$

where $\vec{S}(O)$ is the Stokes vector of incident light, $\vec{\boldsymbol{w}}, \vec{\boldsymbol{u}}$, and $\vec{\boldsymbol{v}}$ are the unite mutually orthogonal vectors in Stokes space:

$$\vec{\boldsymbol{w}} = (n, -(1-n^2)^{1/2}\sin\sigma, (1-n^2)^{1/2}\cos\sigma) \quad (38)$$

$$\vec{\boldsymbol{u}} = ((1-n^2)^{1/2}, n\sin\sigma, -n\cos\sigma) \quad (39)$$

$$\vec{\boldsymbol{v}} = (O, \cos\sigma, \sin\sigma), \quad (40)$$

and

$$f = \frac{\int |v(\omega)|^2 e^{i\omega\tau} p^z \, d\omega}{\int |v(\omega)|^2 \, d\omega} \quad (41)$$

is the mutual correlation function of two eigenpolarization modes, and ($v(\omega)$ is an amplitude spectrum of incident light).

Here we suppose that the principal axes at fiber input and output coincide.

According to (7) and (31)–(33) the degree of polarization at fiber output is equal to

$$P = (A^2 + |M|^2)^{1/2}. \quad (42)$$

The electric field in a real fiber can be expanded in terms of its eigenstates of polarization as

$$\vec{E}(z) = a_1\vec{E}_1 e^{-i\beta_1 z} + a_2\vec{E}_2 e^{-i\beta_2 z}. \tag{43}$$

Then from (20) and (34)–(40) we can find

$$A = \frac{|a_1|^2 - |a_2|^2}{|a_1|^2 + |a_2|^2} \tag{44}$$

$$B = \frac{a_1 a_2^* + a_1^* a_2}{|a_1|^2 + |a_2|^2} f \tag{45}$$

$$C = i\frac{a_1 a_2^* - a_1^* a_2}{|a_1|^2 + |a_2|^2} f \tag{46}$$

We assume that a test real fiber is excited by the quasi-monochromatic linearly polarized light, the angle between the electric-field vector of which and the slow principal axis of PMF is equal to φ_i, and amplitude spectrum $v(\omega)$ has either rectangular, Gaussian, or Lorentzian shape, which means that the mutual correlation function between two eigenpolarization modes is real function [10]. The expansion coefficients a_1 and a_2 are determined by (11) and (12), and according to (10) we have

$$f = \left[\frac{P^2 - \cos^2 2\varphi_i}{\sin^2 2\varphi_i}\right]^{1/2},$$

or

$$f = \frac{\left[\left(\frac{S_1(z)}{S_0(z)}\right)^2 + \left(\frac{S_2(z)}{S_0(z)}\right)^2 + \left(\frac{S_3(z)}{S_0(z)}\right)^2 - \cos^2 2\varphi_i\right]^{1/2}}{\sin 2\varphi_i} \tag{47}$$

By substituting (11) and (12) to (44)–(46), we find

$$A = \cos 2\varphi_i \tag{48}$$

$$B = \sin 2\varphi_i f \tag{49}$$

$$C = 0 \tag{50}$$

and, respectively, from (31)–(33)

$$\frac{S_1(z)}{S_0(z)} = -n\cos 2\varphi_i + (1-n^2)^{1/2} \cdot \cos(\Delta\beta z) f \sin 2\varphi_i \tag{51}$$

$$\frac{S_2(z)}{S_0(z)} = (1-n^2)^{1/2}\sin\sigma\cos 2\varphi_i + [n\sin\sigma\cos(\Delta\beta z) - \cos\sigma\sin(\Delta\beta z)] f \sin 2\varphi_i \tag{52}$$

$$\frac{S_3(z)}{S_0(z)} = -(1-n^2)^{1/2}\cos\sigma\cos 2\varphi_i - [n\cos\sigma\cos(\Delta\beta z) + \sin\sigma\sin(\Delta\beta z)] f \sin 2\varphi_i. \tag{53}$$

Parameter n can be directly determined from (51)

$$n = -\frac{\frac{S_1(z)}{S_0(z)}\cos 2\varphi_i}{\cos^2 2\varphi_i + G^2} + \frac{G\left[\cos^2 2\varphi_i + G^2 - \left(\frac{S_1(z)}{S_0(z)}\right)^2\right]^{1/2}}{\cos^2 2\varphi_i + G^2} \tag{54}$$

where

$$G = \sin 2\varphi_i \cos(\Delta\beta z) f. \tag{55}$$

And finally, from (52) and (53) we obtain

$$\cos\sigma = -\frac{F\frac{S_2(z)}{S_0(z)} + E\frac{S_3(z)}{S_0(z)}}{E^2 + F^2} \tag{56}$$

$$\sin\sigma = \frac{E\frac{S_2(z)}{S_0(z)} - F\frac{S_3(z)}{S_0(z)}}{E^2 + F^2} \tag{57}$$

where

$$E = (1-n^2)^{1/2}\cos 2\varphi_i + nG \tag{58}$$

$$F = \sin 2\varphi_i \sin(\Delta\beta z) f. \tag{59}$$

Note that, for determination of the states of polarization of eigenpolarization modes of fiber having length $z > z_d$, an angle φ_i should not equal $+45°$, since in this case the full depolarization of light ocurrs at the fiber output and information about the state of polarization of eigenpolarization modes will be lost.

B. Second Method

The first method of measurement of the states of polarization is suitable for a fiber of arbitrary length. However, for uniform PMF of sufficiently long length the eigenstates of polarization of PMF can be determined more simply.

As one can conclude from results of the investigation of degree of polarization, when light propagates along a fiber, mutual coherency of eigenpolarization modes decreases, and for fiber length longer than depolarization length these modes become incoherent. Light at $z > z_d$ is partially polarized (except in the case of equal excitation of both eigenpolarization modes), and the fully polarized component of light is one eigenpolarization mode of given uniform fiber.

Indeed, at $z > z_d$ the mutual correlation function $f = 0$: consequently, according to (35)–(37) M = B = C = 0 and taking into account (26)–(28) and (42) we find

$$\frac{S_1(z)}{S_0(z)} = -nA = S_{1e}P \tag{60}$$

$$\frac{S_2(z)}{S_0(z)} = -(1-n^2)^{1/2}\sin\sigma A = S_{2e}P \tag{61}$$

$$\frac{S_3(z)}{S_0(z)} = -(1-n^2)^{1/2}\cos\sigma A = S_{3e}P. \tag{62}$$

Thus, to determine the states of polarization of eigenpolarization modes in a uniform PMF at $z > z_d$, we have to measure Stokes parameters of output light of such fiber and, with the help of (8) and (9), to find the azimuth and ellipicity angle of polarization modes. For this the fiber may be excited by light having an arbitrary amplitude spectrum and an arbitrary state of polarization (except in the case of equal excitation of both eigenpolarization modes).

VI. Eigenstates of Polarization of Radiation in Elliptical Fibers

Eigenstates of polarization of radiation in elliptical fibers were investigated in the installation shown in Fig. 2. It has been noted that an amplitude spectrum of incident light was formed by monochromator and had nearly rectangular shape. This allows us to determine the eigenstates of polarization of the elliptical PMF at length $z > z_d$ by the first method described in the previous section. Coherence time of incident light has remained a constant 6 ps at all wavelengths.

The same orientation of the principal axes at fiber input and output was carried out in that way. First, for sufficiently long fiber ($z > z_d$) the slow and fast principal axes of the fiber input were aligned to the OX and OY axes, respectively, of the laboratory reference frame, as shown in Sec. IV. Then, if this uniform fiber were excited by the linearly polarized light with azimuth in the range $0 < \varphi_i < 45°$, the totally polarized component of outgoing radiation would be the slow eigenpolarization mode. Consequently, to adjust the slow and fast principal axes at the fiber output along OX and OY axes, respectively, it was necessary to twist the output surface of the PMF in the fiber cross-section plane to achieve such a position of fiber output that Stokes parameters $S_2 = 0$ and $S_1 = 0$. As a result, the principal axes of the fiber at length $z > z_d$ coincided with the OX and OY axes of the laboratory reference time at the input and output of the fiber.

For determination of eigenstates of polarization of fibers at length $z < z_d$, first input and output principal axes of sufficient long fiber were oriented the same way, and then without changing the position of the output principal axes necessary length of fiber was cut off on the fiber input end. Finally, the input principal axes of short PMF were aligned to the laboratory reference frame by minimizing the degree of polarization of outgoing radiation at $\varphi_i = 45°$.

To verify proposed methods the azimuth φ_e and ellipticity angle ψ_e of the eigenpolarization modes of the fiber 3 ($z = 0.86$ m) were measured at different azimuths of incident linearly polarized light and at five different wavelengths λ (Fig. 9). The values of φ_e and ψ_e are determined by the first method, where $5° < \varphi_i < 20°$, and by the second method where $25° < \varphi_i < 40°$.

The presented data show that both methods give the same values of φ_e and ψ_e at λ = const. The Azimuth and the ellipicity angle of the eigenpolarization mode do not change when the azimuth of incident linearly polarized light changes at each fixed value of λ. This result confirms the correctness of measurements and is evidence that the angles φ_e and ψ_e shown in Fig. 9 are really the parameters of the eigenpolarization mode.

An ellipticity angle of the eigenpolarization mode of fiber 3 increases monotonically with wavelength so the azimuth is the same for different wavelengths (within the experimental error of 0.3°). Consequently, the orientation of the principal axes of investigated fiber does not depend on wavelength.

Fig. 10 shows the spectral dependencies of ellipticity angle ψ_e of the eigenpolarization states of fibers 1, 2, and 3. The measurements are carried out at two lengths of each fiber, 100 m and $\simeq$3 m. Three typical features of eigenpolarization modes of the investigated PMF can be seen from the presented curves.

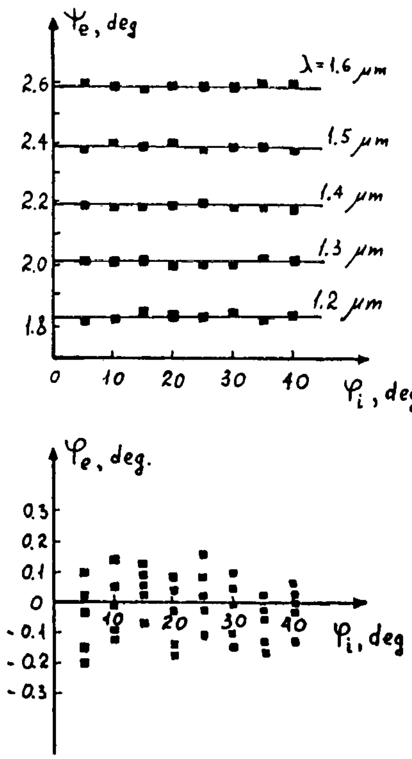

Fig. 9. Dependence of polarization states of eigenpolarization modes of fibers 1, 2, and 3 on azimuth of incident linearly polarized light.

First, the eigenpolarization modes are elliptically polarized, that is, they conformed to the results of investigation of degree of polarization and are the consequence of the internal rotation of the principal axes of these elliptical fibers.

Second, the uniformity of investigated fibers and regular twist along them result in a constant ellipticity angle ψ_e, i.e., the eigenpolarization modes are the same at any cross-section of fiber.

Third, the ellipticity angles of all three investigated PMF increase monotonically with wavelength. The main reasons are the circular birefringence caused by the spectrally independent fiber twist [7] and the linear birefringence montonically decreasing with wavelength in fibers with elliptical stress-induced cladding [16].

The calculation of the ellipticity angle of investigated fibers confirms these conclusions. It is known [8] that for uniform, homogeneously twisted fibers with elliptical stress-induced cladding, the difference between the propagation constants of eigenpolarization modes is equal to

$$\Delta\beta = 2[K_e^2 + (\phi_t - K_t)^2]^{1/2} \tag{63}$$

and the ratio of Cartesian components of eigenpolarization modes is

$$\frac{(E_y)_{1,2}}{(E_x)_{1,2}} = \frac{i(\phi_t - K_t)}{K_e \pm [K_e^2 + (\phi_t - K_t)^2]^{1/2}} \tag{64}$$

where $k_e = 1/2\Delta\beta_e, \Delta\beta_e$ is the difference between propagation constants of eigenpolarization modes caused by only

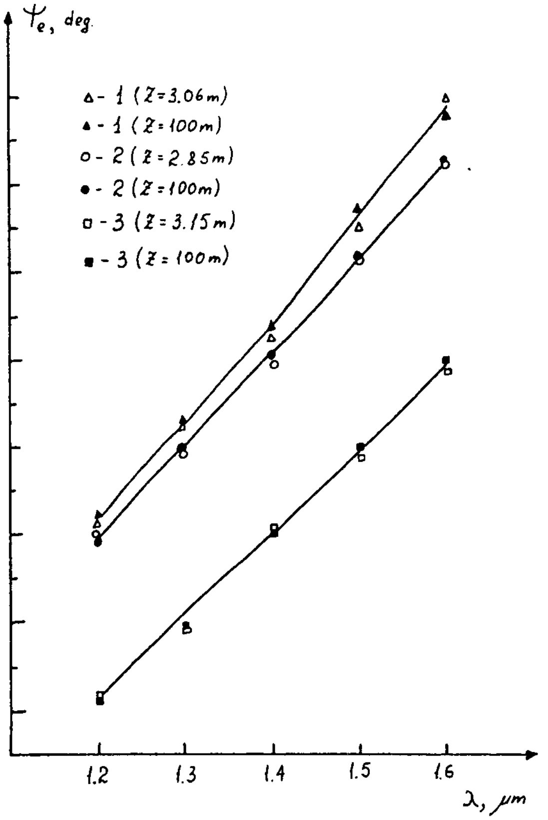

Fig. 10. Spectral dependencies of the ellipticity angle ψ_e of eigen polarization modes of elliptical fibers 1, 2, and 3 of different length.

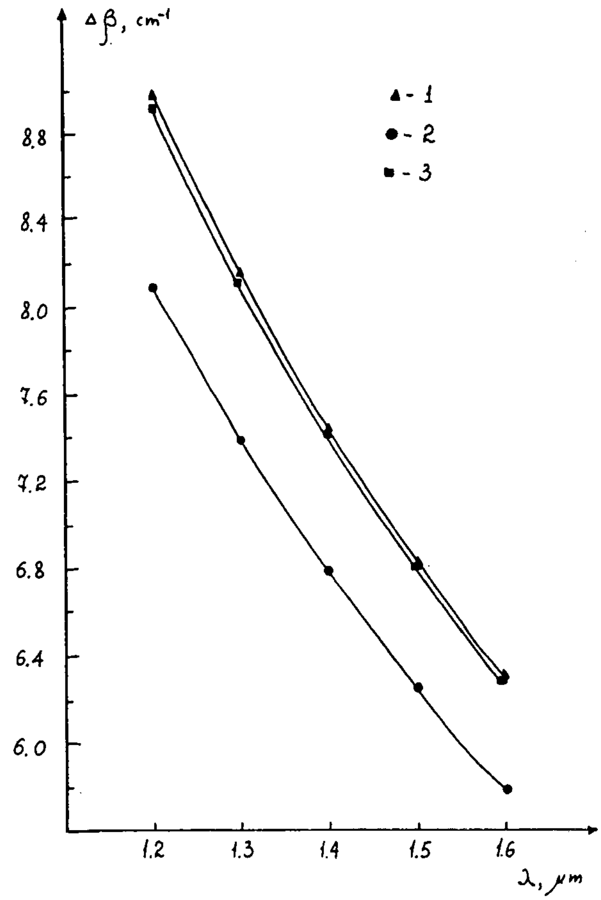

Fig. 11. Spectral dependencies of difference between propagation constants of eigenpolarization modes of elliptical fibers 1, 2, and 3.

the nonsymmetric thermal stress created by stress-induced cladding, ϕ_t is twist rate, $k_t = \alpha\phi_t, \alpha = 0.074$.

The ellipticity angle of eigenpolarization modes of such fiber can be found out from the expression [21]

$$\psi_e = \frac{1}{2}\arcsin\frac{2\,\mathrm{Im}\left[\frac{(E_y)_{1,2}}{(E_x)_{1,2}}\right]}{1+\left[\frac{(E_y)_{1,2}}{(E_x)_{1,2}}\right]} \tag{65}$$

Then substitution of (64) into (65) gives

$$\psi_e = \frac{1}{2}\arcsin\frac{2(\phi_t - K_t)}{\Delta\beta}. \tag{66}$$

The spectral dependencies of the difference between propagation constants of eigenpolarization modes of fibers 1, 2, and 3, which is obtained from elliptical fiber birefringence measurements [16], are shown in Fig. 11. From these data and results of twist rate measurements, presented in Fig. 8, we can calculate the ellipticity angles of eigenpolarization modes of investigated fibers 1, 2, and 3 at different wavelengths. The results of calculations are shown in Fig. 10 by solid curves, and we can see that the measured and calculated values of ψ_e are in good agreement with each other in the whole spectral range.

VII. Conclusions

Thus, the polarization states of eigenpolarization modes of elliptical fibers have been investigated in wide spectral range.

The investigation of dependencies of degree of polarization at elliptical PMF output on wavelength and fiber length shows that these fibers are uniform, regularly twisted fibers.

Two methods have been proposed for measuring the eigenstates of polarization in uniform polarization-maintaining fibers.

The experimental investigation of eigenstates of polarization of elliptical fibers has shown that modes of such PMF are elliptically polarized. Ellipticity angles of eigenpolarization modes increase monotonically with wavelength while the azimuths of eigenpolarization modes do not depend on wavelength. The measured and calculated values of the ellipticity angle are in good agreement across the whole spectral range.

References

[1] T. Kimura, "Coherent optical fiber transmission," *J. Lightwave Technol.*, vol. LT-5, pp. 414–428, Apr. 1987.

[2] D. A. Jackson and J. D. C. Jones, "Fibre optic sensors," *Opt. Acta*, vol. 33, pp. 1469–1503, Dec. 1986.

[3] R. A. Bergh, H. C. Lefevre, and H. J. Shaw, "An overview of fiber-optic gyroscopes," *J. Lightwave Technol.*, vol. LT-2, pp. 91–107, Feb. 1984.

[4] E. A. Golovchenko and A. N. Pilipetskii, "Theoretical analysis of spectrum generation by modulation instability and Raman amplification in high-birefringent fibres," *Sov. Lightwave Commun.*, vol. 1, pp. 271–283, Aug. 1991.

[5] S. V. Chernikov and P. V. Mamyshev, "Effect of polarization on Raman scattering in optical fibres," *Sov. Lightwave Commun.*, vol. 1,

pp. 301–312, Nov. 1991.
[6] R. Ulrich and A. Simon, "Polarization optics of twisted single-mode fibers," *Appl. Opt.*, vol. 18, pp. 2241–2251, July 1979.
[7] J.-I. Sakai and T. Kimura, "Birefringence and polarization characteristics of single-mode optical fibers under elastic deformations," *IEEE J. Quantum Electron.*, vol. QE-17, pp. 1041–1051, June 1981.
[8] J.-I. Sakai, S. Machida, and T. Kimura, "Existence of eigen polarization modes in anisotropic single-mode optical fibers," *Opt. Lett.*, vol. 6, pp. 496–498, Oct. 1981.
[9] J.-I. Sakai and T. Kimura, "Polarization behavior in multiply perturbed single-mode fibres," *IEEE J. Quantum Electron.*, vol. QE-18, pp. 59–65, Jan. 1982.
[10] J.-I. Sakai, S. Machida, and T. Kimura, "Degree of polarization in anisotropic single-mode optical fibres: theory," *IEEE J. Quantum Electron.*, vol. QE-18, pp. 488–495, Apr. 1982.
[11] J.-I. Sakai, "Degree of polarization including the random-mode-conversion effect in anisotropic single-mode optical fibers," *J. Opt. Soc. Amer. A*, vol. 1, pp. 1007–1018, Oct. 1984.
[12] W. K. Burns, R. P. Moeller, and C.-L. Chen, "Depolarization in a single-mode optical fiber," *J. Lightwave Technol.*, vol. LT-1, pp. 44–49, Mar. 1983.
[13] A. B. Grudinin and V. B. Sulimov, "Coherent and polarization properties of radiation in single-mode fibers," *Proc. GPI*, vol. 5, pp. 18–35, 1987.
[14] S. C. Rashleigh, "Origins and control of polarization effects in single-mode fibers," *J. Lightwave Technol.*, vol. LT-1, pp. 312–331, June 1983.
[15] R. H. Stolen, W. Pleibel, and J. R. Simpson, "High-birefringence optical fibers by preform deformation," *J. Lightwave Technol.*, vol. LT-2, pp. 639–641, Oct. 1984.
[16] A. B. Grudinin, A. N. Gurjanov, Z. E. Harutjunian, S. V. Ignatjev, O. B. Smirnov, and S. Ju. Surin, "Polarization characteristics of birefringent fibers with elliptical stress-induced and outer claddings," *Sov. Lightwave Commun.*, vol. 1, pp. 155–167, May 1991.
[17] Z. E. Harutjunian, S. V. Ignatjev, and S. Ju. Surin, "Investigation of modal birefringence in elliptical-stress-cladding fibres by the finite-element method," *Sov. Lightwave Commun.*, vol. 1, pp. 285–289, Aug. 1991.
[18] E. M. Dianov, A. B. Grudinin, A. N. Gurjanov, D. D. Gusovsky, Z. E. Harutjunian, S. V. Ignatjev, and O. B. Smirnov, "Circular core polarization-maintaining optical fibers with elliptical stress-induced cladding," *J. Lightwave Technol.*, vol. 10, pp. 118–124, Feb. 1992.
[19] M. W. Shute and C. S. Brown, "A study of the polarization properties of a rectangular polarization-maintaining fiber," *J. Lightwave Technol.*, vol. 7, pp. 2013–2017, Dec. 1989.
[20] F. M. Sears, "Polarization-maintance limits in polarization-maintaining fibers and measurements," *J. Lightwave Technol.*, vol. 8, pp. 684–690, May 1990.
[21] R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light.* Amsterdam, New York, Oxford: North-Holland, 1977.
[22] Z. E. Harutjunian and A. B. Grudinin, "Stokes parameters measurement by one arbitrary phase plate in wide spectral range," *Sov. Lightwave Commun.*, 1992.
[23] C. G. Askins and M. J. Marrone, "Technique for controlling the internal rotation of principal axes in the fabrication of birefringent fibers," *J. Lightwave Technol.*, vol. 6, pp. 1402–1405, Sept. 1988.

Z. E. Harutjunian, photograph and biography not available at the time of publication.

A. B. Grudinin, photograph and biography not available at the time of publication.