ALL-OPTICAL HIGH GAIN TRANSISTOR ACTION USING SECOND-ORDER NONLINEARITIES

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It is shown that the output stage of a phase-matched frequency doubler is highly sensitive to the phase and amplitude of a weak injected second harmonic signal. Such devices may be regarded as all-optical transistors with very high small signal gain, resulting in strong modulation of the pump light. Gains as high as 60 dB appear to be experimentally feasible in the recently reported highly efficient quasi-phase-matched frequency doublers formed by periodic domain inversion in $LiNbO_3$.

Introduction: There has been a recent resurgence of interest in nonlinear frequency conversion, mainly driven by dramatic advances in high resolution domain inversion of crystals such as $LiNbO_3$, $LiTaO_3$ and KTP. This has permitted realisation of efficient quasiphase-matched (QPM) frequency convertors in both bulk [1] and waveguide [2] forms. These developments have wide practical implications, for example in miniature diode-pumped frequency doublers for blue light generation. More recently, the so-called *cascaded* second-order nonlinear effect, which causes a pump beam to experience a nonlinear phase shift in traversing a phase-mismatched frequency doubler, has been proposed for all-optical switching [3]. It turns out that the phase mismatch condition is unnecessary if the pump is accompanied by a weak injected second harmonic signal, because it is in the course of frequency downconversion that the nonlinear pump phase shift appears [4]. In this Letter, using the results of a detailed and extensive theoretical study of distributed feedback and feedforward parametric frequency conversion in QPM structures (published in 1991 [5]), a further interesting consequence of injecting a weak second harmonic (SH) signal is explored: all-optical transistor action.

Theory: First of all, relevant expressions from the theory are abstracted and adapted. Assuming a collinear, distributed feedforward interaction, the slant parameter s in Reference 5 takes the value $s = 1$. The nonlinear coupling constant then becomes

$$\kappa^{(2)} = \frac{\omega}{4n_p c}\chi_{eq}^{(2)}E_0 \qquad (1)$$

where n_p is the pump index, $\chi_{eq}^{(2)}$ the effective nonlinearity in the device, and E_0 is set by the total input power: $E_0 = \sqrt{\{|E_{p0}|^2 + |E_{s0}|^2\}}$ where E_{p0} and E_{s0} are the pump and second-harmonic electric field amplitudes at the input boundary $x = 0$. The quantities P_p and P_s are the normalised powers in the power and second harmonic waves, i.e. $P_p(x) + P_s(x) = 1$.

Another very useful conserved quantity governs the nonlinear interaction

$$\Gamma = 4\kappa^{(2)}\sqrt{P_s P_p}\cos\psi - \vartheta(P_p - P_s) \qquad (2)$$

where $\psi = \phi_s - 2\phi_p$ is the relative phase between the second harmonic and pump waves and ϑ is the parameter describing dephasing from exact phase matching [5]. The analysis in this Letter is now restricted to exact phase matching (the full solutions are available if desired in Reference 5), and adopting the normalised parameters $\xi = \kappa^{(2)}x$ and $\hat{\Gamma} = \Gamma/2\kappa^{(2)} = 2\sqrt{P_s(1 - P_s)}\cos\psi$, the following differential equation describing the interaction is obtained:

$$\frac{1}{4}\left(\frac{dP_s}{d\xi}\right)^2 = P_s^3 - 2P_s^2 + P_s - (\hat{\Gamma}/2)^2 \qquad (3)$$

Designating the roots of the third order polynomial in P_s by $\beta_1 > \beta_2 > \beta_3$, the general solution takes the form

$$P_s(\xi) = \beta_3 + (\beta_2 - \beta_3)\,\mathrm{sn}^2[2\lambda\xi + F(\Phi_0|q)|q] \qquad (4)$$

where $2\lambda = \sqrt{(\beta_1 - \beta_3)}$, $q = (\beta_2 - \beta_3)/(\beta_1 - \beta_3)$, $\Phi_0 = \arcsin\sqrt{\{(P_{s0} - \beta_3)/(\beta_2 - \beta_3)\}}$, $P_{s0} = P_s(\xi = 0)$ is the value of P_s at the input boundary, sn is a Jacobean elliptic function and F an elliptic integral of the first kind. In the case treated here, the three roots have the simple analytical forms:

$$3\beta_n = 2(1 + \cos\zeta),\ (2 - \cos\zeta) \pm \sqrt{3}\sin\zeta \qquad (5)$$

where $\zeta = 2/3\arccos(3\sqrt{3}\hat{\Gamma}/4)$. The solutions for $P_s(\xi)$ lie in the range $\beta_3 \leq P_s \leq \beta_2$, either oscillating between these limits or tending towards one of them; the behaviour depends on the relative phase $\psi(\xi = 0)$ and the degree of dephasing ϑ from the Bragg condition (zero in this case). The transition between second-harmonic generation and parametric amplification is at $\beta_2 = \beta_3$, which occurs when the discriminant of the cubic polynomial is zero. Exactly at this condition, a nonlinear eigenmode is excited [5] that will progress through the nonlinear grating without growth or fall-off in P_s/P_p.

Discussion: This analysis is now used to explore the behaviour of a second harmonic generator in the presence of a weak injected SH signal. First, the output powers are plotted (Fig. 1) against the injected SH power for a device of physical length L and effective length $\xi_L = \kappa^{(2)}L = 3\pi$ for a relative phase at the input face of $\psi_0 = 0$. At the operating point A in the Figure, the small signal gain is 60 dB. The gain is active only within certain windows of injected power, the highest gain occurring at the lowest average signal power. An obvious question to ask is whether the output state is sensitive to the relative phase ψ_0 at the input. This is also explored in Fig. 1, where for the starting conditions at point A, $P_{sL} = P_s(L)$ and $P_{pL} = P_s(L)$ are plotted against ψ_0. Points A and B correspond to the same conditions. It may be seen that for small phase deviations from zero, there is little effect; it is only when $|\psi_0|$ approaches $\pi/2$ that the convertor is strongly affected, reverting to the solutions expected of a phase-matched frequency doubler. In Fig. 2, the values of P_{s0} needed to ensure an output stage $P_{sL} = P_{pL} = 0{\cdot}5$ are plotted against the effective device length ξ_L, together with the small signal gain that results around this operating point, for three different values of ψ_0: 0, $\pi/4$ and $0{\cdot}49\pi$. For the first two cases, the small signal gain and operating point are only weakly affected by changes in ψ_0; however, the gain falls rapidly as $\psi_0 \to \pi/2$. Above a certain threshold, the gain rises with the effective device length in an approximately linear manner; this means that the gain is approximately proportional to the square-root of the pump power. It is clear that the evolution of the fields is extremely sensitive to the exact boundary condition at the input face; small changes can have dramatic consequences.

The analogy with transistor action is as follows (see Fig. 3). The strong constant pump beam acts as a source of raw power, the 'DC rail'. The small injected SH signal is applied to the 'base' of the transistor, and a large pump signal is developed as a 'voltage' at the 'collector' terminal. An amplified SH signal appears as a 'current' in the 'emitter' lead. It remains to ask whether the predicted behaviour might be realisable in practice. A waveguide SH convertor in periodically domain-inverted $LiNbO_3$ was recently reported [2] with a conversion level of 700%/W cm². A 3 cm long version of this device, at a pump level of 1 W, would yield $\xi_L = \sqrt{(7 \times 1 \times 9)} = 2{\cdot}5\pi$, suggesting that the behaviour predicted in Fig. 1 is already experimentally feasible. The calculations above indicate that a 1 W infra-red pump wave could be controlled by a 1 μW injected SH signal. It is perhaps worth emphasising that this transistor action differs materially from normal parametric amplification in that the device is operating in the strongly driven nonlinear regime where the pump wave is strongly depleted. At high signal bandwidths the parametric interaction will become dephased, causing a fall-off in the efficiency of transistor action, with a gain bandwidth given by the bandwidth of the QPM grating. The stability of the process may be assessed using the phase-plane approach reported recently by Trillo and Wabnitz [6], who have also discussed (using a different approach) the sensitivity of the output state to the phase of a weak injected second harmonic signal [7].

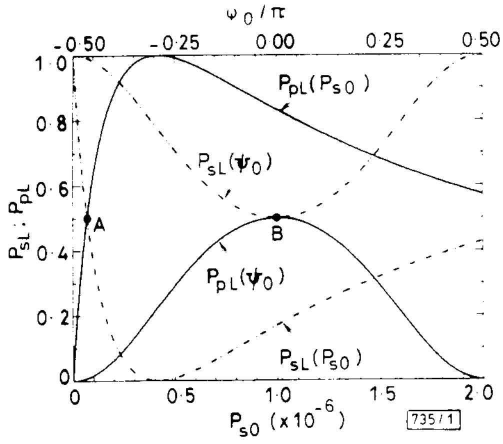

Fig. 1 *Normalised pump and second harmonic power against injected second harmonic power P_{s0} at output of QPM device for exact phase matching and $\kappa^{(2)}L = 3\pi$*

The curves pass through the point A and are for a relative phase $\psi_0 = 0$. Numerical evaluation of the small-signal gain at A yields a value of ~60 dB, where the bias level of the SH signal P_{s0} is only 7×10^{-8}. The sensitivity to changes in ψ_0 for point A is also explored (upper horizontal scale), starting at point B. Although the output state is initially insensitive to small deviations of ψ_0 from zero, it tends to 100% SH for $\psi_0 = \pm\pi/2$ as expected

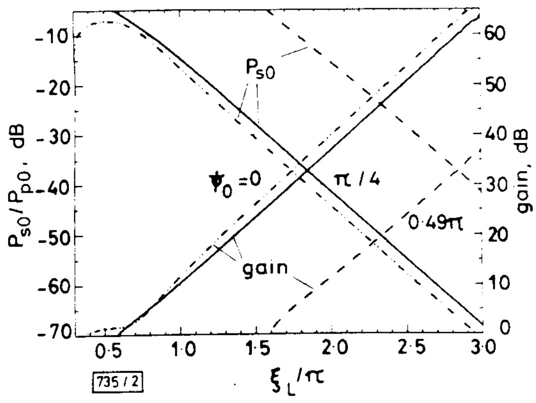

Fig. 2 *Small signal gain and injected second harmonic bias power for $P_{sL} = P_{pL} = 0{\cdot}5$ in a device with variable coupling strength*

Relative phase $\psi_0 = 0$, $\pi/4$ and $0{\cdot}49\pi$ in cases *a*, *b* and *c*, respectively

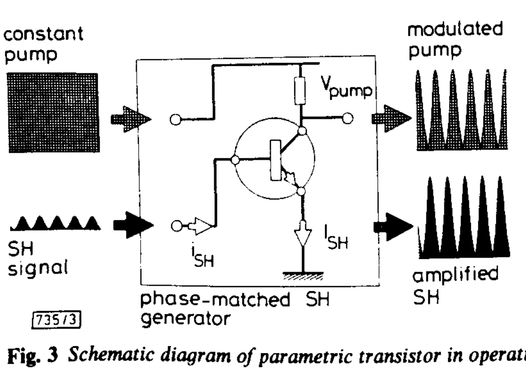

Fig. 3 *Schematic diagram of parametric transistor in operation*

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