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BISTABLE PULSE COLLISIONS OF THE CUBIC-QUINTIC NONLINEAR SCHRODINGER EQUATION

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Abstract:

Making use of numerical collisions simulations we study the stability of the solutions of the nonlinear cubic-quintic Schrodinger equation.A single value and a bistable region of the high order parameter were studied and quasi soliton behaviour was found.

INTRODUCTION

Ultrashort optical solitons are described by the nonlinear Schrodinger equation (NSE) for a medium with Kerr-type nonlinearity and anomalous dispersion.

Since the prediction (1) and experimental observation (2) of soliton propagation in optical fibers there has been an increasing amount of research in this area.Soliton propagation may prove to be of high importance in the development of high bit-rate transmisssion systems(3,12), compression of optical pulses(4),optical switching and bistability(5).

Recently attention has been given to high order effects such as third order dispersion,self frequency shift (6,7),and self steepening,all of which become important when working with high peak power and ultrashort lasers pulses(≈100fs).

When we propagate a high intensity laser pulse down the fiber, the Kerr type nonlinearity alone cannot describe adequatly the field induced change of the refractive index.This is due to the contribution of high order terms.

Pushkarov(8) have obtained solitary-wave solution to the generalized nonlinear Schrodinger equation,including terms up to fourth order in the refractive index expansion:

$$n = ck/\omega_o = n_o + n_2|E|^2 + n_4|E|^4 \tag{1}$$

where c is the speed of light in vacuum,k is the wave number,and ω_0 is the frequency.

Since this first demonstration of solitary-wave behaviour there has been some discussion about the stability of the high order solutions and the bistability associated with them(9,10,14).

In order to study the stability of any solitary wave solution from the generalized NSE(cubic+quintic) we should take in account whether the cubic nonlinear Schrodinger solutions are stable against both small and large pertubations.After the collision of two of these solitons, they emerge with intensity,velocity and profiles unchanged.

This behaviour was observed theoretically and experimentally (2,11) and provide a very strong test of stability.This criterion will be used when comparing with other collisions experiments.

In this paper we examine the stability of solutions of the cubic-quintic nonlinear Schrodinger equation in a single value and bistable region of the parameter by means of numerical collisions simulations .

CALCULATIONS:

The nonlinear Schrodinger equation can be derived by assuming (6,12):

$E(x,t)=\mathrm{Re}(\phi(x,t)\,\mathrm{EXP}(i(kx-\omega t)))$

where ϕ is the electric field amplitude, and expanding the mode propagation constant $k(\omega, |E|^2, |E|^4)$ in a Taylor series about the center frequency. Neglecting damping and higher order dispersion we have:

$$i\frac{\partial u}{\partial \xi}=-\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}+f(|u|^2)\,u \qquad (2)$$

where the following dimensionless units are introduced:

$$A=\sqrt{P_o}\; u \qquad \zeta=z\,/\,L_D \qquad \tau=T/T_o \qquad (3)$$

with

$$P=|\beta_2|/(T_o^2\;\gamma) \qquad L=(T_o)^2\;/|\beta_2| \qquad (4)$$

where P_o is the peak power, T_o is the width of the incident pulse, β_2 is the group velocity dispersion and γ is the nonlinear cofficient(Kerr):

$\gamma = n_2 \ \omega_o \ / \ c \ Aeff$

where Aeff is the fiber effective core area.

In our case the nonlinearity will be given by:

$$f(|u|^2) = \Delta \ |q|^2 \ + \ \theta|q|^4 \tag{5}$$

with

$$\theta = - \ n_4 \ \ |\beta_2| / (T_o)^2 \ \ \gamma^2 \qquad \text{and} \qquad \Delta = -1 \tag{6}$$

In general the values of the coefficients n_2 and n_4 depend strongly on both the medium and frequency selected.

Semiconductor doped glasses have shown high values for the nonlinear susceptibilities χ^5 and χ^7 (13,15).Values of $|\chi^3|$ ranging from $|10^{-7}|$ to $|10^{-12}|$ ESU have been published.For the high order susceptibilities $|\chi^5| \approx 1.8 \ 10^{-14}$ and $|\chi^7| \approx 3.2 \ 10^{-18}$ ESU were obtained in commercial glasses(13,15).Fibres produced from these kind of glasses could lead to the observation of high order effects.

Analytic technique to obtain solutions from equation (2)+(5) can be found elsewhere(8,10,14).Solutions can be obtained taking r to be of the form:

$$s = (r_o \ + (2/3) \ \theta(\ r_0)^2 \ ) \tag{7}$$

$$r(\tau) = \ (2 \ s)/(1 \ + \ (1+(4/3) \ \theta \ r_o \ )\cosh(2\tau\sqrt{s})) \tag{8}$$

and $q(\tau) = \sqrt{r(\tau)}$

$$\text{where} \quad r_o = r(\tau=0) \quad \text{and} \quad q_o = \sqrt{r_o} \tag{9}$$

looking now for solutions where $r(\tau=0.88) = (1/2)r_o$ we have from equation 8:

$$(1+(4/3) \ \theta \ r_o \ )\cosh(1.76 \ \sqrt{s}) \ = \ 3+(8/3) \ \theta \ r_o \tag{10}$$

giving a direct relation between q_o and the parameter θ .

In figure 1 one can see a plot of θ vs q_o derived from equation (10) for $\theta \langle 0$ .In this situation for each value of θ there are two possible values of q_o with different amplitudes for $|\theta|$,up to $|\theta| \approx 0.2$. In this case for the same $|\theta|$ parameter we should have two soliton solutions with different peak powers for the same pulse duration.In figure 2 one can see the plot of function (10) for the case $\theta \rangle 0$. In this instance the solutions are single valued.

The stability of the single and bistable solitons are important for future applications.In the next section we study the stability of these solitons and discuss the stability criterion.

In a laboratory experiment the bistable solutions should be seen in a situation where,having one soliton pulse,a further increase of the laser intensity will lead to another soliton with higher intensity but with the same time duration.

RESULTS:

In order to study the stability of the solitary wave solution from the generalized Schrodinger equation eq(7-8) we have to solve numerically equation (2)+(5) .

In this numerical simulation we used the split-step Fourier technique which has been described elsewhere(6). The accuracy of the numerical procedure was monitored calculating the pulses energy during propagation and using optimum step size(6,16).To study the stability of these soliton pulses we have to study the collision of two solitary waves(eq.7-9) .If the collision is perfectly elastic,(the shapes,amplitude and velocities of the waves are unaltered after the collision) the two pulses are stable solitons.

Initially the collision of two solitons from equation (2) with θ=0 was studied . In this case we have the basic NSE with a perfect elastic scattering(figure 3).After the collision the solitons are perfectly stable,whithout any change from the inicial state(before collision),and without any intensity modulation.

This perfect soliton behaviour will be compared with others collisions experiments.

We can now look for the two regions of the parameter where $\theta\neq0$:the bistable region (figure 1) where $\theta\leq0$and single valued(figure 2) $\theta\geq0$.

Initially we will examine the two branches of $|\theta|=0.1$

(figure 1).The collision of two solitons of the low amplitude branch (θ=-0.1 ,q_o =1.078)is shown in figure (4).In figure (5) one has the initial (a) and final shape (b)of the pulse before and after the collision.Amplitude instabilities after collision are clearly seen.The pulse (b) has higher amplitude compared with (a).The collision is partially inelastic.Looking now for the high amplitude branch (θ=-0.1,q_o =2.622) we found that it is very unstable even in the normal propagation in the fiber.In figure (6) one can see clearly strong amplitude modulation during propagation.Similar behaviour was found for higher values,of the parameter ,in this branch .

If we start increasing $|\theta|$ we will find a growing instability.In figure (7) we have the collision of two solitary waves for θ=-0.2 ($q_o \approx$ 1.305).The amplitude instability is quite clear .

Going out of the solution region (figure 1),taking θ=-0.65 for example,the behaviour is even worse.In figure 8 we have the collision of two of these solitary waves.A strong amplitude modulation after scattering is present with the pulses being very distorted (Figure 9a and figure 9b).

Looking now for the single valued branch(figure 2) with $\theta \geq 0$,we find a very interesting behaviour .In figure (10) we have the collision of two solitary waves with θ=0.1 and q_o=0.952.Looking for the input and output pulses we still find that we have a partially inelastic scattering (figure 11a and 11b).If we keep increasing the parameter the non-soliton behaviour is very strong.In figure 12 we have the collision of two solitary waves with $\theta=+2(q_o=0.685)$.From figure 13a and 13b the pulse distortion and amplitude reduction is quite clear.

The conclusion from these calculations is that the solitary waves solutions from the generalized NSE are not solitons,because of the partially inelastic collisions observed.Nevertheless these pulses could survive collisions with small pulses.In figure 14 we have a collision between a solitary solution ($\theta=-0.1$ $q_o=1.078$)with a pulse of 10% of it's intensity.Even in this case the pulse experiences a reasonable pertubation but after all remains unchanged.These solitary waves are stable under small pertubations.

It was already shown in the literature that in both regions of the parameter($\theta\geq0$ and $\theta\leq0$) we have $dP/d\delta\rangle0$(10).Here P is the energy of the solitary pulse and δ is the propagation parameter.Which according to the Kaplan criterion is a necessary condition to have stability against small pertubations but does not guarantee the stability against strong pertubations(pulse collisions).We can conclude that in this region of the parameter,the solutions present partially inelastic behaviour which caracterize the quasi-soliton. They are stable under small pertubations but shows unstable behaviour in normal pulse collisions. This behaviour is in agreement with the Kaplan criterion(10).

CONCLUSIONS:

In this paper we examine the stability of the cubic-quintic Schrodinger solutions in the single and bistable region of the high order parameter.The solutions are found to be quasi-solitons,presenting partially inelastic collisions,in this region of the parameter.

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FIGURE CAPTIONS:

Figure 1:Soliton amplitude dependence q_o with $|\theta|$ $(\theta<0)$

Figure 2:Soliton amplitude dependence q_o with θ $(\theta>0)$

Figure 3 Soliton collision behaviour $(\theta=0)$ and $q_o=1$.In figures 3,4,6,7,8,10,12,14 distance is in meters (the dimensionless propagation unit (ζ) could be obtained,taking in account that $L_D=64.6$m)

Figure 4:Quasi soliton collision behaviour,$\theta=-0.1$,$q_o=1.078$

Figure 5:Input pulse(a),output pulse(b) extract from figure 4(central pulse)Vertical scales are normalized.

Figure 6:Pulse propagation,$\theta=-0.1$,$q_o=2.622$

Figure 7:Quasi soliton collision $\theta=-0.2$,$q_o=1.305$

Figure 8:Quasi soliton collision,$\theta=-0.65$,$q_o=1$

Figure 9:Input pulse(a),output pulse(b),extract from figure 8(central pulse)(Vertical scale is normalized)

Figure 10:Quasi soliton collision:$\theta=+0.1$,$q_o=0.952$

Figure 11:Input pulse(a),output pulse(b) extract from figure 10(central pulse)(Vertical scale is normalized)

Figure 12:Quasi soliton collision,$\theta=+2.0$,$q_o=0.685$

Figure 13:Input pulse(a),output pulse(b) extract from figure 12(central pulse)(Vertical scale is normalized.

Figure 14:Collision with small perturbation,$\theta=-0.1$,$q_o=1.078$

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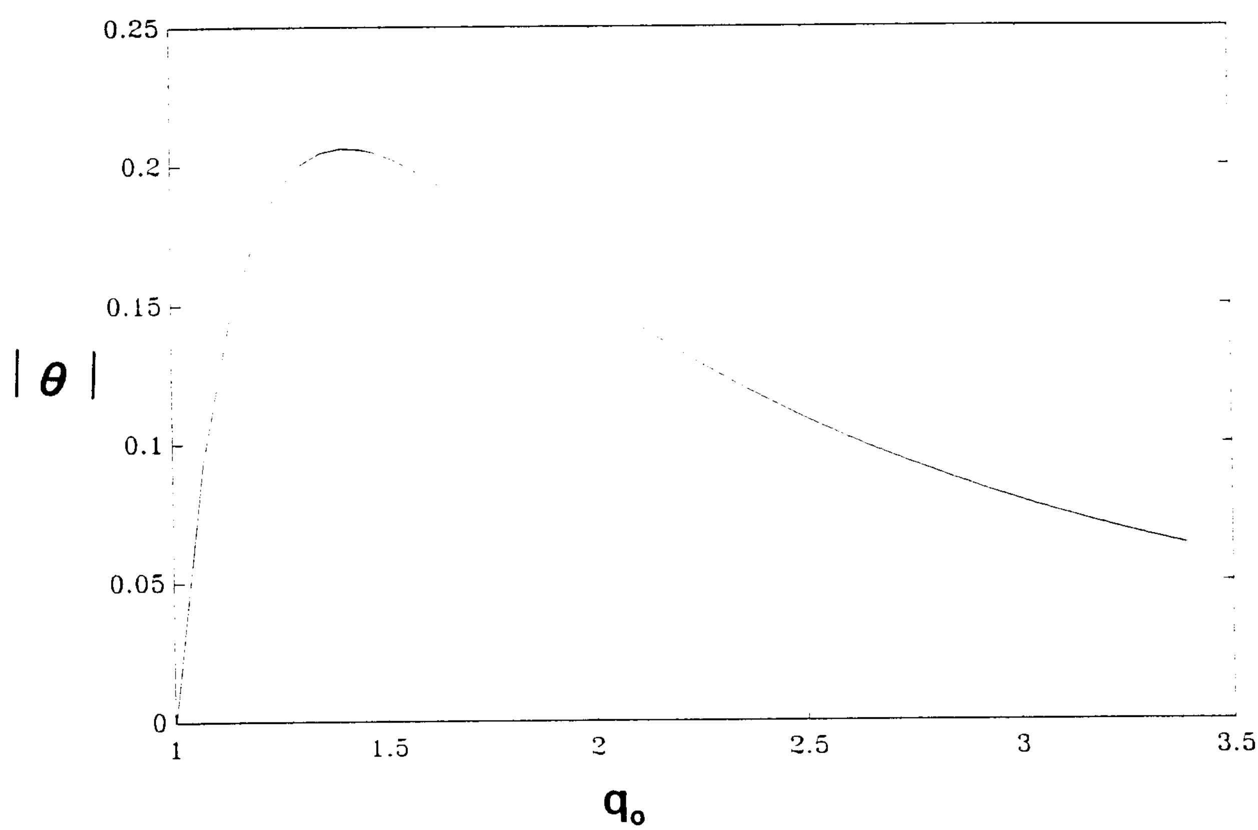

FiG 1 ASB SOMBRA

FIG 1

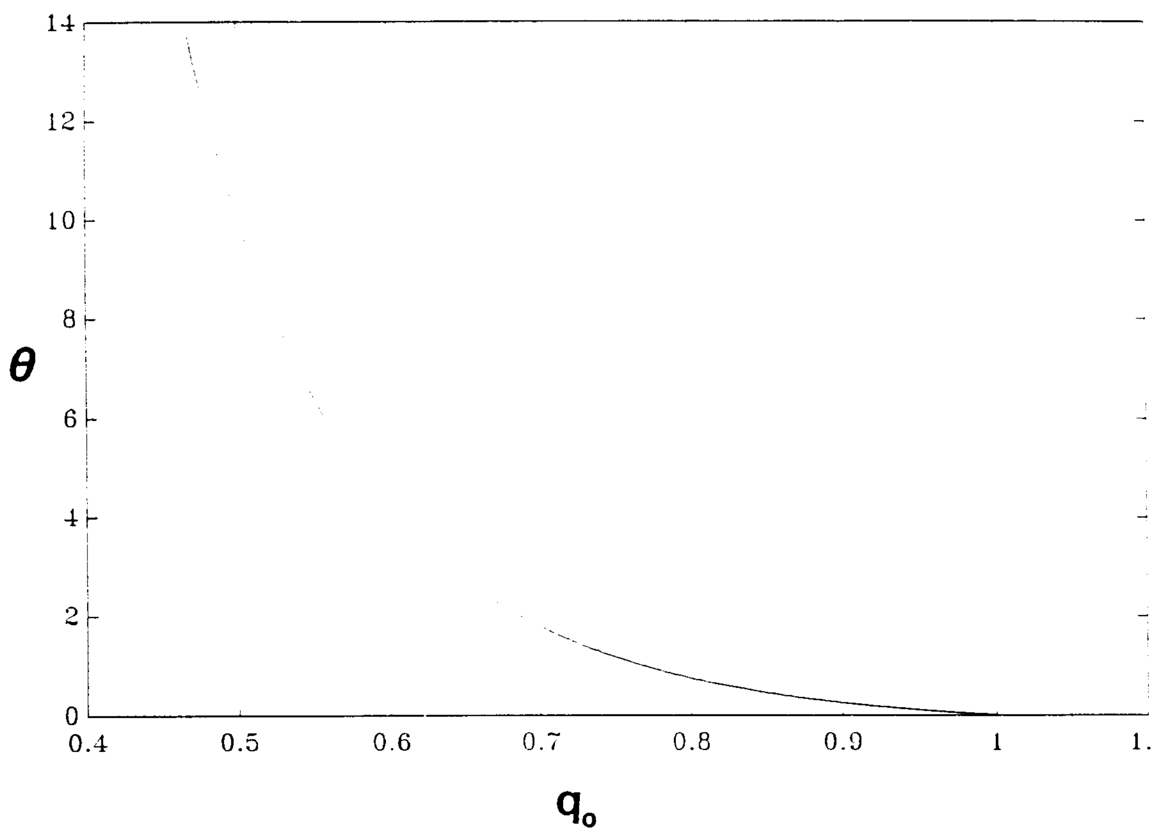

FIGURE 2 ASB SOMBRA

Fig. 2

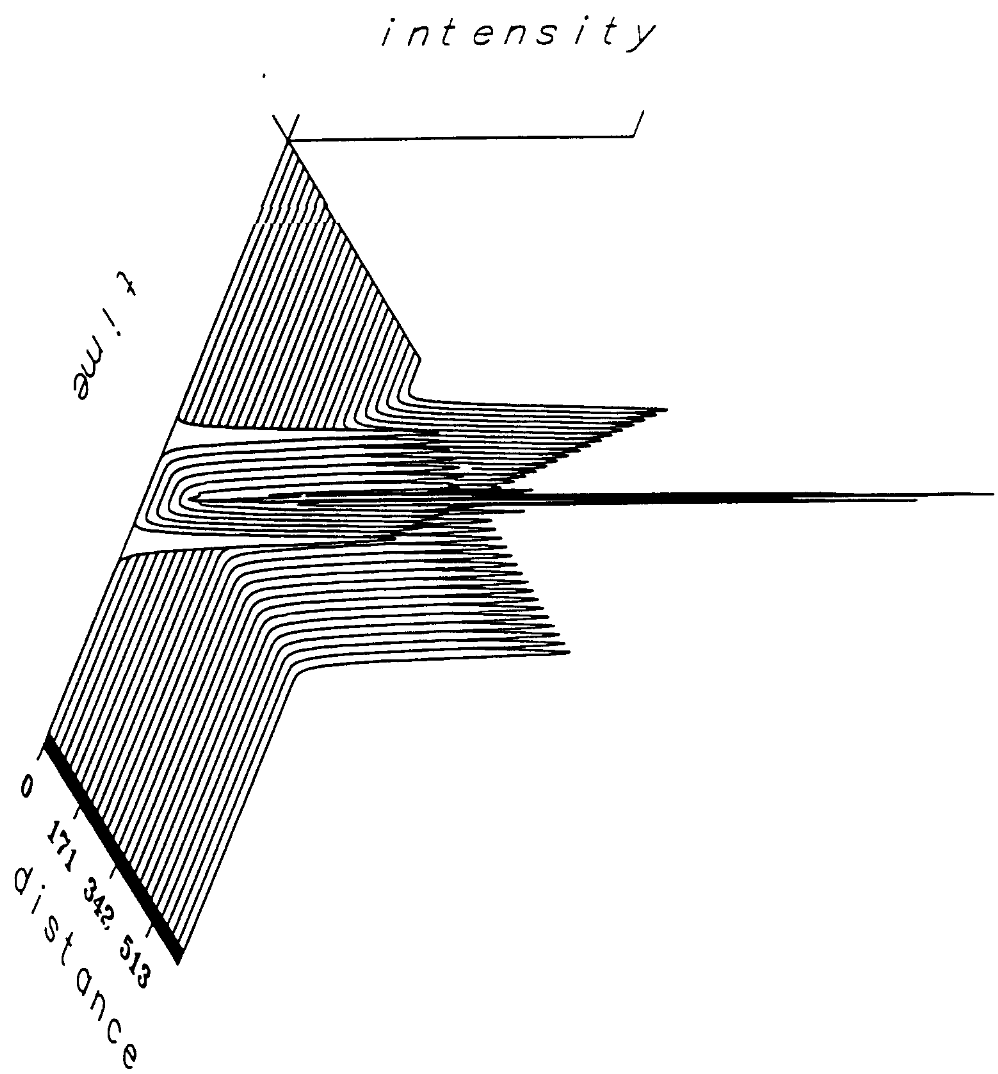

FIGURE 3 ASBSCULBRA

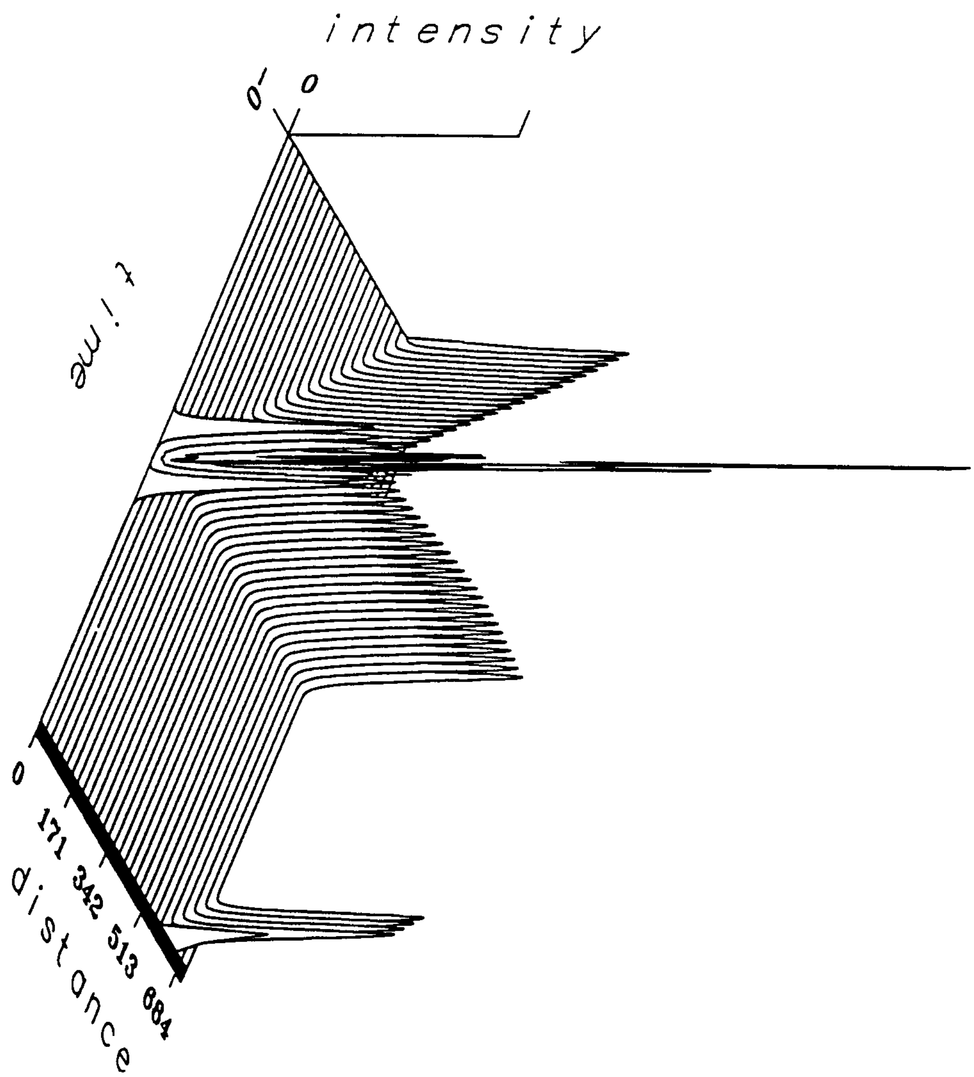

FIGURE 4 ASB SOLLBRA

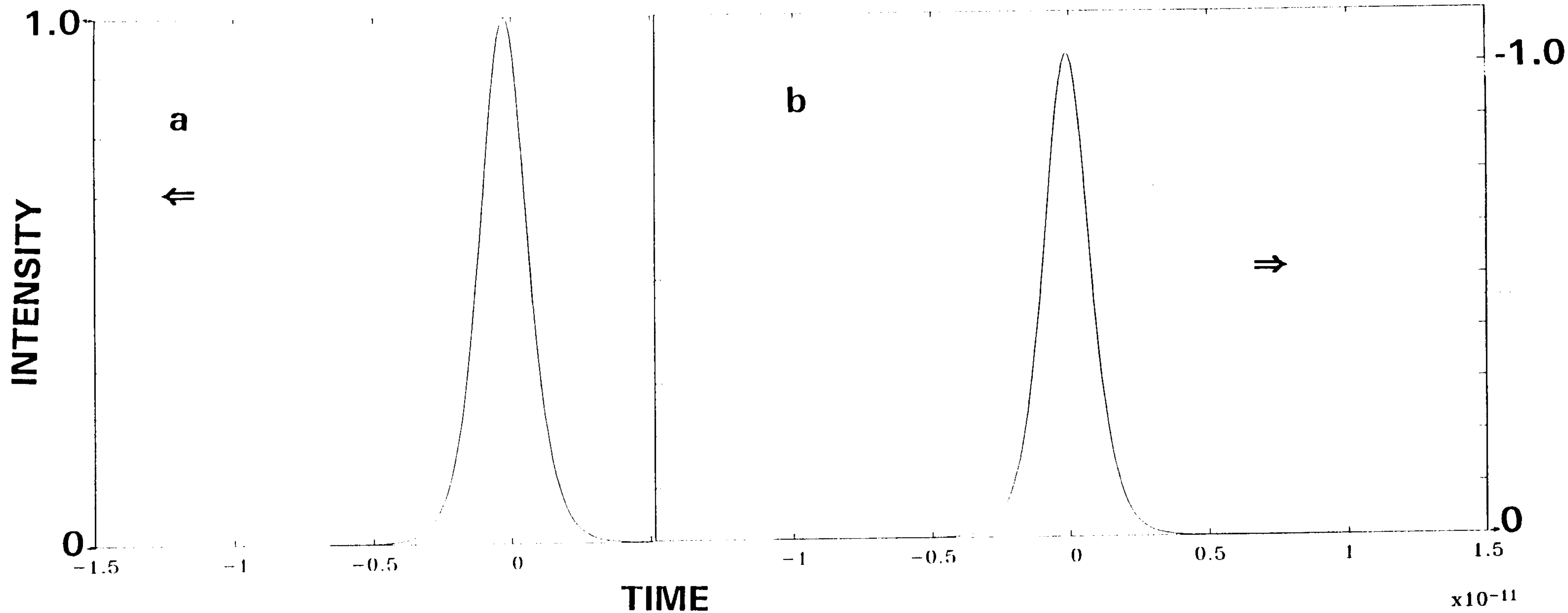

FIGURE 5 ASB SCHURPA

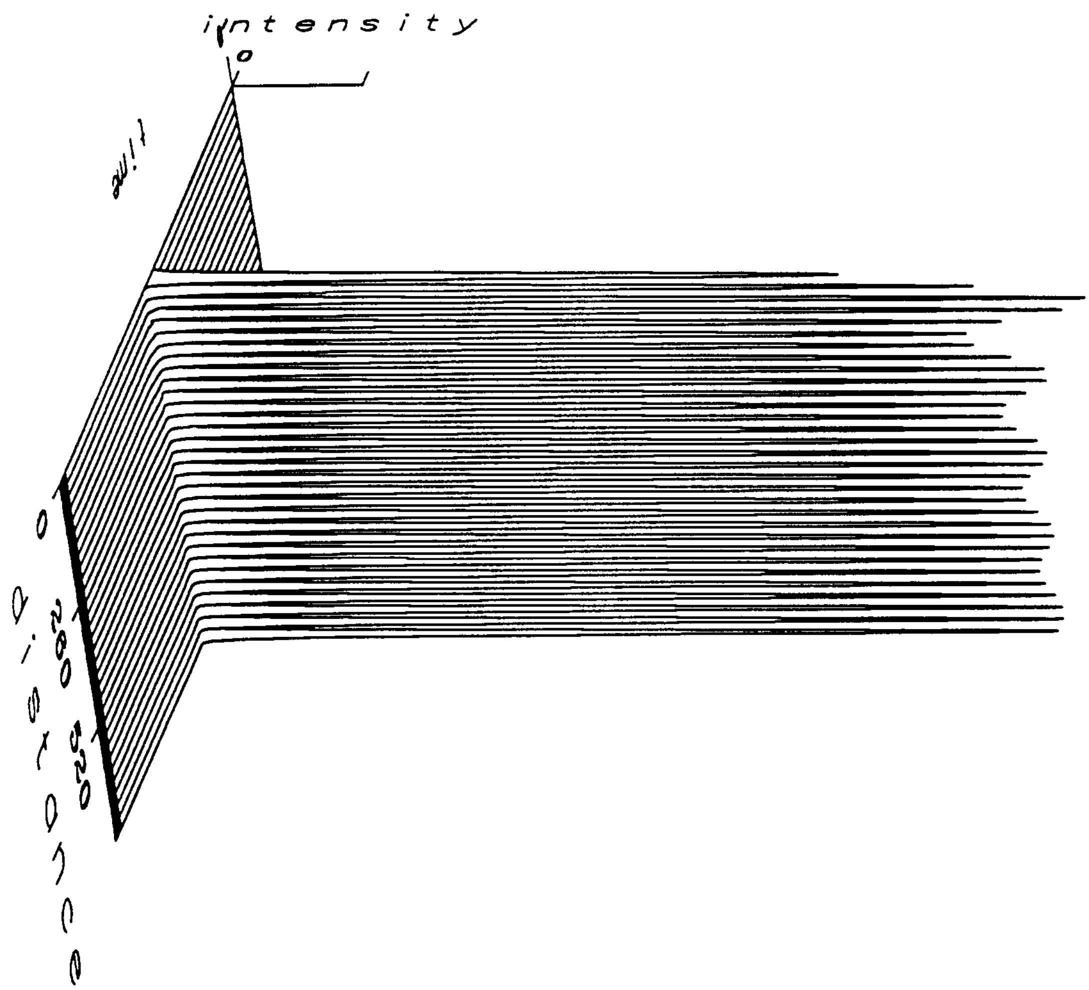

FIGURE 6 ASBSOMBRA

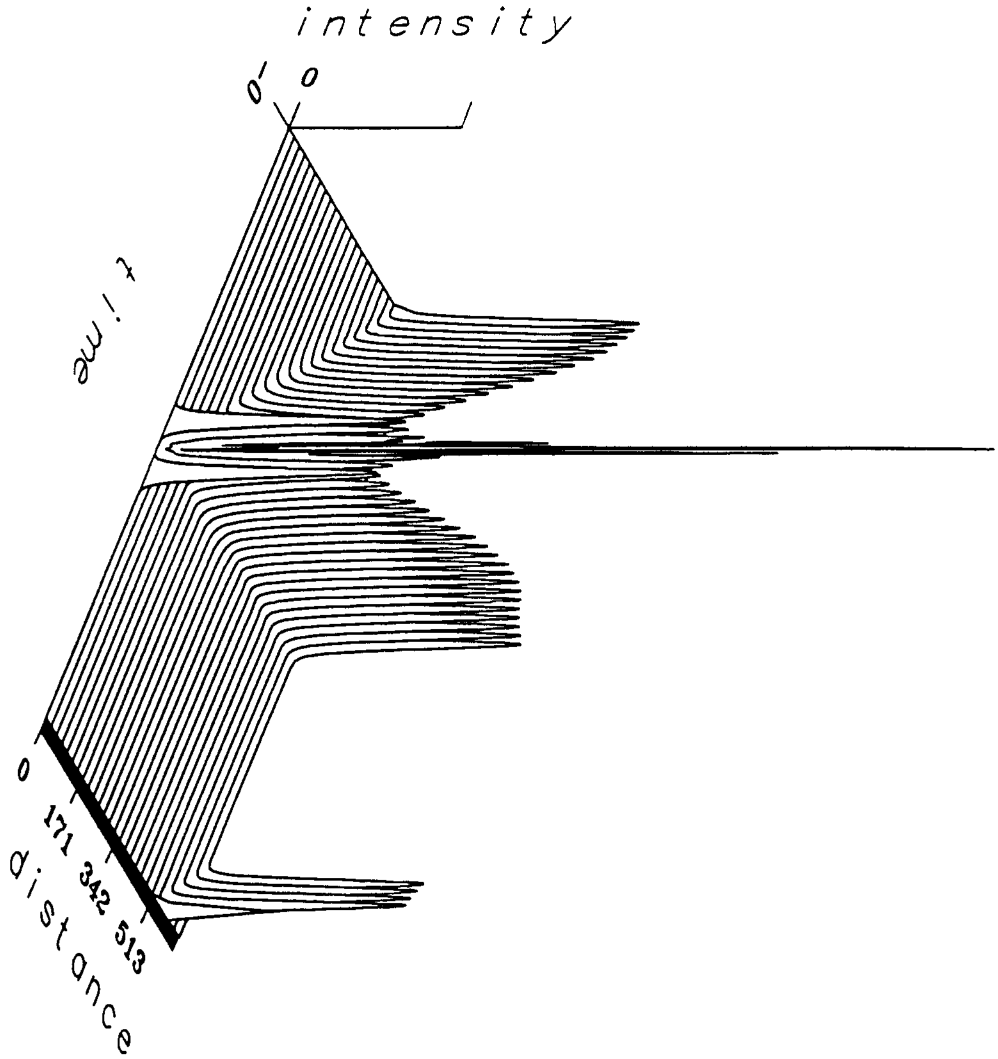

FIGURE 7 ASB SCHUBRA

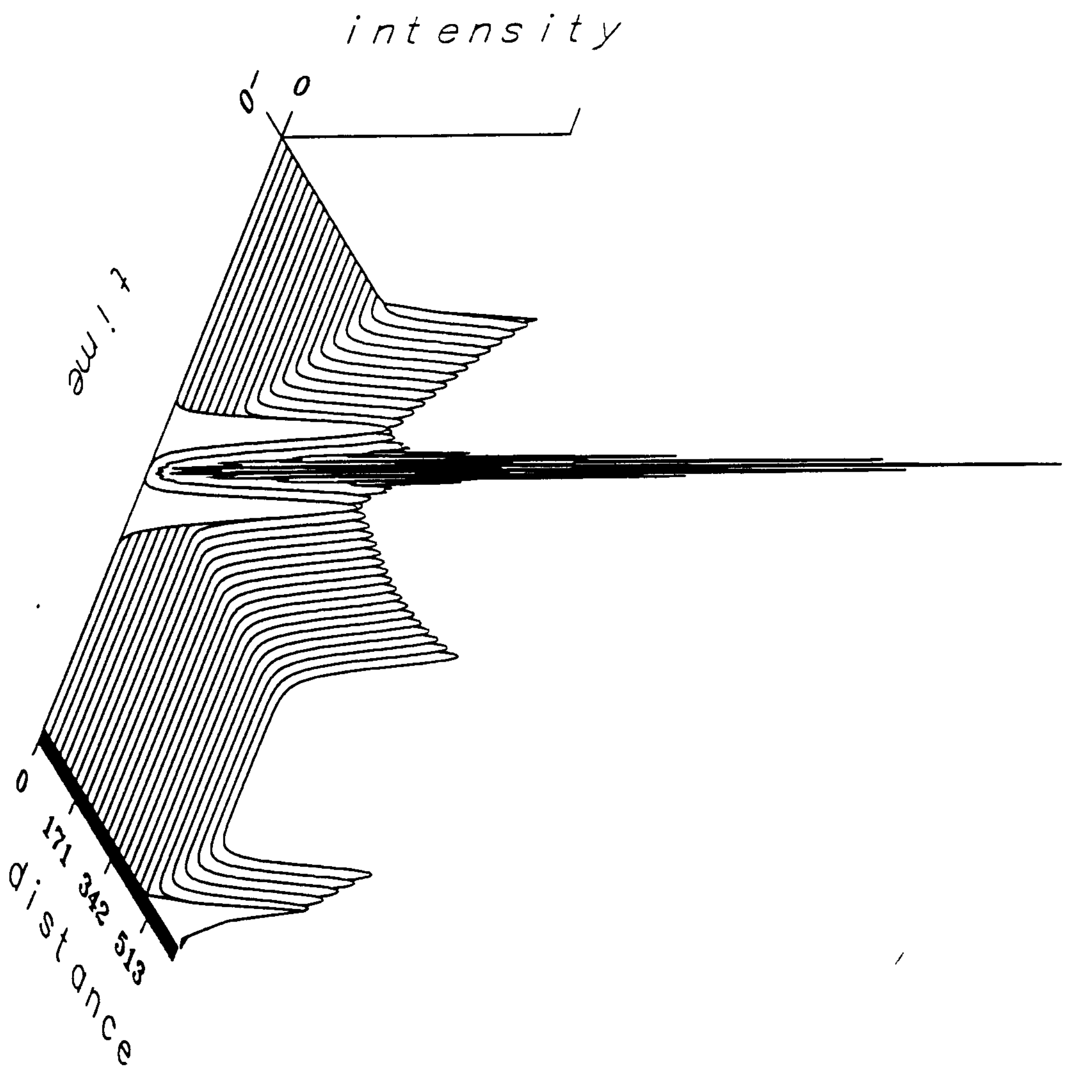

FIGURE 8 ASBSOMBRA

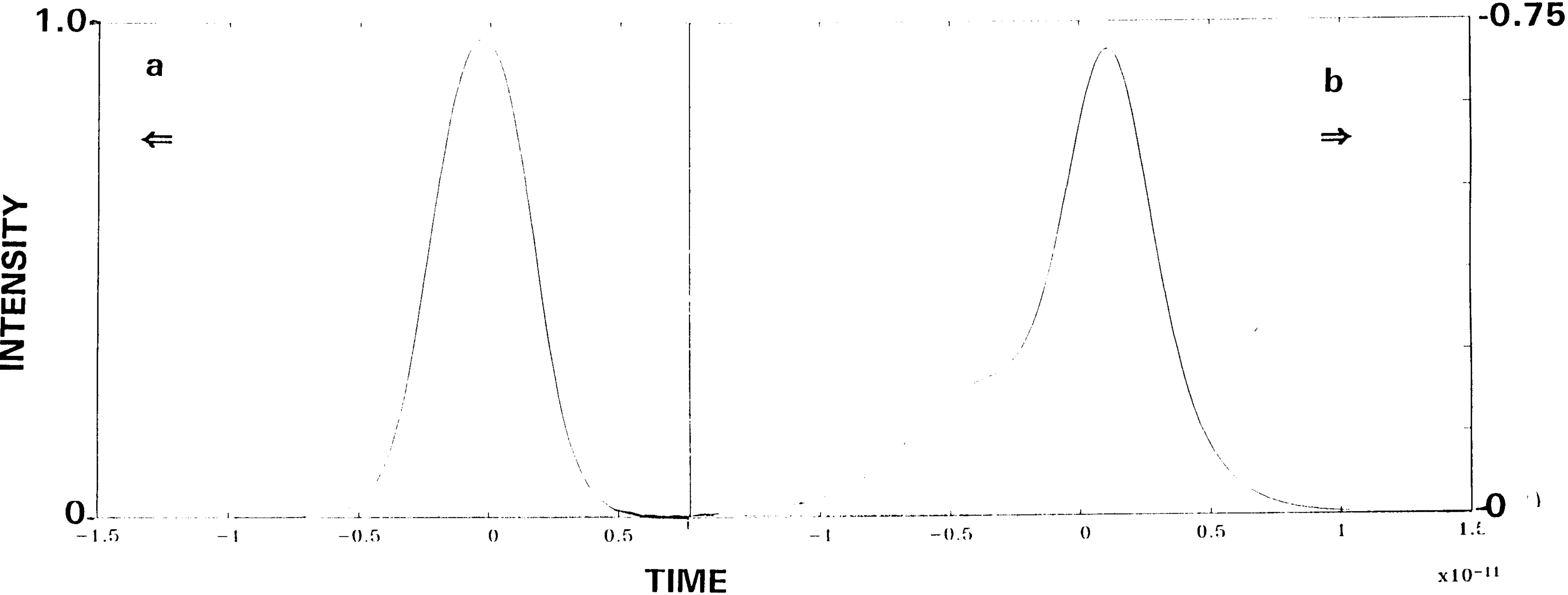

FIG 9 N.G.B. [illegible]

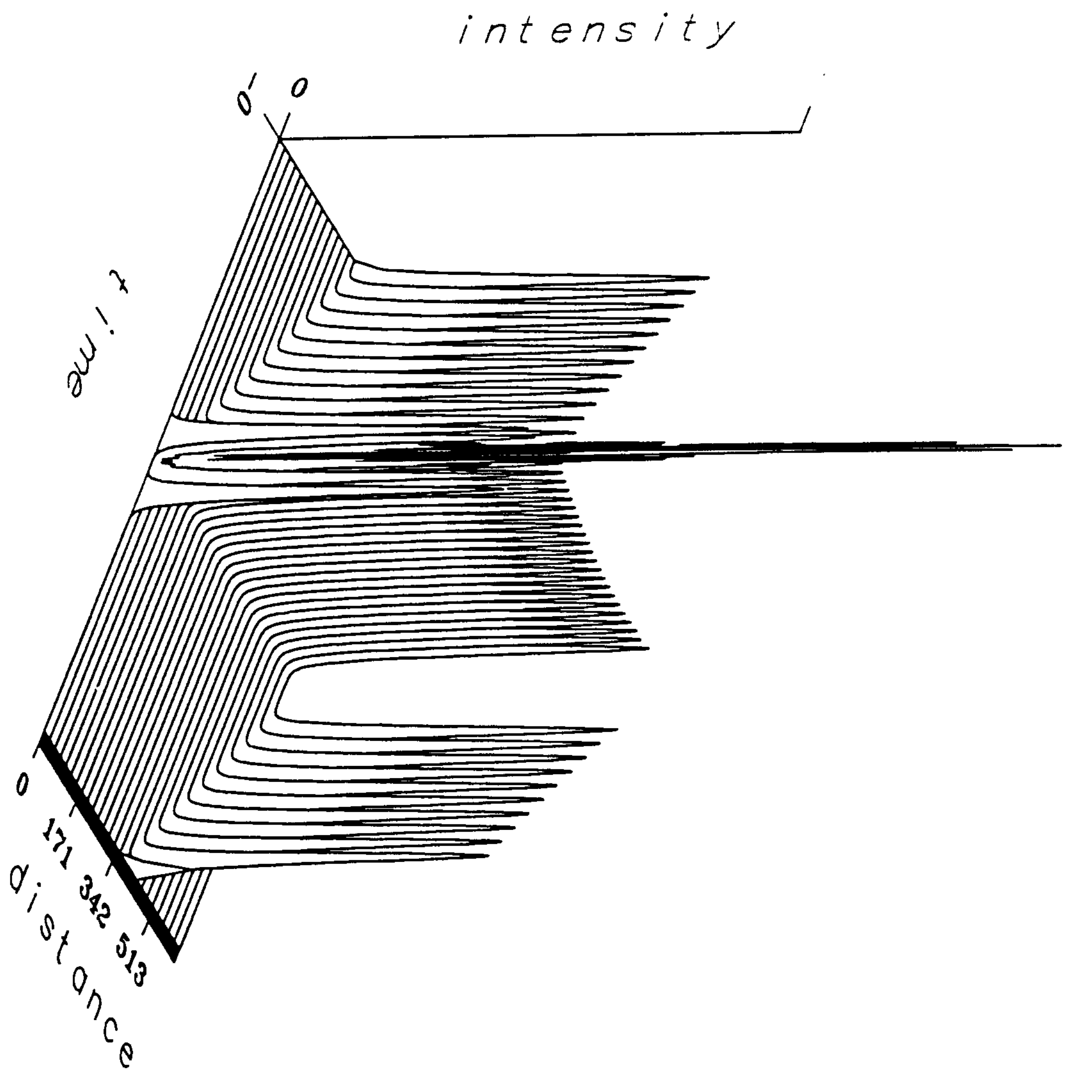

FIG 10 ASB-SCUBRA

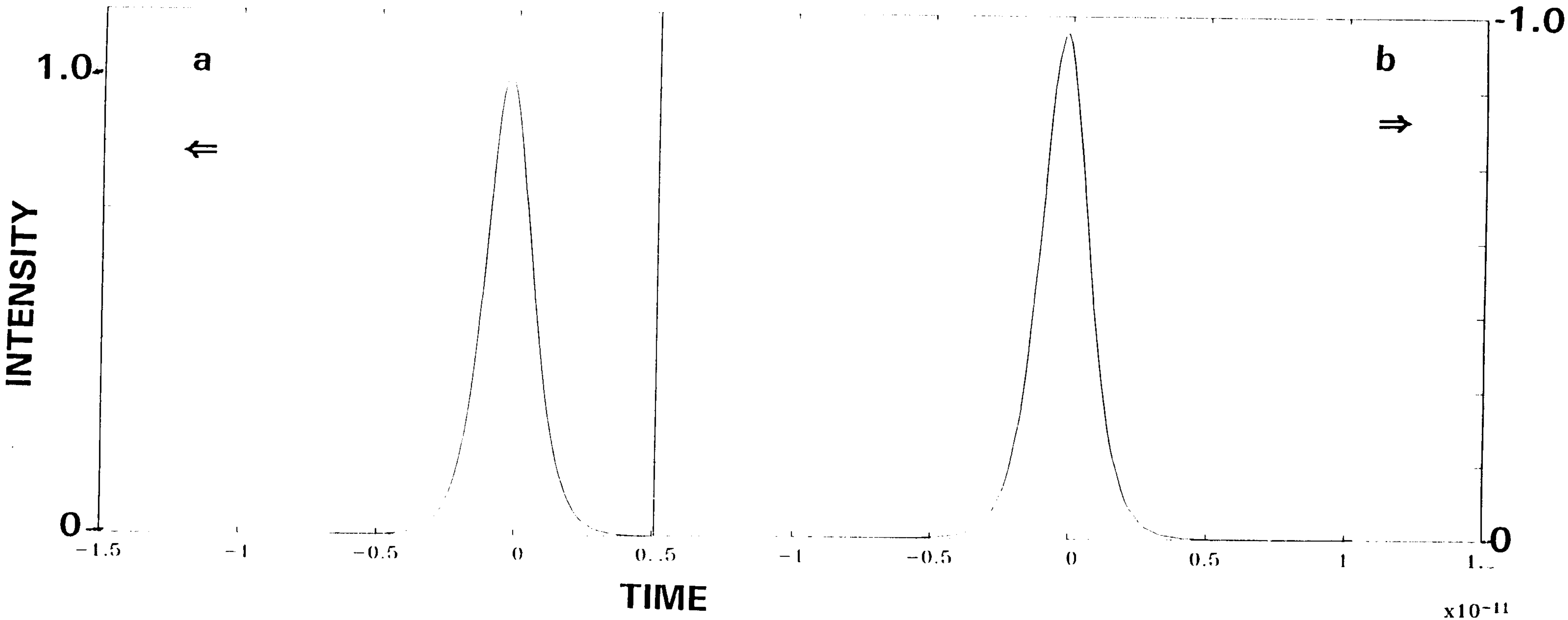

FIG 11

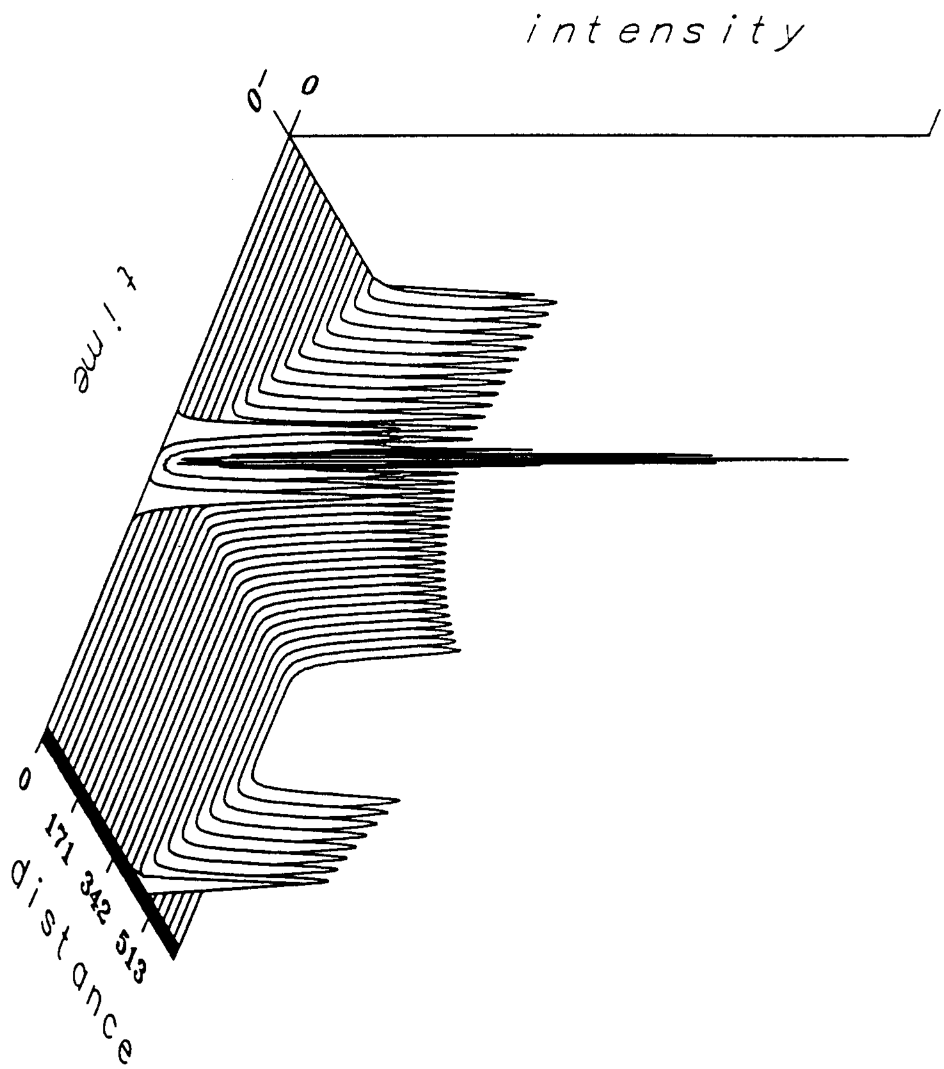

FIG 12 ASB SOMBRA

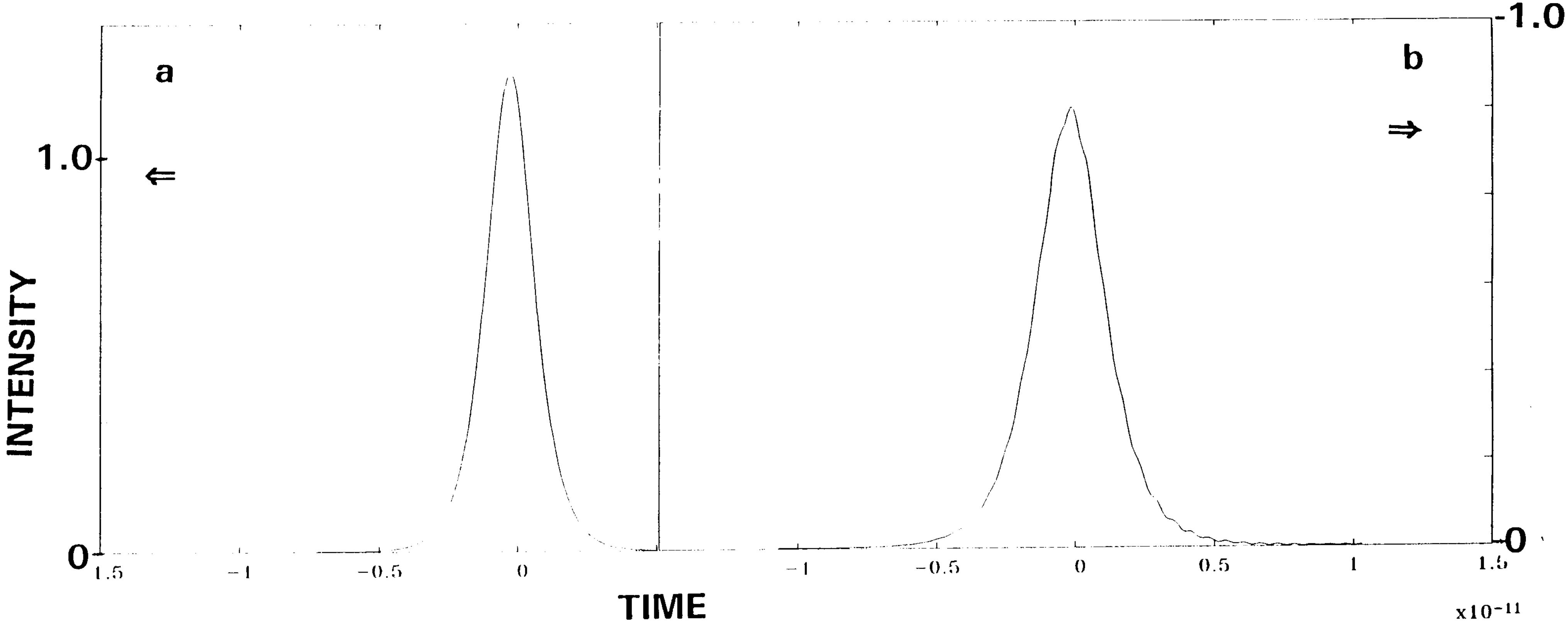

FIG 13 A.S.B. Spectra

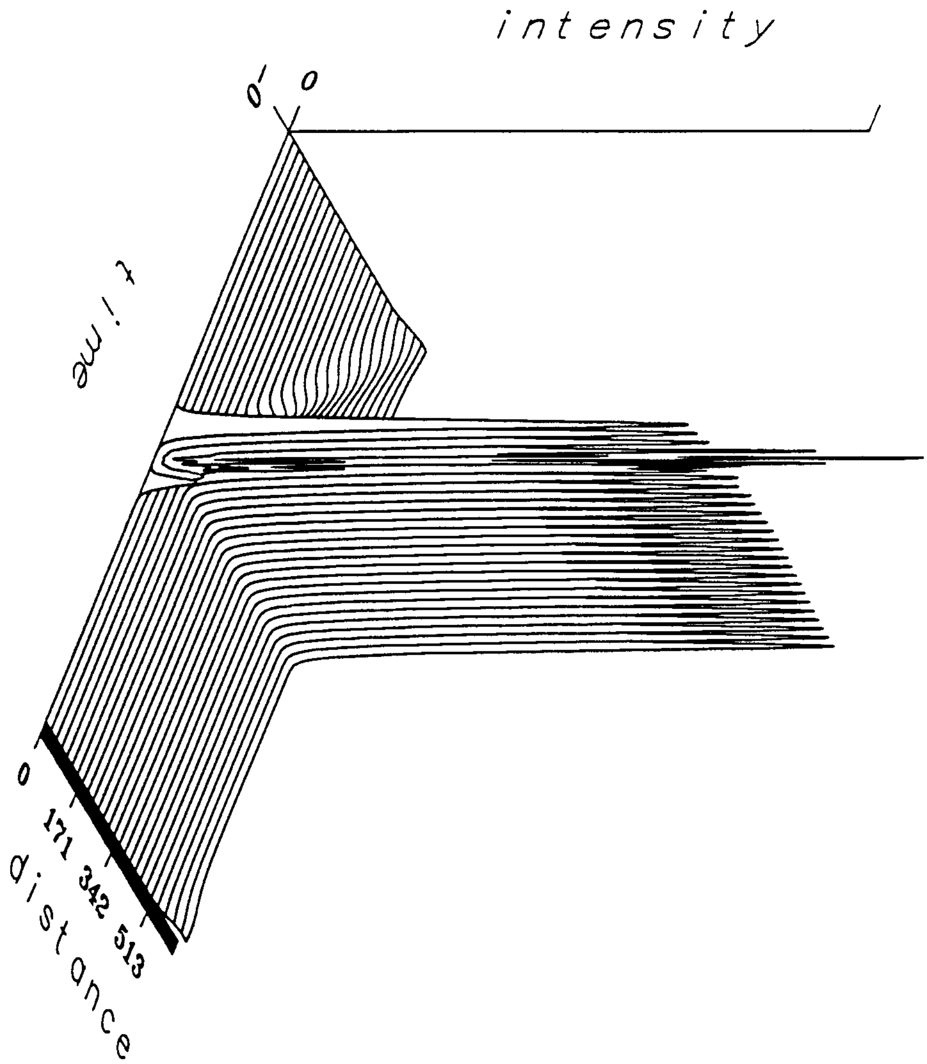

FIG 14 ASB SOMBRA