

Accurate optical frequency-interval measurement by use of nonresonant frequency comb generation

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A novel technique for measuring the separation of widely spaced optical frequencies is demonstrated. It relies on frequency comb generation by use of a laser that incorporates a frequency-shifting element. A Nd:YLF laser is used to produce a frequency comb that has a bandwidth of 140 GHz and that contains in excess of 875 discrete frequencies, accurately spaced by 160 MHz. The longitudinal mode spacing of a dual-frequency laser was measured to an accuracy of ± 5 kHz in 3,733,440.0 kHz by use of the technique described here.

Optical frequency-interval measurements have been made by use of the wavemeter,¹ optical heterodyne techniques,² and reference interferometers.³ The main problem with these schemes is that they can have good accuracy only over restricted frequency intervals; for example, the heterodyne technique is limited by the bandwidth of suitable detectors, or they have poor relative accuracy, as in, for example, the wavemeter. In this Letter a technique is described that enables simple frequency-interval measurements to be made over large frequency ranges, potentially extending to intervals as large as 10–50 THz, depending on the laser host.

The heart of the system is a laser that incorporates frequency-shifted feedback, as shown in Fig. 1, and that will be referred to as the frequency-shifted feedback (FSF) laser. This type of system has previously been studied as a way of generating short pulses from dye lasers⁴ and as a means of broadband operation of laser diodes⁵ and broadband modeless operation of dye lasers.⁶ The FSF laser normally grows from spontaneous noise and generates a broad featureless spectrum. If a narrow-linewidth laser is injected into this laser (ν_L in Fig. 1), then a discrete spectrum results, since the laser spectrum evolves from the seed rather than from spontaneous emission. The individual components of the frequency comb are separated by the round-trip frequency shift ν_{shift} . For the linear cavity configuration shown in Fig. 1, the frequency shift is twice the frequency shift of the traveling-wave acousto-optic modulator. The frequency shift is determined by the rf drive to the modulator, so that $\nu_{\text{shift}} = 2\nu_{\text{rf}}$. The beat frequency δ between an unknown laser of frequency ν_u and the nearest member of the frequency comb may be measured by use of a photodiode and an rf spectrum analyzer, so that the total frequency interval between the reference laser (seed laser) and the unknown laser, $\nu_u - \nu_L$, is given by

$$\nu_u - \nu_L = m\nu_{\text{shift}} + \delta, \quad (1)$$

where m is an integer. Provided that m can be deduced, then the frequency interval can be measured with an accuracy determined by the

measurement of δ . This will be limited by the combined linewidth of the reference and the unknown lasers during the course of the experiment. The power of our proposed technique is that m can be determined in a very simple fashion. The rf drive to the acousto-optic modulator can be tuned in frequency, since frequency comb generation by use of the FSF laser is nonresonant. Alternative techniques for frequency comb generation include frequency-modulated laser,⁷ perhaps incorporating broadband dielectric resonator/optical modulators,⁸ and high-frequency (≈ 20 -GHz) sideband generators that use phase-velocity-matched electro-optic modulators⁹ in conjunction with frequency-interval division in optical parametric oscillator networks.¹⁰ In all these cases, it is necessary to satisfy a cavity resonance condition in order to achieve multicomponent generation at sensible modulator drive powers (approximately 1 W). Consequently the rf frequency must be matched to the cavity resonance frequency to a high degree of accuracy. In contrast, in the FSF laser, ν_{rf} can be tuned over the bandwidth of the acousto-optic modulator, which can be as large as $\pm 25\%$ of the center frequency of the modulator. Hence, by changing the rf frequency and observing the rate of change of δ with ν_{rf} , we may deduce the integer m . Given m and δ , the frequency interval can be calculated by use of Eq. (1). The application of this technique to optical frequency-interval measurements will now be described.

A Nd:YLF laser with FSF was pumped at 800 nm by a Ti:sapphire laser and exhibited a

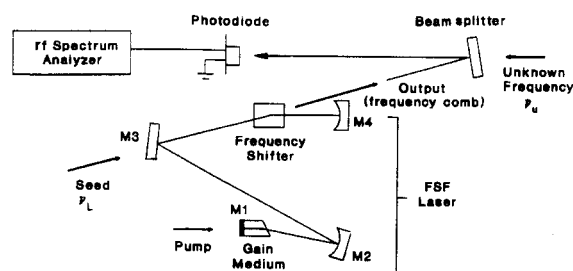


Fig. 1. Schematic of the frequency-interval measurement system: M1–M4, mirrors.

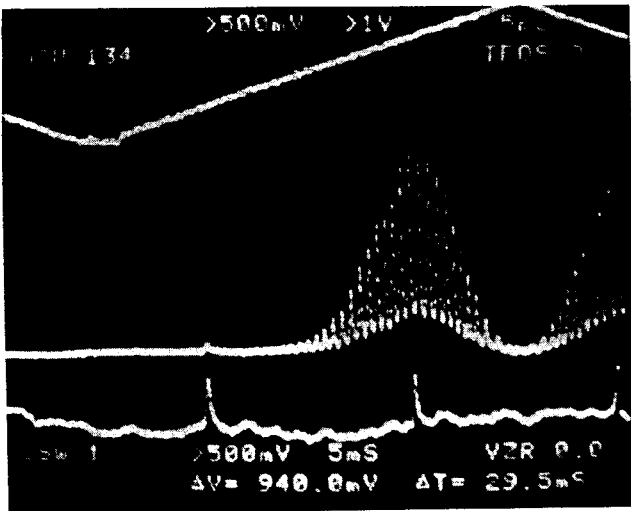


Fig. 2. Optical spectrum of the dual-frequency laser (lower trace) and the frequency comb from the seeded FSF laser (middle trace). A 7.5-GHz free-spectral-range confocal interferometer with a resolution of 60 MHz was used for these measurements, and the frequency scale is 1.1 GHz/division.

200-mW absorbed power threshold and a slope efficiency of 25%.¹¹ The frequency shifter was an acousto-optic Q switch (Isle Optics, Model QS080) operated continuously at a drive frequency of 80 ± 15 MHz. An rf power of 1.6 W produced a maximum single-pass diffraction efficiency of 55%. The frequency shifter was aligned both to attain maximum diffraction efficiency and to increase the frequency of the shifted light on each pass around the resonator during this experiment. The linewidth of the FSF laser could be as large as 140 GHz but, for the purpose of this demonstration, was restricted to 1.9 GHz by insertion of an étalon into the cavity. This allowed us to compare the results obtained from the FSF measurement scheme with a direct frequency-interval measurement by using a broadband photodiode and an rf spectrum analyzer. We designed the cavity to allow the laser to operate over a large range of rf frequencies without needing realignment by placing the modulator at the center of curvature of the 50-cm radius-of-curvature retroreflecting mirror (M4 in Fig. 1). As the rf frequency is changed, the diffraction angle of the frequency-shifted beam changes, but it is always geometrically imaged back onto itself. It was found experimentally that the rf frequency could be tuned over a range of 70–90 MHz.

A dual-frequency laser based on Nd:YLF provided both a reference frequency and an unknown frequency. The longitudinal mode spacing depended on the length of the cavity and was approximately 3.7 GHz. Dual-frequency operation was enforced by placement of the gain medium at the center of the laser cavity so that spatial hole burning allowed only two adjacent longitudinal modes to oscillate. The low-frequency mode was used as the reference frequency and was tuned approximately 3.7 GHz below the center of the FSF laser spectrum. A comb of frequencies was generated from this mode with a sep-

aration of $2\nu_{rf}$, or approximately 160 MHz. Seeding and comb generation, rather than broadband operation, occurred for input signals to the gain medium as small as $20 \mu\text{W}$, and the minimum seed level depended on the detuning from the maximum of the FSF laser output. The second mode was used as the unknown frequency but did not seed the FSF laser, since it was injected too close to the maximum of the FSF laser spectrum. The optical spectrum of the dual-frequency laser is shown in Fig. 2, together with the frequency comb from the seeded FSF laser.

The beat frequency between the unknown frequency and the frequency comb present on the output of the FSF laser could easily be detected by use of a photodiode and an rf spectrum analyzer. It is interesting to note that the rf spectrum analyzer and the photodiode used in this part of the experiment have bandwidths of 1.8 GHz and 500 MHz, respectively, which are significantly lower than the separation between the known and

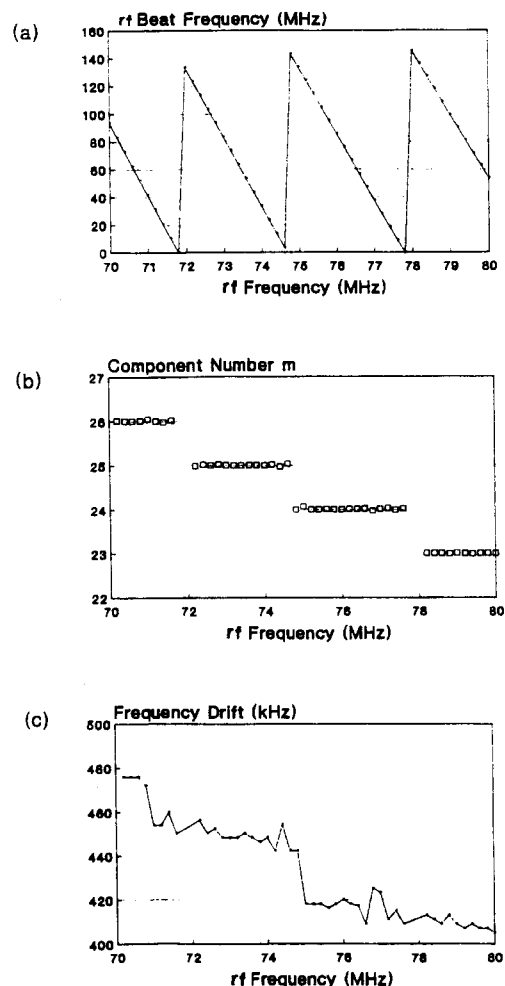


Fig. 3. Measurements that were used to deduce the separation between the known and unknown frequencies. (a) Beat frequency between the unknown laser and the frequency comb as a function of rf frequency, (b) value of m found for each pair of readings, (c) frequency interval deduced by use of Eq. (1). The longitudinal mode spacing of the dual-frequency laser is 3,733,000.0 kHz plus the ordinate reading. Notice that the frequency drifted in a systematic fashion during the course of the experiment.

unknown frequencies. The rf drive frequency was tuned over a range of 70–80 MHz, and the beat frequency δ was recorded between the unknown laser and the nearest member of the frequency comb that was lower in frequency. The frequency of the appropriate beat decreases with the increasing rf drive frequency ν_{rf} , as shown in Fig. 3. This figure also shows the deduced value of m and the calculated frequency interval. The frequency separation drifted systematically during the course of this experiment, which we attribute to a drift in the mode spacing of the dual-frequency laser. Taking the drift into account, the frequency spacing may be deduced to an accuracy of ± 5 kHz in a total frequency interval of 3,733,440 kHz. We confirmed the value directly by using a 30-GHz rf spectrum analyzer and fast photodiode.

This frequency-interval measurement scheme is expected to be extremely powerful, particularly for large frequency intervals that cannot be measured directly by use of heterodyne techniques. The main question that arises in connection with the accuracy of the technique regards the validity of Eq. (1). This in turn depends on the fidelity of the frequency shifting process; for example, is the single-pass frequency shift really equal to the frequency of the applied rf drive? An additional complication comes from the effect of gain pulling as a component traverses the frequency comb. Each frequency component has a finite width owing to the linewidth of the injection laser, so the slope of the gain curve allows the frequencies that are nearer the gain maximum to experience a slightly larger gain. The center of the frequency component will be shifted toward maximum gain by an amount, small compared with its linewidth, on each pass through the gain medium. We calculated the frequency shift that was due to gain pulling by assuming that the gain spectrum and the signal spectrum were Gaussian. An upper limit for the gain pulling, in the worst possible case wherein the change in sign of the shift once the gain maximum is passed is neglected, is given by

$$(\text{number of transits}) \times (\text{single-pass shift}) = \frac{\delta \nu_L^2}{\nu_{\text{shift}}}, \quad (2)$$

where $\delta \nu_L$ is the signal full width at half-maximum. Interestingly, this is independent of the gain bandwidth. Since $\delta \nu_L \ll \nu_{\text{shift}}$, gain pulling of the frequency may be neglected. The relationship between ν_{rf} and the frequency shift ν_{shift} could be analyzed in a similar fashion, but since the rf linewidth is much smaller than the modulator linewidth, it is unlikely that the accuracy of Eq. (1) would be severely compromised, unless extremely large frequency intervals with a large number of frequency shifts were realized.

In this experiment a solid-state laser material with a relatively narrow gain bandwidth was used. Ti:sapphire and other vibronically tunable laser

hosts, as well as dye lasers, semiconductor diode lasers, and rare-earth-doped (e.g., Er^{3+}) fiber lasers are obvious candidates for FSF operation. In these materials linewidths of 1–10 THz are feasible from the FSF laser, allowing optical frequency intervals to be measured over this large range. In some cases linewidths as great as 50 THz could be achieved. It should be remembered that the Nd:YLF laser has already demonstrated linewidths that are 30% of its gain bandwidth. Additional coverage is possible when second-harmonic generation (and sum- and difference-frequency generation) of the FSF laser is considered. If the seed laser is sufficiently stable, then the nonlinear frequency generation can be enhanced by the short pulses that can be generated by matching the length of the laser cavity to the rf frequency.^{4,11} Applications of this source include measurements of large intervals in spectroscopy, calibration of new laser frequency standards, and frequency measurements in coherent optical communications. In the latter context, fiber lasers or semiconductor lasers that incorporate integrated-optic single-sideband frequency shifters with single-pass frequency shifts of the order of 12 GHz can be envisaged.¹² The fiber geometry has the advantage of large gains that can cope with the insertion losses of these frequency shifters and permits compatibility with fiber-optic systems.

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