Propagation characteristics of guided waves in stratified metallic optical waveguides

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An eigenvalue equation is derived for stratified metallic optical slab waveguides with any number of layers. The equation is solved using the numerical zoom analysis (NZA) method. The analysis is applied to various metallic optical slabs and the propagation characteristics of the guided waves are determined, which may be useful for optimizing the design parameters. It is found possible, with proper optimization, to design a polarizer having hundreds of decibels of extinction ratio at the cost of a fraction of a decibel of insertion loss. Examples of optimizations with respect to wavelength, relative refractive indices, core/clad geometries, etc. are demonstrated for slabs with single or double metal layers. Guidelines for determining the metal film thickness have been formulated and tabulated for many useful metals.

I. Introduction

Multilayered metallic waveguides have become increasingly important optical components as polarizers, couplers, modulators, and beam splitters, which are routinely employed in optical fibers, integrated optics, and semiconductor lasers for various purposes. It is well known that compared with the radially stratified metallic waveguide the planar waveguide poses less mathematical complexity and the problem can, in general, be analytically solved. This is the planar or slab waveguide model which can be adopted to approximate a metallic optical fiber waveguide, with which the polarizers having as high as 40 dB of extinction ratio have already been reported. This score has been achieved without any structural optimization and may, therefore, be expected to improve considerably.

The primary objective of this paper is to propose a set of eigenvalue equations (EVE) in an iterative format which can easily be solved for many dielectric and nonferromagnetic metallic waveguides, then to develop a simple computer program for computing the propagation characteristics including phase factor, attenuation, and guided electromagnetic waves for the common TM\(_m\) and TE\(_m\) modes that are closely related to the polarization status evolution; and finally to apply the analysis technique to various practical strataums. In comparison with many existing analysis techniques, e.g., coupled modes method, transfer matrix method, ripple or razor search method, the technique proposed here appears accurate and requires the least computation cost, and it still offers a certain degree of flexibility in applications which is extremely useful for the optimization purposes. Examples of carrying out optimizations are demonstrated, and the design parameters considered include waveguide geometries, refractive indices, number of metal layers, and the operating wavelength.

II. Debye Potentials and Eigenvalue Equation of \(m\)-Layer Stratified Waveguides

Consider the general \(m\)-layer stratified waveguide in Fig. 1 in which \(n_i\) and \(t_i = y_i - y_{i-1}\) denote, respectively, the \(i\)th layer material refractive index and its thickness, where \(y_{i-1}\) and \(y_i\) are its boundary coordinates. The material is assumed homogeneous within each layer; in the cases where this is not actually true, it may be modeled as such using the equivalent-index method.

Suppose the electromagnetic waves are propagating along the \(z\) axis and the time–distance dependence exp\([-j(\omega t - \beta z)]\) may be assumed where \(\omega\) is the light circular frequency and \(\beta = \beta_0 + j \beta_1\) denotes the propagation constant which is complex for lossy media. Using the Debye potential \(y_0 \Phi\) and \(y_0 \Psi\) with \(\Psi = \Psi_y\) \(\exp(j \beta z)\), \(\Phi = \Psi_h\) \(\exp(j \beta z)\) associated with \(E = e\) \(\exp(j \beta z)\), \(H = h\) \(\exp(j \beta z)\), the time-harmonic electromagnetic fields in a general stratified waveguide are well known:

\[ e = -j \beta_0 y_0 \Psi, \quad y_0 \Phi, \quad (\beta / \omega) \nu \Psi - (\beta / \omega) \nu \Phi. \]
\[ h = -j \beta \Phi \chi_0 - (\beta^2/\omega \mu) \varphi \chi_0 + (\delta/\omega \varepsilon \mu \kappa \chi_0) / \partial y. \]  
\[ (1b) \]

where \( \chi_0, \varphi, \delta \) are the unit vectors, and \( \epsilon \) and \( \mu \) are, respectively, the permittivity and the permeability of the materials. \( \varphi \Phi \) and \( \varphi \Psi \) appearing above are also referred to as the electric and magnetic Hertz vectors.

and \( \Phi_y, \Psi_y \) must satisfy the scalar Helmholtz wave equation

\[ (\delta^2/\partial y^2) + U^2 \left( \begin{array}{c} \Phi_y \\ \Psi_y \end{array} \right) = 0, \]  
\[ (2) \]

where \( U^2 = k^2 \eta^2 - \beta^2, \) \( k = 2 \pi / \lambda, \) \( k \) and \( \lambda \) are, respectively, the wavenumber and wavelength in free space. For the TM mode or TE mode, \( \Psi_y = 0 \) or \( \Phi_y = 0 \) may be assumed, but for a hybrid wave a linear combination of both \( \Phi_y \) and \( \Psi_y \) is necessary although they must be defined for each individual layer such that

\[ \tan^{-1}(K_y/K_{m-1}) - U_{m-1} + \tan^{-1}(K_y/K_{m}) \tan \Gamma_{m} \pm \pi n = 0, \]  
\[ (7a) \]

\[ \Gamma_{m} = -U_{m} \tan^{-1}(K_y/K_{m}) \tan \Gamma_{m}, \]  
\[ \Gamma_{m-1} = -U_{m-1} \tan^{-1}(K_y/K_{m-1}) \tan \Gamma_{m}, \]  
\[ \ldots \]

or

\[ \tan^{-1}(j K_{m}/K_{m-1}) + U_{m-1} + \tan^{-1}(K_y/K_{m}) \tan \Gamma_{m} \pm \pi n = 0, \]  
\[ (7b) \]

where \( K_y = U_y/\sqrt{\beta^2 - k^2}, \) \( K_y = 0 \) for the TM mode and \( K_y = U_y/\sqrt{\beta^2 - k^2}, \) \( \beta = m \eta_1 \eta_2/\sqrt{\kappa_1^2 + n_2^2} \) which may be derived alternatively from Brewster's law.\(^{10,16} \) While \( m = 3, 4, \) or \( 5, \) Eq. (7a) may be simplified to be

\[ \tan^{-1}(K_y/K_{m}) - U_{m} + \tan^{-1}(K_y/K_{m-1}) \pm \pi n = 0, \]  
\[ (8a) \]

\[ \tan^{-1}(K_y/K_{m}) - U_{m} + \tan^{-1}(K_y/K_{m-1}) \tan(-U_{m} + \tan^{-1}(K_y/K_{m})) \pm \pi n = 0, \]  
\[ (8b) \]

\[ \tan^{-1}(K_y/K_{m}) - U_{m} + \tan^{-1}(K_y/K_{m-1}) \tan(-U_{m} + \tan^{-1}(K_y/K_{m})) \tan(-U_{m} + \tan^{-1}(K_y/K_{m})) \pm \pi n = 0, \]  
\[ (8c) \]

respectively.\(^{5,6,12} \)

III. Computational Scheme for Solving EVE

The eigenvalue equations in the format of Eqs. (7) and (8) are transcendental and can only be solved numerically. There are a few numerical algorithms including the usual computational methods for solving strongly nonlinear equations which may be adopted to solve Eqs. (7) and (8). These methods, however, may easily miss solutions of high-order modes and are therefore not particularly suitable for solving Eqs. (7) and (8). The so-called ripple search method or its improved version razor search\(^{11,29} \) is considered to be a good method for optimizing microwave and lightwave networks, although the program often demands an

![Fig. 1. Geometry of an m-layer stratified metallic waveguide.](image)

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excessive amount of computational effort which is a drawback. Analytic approximations,\textsuperscript{12,14} on the other hand, are desirable and may be utilized to solve the EVE for a specific stratified structure, but their merits are rather restricted since any alteration of the structural overlay requires updating the approximate formulas; and the accuracy may still remain poor near the possible resonances of the waveguide structure. Having experienced much inconvenience and even difficulty with these alternative methods, it was felt necessary to develop a more applicable computation scheme, referred to as the numerical zoom analysis (NZA) method later, with which the EVE of any stratified overlay may easily be solved at a fraction of the computation cost required otherwise.

In essence the so-called NZA method encourages us to compute the left-hand side (lhs) of Eqs. (7) and (8) to see if its modulus is zero for an arbitrarily selected propagation constant $\beta = \beta_0 + j\beta_1$. If it is zero, or more precisely, nearly so, e.g., $10^{-6}$ as chosen here, the root is found. Unless a miracle happens, a random guess will often end up with a disappointment; then a search covering an expected region for both $\beta_0$ and $\beta_1$ is required. Not an abstruse approach is it, but the method could easily prove onerous owing to the horrendous amount of computations involved which is frustrating and unacceptable. Much of the ado may be greatly palliated if Eqs. (7) and (8) are first reformed to include the variable of the square of the equivalent refractive index, defined as $s = (8/k)^2 = s_0 + j s_1$, here $s_0 < n_{eq}^2$ and $s_1 \ll s_0$. Second, the solution is then sought through minimizing the modulus of the lhs of Eqs. (7) and (8) whereas $s_0$ and $s_1$ are alternatively scanned. Moreover, the resolvability of each scan decreases as the search progresses in such a manner that only a few loops will rapidly produce a high resolvability, for example, of the order of $10^{-9}$ for $s_0$ as often observed. In other words the search is continuously zoomed on a very confined area to achieve at the same time both accuracy and efficiency. A similar zoom routine applies to the $s_1$ scan which is initiated only after the resolvability of $s_0$ reaches, say, $10^{-4}$-$10^{-5}$ and the search range for $s_0$ is properly established. This scheme has been authenticated by other methods\textsuperscript{1-7,12} and it turns out to be a very fast and accurate method indeed.

One ought to be somewhat careful with this straightforward NZA method and the reason is explained in what follows. Let $z_1$ and $z_2$ be a pair of conformal mapping variables and $z_0 = \tan^{-1}z_0$. The planes $z_1$ and $z_2$ are shown in Fig. 2 where $z_1 = \xi_1 + j\eta_1$, $z_2 = \xi_2 + j\eta_2$, and $\xi_1, \eta_1, \xi_2, \eta_2$ are the axes. It is easily seen that while the axes $\xi_1/\eta_1$ are mapped to $\xi_0/\eta_0$, the straight lines $\xi_1 = \pm \pi/4$ are then mapped to be the unit circle $\xi_1^2 + \eta_1^2 = 1$ (see Fig. 2). However, it must be noted that due to the periodicity of the function $\tan z_0$, the $z_0$ plane may be mapped onto the $z_1$ plane repeatedly as shown in Fig. 2, in which the relevant mapping sections are also numbered. The question arises as to which section is to be mapped while $\tan^{-1}K_1/JK_2$ and $\tan^{-1}(K_3/JK_2)\tan\Gamma_3$, etc. are evaluated in solving Eq. (7a). It is proposed to select the principal value $-\pi/2 < \text{Re}[z_1] < \pi/2$ for $\tan^{-1}z_0$ to maintain the integer $m$ in Eqs. (7a) and (8a) to be, whenever possible, associated with the mode order, and if not, $-\pi/2 < \text{Re}[z_1] < 3\pi/2$ is considered next and so on. As our experience shows this is a quite viable choice as far as a metallic stratified waveguide is concerned since, for a given waveguide geometry, a higher-order mode always possesses a lower $\beta_0$ but a higher $\beta_1$ and $\tan^{-1}K_1/JK_2$ and $\tan^{-1}(K_3/JK_2)\tan\Gamma_3$ may be therefore adjusted according to the fundamental mode ($m = 0$) cutoff conditions. This point has been fully considered in the program and, as a result, the scheme allows determination of the propagation constants of any other modes.

IV. Field Distributions

The modal field distribution is often of interest and its detail can be derived directly from Eqs. (1), (3), and (4) in which all the amplitude coefficients $A_i$ and $B_i$ ($i \geq 2$) are expressed in terms of $A_1$ and $B_1$ using Eqs. (5a) and (6a); and the phase coefficients are eliminated through the following relations:

\begin{align}
\phi_{m-1} &= -U_{m-1}Y_{m-1} + \tan^{-1}(K_{m-1}/J_{m-1}), \\
\phi_{i-1} &= -U_{i-1}Y_{i-1} + \tan^{-1}(K_{i-1}/J_{i-1}) \tan(U_{i-1} + \phi_i) \\
&\quad \text{(for } i = m - 1, m - 2, \ldots, 3), \\
\phi_2 &= -\tan^{-1}(K_2/J_2), \\
\psi_{m-1} &= -U_{m-1}Y_{m-1} + \tan^{-1}(U_{m-1}/U_{m-1}) \\
\psi_{i-1} &= -U_{i-1}Y_{i-1} + \tan^{-1}(U_{i-1}/U_{i-1}) \tan(U_{i-1} + \psi_i) \\
&\quad \text{(for } i = m - 1, m - 2, \ldots, 3), \\
\psi_2 &= -\tan^{-1}(U_2/J_2).
\end{align}

These formulas are essential for calculating the field distributions and the polarization evolutions.

V. Polarization Parameters and Attenuation Rates $\alpha_{TM}$ and $\alpha_{TE}$

In optical waveguides the status of polarization (SOP) is often of practical importance and its evolution along a metallic optical slab may now be established. Without loss of generality we shall consider the fundamental mode ($m = 0$) and assume that both the TM and TE modes may be excited. The fields are obviously two-dimensional. Since the traditional $2 \times 2$ matrix description is convenient, it is adopted in the following discussions.
Suppose the TM and TE modes correspond to y and x polarizations, respectively, and the propagation constants of these two orthogonal modes are $\beta_x$, $\beta_y$; and

$$\beta_y = \beta_x + j\beta_{xy},$$  \hspace{1cm} (11a)  

$$\beta_x = \beta_x + j\beta_{yx}.$$  \hspace{1cm} (11b)

Suppose further that the input light is expressed in vector notation, i.e., the output light from the endface would then be

$$
\begin{bmatrix}
E_x' \\
E_y'
\end{bmatrix} = \exp(-\beta_x z) \begin{bmatrix}
\exp(-j\beta_{xy} z) \\
\exp(-j\beta_{yx} z)
\end{bmatrix} 
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix}.
$$

Here $z$ is the fiber length, $\beta_y = \beta_x + j\beta_{xy}$ is the averaged propagation phase factor, and $\Delta \beta = \beta_x + j\beta_{yx}$ is the linear birefringence between the y- and x-polarized lights.

The phase involved in the term outside the square bracket on the rhs of Eq. (12) does not at all contribute to the polarization status and therefore may be neglected. The output field vector takes the form

$$
\begin{bmatrix}
E_x' \\
E_y'
\end{bmatrix} = \exp(-\beta_x z) \begin{bmatrix}
\exp(-j\beta_{xy} z) E_x \\
\exp(-j\beta_{yx} z) E_y
\end{bmatrix},
$$

from which the output polarization ellipse may be determined.36

More often the attenuation rates are important and may be defined for the two orthogonal modes as

$$\sigma_{TE} \text{ (dB)} = 8.6866\beta_{xy},$$  \hspace{1cm} (14a)  

$$\sigma_{TM} \text{ (dB)} = 8.6866\beta_{yx},$$  \hspace{1cm} (14b)

which describe the losses of the two modes if launched individually. When both the x and y components are presented at the input, they must exist at the output aso, although their magnitudes will differ. According to Eqs. (12) and (13), the measure $\sigma_{TM} - \sigma_{TE}$ actually indicates how fast the TM component decays against the TE component, which is required to be infinite for an ideal polarizer. In practice, $\sigma_{TE}$ and $\sigma_{TM}$ are not equal, respectively, the insertion loss and the extinction ratio; so the ratio $\sigma_{TM}/\sigma_{TE}$ should also be infinite for a good polarizer since the insertion loss is minimized. Both $\sigma_{TM} - \sigma_{TE}$ and $\sigma_{TM}/\sigma_{TE}$ are important and to be examined in the following analyses.

VI. Four-Layer Overlay Metallic Fiber

The simplest metallic slab is the one modeled with four layers for which $n_1 = n_2 = n_{cl}$, $K_1 = K_2 = K_{cl}$, $U_1 = U_2 = U_{cl}$, $n_2 = n_{co}$, $K_2 = K_{co}$, $U_2 = U_{co}$, $n_4 = n_{me}$, $K_2 = K_{me}$, $U_4 = U_{me}$. $t_2 = A_2$, $t_4 = b_1$ may be assumed for Fig. 1, where $A_2$ is the core thickness, $b_1$ is the buffer layer thickness; and $n_{co}$, $n_{cl}$, and $n_{me}$ are the core, clad, and metal refractive indices, respectively. To gain insight into the general behavior of this type of structure Eq. (8a) is to be solved first.

A. Buffer Layer Dependence

Let $b_1$ vary from 0 to 10 $\mu$m while other parameters are fixed: $n_{cl} = 1.4585$ (silica), $n_{co} = 1.46583$ (germanium-doped core), $A_2 = 45$ $\mu$m, $\lambda = 0.53$ $\mu$m, $n_{me} = -73.0 + j 40.0$ (liquid gallium at the stated wavelength). The phase factor $\beta$ and the attenuation rate may be computed and the results for the typical guided fundamental mode are illustrated in Fig. 3, where $\sigma_{TM}$ and $\sigma_{TE}$ are the effective refractive indices associated with the phase factors ($\beta = \beta_x$, $n_{TM} = \beta_{TM}/k$, $n_{TE} = \beta_{TE}/k$). It is readily seen that while $b_1 > 6$ $\mu$m both $\sigma_{TM}$ and $\sigma_{TE}$ are approaching an asymptotic maximum of 1.462925. The surface plasmon excited in the metal is believed responsible for it and the guided light appears to be dragged by them, in particular, when the buffer is thin.

Interestingly, the fundamental TE mode always seems slower than the TM mode and this can be observed for other metals too as long as their real part of the permittivity remains negative.13 The TM mode is faster but suffers more loss which suggests that more electromagnetic waves may have been coupled to the surface plasmons. As expected a thinner buffer shows more attenuation for both the TM and TE modes as our example illustrates, $\sigma_{TE} = 10^{-7.4356}$ (3.85 $\mu$m) and $\sigma_{TM} = 10^{-7.7282}$ (6.7008 $\mu$m). $\sigma_{TM}/\sigma_{TE}$ looks like a step function and its jump takes place at $b_1 = 0.75$ $\mu$m which was previously ascribed due to the coupling resonance between the guided light and the surface plasmons.4,7,12,21,37,38 This is quite a common phenomenon and has been observed in many waveguide designs.1,7,12,21,22 From the polarizer point of view, 4 $\mu$m > $b_1 > 0.5$ $\mu$m seems appropriate to ensure both large $\sigma_{TM} - \sigma_{TE}$ (> 100 dB) and $\sigma_{TM}/\sigma_{TE}$ (< 140) in this example although the detailed design should also take into account other parameters such as waveguide length and tolerable insertion losses.

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B. Maximal Buffer Layer Thickness

It is seen from Fig. 3 that, while $b_1 > 6 \mu m$, the existence of metal (gallium) becomes gradually negligible since $\eta_{TM}$ and $\eta_{TE}$ reach the asymptotic value, and both modes suffer only insignificant losses. The question arises as to how thick the buffer layer must be to ensure a negligible metal effect or vice versa how close to the core a metal layer must be to achieve the desired functions. The problem is very practical and is now examined using the EVD derived in Sec. II.

Returning to Eqs. (8a) and (8b), after a brief inspection of the difference between the stratified waveguides with or without a metal layer, one may conclude at once that

$$\tan [-U_{cb} + \tan^{-1}(K_{me}/jK_{ed})] = -j.$$  \hspace{1cm} (15)

is a sufficient condition if a metal waveguide can be treated as though the metal does not exist. This condition can easily be met, for example, by letting $b_1 \rightarrow \infty$, because in this case the waveguide may be simply modeled as a three-layer dielectric waveguide. A few microns may be quite enough to justify an infinity and the detailed derivation is now established through solving Eq. (15).

Rewrite the following for Eq. (15):

$$\tan(\xi_1 + j\eta_1) = \xi_0 + j\eta_0,$$

where $\xi_0 = 0$, $\eta_0 = -1$, $\xi_i = \text{Re}[-U_{cb} + \tan^{-1}(K_{me}/jK_{ed})]$, $\eta_i = \text{Im}[-U_{cb} + \tan^{-1}(K_{me}/jK_{ed})]$, and $\beta$ involved in $U_{cb}$, $K_{ed}$, and $K_{me}$ is the solution of Eq. (8a). With given $\xi_i$ and $\eta_i$, $\xi_0$ and $\eta_0$ are determined by Eqs. (16):

$$\xi_0 = \frac{\sin 2\xi_i}{\cosh 2\eta_i + \cos 2\xi_i},$$  \hspace{1cm} (16a)

$$\eta_0 = \frac{\sinh 2\xi_i}{\cosh 2\xi_i + \cos 2\xi_i}. $$  \hspace{1cm} (16b)

The validity of Eq. (15) would require $\eta_0 \rightarrow \infty$, $\xi_0 \rightarrow -\infty$, and therefore $\eta_i \rightarrow -\infty$ or $b_1 \rightarrow \infty$ as intuitively expected. From an engineering point of view, a negligible number $\xi_0$ referred to as accuracy or error, may be conveniently treated as zero ($\xi_0 \approx 0$). In view of Eq. (16a), $\eta_0 = -\frac{1}{2} \ln 2/\xi_0$ would be accurate enough to ensure Eq. (15). In practice $\eta_0$ appears to be somewhere between $-5$ and $-11$ if $\xi_0$ ranges from $10^{-4}$ to $10^{-9}$. It is quite obvious that a higher accuracy demands a thicker buffer layer, and the following equation

$$\sin \theta \approx \theta \approx \frac{\sin \theta}{\cos \theta}.$$

seems appropriate to most applications since $\tan^{-1}(K_{me}/jK_{ed})$ in Eq. (15) has little importance in determining maximal $b_1$. According to Eq. (15a), if $b_1 > 8.7 \mu m$ the metallic fiber discussed in Sec. VI.A may be treated as a pure dielectric fiber. This is indeed clearly seen from Fig. 3.

C. Other Metal

Apart from gallium many other metals including aluminum, mercury, etc. may also be employed in slabs; and their effects on the guided waves are of interest. The waveguide parameters are computed for various metallic slabs which are characterized in Sec. VI.A with the buffer layer $b_1 = 1 \mu m$. The results are tabulated in Table I. It is observed that despite huge differences in the permittivities $\varepsilon_i' = \varepsilon_i + j\varepsilon_i''$ $\eta_{TM}$ experiences little variation among these metals even if $\varepsilon$ becomes positive.13 Regarding the attenuations it appears to us that, while $\alpha_{TM}$ is closely correlated to the imaginary part of the permittivity $\varepsilon_i''$, $\alpha_{TM}/\alpha_{TE}$ on the other hand, monotonically increases as the real part of the permittivity $\varepsilon$ decreases [Figs. 4(a) and (b)]. This may provide some reason why gallium and aluminum are more suitable for the polarizers although the performance of other metals such as silver and mercury can be noticeably improved through individual optimization.

D. Dispersive Characteristics

The typical stratified optical slab waveguide is dispersive because Eqs. (7) and (8) clearly include wavelength. The detailed dispersive property of a metallic fiber must be analyzed by accounting for the metal; in particular, the metal dispersion. As an example, the gallium metallic waveguide has been analyzed and the propagation characteristics are obtained as a function of wavelength (Fig. 5). The wavelength dependence on the refractive index of gallium is taken from Ref. 35 and is further extrapolated and fitted into the following analytic forms:

$$\varepsilon = -19.9 + 111.8\lambda^2 + 121.56\lambda^4 - 855.22\lambda^4$$
$$+ 1593.8\lambda^6 - 970.97\lambda^8,$$

$$\varepsilon' = 5.31 + 39.21\lambda^2 + 61.26\lambda^4 + 162.66\lambda^6$$
$$- 275.98\lambda^8 + 151.95\lambda^{10},$$

where $\lambda' = \lambda - 0.4$ and both $\lambda$ and $\lambda'$ are measured in microns. It is noted from Fig. 5 that $\alpha_{TM}$ increases much faster than $\alpha_{TE}$, as the wavelength increases. This would mean a better extinction ratio for the produced slab if it is measured at a longer wavelength; and this has been observed in our laboratory. In other words, gallium or metal in general has more effects at longer wavelengths in terms of producing higher attenuation $\alpha_{TM}$, $\alpha_{TE}$ and smaller phase factor $\eta_{TM}$, $\eta_{TE}$. 

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E. Relative Refractive-Index Difference

The propagation parameter $n, \alpha$ are seriously affected by the slab parameters such as $A_2, n_{co}, n_{cl}$, etc. and the dependences must be studied if the stratified structure is to be optimized. One of the examples concerning the core dimension dependence is illustrated in Fig. 6 in which $b_1 = 1.0 \mu m$, $n_{co} = 1.46583$, $n_{cl} = 1.4585$, $\lambda = 0.83 \mu m$ have been taken. As expected, the $A_2$ dependence on the propagation constant is rather moderate and when $A_2$ is sufficiently thick, e.g., $A_2 > 6-7 \mu m$ in this example, the polarization parameters including $\alpha_{TM}, \alpha_{TE}$ appear unchanged.

Unlike the core dimension, the relative refractive-index difference defined by convention as $\Delta = (n_{co} - n_{cl})/n_{co}$ plays a vital role in terms of guiding electromagnetic waves as described by Eqs. (7) and (8). The $\Delta$ dependence has also been analyzed using the zoom technique and it is observed that this dependence is far from moderate which is interesting. In fact, $\alpha_{TM}/\alpha_{TE}$ increases rapidly to hundreds or even thousands when $\Delta$ increases from zero which follows with another rapid decrease when $\Delta$ is too large. Our close analysis shows that the peak value of $\alpha_{TM}/\alpha_{TE}$ as well as the $\Delta$ value at which the peak occurs may vary considerably depending on $n_{co}$, even when all other slab parameters such as $\lambda$, $b_1$, $A_2$ are unchanged. One of the examples of the $\Delta$ dependence is demonstrated in Fig. 7 in which $n_{cl} = 1.4585$, $b_1 = 1 \mu m$, $\lambda = 0.83 \mu m$, $A_2 = 4.5 \mu m$ are taken, and the peak occurs at $\Delta = 1.2\%$. This phenomenon is not unexpected because the guided waves are more
confined within the core region when $\Delta$ is large, and only a minor part of the wave may interact with the surface plasmons in the metal. As a result, smaller attenuation is expected if $\Delta$ is large. On the other hand, if $\Delta$ is too small the guidance mechanism formed between the dielectric core and clad may be seriously damaged and in real terms the consequence is to degrade the structure to a dielectric-metal two-layer structure. Now the TM and TE modes both have to suffer extremely heavy losses since the waves are no longer guided and much of the fields is absorbed into the metal. In Fig. 7 $\sigma_{TE} > 14630$ dB and $\sigma_{TM} > 75404$ dB are estimated when $\Delta < 0.3\%$; and these heavy losses should be avoided in any waveguide designs.

VII. Five-Layer Overlay Metallic Fiber

A. Thin Metal Film

For the stratified slab with finite metal film the structure should be modeled as being five layered. Let $m = 5$ in Fig. 1 and assume $n_1 = n_3 = n_5 = n_{clad}, n_2 = n_{core}, n_4 = n_{metal}, t_2 = A_2, t_3 = b_1$. The phase factor and the attenuation rates of the propagating modes may now be computed through solving the EVE [Eq. (8c)]. The results of such typical fibers are illustrated in Fig. 8, for which $n_{metal} = 1.4585, \Delta = 0.5\%, A_2 = 4.5 \mu m, b_1 = 1 \mu m, t_4 = 0.1 \AA - 10 \mu m$. It is not surprising to observe that a good polarizer does not require very thick metal film and in most cases a fraction of a micron will be sufficient. This is largely credited to the well-known skin effects and Fig. 8 seems to confirm this. Figure 8 also shows the behavior of the propagation characteristics when the metal film is gradually reduced to zero. However, it is not quite clear if the macroscopic parameter refractive index can still make sense when the metal film is thinner than a few angstroms. What seems clear is that when the metal film is sufficiently thin (e.g., 10 $\AA$ or less in our example), the TE mode will unexpectedly suffer more loss. This phenomenon may be attributed to a coupling mechanism between the guided waves and the longitudinal and lateral plasmons which may change as the metal film thickness is continuously reduced. Such a property may prove valuable in integrated optics whereas the technology may be more suitable to produce controllable very thin metal films. From a fiber fabrication viewpoint moderately thick film may be desired and the conclusion is to favor a high extinction ratio facilitated by a thick film.

B. Thick Metal Layer Criteria

It has been demonstrated that the waveguide performance improves continuously while the metal film is thickening. The question arises as to how thick the film is to be considered thick enough in a good fiber

![Graph](image-url)
polarizer. The problem is indeed worth investigating if any waste of metal is to be avoided. The study pursued thus far in this paper allows the establishment of the criteria of the so-called thick metal.

Returning to Eqs. (8b) and (9c) one may see that the following relation justifies an infinitely thick metal:

\[ \tan^{-1} \left( \frac{K_{OE}}{g} \right) = C. \]  

Equation (17) looks like Eq. (15) and can be solved in much the same way. In view of the fact that \( \tan^{-1} \left( \frac{K_{OE}}{jK_{ME}} \right) \) plays no important part in almost all practical circumstances, the thick metal film is then judged by

\[ \ln |-U_{ME} - C| < - \frac{1}{2} \ln (2/\epsilon_0). \]  

where \( \epsilon_0 \) is once again the measure of accuracy to be decided in computations. So, \( t_{ME} \) can be computed and predicted for a structure provided \( \epsilon_0 \) is chosen. For example, for a weak or moderately strong waveguide (\( A < 10-20\% \)) \( \epsilon_0 = 10^{-4} \) may be accepted, and \( \tau_c \) is \( t_{ME} \) at the wavelength ranging from 0.4 to 12.0 \( \mu m \) may be calculated. The results are tabulated in Table II for many practical metals. It is seen from Table II that the metal film of 0.07-0.4 \( \mu m \) can ultimately be treated as being infinitely thick for most metals except silicon for which the criterion is relaxed to tolerate 3 \( \mu m \).

C. Double Metal Films

Consider that the dielectric optical waveguide is sandwiched between two metal films. Suppose the metal films are quite thick (Table II) and the structural overlay of this doubly metallic fiber remains five layered. Returning to Fig. 1, we now may note \( \tau_2 = A_2 \), \( \tau_2 = \tau_0 = \beta_1, \) \( \tau_1 = \eta_1 = \eta_{ME}, \) \( \tau_2 = \eta_{CO}, \) \( \tau_1 = \eta_{CL} \). (When the metal films are finite the model is modified to be seven layered and the following analysis can still be carried out.) To such a structure the NZA still applies even when \( \tau_2 \neq \tau_0, \) \( \eta_1 \neq \eta_5. \) The performance of a typical doubly metallic fiber is illustrated in Fig. 9 in which \( \eta_{CL} = 1.4585, \) \( \eta_{CO} = 1.46583, \) \( \lambda = 0.83 \mu m, \) \( A_2 = 4.5 \mu m \) are once again assumed. The relationships between \( \alpha_{TM}, \alpha_{TE}, \) and \( b_1 \) remain fairly linear in the logarithm scale so long as the metals are not placed too close to the core. In our example \( b_1 > 1 \mu m \) is necessary to retain this linearity and moreover \( \alpha_{TM} = 10.86145.9033 - b_1 \) and \( \alpha_{TE} = 10.67335.7725 - b_1 \) are observed accurate (within 5% error). In comparison with the single metal layer, the double-layer structure tends to have smaller attenuation but higher propagation indices for both the TM and TE modes; and the polarization measure \( \alpha_{TM}/\alpha_{TE} \) is improved a little but not doubled as might be intuitively expected. This is somewhat disappointing if one infers that the double
metal layers could offer a double chance to couple the lightwaves into the plasmons. However, the theoretical analysis shown here agrees quite well with our experimental results. There must be some other factors which counteract the above intuition. This is the field redistribution which is further confined within the core after the second metal is introduced. The detailed magnitude of this redistribution may be calculated using Eqs. (9) and (10) and the result seems to have provided the reason for this disappointment. In general, the ratio $\alpha_{TM}/\alpha_{TE}$ may, nevertheless, still be improved and a typical order of 26% increase is gained in our example.

**VIII Concluding Remarks**

The iterative EVE has been derived for any multi-layered nonferromagnetic metallic optical slab waveguides and subsequently solved using the numerical zoom analysis technique. The method is very useful and may be exclusively employed to optimize a stratified waveguide design for which the dielectric refractive indices, fiber geometries, choice of metal, thickness, and number of metal layers are of importance. A study of this problem has shown that hundreds of decibels of extinction ratio are readily achievable at the cost of only a fraction of decibel insertion loss if optimization is carried out. This should prove valuable in designing an ideal polarizer. Examples of optimizations with respect to parameters such as wavelength have been demonstrated and propagation characteristics of the guided waves in a typical stratified optical waveguide have been investigated. The guidelines and the basic formulas about how close to the core a metal layer should be placed and how thick a metal film is judged as being thick enough, etc., have also been discussed.

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