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Theoretical analysis of mechanism of photorefractive enhancement of photochromic gratings in BSO

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We present the results of theoretical modelling of the increase in diffraction efficiency in photorefractive BSO under conditions of photorefractive enhancement via permanent photochromic gratings. The results are modelled using coupled wave theory, including the anisotropic nature of the secondary refractive index grating, optical activity and electric field induced birefringence. The dependence of the enhancement of the diffraction efficiency as a function of the input polarisation of the diffracted beam, and of the applied electric field is presented for two different crystal geometries.

1. Introduction

Bismuth silicon oxide (BSO) and other crystals from the sillenite family (BGO and BTO) have found many applications in the area of optical phase conjugation and holographic information processing. The ability to store information permanently in BSO would open up possibilities of using photorefractive crystals for information storage and retrieval applications. The writing of permanent photochromic gratings in BSO multiplexed with real time photorefractive gratings allows the simultaneous storage of two essentially different holographic gratings. The long term behaviour of the diffraction efficiency of combined permanent and real time holograms has been previously reported [1] along with applications of the two multiplexed gratings [2]. Recently the spatial redistribution of some centres that are different from photorefractive centres has also been demonstrated in BTO [3].

In ref. [4] we presented experimental results and phenomenological modelling of the enhancement in diffraction efficiency, observed by uniformly illuminating photochromic gratings written in BSO with 488 nm light, for which an increase in the diffraction efficiency >100 was observed. Controllable enhancement was demonstrated by simply varying the applied electric field or the polarisation of the read-out beam. The enhancement of the diffraction efficiency in BSO, which is relevant to optical switching and processing schemes, depends upon the mechanism of photorefractive enhancement of photochromic gratings in BSO. In this paper we further investigate these effects by considering a coupled wave theory approach which includes the anisotropic nature of the secondary refractive index grating, optical activity and electric field induced birefringence.

2. Vectorial wave mixing model

To model the enhancement of the diffraction efficiency in a material such as BSO, requires a coupled wave theory that includes the vectorial nature of the beams. It is then possible to include the anisotropic nature of induced refractive index grating, optical activity and electric field induced birefringence [5]. The absorption modulation that creates the photorefractive grating is included in a straightforward manner if it is assumed isotropic. The coupled wave equations for the readout and diffracted beam can then be written as:

$$\frac{\mathrm{d}\boldsymbol{B}}{\mathrm{d}z}+\frac{\alpha}{2}\boldsymbol{B}+\mathrm{i}\mathbf{H}\boldsymbol{B}=(k_\alpha\mathbf{I}+\mathrm{i}k_\mathrm{n}\mathbf{S})\boldsymbol{C}, \quad (1)$$

$$\frac{\mathrm{d}\boldsymbol{C}}{\mathrm{d}z}+\frac{\alpha}{2}\boldsymbol{C}+\mathrm{i}\mathbf{H}\boldsymbol{C}=(k_\alpha^*\mathbf{I}+\mathrm{i}k_\mathrm{n}^*\mathbf{S})\boldsymbol{B}. \quad (2)$$

The vectors $\boldsymbol{B}$ and $\boldsymbol{C}$ represent the orthogonal polarisation components of the diffracted and readout beams, respectively. These vectors are Jones vectors and have the following form:

$$\boldsymbol{B}=\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \quad (3)$$

$$\boldsymbol{C}=\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}. \quad (4)$$

Each element represents the amplitude of an orthogonal polarisation mode. The scalar terms $k_\alpha(z)$ and $k_\mathrm{n}(z)$ are the grating terms for the absorptive and refractive grating, respectively. These terms essentially express the strength of these gratings and their variation with z, the propagation direction. The constant α is the linear absorption coefficient.

In eqs. (1), (2) there are also two matrix terms. These are a scattering matrix S and a perturbation matrix H. The matrix S allows for the anisotropic nature of the refractive index grating. The elements of this two by two matrix are given by

$$S_{ij}=(\boldsymbol{e}_\mathrm{i}\cdot\hat{\boldsymbol{\epsilon}}\cdot\hat{\boldsymbol{R}}\cdot\hat{\boldsymbol{k}}_\mathrm{g}\cdot\hat{\boldsymbol{\epsilon}}\cdot\boldsymbol{e}_\mathrm{j}) \quad (5)$$

where $\hat{\boldsymbol{k}}_\mathrm{g}$ is the unit grating wavevector, $\hat{\boldsymbol{R}}$ the suitably normalised electro-optic tensor, $\hat{\boldsymbol{\epsilon}}$ the suitably normalised relative permittivity tensor.

The vectors $\boldsymbol{e}_i$ and $\boldsymbol{e}_j$ are unit vectors in the direction of the allowed orthogonal polarisation modes. In the case of the cubic photorefractive material BSO, in its two most commonly used orientations, the scattering matrix is

(i)

$$\mathbf{S}=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (6)$$

for the $\hat{\boldsymbol{k}}_\mathrm{g}\parallel\langle 110\rangle$ crystal direction.

(ii)

$$\mathbf{S}=\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad (7)$$

for the $\hat{\boldsymbol{k}}_\mathrm{g}\parallel\langle 001\rangle$ crystal direction.

In eqs. (1)–(2) it may be noted that no scattering matrix is associated with the absorption grating term $k_\alpha(z)$. As stated earlier, the absorption grating is assumed isotropic. Therefore, the identity matrix **I** appears with this grating term. The perturbation matrix **H** is used to introduce the effects of optical activity and electric field induced birefringence. Optical activity is represented as

$$\mathbf{H}=\rho\begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \quad (8)$$

where ρ is the rotary power (rad m^{-1}).

To see that **H** does in fact represent a rotation of the polarisation is easily shown. Considering no absorptive or refractive perturbations to the medium, the coupled equation for a single wave is

$$\mathrm{d}\boldsymbol{A}/\mathrm{d}z+\mathrm{i}\mathbf{H}\boldsymbol{A}=0, \quad (9)$$

as **H** is constant this is immediately integrated to give

$$\boldsymbol{A}(z)=\boldsymbol{A}(0)\exp(\mathrm{i}\mathbf{H}z). \quad (10)$$

The operator **H** is hermitian and traceless, therefore the exponential operator is unitary. Hence, if

$$\boldsymbol{A}(0)=\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (11)$$

it is straight forward to show that

$$\boldsymbol{A}(z)=\begin{pmatrix} \cos(\rho z) \\ \sin(\rho z) \end{pmatrix}. \quad (12)$$

Thus the polarisation vector is rotated as the wave propagates through the material. In the case of electric field induced birefringence the matrix **H** is given by

$$\mathbf{H}=\beta\mathbf{S}, \quad (13)$$

where **S** is the scattering matrix.

This is not necessarily the same as the scattering matrix of the induced refractive index grating, but if the electric field is applied in the same direction as the grating wavevector, then the two are equivalent. The constant β is given by

$$\beta=\tfrac{1}{2}k_0 n^3 r_{41}|E_\mathrm{A}|, \quad (14)$$

where $|E_\mathrm{A}|$ is the modulus of the applied electric field and r_{41} the relevant electro-optic coefficient.

3. The solution of the coupled wave equations

The solution of the coupled equations given in the previous section is in general very difficult. A number of approximations are made so that progress can be made. The first approximation is to assume an undepleted read beam $\boldsymbol{C}$. This means wave $\boldsymbol{C}$ loses a negligible amount of energy through diffraction into wave $\boldsymbol{B}$. Then

$$\mathrm{d}\boldsymbol{B}/\mathrm{d}z+\tfrac{1}{2}\alpha\boldsymbol{B}+\mathrm{i}\mathbf{H}\boldsymbol{B}=(k_\alpha\mathbf{I}+\mathrm{i}k_\mathrm{n}\mathbf{S})\boldsymbol{C}\,, \tag{15}$$

$$\mathrm{d}\boldsymbol{C}/\mathrm{d}z+\tfrac{1}{2}\alpha\boldsymbol{C}+\mathrm{i}\mathbf{H}\boldsymbol{C}=0\,. \tag{16}$$

The matrix $\mathbf{H}$ can be, for the cases that will be investigated, considered a hermitian matrix. This can be decomposed as

$$\mathbf{H}=h_0\mathbf{I}+\mathbf{H}'\,, \tag{17}$$

where $\mathbf{H}'$ is hermitian and traceless.

Therefore, the introduction of the following transformation upon the beam vectors

$$\boldsymbol{B}'=\boldsymbol{B}\exp[-(\alpha/2+h_0)z]\,, \tag{18}$$

$$\boldsymbol{C}'=\boldsymbol{C}\exp[-(\alpha/2+h_0)z]\,, \tag{19}$$

leaves the coupled equations as

$$\mathrm{d}\boldsymbol{B}'/\mathrm{d}z=\mathrm{i}\mathbf{H}'\boldsymbol{B}=(k_\alpha\mathbf{I}+\mathrm{i}k_\mathrm{n}\mathbf{S})\boldsymbol{C}'\,, \tag{20}$$

$$\mathrm{d}\boldsymbol{C}'/\mathrm{d}z+\mathrm{i}\mathbf{H}'\boldsymbol{C}=0\,. \tag{21}$$

A unitary transformation can now be performed upon the beam vectors so that

$$\boldsymbol{B}''=\mathbf{U}\boldsymbol{B}'\,, \tag{22}$$

$$\boldsymbol{C}''=\mathbf{U}\boldsymbol{C}' \tag{23}$$

and the coupled equations become

$$\mathrm{d}\boldsymbol{B}''/\mathrm{d}z=(k_\alpha\mathbf{I}+\mathrm{i}k_\mathrm{n}\mathbf{U}\mathbf{S}\mathbf{U}^\dagger)\boldsymbol{C}''\,, \tag{24}$$

$$\mathrm{d}\boldsymbol{C}''/\mathrm{d}z=0\,, \tag{25}$$

where

$$\mathbf{U}=\exp(\mathrm{i}\mathbf{H}'z)\,. \tag{26}$$

Therefore, $\boldsymbol{C}''$ is a constant and with the boundary condition for the readout beam $\boldsymbol{B}(0)=0$, the solution for $\boldsymbol{B}''(L)=0$ is given by

$$\boldsymbol{B}''(z)=\int_0^L(k_\alpha\mathbf{I}+\mathrm{i}k_\mathrm{n}\mathbf{U}\mathbf{S}\mathbf{U}^\dagger)\,\mathrm{d}z\,\boldsymbol{C}''\,. \tag{27}$$

In all cases of interest the scattering matrix $\mathbf{S}$ is also hermitian, and can be decomposed to

$$\mathbf{S}=s_0\mathbf{I}+\mathbf{S}'\,, \tag{28}$$

where $\mathbf{S}'$ is now hermitian and traceless. Therefore

$$\boldsymbol{B}''(L)=\int_0^L\{[k_\alpha+\mathrm{i}s_0k_\mathrm{n}]\mathbf{I}+\mathrm{i}[k_\mathrm{n}\mathbf{U}\mathbf{S}'\mathbf{U}^\dagger]\}\,\mathrm{d}z\,\boldsymbol{C}'' \tag{29}$$

or

$$\boldsymbol{B}''(L)=(u\mathbf{I}+v\mathbf{T})\boldsymbol{C}''\,. \tag{30}$$

The experimentally observable is the diffraction efficiency, and this is given by

$$\eta=\frac{(\boldsymbol{B}(L),\boldsymbol{B}(L))}{(\boldsymbol{C}(0),\boldsymbol{C}(0))}=\frac{(\boldsymbol{B}''(L),\boldsymbol{B}''(L))}{(\boldsymbol{C}''(0),\boldsymbol{C}''(0))}\times\exp[-(\alpha L+2h_0)]\,, \tag{31}$$

where $(.,.)$ is the hermitian inner product with definition $(X,\ Y)=\Sigma_iX_iY_i^*$.

Taking $(\boldsymbol{C}''(0),\ \boldsymbol{C}''(0))=1$ gives the diffraction efficiency η in the following form

$$\eta=\exp[-(\alpha L+2h_0)]\,[|u|^2+uv^*(\boldsymbol{C}'',\mathbf{T}\boldsymbol{C}'')+u^*v(\mathbf{T}\boldsymbol{C}'',\boldsymbol{C}'')+|v|^2(\mathbf{T}\boldsymbol{C}'',\mathbf{T}\boldsymbol{C}'')]\,. \tag{32}$$

To find analytical solutions it is assumed that the grating term k_n is constant in z. Then

$$u=\int_0^L k_\alpha\,\mathrm{d}z+\mathrm{i}s_0k_\mathrm{n}L=\bar{k}_\alpha L+\mathrm{i}s_0k_\mathrm{n}L\,, \tag{33}$$

where $\bar{k}_\alpha$ is the space averaged absorption grating term.

$$v=\mathrm{i}k_\mathrm{n}L \tag{34}$$

and

$$\mathbf{T}=\frac{1}{L}\int_0^L\mathbf{U}\mathbf{S}'\mathbf{U}^\dagger\,\mathrm{d}z\,. \tag{35}$$

To obtain explicit expressions for diffraction efficiency specific cases will have to be considered. This is shown in the next section.

4. Solution for specific cases

The two cases to be considered are for the crystal orientated in its two most common orientations, these are given below as cases (A) and (B).

(A) Applied field and grating wavevector $\hat{k}_g \parallel \langle 110 \rangle$ crystal direction (fig. 1a).

(B) Applied field $\parallel \langle 110 \rangle$ crystal direction, and grating wavevector $\hat{k}_g \parallel \langle 001 \rangle$ crystal direction (figure 1b).

In case (B) the applied field being perpendicular to the grating wavevector is a slight departure from the usual case, but this is still easily accommodated within the theory. To manipulate the equation further, a convenient decomposition of the hermitian

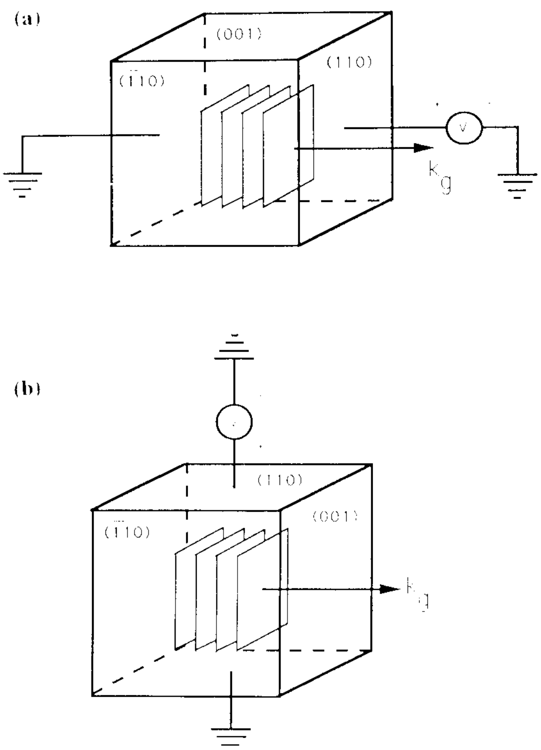

Fig. 1. Principal crystal orientations. (a) $\hat{k}_g$ and applied field $\parallel \langle 110 \rangle$ direction. (b) $\hat{k}_g \parallel \langle 001 \rangle$, and applied field $\parallel \langle 110 \rangle$ direction.

traceless matrices $\mathbf{H}'$ and $\mathbf{S}'$ in terms of the Pauli spin matrices is adopted. The Pauli spin matrices are defined here as:

$$\boldsymbol{\sigma}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{36}$$

$$\boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{37}$$

$$\boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}. \tag{38}$$

These are supplemented with the identity matrix, which is now renamed $\boldsymbol{\sigma}_0$. The matrices $\mathbf{S}$ and $\mathbf{H}$ can now be expanded in terms of these composite matrices:

$$\mathbf{S} = s_0 \boldsymbol{\sigma}_0 + \mathbf{S}' = s_0 \boldsymbol{\sigma}_0 + \underline{s} \cdot \underline{\sigma}$$

$$= s_0 \boldsymbol{\sigma}_0 + s_1 \boldsymbol{\sigma}_1 + s_2 \boldsymbol{\sigma}_2 + s_3 \boldsymbol{\sigma}_3 \tag{39}$$

and similarly for $\mathbf{H}$:

$$\mathbf{H} = h_0 \boldsymbol{\sigma}_0 + h_1 \boldsymbol{\sigma}_1 + h_2 \boldsymbol{\sigma}_2 + h_3 \boldsymbol{\sigma}_3 . \tag{40}$$

In case (A) with the electric field and grating wave vector parallel to the $\langle 110 \rangle$ crystal direction, $\mathbf{S}$ and $\mathbf{H}$ are given by

$$\mathbf{S} = \mathbf{S}' = s_1 \boldsymbol{\sigma}_1 , \tag{41}$$

with $s_1 = 1$

$$\mathbf{H} = \mathbf{H}' = h_1 \boldsymbol{\sigma}_1 + h_3 \boldsymbol{\sigma}_3 , \tag{42}$$

with $h_1 = \beta$ and $h_3 = \rho$.

In case (B) with the electric field parallel to the $\langle 110 \rangle$ crystal direction, and the grating wave vector parallel to the $\langle 001 \rangle$ direction $\mathbf{S}$ and $\mathbf{H}$ are given by

$$\mathbf{S} = s_0 \boldsymbol{\sigma}_0 + s_1 \boldsymbol{\sigma}_1 , \tag{43}$$

with $s_0 = s_1 = \frac{1}{2}$. This gives $\mathbf{S}' = \frac{1}{2}\sigma_1$, and

$$\mathbf{H} = \mathbf{H}' = h_1 \boldsymbol{\sigma}_1 + h_3 \boldsymbol{\sigma}_3 , \tag{44}$$

again with $h_1 = \beta$ and $h_3 = \rho$.

Therefore the expression of u in eq. (33) is easily found in each case. It is then only the expression for matrix $\mathbf{T}$ in eq. (35) that requires evaluation.

$$\mathbf{T} = \frac{1}{L} \int_0^L \mathbf{U}\mathbf{S}'\mathbf{U}^\dagger \, \mathrm{d}z , \tag{45}$$

$$\mathbf{U} = \exp(\mathrm{i}\mathbf{H}'z) . \tag{46}$$

As $\mathbf{H}'$ is hermitian and traceless, the matrix $\mathbf{U}$ is unitary. Also $\mathbf{S}'$ is hermitian and traceless. Therefore, $\mathbf{U}\mathbf{S}'\mathbf{U}^\dagger$ represents a "rotation" of matrix $\mathbf{S}'$. This can be seen if a three space vector approach is taken. To show this, $\mathbf{U}$ is expanded into its linear form:

$$\mathbf{U}=\cos(hz)\,\mathbf{I}+\mathrm{i}\,\sin(hz)\,\mathbf{H}'/h\,, \tag{47}$$

where h is a positive eigenvalue of the matrix $\mathbf{H}'$.

It has already been shown that matrices $\mathbf{S}'$ and $\mathbf{H}'$ can be expanded in terms of the Pauli spin matrices. Therefore, $\mathbf{S}'$ and $\mathbf{H}'$ can be represented as

$$\mathbf{S}'=\underline{s}\cdot\underline{\sigma}\,,$$

$$\mathbf{H}'=\underline{h}\cdot\underline{\sigma}\,, \tag{48}$$

with

$$\underline{s}=\begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}, \tag{49}$$

$$\underline{h}=\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}, \tag{50}$$

$$\underline{\sigma}=\begin{pmatrix} \boldsymbol{\sigma}_1 \\ \boldsymbol{\sigma}_2 \\ \boldsymbol{\sigma}_3 \end{pmatrix}. \tag{51}$$

If $\mathbf{U}\mathbf{S}'\mathbf{U}^\dagger$ is multiplied out, and the following relations for commutator and anticommutator are used, a geometrical interpretation of $\mathbf{U}\mathbf{S}'\mathbf{U}^\dagger$ is found.

$$[\hat{\mathbf{H}},\mathbf{S}']=2\mathrm{i}(\hat{\underline{h}}\times\underline{s})\cdot\underline{\sigma}\,, \tag{52}$$

$$(\hat{\mathbf{H}},\mathbf{S})=2(\hat{\underline{h}}\cdot\underline{s})\cdot\boldsymbol{\sigma}_0\,, \tag{53}$$

where $\hat{\mathbf{H}}=\mathbf{H}/h$ (the unit operator) and $\hat{\underline{h}}=\underline{h}/h$ (the unit vector).

Therefore,

$$\mathbf{U}\mathbf{S}'\mathbf{U}^\dagger=\{(\hat{\underline{h}}\cdot\underline{s})\hat{\underline{h}}+[\underline{s}-(\hat{\underline{h}}\cdot\underline{s})\hat{\underline{h}}]\cos(2hz)$$
$$-(\hat{\underline{h}}\times\underline{s})\sin(2hz)\}\cdot\underline{\sigma}\,. \tag{54}$$

The vector within brackets $\{\}$ is shown in fig. 2. This figure shows that vector $\underline{s}$ is rotated to vector $\underline{s}'$ at a rate $\theta=2hz$. Hence h, which depends upon the material anisotropy, the field induced birefringence and the optical activity controls the rate at which scattering of the readout beam diffracts into each polarisation mode of diffracted beam $\boldsymbol{B}$. The matrix $\mathbf{T}$

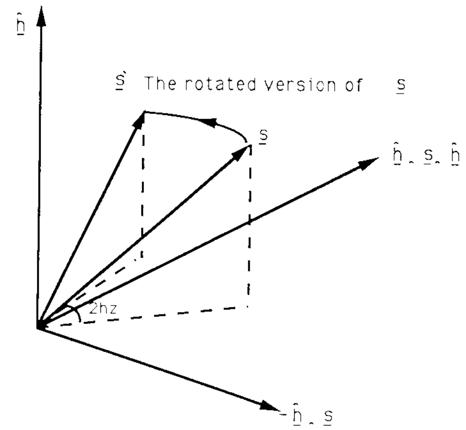

Fig. 2. The rotation of the vector $\underline{s}$ by the amount $2hz$. This pictorially represents the rate at which each polarisation element of the diffracted beam is altered by propagation through the medium.

then represents a space averaged form of this scattering. The integration is easily performed and $\mathbf{T}$ is given by

$$\mathbf{T}=\{(\hat{\underline{h}}\cdot\underline{s})\hat{\underline{h}}+[\underline{s}-(\hat{\underline{h}}\cdot\underline{s})\hat{\underline{h}}]\,\mathrm{sinc}(2hL)$$
$$-(\hat{\underline{h}}\times\underline{s})[(1-\cos(2hL))/2hL]\}\cdot\underline{\sigma}\,. \tag{55}$$

If $\mathbf{T}$ is represented as $\mathbf{T}=\underline{t}\cdot\underline{\sigma}$ then is case (A) where

$$\underline{s}=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \tag{56}$$

and

$$\underline{h}=\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}=\begin{pmatrix} \beta \\ 0 \\ \rho \end{pmatrix}, \tag{57}$$

then $\underline{t}=\underline{t}_\mathrm{A}$ is given by

$$\underline{t}_\mathrm{A}=\begin{pmatrix} t_{1\mathrm{A}} \\ t_{2\mathrm{A}} \\ t_{3\mathrm{A}} \end{pmatrix}$$
$$=\begin{bmatrix} \dfrac{h_1^2}{h^2}+\left(1-\dfrac{h_1^2}{h^2}\right)\mathrm{sinc}(2hL) \\ \dfrac{h_1h_2}{h^2}\,[1-\mathrm{sinc}(2hL)]-\dfrac{h_3}{h}\left(\dfrac{1-\cos(2hL)}{2hL}\right) \\ \dfrac{h_1h_3}{h^2}\,[1-\mathrm{sinc}(2hL)]+\dfrac{h_2}{h}\left(\dfrac{1-\cos(2hL)}{2hL}\right) \end{bmatrix}. \tag{58}$$

In case (B) where

$$\underline{s}=\begin{pmatrix}\frac{1}{2}\\0\\0\end{pmatrix}, \quad (59)$$

$\underline{t}=\underline{t}_B$ is simply given by

$$\underline{t}_B=\begin{pmatrix}t_{1B}\\t_{2B}\\t_{3B}\end{pmatrix}=\tfrac{1}{2}\underline{t}_A . \quad (60)$$

In both cases the eigenvalue h is given by

$$h=(h_1^2+h_2^2+h_3^2)^{1/2}=\sqrt{\beta^2+\rho^2} . \quad (61)$$

The readout beam $\boldsymbol{C}$ is chosen so that the expression for the diffraction efficiency in eq. (32) can be evaluated. At $z=0$ the vector $\boldsymbol{C}=\boldsymbol{C}''$, and can be written as

$$\boldsymbol{C}=\begin{pmatrix}\cos\phi\\\sin\phi\end{pmatrix}, \quad (62)$$

where ϕ is the angle that linearly polarised light of the readout beam makes with the fast induced axis of the crystal.

Since $\mathbf{T}$ is hermitian and $\mathbf{T}^2=t^2\mathbf{I}$, where t is the positive eigenvalue of $\mathbf{T}$, the diffraction efficiency expression of eq. (32) can now be written as

$$\begin{aligned}\eta=\exp(-\alpha L)\{&|\bar{k}_\alpha L+\mathrm{i}s_0k_nL|^2\\&+[(\bar{k}_\alpha L+\mathrm{i}s_0k_nL)(-\mathrm{i}k_n^*L)\\&+(\bar{k}_\alpha^*L-\mathrm{i}s_0k_n^*L)\,\mathrm{i}k_nL](\mathbf{T}\boldsymbol{C}'',\boldsymbol{C}'')\\&+|\mathrm{i}k_nL|^2t^2(\boldsymbol{C}'',\boldsymbol{C}'')\} . \end{aligned}\quad (63)$$

The inner product $(\boldsymbol{C}'',\boldsymbol{C}'')=1$, and $(\mathbf{T}\boldsymbol{C}'',\boldsymbol{C}'')$ can be evaluated as follows

$$\begin{aligned}(\mathbf{T}\boldsymbol{C}'',\boldsymbol{C}'')&=t_1(\boldsymbol{\sigma}_1\boldsymbol{C}'',\boldsymbol{C}'')\\&\quad+t_2(\boldsymbol{\sigma}_2\boldsymbol{C}'',\boldsymbol{C}'')+t_3(\boldsymbol{\sigma}_3\boldsymbol{C}'',\boldsymbol{C}'')\\&=t_1\cos2\phi+t_2\sin2\phi . \end{aligned}\quad (64)$$

In case (A) $s_0=0$ and $\mathbf{T}=\underline{t}_A\cdot\underline{\sigma}$, therefore

$$\begin{aligned}\eta_A=\mathrm{e}^{-\alpha L}\{&|\bar{k}_\alpha L|^2+[\mathrm{i}(-\bar{k}_\alpha k_n^*L^2+\bar{k}_\alpha^*k_nL^2)\\&\times(t_{1A}\cos2\phi+t_{2A}\sin2\phi)+|\mathrm{i}k_nL|^2t_A\} , \end{aligned}\quad (65)$$

where t_A is the length of vector $\underline{t}_A$ and t_{1A}, t_{2A} are the first two components of vector $\underline{t}_A$.

In case (B) where $s_0=\frac{1}{2}$ and $\mathbf{T}=\underline{t}_B\cdot\sigma$:

$$\begin{aligned}\eta_B=\mathrm{e}^{-\alpha L}\{&|\bar{k}_\alpha L+\mathrm{i}k_nL/2|^2\\&+[(\bar{k}_\alpha L+\mathrm{i}k_nL/2)(-\mathrm{i}k_n^*L)+(\bar{k}_\alpha^*-\mathrm{i}k_n^*L/2)]\\&\times(t_{1B}\cos2\phi+t_{2B}\sin2\phi)+|\mathrm{i}k_nL|^2t_B\} , \end{aligned}\quad (66)$$

where t_B is the length of vector $\underline{t}_B$ and t_{1B}, t_{2B} are the first two components of vector $\underline{t}_B$.

These are the two analytical expressions required for the predictions of diffraction efficiencies from the photorefractive enhanced photochromic grating. All that is required are estimates for $\bar{k}_\alpha$ and k_n.

5. The evaluation of $\bar{k}_\alpha$ and k_n

The grating terms can be written as

$$k_n=(\gamma_{nr}+\mathrm{i}\gamma_{ni})m_n/2 , \quad (67)$$

$$\bar{k}_\alpha=(\gamma_{\alpha r}+\mathrm{i}\gamma_{\alpha i})\bar{m}_\alpha/2 , \quad (68)$$

where m_n is the modulation of the refractive grating and $\bar{m}_\alpha$ the space averaged modulation of the absorption grating.

The γ terms represent the real and imaginary parts of the coupling constants for each grating. Indices n and α represent refractive and absorptive parts, respectively, and indices r and i represent real and imaginary parts, respectively. It can immediately be assumed that the absorptive grating will be in phase with the illumination, therefore $\gamma_{\alpha i}=0$. To find values for the quantity $\gamma_{\alpha r}$ is difficult, and will be left as a fitting parameter. The photorefractive part of the grating will in general be out of phase with the illumination. Hence

$$k_n=\Gamma\mathrm{e}^{\mathrm{i}\psi}m_n/4 , \quad (69)$$

where Γ is the intensity coupling constant and ψ the grating phase shift.

The modulation m_n can be connected to the modulation $\bar{m}_\alpha$, therefore, it is only Γ and ψ that need be evaluated. This can be achieved by solving the band transport equations in the usual manner [6]:

$$\gamma=2\Gamma\mathrm{e}^{\mathrm{i}\psi}=\tfrac{1}{2}kn^3r_{41}E_1 , \quad (70)$$

where

$$E_1 = \frac{-\frac{1}{2}(i\nu + E_A)}{1 + i\nu(E_A + i\nu)}, \tag{71}$$

where ν is the normalised grating spatial frequency and E_A the normalised applied field.

In the case under consideration the space charge field can generally be assumed unsaturated. Therefore

$$E_1 \simeq -\tfrac{1}{2}(i\nu + E_A) . \tag{72}$$

This expression may need some modification because it is possible that the photochromic grating causes a redistribution of the available donor sites, which may cause an asymmetry in the size of the space charge field with respect to the applied electric field.

The expressions (65) and (66) were used to generate theoretical curves for the enhancement factor, which is defined as the diffraction efficiency with a photorefractive grating divided by the diffraction efficiency with only the photochromic grating, versus the input polarisation ϕ and the applied electric field E_A. The refractive part of the grating, without any applied field, was assumed to have a strength of $\Gamma = 100\ \mathrm{m}^{-1}$, and the absorptive part of the grating was assumed to have a strength $\gamma_{\alpha r} = 20\ \mathrm{m}^{-1}$. The optical activity parameter was $\rho = 30°\ \mathrm{mm}^{-1}$, the linear absorption at 633 nm $\alpha = 30\ \mathrm{m}^{-1}$. The results of this analysis can be seen in figs. 3 to 6. In figures 3 and 4 the variation of enhancement with input polarisation is seen. Figure 3 shows the case of the

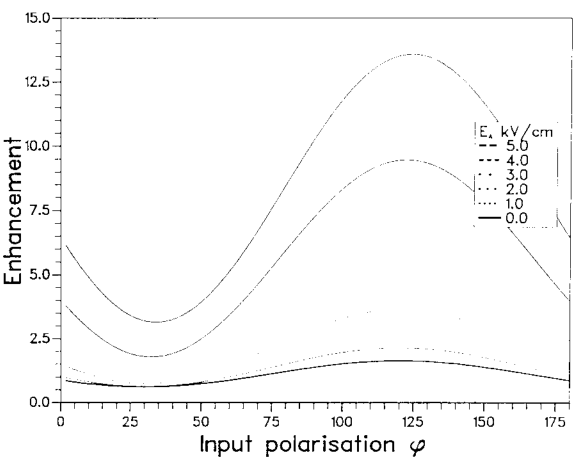

Fig. 3. Enhancement factor as a function of input polarisation for the ⟨001⟩ geometry, for various applied fields.

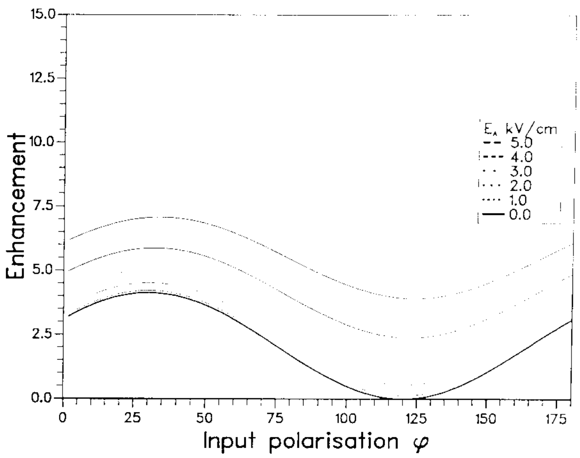

Fig. 4. Enhancement factor as a function of input polarisation for the ⟨110⟩ geometry, for various applied fields.

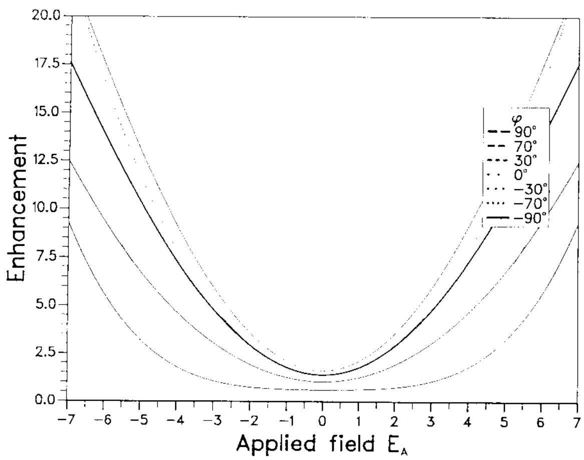

Fig. 5. Enhancement factor as a function of applied field for the ⟨001⟩ geometry, for various input readout beam polarisations.

⟨001⟩ geometry. Figure 4 shows the ⟨110⟩ geometry. It can be seen that the enhancements are π out of phase, and that enhancements are generally better for the ⟨001⟩ geometry. Figures 5 and 6 show the variation with applied electric field. Figure 5 shows the ⟨001⟩ geometry and fig. 6 shows the ⟨110⟩ geometry. This again shows that enhancements are greater for the ⟨001⟩ geometry.

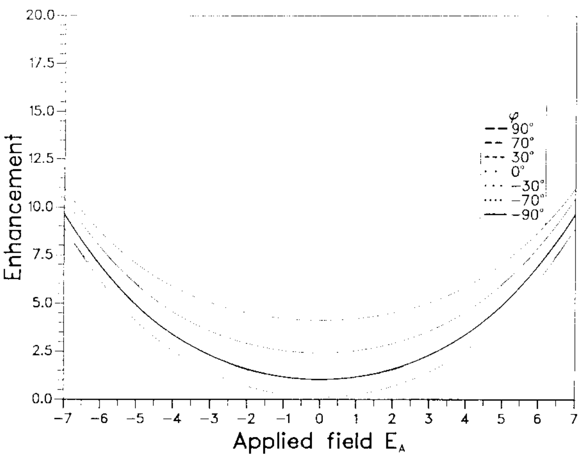

Fig. 6. Enhancement factor as a function of applied field for the ⟨110⟩ geometry, for various input readout beam polarisations.

6. Conclusion

The results of the theoretical modelling, in particular the enhancement as a function of the input polarisation, shows good qualitative agreement with experimental observations [4]. The quantitative agreement is not exact, but no systematic attempt has been made at direct comparison. The quantitative agreement could be improved by including into the model the effect of the asymmetric density of donor/acceptor sites induced by the photochromic mechanism. Further quantitative agreement may be achievable by measuring accurately the experimental values of absorption coefficient, grating strength and other experimental parameters. From these results it is clear that a permanent photochromic grating does induce a secondary photorefractive grating upon uniform illumination of the crystal containing the photochromic gratings. The induced photorefractive grating produced by uniformly illuminating these photochromic gratings is essentially equivalent to the interference pattern produce by interfering two beams. Further improvement of this coupled wave equation model is possible, but is not being pursued at this point.

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