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Optical logic operations via polarization encoding in a phase conjugate Michelson interferometer

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We demonstrate a technique of parallel optical logic operations performed on two two-dimensional input variables. The technique is based upon a cascaded phase conjugate Michelson interferometric (PCMI), arrangement, in which multiple signal beams, sharing a common reference beam, are used to record angularly multiplexed holograms in a single crystal of BSO. The logic operations are performed at the output port of a beamsplitter via polarization manipulation of the two phase conjugate outputs. When the PCMI is working in polarization preserving mode, complete subtraction is achieved. However, when the PCMI operates in non-polarization preserving mode, and the two phase conjugate beams are of nearly orthogonal polarizations to that of input polarizations, coherent addition is observed. By varying the polarization angle of the two phase conjugate outputs, continuous logic operations are achieved. We present the experimental results of all sixteen basic logic operations on two variables.

1. Introduction

In recent years digital electronic processing of images has proved to be increasingly reliable and versatile. Its speed, however, is fundamentally limited by its serial nature. During the last decade it has been realized that optical techniques may have an important role in implementing logic schemes and other two-dimensional processing operations, in ways that are fundamentally different from those of electronic techniques. In optical systems the inputs and outputs are usually two-dimensional spatial patterns, and therefore have an inherent speed advantage due to the inbuilt parallelism.

Optical Boolean logic gates may be considered as the basic elements of a digital optical processor. An early implementation of optical logic demonstrating the technique of image subtraction of complex amplitudes was performed by Gabor *et al.* [1] who used a double exposure holographic method. This technique however, can only be applied on permanently recorded data. Yu *et al.* [2] have reported a technique of optical logic based upon two magneto-optic spatial light modulators (MOSLM). This requires a high degree of collimation and critical alignment of the beams to preserve the one-to-one pixel relationship required. The reported results were shown to contain more than two grey levels.

Yu *et al.* [3] have also demonstrated Boolean operations via an optoelectronic technique which relies on a micro-channel spatial light modulator and liquid crystal TVs. In their technique various modes of operation of a MOSLM are controlled by computer and associated electronics. Their results showed image distortion, arising mainly from imperfection of the MOSLM operation. A single pixel logic operation, to provide both the sum and carry outputs of full addition, based upon the reflection and transmission characteristics of nonlinear Fabry–Perot etalons has been demonstrated by Tooley *et al.* [4]. In this technique the discrimination between low and high levels of the input and output is critical, and certain problems concerning alignment and the requirement of gain in the system need to be addressed.

A different technique which is based upon spatial filtering has been demonstrated by Weigelt [5]. In this scheme the input logical value is encoded by statistical structures, theta modulation, fringe speckles etc. and logical value 0, encoded as transparency, are placed in a cascaded spatial filtering set up. The technique, however, requires a complex and time consuming encoding of the input and needs different combinations of Fourier plane masks. Image resolution and light

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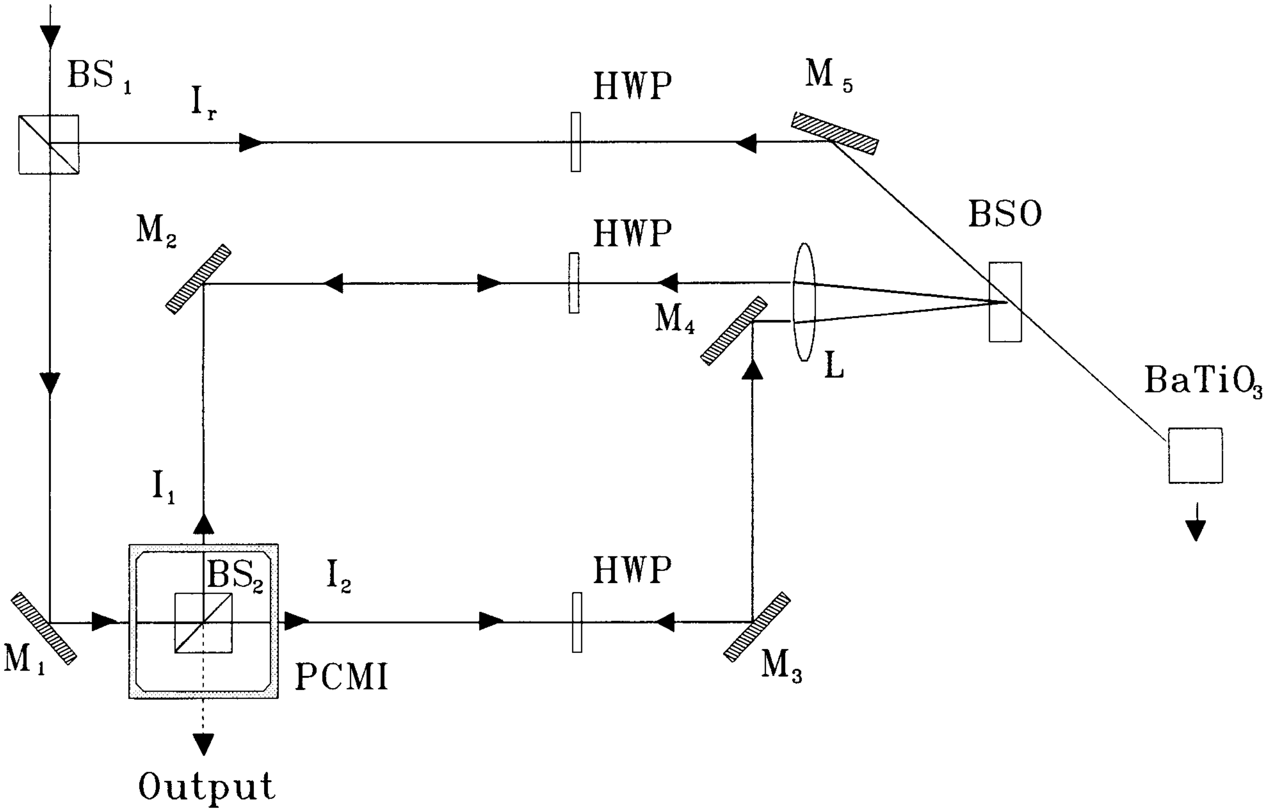


Figure 1. A basic configuration of PCMI is shown where a BSO crystal is used as a phase conjugate mirror and a self-pumped $BaTiO_3$ is used to provide a readout beam for the two holograms: BS beamsplitter, M mirror, HWP half-wave plate.

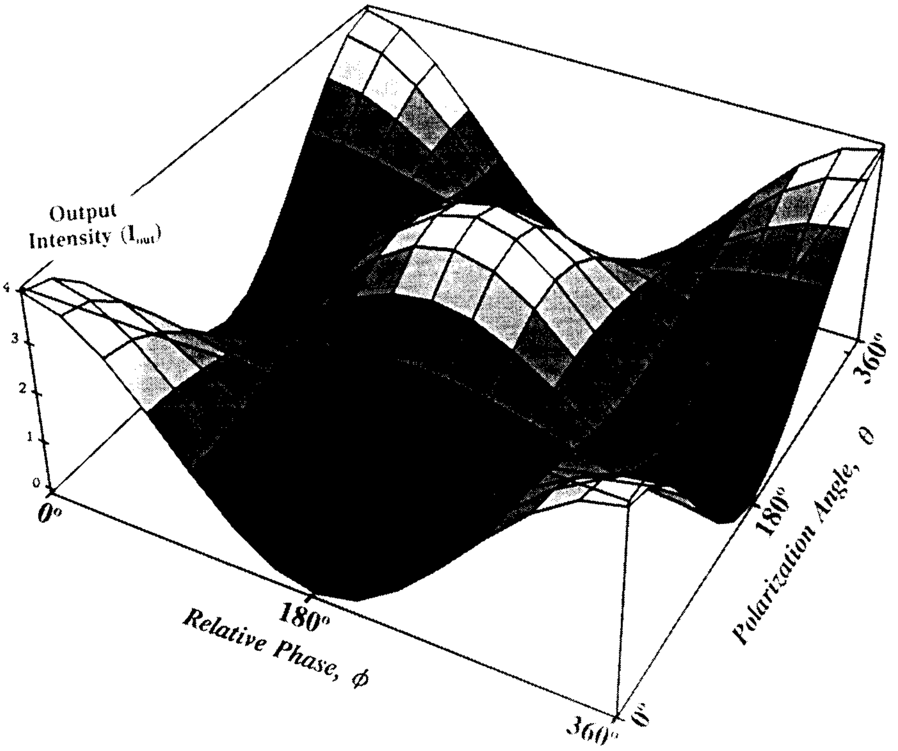


Figure 2. A computed three-dimensional graph of output intensity is shown as a function of phase angle ϕ and polarization angle θ between the two phase conjugate outputs at the beamsplitter.

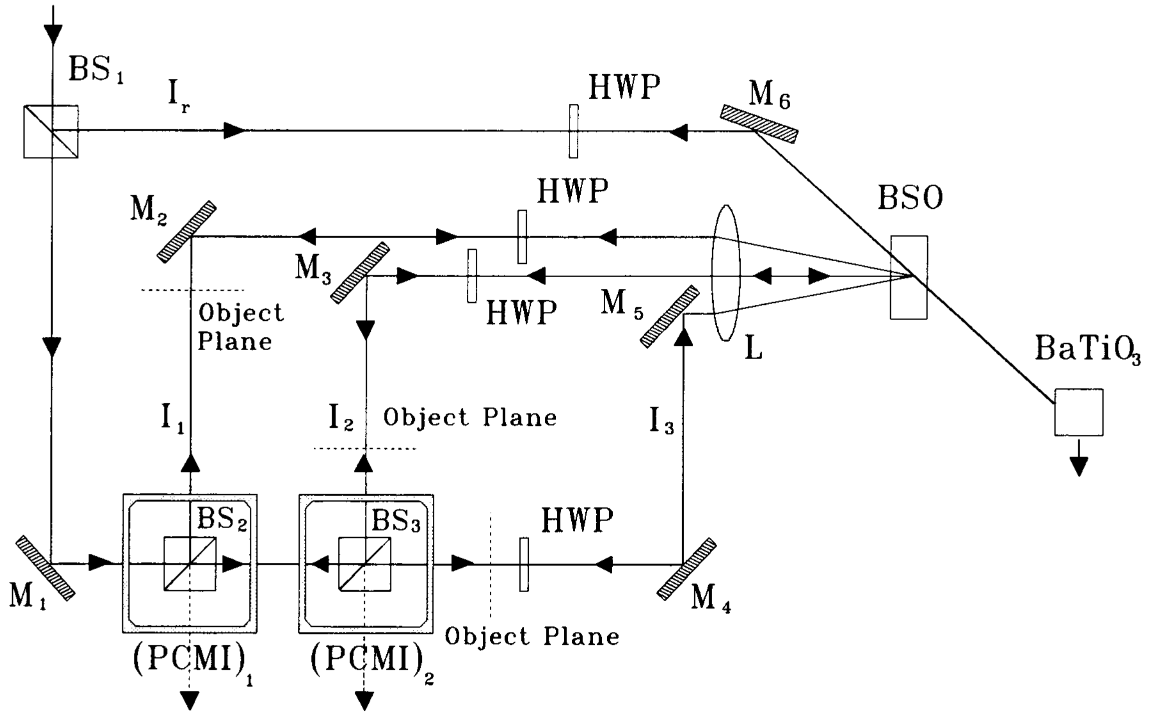


Figure 3. A cascaded PCMI scheme is shown where the phase conjugate output at the input port of $(PCMI)_2$ serves as one of the inputs to $(PCMI)_1$. All 15 basic logic operations can be performed using this system: BS beamsplitter, M mirror, HWP half-wave plate.

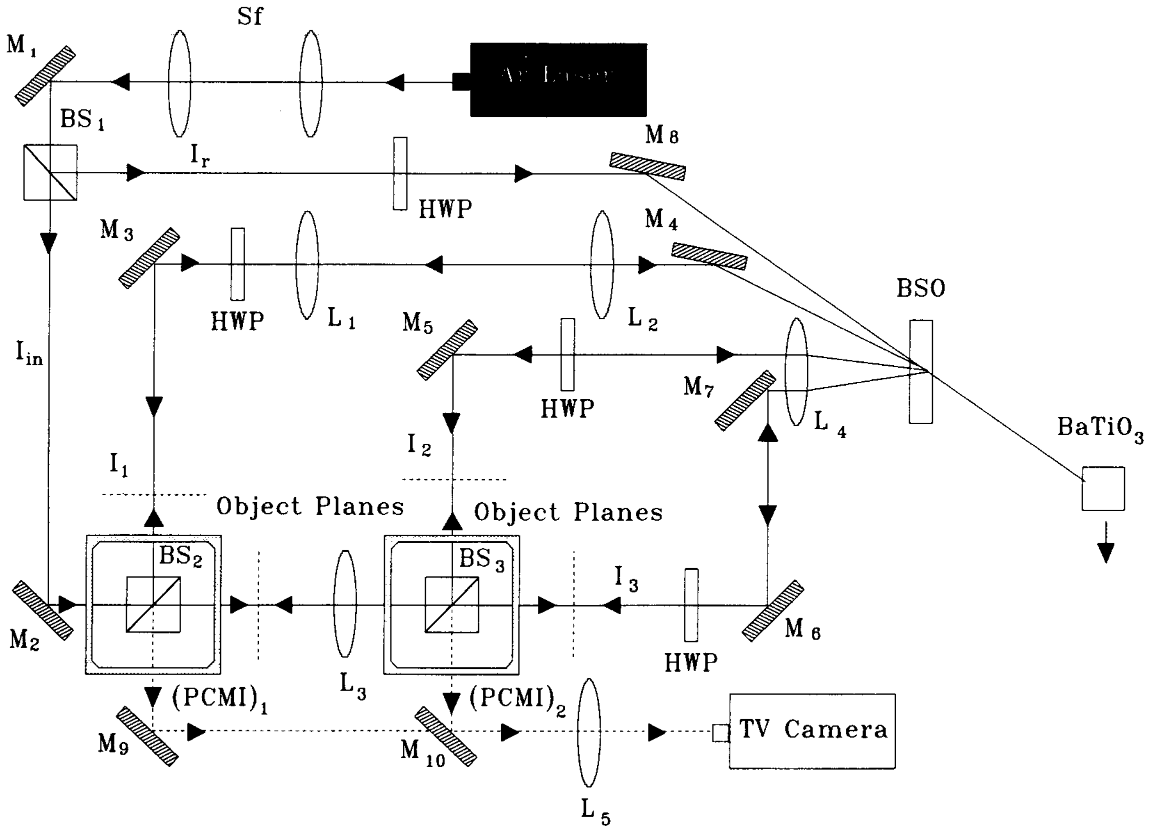


Figure 4. This schematic diagram shows the experimental arrangement for the optical logic operations.

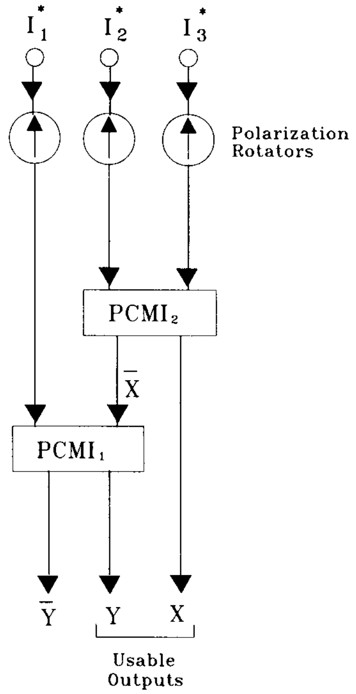


Figure 5. A block diagram is shown of cascaded PCMI. X or Y are the useable outputs depending on the operation performed.

intensity is also lost in every stage of the spatial filtering. Different Fourier plane spatial filtering masks are also required for different logic operations making the technique rather inflexible. A technique of polarization based logic has also been reported by Lohmann *et al.* [6] in which the logic operations are performed at non-real time rates because of the slow nature of encoding the input and output data.

Optical subtraction (XOR), and addition has also been implemented by the use of classical interferometric techniques such as Mach–Zender or Michelson interferometry [7]. However, it is well known that such interferometers require a high degree of stability, and are extremely difficult to adjust (and maintain) the fixed path length between the two arms. In addition only the central fringe is useful for image subtraction and addition purposes which in many cases is not enough to cover the full image field.

In recent years optical phase conjugation (OPC), by degenerate four-wave mixing (DFWM) in nonlinear media such as photorefractive crystals, has emerged as a versatile low power technique for implementing optical data processing schemes. Optical image subtraction has been achieved by OPC via double exposure methods by Huignard *et al.* [8], who reported image subtraction and parallel optical logic in $LiNbO_3$. A similar technique was also reported by Ja [9] via a reflection grating configuration in $Bi_{12}GeO_{20}$. These techniques, however, exploit sequential grating recording and erasing cycles which limits their general application or usefulness.

By replacing the conventional mirrors in a Michelson interferometer with phase conjugate mirrors, we can combine the advantages of both, to construct a phase conjugate Michelson interferometer (PCMI). Because of the dynamic nature of the photorefractive recording and readout process, we therefore ensure temporal stability of the output against variations in the optical path length of the two arms, thermal drifts etc., and any phase irregularities due to the optical components used are exactly cancelled because of the double pass nature of OPC. Furthermore, the interferometer is also self aligning. Normally, under conditions of polarization preserving phase conjugation destructive interference occurs at the output of such a PCMI. Using such a scheme, some (but not the complete set), of the basic logic operations have already been demonstrated [10–12].

2. Theoretical considerations

The basic PCMI is illustrated in figure 1, in which BSO is used as the phase conjugate mirror (PCM). An input beam is split via a beam-splitter BS_2, to form separate inputs I_1 and I_2. Beam I_1 subsequently passes through a half-wave plate, and is directed towards the BSO. Beam I_2 passes through a second half-wave plate, and is also directed towards the BSO. A reference beam I_r interferes with these two beams inside the BSO, to record two angularly multiplexed gratings. The transmitted reference beam is phase conjugated via a self-pumped $BaTiO_3$ crystal, to become the horizontally polarized readout beam.

The BSO crystal was used in the $K_g \perp \langle 001 \rangle$ configuration where it performs as a half wave plate with respect to the polarization direction of the readout beam (for the ideal thin crystal case) [14]. In such a set up as in figure 1, it is easy to implement a polarization preserving phase conjugator. When the input polarizations of the two signal beams I_1 and I_2 are set in a horizontal plane before passing through the half wave plates, and are both rotated to a vertical direction on passing through the half wave plates, they will interfere with a vertically polarized reference beam I_r inside the BSO.

Because of the optical activity in BSO, which is of the order $\sim 45° \, mm^{-1}$ for $\lambda = 514$ nm [15], the polarization of the transmitted beam I_r can be adjusted to be horizontally polarized by changing the path length in the crystal. The transmitted beam is phase conjugated to produce a readout beam which is also horizontally polarized. The polarization of the diffracted beam from

Table 1. All the 16 basic logic operations are shown in terms of X̄, X, and Y. (For the operations A OR B and A XOR B under the column for X, only X is used as the output.)

Operation	X̄	X	Y
A OR B	(A + B)▲	A OR B	(A + B)▲
A XOR B	(A + B)●	(A − B)	(A + B)●
Ā OR B	A OR B	(A + B)▲	(A − 1) OR B
A OR B̄	A OR B	(A + B)▲	A OR (B − 1)
A AND B	A AND B	A AND B	A AND B
A NAND B	A AND B	A AND B	(A AND B) − 1
Ā AND B	A OR B	(A + B)▲	(A OR B) − A
A AND B̄	A AND B	A AND B	(A AND B) − A
A NOR B	A OR B	(A + B)▲	(A OR B) − 1
A XNOR B	(A + B)●	A − B	(A + B)● − 1
A	B − B	(B + B)●	(B − B) + A
Ā	*	*	A − (B OR 1)
B	(A + B)■	(A + B)■	(A + B)■ − A
B̄	*	*	(A OR 1) − B
1	A OR B	(A + B)▲	(A OR B) OR 1
0	B − B	(B + B)●	(B − B)+(A − A)

■ : Incoherent addition i.e, equal to $2I_0$
● : These additions are fully coherent and equal to $4I_0$
▲ : These additions are partially coherent and do not yield the maximum value for addition i.e, $2I_0 < I < 4I_0$
* : These operations were not recorded

the two recorded gratings will be vertically polarized because of the BSO configuration adopted. The polarizations of these diffracted beams I_1^* and I_2^* will subsequently be rotated to the horizontal direction after passing through the half-wave plates, and finally recombine at the beamsplitter BS$_2$. Under these conditions and for a lossless beamsplitter, the Stokes principle of reversibility of light is fulfilled and total subtraction is achieved at the output port of BS$_2$ and a complementary output, that is an addition of I_1^* and I_2^* will be achieved at the input port of BS$_2$. We may consider this therefore as a polarization preserving mode of our PCMI.

However, when the half-wave plates in the two arms of the interferometer are rotated a change occurs in the polarization directions of the input light to the BSO crystal. The vertical components of these two beams interfere with the vertically polarized reference beam I_r as before, to record the two multiplexed gratings. The reflected (phase conjugate) beams, in this set up, will be vertically polarized. On the return path, the vertically polarized beam I_1^* passes back through the half-wave plate and is rotated to a different angle, which depends upon the orientation of the fast axis of the half-wave plate. The resultant polarization direction of the phase conjugate beam after re-traversing the half-wave plate, in this case, will be different from that of the input light, which was horizontally polarized. In addition, there may be other components such as metal mirrors which will introduce a non-reciprocal phase

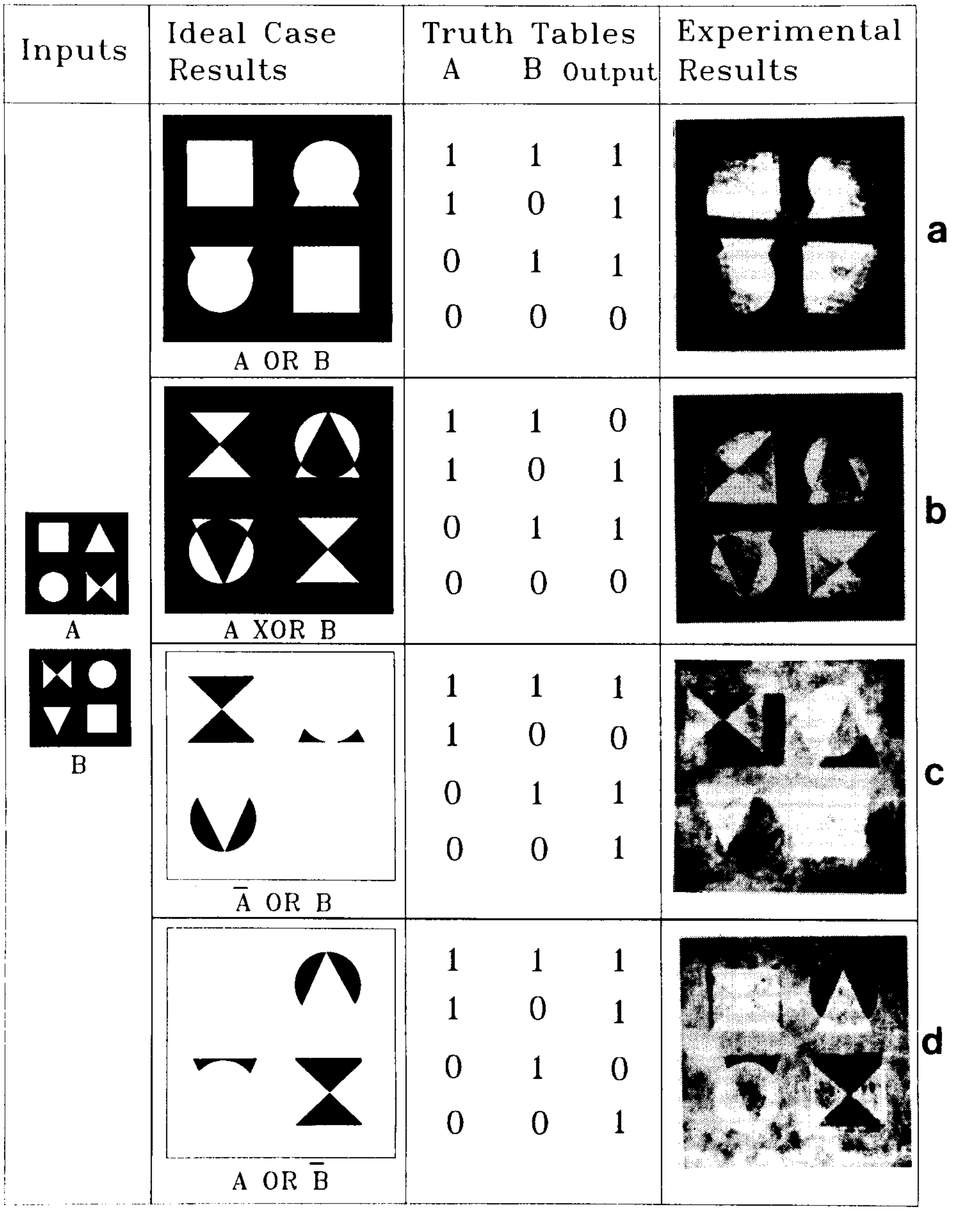


Figure 6. The experimental results of all 16 basic logic operations are shown. For all cases, we show the simulated results to be expected for comparison with the experimentally achieved results: (*a*) A OR B, (*b*) A XOR B, (*c*) $\bar{A}$ OR B, (*d*) A OR $\bar{B}$, (*e*) A AND B, (*f*) A NAND B, (*g*) A AND $\bar{B}$, (*h*) $\bar{A}$ AND B, (*i*) A NOR B, (*j*) A XNOR B, (*k*) 1, (*l*) 0, (*m*) A, (*n*) B, (*o*) A, (*p*) $\bar{B}$.

shift for the phase conjugate non-horizontally polarized beam. This mode of operation therefore is clearly not polarization preserving.

The above description shows that the phase conjugation process may not, in general, be polarization or phase preserving. When two such phase conjugate beams recombine at the beamsplitter BS_2, their interference behaviour will depend both on the polarization angle and on the relative phase difference. The output intensity can therefore be expressed by

$$I_{out} = 2I_o(1 + \cos\phi \cos\theta), \qquad (1)$$

where ϕ is the relative phase between the two linearly polarized beams, θ is the polarization angle between the two beams at the BS_2 and I_o is the intensity of the individual phase conjugate beams at the output port of the BS_2. In this case, the polarization angle can be varied by rotating one or both of the wave plates. The phase difference is less controllable, however, as this depends on relative phase shifts introduced for vertical and horizontal polarized components on reflection from metal coated mirrors for example.

3. Applications to logic operations

The PCMI can be used quite generally for the construction of optical logic gates. A three-dimensional plot of equation (1) shown in figure 2, illustrates that a continuous level of output intensities can be achieved from

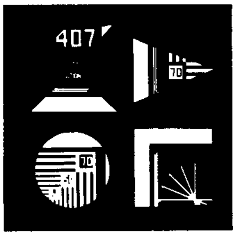
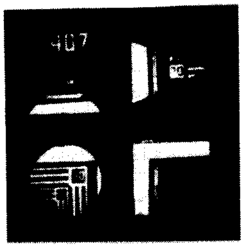
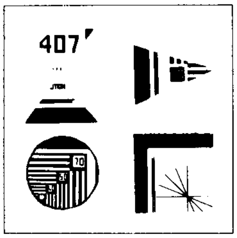
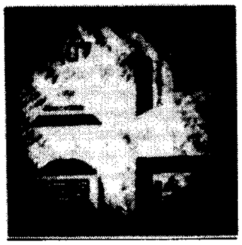
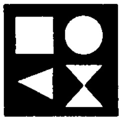
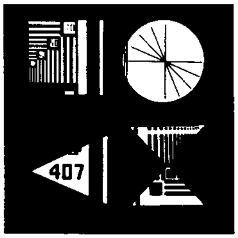
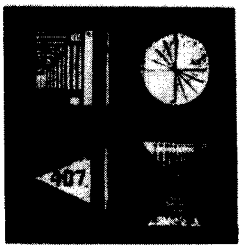
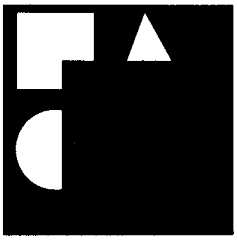
Inputs	Ideal Case Results	Truth Table A	 B	 Output	Experimental Results	
 A	 A AND B	1 1 0 0	1 0 1 0	1 0 0 0		e
Resolution Chart B	 A NAND B	1 1 0 0	1 0 1 0	0 1 1 1		f
 A	 A AND $\bar{B}$	1 1 0 0	1 0 1 0	0 1 0 0		g
 A  B	 $\bar{A}$ AND B	1 1 0 0	1 0 1 0	0 0 1 0		h

Figure 6—continued

0 to $4I_0$. Clearly the same output intensity can be achieved for various different combinations of ϕ and θ for two inputs of initially equal intensities. However, this present two-beam arrangement cannot achieve all of these combinations. Binary logic operations such as XOR, OR, addition (coherent and incoherent) and non-binary intermediate operations are achievable from a range of combinations of ϕ and θ. The three-dimensional plot also shows that the non-polarisation preserving quality is an added advantage for our PCMI. By way of illustration we describe here the four combinations of phase angle ϕ and polarization angle θ between the two phase conjugate beams to achieve the logic operations mentioned above.

Case I—Operation XOR: When the interferometer works in polarization preserving mode, that is the input and output polarizations and phases are preserved. In such a situation $\phi = 180°$, $\theta = 0°$ and according to equation (1) zero output is achieved at the output port of BS_2.

Case II—Coherent addition: When the relative phase $\phi = 0°$ and polarization angle $\theta = 0°$ between the beams I_1^* and I_2^* constructive interference occurs at the output port of BS_2 and the intensity in this case will be $4I_0$.

Case III—Incoherent addition: Addition of the two outputs can also be achieved when the relative phase has any arbitrary value from 0° to 360° and the two

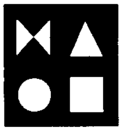
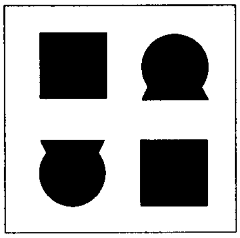
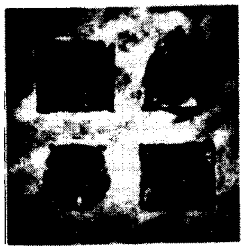
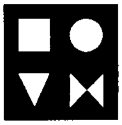
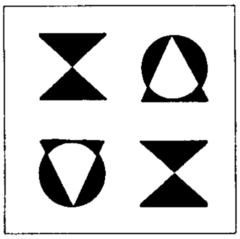
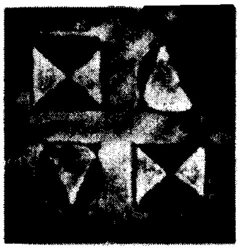
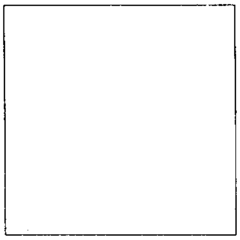
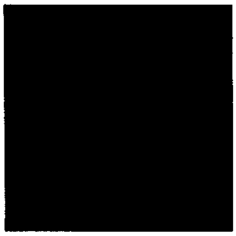
Inputs	Ideal Case Results	Truth Tables			Experimental Results	
		A	B	Output		
 A	 A NOR B	1	1	0		i
		1	0	0		
		0	1	0		
		0	0	1		
 B	 A XNOR B	1	1	1		j
		1	0	0		
		0	1	0		
		0	0	1		
 A	 1	1	1	1		k
		1	0	1		
		0	1	1		
		0	0	1		
 B	 0	1	1	0		l
		1	0	0		
		0	1	0		
		0	0	0		

Figure 6—continued

beams are of orthogonal polarization, that is $\theta = 90°$ and hence will not interfere at all. Therefore the addition in this case is an incoherent one and the output will always be $2I_0$.

Case IV—Operation OR: An OR operation is achieved when the phase angle $\phi = 180°$ and $\theta = 60°$ or $\phi = 0$ and $\theta = 120°$. At these combinations of ϕ and θ the output is equal to I_0 which corresponds to an OR operation. The complementary output from the above operation is observed at the input port of a second PCMI.

Taking advantage of these basic operations of the PCMI and multiplexed holographic recording in BSO, it is simple to cascade another PCMI with the existing one. To construct such a system an additional beam, and an additional half-wave plate, is added to the BSO set up as shown in figure 3. This interferometer consists of two PCMIs, the phase conjugate output from the second interferometer serves as a direct input to the first one. Both the PCMIs can equally perform operations of XOR, OR and addition (coherent and incoherent) between the outputs from all three arms of the cascaded interferometers. Using such a system all 16 basic logic operations on the two variables can be achieved. These logic operations can be performed in two stages. For the first stage, $(\mathrm{PCMI})_2$ performs one of the above four logic operations between outputs I_2^* and I_3^* at its input port (a complementary operation being

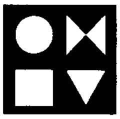
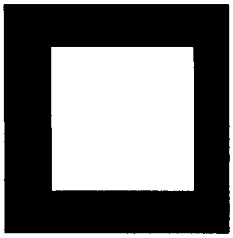
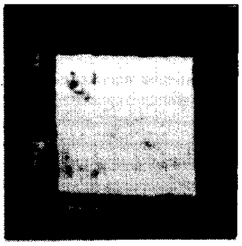
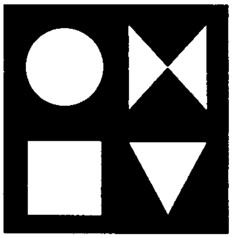
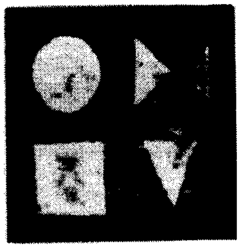
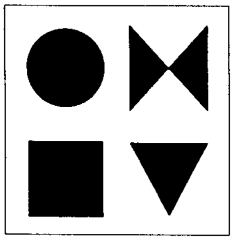
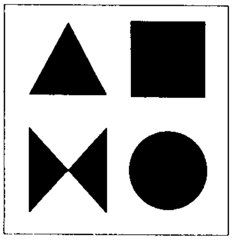
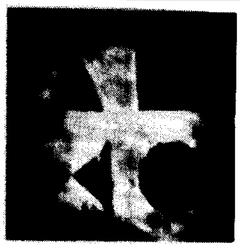
Inputs	Ideal Case Results	Truth tables A	B	Output	Experimental Results	
 A  B	 A	1 1 0 0	1 0 1 0	1 1 0 0		m
	 B	1 1 0 0	1 0 1 0	1 0 1 0		n
 A  B	 $\bar{A}$	1 1 0 0	1 0 1 0	0 0 1 1		o
	 $\bar{B}$	1 1 0 0	1 0 1 0	0 1 0 1		p

Figure 6—continued

achieved at the output port of (PCMI)$_2$). Secondly, the output from (PCMI)$_2$ and I_1^* again perform one of the above four logic operations.

4. Experimental arrangement

Figure 4 shows an Ar$^+$ laser used at 514·5 nm in multilongitudinal mode. A single crystal of BSO with dimensions 2 × 10 × 10 mm, was used to record the angularly multiplexed gratings. The laser output was spatially filtered and divided by a beam-splitter BS$_1$ into a reference beam I_r and input beam I_{in} to the interferometer. Beam I_{in} was sub-divided by a beam-splitter, BS$_2$, and the transmitted beam was further divided by BS$_3$. Beams I_1, I_2 and I_3 pass through the separate half-wave plates and imaging lens systems before entering the BSO crystal. Lenses L_1, L_2 and L_3 were each of 30 cm focal length, while lens L_4 was of 25 cm focal length. The polarizations of the reference and all three signal beams were horizontal before they pass through any half-wave plates. The intensity of I_r at the BSO crystal was 32 mW while the intensities of beams I_1, I_2 and I_3 (in our test case), were 25 mW, 16 mW and 14·5 mW, respectively. The reference beam transmitted through the BSO crystal was phase conjugated by a self-pumped BaTiO$_3$ crystal. This configuration which provides a very precise pair of phase conjugate pump beams for the FWM interaction is an important aspect of the PCMI described here. We have already used this configuration for multiplexed novelty filtering experiments [16], as it ensures fringe-free images at the beamsplitter output port. We now consider the implementation of the two modes of polarization preserving and polarization non-preserving PCMI.

4.1 *Case 1—Polarization preserving mode*

For this case we consider only one PCMI and therefore block beam I_1. The polarization of beam I_r before the BSO crystal was rotated to the vertical plane by a half-wave plate. Similarly the polarization directions of the input signal beams I_2 and I_3 were also rotated to the vertical. Because of the intrinsic optical activity, the polarization of the transmitted reference beam was rotated to the horizontal direction. This beam was phase conjugated by the self-pumped $BaTiO_3$, and subsequently read out of the two multiplexed gratings. The phase conjugate beams I_2^* and I_3^* were vertically polarized; tracing back their original paths these beams passed back through the half-wave plates which again rotate their polarizations to the horizontal direction. At the beam splitter BS_3 they interfere destructively because of the 180° relative phase shift introduced between the two beams on reflection from BS_3. It is worth noting that because of the preservation of polarization, the relative phase shift of 180° between the two beams was also preserved on phase conjugation.

The fulfilment of these conditions ensured that the Stoke's principle of reversibility holds and we observe no light at the output port of BS_3 (XOR operation). A complementary output was observed at the input port of BS_3 which corresponds to coherent addition.

4.2 *Case II—Non-polarization preserving mode*

As in case I, the polarization of beams I_2, I_3 and the readout beam was initially horizontal. After passing through the half-wave plates, the polarizations of the two beams were at +25° and +155° (from the horizontal plane). The vertically polarized phase conjugate beams I_2^* and I_3^* passes through the half-wave plates and their polarizations were rotated to +102° and +70° (from the horizontal plane). Subsequent reflection of beam I_2^* from a metallic mirror, M_5, and entry to the output port of BS_3, combined with beam I_3^* lead to constructive interference on BS_3. The measured polarizations of beams I_2^* and I_3^* at the output port of BS_3 were +95° and +75° (with respect to the horizontal). Therefore both the beams now have vertical components (s-polarized) which were not present originally in the input beams. We observe here coherent addition at the output port of BS_3 and a complementary output (XOR operation) at the input port of BS_3. These s-polarized components may also have undergone an additional relative phase shift. Any such phase shift depends on the exact geometrical and material arrangement of the reflecting surfaces, and therefore cannot be compensated or reversed during phase conjugation because of the initial absence of any such s-polarised light (as discussed earlier).

The operations of coherent addition and subtraction represent the two extremes of the system. By selecting any specific polarization direction, we can achieve a continuous operation between the two extremes. The OR operation is achieved when the polarization angle of the output phase conjugate beams I_2^* and I_3^* were +20° and +322°, respectively (assuming a relative phase shift of 180° between the two beams, which is consistent with equation (1)).

By exactly similar reasoning the operation of a series of cascaded PCMI can be considered. Figure 5 shows a block diagram of a two-stage cascaded PCMI of figure 4. The phase conjugate outputs I_1^*, I_2^* and I_3^* interfere in $PCMI_1$ and $PCMI_2$ to give X, $\bar{X}$, Y and $\bar{Y}$ logic outputs. The outputs X from $PCMI_2$ and Y from $PCMI_1$ are the usable logic outputs. While $\bar{X}$ and $\bar{Y}$ represent the complementary logic outputs at the input ports of these PCMIs. We use X as the logic output for (A OR B) and (A XOR B), see figure 6(*a*, *b*), while Y is used for the rest of the 16 logic operations. All 16 logic operations are summarised in table 1, in which it is seen that the operation (A AND B) was performed by using only one beam, operations of (A OR B), (A XOR B), (A NAND B) and (A AND $\bar{B}$), see figure 6(*a*, *b*, *f*, *g*), were performed using two of the three beams of cascaded PCMI arrangement, and operations of ($\bar{A}$ AND B), ($\bar{A}$ OR B), (A OR $\bar{B}$), (A NOR B), (A XNOR B), A, $\bar{A}$, B, $\bar{B}$, and 1, as shown in figure 6, were performed using all three beams. For completeness only operation 0 was performed using four beams in a cascaded PCMI configuration of two individual PCMIs.

5. Conclusion

A technique of optical logic on two variables has been described using PCMI in cascaded form with a single crystal of BSO. Multiple signal beams and a single reference beam are used to record angularly multiplexed gratings. Subsequently, the reference beam is phase conjugated via a self-pumped $BaTiO_3$ crystal to readout all the gratings. The technique relies upon polarization selection of the two input signal beams. So that their mutual interference at the output, is a function of their output polarizations, and relative phase. Complete subtraction is achieved when PCMI is working in polarization preserving mode. However, when the PCMI operates in non-polarisation preserving mode and the two phase conjugate beams were s-polarized coherent addition was achieved. Other continuous intermediate operations are observed when the polarization of the phase conjugate beams were set between these two extremes.

Using these basic operations in a two-stage cascaded PCMI we have demonstrated all 16 logic operations on two variables. We feel the main advantage of our technique is that it is simple, stable and versatile since this type of cascading is almost impossible to implement via previous techniques. Operations of cascaded PCMI can also be controlled much more easily if we rotate the polarization of the readout beam at specific positions of the axes of half-wave plates in the two arms of PCMI. This technique may also be useful in a cascaded PCMI system.

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